Sneaking up on dense QCD using large N methods

Aleksey Cherman

UNIVERSITY OF CAMBRIDGE

Based on work with M. Hanada, D. Robles-Llana, B. Tiburzi...

at YITP, 16 February 2012

Dense matter is fascinating!

$$n_B \gtrsim \Lambda_{QCD}^{-3}$$

Intrinsically interesting probe of QCD

Very important for neutron star physics

Finite density driven by a chemical potential for quark (~baryon) number

$$\mathcal{L}_{\rm QCD} \to \mathcal{L}_{\rm QCD} + \mu_B \bar{\psi}\gamma^0\psi$$

Many spectacular phenomena seen using weak-coupling methods, which apply for $\mu_B/\Lambda_{QCD} \to \infty$

For $\mu_B/\Lambda_{QCD} \sim 1$, not much is known reliably from first principles.
Normally, this is where one would turn to lattice Monte Carlo methods.

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Lattice does not work at finite μ_B !

What makes Monte Carlo methods tick

$$\langle \mathcal{O} \rangle = \frac{\int dA_\mu d\psi d\bar{\psi} e^{-S[A_\mu,\psi,\bar{\psi}]} \mathcal{O}[A_\mu,\psi,\bar{\psi}]}{\int dA_\mu d\psi d\bar{\psi} e^{-S[A_\mu,\psi,\bar{\psi}]}}$$

$$= \frac{1}{Z} \int dA_\mu \det(\not{D}) e^{-S[A_\mu]} \mathcal{O}[A_\mu]$$

Monte Carlo method: generate random A_μ configurations using

$$\det(\not{D}) e^{-S[A_\mu]}$$

as a probability distribution, then evaluate the integral.

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QCD at μ_B=0: $\gamma_5 \not{D} \gamma_5 = \not{D}^\dagger$ → Eigenvalues of $\not{D}$ come in λ, λ* pairs

So then $\det(\not{D}) = \prod_i \lambda_i > 0$

The ~~sign~~ phase problem

Once $\mu_B > 0$, γ^5 symmetry breaks, and $\det(\not{D})$ becomes complex, with a rapidly fluctuating phase.

Can't use importance sampling anymore!

No known way to generically dodge this kind of problem.

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But maybe one just needs a clever algorithm to sum up the fluctuating phases?

Well...

Computational Complexity and Fundamental Limitations to Fermionic Quantum Monte Carlo Simulations

Matthias Troyer[1] and Uwe-Jens Wiese[2]
[1]*Theoretische Physik, ETH Zürich, CH-8093 Zürich, Switzerland*
[2]*Institut für theoretische Physik, Universität Bern, Sidlerstrasse 5, CH-3012 Bern, Switzerland*
(Received 11 August 2004; published 4 May 2005)

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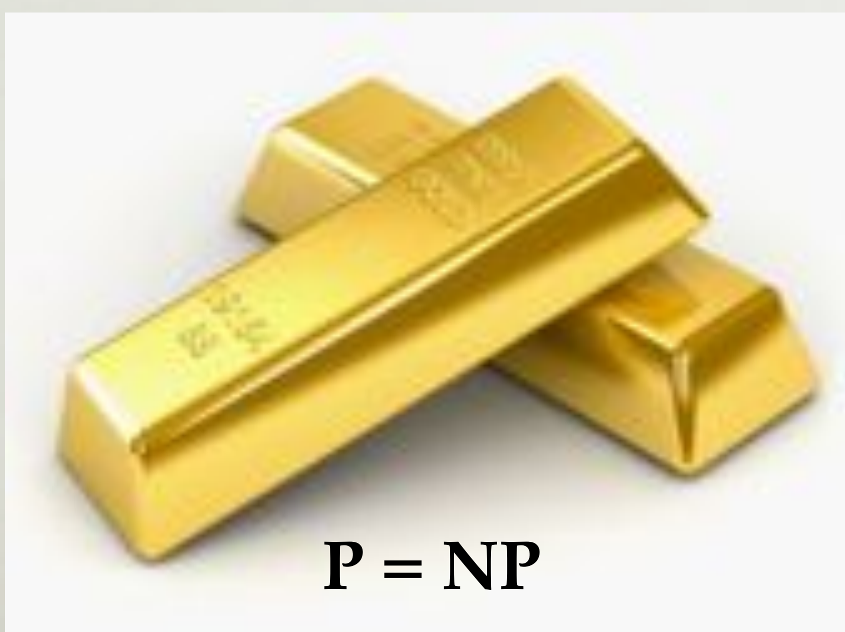

Clay Institute Prize

or

So how to make progress?

(1) Do **not** look for general solutions: exploit specifics of theory.

(2) Our approach: Exploit QCD details, but not in $N_c = 3$ world - too hard!

Go to the large N limit!

Good (10-30%) approx. to real world for many observables at $\mu_B = 0$.

Probably much less close to our world for $\mu_B > 0$, but such is life.

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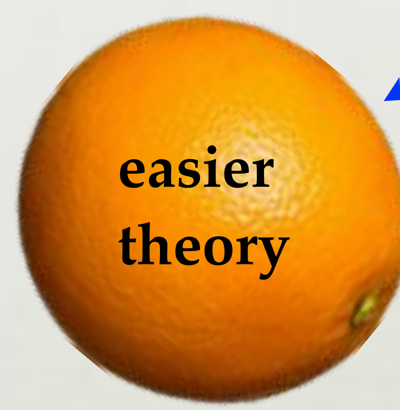
easier to `squeeze'...

First glance:

$=$



$+ \, 1/N$

The idea: find sign-problem-free theory which is `orbifold-equivalent' to large N QCD at μ_B>0.

