Single flavor staggered fermions

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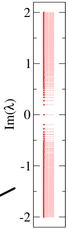
New types of Fermions Workshop
YITP, Feb. 9, 2012

☞Phys. Lett. B696 (2011) 422

SFB TR55

Lattice fermions



Naive fermions

$D_N = \gamma_\mu D_\mu$

16 species

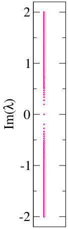
remove 4-fold degeneracy

add Wilson term

Wilson fermions

$D_W = \gamma_\mu D_\mu + rW$

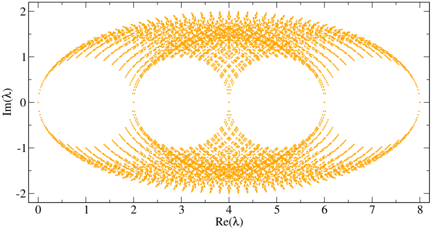
1+4+6+4+1 species



Staggered fermions

$D_{st} = \eta_\mu D_\mu$

4 species

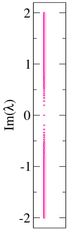


Chiral symmetry

Wilson fermions D_W

1+4+6+4+1 species

↓



Staggered fermions

$$D_{st} = \eta_\mu D_\mu$$

4 species, no anomaly

$$U(1)_\epsilon : \{D_{st}, \epsilon\} = 0$$

$$\epsilon = (-1)^{x_1+x_2+x_3+x_4}$$

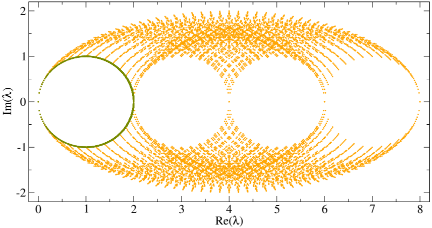
$$\sim (\gamma_5 \otimes \xi_5)$$

Overlap fermions

$$D_{ov} = \rho\,(1 + \gamma_5 \text{sign}\,(\gamma_5\,(D_W - \rho)))$$

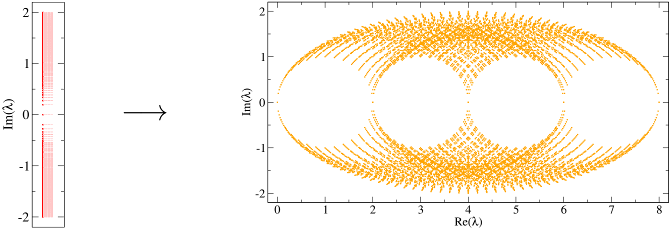
1 species, correct anomaly

Full $SU(N_f)$ chiral Symetry



Wilson term

Add a Wilson term to naive operator:



Momentum dependent mass (remove doublers)

Positivity of $\det(D_{\mathrm{W}})$:

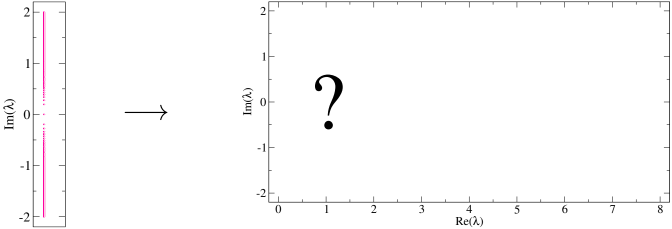
- Naive operator: antihermitean, $\{D_{\mathrm{N}}, \gamma_5\} = 0$
- Wilson term W: hermitean, $[W, \gamma_5] = 0$

➛ $D_{\mathrm{W}}\gamma_5 = \gamma_5 D_{\mathrm{W}}^{\dagger}$

➛ eigenvalues real or in complex conjugate pairs $\lambda_i = \lambda_{i^*}^*$

Staggered Wilson term

Add a Wilson term to staggered operator:



Momentum (taste) dependent mass (remove doublers)

Positivity of $\det(D_{\mathrm{A}})$:

- Staggred operator: antihermitean, $\{D_{\mathrm{N}}, \epsilon\} = 0$
- Wilson term A: hermitean, $[A, \epsilon] = 0$

➛ $D_{\mathrm{A}}\epsilon = \epsilon D_{\mathrm{A}}^{\dagger}$

➛ eigenvalues real or in complex conjugate pairs $\lambda_i = \lambda_{i^*}^*$

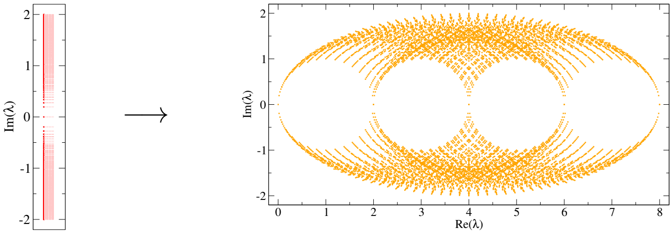
Wilson term construction

Usual Wilson term:

$$W = \sum_\mu (C_\mu + 1)$$

$$C_\mu := \frac{1}{2}\left(V_\mu + V_\mu^\dagger\right) \qquad (V_\mu)_{xy} := U_\mu(x)\delta_{x+\hat\mu,y}$$

$$W^\dagger = W \checkmark \qquad [W, \gamma_5] = 0 \checkmark$$



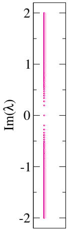
Staggered Wilson term construction

Staggered Wilson term: (Adams, 2010)

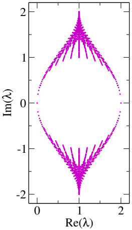
$$A = \epsilon\eta_5\,(C_1 C_2 C_3 C_4)_{\text{sym}}$$

$$C_\mu := \frac{1}{2}\left(V_\mu + V_\mu^\dagger\right) \qquad (V_\mu)_{xy} := U_\mu(x)\delta_{x+\hat{\mu},y}$$

$A^\dagger = A$ ✓ $\qquad [A,\epsilon] = 0$ ✓



⟶



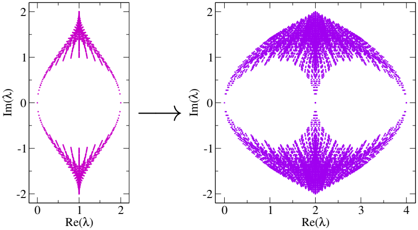
- $\eta_5 = \eta_1\eta_2\eta_3\eta_4 = (-1)^{x_1+x_3}$
- $\eta_\mu = (-1)^{\sum_{\nu<\mu} x_\nu} \sim (\gamma_\mu \otimes 1)$
- $\epsilon = (-1)^{x_1+x_2+x_3+x_4} \sim (\gamma_5 \otimes \xi_5)$
- $\{C_\mu, \epsilon\} = 0$
- $A \sim (1 \otimes \xi_5) + O(a)$

