Adams and Hoelbling fermions: numerical properties

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Motivation

- Light u,d quarks needed to simulate correct physics $\to$ expensive
- Cost-saving: **staggered fermions** (1/4 d.o.f.)

$$S_F = \sum_x \bar{\chi}(x) \sum_\mu \eta_\mu(x)(U_\mu(x)\chi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\chi(x-\hat{\mu})) + m_q \sum_x \bar{\chi}(x)\chi(x)$$

$$\eta_\mu = \pm 1;\ \{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu} \ \to\ \prod_4 \eta = -1 \text{ around any plaquette}$$

- Drawback: $N_f = 4$ (degenerate when $a = 0$) "tastes" $\to \sqrt{\det(D_{st})}$

"rooting is evil" **Mike Creutz**

- non-locality?
- 't Hooft vertex, $U(1)_A$ breaking?
- staggered fermions "don't feel the topology"

- No quartet of low-lying eigenvalues $\leftrightarrow$ no index theorem

- Adams to the rescue: $N_f = 2$ staggered overlap fermions

0912.2850, 1008.2833

- Hoelbling: $N_f = 1$ 1009.5362

Construction

Two ingredients:

1: taste-dependent mass term (cf. Wilson term)

$\Longrightarrow$ split tastes into several branches – fine-tune for massless limit

2: plug into overlap (cf. Neuberger)

$\Longrightarrow$ eliminate fine-tuning, etc.. – large computational overhead

Look at **1** then **2**

Possible mass terms — Golterman & Smit (1984)

$\mathbf{1}$	0-link
γ_μ	1-link
$\gamma_\mu\gamma_\nu$	2-link
$\gamma_\mu\gamma_\nu\gamma_\rho$	3-link
$\gamma_1\gamma_2\gamma_3\gamma_4$	4-link

- 1-link and 3-link do not satisfy γ_5-hermiticity $\rightarrow$ determinant *complex*
  - can be fixed with 2 conjugate copies, ie. doubling N_f
  - 1-link: cancellations between kinetic & mass terms
    free spectrum on a circle!

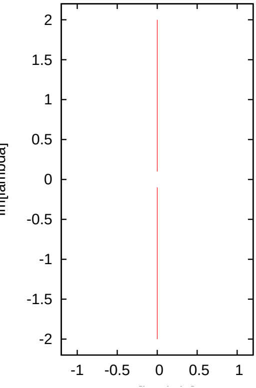
0-link = **KS**; 2-link = **Hoelbling**; 4-link = **Adams**

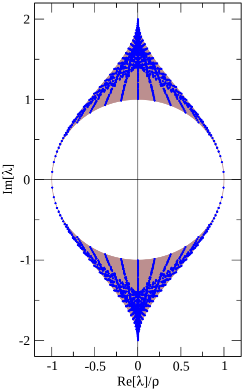
Notation: 0-link $\rightarrow m$, 2-link $\rightarrow M_{Hoelbling}$, 4-link $\rightarrow M_{Adams} \propto \Gamma_{55}\Gamma_5$

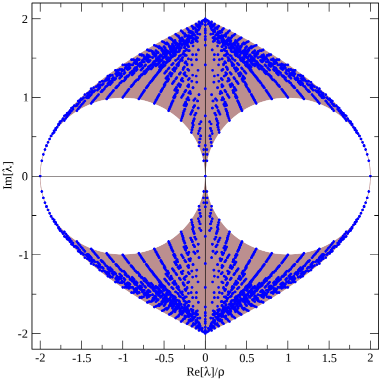
- NB. Also can multiply above mass terms by $\Gamma_{55} \equiv (-)^x$?

