Quantum dynamics of intrinsic topological magnetic defects

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What are topological defects?

3D: Pyrochlore: monopoles
from classical Coulomb gas to quantum liquid
monopole condensates, Higgs transition

2D: Skyrmion Hall effect

1D: dynamic defects of vector chirality

Furukawa-Sato-SO, JPSJ77, 123712 (2008); PRL 105, 257205 (2010)
Furukawa-Sato-SO-Furusaki, Haldane-dimer phase in J1-J2 spin-1/2 chain.

Topological defects in D(=d+1)-dimensions

$\pi_n(M)$ Homotopy group

$\vec{n}(\mathbf{r},t): R^n = S^n \mapsto M$ [target (order parameter) space]

$S^D \mapsto M$: texture $(n = D)$ — Associated with a degenerate manifold of the symmetry broken states

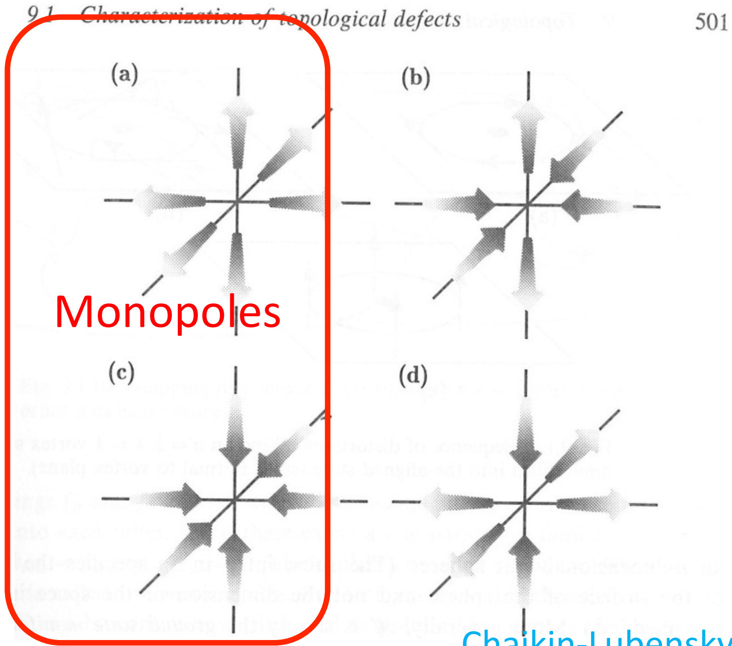
$S^{D-1} \mapsto M$: point defect $(n = D-1)$

$S^{D-2} \mapsto M$: line defect $(n = D-2)$

$S^{D-3} \mapsto M$: surface defect $(n = D-3)$

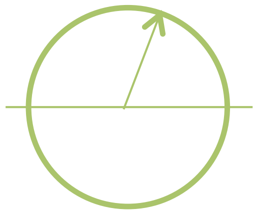
$S\uparrow D$:a D−dimensional sphere or a sphere in $(D+1)$ dimensions

Hedge hogs in 3d



Chaikin-Lubensky
Mermin

Ex. 3d finite-T superconductors
order parameter space: O(2)



