Flavored-mass terms for naive and staggered fermions

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M. Creutz, T. Kimura, T. Misumi, *JHEP* **1012**:041 (2010)

M. Creutz, T. Kimura, T. Misumi, *PRD* **83**:094506 (2011)

T. Kimura, S. Komatsu, T. Misumi, T. Noumi, S. Torii, S. Aoki, *JHEP* **1201**:048 (2012)

T. Misumi, *Ph.D Thesis*, Kyoto University (2012)

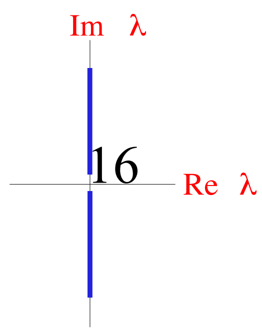
02/09/2012 NTFL workshop@Yukawa Institute, Kyoto

Introduction

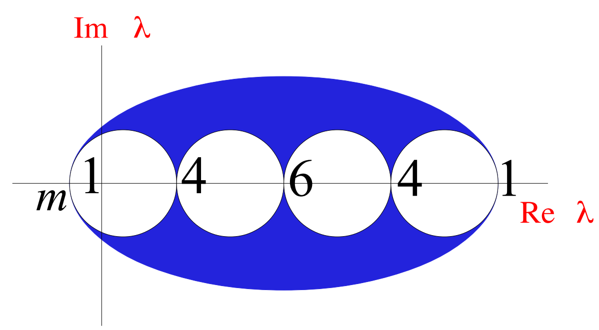
◆ Wilson fermion

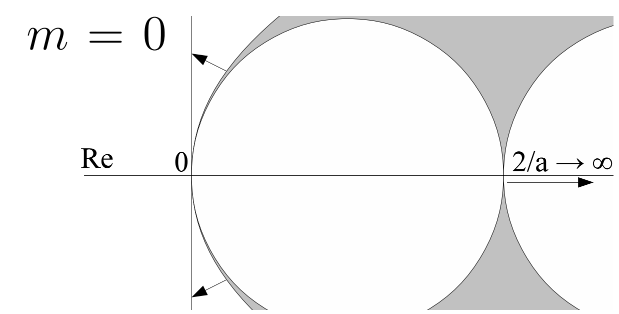
$$S = S_{nf} + S_W \quad \text{with} \quad S_W = -\frac{ar}{2}\sum_{n,\mu} a^4 \bar{\psi}_n \frac{(\psi_{n+\hat{\mu}} - 2\psi_n + \psi_{n-\hat{\mu}})}{a^2} \quad \sim \quad a\int d^4x \bar{\psi}(x) D_\mu^2 \psi(x)$$

D_{naive}



D_{wilson}





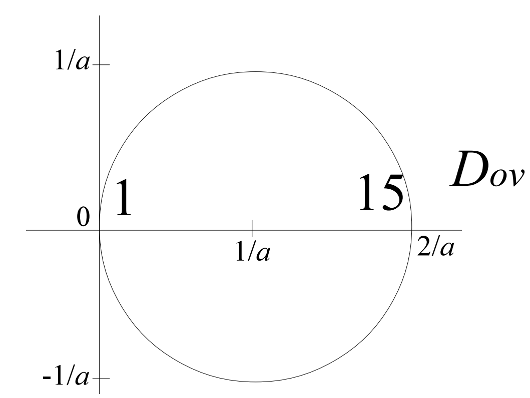
§ <u>15 species are decoupled → doubler-less</u>

§ <u>additive mass renormalization → Fine-tune for chiral limit</u>

Overlap & Domain-wall fermion

$$D_{ov} = 1 + \gamma_5 \frac{H_W(m)}{\sqrt{H_W^2(m)}} = 1 + \frac{D_W(m)}{\sqrt{D_W^\dagger(m) D_W(m)}}$$

Ginsparg-Wilson : $\{\gamma_5, D_{ov}\} = a D_{ov} \gamma_5 D_{ov}$



◆Staggered fermion

Spin diagonalization : $\psi_n = \gamma_1^{n_1}\gamma_2^{n_2}\gamma_3^{n_3}\gamma_4^{n_4}\chi_n, \quad \bar{\psi}_n = \bar{\chi}_n\gamma_4^{n_4}\gamma_3^{n_3}\gamma_2^{n_2}\gamma_1^{n_1}$

$$S_{\rm nf} = 4S_{\rm st} = 4[\frac{1}{2}\sum_{n,\mu}\eta_\mu(n)\bar{\chi}_n\,(\chi_{n+\hat{\mu}} - \chi_{n-\hat{\mu}}) + \frac{m}{2}\sum_n \bar{\chi}_n\chi_n] \qquad \eta_\mu(n) = (-1)^{\sum_{\nu<\mu} n_\nu}$$

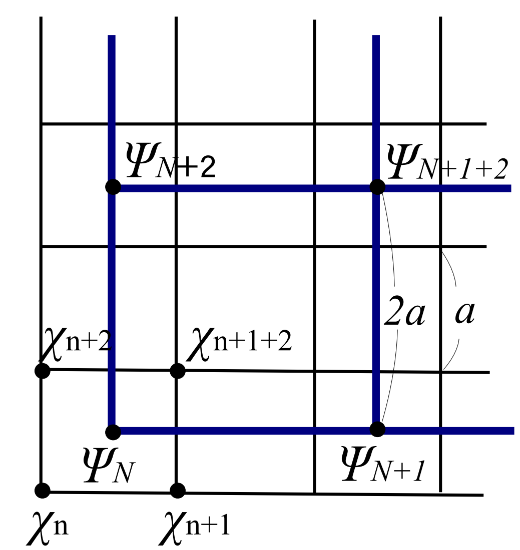
One naive fermion → 4 Staggered fermions

Properties

- 4-flavor Dirac fermions
- Flavored chiral symmetry

$$\epsilon_n = (-1)^{n_1+n_2+n_3+n_4}$$

$$\sim \ \Gamma_{55} = \underset{\text{spin}}{\gamma_5} \otimes \underset{\text{flavor}}{\gamma_5}$$



§ chiral symmetry + one-component → suitable for calculations

§ 4 species → more than 3.....

Naive $\xrightarrow[\text{Wilson term}]{\text{Chiral broken}}$ Wilson $\xrightarrow[\text{Overlap form.}]{\text{GW symmetry}}$ Overlap