Free spectra and taste reduction I

$\not{D}_{st}$ | $\not{D}_{st} + M_{Adams}$ | $\not{D}_{st} + M_{Hoelbling}$



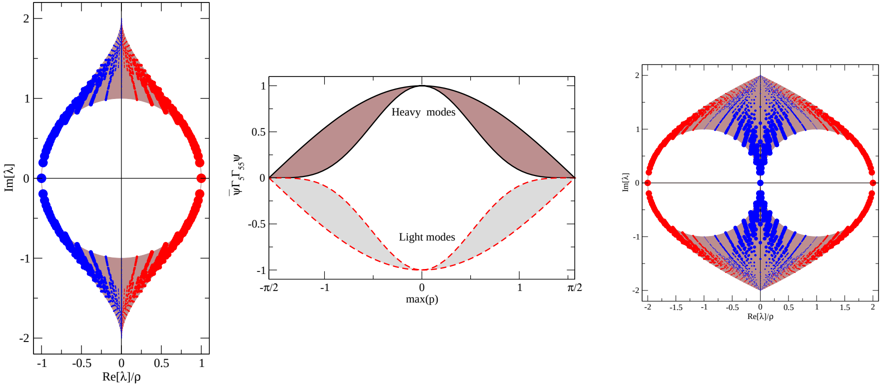




$N_f = 4$ | $N_f = 2 + 2$ | $N_f = 1 + 2 + 1$

Free spectra and taste reduction II

- Select one branch by x-shift, ie. additive mass renormalization (cf. Wilson)
  Other branch(es) have masses $O(1/a)$, ie. doublers



- Real (low-momentum) modes are *taste-chiral* in each branch $\rightarrow\ \Gamma_{55}\Gamma_5 \approx \pm 1$

  $\Rightarrow$ can trade $\Gamma_5 = \gamma_5 \otimes \mathbf{1} + O(a)$ for $\Gamma_{55} = \gamma_5 \otimes \gamma_5$ *exactly* $\rightarrow$ index theorem

Index from eigenvalue flow

- Index from flow of eigenvalues $\lambda(m)$ of $H(m) = \gamma_5(\not{D} + m)$
  - $(\not{D} + m_0)$ has zero-mode $|\Psi_0\rangle \;\rightarrow\;$ eigenvalue $\lambda(m_0) = 0$ for $H(m_0)$
  - Perturb m away from $m_0 \;\rightarrow\;$ eigenvalue displaced by $\langle\Psi_0|\gamma_5(m - m_0)|\Psi_0\rangle$

    $\lambda(m) = \pm(m - m_0)$ (ie. crossing) **IF** $\langle\Psi_0|\gamma_5|\Psi_0\rangle = \pm 1$, ie. $|\Psi_0\rangle$ chiral
  - Works with $\not{D} = \not{D}_{Wilson}$, or $\not{D}_{st} + M_{Adams}, \not{D}_{st} + M_{Hoelbling}$ with $\gamma_5 \rightarrow \Gamma_{55}$

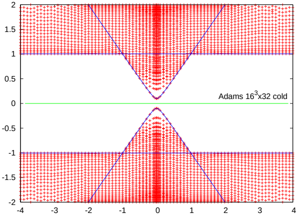
- Alternative: can also consider flow for $\hat{H}(\rho) = \Gamma_{55}(\not{D}_{st} + \rho M_{Adams})$

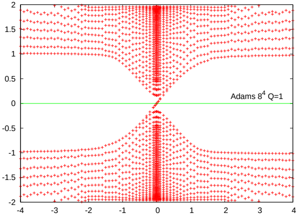
  Adams' original proposal
  - $\not{D}_{st}$ has near-chiral, near-zero modes
  - $M_{Adams} = \Gamma_{55}\Gamma_5 \;\rightarrow\;$ perturbation $\rho\Gamma_5 \approx \rho\gamma_5 \otimes \mathbf{1}$

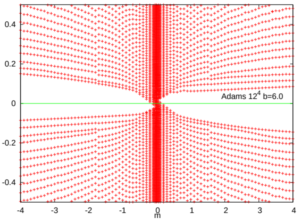
"Classic" versus "original" ?

Comparing two eigenvalue flows

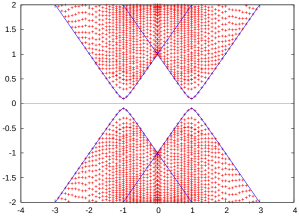
- Adams' **original**

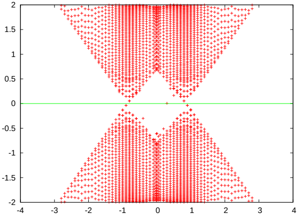


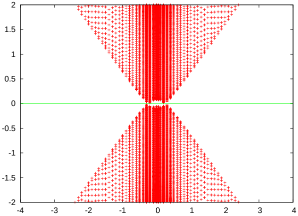




- Adams' **classic**



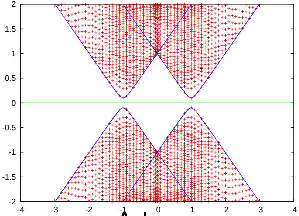


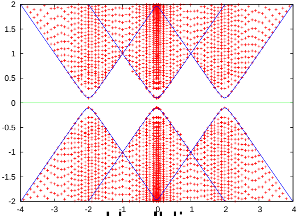


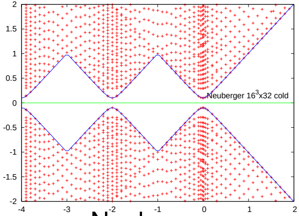
Cold config. $16^3 x32$ Cooled $Q = 1, 8^4$ $Q = -1, \beta = 6.0, 12^4$

