Flavored-mass terms and symmetries on the lattice

Tatsuhiro MISUMI *YITP/BNL*

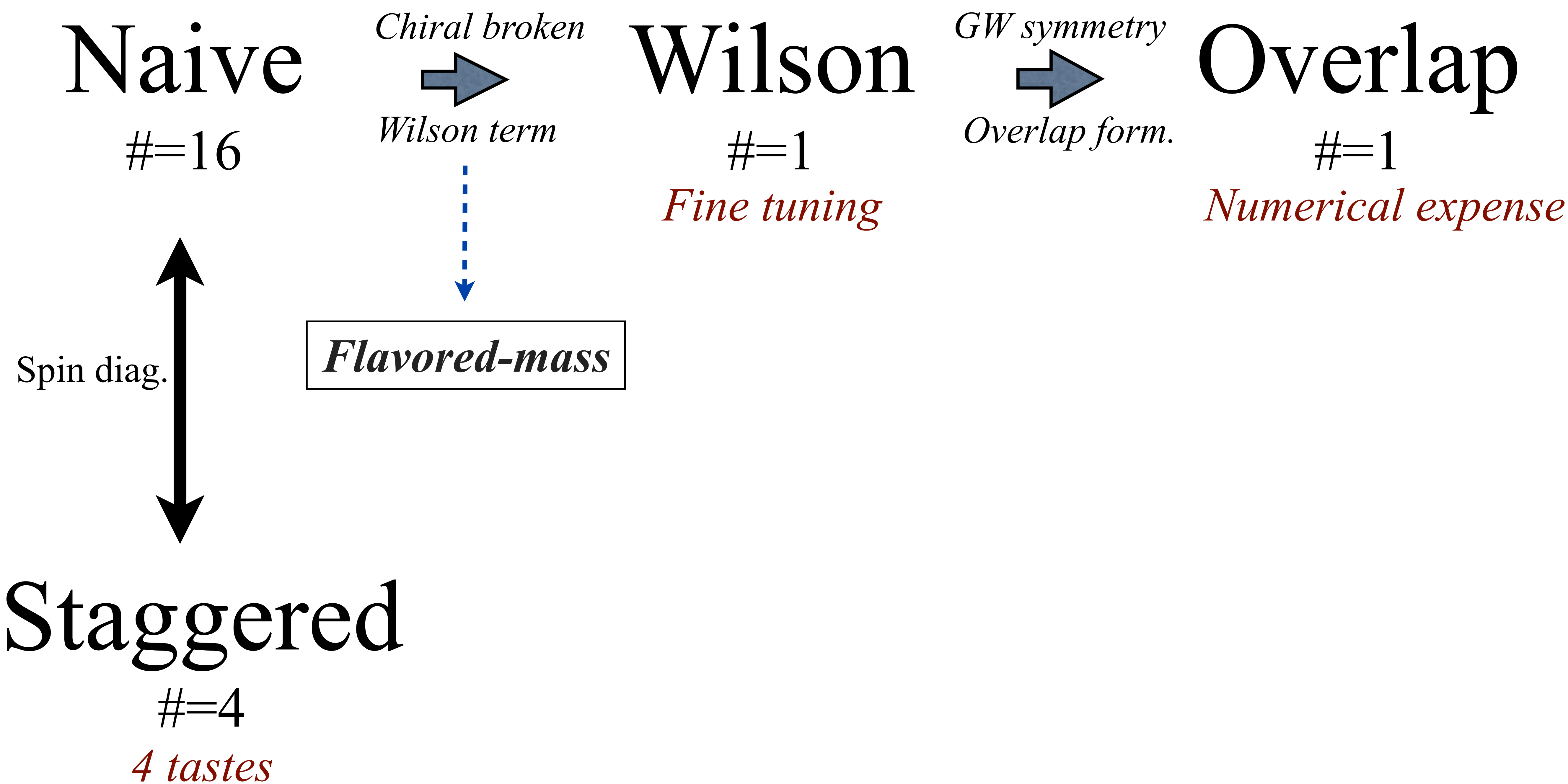
M. Creutz, T. Kimura, T. Misumi, *JHEP* **1012**:041 (2010)