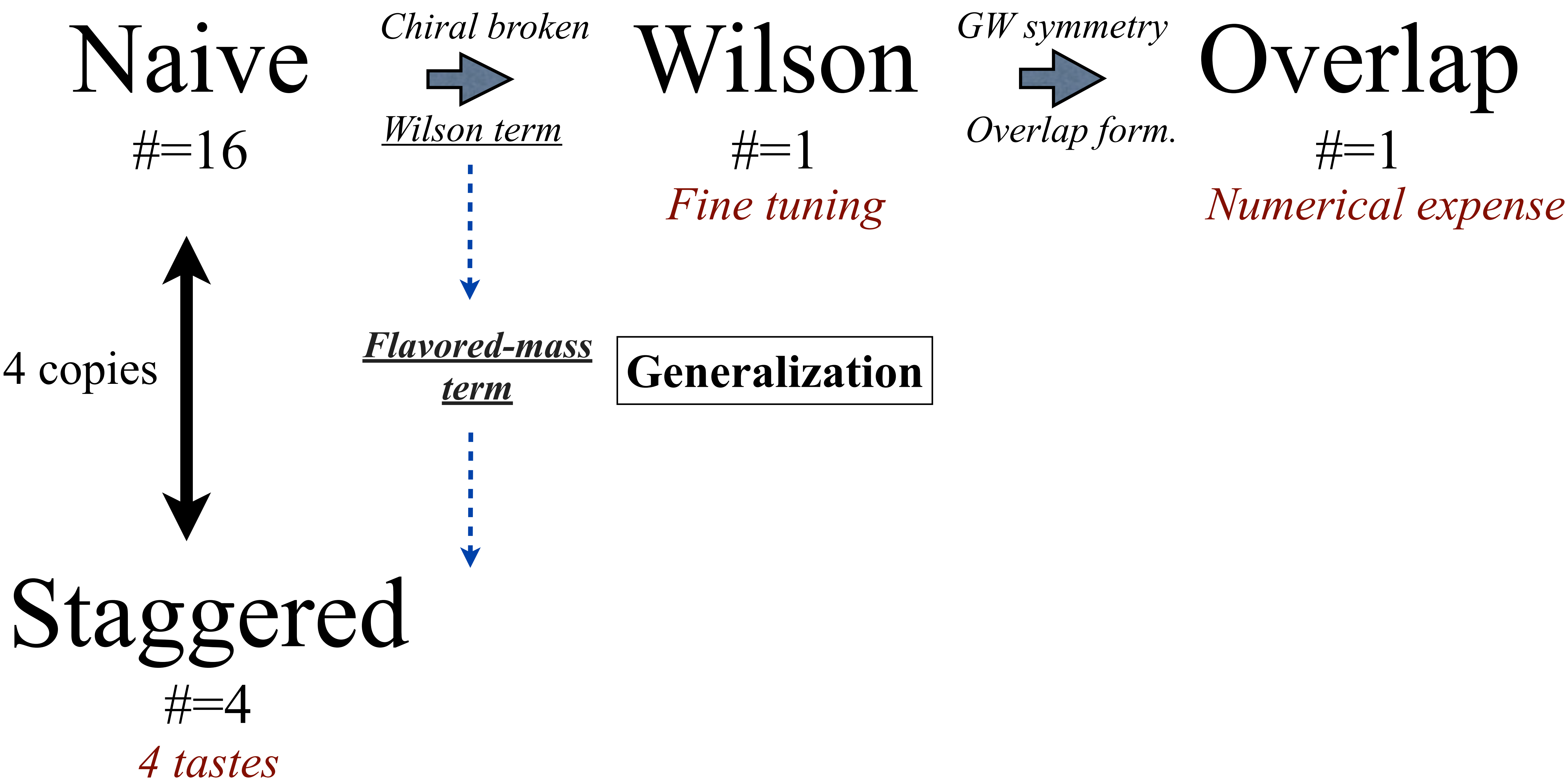
Naive	Wilson	Overlap
#=16	#=1	#=1
	Fine tuning	*Numerical expense*

4 copies

Staggered

#=4

4 tastes


Naive
#=16
Chiral broken
Wilson term
Wilson
#=1
Fine tuning
GW symmetry
Overlap form.
Overlap
#=1
Numerical expense
4 copies
Flavored-mass term
Generalization
Staggered
#=4
4 tastes

Flavored mass terms

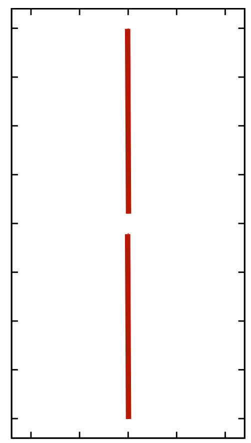
~ Generalized Wilson terms ~

e.g.) 2-split flavored mass

Naive

D_{nf}

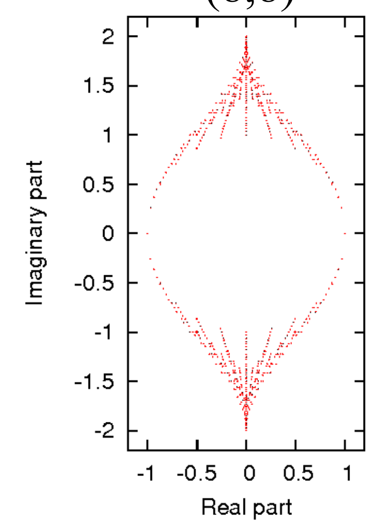
Im [λ]



Re [λ]

$$M_{\mathrm{P}} = \sum_{sym.} \prod_{\mu=1}^{4} C_\mu$$

(8,8)



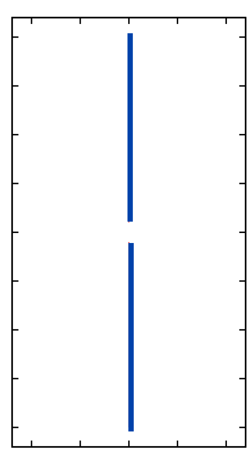
Creutz, Kimura, TM, JHEP1012,041 [1011.0761]

4 copies

Staggered

Golterman, Smit (1984)

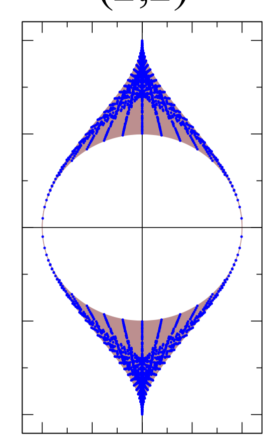
D_{st}



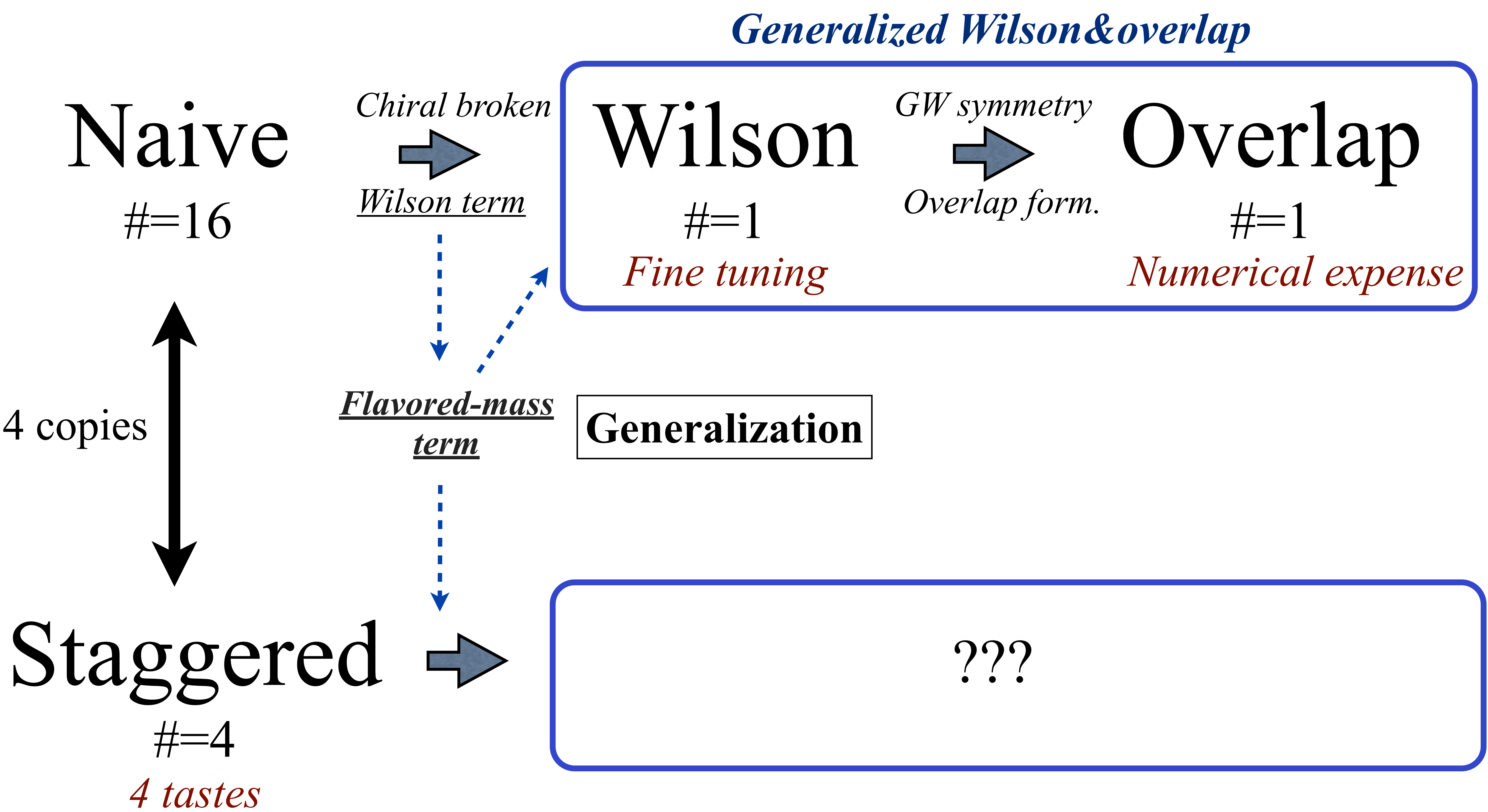
$$M_{\mathcal{A}} = \zeta_5 \sum_{sym.} \prod_{\mu=1}^{4} C_\mu$$