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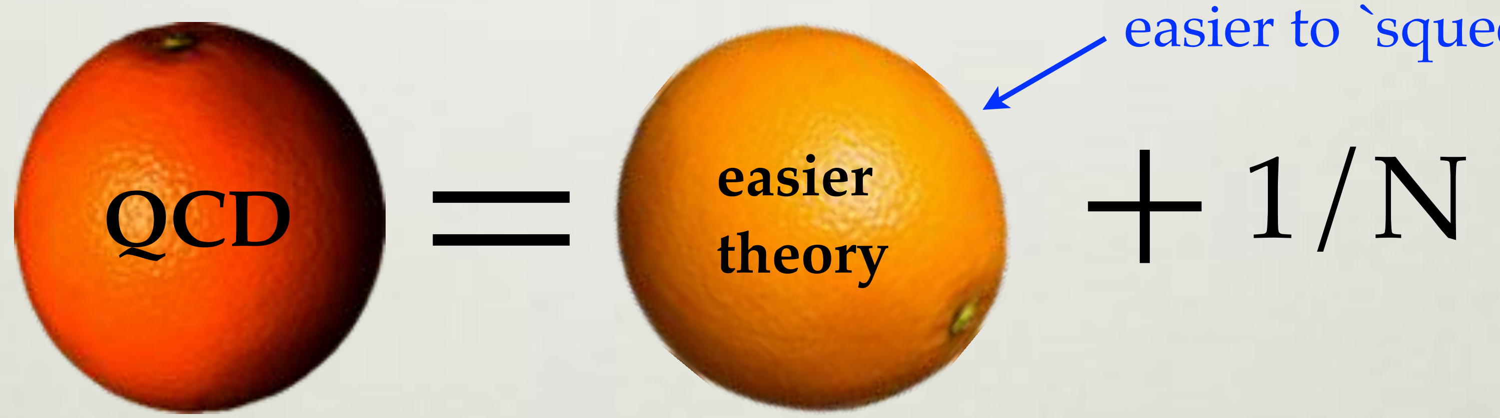
Better picture



The idea: find sign-problem-free theory which is `orbifold-equivalent' to large N QCD at μ_B>0.

e.g.: Hands et al + many others

First: Do sign-problem-free theories exist?

Yes!

1. QCD with N=2 colors, and
2. QCD with adjoint representation quarks.

$\gamma_5 \not{D}\, \gamma_5 = \not{D}^\dagger$ still broken when $\mu_B > 0$

But now fermion representation is (pseudo)-real...

$\Rightarrow$ additional symmetry: even when $\mu_B > 0$!

$$C\gamma_5 \not{D}\, (C\gamma_5)^{-1} = \not{D}^*$$

No sign problem!

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But 1 & 2 have a number of major differences from N=3 QCD...

Goal is to use large N to get something equivalent to QCD.

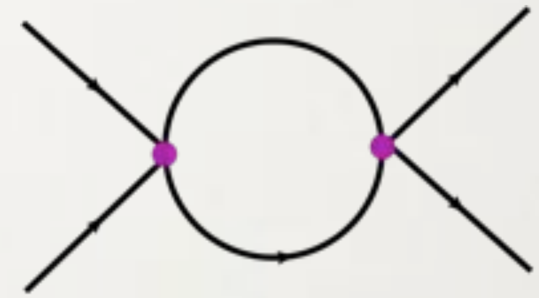
Second: lightning review of large N

't Hooft large N limit: $N \to \infty$, keeping $g^2 N$ fixed, N_f fixed

Non-planar diagrams and quark loops suppressed

$\sim 1/N^{1/2}$

$\sim 1/N$



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Mesons are stable, weakly-interacting; meson loops suppressed.

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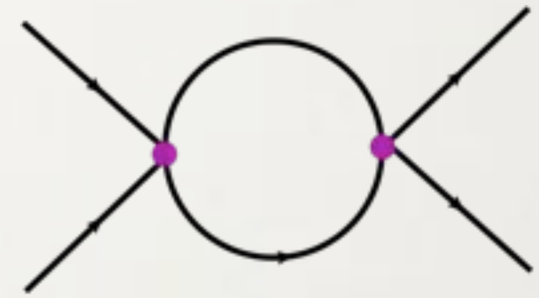

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Sign problem still present at large N.

Folklore says large N means we can set $\det(\not{D}) = 1$ OK at $\mu_B=0$.

But at finite μ_B this is **known** to give wrong answers: spurious phase transitions!

e.g. Barbour et al, 1986, Stephanov 1996

Setting $\det(\not{D}) = 1$ by hand is a mutilation of the theory...

Expect $\det(\not{D})$ to continue to have a fluctuating phase even at large N, so sign problem is still there...

AC, Hanada, Robles-Llana, PRL 2011

AC, Tiburzi 1103.1639

The proposal

1. SU(N) gauge theory + N_f fundamental fermions $\cong$ SO(2N) gauge theory + N_f fundamental fermions

QCD ↑ — Orbifold equivalence ↑ — easier theory ↑

2. Equivalence can be made to hold even when $\mu_B > 0$.

Use deformation approach due to Unsal+Yaffe

3. The SO(2N) theory does not have a sign problem at finite μ_B.

Make sure D has enough symmetry, e.g. $C\gamma_5 \not{D} \left(C\gamma_5\right)^{-1} = \not{D}^*$

A quick look at SO gauge theories

$$\mathcal{L} = \frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \sum_{a=1}^{N_f}\bar{\psi}_a(\not{D} + m + \mu_B\gamma^4)\psi_a$$

Looks a lot like QCD: has both mesons and baryons

Still have $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ symmetry.

Witten & Coleman, Peskin, 1980

But SO is real, so all fermion reps are real

Flavor symmetry enhanced to $SU(2N_f)$

$$SU(2N_f) \xrightarrow{\langle\bar{\psi}\psi\rangle \neq 0} SO(2N_f)$$

$$N_f^2 - 1 \; + \; N_f(N_f - 1)$$

NG bosons

Two ways to make color singlets in SO(2N)

QCD: $\bar{\psi}_a \gamma^5 \psi_b$ $N_f^2 - 1$ pions, P=-1

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Baryonic pions, P=+1

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In what sense can such a weird theory be `equivalent' to QCD?

 $\neq$ 

Kachru, Silverstein 1998

Orbifold Equivalence

Kovtun, Unsal, Yaffe, 2003-4

Pick "mother" theory with a global symmetry G. $+$ Pick a discrete cyclic subgroup $\mathbb{Z}_\Gamma \subset G$

The orbifold projection: Set to zero all degrees of freedom in the mother not invariant under $\mathbb{Z}_\Gamma \subset G$

$\mathbb{Z}_\Gamma$ orbifold "daughter theory"

If $\mathbb{Z}_\Gamma$ symmetry is not spontaneously broken

Correlation functions of `neutral' operators in mother and daughter theories will coincide in the large N limit.

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Truth in advertising:

Existing proofs of large N equivalence require some generalizations for this application: no general proof yet that necessary conditions above are also sufficient for *fund.* fermion case.

From SO(2N) to SU(N) QCD in one slide

How does one connect an SO(2N) gauge theory to an SU(N) theory?