$\vec{n}(\mathbf{r}): R^3 \mapsto M = \mathrm{S}^1$

$S^3 \mapsto S^1$: no texture

$S^{3-1} \mapsto S^1$: no point defect

$S^{3-2} \mapsto S^1$: **line defect (SC vortex)**, Z

$S^{3-3} \mapsto S^1$: no surface defect

Ex. 3d Heisenberg antiferromagnet

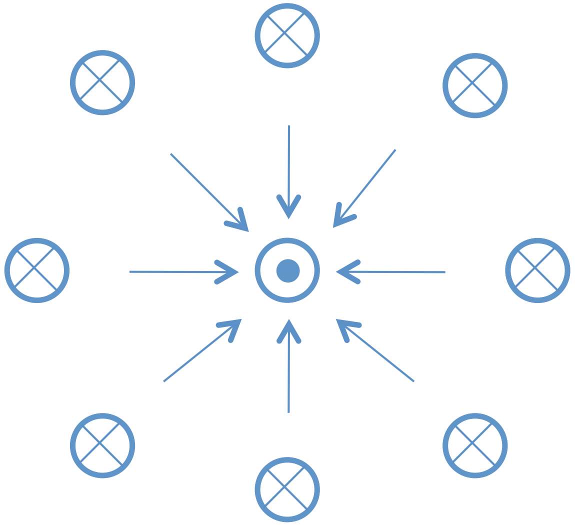
$S^3 \mapsto S^2$: no texture

$S^{3-1} \mapsto S^2$: **point defect (hedge hogs)**, Z

$S^{3-2} \mapsto S^2$: no line defect

$S^{3-3} \mapsto S^2$: no surface defect

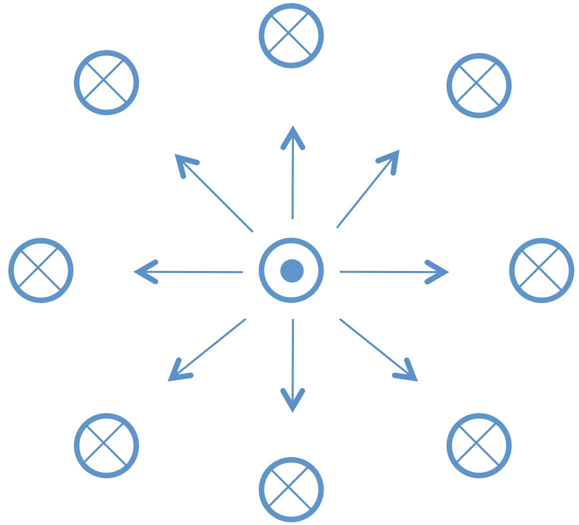
Skyrmion defects in 2d



$S^2 \mapsto S^2$: **defect texture,** Z

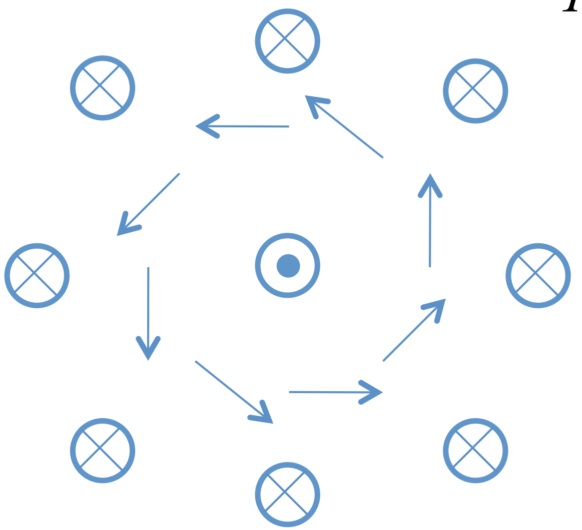
$S^{2-1} \mapsto S^2$: no point defect

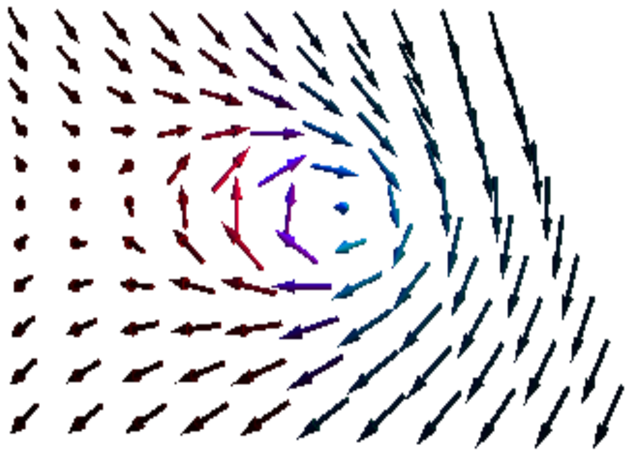
$S^{2-2} \mapsto S^2$: no line defect

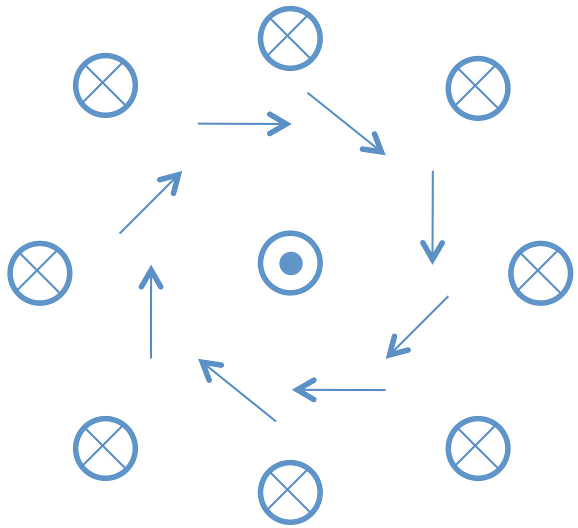


Single skyrmions

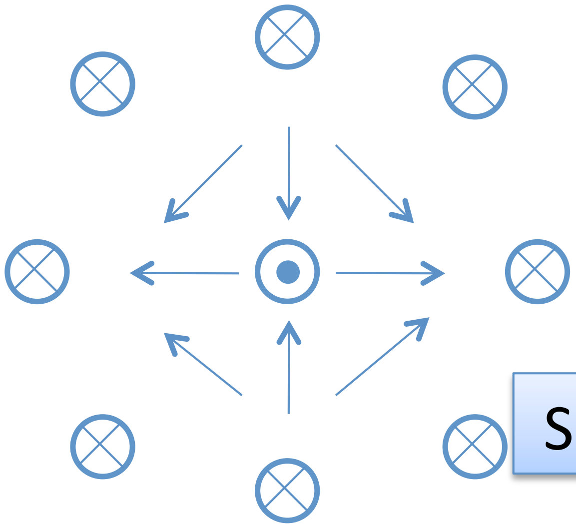
$$q = \frac{\varepsilon_{ij}}{4\pi} \int d^2\mathbf{r}\, \vec{S} \cdot \frac{\partial \vec{S}}{\partial r_i} \times \frac{\partial \vec{S}}{\partial r_j} = 1$$

