Lattice QCD in and out of the epsilon regime

(with dynamical overlap fermions)

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for JLQCD Collaboration

JLQCD collaboration, arXiv:1111.0417

S. Aoki (Tsukuba) & HF, PRD 84, 014501 (2011) [arXiv:1105.1606]

[P.H. Damgaard (NBI,NBIA) & HF, JHEP0901:052, 2009]

1. Introduction

Q.

What is the “epsilon regime” ?

A.

Quantum Chromo Dynamics (QCD)

in (vicinity of) the $m_{\rm quark} = 0$ limit

in a finite volume.

1. Introduction

[Nambu, 1961] (2008 Nobel Prize)

Chiral symmetry is important near $m_{\rm quark} = 0$

- Chiral symmetry breaking and constituent mass

$$\langle \bar{q}q \rangle \neq 0$$

$\rightarrow$ "effective" quark action

acquires constituent mass

$\sim \Lambda_{QCD} \sim 300\mathrm{MeV}$

$$\mathcal{L} \rightarrow \bar{q}(D+m)q + C\bar{q}q\bar{q}q + \cdots$$

$$\rightarrow \bar{q}(D+m+2C\langle \bar{q}q \rangle)q + \cdots$$

$\rightarrow$ hadron masses are ~ O(1) GeV.

- Pion effective theory

(pseudo) Nambu-Goldstone boson = pion

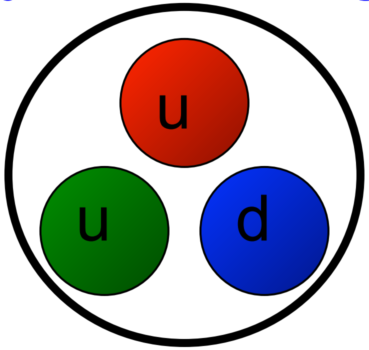
described by Chiral perturbation theory (ChPT)

[Weinberg 1979]

1. Introduction

Origin of mass = chiral symmetry breaking.

- Hadrons consist of quarks.



- But

  proton mass >> quark mass ×3

  (1GeV) (3-6MeV)

- Chiral symmetry breaking generate ~90% of mass.

1. Introduction

JLQCD (+ TWQCD) collaboration

is simulating lattice QCD

with exact chiral symmetry

using overlap fermion action.

1. Introduction

JLQCD (+ TWQCD) collaboration

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大学共同利用機関法人 KEK

Press Release

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last update: 07/04/24

Press Release

Spontaneous symmetry breaking in QCD reproduced on supercomputer

April 24, 2007
High Energy Accelerator Research Organization (KEK)
Kyoto University

A research group led by Shoji Hashimoto, Ph. D., an associate professor at KEK, succeeded for the first time to reproduce the phenomena of spontaneous symmetry breaking in Quantum Chromo-Dynamics (QCD) using numerical simulations.

1. Introduction

Chiral symmetry is expensive.

differenct action = different errors and cost.

Fermion action	Chiral symmetry	Discretization error	Numerical cost
Overlap	**Exact**	**$O(a^2)$**	**Very expensive**
Domain-wall	Weakly broken	$O(a^2)$	Expensive
Wilson	Broken	$O(a)$	Marginal
Staggered	Broken (U(1) remains)	$O(a^2)$	Cheap

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New-type (min. doubling)	**Exact**	**O(a^2)**	Cheap

Lattice size
< 2.4 fm
~4fm
~5fm
~6fm

JLQCD = Small volume QCD…

1. Introduction

Finite volume = Pion physics.

Correlation length (1/M) of QCD particles

Pions(~140MeV) ~ 1.4fm

Kaons(~500MeV)~ 0.4fm

Rho (~800MeV)~0.26fm

Proton (~1GeV) ~0.2fm

At $E \sim p \lesssim 200 \text{ MeV}$, QCD = pion (+ kaon) theory.

Finite volume correction in QCD =

chiral perturbation theory (ChPT)

weakly coupled = analytically calculable.

1. Introduction

Our strategy

Numerical calculation

Small lattice QCD
with exact chiral symmetry

→

Analytic calculation

Finite volume correction by pion theory

↓

QCD at V=∞

Chiral symmetry is important but expensive.

Contents

✓ 1. Introduction
2. Analytic calculation of finite V effects (on the pions)
3. Numerical lattice QCD results
4. Summary

2. Analytic calculation of finite V effects

Pion correlator at V= ∞

(in Euclidean space-time,)

$$\int d^3x \langle P^a(x) P^b(0) \rangle = A\delta^{ab} \int \frac{dp_4}{2\pi} \frac{e^{ip_4 t}}{p_4^2 + M_\pi^2}$$

$$\propto \exp(-M_\pi t)$$

2. Analytic calculation of finite V effects

Pion correlator at finite V (in the p-expansion)