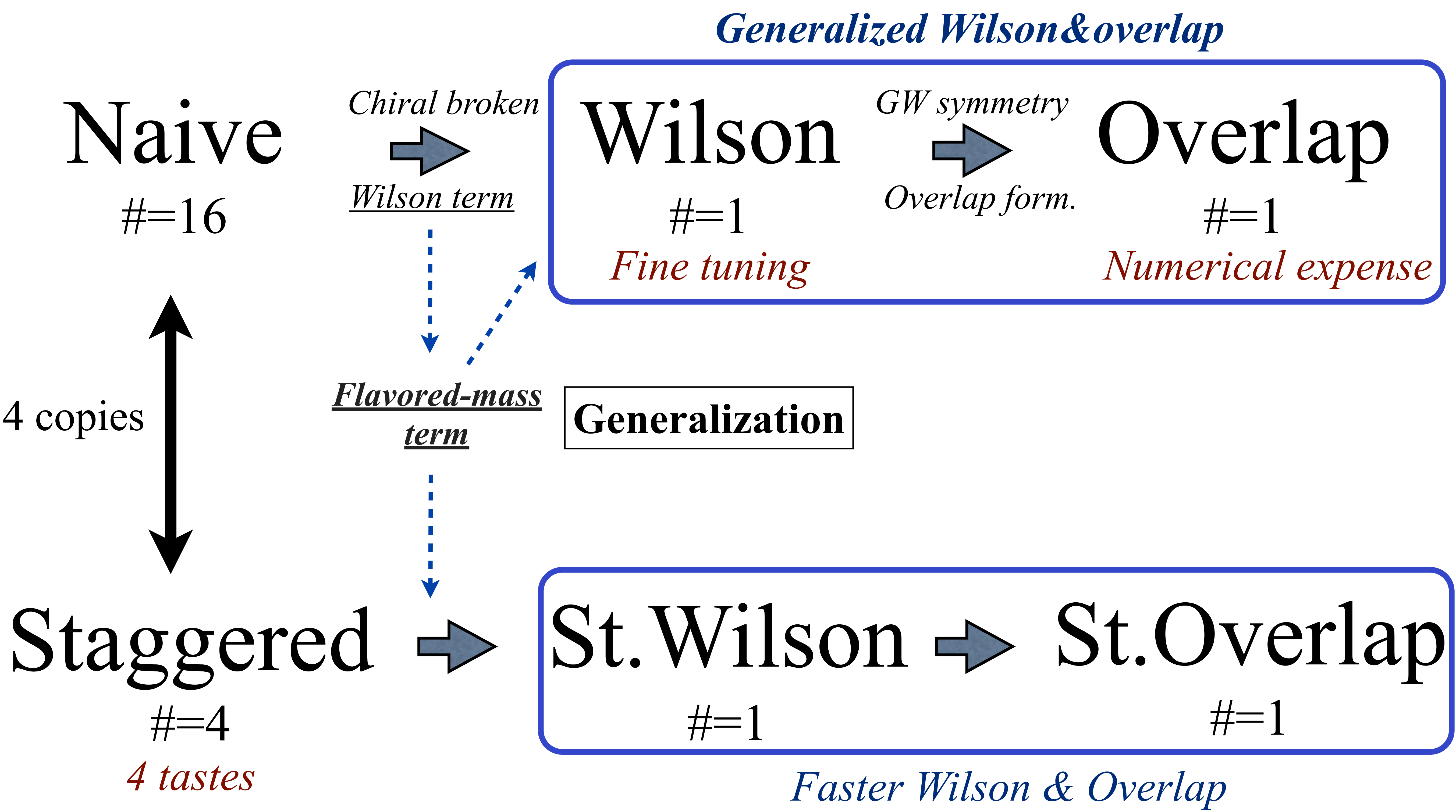
Adams, PRL104, 141602 [0912.2850]

(2,2)



de Forcrand, Kurkela, Panero, [1102.1000]


Generalized Wilson&overlap
Naive
#=16
Chiral broken
Wilson term
Wilson
#=1
Fine tuning
GW symmetry
Overlap form.
Overlap
#=1
Numerical expense
4 copies
Flavored-mass term
Generalization
Staggered
#=4
4 tastes
???


Generalized Wilson&overlap
Naive
#=16
Chiral broken
Wilson term
Wilson
#=1
Fine tuning
GW symmetry
Overlap form.
Overlap
#=1
Numerical expense
4 copies
Flavored-mass term
Generalization
Staggered
#=4
4 tastes
St.Wilson
#=1
St.Overlap
#=1
Faster Wilson & Overlap

1. Flavored-mass terms

~ general terms to lift degenerate species ~

◆ Naïve fermion M. Creutz, T. Kimura, TM, *JHEP*1012:041 (2010)

- 16 species $\Gamma_{(i)}^{-1}\gamma_\mu\Gamma_{(i)} = \gamma_\mu^{(i)}$

$$\downarrow$$

- 16-flavor multiplet

$$\Psi(p) = \begin{pmatrix} \psi_{(1)}(p-p_{(1)}) \\ \psi_{(2)}(p-p_{(2)}) \\ \vdots \\ \psi_{(16)}(p-p_{(16)}) \end{pmatrix}$$