Remnant flavor degeneracy

- Staggered flavor basis: $A \sim \xi_5 = \mathrm{diag}(1, 1, -1, -1)$
- Twofold degeneracy left!
- Let us take

$$M_{\mu\nu} = i\epsilon_{\mu\nu}\eta_\mu\eta_\nu \left(C_\mu C_\nu\right)_{\mathrm{sym}}$$

$$M_{\mu\nu}^\dagger = M_{\mu\nu} \checkmark \qquad [M_{\mu\nu}, \epsilon] = 0 \checkmark$$



- $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu} = (-1)^{x_\mu + x_\nu}, \ \mu < \nu$
- $\eta_\mu = (-1)^{\sum_{\nu<\mu} x_\nu}$
- $M_{\mu\nu} \sim (1 \otimes \sigma_{\nu\mu}) + O(a)$
- $\sigma_{\nu\mu} = \mathrm{diag}(1, -1, -1, 1)$

Discrete staggered symmetries

Remnants of Poincare symmetry:

	D_{st}	A	$M_{\mu\nu}$
shift (translation)	+	-	$\pm$
axis reversal	+	-	$\pm$
rotation	+	+	$M_{\alpha\beta}$

- Preserved by staggered operator
- Preserved up to a sign flip by A
- Rotation introduces new terms for $M_{\mu\nu}$

➛ Bad: new counterterms

! Search for more symmetric construction

Symmetrized staggered Wilson

$$M_s = \frac{1}{\sqrt{3}}\Big(s_{12}\left(s_1 s_2 M_{12} + s_3 s_4 M_{34}\right) + s_{13}\left(s_1 s_3 M_{13} + s_4 s_2 M_{42}\right) + s_{14}\left(s_1 s_4 M_{14} + s_2 s_3 M_{23}\right)\Big)$$

- Shift or axis reversal in ρ: $s_\rho \to -s_\rho$

rotation (ρ, σ)				sign flip
(1,4)	(2,3)	(3,1)	(2,4)	$s_{12} \to -s_{12}$
(1,2)	(3,4)	(4,1)	(3,2)	$s_{13} \to -s_{13}$
(1,3)	(4,2)	(2,1)	(4,3)	$s_{14} \to -s_{14}$

- $M_s \sim (1 \otimes \xi^{(s)}) + O(a) \qquad \xi^{(s)} = \mathrm{diag}(0, 0, 2, -2)$

New symmetries

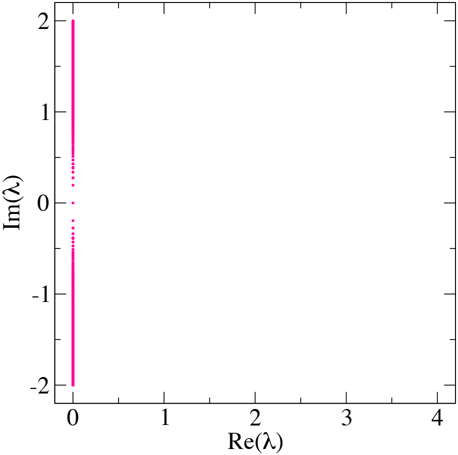
- Symmetries of the action:

D_{st}	A	M_S
$x \to x + \hat{\mu}$	$x \to x + \hat{\mu} \pm \hat{\nu}$	$x \to x + \hat{1} \pm \hat{2} \pm \hat{3} \pm \hat{4}$
$x_\mu \to -x_\mu$	$x_\mu \to -x_\mu + 1$	$x_\mu \to -x_\mu + 1$
$x \to R^{(\mu\nu)} x$	$x \to R^{(\mu\nu)} x$	$x \to R^{(\mu\nu)} R^{(\sigma\tau)} x$

Leading (a^3) terms:

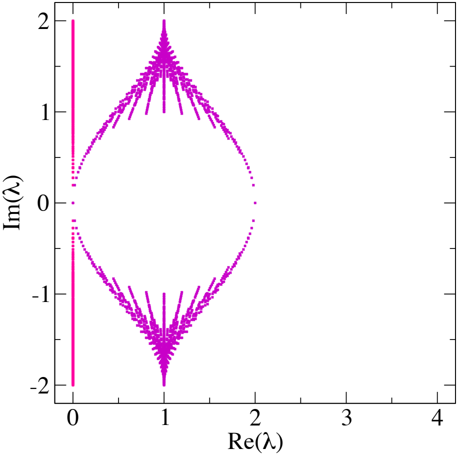
- Staggered symmetries:none
- 2-flavor symmetries:A
- 1-flavor symmetries:A and all M_S
  - ✗ Loop corrections renormalize flavor structure
  - ✓ Flavor assignment is arbitrary → no problem?

Single flavor staggered operator



- D_{st}
- $D_{st} + 1 + A$
- $D_{st} + 2 + A + M_{\mu\nu}$
- $D_s = D_{st} + (2 + M_s)$
- $D_1 = 1 + \epsilon \text{sign}(\epsilon(D_s - 1))$

Single flavor staggered operator



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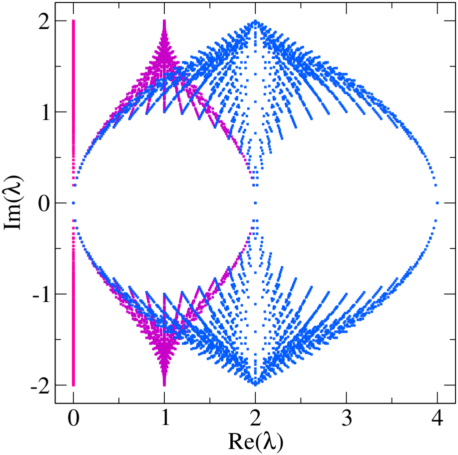

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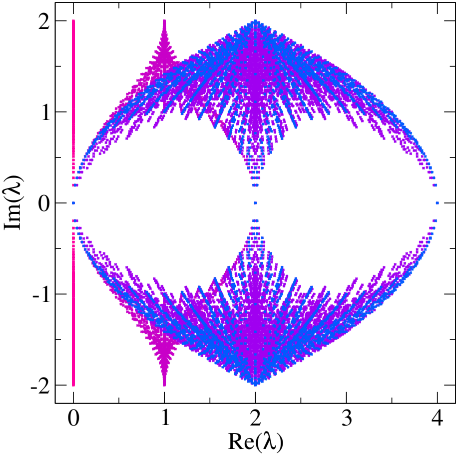

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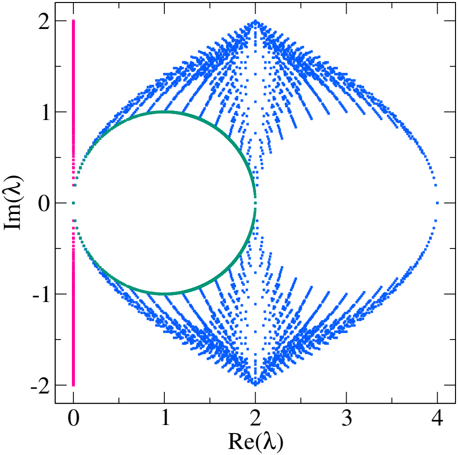