(1) Change the gauge group: project onto SU(N) subgroup

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \in SO(2N) \qquad A_\mu \longrightarrow J A_\mu J^T = A_\mu$$

(2) The bmesons better get killed by projection...

$$\omega = e^{i\pi/2} \in U(1)_B \qquad \psi \longrightarrow \omega J \psi = \psi$$

Result of orbifold: $\mathcal{L}^{SO} \longrightarrow \mathcal{L}^{SU}$

Survivors of projection

All gauge-invariant operators in pure-glue sector of SO theory

All meson operators

neutral sector in SO

Victims of projection

All bmeson operators

non-neutral sector

Operators of the form $\psi^T \psi$ have $\mathbb{Z}_2$ charge -1

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AC, Mike Blake, 1203.XXXX

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Cartoon picture of orbifold equivalence

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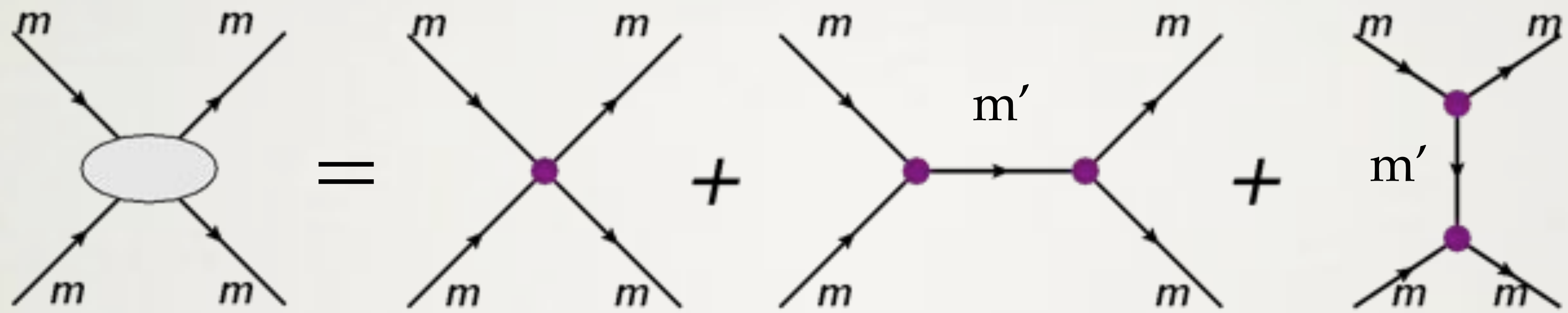


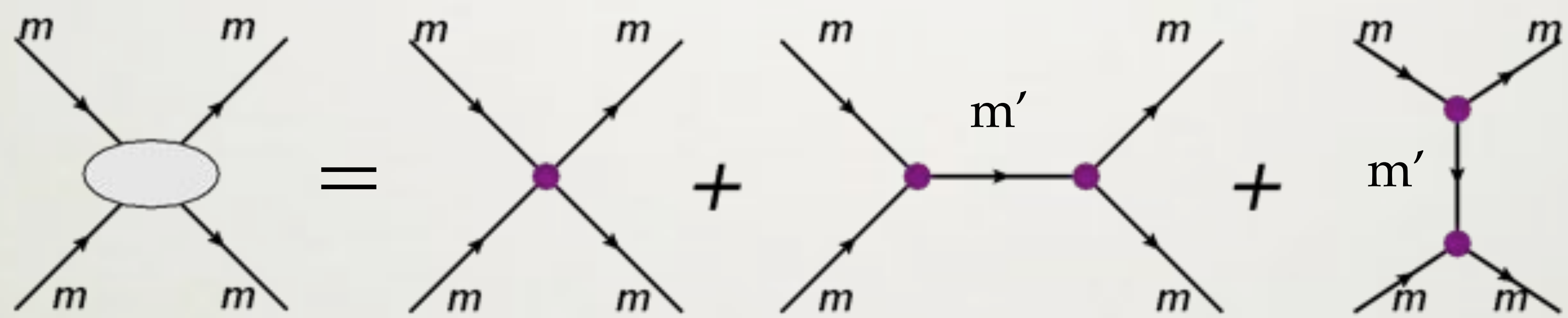
$\sim 1/N$ $\sim 1/N$ $\sim 1/N$

m = meson

b = bmeson

Discard bmesons

Daughter:



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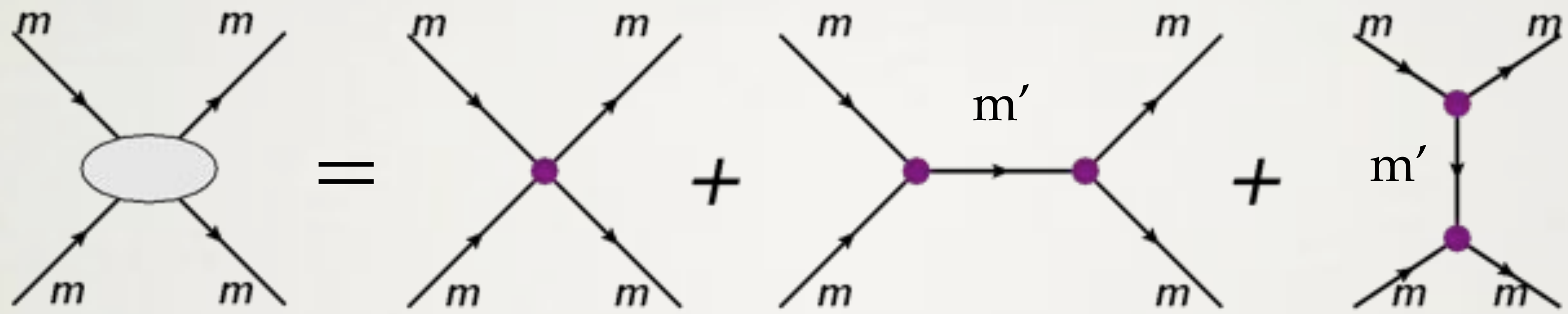