(periodic boundary for t-direction)

$$\int d^3x \langle P^a(x) P^b(0) \rangle = B\delta^{ab} \frac{1}{T} \sum_{p_4} \frac{e^{ip_4 t}}{p_4^2 + M_\pi^2}$$

$$p_4 = 2\pi n_t / T \quad (n_t : \text{integer})$$

$$= B\delta^{ab} \int dp \sum_n \delta(p - 2\pi n/T) \frac{1}{T} \frac{e^{ipt}}{p^2 + M_\pi^2} \qquad \left(\sum_k \delta(p - 2\pi k/T) = \sum_n \frac{T e^{ipnT}}{2\pi} \right)$$

$$\propto \frac{\cosh(M_\pi(t - T/2))}{\sinh(M_\pi T/2)}$$

2. Analytic calculation of finite V effects

BUT… In the limit $M_\pi \to 0$,

$$V = \infty : \quad \exp(-M_\pi t) \to 1$$

$$V \neq \infty : \quad \frac{\cosh(M_\pi(t - T/2))}{\sinh(M_\pi T/2)} \to \frac{2}{M_\pi T} \to \infty$$

Infra-red divergence due to finite V ???

despite we have IR cut-off $1/V^{1/4}$?

Something wrong ! Exp->Cosh is not enough !

2. Analytic calculation of finite V effects

Many vacua contribute at finite V

This fake IR divergence is due to a fixed vacuum:

$$U(x) = 1 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right) \in SU(N_f)$$

but at finite V, the vacuum is not uniquely determined: vacuum= moduli = dynamical variable

$$U(x) = U_0 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right),$$

U_0 should be non-perturbatively treated.

→ ε-expansion (is needed for $M_\pi V^{1/4} \ll 1.$)

2. Analytic calculation of finite V effects

ε expansion of Chiral Lagrangian

$$\mathcal{L} = -\frac{\Sigma}{2}\mathrm{Tr}\left[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}\right]$$
$$+ \frac{1}{2}\mathrm{Tr}(\partial_\mu \xi)^2$$
$$+ \frac{\Sigma}{2F^2}\mathrm{Tr}[\mathcal{M}^\dagger U_0 \xi^2 + \xi^2 U_0^\dagger \mathcal{M}] + \cdots ,$$

Zero-mode = SU(N) matrix model

Non-zero-mode = massless bosons

(perturbative) interactions

= a hybrid system of
matrix model and massless bosonic fields

2. Analytic calculation of finite V effects

ε and p expansions are really useful ?

ε expansion

$$M_\pi L \ll 1.$$

p expansion

$$M_\pi L \gg 1.$$

Group	Nf	Action	a(fm)	L	Mπ (MeV)
ETMC	2	Twisted mass	0.05-0.100	~3fm	280
MILC	2+1	Staggered	0.045-0.12	3~6 fm	250
RBC/UKQCD	2+1	Domain wall	0.085-0.11	3~4fm	290
JLQCD	2+1	Overlap	0.11	1.8fm	310
PACS-CS	2+1	Wilson	0.09	~3fm	140
BMW	2+1	Wilson	0.065-0.125	3~5fm	190
ALV	2+1	DW on MILC	0.06-0.12	3~4fm	250
HPQCD	2+1	HISQ	0.045-0.15	3~4fm	360

But on the lattice,

$$M_\pi L = 2 \sim 5.$$

No requirement in original theory (if $E, p \ll m_\rho$):

both expansions are bad. $\rightarrow$ Better way of expansion (covering both regimes) ?

2. Analytic calculation of finite V effects

New expansion

p-expansion

$$U(x) = 1 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right), \qquad M_\pi \sim \mathrm{LO}$$

ε-expansion

$$U(x) = U_0 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right), \qquad M_\pi \sim \mathrm{NLO}$$

2. Analytic calculation of finite V effects

New expansion

p-expansion

$$U(x) = 1 \exp\left(i \frac{\sqrt{2}\pi(x)}{F} \right), \qquad M_\pi \sim \text{LO}$$

ε-expansion

$$U(x) = U_0 \exp\left(i \frac{\sqrt{2}\pi(x)}{F} \right), \qquad M_\pi \sim \text{NLO}$$

New *i* (interpolating)- expansion

$$U(x) = U_0 \exp\left(i \frac{\sqrt{2}\pi(x)}{F} \right), \qquad M_\pi \sim \text{LO}$$

[Damgaard & HF, 2008]

2. Analytic calculation of finite V effects

The pion (chiral) Lagrangian at finite V

$$\mathcal{L} = -\frac{\Sigma}{2}\mathrm{Tr}\left[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}\right]$$

Zero-mode = SU(N) matrix model

$$+\frac{1}{2}\mathrm{Tr}(\partial_\mu \xi)^2 + \frac{1}{2}\sum_a M_a^2 (\xi^a)^2$$

Non-zero-mode = massive bosons

$$+\frac{\Sigma}{2F^2}\mathrm{Tr}[\mathcal{M}^\dagger (U_0 - 1)\xi^2 + \xi^2 (U_0 - 1)^\dagger \mathcal{M}]$$

(perturbative) interactions

$$+\cdots$$

[Damgaard & HF, 2008]

= a hybrid system of

matrix model and massive bosonic fields

2. Analytic calculation of finite V effects

Pion correlator at 1-loop

[Aoki & HF, 2011]