Index theorem

- Cold configuration: agreement with analytic result

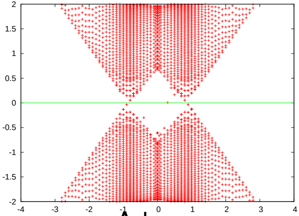

Adams

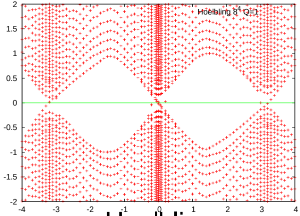

Hoelbling



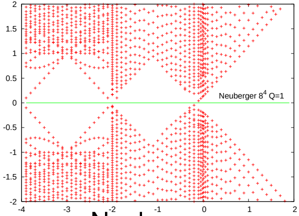
Neuberger

- Cooled $Q = 1$ instanton: $N_f \times Q$ crossings


Adams



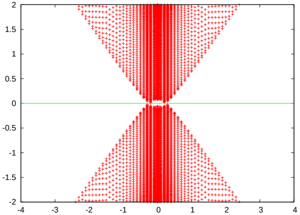
Hoelbling



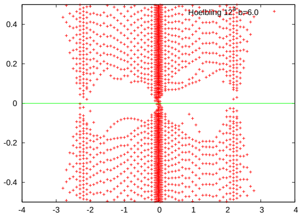
Neuberger

Effects of gauge field fluctuations

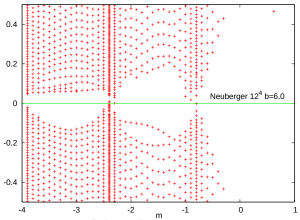
- $\beta = 6.0$, $Q = 1$: eigenvalue gap closes, esp. Adams



Adams



Hoelbling

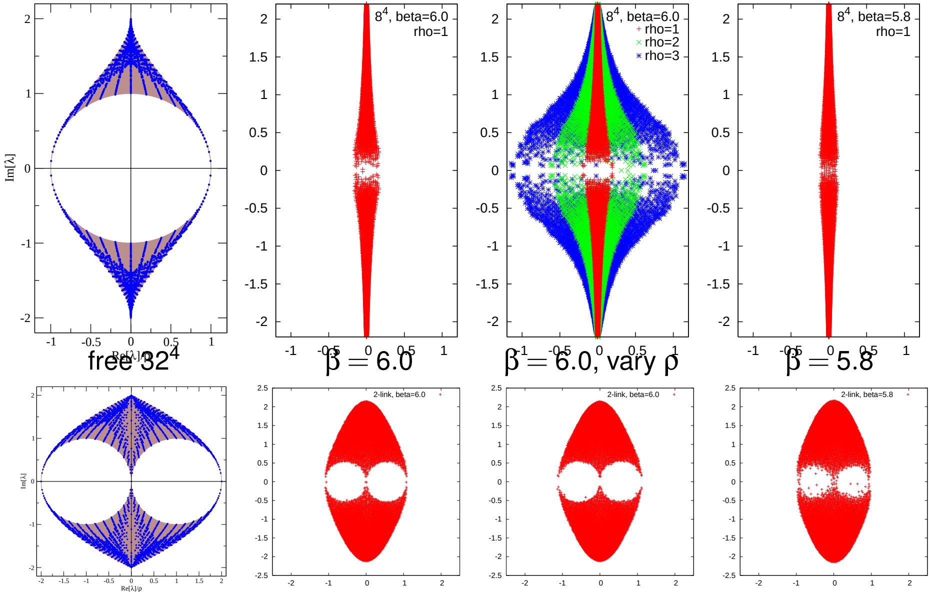


Neuberger

Effects of gauge field fluctuations

- The width of the spectrum fluctuates (shrinks)
  $\rightarrow$ fine-tuning for massless quarks
- The gap in the spectrum fills up
  $\rightarrow$ distinction between light modes and doublers blurred