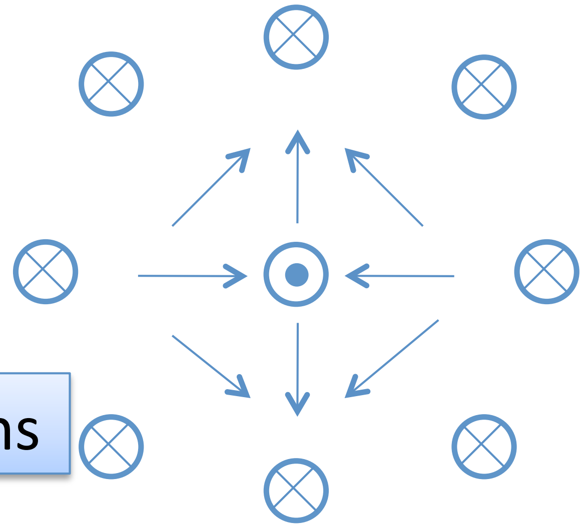






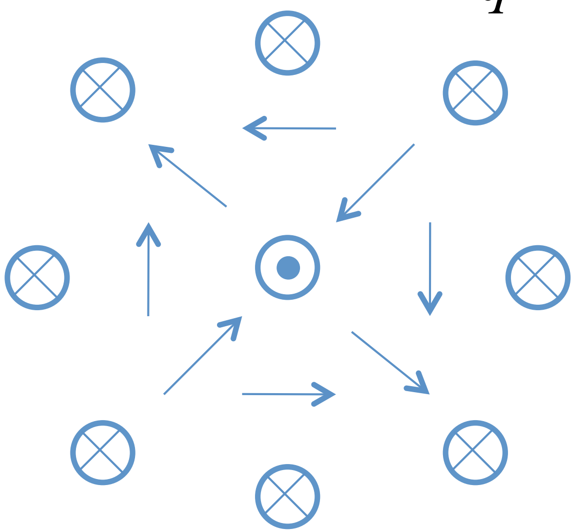
Anti-skyrmion defects in 2d

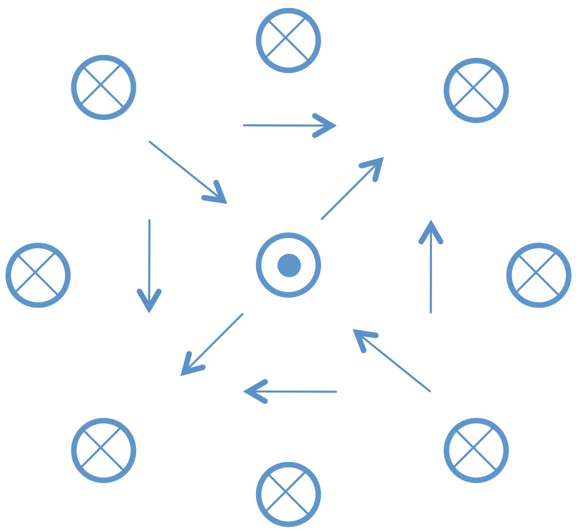




Single anti-skyrmions

$$q = \frac{\varepsilon_{ij}}{4\pi}\int d^2\mathbf{r}\,\vec{S}\cdot\frac{\partial\vec{S}}{\partial r_i}\times\frac{\partial\vec{S}}{\partial r_j} = -1$$





Collaborators on pyrochlore

Y. Machida, S. Nakatsuji, SO et al., Nature **463**, 210 (2010).
SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).
SO, J. Phys.: Conf. Series 320, 012065 (2011).
L.-J. Chang, SO et al., arXiv:1111.5406.
S.B. Lee, SO, L. Balents, unpublished.

Theory:

Y. Tanaka (RIKEN)

L. Balents, S.B. Lee (KITP, UCSB)

Y.-J. Kao (Natl. Taiwan Univ.)

Experiment:

L.-J. Chang (Cheng Kung Univ. in Taiwan)

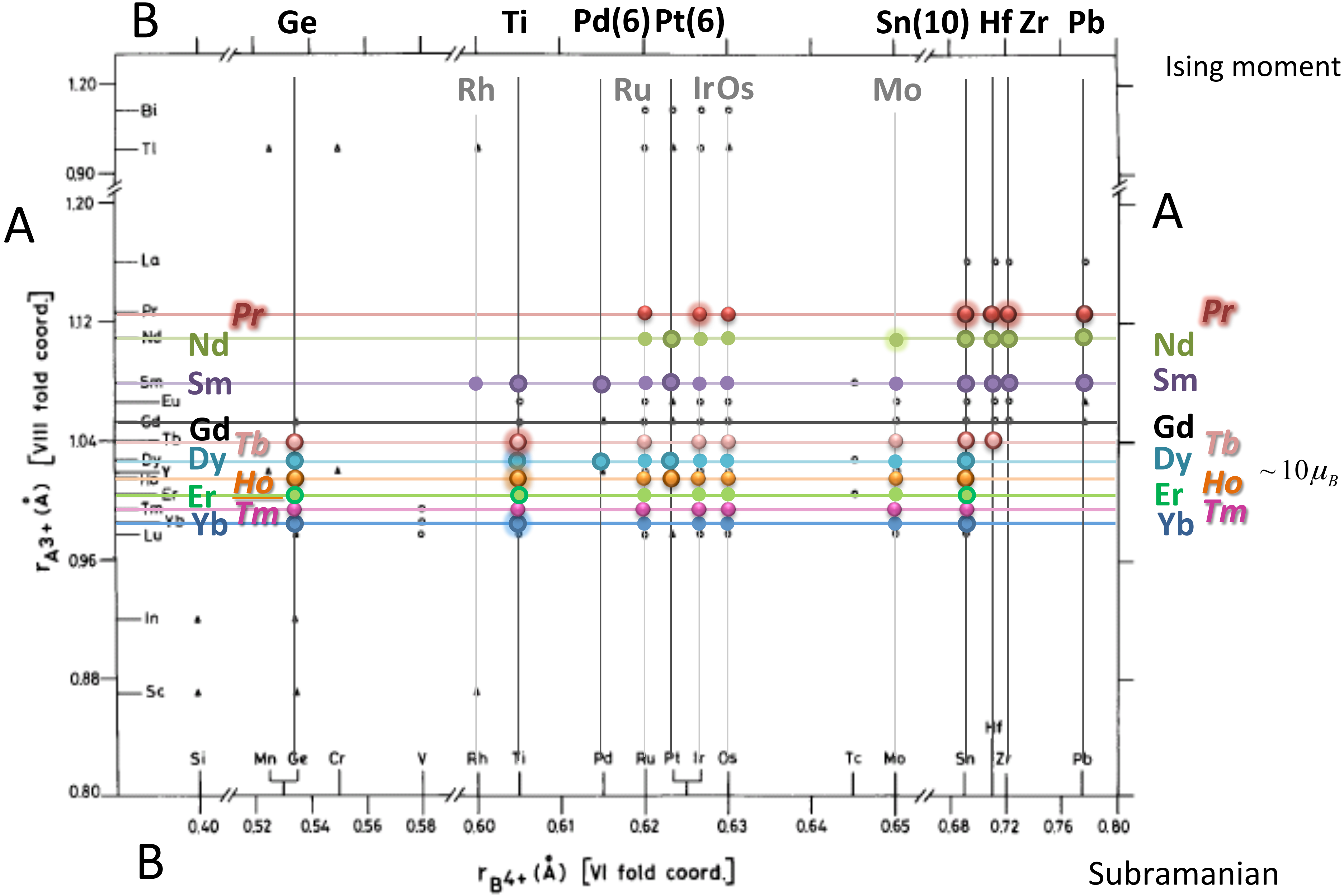
Y. Su (Julich Centre for Neutron Science)