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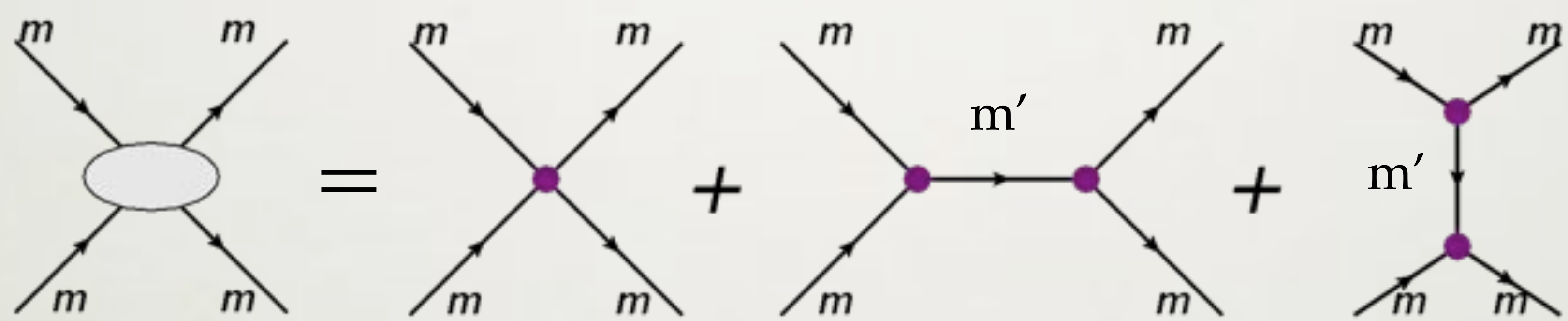

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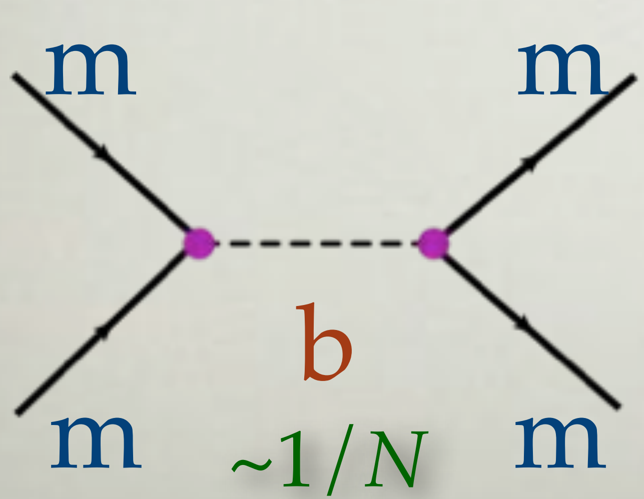

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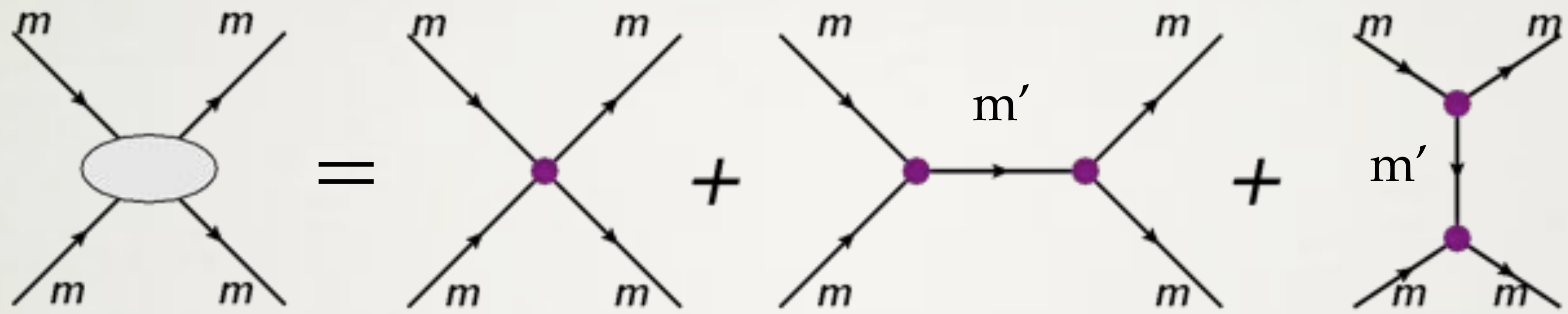

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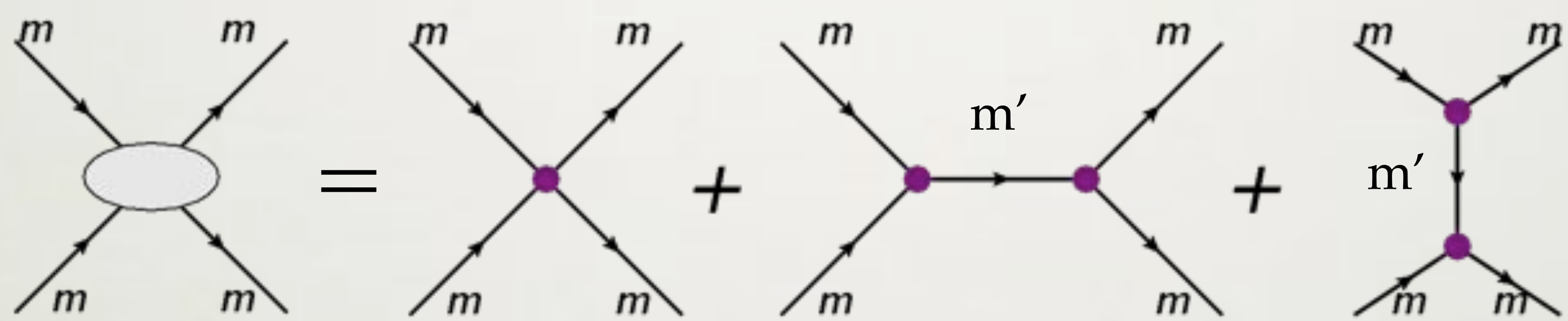

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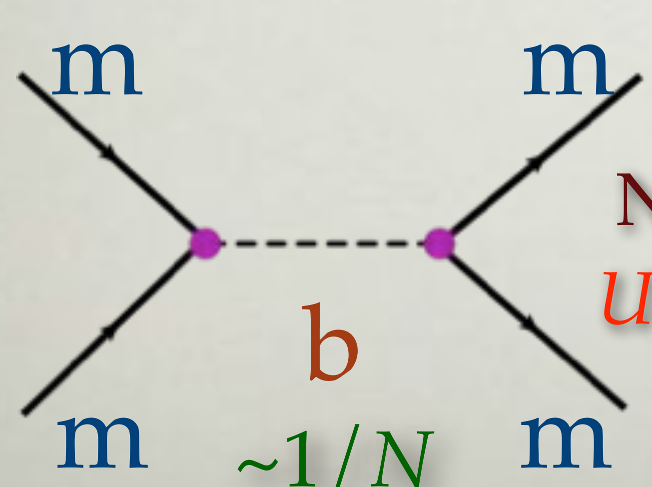