M. Creutz, T. Kimura, T. Misumi, *PRD* **83**:094506 (2011)

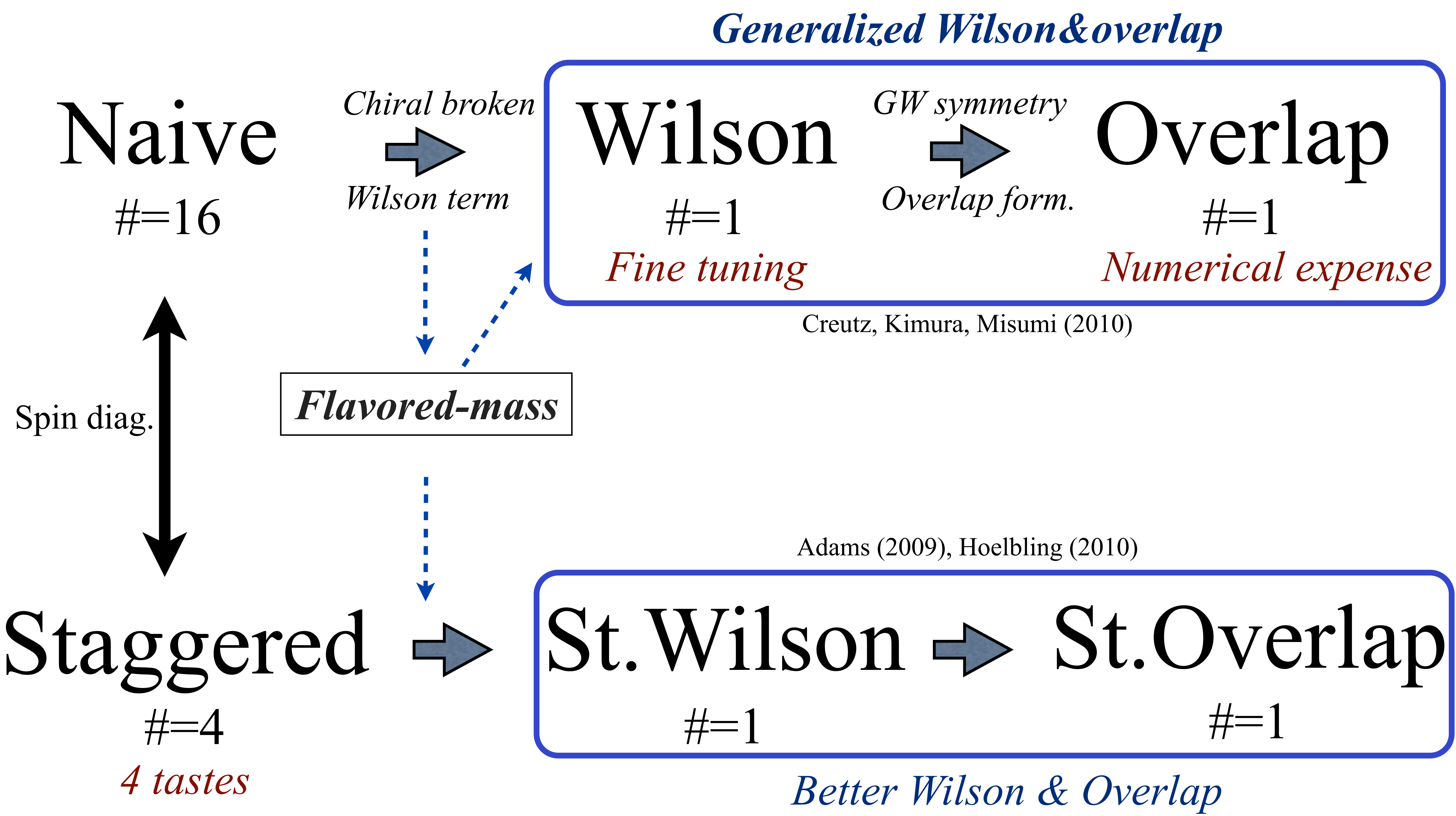
T. Kimura, S. Komatsu, T. Misumi, T. Noumi, S. Torii, S. Aoki, *JHEP* **1201**:048 (2012)