Eigenvalue spectra: Adams vs Hoelbling, 8^4

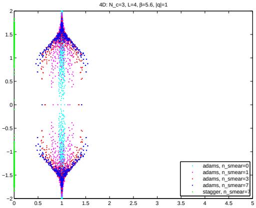


free 32^4 | $\beta = 6.0$ | $\beta = 6.0$, vary ρ | $\beta = 5.8$

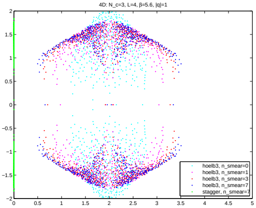
Usefulness

Superior robustness of Hoelbling fermions:
2-link vs **4**-link transporters

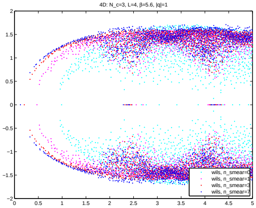
- Can always smear links to suppress fluctuations: Thanks Stephan Dürr



Adams



Hoelbling



Wilson

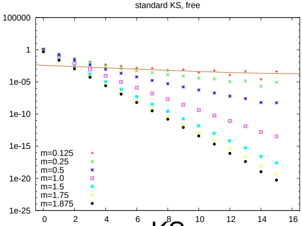
$4^4, \beta = 5.6$

Pion mass

- Compute quark propagator $G(x, y, z, t)$ from point source:
  - $16^3 \times 32$ lattices, $\beta = 6.0$ quenched
  - Form $\vec{p} = \vec{0}$ meson correlator (connected diagram only)
    $C(t) = \sum_{xyz} G(x, y, z, t)\Gamma_{55} G(x, y, z, t)^\dagger \Gamma_{55} = \sum_{xyz} |G(x, y, z, t)|^2$
  - Look for effective mass plateau $\rightarrow$ (lightest) pion mass (am_π)
- Monitor $(am_\pi)^2$ vs (am_q): PCAC + additive mass renormalization

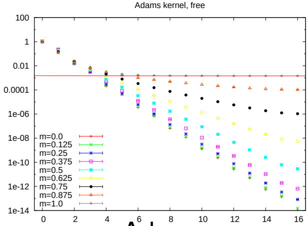
Pion correlator

- Free field:



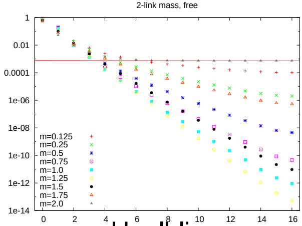
KS

$m_\pi = 0$ for $m_0 = 0$



Adams

$m_0 = 1$

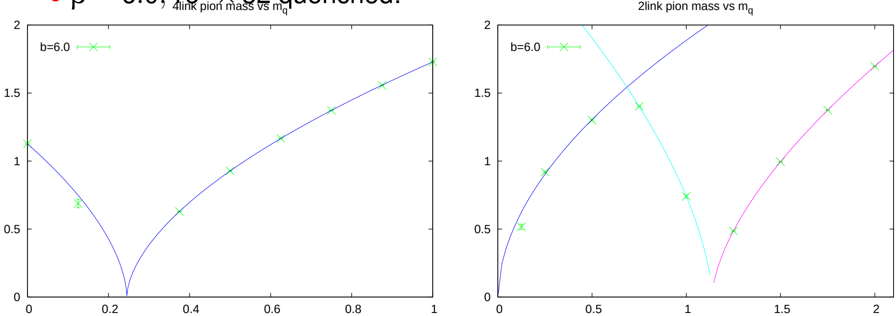


Hoelbling

$m_0 = 2$ **and** 0

Pion mass vs m_q

- $\beta = 6.0, 16^3 \times 32$ quenched:



Adams — Hoelbling

sqrt behaviour for $N_f = 2$ and $N_f = 1$??

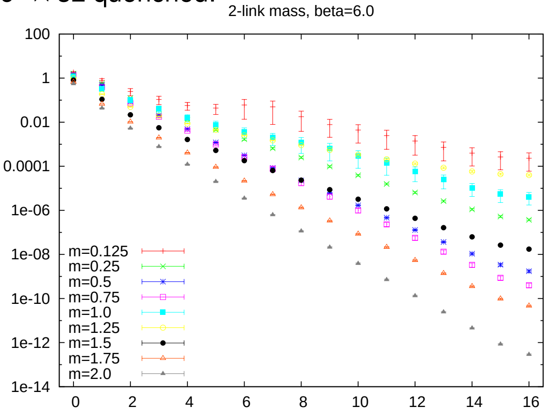
$m_0 \sim 0.25$ instead of 1 — $m_0 \sim 1.15$ instead of 2 **or** 0

4 links vs 2 links: $0.25/1 \sim (1.15/2)^2$

Use Hoelbling fermions for $N_f = 2$: *no need to tune* m_0 !

