Flash Talk: Diagrammatic Monte Carlo for Strong Coupling LQCD

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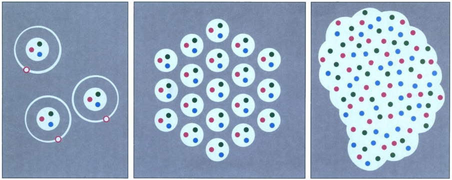

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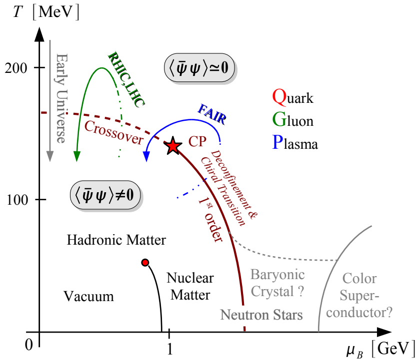
1. Motivation for Continuous Time Strong Coupling LQCD
2. Diagrammatic Monte Carlo
3. Outlook



What is Strong Coupling Lattice QCD?

Look at QCD in a regime where the **sign problem** can be made mild:
This is obtained by changing the nature of integration variables:
— no sampling of gauge fields $\{U\}$!
— no fermion determinant (no HMC)!

Staggered QCD in the strong coupling limit:

- start from the "1-flavor" staggered QCD Lagrangian in Euclidean time: $\mathcal{L}_{\rm QCD} = \sum_\nu \frac{1}{2}\eta_\nu(x)\left(e^{\mu\delta_{\nu 0}}\bar{\chi}(x)U_\nu(x)\chi(x+\hat{\nu}) - e^{-\mu\delta_{\nu 0}}\bar{\chi}(x+\hat{\nu})U_\nu^\dagger(x)\chi(x)\right) + am_q\bar{\chi}\chi - \frac{\beta}{2N_c}\sum_P\left(\mathrm{tr}U_P + \mathrm{tr}U_P^\dagger\right) + \mathcal{O}(a^2)$

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- take limit of **infinite gauge coupling:** $g \rightarrow \infty,\ \beta = \frac{2N_c}{g^2} \rightarrow 0$
- allows to integrate out the gauge fields completely, as **link integration factorizes**

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Drawback: strong coupling limit is converse to asymptotic freedom:

- degrees of freedom in SC-QCD live on a **crystal**
- fermions have **no spin** (taste splitting maximal)

Why Strong Coupling Lattice QCD?

1-flavor strong coupling QCD might appear crude, but

- exhibits **confinement**, i.e. only color singlet degrees of freedom survive:
  - **mesons** (rep. by monomers and dimers)
  - **baryons** (rep. by oriented self-avoiding loops)
- and **chiral symmetry breaking/restoration:**

$$\mathrm{U}_A(1): \qquad \chi(x) \mapsto e^{i\eta_5(x)\theta_A}\chi(x),\ \bar{\chi}(x) \mapsto \bar{\chi}(x)e^{i\eta_5(x)\theta_A}, \qquad \eta_5(x) = (-1)^{\sum_{\nu=0}^{d} x_\nu}$$

  is spontaneously broken below T_c,

hence its phase diagram might be **similar to the QCD phase diagram**

SC-LQCD as effective theory for **nuclear matter**:

- derive nuclear interactions between hadrons from (Lattice) QCD
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SC-LQCD as effective theory for **nuclear matter**:

- derive nuclear interactions between hadrons from (Lattice) QCD
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Just another effective model for QCD?

- yes and no: think of SC-LQCD rather as a **1-parameter deformation of QCD**

SC-QCD Partition Function

Exact rewriting of SC-QCD partition function (no approximation!):

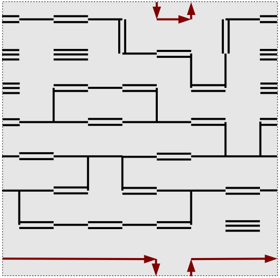
$$\mathcal{Z}(m_q, \mu, N_\tau) = \sum_{\{k,n,l\}} \prod_{b=(x,\hat{\mu})} \frac{(3-k_b)!}{3!k_b!} \prod_x \frac{3!}{n_x!} (2am_q)^{n_x} \prod_l w(\ell, \mu)$$

- **Grassmann constraint:**
  color neutral states at each site

$$n_x + \sum_{\hat{\mu}=\pm\hat{0},\ldots\pm\hat{d}} k_{\hat{\mu}} = 3 \qquad \forall x \in {N_\sigma}^3 \times N_\tau$$

- weight for baryon loop l (sign $\sigma(\ell)$):

$$w(\ell, \mu) = \frac{1}{\prod_{x\in\ell} 3!} \sigma(\ell) e^{3N_\tau r_l a_\tau \mu}$$



SC-LQCD at finite Temperature and Continuous Time:

How to vary the temperature?