➡ *Flavor mass matrix*

$$\bar{\Psi}(\mathbf{1}\otimes \underline{X})\Psi$$

Mass matrix

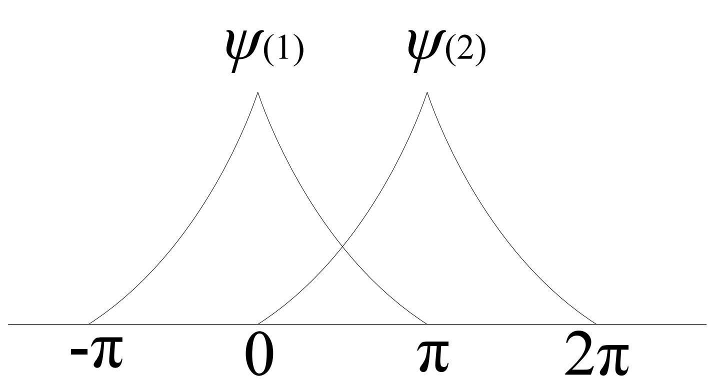
label	position	χ charge	Γ	type
1	$(0,0,0,0)$	$+$	$\mathbf{1}$	S
2	$(\pi,0,0,0)$	$-$	$i\gamma_1\gamma_5$	A
3	$(0,\pi,0,0)$	$-$	$i\gamma_2\gamma_5$	A
4	$(\pi,\pi,0,0)$	$+$	$i\gamma_1\gamma_2$	T
5	$(0,0,\pi,0)$	$-$	$i\gamma_3\gamma_5$	A
6	$(\pi,0,\pi,0)$	$+$	$i\gamma_1\gamma_3$	T
7	$(0,\pi,\pi,0)$	$+$	$i\gamma_2\gamma_3$	T
8	$(\pi,\pi,\pi,0)$	$-$	γ_4	V
9	$(0,0,0,\pi)$	$-$	$i\gamma_4\gamma_5$	A
10	$(\pi,0,0,\pi)$	$+$	$i\gamma_1\gamma_4$	T
11	$(0,\pi,0,\pi)$	$+$	$i\gamma_2\gamma_4$	T
12	$(\pi,\pi,0,\pi)$	$-$	γ_3	V
13	$(0,0,\pi,\pi)$	$+$	$i\gamma_3\gamma_4$	T
14	$(\pi,0,\pi,\pi)$	$-$	γ_2	V
15	$(0,\pi,\pi,\pi)$	$-$	γ_1	V
16	(π,π,π,π)	$+$	γ_5	P

◆ Point-split fields

M. Creutz (2010), for minimally doubled fermions.

$$
\begin{aligned}
\psi_{(1)}(p-p_{(1)}) &= \frac{1}{2^4}(1+\cos p_1)(1+\cos p_2)(1+\cos p_3)(1+\cos p_4)\Gamma_{(1)}\psi(p), \\
\psi_{(2)}(p-p_{(2)}) &= \frac{1}{2^4}(1-\cos p_1)(1+\cos p_2)(1+\cos p_3)(1+\cos p_4)\Gamma_{(2)}\psi(p), \\
\psi_{(3)}(p-p_{(3)}) &= \frac{1}{2^4}(1+\cos p_1)(1-\cos p_2)(1+\cos p_3)(1+\cos p_4)\Gamma_{(3)}\psi(p), \\
&\vdots \\
\psi_{(16)}(p-p_{(16)}) &= \frac{1}{2^4}(1-\cos p_1)(1-\cos p_2)(1-\cos p_3)(1-\cos p_4)\Gamma_{(16)}\psi(p)
\end{aligned}
$$

$\longrightarrow$ Independent fields in low energy limit



$$
\Psi(p) = \begin{pmatrix} \psi_{(1)}(p-p_{(1)}) \\ \psi_{(2)}(p-p_{(2)}) \\ \vdots \\ \psi_{(16)}(p-p_{(16)}) \end{pmatrix} \Rightarrow \bar{\Psi}(\mathbf{1}\otimes \underbrace{X}_{\text{Mass matrix}})\Psi
$$

16-flavor multiplet

• *Conditions on flavored-mass terms*

(1) *gamma-5 hermiticity* : $D^\dagger = \gamma_5 D \gamma_5$

$\longrightarrow$ $\det(D) \geq 0$ essential for euclidian vector-like theory

※ $\underbrace{\gamma_5}_{\text{spin}} \otimes \underbrace{(\tau_3 \otimes \tau_3 \otimes \tau_3 \otimes \tau_3)}_{\text{flavor}}$ for $\Psi(p) = \begin{pmatrix} \psi_{(1)}(p - p_{(1)}) \\ \psi_{(2)}(p - p_{(2)}) \\ \vdots \\ \psi_{(16)}(p - p_{(16)}) \end{pmatrix}$

(2) O(a) *irrelevant term*

$\sim \quad a \int d^4x \bar{\psi}(x) D_\mu^2 \psi(x)$ dim-5 operator vanishes in a→0

- Physical modes in the continuum limit
- Rotational symmetry

◆ Flavored-mass terms

$$V : \bar{\Psi}\,(\mathbf{1}\otimes(\tau_3\otimes\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1}))\,\Psi = \cos p_1\bar{\psi}\psi$$

$$T : \bar{\Psi}\,(\mathbf{1}\otimes(\tau_3\otimes\tau_3\otimes\mathbf{1}\otimes\mathbf{1}))\,\Psi = \cos p_1\cos p_2\bar{\psi}\psi$$

$$A : \bar{\Psi}\,(\mathbf{1}\otimes(\mathbf{1}\otimes\tau_3\otimes\tau_3\otimes\tau_3))\,\Psi = \left(\prod_{\mu=2}^{4}\cos p_\mu\right)\bar{\psi}\psi$$

$$P : \bar{\Psi}\,(\mathbf{1}\otimes(\tau_3\otimes\tau_3\otimes\tau_3\otimes\tau_3))\,\Psi = \left(\prod_{\mu=1}^{4}\cos p_\mu\right)\bar{\psi}\psi$$

$$\Rightarrow$$

$$M_{\rm V} = \sum_{\mu} C_\mu,$$

$$M_{\rm T} = \sum_{perm.}\sum_{sym.} C_\mu C_\nu,$$

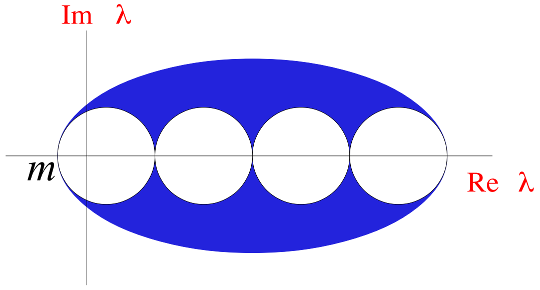
$$M_{\rm A} = \sum_{perm.}\sum_{sym.}\prod_{\nu} C_\nu,$$

$$M_{\rm P} = \sum_{sym.}\prod_{\mu=1}^{4} C_\mu,$$

- O(a) *irrelevant terms* $\sum_n \bar{\psi}_n(M_P-1)\psi_n \;\rightarrow\; -a\int d^4x\bar{\psi}(x)D_\mu^2\psi(x)+O(a^2)$