Pion correlator Hoelbling

- $\beta = 6.0, 16^3 \times 32$ quenched:



Overlap construction

- Just like Neuberger: $D_{sov} = 1 + \text{``}\gamma_5\text{''}\,\mathrm{sign}(H(-m_0))$

with $\text{``}\gamma_5\text{''} = \Gamma_{55} = (-)^{x+y+z+t}$ (need $\text{``}\gamma_5\text{''}^2 = 1$)

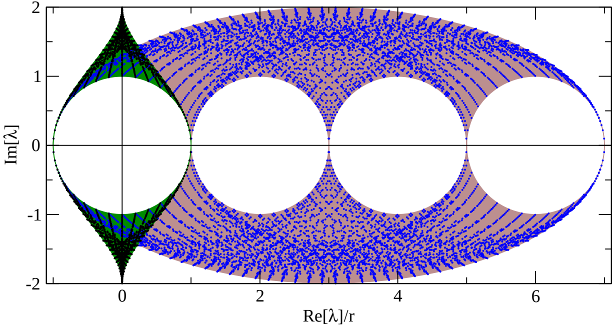
- Potential advantages:
  - cheaper (4 times fewer d.o.f. per site)
  - more robust ?

And reduces $N_f = 4$ to $N_f = 2$ tastes *without* fine-tuning

N.B. m_0 is really ρ in Adams' proposal [no mass shift]

Free field: $U_\mu(x) = \mathbf{1}\ \forall \mathbf{x}, \mu$

Spectrum of kernel: $\gamma_5 H_W(m_0 = -1)$ and $\gamma_5 H_{Adams}(m_0 = -1)$



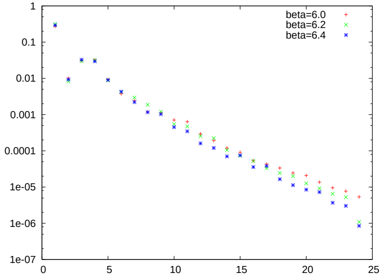
$\gamma_5 \mathrm{sign}(H) = \frac{D}{\sqrt{D^\dagger D}}$ projects eigenvalues of $D = \gamma_5 H$ on unit circle

Adams: two low-p eigenmodes projected to -1, two projected to +1 $\Rightarrow$ $N_f = 2$

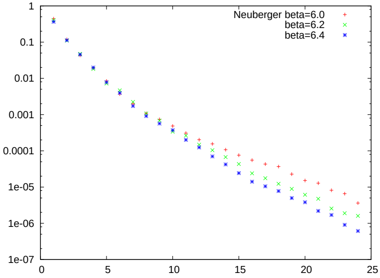
Locality of operator?

$\mathrm{Max}_y |M_{xy}|$ versus $|x-y|$ (Manhattan distance) cf. hep-lat/9808010

Adams



Neuberger

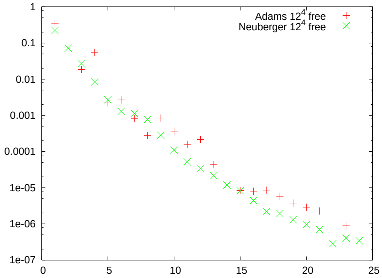


Adams comparable to Neuberger although kernel less local (4-link)

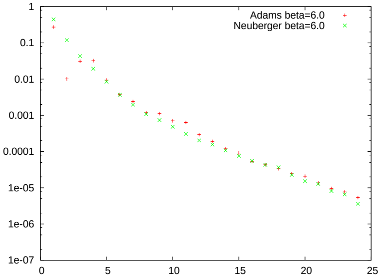
Locality of operator?