T. Misumi, *Ph.D Thesis*, Kyoto University (2012)

02/17/2012 NTFL workshop@Yukawa Institute, Kyoto



Naive
#=16
Chiral broken
Wilson term
Wilson
#=1
Fine tuning
GW symmetry
Overlap form.
Overlap
#=1
Numerical expense
Spin diag.
Flavored-mass
Staggered
#=4
4 tastes



Generalized Wilson&overlap
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Creutz, Kimura, Misumi (2010)
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Adams (2009), Hoelbling (2010)
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St.Overlap
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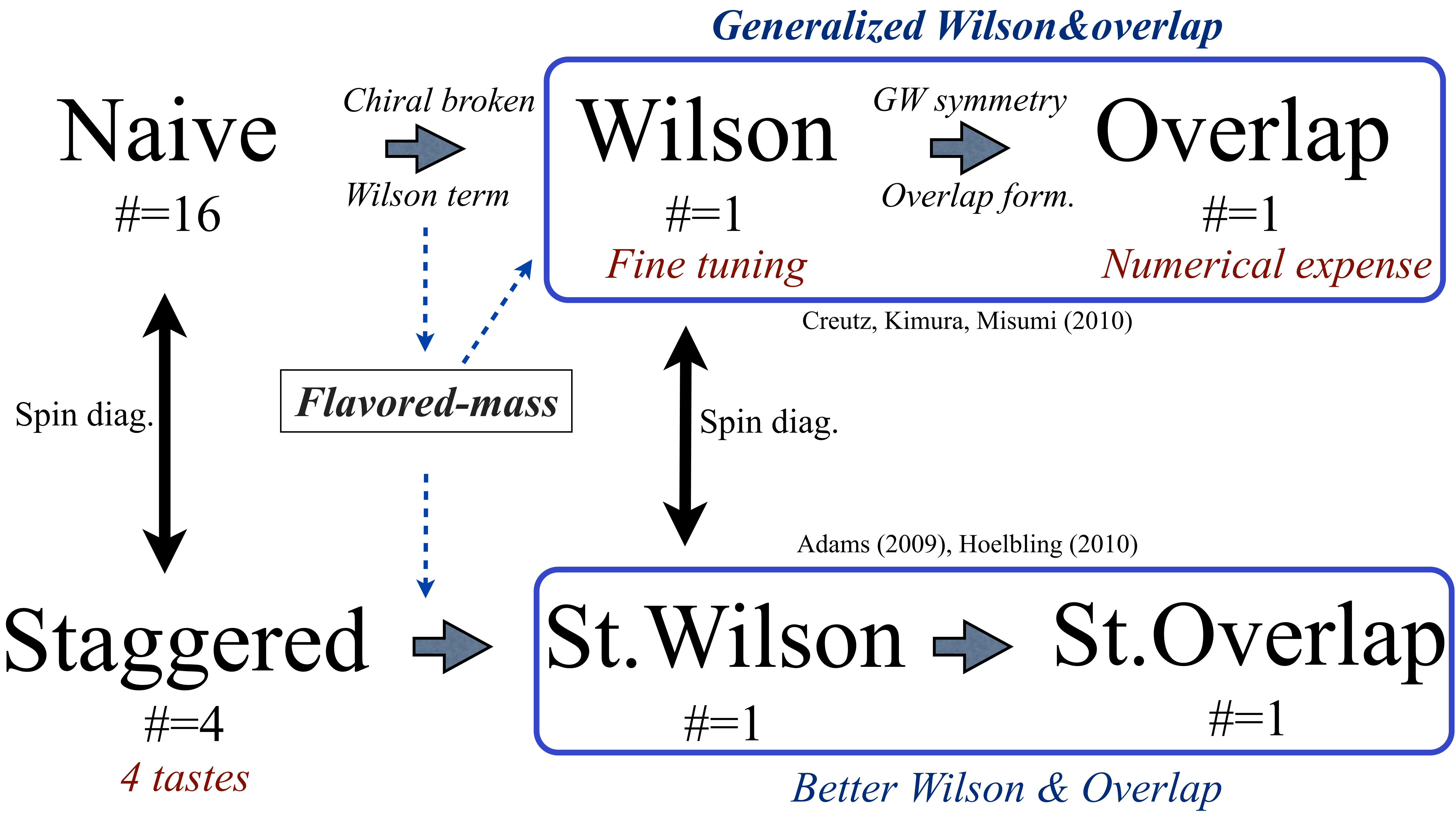
Better Wilson & Overlap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◆ Flavored-mass terms *~ generalized Wilson terms ~*