Because of the mixing of zero and non-zero modes, the calculation is fairly tedious :

$$
\begin{aligned}
\langle P(x)P(0)\rangle = & -\frac{\Sigma^2}{4}(Z_M^{12}Z_F^{12})^4\mathcal{C}^{0a} + \frac{\Sigma^2}{\mu_1+\mu_2}\left(\frac{\Sigma_{\rm eff}}{\Sigma} - (Z_M^{12}Z_F^{12})^2\right)\mathcal{C}^{0b} \\
& + \frac{\Sigma^2}{2}(\Delta Z_{11}^{\Sigma} - \Delta Z_{22}^{\Sigma})\mathcal{C}^{0c} + \frac{\Sigma^2}{2F^2}\Big[(Z_F^{12}(Z_M^{12})^2)^2\mathcal{C}^1\bar{\Delta}(x, M_{12}'^2) \\
& + \mathcal{C}^2\left(\frac{\Sigma}{F^2}\partial_{M^2}\right)\bar{\Delta}(x,M^2)\Big|_{M^2=M_{12}^2} \\
& + \mathcal{C}_{12}^4\left(\bar{\Delta}(x,M_{11}^2) - \bar{\Delta}(x,M_{12}^2)\right) + \mathcal{C}_{21}^4\left(\bar{\Delta}(x,M_{22}^2) - \bar{\Delta}(x,M_{12}^2)\right) \\
& + \sum_{j\neq 1}\mathcal{C}_{1j}^5\left(\bar{\Delta}(x,M_{2j}^2) - \bar{\Delta}(x,M_{12}^2)\right) + \sum_{i\neq 2}\mathcal{C}_{2i}^5\left(\bar{\Delta}(x,M_{1i}^2) - \bar{\Delta}(x,M_{12}^2)\right) \\
& + \mathcal{C}^6\bar{G}(x,M_{11}^2,M_{22}^2) \\
& + \mathcal{C}_{12}^7\left(\bar{G}(x,M_{11}^2,M_{22}^2) - \bar{G}(x,M_{11}^2,M_{11}^2)\right) \\
& + \mathcal{C}_{21}^7\left(\bar{G}(x,M_{11}^2,M_{22}^2) - \bar{G}(x,M_{22}^2,M_{22}^2)\right)\Big],
\end{aligned}
$$

2. Analytic calculation of finite V effects

where

[Aoki & HF, 2011]

$$\mathcal{C}^{0a} \equiv \left\langle ([U_0]_{12} - [U_0^\dagger]_{21})([U_0]_{21} - [U_0^\dagger]_{12}) + \frac{1}{2}([U_0]_{12} - [U_0^\dagger]_{21})^2 + \frac{1}{2}([U_0]_{21} - [U_0^\dagger]_{12})^2 \right\rangle_{U_0},$$

$$\mathcal{C}^{0b} \equiv \left\langle \frac{[U_0 + U_0^\dagger]_{11}}{2} + \frac{[U_0 + U_0^\dagger]_{22}}{2} \right\rangle_{U_0},$$

$$\mathcal{C}^{0c} \equiv \frac{1}{4}\langle ([U_0]_{12} - [U_0^\dagger]_{21})^2 - ([U_0]_{21} - [U_0^\dagger]_{12})^2 \rangle_{U_0},$$

$$\begin{aligned} U_0 &\in SU(N) \text{ in } \theta = 0 \text{ vacuum}, \\ U_0 &\in U(N) \text{ in a fixed } Q \text{ sector} \end{aligned}$$

$$\mathcal{C}^1 \equiv \left\langle ([U_0]_{11} + [U_0^\dagger]_{22})([U_0]_{22} + [U_0^\dagger]_{11}) + \sum_{j\neq 1}^{N_f} [U_0]_{1j}[U_0^\dagger]_{j1} + \sum_{i\neq 2}^{N_f} [U_0]_{2i}[U_0^\dagger]_{i2} \right.$$

$$\mathcal{C}^2 \equiv \left\langle 2([\mathcal{R}]_{11} + [\mathcal{R}]_{22}) - \sum_{j\neq 1} \frac{[\mathcal{R}]_{1j}[\mathcal{R}]_{j1}}{m_j - m_1} - \sum_{i\neq 2} \frac{[\mathcal{R}]_{2i}[\mathcal{R}]_{i2}}{m_i - m_2} \right\rangle_{U_0},$$

$$\mathcal{C}^3_{ij} \equiv \frac{1}{2}\langle ([U_0]_{ji})^2 + ([U_0^\dagger]_{ij})^2 \rangle_{U_0} + \frac{\langle [\mathcal{R}]_{ij}[U_0^\dagger]_{ij} + [U_0]_{ji}[\mathcal{R}]_{ji} \rangle_{U_0}}{m_i - m_j} + \frac{\langle ([\mathcal{R}]_{ij})^2 + ([\mathcal{R}]_{ji})^2 \rangle_{U_0}}{2(m_i - m_j)^2},$$