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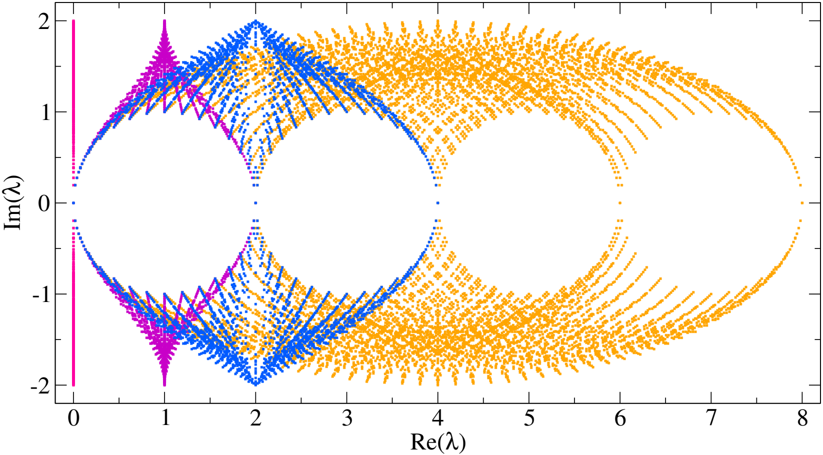
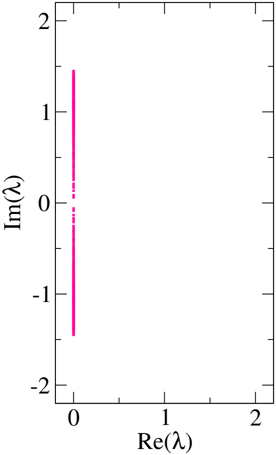


Some tests in 2D



- Only 2-fold degeneracy in 2D
- M_{12} uniquely lifts this degeneracy

- D_{st}
- $D_{\mathrm{st}} + 1 + M_{12}$ $\quad Q = 0$
- $1 + \epsilon\mathrm{sign}(\epsilon(D_{\mathrm{st}} + M_{12}))$

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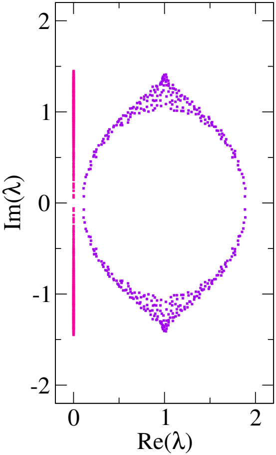

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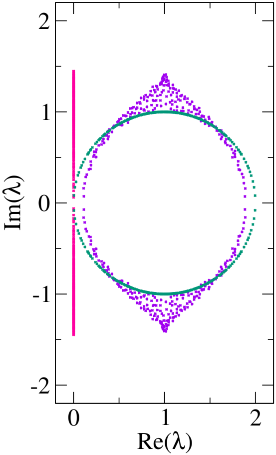

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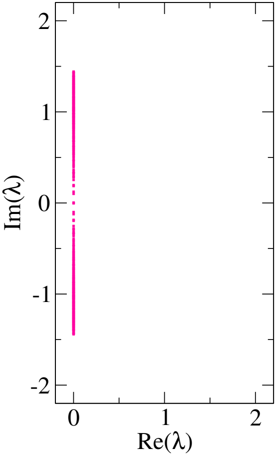

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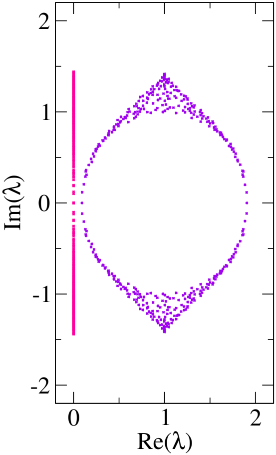

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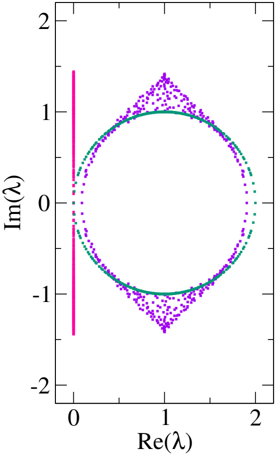

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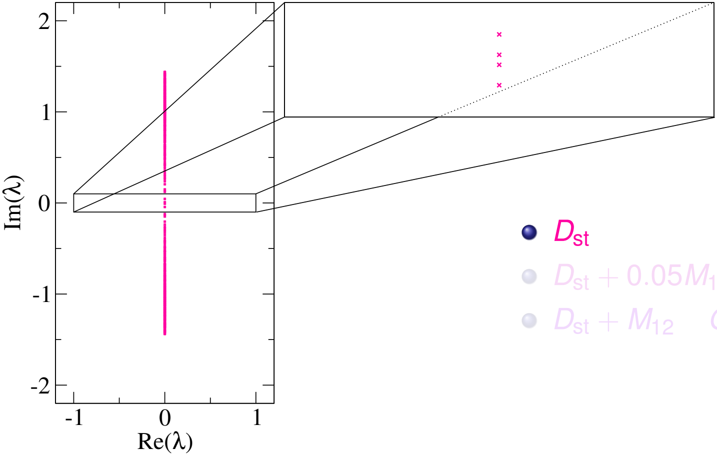
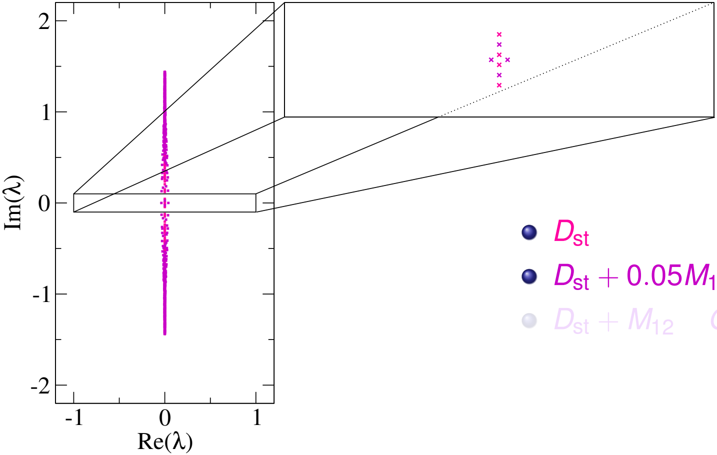


- D_{st}
- $D_{st} + 0.05M_{12}$
- $D_{st} + M_{12}$ $\quad Q = 2$

Some tests in 2D



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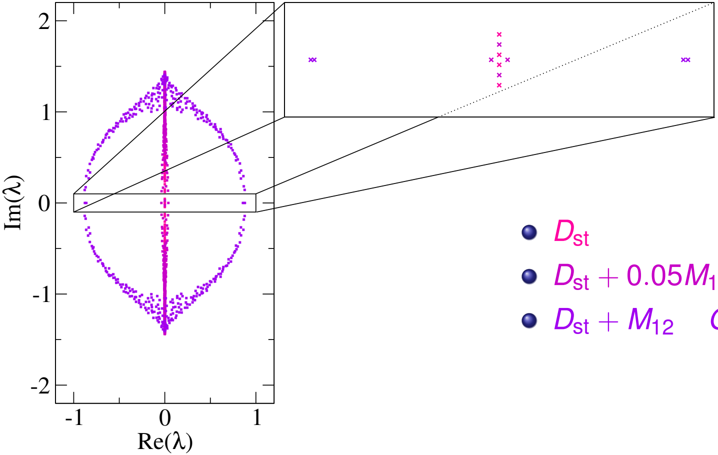

