The condition for the existence the gapless modes in topological defects from Green's functions

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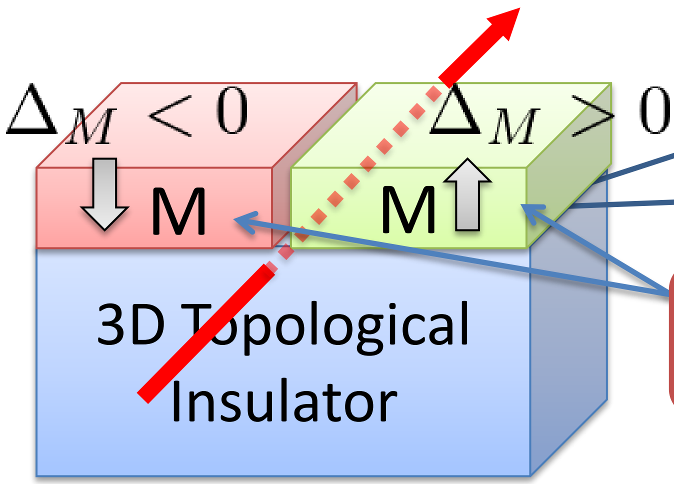
K. Shiozaki and S. Fujimoto, arXiv:1111.1685

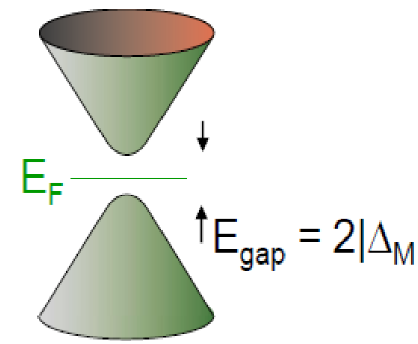
Surface Quantum Hall effect

X. -L. Qi, T. L. Hughes, and S. -C. Zhang, Phys. Rev. B 78, 195424 (2008).



Surface helical Dirac fermion

$$H = -iv\boldsymbol{\sigma}\cdot\boldsymbol{\nabla} - \mu + \Delta_M\sigma_z$$

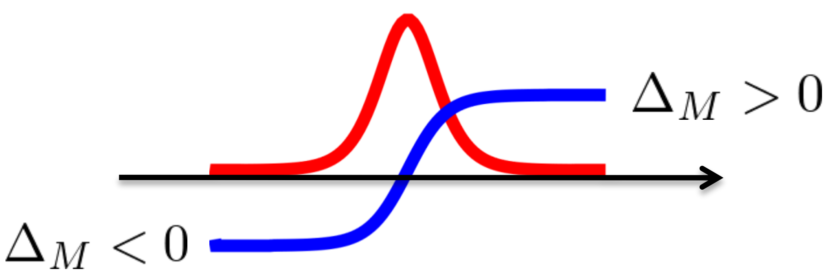


Magnetic insulator

Break the time reversal symmetry

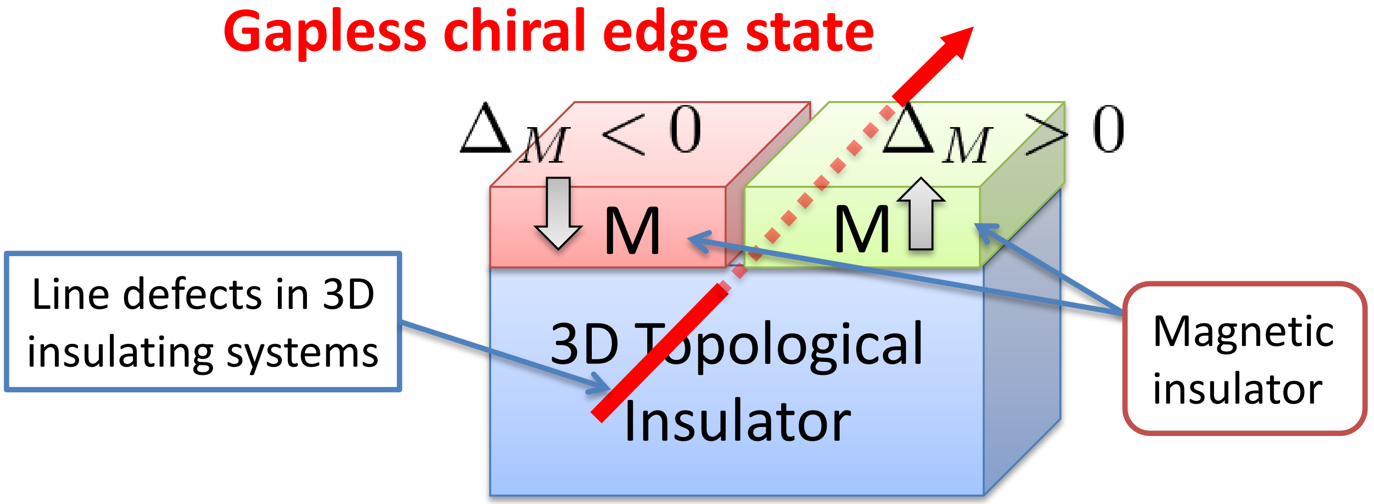
induces the mass gap Δ_M to the helical Dirac fermion