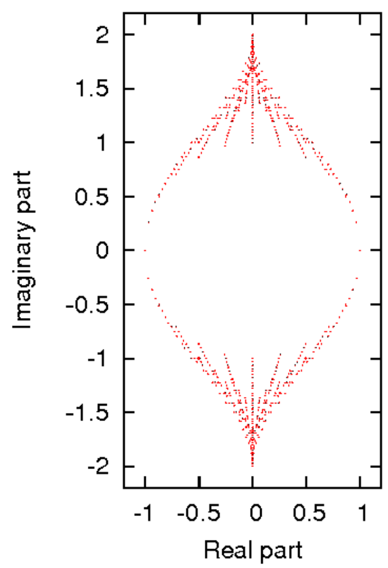
- *low-energy species-splitting terms*

- $M_{\rm V}$ ($M_{\rm A}$) $\longrightarrow$ *Wilson term*



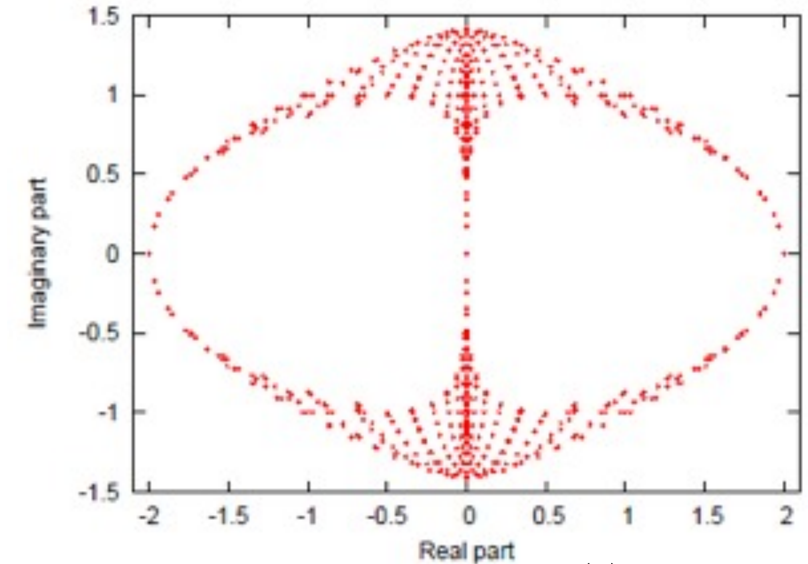
Dirac spectra with flavored mass terms

(8,8)



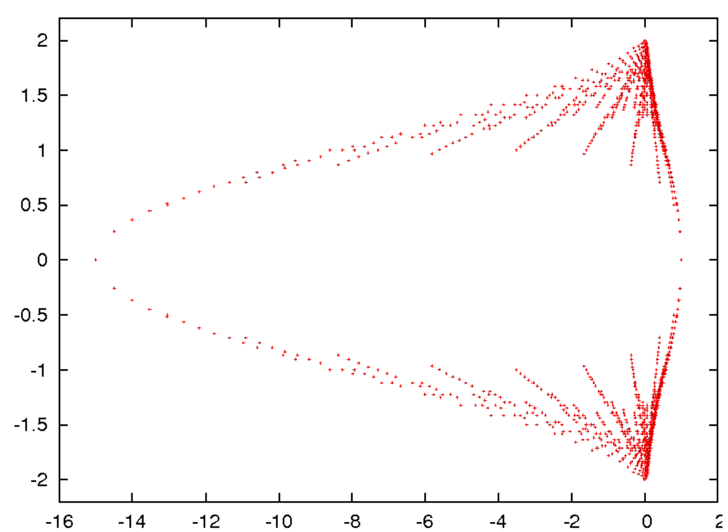
$D_{\rm nf} - M_P$

(4,8,4)



$D_{\rm nf} - M_{\rm T}^{(i)}$

(1,15)



$D_{\rm nf} - (M_V + M_T + M_A + M_P)$

M_V+M_A : (1,14,1)

M_P+M_T : (4,12)

M_P+M_V : (5,1,10)

M_T+M_V : (10,5,1)

$M_A+M_V+M_{T1}+M_{T2}$: (3,12,1)

$\vdots$

$\longrightarrow$ *Multi-flavor Wilson & Overlap* (although we need care about renormalization)

◆ Pseudo-scalar type

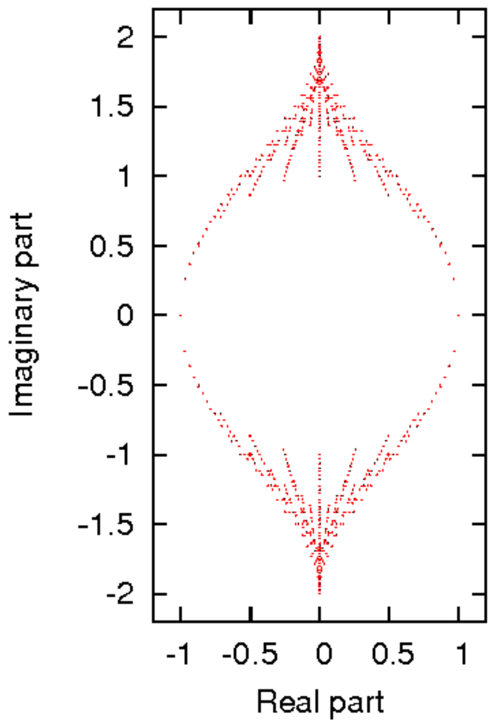
$$M_{\mathrm{P}} = \sum_{sym.} C_1 C_2 C_3 C_4$$

$$\Rightarrow \quad \bar{\Psi}\left(\mathbf{1} \otimes (\tau_3 \otimes \tau_3 \otimes \tau_3 \otimes \tau_3)\right)\Psi$$

8 (+) and 8 (-) masses

⟷ consistent

(8,8)



$D_{\mathrm{nf}} - M_P$

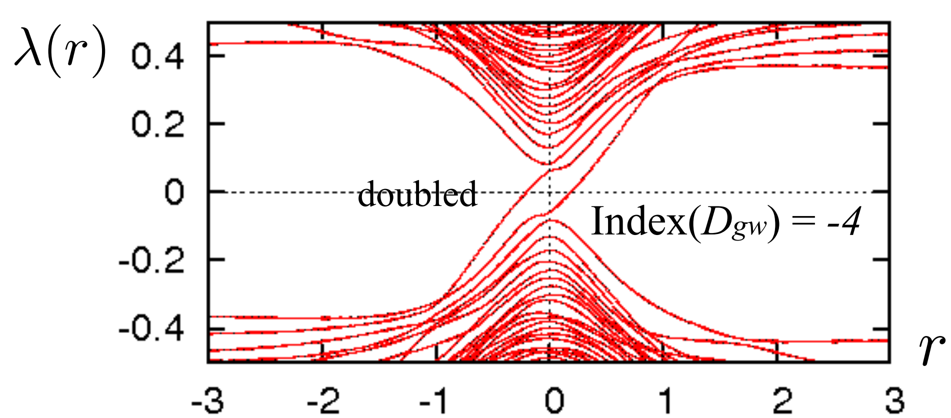
- ***Index theorem from spectral flow*** cf.) For staggered, Adams (2009)

$$H = \gamma_5(D_{\mathrm{nf}} - rM_{\mathrm{P}})$$

$$\boxed{\mathrm{Index}(D_{\mathrm{nf}}) \ = \ -\ \mathrm{Spectral\ flow}(H)}$$

$$\Rightarrow \quad \mathrm{Index}(D_{\mathrm{nf}}) = 2^d(-1)^{d/2}Q$$

36×36 lattice, randomness $\delta=0.25$, $Q=1$



Adams-type flavored mass

D. Adams (2009)

- *spin diagonalization*

$$\bar{\psi}_x \psi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}} = \bar{\chi}_x \gamma_4^{x_4}\gamma_3^{x_3}\gamma_2^{x_2}\gamma_1^{x_1}\gamma_1^{x_1+1}\gamma_2^{x_2+1}\gamma_3^{x_3+1}\gamma_4^{x_4+1}\chi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}}$$

$$= (-1)^{x_2+x_4}\bar{\chi}_x\gamma_5\chi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}} \qquad (\gamma_5 \ \ \text{diagonalized})$$

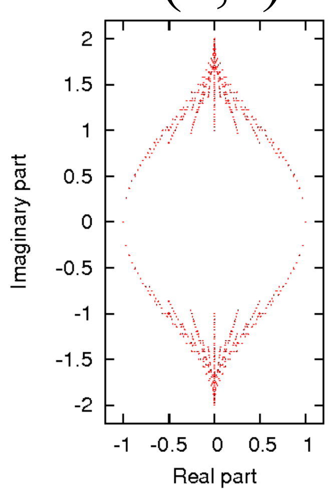
$$\rightarrow \pm\bar{\chi}_x \epsilon\eta_1\eta_2\eta_3\eta_4\chi_{x+\hat{1}+\hat{2}+\hat{3}+\hat{4}}$$