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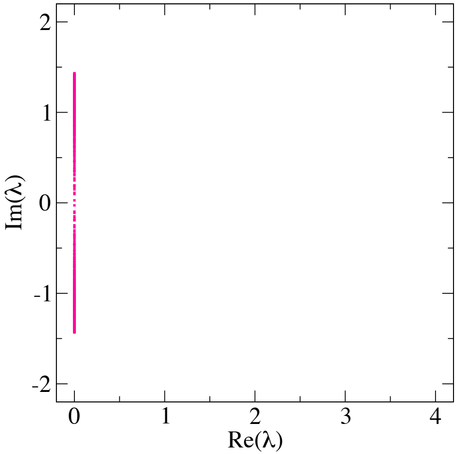

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- D_W

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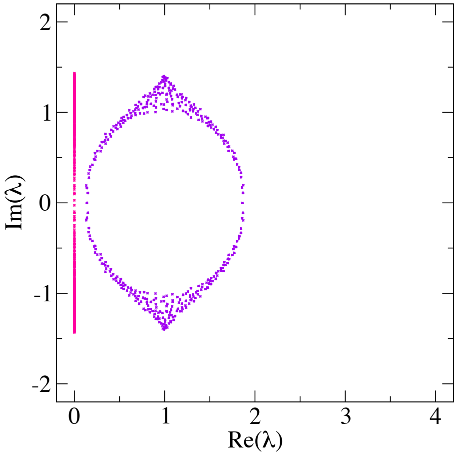

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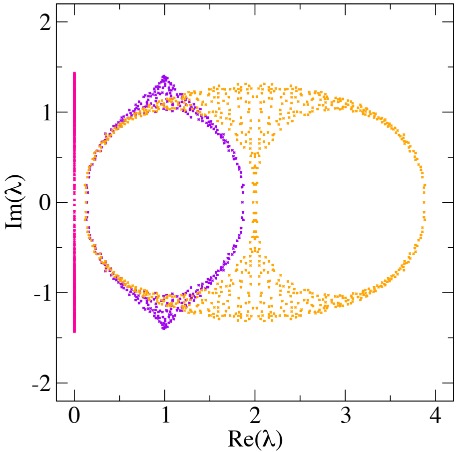

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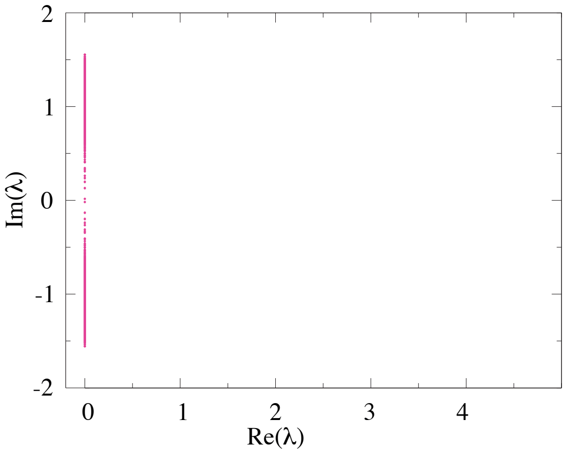
Spectrum 6^4, $\beta = 5.6$



- D_{st} 7-APE
- $D_{st} + 1 + A$
- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + M_s$

(Data courtesy S. Dürr)

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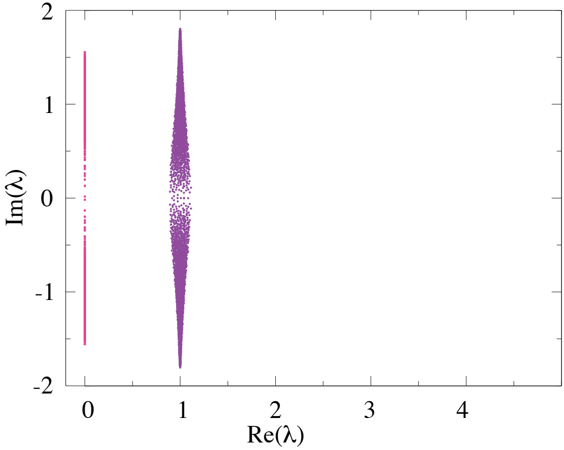

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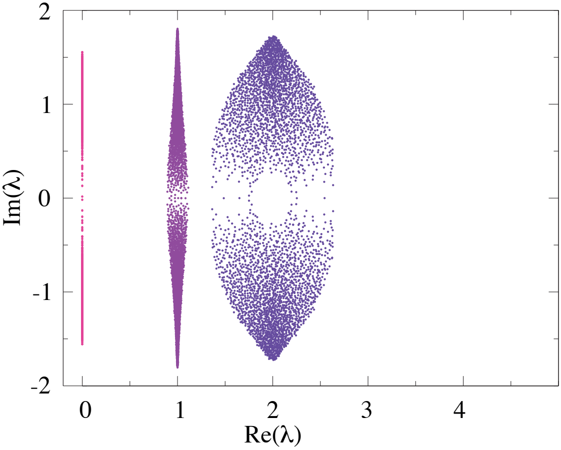

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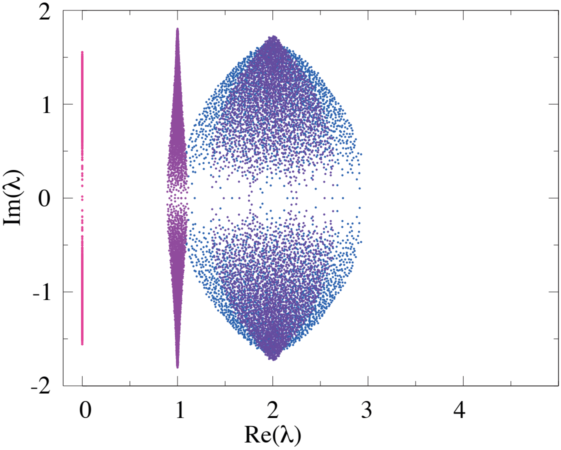