Not allowed if $U(1)_B$ unbroken

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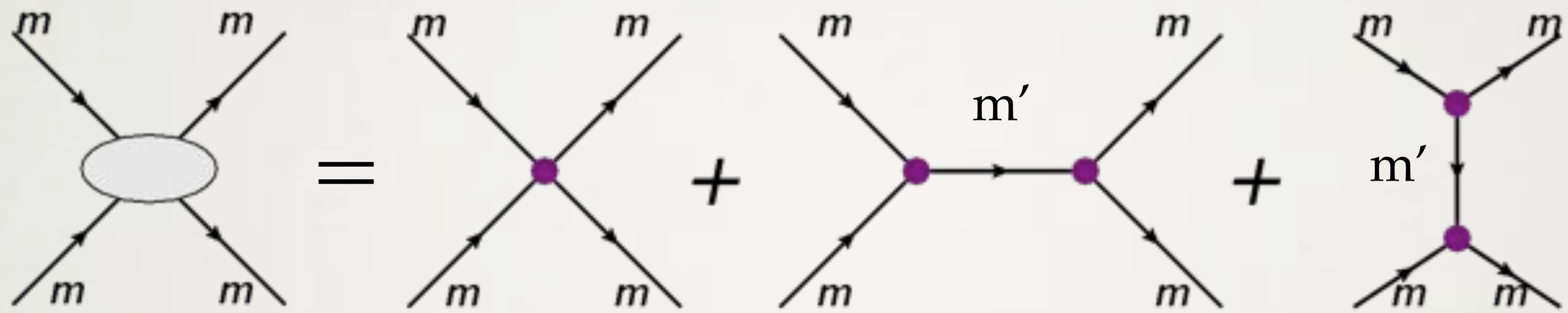

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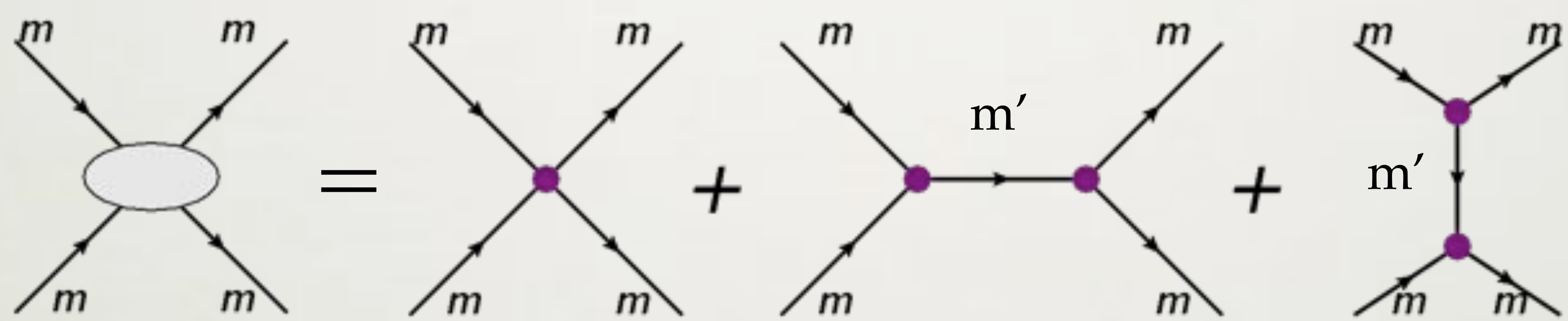

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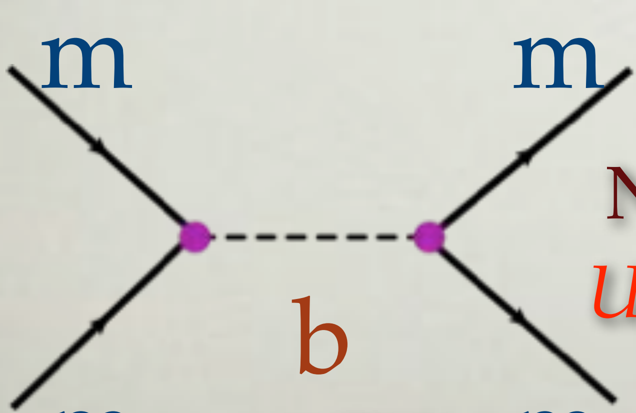

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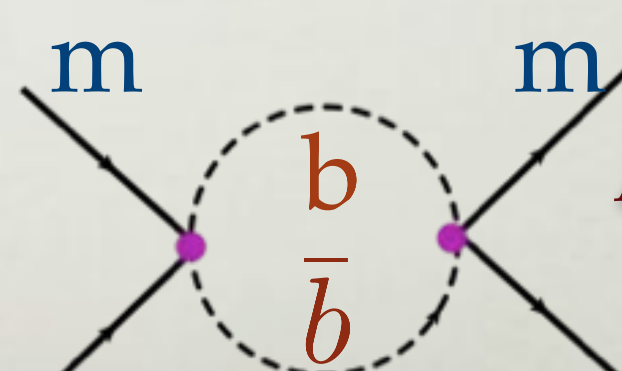

$\sim 1/N^2$

Allowed but suppressed

The good news

No bmeson condensation at μ_B=0. Vafa-Witten theorem

SO theory should be large N equivalent to QCD at μ_B=0

In fact, can show that there is no bmeson condensation at least for $\mu_B < m_\pi/2$. Using XPT analysis

So at least up to $\mu_B<m_\pi/2$, expect equivalence to hold.

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So at least up to $\mu_B < m_\pi/2$, expect equivalence to hold.

But (in principle) large N QCD has a sign problem for *any* $\mu_B > 0$!

So orbifold equivalence gives a way to dodge the sign problem at least for $\mu_B < m_\pi/2$.

Already enough to think about physics at small μ_B/T - see Hanada-Yamamoto 2011

But we need to go past $\mu_B < m_\pi/2$ to study nuclear matter...

The bad news

Once $\mu_B > m_\pi/2$ bpions condense : $\langle \psi^T C \gamma^5 \psi \rangle \neq 0$

Equivalence is lost for $\mu_B > m_\pi/2$!