Domain wall gapless chiral edge state



・・・There exist gapless chiral edge states at domain wall : $\Delta_M \leftrightarrow -\Delta_M$

Topological invariants from Green's functions



our study...

Construction topological invariants characterizing the existence of gapless modes at line defects in heterostructure systems with broken time-reversal symmetry by using the Green's functions.

cf. classification of topological defects
[J. C. Y. Teo and C. L. Kane, Phys. Rev. B **82**, 115120 (2010).]

- **Green's functions for 3-spatial dimensional systems**
- **Full quantum formulation (not semi-classical approximation)**

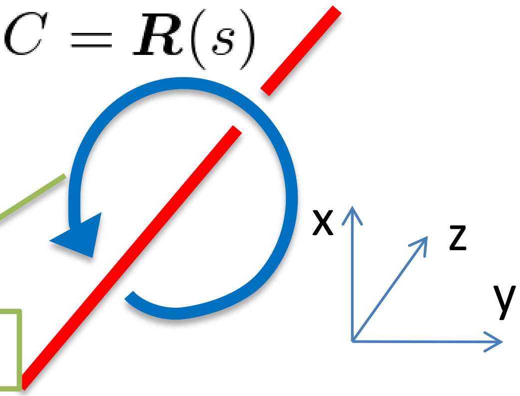
Topological invariants from Green's functions

cf. Volovik "*The Universe in a Helium Droplet*"

Consider the closed path surrounding the line defect

$$G(i\omega, \boldsymbol{p}, \boldsymbol{R}) \mapsto G(i\omega, \boldsymbol{p}, \boldsymbol{R}(s))$$

$$C = \boldsymbol{R}(s)$$



if the Green's function is not singular for all $(\omega, \boldsymbol{p})$ on the closed path $C = \boldsymbol{R}(s)$,
We can define the topological invariant by

Singularity free on the path

$$N_5 = -\frac{i}{96\pi^3} \oint_C d\boldsymbol{R} \cdot \int d\omega d^3\boldsymbol{p}\, \epsilon^{\mu\nu\rho\sigma} \mathrm{tr}\left[G^{-1} \frac{\partial}{\partial \boldsymbol{R}} G G^{-1} \partial_\mu G G^{-1} \partial_\nu G G^{-1} \partial_\rho G G^{-1} \partial_\sigma G \right]$$

$$\mu, \nu, \rho, \sigma \in (\omega, p_x, p_y, p_z)$$

・・・Winding number of the map $G : (\omega, p_x, p_y, p_z, s) \mapsto \mathrm{GL}\,(n, \mathbb{C})$