- $aT = 1/N_\tau$ is discrete with N_τ even
- $aT_c \simeq 1.5 \qquad \Rightarrow \qquad$ we cannot address the phase transition!

Solution: introduce an **anisotropy** γ in the Dirac couplings:

Strategy for **unambiguous** answer: the **continuous Euclidean time limit** (CT-limit):

$$N_\tau \to \infty, \qquad \gamma \to \infty, \qquad \gamma^2/N_\tau \quad \text{fixed}$$

- $a/a_t = \gamma^2$ inspired by mean field, in contrast to weak coupling $a/a_t = \gamma$
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Several **advantages** of continuous Euclidean time approach:

- ambiguities arising from the functional dependence of observables on the anisotropy parameter will be circumvented, **only one parameter** setting the temperature
- no need to perform the continuum extrapolation $N_\tau \to \infty$
- baryons become static in the CT-limit, the **sign problem is completely absent!**

Continuous Time Partition Function

Final goal: partition function in $\beta = 1/aT$:

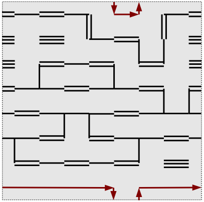
- expansion in powers of γ^{-2}, i. e. in total number of spatial hoppings

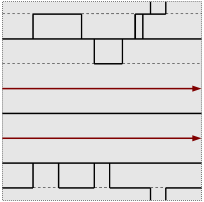
$$\kappa = \frac{1}{2} \sum_{x \in V_M} (n_L(x) + n_T(x))$$

- sum over all spatial hopping positions $\sim N_\tau/2$ $\Rightarrow$ expansion in inverse temperature $\beta = N_\tau/\gamma^2$:

$$\mathcal{Z}(\beta) = \sum_{\kappa \in 2\mathbb{N}} \frac{(\beta/2)^\kappa}{\kappa!} \sum_{\mathcal{C} \in \Gamma_\kappa} v_T^{n_T(\mathcal{C})}$$

- Γ_κ is the set of equivalence classes of configurations with κ spatial hoppings and $\mathcal{C} \in \Gamma_\kappa$ only differ in their topology





successfully used so far: **Worm algorithm in continuous time**

here: attempt **diagrammatic MC** based on the diagrammatic expansion in κ

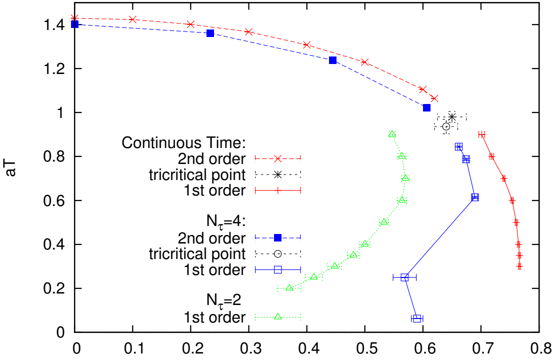
SC-QCD Phase Diagram

Studied via CT-Worm algorithm (arXiv:1111.1434 [hep-lat])
Comparison of phase diagram with $N_\tau = 4$ data (M. Fromm, 2010):

- CT-data compared to $N_\tau = 4$ data for identification

$$a\mu = \gamma^2 a_\tau \mu$$

- behavior at low μ agrees well, location of TCP agrees within errors
- no re-entrance is seen at small temperatures



Motivation: Generalization of SC-QCD to 2 chiral flavors

At present, no 2-flavor formulation for SC-QCD suitable for the Worm algorithm present:

- already the mesonic sector has a severe (unphysical) **sign problem** in dimer representation
- new type of 2-flavor dimer give negative sign in mesonic loops already for U(2).