$$\mathcal{C}^4_{ij} \equiv \langle [U_0]_{ij}[U_0^\dagger]_{ji} \rangle_{U_0} + \frac{\langle [\mathcal{R}]_{ji}[U_0]_{ij} + [\mathcal{R}]_{ij}[U_0^\dagger]_{ji} \rangle_{U_0}}{m_j - m_i} + \frac{\langle [\mathcal{R}]_{ij}[\mathcal{R}]_{ji} \rangle_{U_0}}{(m_j - m_i)^2},$$

$$\mathcal{C}^5 \equiv -\left\langle ([U_0]_{12} + [U_0^\dagger]_{21})([U_0]_{21} + [U_0^\dagger]_{12}) + \frac{1}{2}([U_0]_{12} + [U_0^\dagger]_{21})^2 + \frac{1}{2}([U_0]_{21} + [U_0^\dagger]_{12})^2 \right\rangle_{U_0},$$

$$\mathcal{C}^6_{ij} \equiv \frac{1}{2}\langle ([U_0]_{ji} + [U_0^\dagger]_{ij})^2 \rangle_{U_0} + \frac{\langle ([\mathcal{R}]_{ij} + [\mathcal{R}]_{ji})([U_0]_{ji} + [U_0^\dagger]_{ij}) \rangle_{U_0}}{m_i - m_j}$$

2. Analytic calculation of finite V effects

The zero-mode integrals

"simplest" example (in 2+1 flavor theory)

$$\mathcal{S}_v \equiv \left\langle \frac{[U_0 + U_0^\dagger]_{vv}}{2} \right\rangle_{U_0} = -\frac{1}{(\mu^2-\mu_v^2)^2(\mu_s^2-\mu_v^2)} \times \frac{\det\begin{pmatrix} \partial_{\mu_v} K_\nu(\mu_v) & I_\nu(\mu_v) & I_\nu(\mu) & \mu^{-1}I_{\nu-1}(\mu) & I_\nu(\mu_s) \\ -\partial_{\mu_v}(\mu_v K_{\nu+1}(\mu_v)) & \mu_v I_{\nu+1}(\mu_v) & \mu I_{\nu+1}(\mu) & I_\nu(\mu) & \mu_s I_{\nu+1}(\mu_s) \\ \partial_{\mu_v}(\mu_v^2 K_{\nu+2}(\mu_v)) & \mu_v^2 I_{\nu+2}(\mu_v) & \mu^2 I_{\nu+2}(\mu) & \mu I_{\nu+1}(\mu) & \mu_s^2 I_{\nu+2}(\mu_s) \\ -\partial_{\mu_v}(\mu_v^3 K_{\nu+3}(\mu_v)) & \mu_v^3 I_{\nu+3}(\mu_v) & \mu^3 I_{\nu+3}(\mu) & \mu^2 I_{\nu+2}(\mu) & \mu_s^3 I_{\nu+3}(\mu_s) \\ \partial_{\mu_v}(\mu_v^4 K_{\nu+4}(\mu_v)) & \mu_v^4 I_{\nu+4}(\mu_v) & \mu^4 I_{\nu+4}(\mu) & \mu^3 I_{\nu+3}(\mu) & \mu_s^4 I_{\nu+4}(\mu_s) \end{pmatrix}}{\det\begin{pmatrix} I_\nu(\mu) & \mu^{-1}I_{\nu-1}(\mu) & I_\nu(\mu_s) \\ \mu I_{\nu+1}(\mu) & I_\nu(\mu) & \mu_s I_{\nu+1}(\mu_s) \\ \mu^2 I_{\nu+2}(\mu) & \mu I_{\nu+1}(\mu) & \mu_s^2 I_{\nu+2}(\mu_s) \end{pmatrix}},$$

$$\mu_v = m_v \Sigma V,\ \mu = m_{ud}\Sigma V,\ \mu_s = m_s \Sigma V.$$

2. Analytic calculation of finite V effects

Pion correlator at 1-loop [Aoki & HF, 2011]

$$\int d^3x \langle P(x)P(0)\rangle = C_{PP}\frac{\cosh(M_\pi^{NLO}(t-T/2))}{\sinh(M_\pi^{NLO}T/2)} + D_{PP}$$

D_{PP} cancels IR divergence: $D_{PP} \underset{M\to 0}{\sim} -2\frac{C_{PP}}{M_\pi^{NLO}T} + E + \cdots ,$

disappears in the p-regime: $\lim_{M_\pi \to \text{large}} D_{PP} \sim \exp(-m\Sigma V) \to 0.$

($C_{PP}, D_{PP}, M_\pi^{NLO}$: functions of Σ and F_π)

$$\to_{V\to\infty} \exp(-M_\pi t)$$

2. Analytic calculation of finite V effects

Summary of Sec.2

1. Finite V effects = Pion physics

   (Other hadrons are heavy and insensitive to finite V.)

2. Pion physics = Chiral perturbation theory (ChPT)

   weakly coupled → analytically calculable.

3. ChPT in finite V = matrix model + bosonic fields

   computations are tedious… but the result is beautiful:

$$\int d^3x \langle P(x)P(0)\rangle = C_{PP}\frac{\cosh(M_\pi^{NLO}(t-T/2))}{\sinh(M_\pi^{NLO}T/2)} + D_{PP}$$

Smooth interpolation to infinite V in a infra-red finite way.