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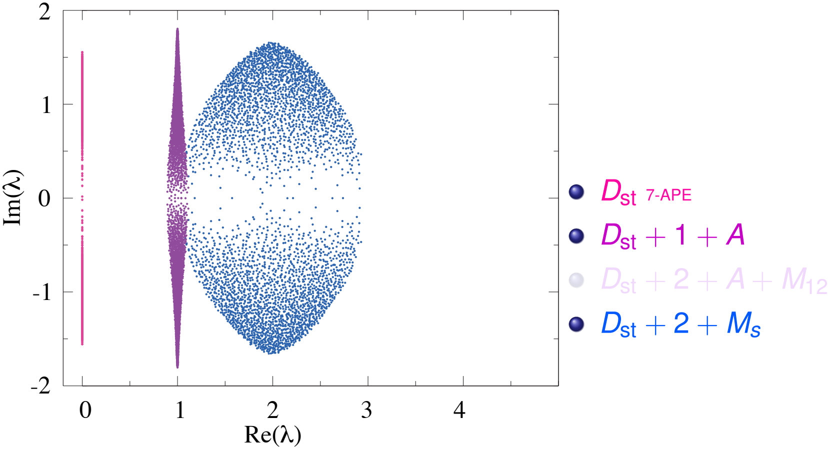

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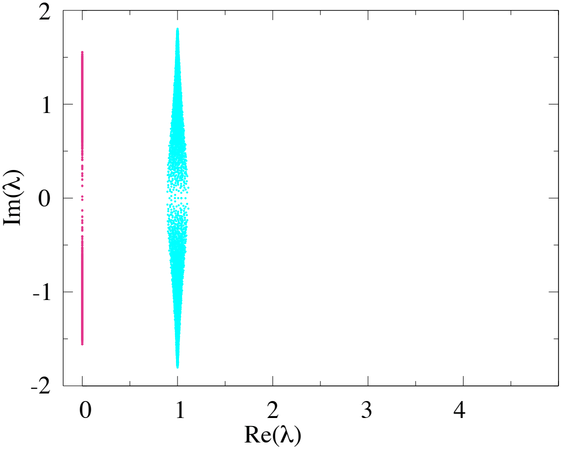
Adams operator 6^4, $\beta = 5.6$



- $D_{\mathrm{st}} + 1 + A$
- $D_{\mathrm{st}} + 1 + A$, 1-APE
- $D_{\mathrm{st}} + 1 + A$, 3-APE
- $D_{\mathrm{st}} + 1 + A$, 7-APE
- D_{st}, 7-APE

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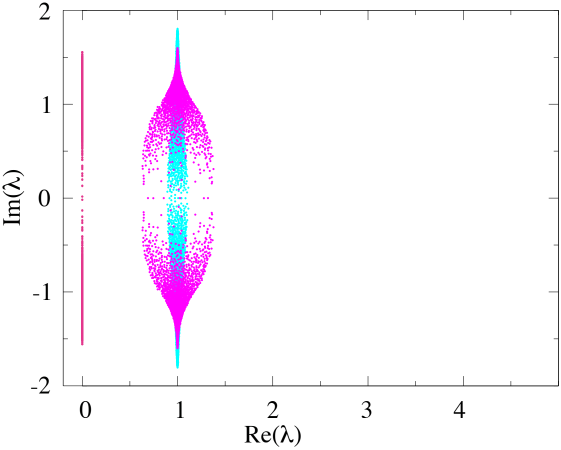

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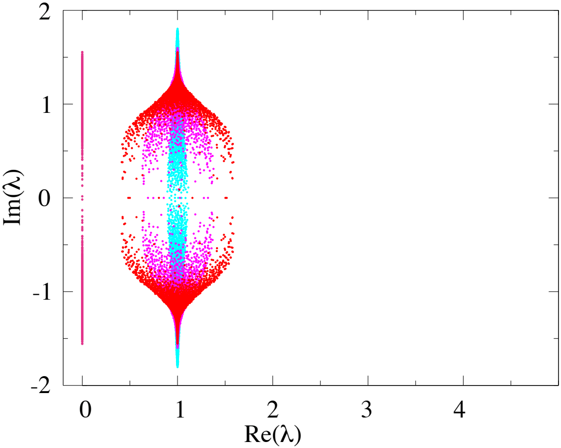

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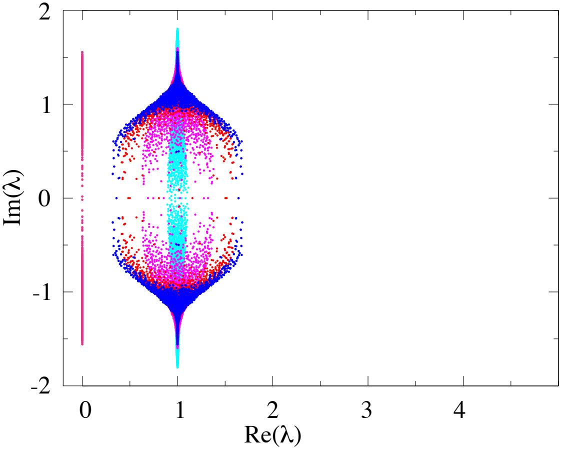

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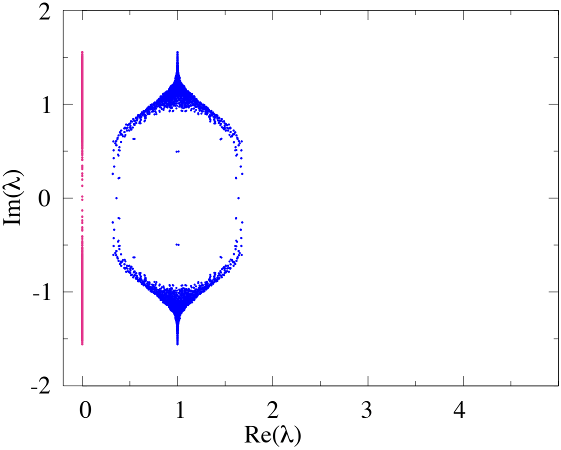

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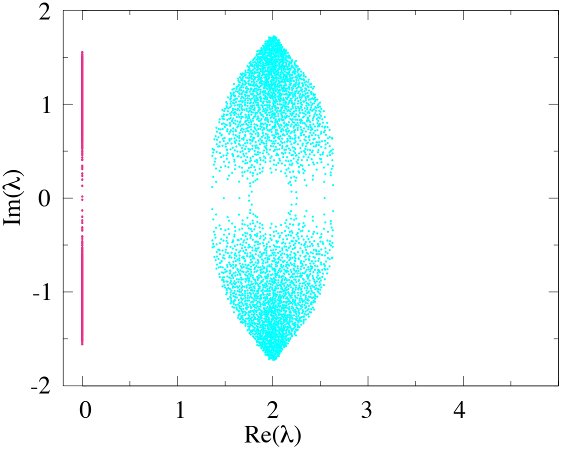
(Data courtesy S. Dürr)

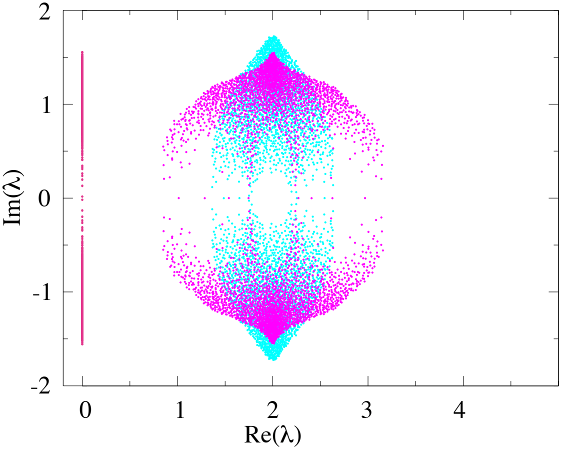
Unsymmetrized operator 6^4, $\beta = 5.6$



- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + A + M_{12}$, 1-APE
- $D_{st} + 2 + A + M_{12}$, 3-APE
- $D_{st} + 2 + A + M_{12}$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

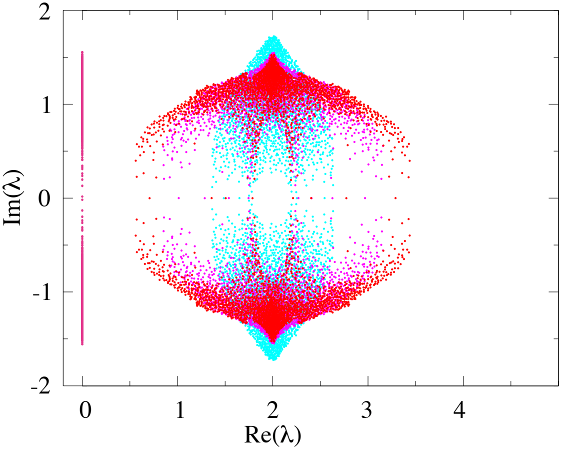
Unsymmetrized operator 6^4, $\beta = 5.6$



- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + A + M_{12}$, 1-APE
- $D_{st} + 2 + A + M_{12}$, 3-APE
- $D_{st} + 2 + A + M_{12}$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

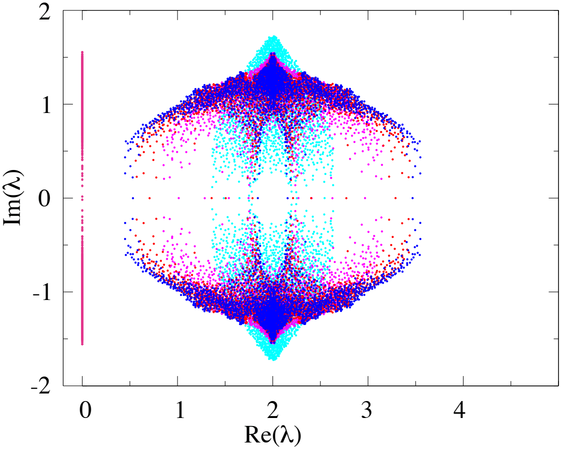
Unsymmetrized operator 6^4, $\beta = 5.6$



- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + A + M_{12}$, 1-APE
- $D_{st} + 2 + A + M_{12}$, 3-APE
- $D_{st} + 2 + A + M_{12}$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

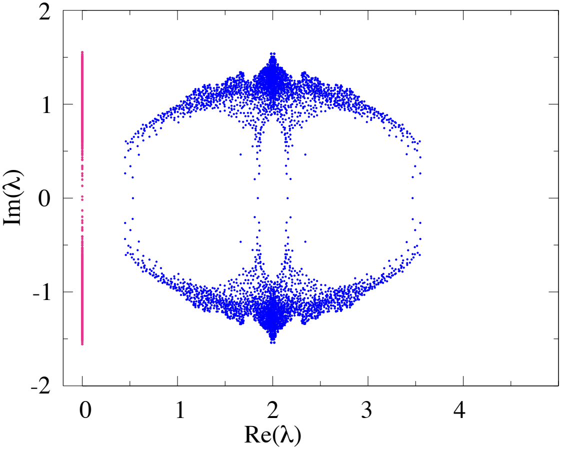
Unsymmetrized operator 6^4, $\beta = 5.6$



- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + A + M_{12}$, 1-APE
- $D_{st} + 2 + A + M_{12}$, 3-APE
- $D_{st} + 2 + A + M_{12}$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

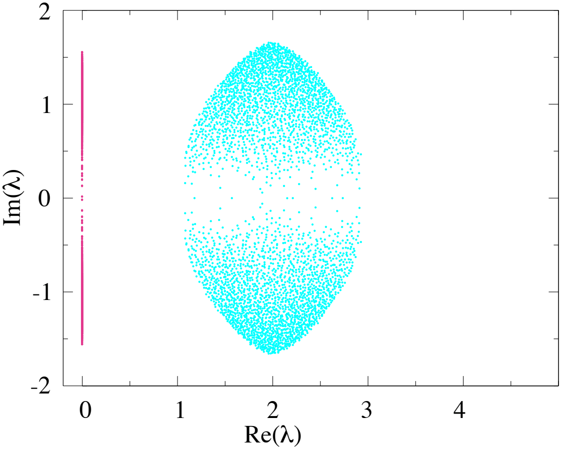
Unsymmetrized operator 6^4, $\beta = 5.6$



- $D_{\text{st}} + 2 + A + M_{12}$
- $D_{\text{st}} + 2 + A + M_{12}$, 1-APE
- $D_{\text{st}} + 2 + A + M_{12}$, 3-APE
- $D_{\text{st}} + 2 + A + M_{12}$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

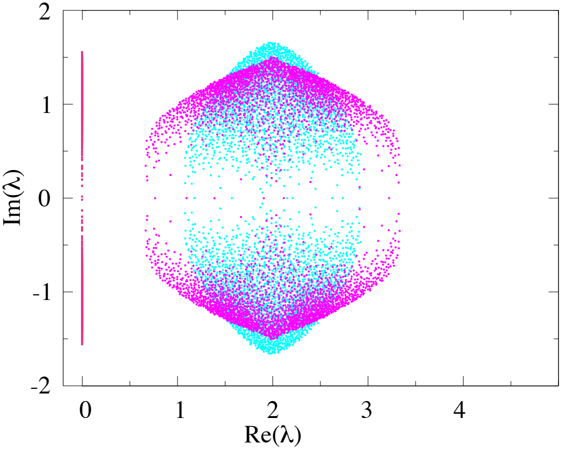
Symmetrized operator 6^4, $\beta = 5.6$



- $D_{st} + 2 + M_s$
- $D_{st} + 2 + M_s$, 1-APE
- $D_{st} + 2 + M_s$, 3-APE
- $D_{st} + 2 + M_s$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

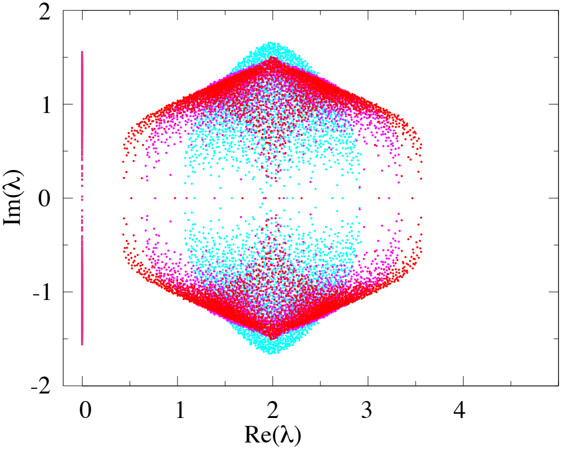
Symmetrized operator 6^4, $\beta = 5.6$



- $D_{\rm st} + 2 + M_s$
- $D_{\rm st} + 2 + M_s$, 1-APE
- $D_{\rm st} + 2 + M_s$, 3-APE
- $D_{\rm st} + 2 + M_s$, 7-APE
- $D_{\rm st}$, 7-APE