Y. Yasui (Nagoya Univ),

K. Kakurai (JAEA)

M. R. Lees (Univ. of Warwick)

Magnetic pyrochlore oxides $A_2B_2O_7$



Subramanian

Comments:

Assumptions:

(i) A large amplitude of magnetic moments.

(ii) Spins obey the classical statistics.

$$\hat{H}_{\rm Ising} = -D_{\rm Ising} \sum_{\boldsymbol{r}} (\boldsymbol{n}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{J}}_{\boldsymbol{r}} / J)^2$$

→ This is taken to infinity!

$$\hat{H}_{\rm D} = \frac{\mu_0}{4\pi} \sum_{\langle \boldsymbol{r}, \boldsymbol{r}' \rangle} \left[\frac{\hat{\boldsymbol{m}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{m}}_{\boldsymbol{r}'}}{(\Delta r)^3} - 3 \frac{(\hat{\boldsymbol{m}}_{\boldsymbol{r}} \cdot \Delta \boldsymbol{r})(\Delta \boldsymbol{r} \cdot \hat{\boldsymbol{m}}_{\boldsymbol{r}'})}{(\Delta r)^5} \right]$$

$$\hat{H}_{\rm H} = -3 J_{\rm n.n.} \sum_{\langle \boldsymbol{r}, \boldsymbol{r}' \rangle}^{\rm n.n.} \hat{\boldsymbol{J}}_{\boldsymbol{r}} \cdot \hat{\boldsymbol{J}}_{\boldsymbol{r}'} / J^2.$$

When we have a relatively smaller amplitude of moments and/or a D_{3d} crystalline electric field, these assumptions do not hold in general.