4 Adams fermions derived up to sign

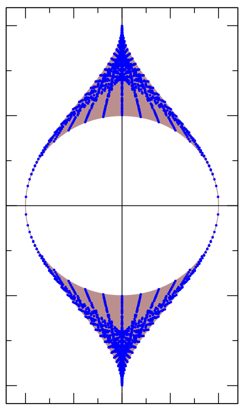
$$\bar{\psi}_x C_1C_2C_3C_4\psi_x \qquad \rightarrow \qquad \pm\bar{\chi}_x(\epsilon\eta_1\eta_2\eta_3\eta_4 C_1C_2C_3C_4)\chi_x$$

(8,8) (2,2)

M_{P}



$\longleftrightarrow$



$M_{\mathcal{A}}$

de Forcrand, Kurkela, Panero, [1102.1000]

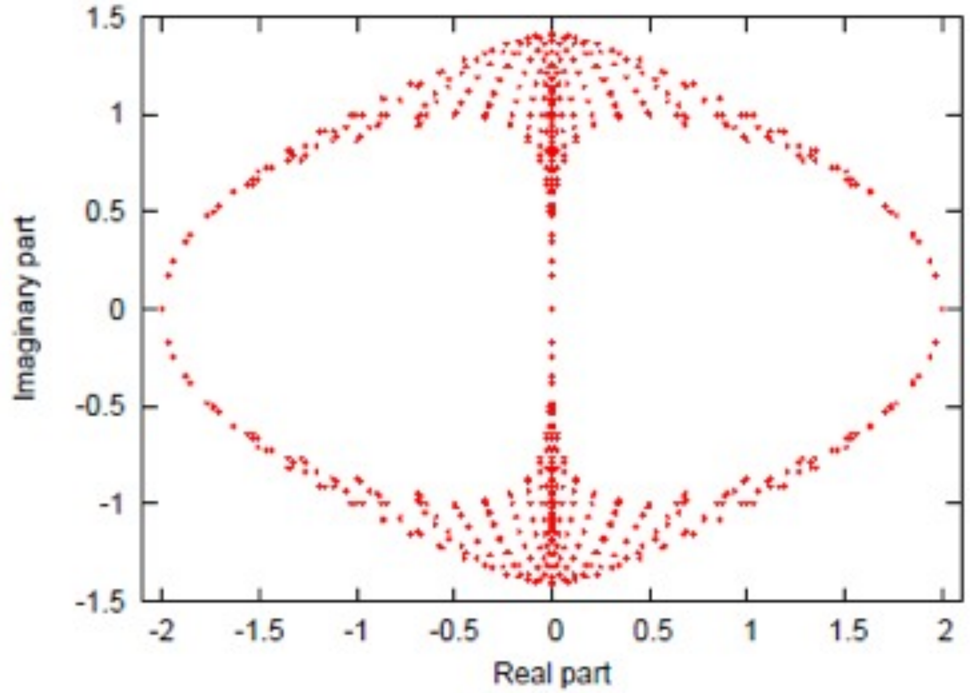
$$S_{\mathrm{nf}}(M_{\mathrm{P}}) \qquad \rightarrow \qquad S_{\mathrm{st}}(M_{\mathcal{A}})$$

◆ Tensor type

$$M_{\mathrm{T}} = M_{\mathrm{T}}^{(1)} + M_{\mathrm{T}}^{(2)} + M_{\mathrm{T}}^{(3)},$$
$$M_{\mathrm{T}}^{(1)} = \frac{1}{2}(C_1C_2 + C_2C_1) + \frac{1}{2}(C_3C_4 + C_4C_3),$$
$$M_{\mathrm{T}}^{(2)} = \frac{1}{2}(C_1C_3 + C_3C_1) + \frac{1}{2}(C_2C_4 + C_4C_2),$$
$$M_{\mathrm{T}}^{(3)} = \frac{1}{2}(C_1C_4 + C_4C_1) + \frac{1}{2}(C_2C_3 + C_3C_2).$$

Double rotation symmetric units : $x \rightarrow R^{(\mu\nu)}R^{(\rho\sigma)}x$

(4,8,4)



$D_{\mathrm{nf}} - M_{\mathrm{T}}^{(i)}$

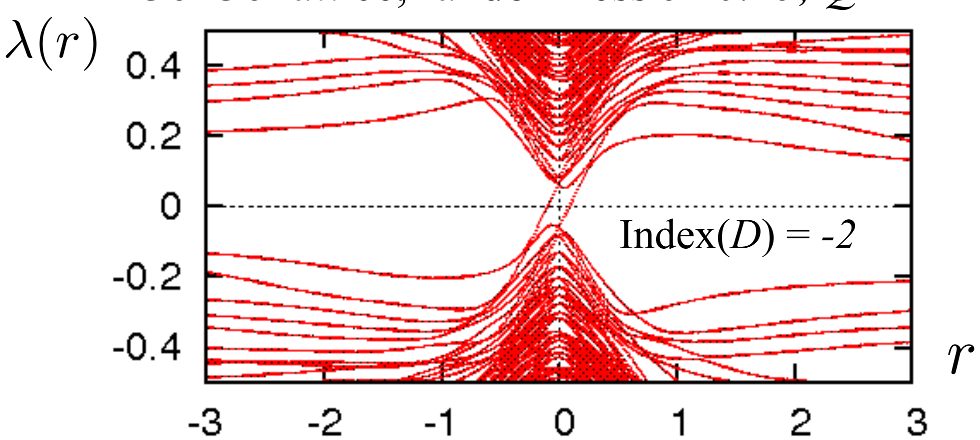
- *Index theorem from spectral flow*

$$H = \gamma_5(D_{\mathrm{nf}} - rM_{\mathrm{T}}^{(i)})$$

Index(D) = - Spectral flow(H)

$$\Rightarrow \quad \mathrm{Index}(D) = 2^{d-1}(-1)^{d/2}Q$$

36×36 lattice, randomness δ=*0.25, Q=1*



Hoelbling-type flavored mass

Hoelbling PLB696, 422(2011) [1009.5362]. de Forcrand (2010)

- *spin diagonalization*

$$\bar{\psi}_x\psi_{x+\hat{1}+\hat{2}} + \bar{\psi}_x\psi_{x+\hat{3}+\hat{4}} = (-1)^{x_2}\bar{\chi}_x\gamma_1\gamma_2\chi_{x+\hat{1}+\hat{2}} + (-1)^{x_4}\bar{\chi}_x\gamma_3\gamma_4\chi_{x+\hat{3}+\hat{4}}$$
$$\rightarrow \pm\bar{\chi}_x i\epsilon_{12}\eta_1\eta_2\chi_{x+\hat{1}+\hat{2}} \pm \bar{\chi}_x i\epsilon_{34}\eta_3\eta_4\chi_{x+\hat{3}+\hat{4}}$$

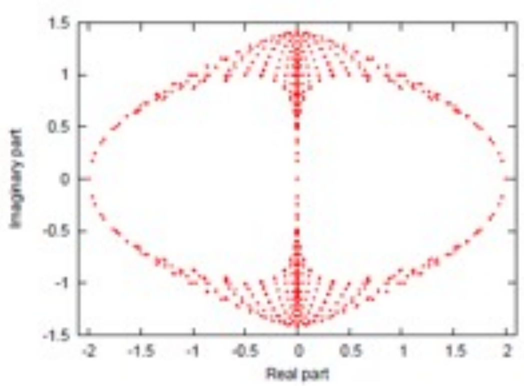
※ two terms simultaneously diagonalizable : $[\sigma_{12}, \sigma_{34}] = 0$