(Data courtesy S. Dürr)

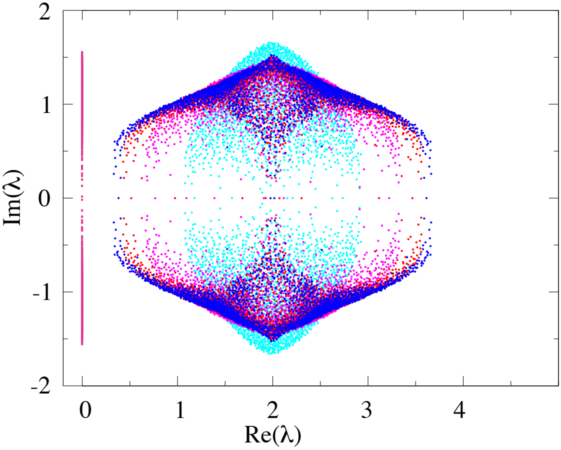
Symmetrized operator 6^4, $\beta = 5.6$



- $D_{st} + 2 + M_s$
- $D_{st} + 2 + M_s$, 1-APE
- $D_{st} + 2 + M_s$, 3-APE
- $D_{st} + 2 + M_s$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

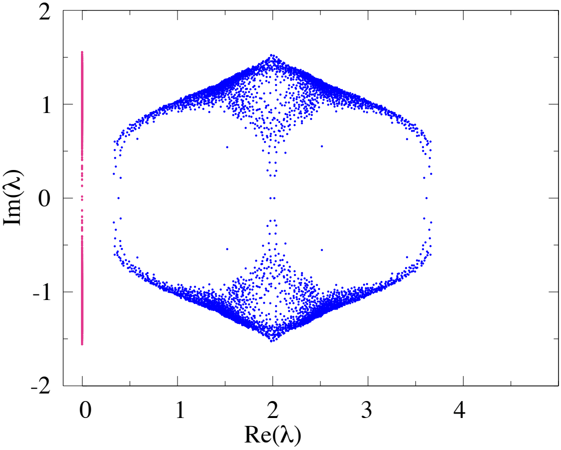
Symmetrized operator 6^4, $\beta = 5.6$



- $D_{st} + 2 + M_s$
- $D_{st} + 2 + M_s$, 1-APE
- $D_{st} + 2 + M_s$, 3-APE
- $D_{st} + 2 + M_s$, 7-APE
- D_{st}, 7-APE

(Data courtesy S. Dürr)

Symmetrized operator 6^4, $\beta = 5.6$



$D_{st} + 2 + M_s$

$D_{st} + 2 + M_s$, 1-APE

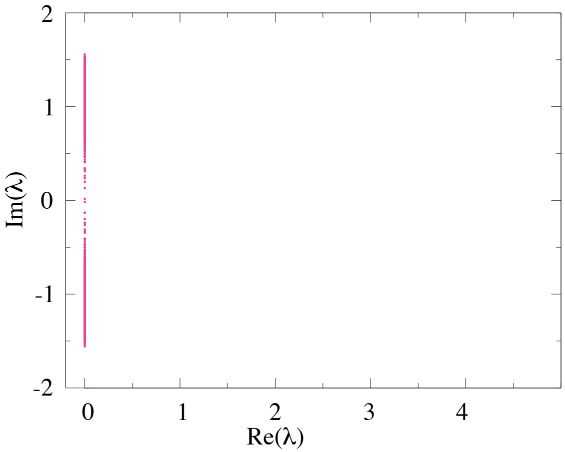
$D_{st} + 2 + M_s$, 3-APE

$D_{st} + 2 + M_s$, 7-APE

D_{st}, 7-APE

(Data courtesy S. Dürr)

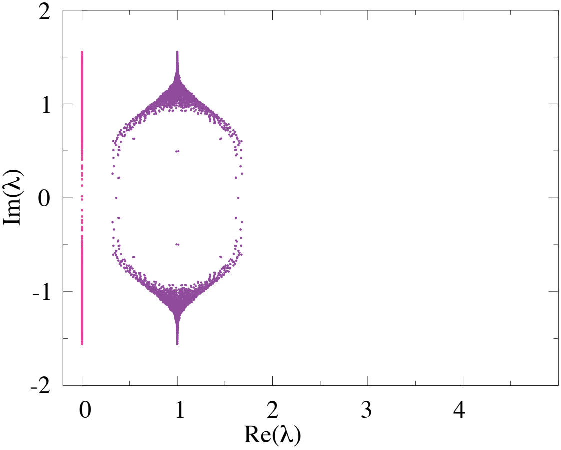
Spectrum 6^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + M_s$

(Data courtesy S. Dürr)

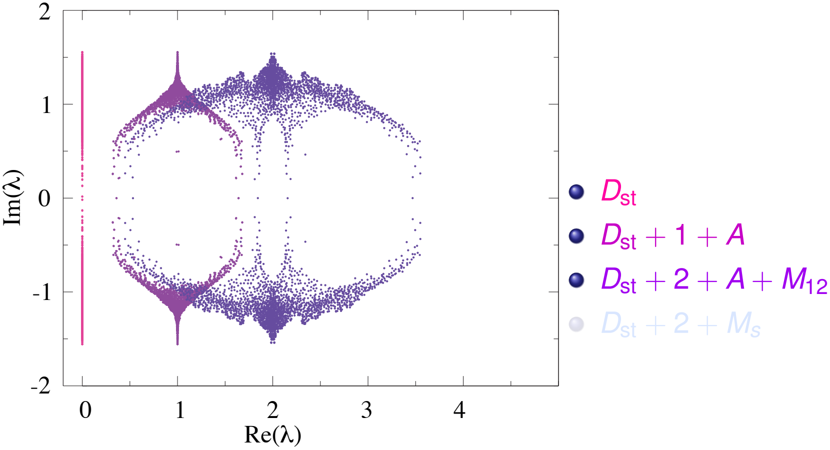
Spectrum 6^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + M_s$

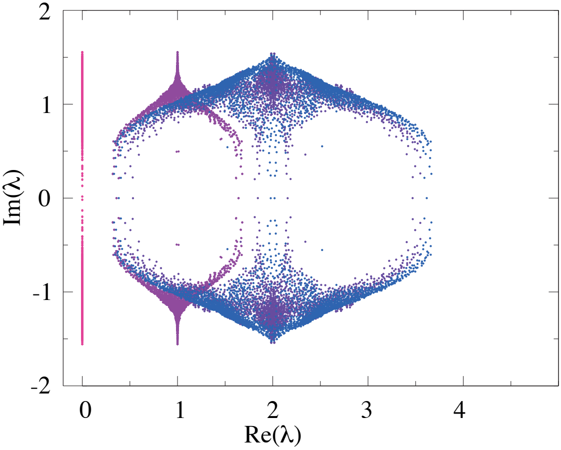
(Data courtesy S. Dürr)