$Tb_2\mathit{TM}_2O_7$, $Pr_2\mathit{TM}_2O_7$, $Yb_2\mathit{TM}_2O_7$ *(TM=Ti, Zr, Sn, Hf, Ir, …)*

Compact U(1) gauge theory

Quantum pseudospin-1/2 Hamiltonian

Hermele-Fisher-Balents, PRB 69, 64404

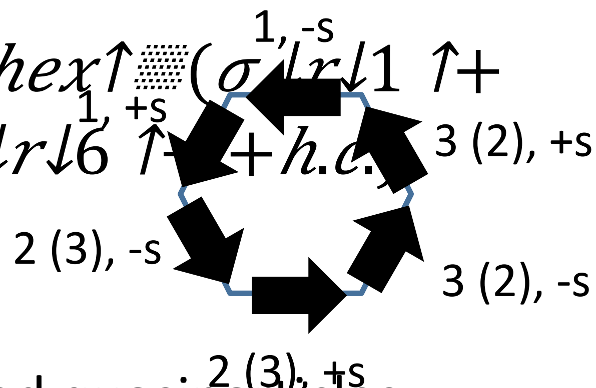
$$H = J_{n.n.} \sum_{\langle r,r' \rangle}^{n.n.} \left[g_{\parallel} \sigma_r^z \sigma_{r'}^z + g_{\perp} \left(\sigma_r^x \sigma_{r'}^x + \sigma_r^y \sigma_{r'}^y \right) \right]$$

1. Assume $J_{n.n.} > 0$, $g_{\parallel} > 0$.
2. Start from degenerate spin-ice ground states
3. 3rd-order perturbation in $g_{\perp}$ →

$$H_{Ring} = 12 J_{n.n.} (2 g_{\perp})^3 / (4 g_{\parallel})^2 \sum_{hex} \left(\sigma_{r_1}^{+} \sigma_{r_2}^{-} \sigma_{r_3}^{+} \sigma_{r_4}^{-} \sigma_{r_5}^{+} \sigma_{r_6}^{-} + h.c. \right)$$

$$\sigma_r^{\pm} = (\sigma_r^x \pm i \sigma_r^y)/2$$

6×2 ways



pi-flux ($g_{\perp} > 0$)

Magnetic monopoles: well-defined deconfined quasiparticles.

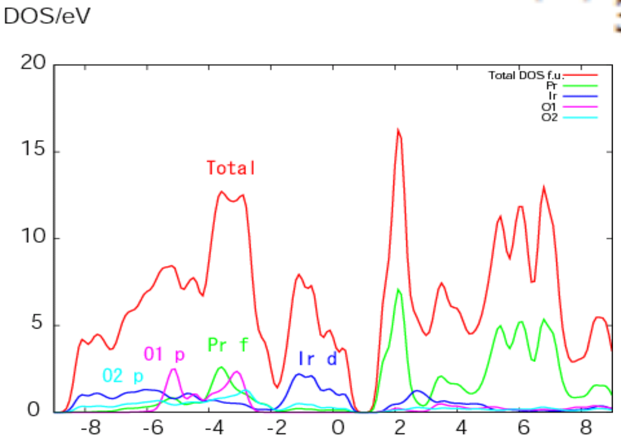
However, the model might be oversimplified.

What is an effective model for Quantum spin ice systems?