3. Numerical lattice QCD results (preliminary)

QCD simulation with exact chiral symmetry

[JLQCD & TWQCD collaborations, 2006-2011]

2+1-flavor overlap Dirac fermions [Neuberger 98]

Iwasaki gauge action, β=2.3, 1/a ~ 1.759 GeV.

Lattice size : L=16 [1.8 fm], T=48.

Topology fixed: Q=0 (or 1)

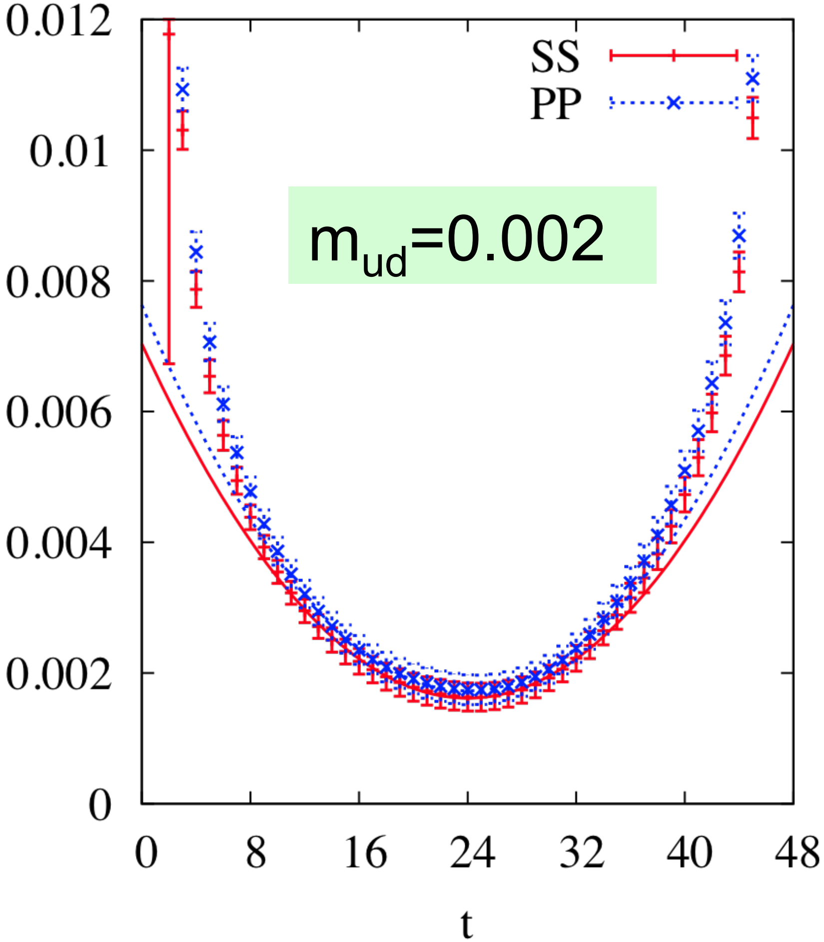
Quark masses : m_s=0.08, 0.100,

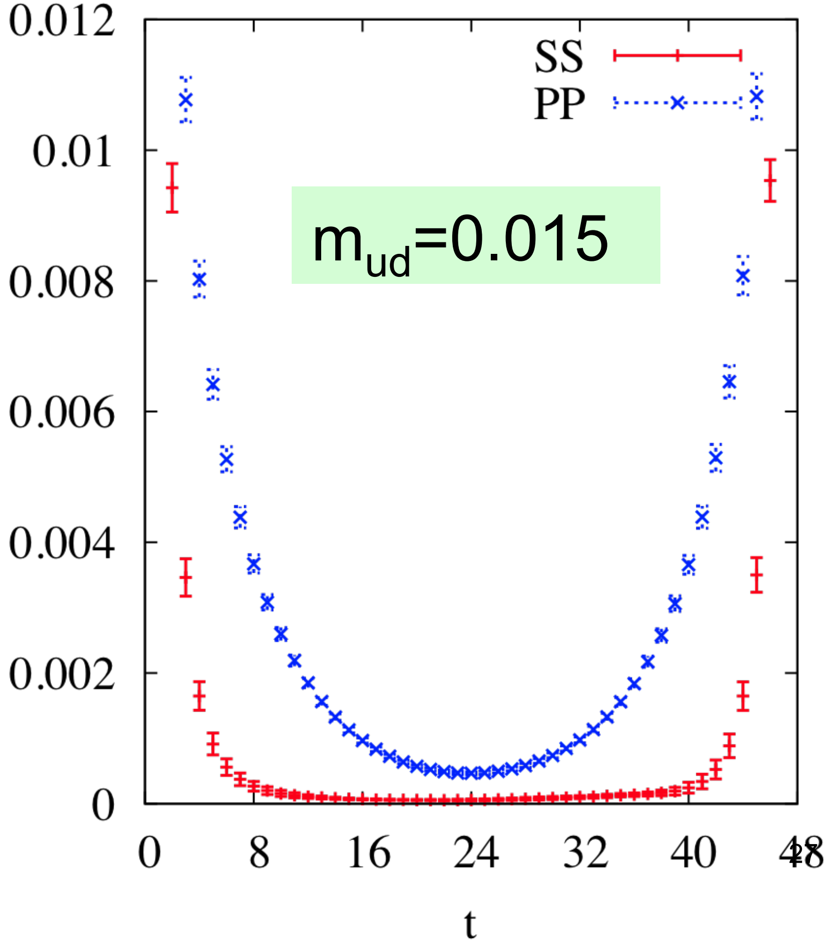
m_{ud}= 0.002, 0.015,0.025,0.035,0.050,0.080,0.100