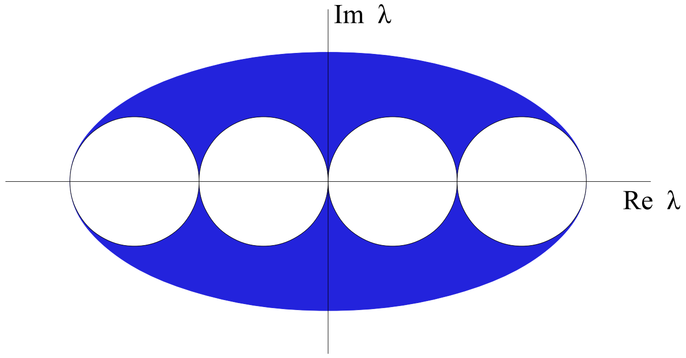
$$M_{\mathrm{V}} = \sum_{\mu} C_{\mu},$$ *Vector (1-link)*

$$M_{\mathrm{T}} = \sum_{perm.} \sum_{sym.} C_{\mu} C_{\nu},$$ *Tensor (2-link)*

$$M_{\mathrm{A}} = \sum_{perm.} \sum_{sym.} \prod_{\nu} C_{\nu},$$ *Axial-V (3-link)*

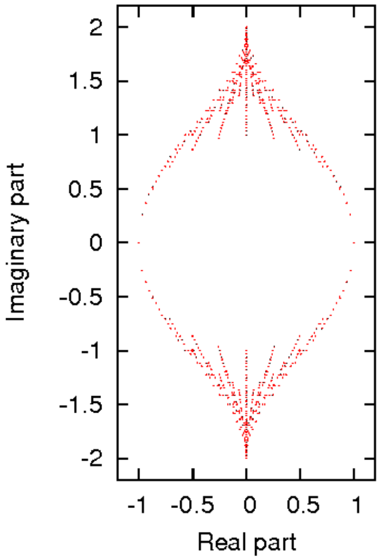
$$M_{\mathrm{P}} = \sum_{sym.} \prod_{\mu=1}^{4} C_{\mu},$$ *Pseudo-S (4-link)*

- O(a) *irrelevant terms* $\sum_{n} \bar{\psi}_n (M_P - 1) \psi_n \;\rightarrow\; -a \int d^4 x \bar{\psi}(x) D_{\mu}^2 \psi(x) + O(a^2)$
- M_{V} $\rightarrow$ *Wilson term*



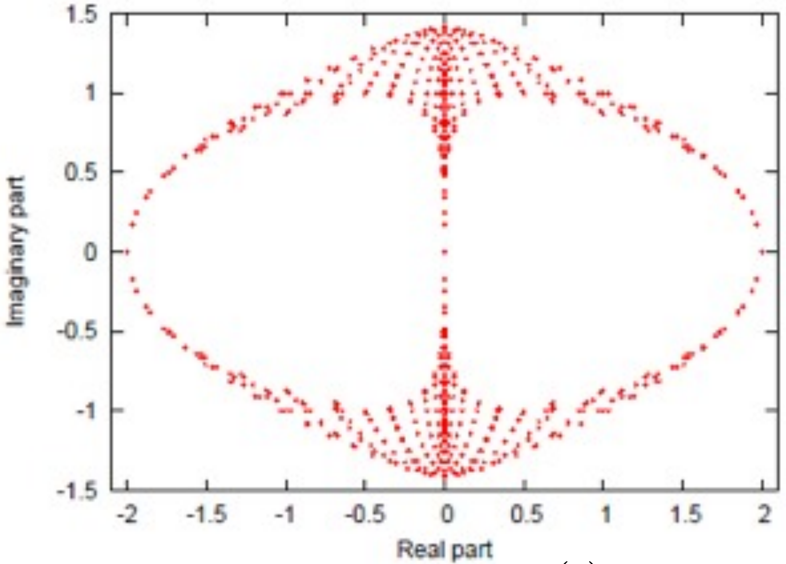
Dirac spectra with flavored mass terms

(8,8)