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The proposal

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1. SO(2N) gauge theory with N_f flavors of fundamental Dirac fermions $\cong$ SU(N) gauge theory with N_f flavors of fundamental Dirac fermions

Orbifold equivalence   Large N QCD

2. Equivalence can be made to hold even when $\mu_B > m_\pi/2$.

→ Use deformation approach due to Unsal+Yaffe

3. The SO(2N) theory does not have a sign problem at finite μ_B.

→ Make sure D has enough symmetry, e.g.

$$C\gamma_5 \not{D} \, (C\gamma_5)^{-1} = \not{D}^*$$

Protecting $U(1)_B$

inspired by double-trace deformations of Unsal and Yaffe, 2008.

We deform the SO(2N) theory so that

(1) the modified theory still maps to QCD, and

(2) prevent bpion $S_{ab} = \psi_a^T C \gamma_5 \psi_b$ condensation.

$$\mathcal{L}_{SO} \to \mathcal{L}_{SO} + \frac{\mathfrak{C}^2 a^2}{N} S^\dagger_{ab} S^{ab}$$

Note: deformation term orbifolds to zero.

Cartoon picture: should act like a mass term for bpions.

So system pays extra cost for condensing when C > 0...

So use deformations to discourage bpion condensation.

Next step: make sure this is more than a cartoon.

Sometimes irrelevant operators are quite relevant

Original theory: YM on lattice + naive fermions

Natural scale for physical m_q on lattice: ***m~$1/a$***

Symmetries: Chiral sym, doubler sym

Consequences: Doubler sym locks m_d of 2^D-1 tastes to m_{phys}, χ-sym keeps $m_{\text{phys}} = m_{\text{bare}}$

Deformed theory: YM + naive fermions + Wilson term

$$\mathcal{L}_{\text{naive}} \to \mathcal{L}_{\text{naive}} + ra\,\bar{\psi}\not{D}^2\psi$$

Deformation breaks doubler symmetry → Doubler masses zoom off to the natural scale m~$1/a$ when r~1

(χ-sym broken too, but by tuning m_{bare} can tune m_{phys} to anything.)

Sometimes irrelevant operators are quite relevant

Original theory: YM on lattice + naive fermions

Natural scale for physical m_q on lattice: ***m~1/a***

Symmetries: Chiral sym, doubler sym

Consequences: Doubler sym locks m_d of 2^D-1 tastes to m_{phys}, χ-sym keeps $m_{\text{phys}} = m_{\text{bare}}$

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Deformation breaks doubler symmetry → Doubler masses zoom off to the natural scale $m\sim 1/a$ when r~1

(χ-sym broken too, but by tuning m_{bare} can tune m_{phys} to anything.)

Natural scale for meson masses: $\boldsymbol{m_{\text{hadron}} \sim \Lambda_{QCD}}$

In SO theory, deformation breaks $SU(2N_f)$ symmetry keeping m_{bpion} locked to m_π → For $\mathfrak{C} \sim 1$ expect m_{bpion} to zoom off to $m_{\text{bpion}} \sim \Lambda_{\text{QCD}}$

Of course, lattice simulations critical to better understand deformed theory

Deformations and Effective Field Theory

AC, B. Tiburzi, 1103.1639

Hard to understand deformed theory analytically in general.

But if m_q, $\mu_B << \Lambda_{QCD}$ and $\mathfrak{C} \ll 1$, low-energy physics can be systematically describable using effective field theory.

Here EFT is just chiral perturbation theory adapted for SO gauge theory.

In *XPT* it is easiest to work with the deformations

$$V_\pm = \frac{\mathfrak{C}^2 a^2}{N} \sum_{a,b=1}^{N_f} \left(S^\dagger_{ab} S_{ab} \pm P^\dagger_{ab} P_{ab} \right) \qquad \begin{aligned} P_{ab} &= \psi_a^T C \psi_b \\ S_{ab} &= \psi_a^T C \gamma^5 \psi_b \end{aligned}$$

Without deformations, the EFT has the Lagrangian

$$\mathcal{L} = \frac{F_\Pi^2}{4} \mathrm{tr}\left[D_\mu \Sigma D_\mu \Sigma^\dagger \right] - \frac{\lambda F_\Pi^2}{4} \mathrm{tr}\left[\Sigma \mathcal{M} + \Sigma^\dagger \mathcal{M}^\dagger \right]$$

Deformations induce new terms in the low-energy action...

Just have to work them out...

Two deformations

To capture effects of deformations, use spurion analysis.

Deformation is 4-quark operator, so can borrow standard techniques used in XPT to understand e.g. finite lattice-spacing effects

V_+ produces just one new term in the EFT

$$c_+ \, F_\Pi^2 \sum_{a,b=1}^{N_f} \left(\mathrm{tr}\left[\Sigma L^{(ab)}\right] \mathrm{tr}\left[\Sigma^\dagger L^{(ab)\dagger}\right] + \mathrm{tr}\left[\Sigma R^{(ab)}\right] \mathrm{tr}\left[\Sigma^\dagger R^{(ab)\dagger}\right] \right)$$

V_- produces two new terms in the EFT

$$c_- F_\Pi^2 \sum_{a,b=1}^{N_f} \left(\mathrm{tr}[\Sigma L^{(ab)}]\mathrm{tr}[\Sigma R^{(ab)}] + \mathrm{tr}[\Sigma^\dagger L^{(ab)\dagger}]\mathrm{tr}[\Sigma^\dagger R^{(ab)\dagger}] \right)$$

$$+ \, d_- F_\Pi^2 \sum_{a,b=1}^{N_f} \left(\mathrm{tr}[\Sigma L^{(ab)} \Sigma R^{(ab)}] + \mathrm{tr}[\Sigma^\dagger L^{(ab)\dagger} \Sigma^\dagger R^{(ab)\dagger}] \right)$$

New low-energy constants $\boxed{c_+ , c_- , d_-}$

Spectrum of the deformed theory

Without symmetry breaking:

Mode	Mass with V_- deformation	Mass with V_+ deformation
π	$(m_\pi^2 + 4d_-)^{1/2}$	m_π
η'	$(m_\pi^2 + 4d_-)^{1/2}$	m_π
b	$(m_\pi^2 + 4c_-)^{1/2} + 2\mu$	$(m_\pi^2 + 4c_+)^{1/2} + 2\mu$
$b^\dagger$	$(m_\pi^2 + 4c_-)^{1/2} - 2\mu$	$(m_\pi^2 + 4c_+)^{1/2} - 2\mu$

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Matching to microscopic theory gives N_c scaling of the new LECs

$c_-, c_+ \sim N_c^0$, $d_- \sim N_c^{-1}$

Can also show that the sign of C in microscopic theory controls the signs of the LECs in the EFT.