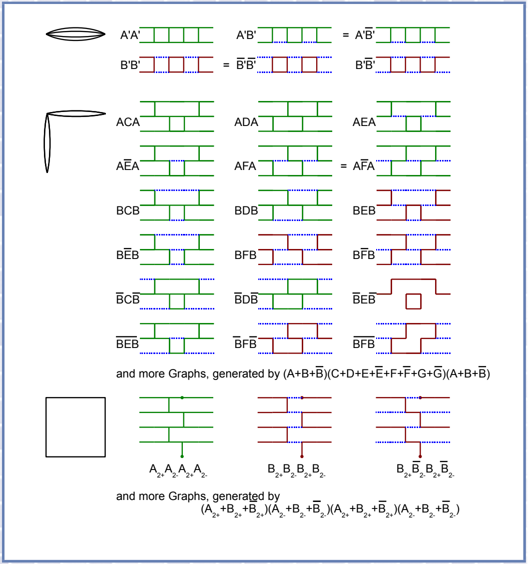
Aim: obtain phase diagram for 2-flavor SC-QCD, where **pion exchange** may play a crucial role for nuclear transition

- observation: box representation for 2 different flavors can be compose such that cancellations appear
- first step: Monte Carlo for insertion/removal of boxes rather than Worm algorithm

Diagrammatic Monte Carlo (1)

In principle, diagrammatic expansion amounts to **hopping parameter expansion** in κ on N_s^3 with time ordering of L- and T-vertices:

- label spatial dimers with time index $i = 1, \ldots \kappa$
- Γ_κ: enumeration of all valid configurations consistent with location not feasible
- diagrammatic MC algorithm in progress



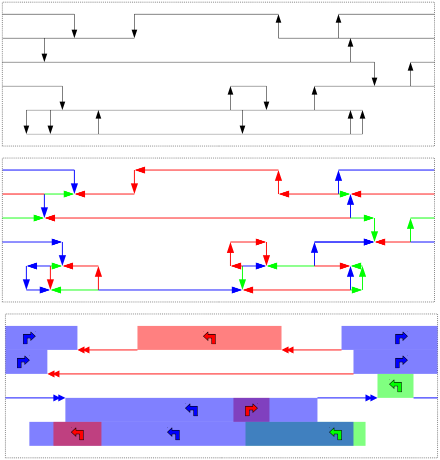
Diagrammatic Monte Carlo (2)

Combinatorics of diagrams in Γ_κ govered by assignment of

- emission/absorption events to vertices, or
- quivalently: even/odd lengths of time intervals
- this is remnant of Grassmann constraint

Observation: emission-absorption ordering induces **orientation on boxes**:

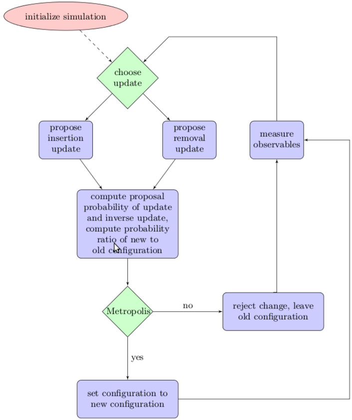
- spatial dimers have orientation from emission to absorption site
- solid lines can be consistently oriented (colors for illustration)
- SC-QCD partition function can be conceived as composed of static lines and oriented rectangles



Diagrammatic Monte Carlo (3)

DMC flow chart:

insertion and **removal** updates for **pairs** of oppositely oriented spatial dimers

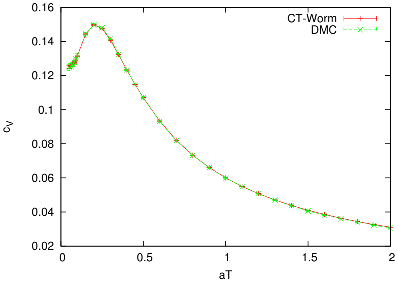


Besides insertion/removal update:

- additional shift and static line updates improve on ergodicity
- for $d > 1$: swap update needed

First measurement:

- specific heat (can be obtained from spatial dimers) for U(1) on 1+1-dimensional lattice:



Conclusions

Prospects:

- CT partition function: new formulation for analytic treatment
- hope: extend formulation for two flavors (incorporates pion exchange)
- extension to SU(3) with finite baryon chem. potential straight forward

Drawbacks

- generalization to higher dimensions turns out to be very difficult
- not yet possible to study periodic boundary conditions
- in contrast to worm: no 2-point function for free