4 Hoelbling fermions (3 units) up to sign

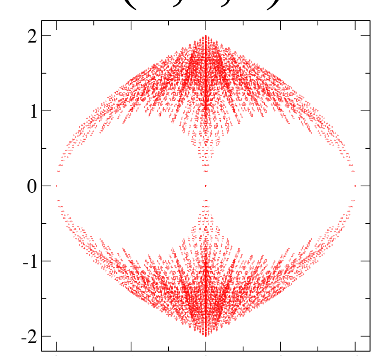
$$\bar{\psi}_x[(C_1C_2 + C_2C_1) + (C_3C_4 + C_4C_3)]\psi_x$$
$$\rightarrow \quad \pm\bar{\chi}_x[i\epsilon_{12}\eta_1\eta_2(C_1C_2 + C_2C_1) \pm i\epsilon_{34}\eta_3\eta_4(C_3C_4 + C_4C_3)]\chi_x$$

(4,8,4) (1,2,1)

$M_{\mathrm{T}}^{(i)}$



$\longleftrightarrow$



$M_{\mathcal{H}}^{(i)}$

Hoelbling, PLB696, 422(2011) [1009.5362].

$S_{\mathrm{nf}}(M_{\mathrm{T}}^{(i)}) \quad \rightarrow \quad S_{\mathrm{st}}(M_{\mathcal{H}}^{(i)})$

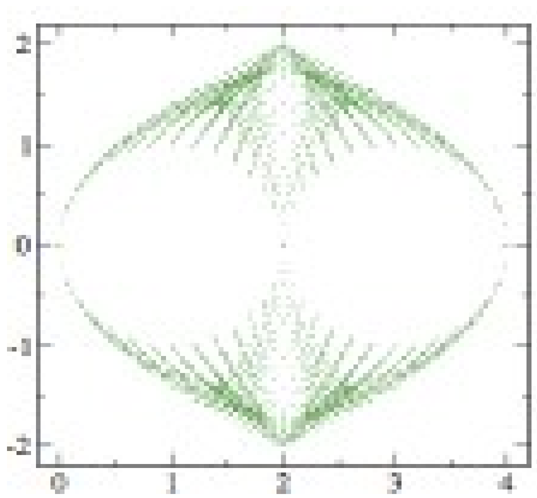
Three units of Hoelbling flavored mass $M_{\mathrm{T}}^{(i)} \rightarrow M_{\mathcal{H}}^{(i)}$

$$
\begin{aligned}
M_{\mathcal{H}} &= M_{\mathcal{H}}^{(1)} + M_{\mathcal{H}}^{(2)} + M_{\mathcal{H}}^{(3)}, \\
M_{\mathcal{H}}^{(1)} &= \frac{i}{2\sqrt{3}}[\epsilon_{12}\eta_1\eta_2(C_1C_2 + C_2C_1) + \epsilon_{34}\eta_3\eta_4(C_3C_4 + C_4C_3)], \\
M_{\mathcal{H}}^{(2)} &= \frac{i}{2\sqrt{3}}[\epsilon_{13}\eta_1\eta_3(C_1C_3 + C_3C_1) + \epsilon_{42}\eta_4\eta_2(C_4C_2 + C_2C_4)], \\
M_{\mathcal{H}}^{(3)} &= \frac{i}{2\sqrt{3}}[\epsilon_{14}\eta_1\eta_4(C_1C_4 + C_4C_1) + \epsilon_{23}\eta_2\eta_3(C_2C_3 + C_3C_2)].
\end{aligned}
$$

※ *Direct decomposition is impossible, unlike Adams' case.*

$[\sigma_{\mu\nu}, \sigma_{\nu\rho}] \neq 0 \quad \Rightarrow \quad M_{\mathrm{T}} \not\rightarrow M_{\mathcal{H}}$

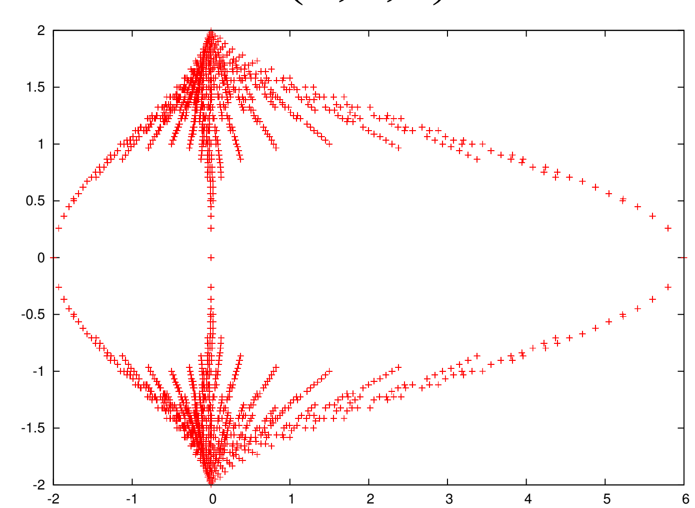
(1,2,1)



$M_{\mathcal{H}} = M_{\mathcal{H}}^{(1)} + M_{\mathcal{H}}^{(2)} + M_{\mathcal{H}}^{(3)}$

Hoelbling, PLB696, 422(2011) [1009.5362].

(6,8,2)



$M_{\mathrm{T}} = M_{\mathrm{T}}^{(1)} + M_{\mathrm{T}}^{(2)} + M_{\mathrm{T}}^{(3)}$

2. Symmetries of St.Wilson

- ***Shift symmetry*** $\longrightarrow$ broken to *2-link shift for S_A*
broken to *4-link shift for S_H*

$$\mathcal{S}_\rho: \ \chi_x \to \zeta_\rho(x)\chi_{x+\hat{\rho}}, \quad \bar{\chi}_x \to \zeta_\rho(x)\bar{\chi}_{x+\hat{\rho}}, \quad U_{\mu,x} \to U_{\mu,x+\hat{\rho}}$$

- ***Axis reversal*** $\longrightarrow$ broken to *shifted axis reversal*

$$\mathcal{I}_\rho: \ \chi_x \to (-1)^{x_\rho}\chi_{Ix}, \quad \bar{\chi}_x \to (-1)^{x_\rho}\bar{\chi}_{Ix}, \quad U_{\mu,x} \to U_{\mu,Ix}$$

- ***Rotation*** $\longrightarrow$ remain *in S_A*
broken to *subgroup in S_H*

$$\mathcal{R}_{\rho\sigma}: \ \chi_x \to S_R(R^{-1}x)\chi_{R^{-1}x}, \quad \bar{\chi}_x \to S_R(R^{-1}x)\bar{\chi}_{R^{-1}x}, \quad U_{\mu,x} \to U_{\mu,Rx}$$

- ***Conjugation*** $\longrightarrow$ remain *in S_A*
broken *in S_H*

$$\mathcal{C}: \ \chi_x \to \epsilon_x\bar{\chi}_x^T, \ \bar{\chi}_x \to -\epsilon_x\bar{\chi}_x^T, \ U_{\mu,x} \to U^*_{\mu,x}$$

§ Separating spinor & taste in momentum space

Γ_μ : spinor-space gamma

Ξ_μ : taste-space gamma

$$\phi(p)_A \equiv \chi(p + \pi_A) \ (-\pi/2 \le p_\mu < \pi/2)$$

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}, \ \{\Xi_\mu, \Xi_\nu\} = 2\delta_{\mu\nu} \text{ and } \{\Gamma_\mu, \Xi_\nu\} = 0$$