$\mathrm{Max}_y |M_{xy}|$ versus $|x - y|$ (Manhattan distance)   cf. hep-lat/9808010

Both, cold



Both, $\beta = 6.0$



Adams comparable to Neuberger although kernel less local (4-link)

Cost of applying operator

- Multiplication by D: about **2** times faster for Adams (no Dirac indices)
- Sign(H) [using CG, no deflation]:
  - about **8** times faster for Adams on easy cases
  - about **2-3** times faster on hard cases

Can optimize m_0 and ρ in Adams' operator: not exploited yet

Also:

improved kinetic operator, link smearing (kinetic and/or mass), deflation, preconditioning, ...

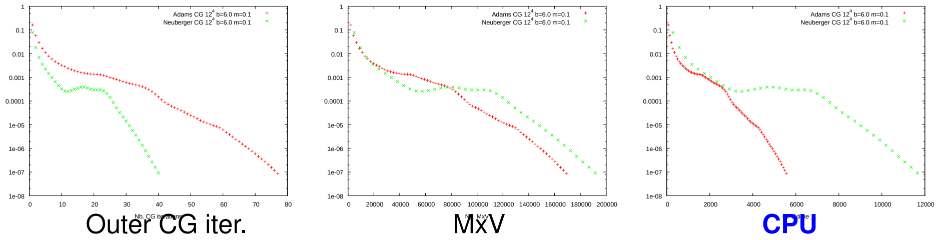
Cost of inversion: compare with Neuberger

Apples with apples:

- same gauge field ($12^4, \beta = 6.0$)
- same basic algorithm (CG inner, CG outer)

Adams versus Neuberger

- $\beta = 6.0$

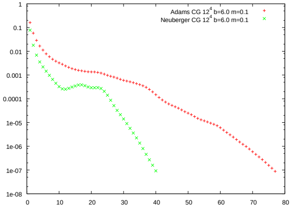


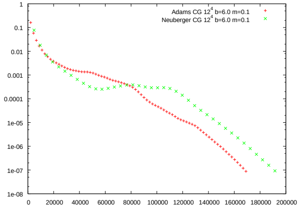
Net CPU gain: factor 2-3 over Neuberger...

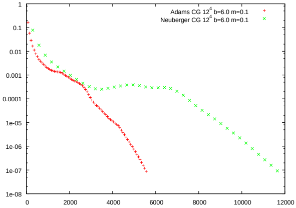
Cost of inversion: compare with Neuberger

Adams versus Neuberger

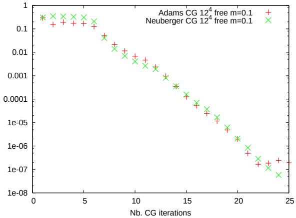
- $\beta = 6.0$

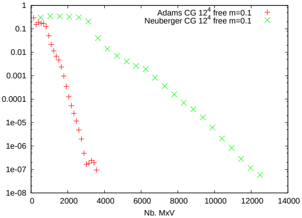


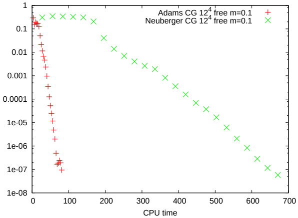




Outer CG iter. MxV **CPU**







- Free field: now factor 8+ $\rightarrow$ try to keep the free spectrum

Conclusions

- Index theorem, overlap etc.. work as advertised

- Compare taste-dependent mass terms *exhaustively* first – then try overlap

- Here: 2-link (Hoelbling) more robust than 4-link (Adams)

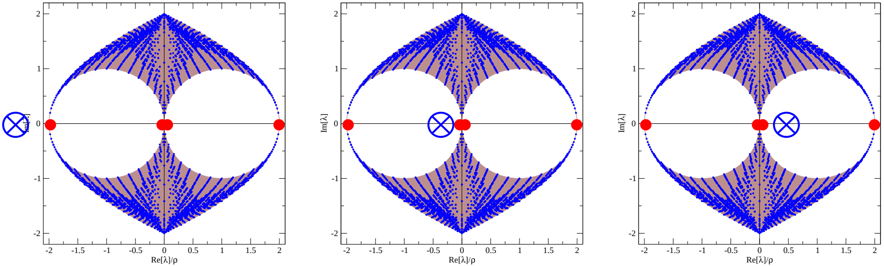
- **2-link** $\rightarrow N_f = 2$ **without fine-tuning ?**

- Other possibilities not yet considered?
  eg. start with 8 tastes, split 1-3-3-1

Sign problem?

Thanks Mike, Tatsu, Taro

Take $Q = 1$ configuration $\rightarrow$ 1+2+1 real eigenvalues. Vary m_0 : $\otimes$



det > 0 det < 0 det < 0