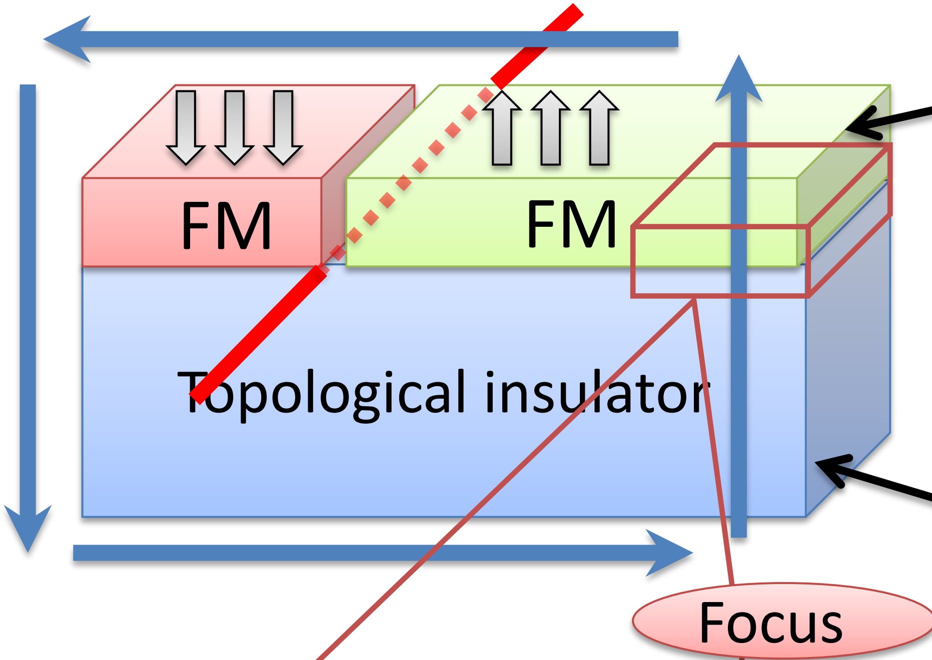
$$\pi_5(\mathrm{GL}\,(n, \mathbb{C})) = \mathbb{Z}$$

Singularity of Green's function ・・・

Fermi surface $\det G(i\omega, \boldsymbol{p}, \boldsymbol{R}) = \infty$

(Mott insulator) $\det G(i\omega, \boldsymbol{p}, \boldsymbol{R}) = 0$

A model for TI–ferromagnet tri-junction systems

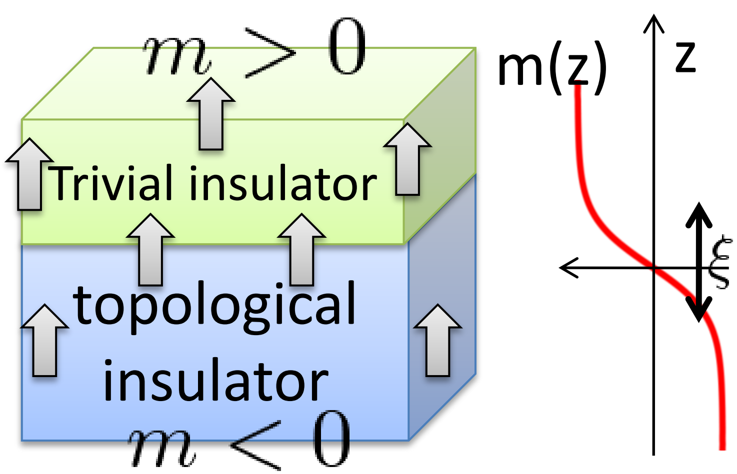


$$H = v\mu_1 \boldsymbol{k}\cdot\boldsymbol{\sigma} + \left(-m + \epsilon \boldsymbol{k}^2\right)\mu_3 + h\sigma_3$$

$\boldsymbol{\sigma}$ ･･･ spin $\boldsymbol{\mu}$ ･･･ orbital

Low energy effective Hamiltonian of two-orbital cubic lattice model for TI

$$H = v\mu_1 \boldsymbol{k}\cdot\boldsymbol{\sigma} + \left(m + \epsilon \boldsymbol{k}^2\right)\mu_3$$

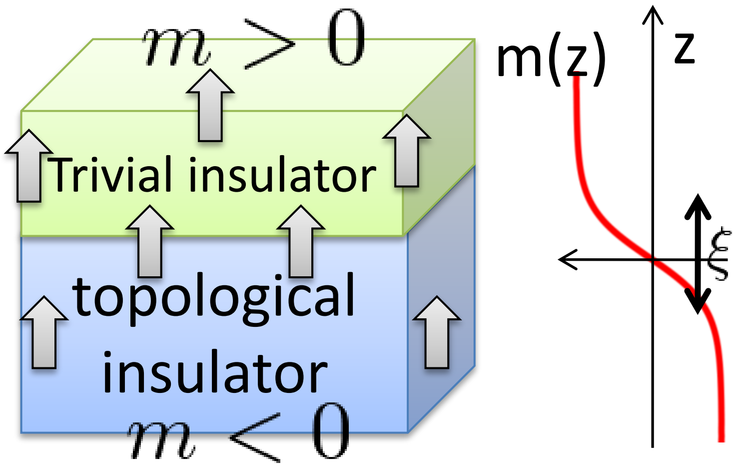


$$H = v\mu_1(k_x\sigma_1 + k_y\sigma_2 - i\partial_z\sigma_3) + m(z)\mu_3 \quad + h\sigma_3$$