⇨

Shift $\mathcal{S}_\mu : \ \phi(p) \ \to \ \exp(ip_\mu)\Xi_\mu \, \phi(p)$

Axis inv. $\mathcal{I}_\rho : \phi(p) \ \to \ \Gamma_\rho \Gamma_5 \Xi_\rho \Xi_5 \, \phi(Ip)$

Rotation $\mathcal{R}_{\rho\sigma} : \ \phi(p) \ \to \ \exp(\frac{\pi}{4}\Gamma_\rho\Gamma_\sigma) \exp(\frac{\pi}{4}\Xi_\rho\Xi_\sigma) \, \phi(R^{-1}p)$

Conjugation $\mathcal{C} : \ \phi(p) \ \to \ \bar{\phi}(-p)^T$

◆ Discrete symmetries in Staggered Wilson

Re-interpretation of Golterman-Smit (1984)

- **Parity** $\rightarrow$ 4th-*shift* × spatial *axis*

$$\mathcal{I}_s\mathcal{S}_4 \sim \exp(ip_4)\underline{\Gamma_4\,\phi(-\mathbf{p},p_4)}$$

- **Charge conjugation** $\rightarrow$ triple-rotation × conjugation for Hoelbling-type

- **Shifted square rotation** → *μν-rot* × *νμ-rot* × *μ-shift* × *ν-shift*

$$\mathcal{S}_\nu\mathcal{S}_\mu\mathcal{R}_{\nu\mu}\mathcal{R}_{\mu\nu} \sim \exp(ip_\mu + ip_\nu)\underline{\Gamma_\mu\Gamma_\nu\,\phi(\tilde{p})}$$

These symmetries hold for $M_\mathcal{A},\ M_\mathcal{H},\ M_\mathcal{H}^{(i)}$.

Indicates restoration of essential symmetries in the continuum limit.

3. Central cusps

Creutz, Kimura, Misumi, *PRD* **83**:094506 (2011),
Kimura, Komatsu, Misumi, Noumi, Torii, Aoki, *JHEP* **1201**:048 (2012)

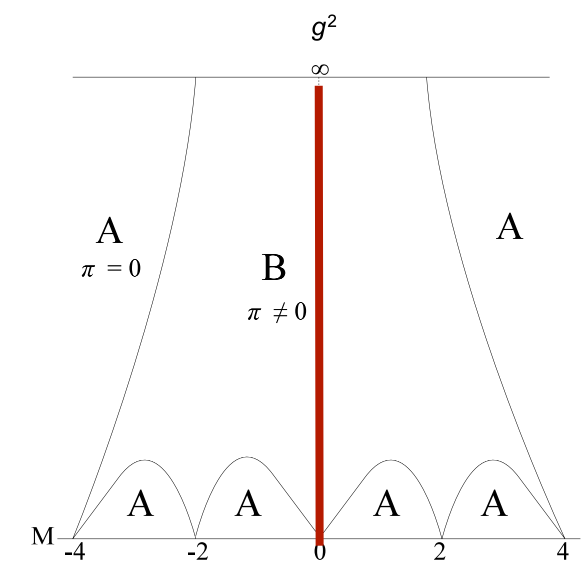
◆ Wilson fermion without on-site terms $M_W \equiv m + 4r = 0$

$$S = \frac{1}{2}\sum_{x,\mu}\bar{\psi}_x[\gamma_\mu(\psi_{x+\mu}-\psi_{x-\mu})-(\psi_{x+\mu}+\psi_{x-\mu})]$$

Extra U(1)$_V$ symmetry emerge !

$$\psi_x \;\rightarrow\; e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}, \quad \bar{\psi}_x \;\rightarrow\; \bar{\psi}_x e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}$$

- prohibits additive mass renormalization !
- will be spontaneously broken due to pion condensation ! $\langle\bar{\psi}\gamma_5\psi\rangle$



Aoki phase

§ Strong-coupling meson potential $p = (\pi, \pi, \pi, \pi + im_{SPA})$

$$\cosh(m_{SPA}) = 1 + \frac{2M_W^2(16+M_W^2)}{16-15M_W^2}$$

Massless NG boson

It is expected to describe *6-flavor Twisted-mass QCD*. $\bar{\psi}\psi \;\leftrightarrow\; \bar{\psi}\gamma_5\psi$

different bases

◆ For other naive flavored mass terms

M_{A} : U(1)v restored

M_{T} : None

M_{P} : None

◆ For staggered flavored mass terms

M_{A} : None

M_{H} : Naive conjugation

$$\mathcal{C}:\ \chi_x \to \bar{\chi}_x^T,\quad \bar{\chi}_x \to \chi_x^T,\quad U_{\mu,x} \to U_{\mu,x}^*$$

Restoration of U(1)v is peculiar to odd-link flavored mass.

Odd-link flavored mass for staggered fermions possible ?

$$M_{1\mathrm{L}} = \sum_{\mu} \xi_\mu C_\mu \sim \sum_{\mu} (1 \otimes \gamma_\mu) + O(a)$$

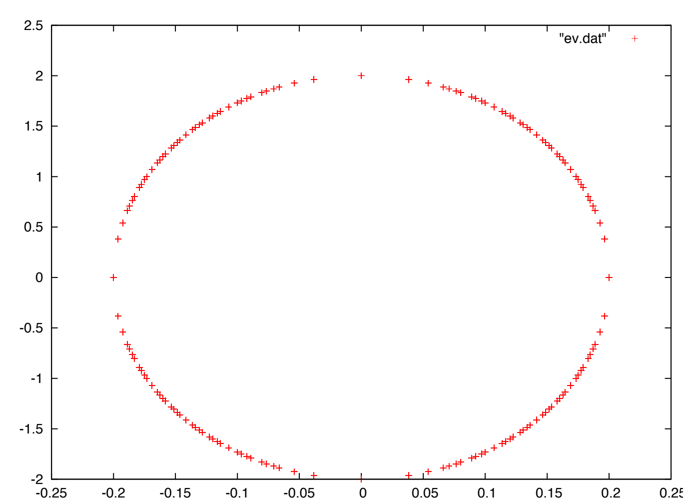
- gamma5-hermiticity breaks down.

  Isospin-type possible?

  in communication with de Forcrand (2011)

- chiral symmetry remains............

  Automatically overlap !?



T.Kimura and TM (2011)

4. Summary

1. Flavored-mass terms give us new types of Wilson and overlap fermions.

2. Staggered-Wilson can be derived from generalized Wilson fermions through spin-diagonalization.

3. Central cusps are expected to describe twisted-mass QCD without any parameter tuning.

◆gauge configuration

case [13]: we start with a smooth $U(1)$ gauge field with topological charge Q,

$$U_{x,x+e_1} = e^{i\omega x_2}, \qquad U_{x,x+e_2} = \begin{cases} 1 & (x_2 = 1, 2, \cdots, L-1) \\ e^{i\omega L x_1} & (x_2 = L) \end{cases}, \tag{30}$$

where L is the lattice size and ω is the curvature given by $\omega = 2\pi Q$. Then, to emulate a typical gauge configuration of a practical simulation, we introduce disorder effects to link variables by random phase factors, $U_{x,y} \to e^{ir_{x,y}} U_{x,y}$, where $r_{x,y}$ is a random number uniformly distributed in $[-\delta\pi, \delta\pi]$. The parameter δ determines the magnitude of disorder.

$M_{\mathcal{A}},\ M_{\mathcal{H}}$

cf.) $$1 - \cos ap_1 \cos ap_2 = \frac{a^2p_1^2 + a^2p_2^2}{2} + O(a^3)$$

$$2 - (\cos ap_1 + \cos ap_2) = \frac{a^2p_1^2 + a^2p_2^2}{2} + O(a^3)$$