i) Pr case

ii) Yb case

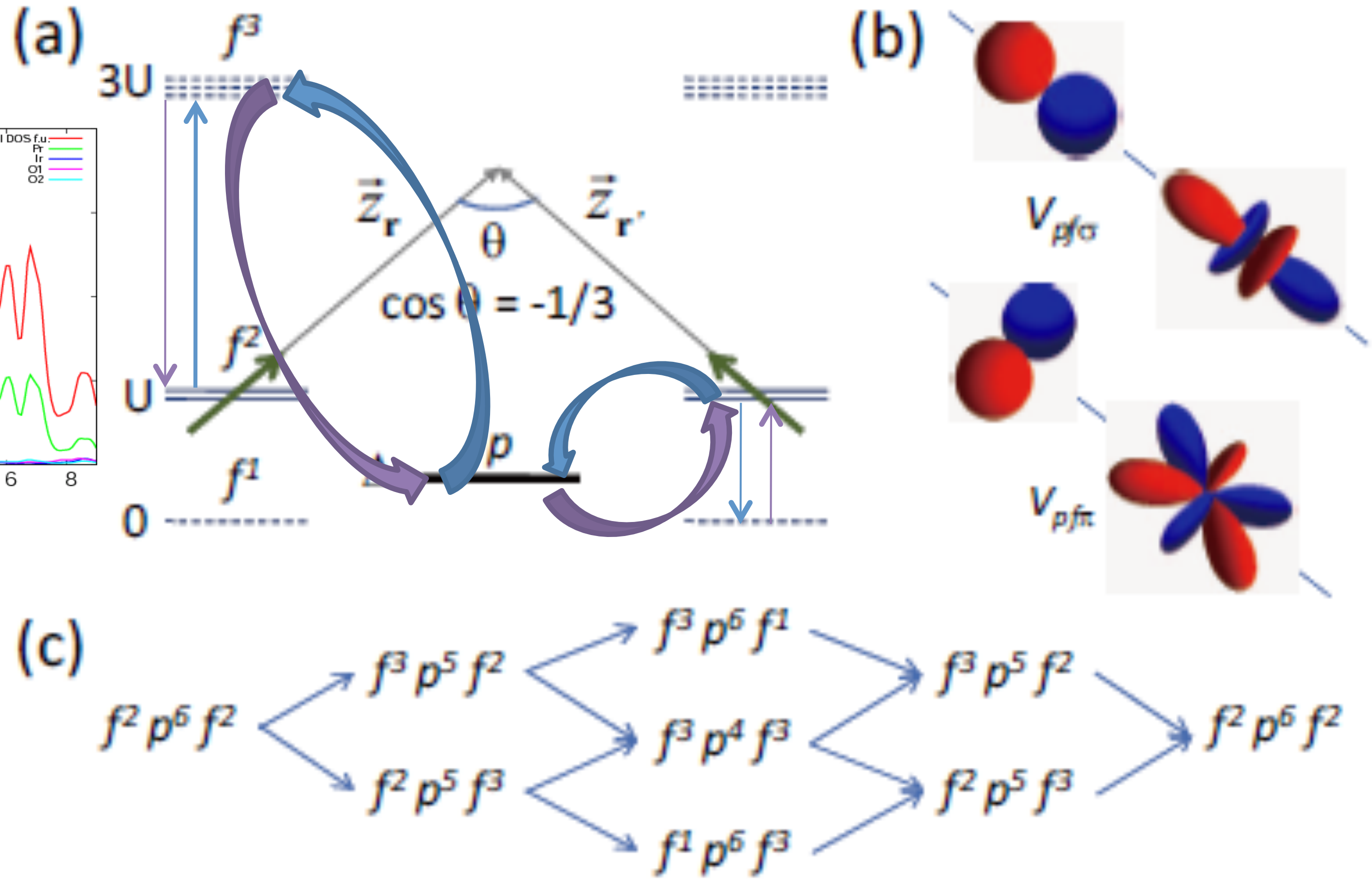
Superexchange Hamiltonian (Pr)



LDA+U calc.

Local level scheme

2 transfer integrals



4th-order perturbation expansion of transfer integrals

(d) Projection of the superexchange Hamiltonian
onto the subspace of local ground doublets

SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).

Effective quantum pseudospin-1/2 Hamiltonian (Pr)

$$H_{eff} = J\sum_{\langle r,r'\rangle}^{\text{n.n}}\left[4s_r^z s_{r'}^z + 2\delta\left(s_r^+ s_{r'}^- + s_r^- s_{r'}^+\right) + 2q\left(e^{i2\varphi_{r,r'}}\sigma_r^+\sigma_{r'}^+ + \text{h.c.}\right)\right]$$