m(z) ･･･ kink structure

Uniform ferromagnetic perturbation

An analytically solvable model



$$H = v\mu_1(k_x\sigma_1 + k_y\sigma_2 - i\partial_z\sigma_3) + m(z)\mu_3 + h\sigma_3$$

$$m(z) = m \tanh\left(\frac{z}{\xi}\right)$$

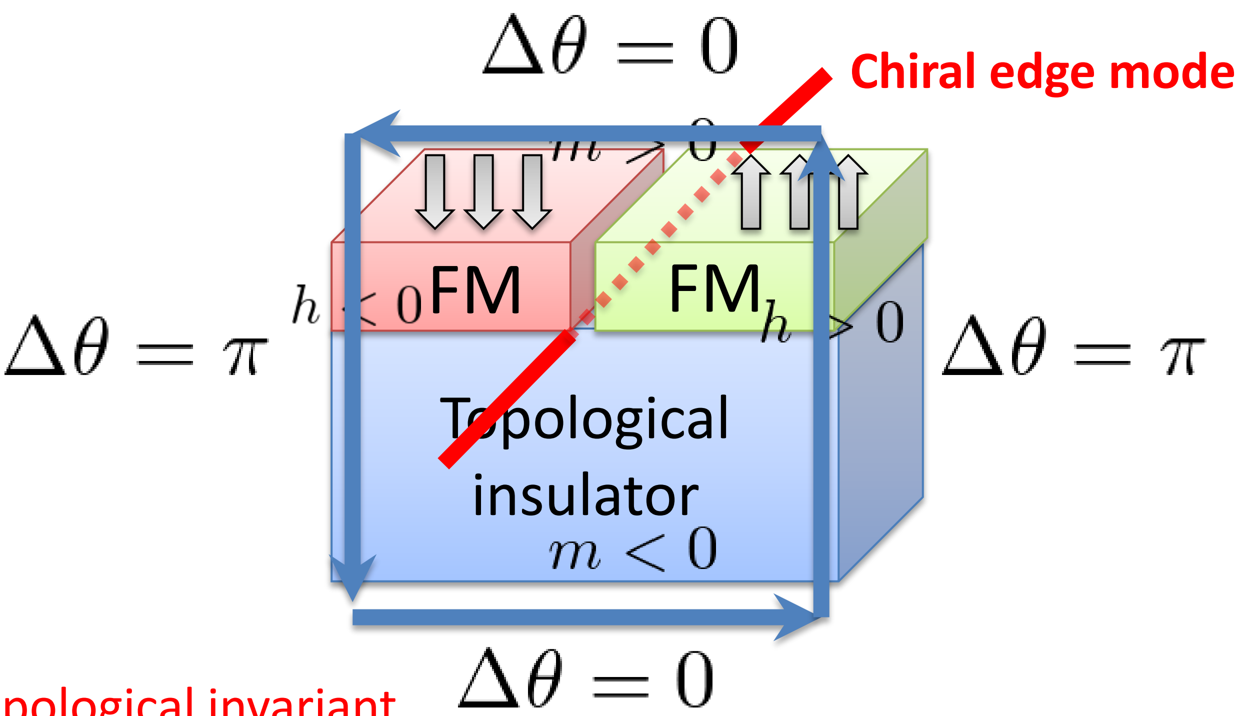
$$H = v\mu_1(k_x\sigma_1 + k_y\sigma_2 - i\partial_z\sigma_3) + m\tanh(z/\xi)\mu_3 + h\sigma_3$$

This Hamiltonian is **analytically solvable**

- polyacetylene [Takayama, Liu and Maki (1980)]
- Inhomogeneous Superconductor [Bar-Sagi and Kuper (1972)]

・・・ all energy eigenstates can be analytically represented

Numerical results ($\xi = v/m$)



Nontrivial topological invariant

$$N_5 = \frac{1}{2\pi}\oint_C d\theta(\boldsymbol{R}) = 1$$

⇨ The existence of the singularity

consistent with the existence of chiral edge mode