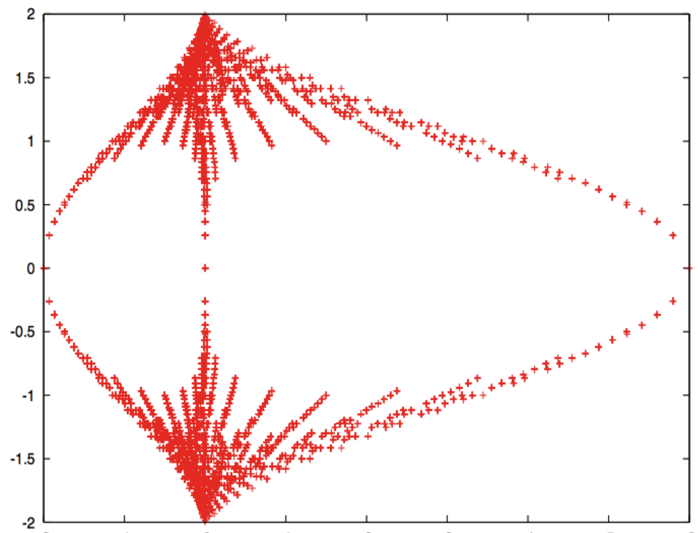
$D_{\mathrm{nf}} - M_P$

(4,8,4)



$D_{\mathrm{nf}} - M_{\mathrm{T}}^{(i)}$

(6,8,2)



$D_{\mathrm{nf}} - M_T$

cf.) some combinations

M_V+M_A : (1,14,1)

M_P+M_T : (4,12)

M_P+M_V : (5,1,10)

M_T+M_V : (10,5,1)

M_A+M_V +M_T : (3,12,1)

M_A+M_V +M_T+M_P : (1,15)

$\vdots$

→ *Multi-flavor Wilson & Overlap*

Adams-type staggered flavored mass

Adams, PRL104, 141602 [0912.2850]

- *spin diagonalization*

$$\bar{\psi}_x \psi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}} = \bar{\chi}_x \gamma_4^{x_4}\gamma_3^{x_3}\gamma_2^{x_2}\gamma_1^{x_1}\gamma_1^{x_1+1}\gamma_2^{x_2+1}\gamma_3^{x_3+1}\gamma_4^{x_4+1}\chi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}}$$

$$= (-1)^{x_2+x_4}\bar{\chi}_x \gamma_5 \chi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}} \qquad (\gamma_5 \;\; \text{diagonalized})$$

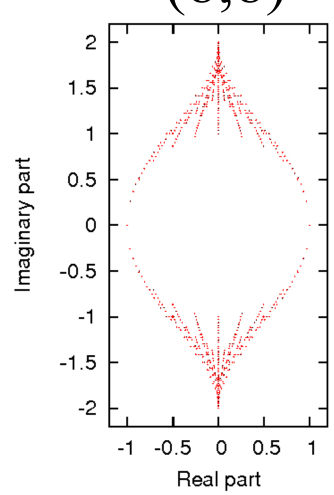
$$\rightarrow \pm\bar{\chi}_x \epsilon\eta_1\eta_2\eta_3\eta_4 \chi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}}$$

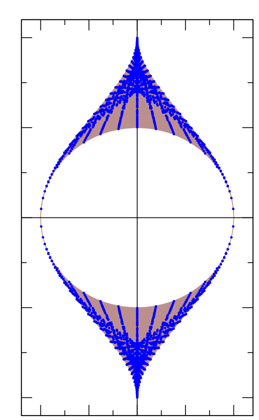
Adams fermions derived

$$\bar{\psi}_x C_1 C_2 C_3 C_4 \psi_x \qquad \rightarrow \qquad \pm\bar{\chi}_x(\epsilon\eta_1\eta_2\eta_3\eta_4 C_1 C_2 C_3 C_4)\chi_x$$

(8,8) (2,2)

M_{P}





$M_{\mathcal{A}}$

de Forcrand, Kurkela, Panero, [1102.1000]

$$S_{\mathrm{nf}}(M_{\mathrm{P}}) \qquad \rightarrow \qquad S_{\mathrm{st}}(M_{\mathcal{A}})$$