(~3 MeV) ε-regime, (30MeV <) p-regime

3. Numerical lattice QCD results (preliminary)

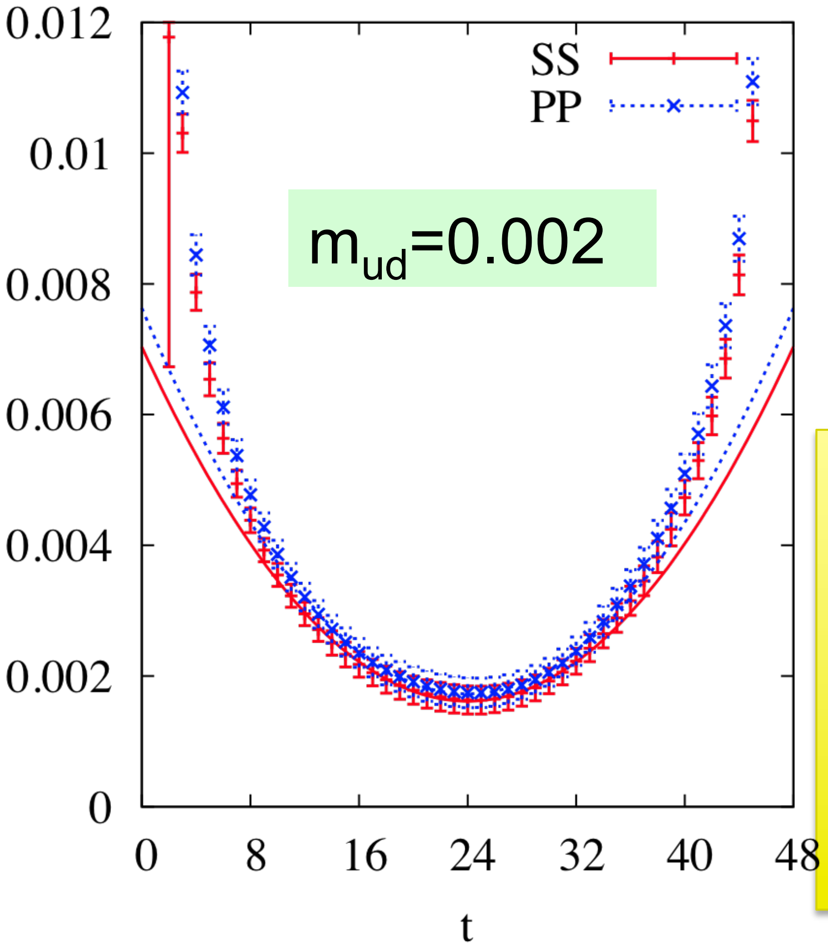
Really inside the ε-regime?→ Scalar channel knows.





3. Numerical lattice QCD results (preliminary)

Really inside the ε-regime? → Yes.



ε expansion analysis : [Damgaard et al, 2002]

PP → $\Sigma_{\rm eff} = 0.001936(66)$

SS → $\Sigma_{\rm eff} = 0.001938(70)$

consistent with our previous value

$$\Sigma_{\rm eff} = 0.002041(70) \sim [241\mathrm{MeV}]^3$$

from Dirac spectrum.