So deformations work by raising the bpion masses, while leaving neutral-sector stuff alone.

To nail down symmetry realization pattern, minimize effective potential in deformed theory

AC, M. Hanada, D. Robles-Llana, 2010

AC, B. Tiburzi 2011

The proposal

1. SO(2N) gauge theory with N_f flavors of fundamental Dirac fermions $\cong$ SU(N) gauge theory with N_f flavors of fundamental Dirac fermions

Orbifold equivalence   Large N QCD

2. Equivalence can be made to hold even when $\mu_B > m_\pi/2$.

→ Use deformation approach due to Unsal+Yaffe

3. The SO(2N) theory does not have a sign problem at finite μ_B.

→ Make sure D has enough symmetry, e.g.

$$C\gamma_5 \not{D} \, (C\gamma_5)^{-1} = \not{D}^{\,*}$$

Sign-free implementation of deformations

Deformations are four-quark operators, so must use auxiliary fields to put them on the lattice.

Sign problem reappears if aux field implementation breaks enough symmetries!

Aux fields coupling to $S_{ab} = \psi_a^T C \gamma_5 \psi_b$ must be complex, **sign problem**

For V_-, we found a rather baroque way to implement auxiliary fields that avoids reintroducing the sign problem

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Fierz rearrangement: $S^{\dagger\, ab} S_{ab} = \sum_\Gamma (\bar{q}_a^i \Gamma q_a^j)^2$

Can couple real auxiliary fields f_{ij} to

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$U(1)_B$ singlet, color tensor

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Integration over f_{ij} gives original 4-quark terms

$$S^{\dagger\, ab} S_{ab} \longrightarrow \sum_\Gamma \left[\frac{1}{2} f_{ij}^\Gamma f^{ij\,\Gamma} + i c_\Gamma f_{ij}^\Gamma \bar{q}_a^i \Gamma q_a^j \right]$$

+ similar terms for $P^{\dagger\, ab} P_{ab}$

Sign-free implementation of V_- deformations

Result of integrating in auxiliary fields in flavor-singlet channel:

$$\frac{c^2}{\Lambda^2}\left(S^{\dagger\,ab}S_{ab} - P^{\dagger\,ab}P_{ab}\right) \Rightarrow \begin{aligned}&(f_{ij})^2/2 + (g_{ij})^2/2 + (h_{\mu\nu,ij})^2/2 \\ &+ ic_1 f_{ij}\bar{\psi}^i_a\psi^j_a + ic_2 g_{ij}\bar{\psi}^i_a\gamma^5\psi^j_a \\ &+ ic_3 h_{\mu\nu,ij}\bar{\psi}^i_a\gamma^{\mu\nu}\psi^j_a\end{aligned}$$

Factors of *i* **break** $C\gamma^5$ symmetry.

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Factors of *i* **break** $C\gamma^5$ symmetry.

But for $m_q = 0$, aux fields preserve $CD(\mu_B, c)C^{-1} = -D(\mu_B, c)^*$

Enough symmetry to ensure positivity as $m_q \to 0$, even when $c > 0$

Finally:

✔No sign problem in the chiral limit.

✔Large N equivalence to QCD kept past $\mu_B = m_\pi/2$

The same trick does not work for V_+. Are there other tricks that do?

Summary and open questions

Using SO theory, we can dodge sign problem even past $m_\pi/2$.

Vanishing of sign problem as $m_q \to 0$ $\Rightarrow$ Sign-quenching should be a good approximation for light quarks.

Summary and open questions

Using SO theory, we can dodge sign problem even past $m_\pi/2$.

Vanishing of sign problem as $m_q \to 0$ ➡ Sign-quenching should be a good approximation for light quarks.

Does equivalence hold through nuclear matter transition?

- Do bmesons with charge/mass less than lightest baryons exist, even in deformed theory?

- If so, expect condensation for big enough μ_B, killing equivalence.

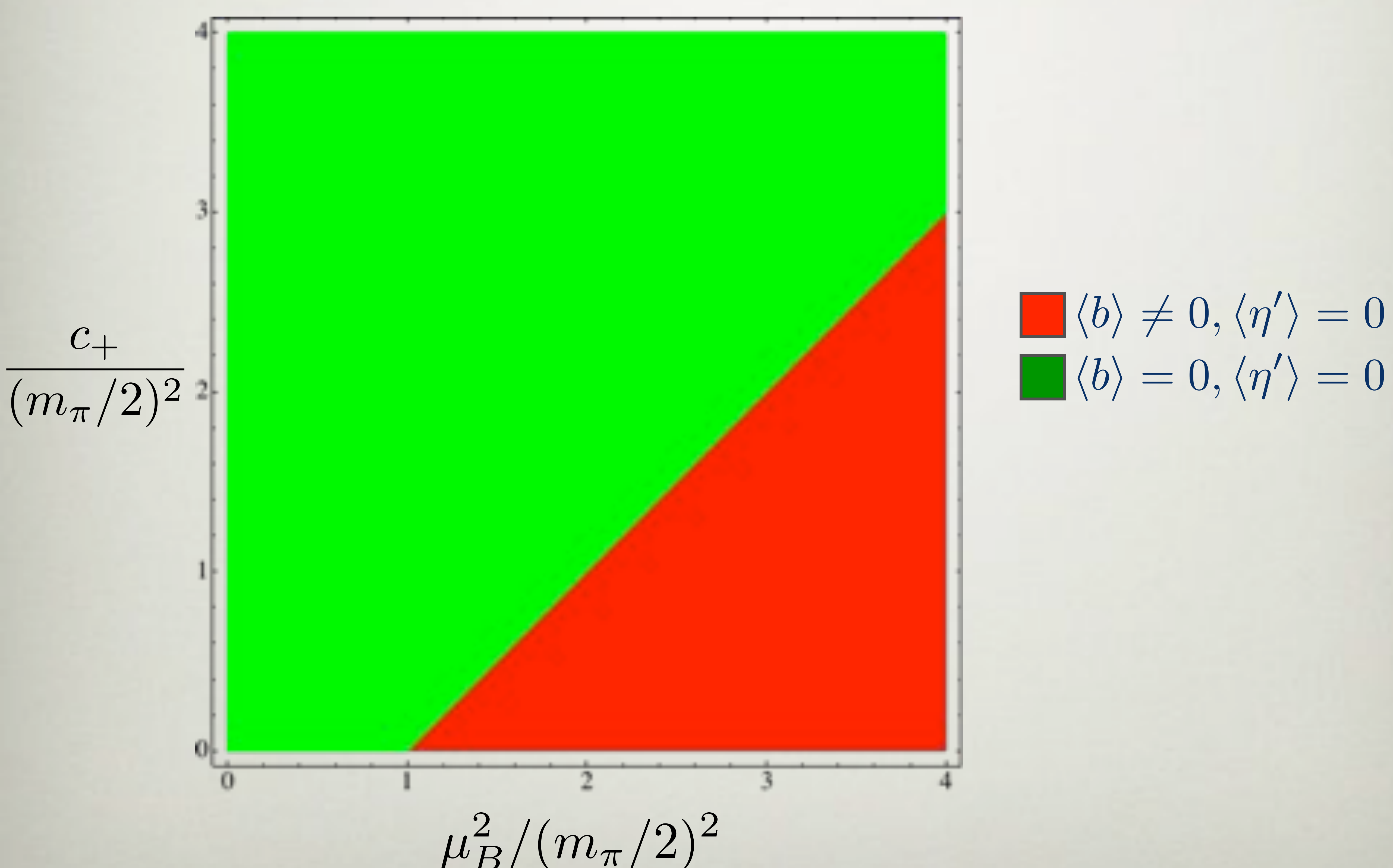
We need non-perturbative tests!