Hoelbling-type flavored mass

Hoelbling PLB696, 422(2011) [1009.5362], de Forcrand (2010)

- *spin diagonalization*

$$\bar{\psi}_x \psi_{x+\hat{1}+\hat{2}} + \bar{\psi}_x \psi_{x+\hat{3}+\hat{4}} = (-1)^{x_2} \bar{\chi}_x \gamma_1 \gamma_2 \chi_{x+\hat{1}+\hat{2}} + (-1)^{x_4} \bar{\chi}_x \gamma_3 \gamma_4 \chi_{x+\hat{3}+\hat{4}}$$
$$\to \pm \bar{\chi}_x i\epsilon_{12} \eta_1 \eta_2 \chi_{x+\hat{1}+\hat{2}} \pm \bar{\chi}_x i\epsilon_{34} \eta_3 \eta_4 \chi_{x+\hat{3}+\hat{4}}$$

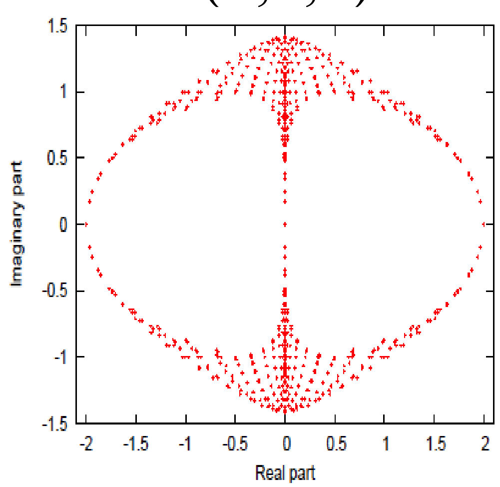
※ two terms simultaneously diagonalizable : $[\sigma_{12}, \sigma_{34}] = 0$

parts of Hoelbling fermions derived

$$\bar{\psi}_x [(C_1 C_2 + C_2 C_1) + (C_3 C_4 + C_4 C_3)] \psi_x$$
$$\to \quad \pm \bar{\chi}_x [i\epsilon_{12} \eta_1 \eta_2 (C_1 C_2 + C_2 C_1) \pm i\epsilon_{34} \eta_3 \eta_4 (C_3 C_4 + C_4 C_3)] \chi_x$$

(4,8,4)

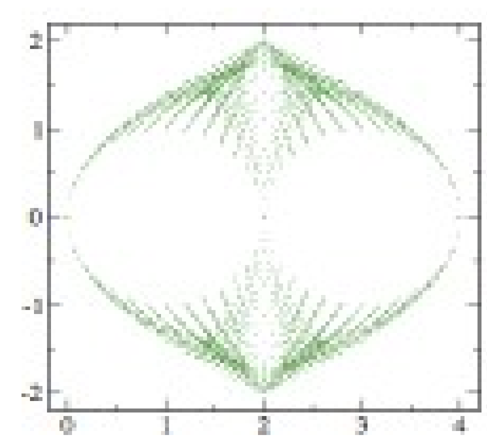
$M_{\mathrm{T}}^{(i)}$



Spin diag. & Sum of 3 parts

$\longrightarrow$

(1,2,1)



M_H

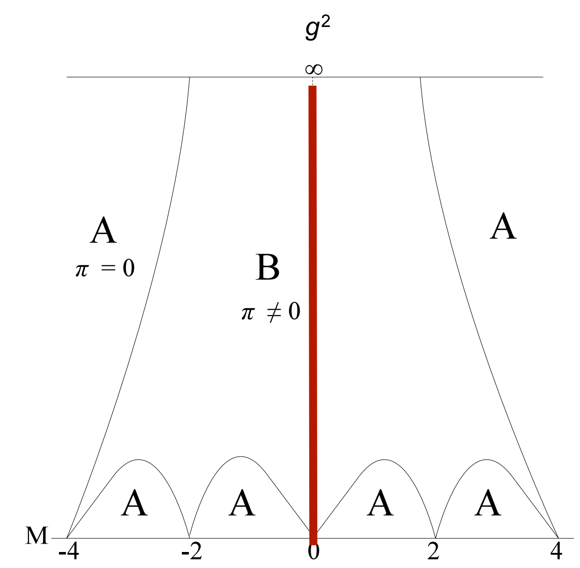
Hoelbling, PLB696, 422(2011) [1009.5362].