Spectrum 6^4, $\beta = 5.6$, 7-APE smeared



(Data courtesy S. Dürr)

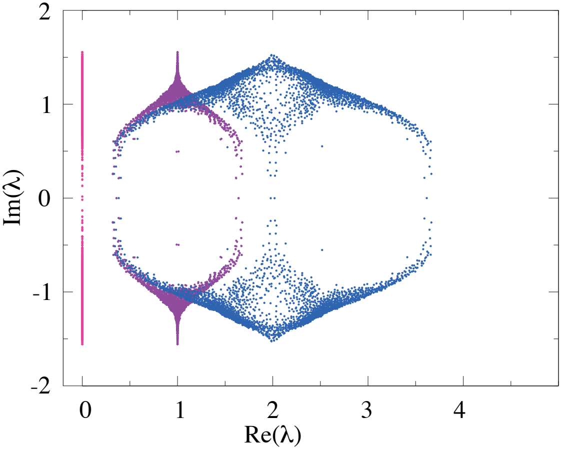
Spectrum 6^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + M_s$

(Data courtesy S. Dürr)

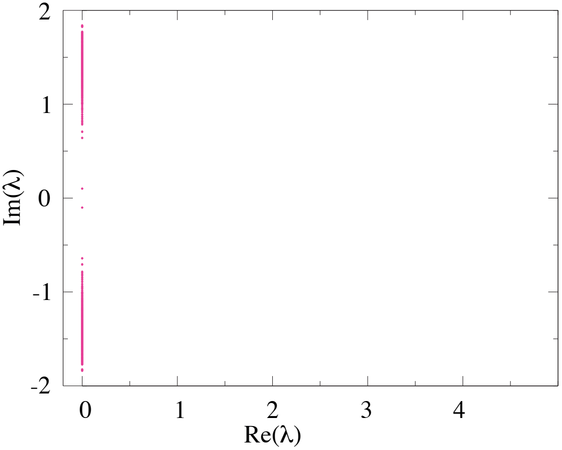
Spectrum 6^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_{st} + 2 + A + M_{12}$
- $D_{st} + 2 + M_s$

(Data courtesy S. Dürr)

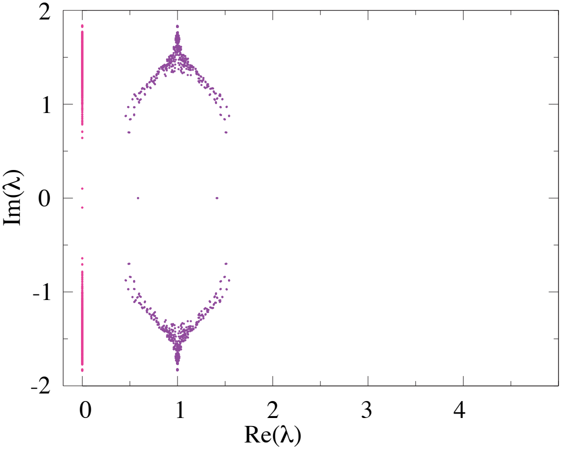
Spectrum 4^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_s = D_{st} + 2 + M_s$
- D_W

(Data courtesy S. Dürr)

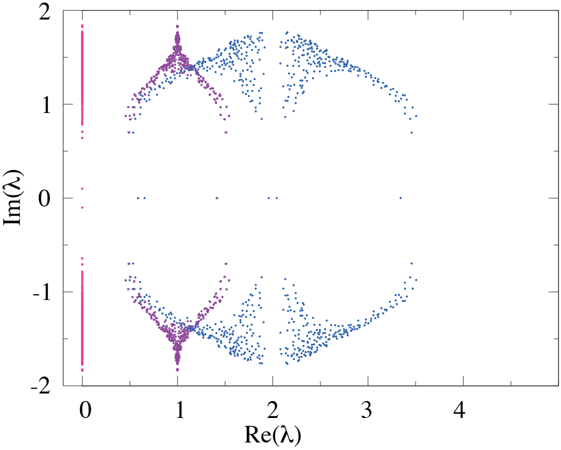
Spectrum 4^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_s = D_{st} + 2 + M_s$
- D_W

(Data courtesy S. Dürr)

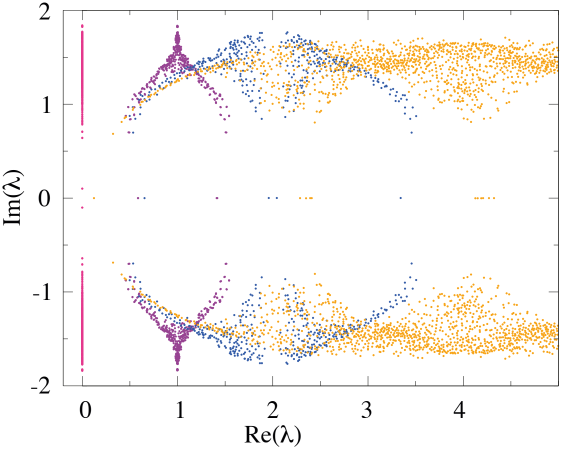
Spectrum 4^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_s = D_{st} + 2 + M_s$
- D_W

(Data courtesy S. Dürr)

Spectrum 4^4, $\beta = 5.6$, 7-APE smeared



- D_{st}
- $D_{st} + 1 + A$
- $D_s = D_{st} + 2 + M_s$
- D_W

(Data courtesy S. Dürr)

Conclusion

Single flavor staggered operator is possible

Wilson fermions without remnants of spurious naive degeneracy exist

But is it useful?

- ✓ Better condition number
- ✓ Smaller matrix

- ✗ Staggered spinor structure
- ✗ 2-hop Wilson term

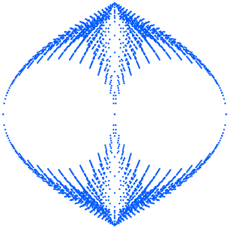
Essential: Check renormalization, exceptionals, scaling

Similar construction: (De Forcrand, Kurkela, Panero)
Flavored mass term for naive fermions: (Creutz, Kimura, Misumi)

Staggered Wilson

To do list:

- Find counterterm structure
- Construct mesons, baryons
- $O(a)$ improvement
  - Clover "for free" due to 2-hop Wilson term? (some CPU, but no additional bandwidth)
- Optimize algorithms for the structure
- Check flavor breaking
- Study scaling
- Apply to real problem
  - Insensitive to flavor breaking
  - → Ground states, bulk properties, spectral quantities
  - Hadron/quark masses? Thermodynamics?



Staggered overlap

To do list:

- Find counterterm structure
- Check locality
- Check flavor breaking
- Study scaling
- Apply to real problem
  - Insensitive to flavor breaking
  - Chiral symmetry essential
  - ➛ Spectral quantities?