Lattice, AdS/CFT, ...

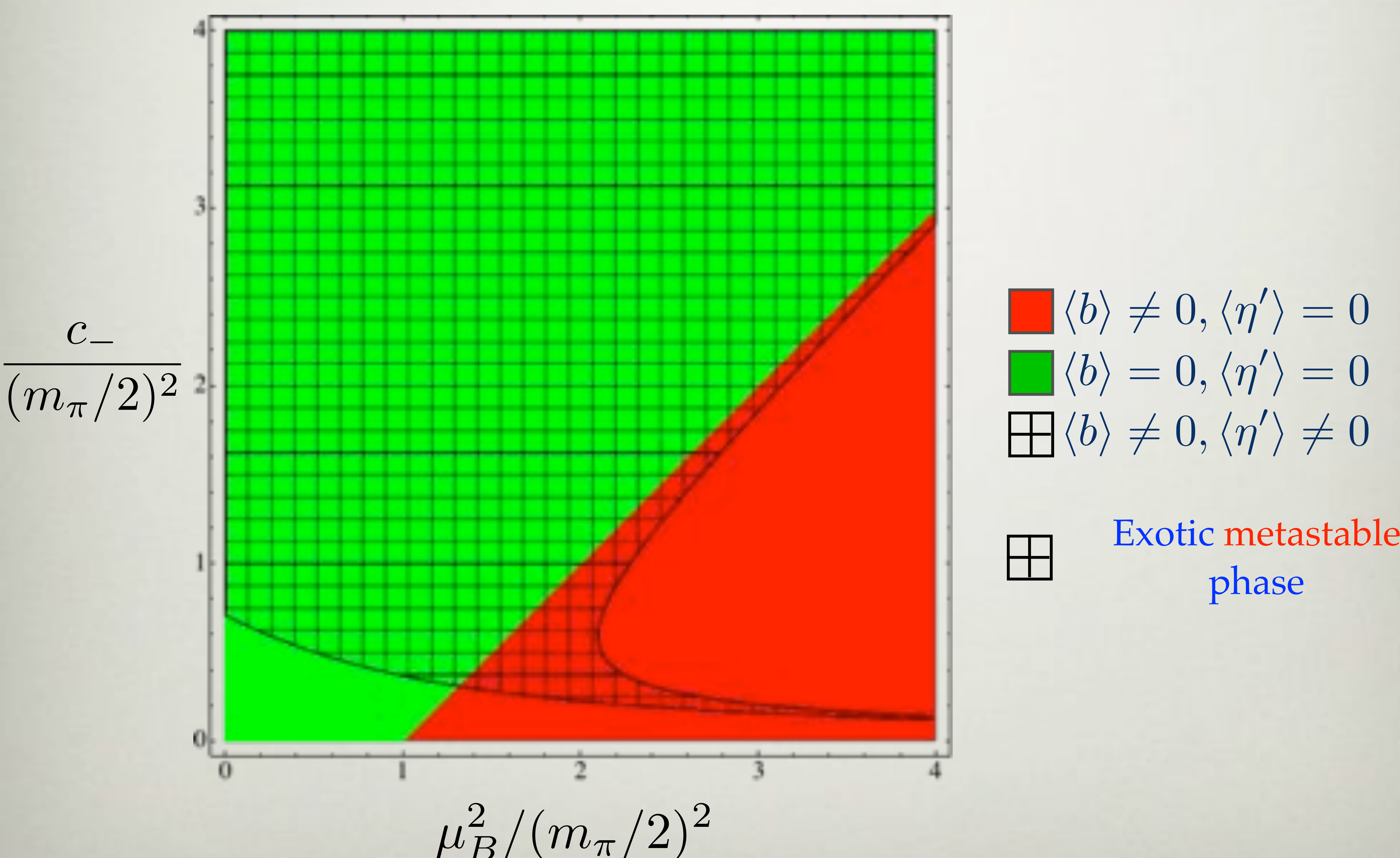
To do:

Extend equivalence proofs, look for sign-free way to work with V_+, try to get away from chiral limit, try to dodge other sign problems,...

Phase diagram of the V_+-deformed theory



Phase diagram of the V_{-}-deformed theory



Orbifold equivalence past $\mu_B = m_\pi/2$

With both deformations, the SO theory can be forced to stay in a $U(1)_B$-unbroken phase past $\mu_B = m_\pi/2$.

The correlation functions of neutral operators are identical with both deformations in the normal phase.

The V_--deformed theory has an exotic phase with η'-condensation. This phase is always metastable in our analysis.

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At level of EFT, large N-equivalence is `obvious':

$$\frac{U(N_f)_L \times U(N_F)_R}{U(N_f)_V} \subset \frac{SU(2N_f)}{SO(2N_f)}$$

At large N, neutral correlators in $SU(2N_f)/SO(2N_f)$ EFT with given LECs trivially coincide with correlators computed in an $SU(N_f)$ EFT with the same LECs, so long as $U(1)_B$ is not broken.