◆ Central cusps

Creutz, Kimura, Misumi, *PRD* **83**:094506 (2011),
Kimura, Komatsu, Misumi, Noumi, Torii, Aoki, *JHEP* **1201**:048 (2012)

- Wilson fermion without on-site terms $M_W \equiv m + 4r = 0$

$$S = \frac{1}{2}\sum_{x,\mu} \bar{\psi}_x [\gamma_\mu(\psi_{x+\mu} - \psi_{x-\mu}) - (\psi_{x+\mu} + \psi_{x-\mu})]$$



Aoki phase

⇨ *Extra U(1)v symmetry emerge !* (works as chiral symmetry)

$$\psi_x \;\rightarrow\; e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}, \quad \bar{\psi}_x \;\rightarrow\; \bar{\psi}_x e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}$$

- prohibits additive mass renormalization !
- will be spontaneously broken due to pion condensation ! $\langle\bar{\psi}\gamma_5\psi\rangle$

§ Strong-coupling meson potential $p = (\pi, \pi, \pi, \pi + im_{SPA})$

$$\cosh(m_{SPA}) = 1 + \frac{2M_W^2(16 + M_W^2)}{16 - 15M_W^2}$$

Massless NG boson

It is expected to describe *6-flavor Twisted-mass QCD*. $\bar{\psi}\psi \;\leftrightarrow\; \bar{\psi}\gamma_5\psi$

different bases

◆ Central cusps for other flavored masses

progress in NTFL workshop (2012)

- For other naive flavored mass terms

M_{A} : U(1) restored

$M_{\mathrm{T}}^{(i)}$: U(2) restored

M_{P} : N/A

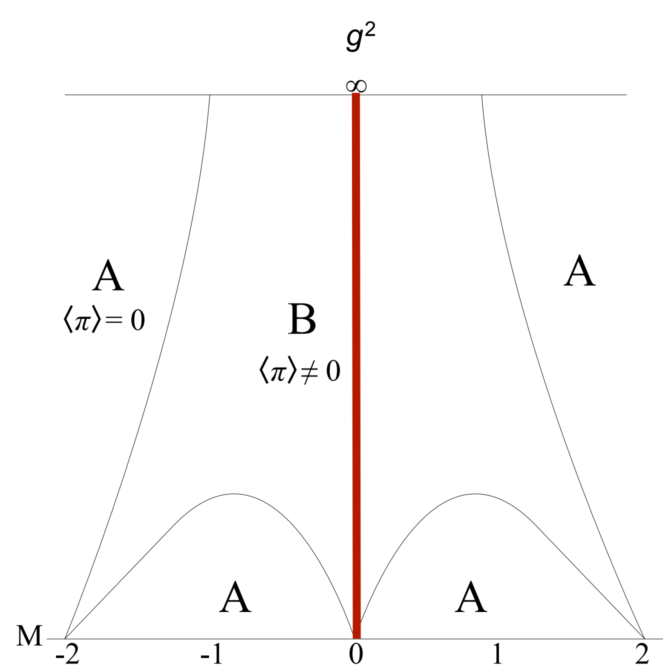
- For staggered flavored mass terms

$M_{\mathcal{A}}$: N/A

$M_{\mathcal{H}}$: C-like symmetry restored

$$\mathcal{C}: \ \chi_x \rightarrow \bar{\chi}_x^T, \quad \bar{\chi}_x \rightarrow \chi_x^T, \quad U_{\mu,x} \rightarrow U_{\mu,x}^*$$

$\rightarrow$ 2-flavor twisted-mass QCD!? cf.) de Forcrand, et.al. [1202.1867]



Alternative use of Wilson-type fermions....?

4. Summary

1. Flavored-mass terms give us new types of Wilson and overlap fermions.

2. Staggered-Wilson can be derived from generalized Wilson fermions through spin-diagonalization.

3. Central cusps are expected to describe twisted-mass QCD without any parameter tuning.