Next, let us extract

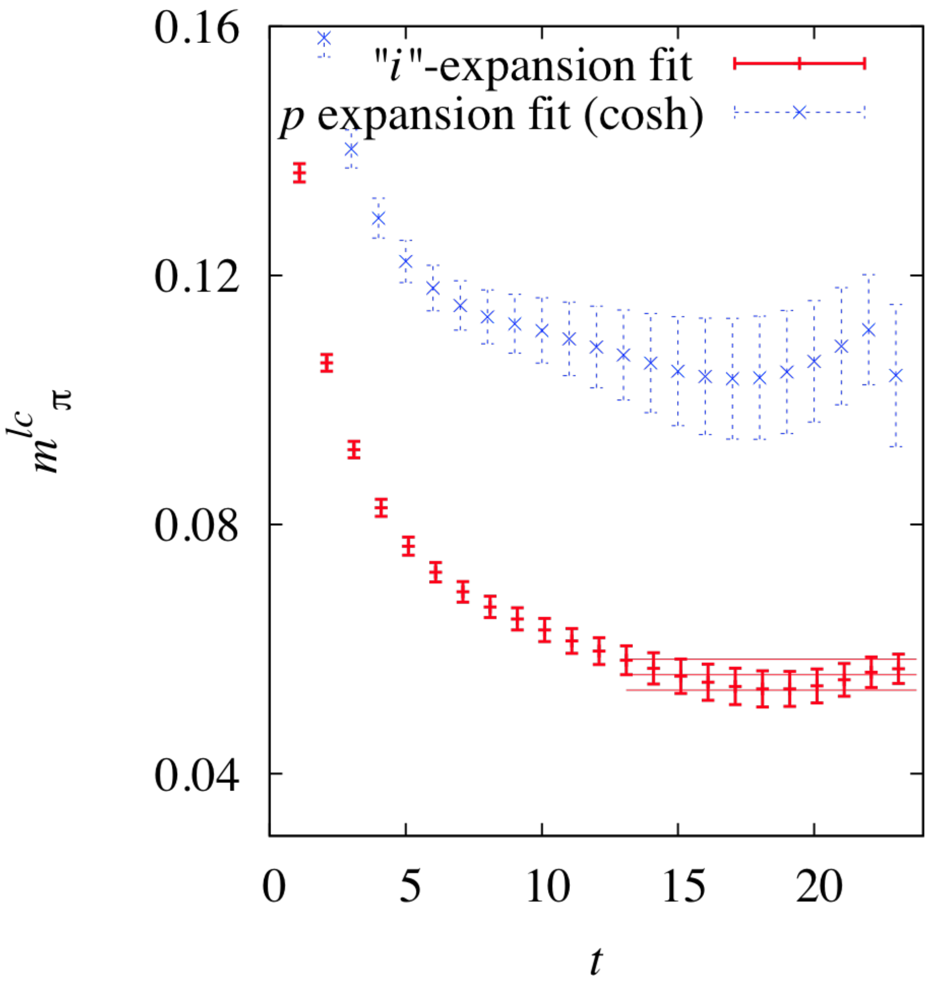
$$M_\pi \quad \text{and} \quad F_\pi .$$

with the "i" expansion formula.

(Naïve GMOR relation suggests $M_\pi \sim 100\mathrm{MeV}.$)

3. Numerical lattice QCD results (preliminary)

Pion mass and decay constant in the ε regime



Fitting with

$$C_{PP}\frac{\cosh(M_\pi^{NLO}(t-T/2))}{\sinh(M_\pi^{NLO}T/2)}+D_{PP}$$

$$[\Sigma_{\rm eff}=0.002041(70)\ \ \text{input}]$$

after 1-loop (of non-zero modes) volume correction, we obtain

$$M_\pi^{V=\infty}=97.6(4.2)\ \ \text{MeV},$$

$$F_\pi^{V=\infty}=128.6(5.6)\ \ \text{MeV}.$$

$$(1/a=1.759\ \text{GeV})$$

3. Numerical lattice QCD results (preliminary)

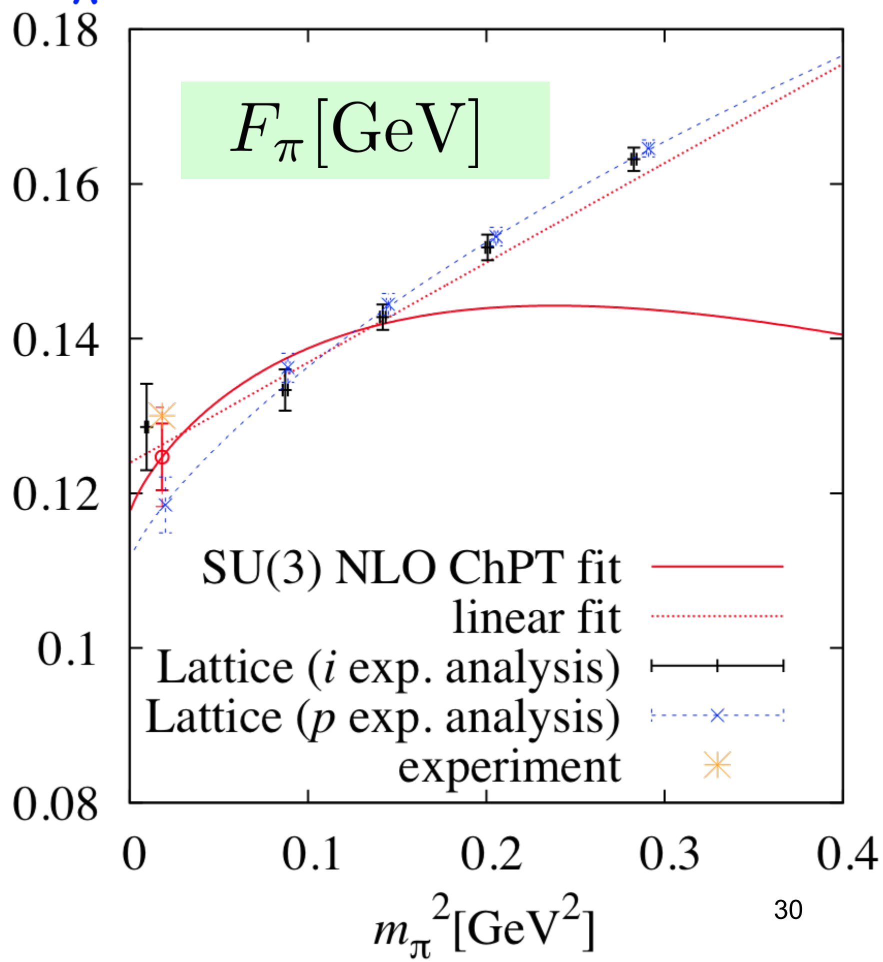
Chiral “interpolation” for F_π

$$F_\pi = 125(4)(^{+5}_{-0})\ \ \mathrm{MeV}$$

bigger than our previous analysis using p-regime data only :

$$F_\pi = 119(4)\ \ \mathrm{MeV}$$

Linear fit looks better than NLO ChPT fit, though…



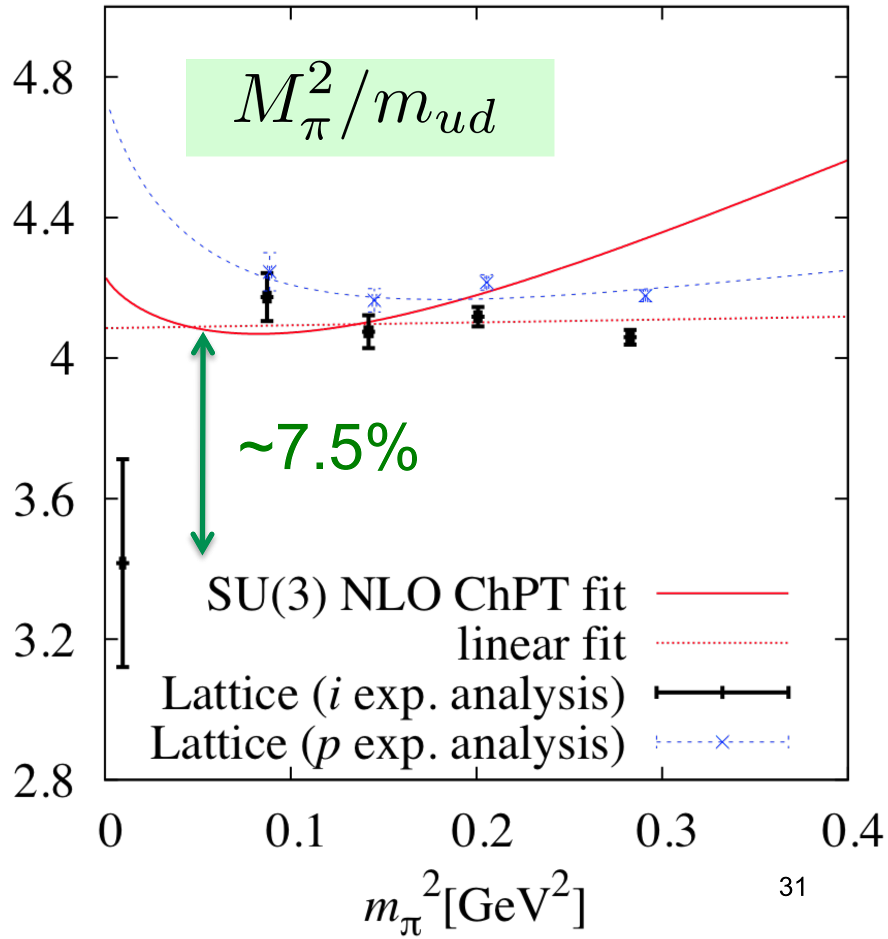
3. Numerical lattice QCD results (preliminary)

Chiral “interpolation” for M_π

ChPT fit looks bad :

$2.5\sigma\ [7.5\%]$ deviation.

2-loop (1/V) effects from non-zero modes under control ?



3. Numerical lattice QCD results (preliminary)

Bad convergence of ChPT for M_π ?

In the limit $M_\pi \to 0$,

$$M_\pi^V = \underbrace{M_\pi}_{\to 0} + \underbrace{\delta M_{\rm NLO}}_{\mathcal{O}(1/L^2)} + \underbrace{\delta M_{\rm NNLO}}_{\mathcal{O}(1/F^2L^4)} + \cdots$$

Bad.
(big correction/LO)

$$F_\pi^V = \underbrace{F_\pi}_{\rm finite} + \underbrace{\delta F_{\rm NLO}}_{\mathcal{O}(1/L^2)} + \underbrace{\delta F_{\rm NNLO}}_{\mathcal{O}(1/F^2L^4)} + \cdots$$

Good.

4. Summary

Small lattice QCD with exact chiral symmetry

+

Finite V correction from pion effective theory (ChPT)

$$\int d^3x \langle P(x)P(0)\rangle = C_{PP}\frac{\cosh(M_\pi^{NLO}(t-T/2))}{\sinh(M_\pi^{NLO}T/2)} + D_{PP}$$

can extract physics at V=∞.