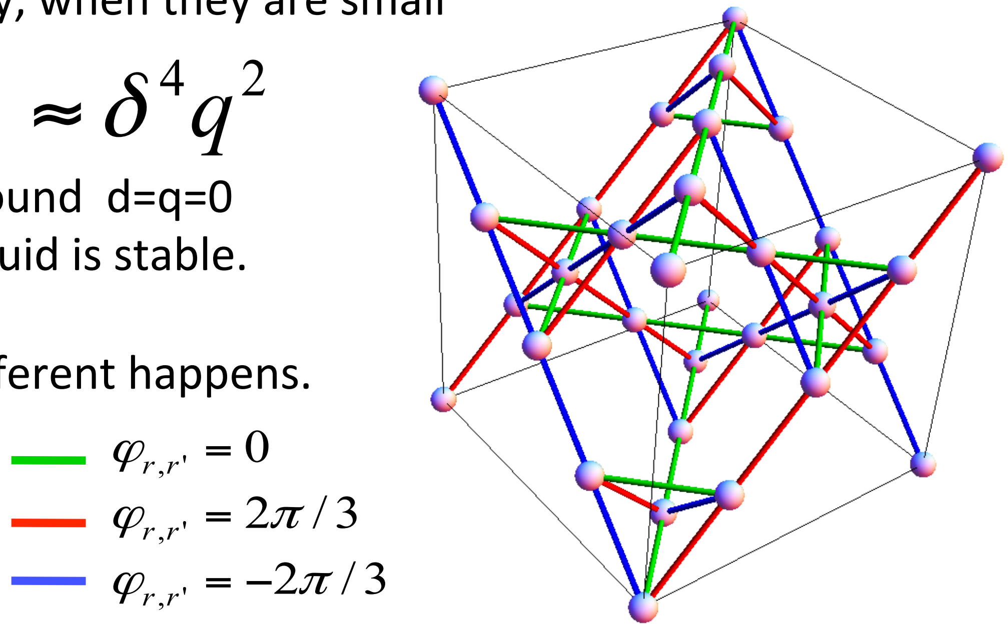
Ising   *exchange*   *double spin-flip*

Partially lift the degeneracy of the ice manifold
in degenerate perturbation theory, when they are small

Ring exchanges $\approx \delta^3$ $\approx \delta^4 q^2$

There would be a finite region around d=q=0
where the spin ice or U(1) spin liquid is stable.

But, they are large, something different happens.



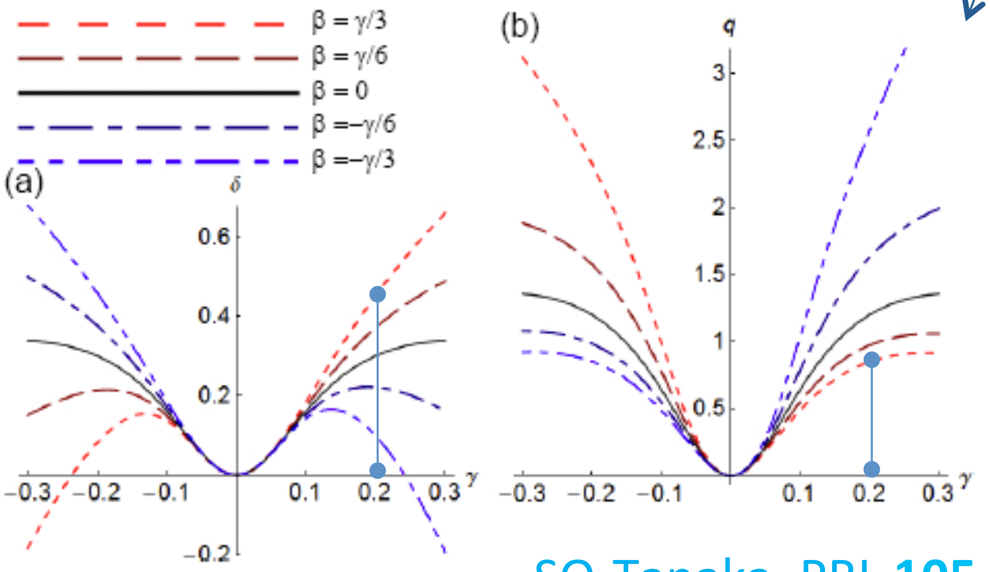
SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).

Effective quantum psuedospin-1/2 Hamiltonian (Pr)

$$H_{eff} = J\sum_{\langle r,r'\rangle}^{\mathrm{n.n}}\left[4s_r^z s_{r'}^z + 2\delta\left(s_r^+ s_{r'}^- + s_r^- s_{r'}^+\right) + 2q\left(e^{i2\varphi_{r,r'}} s_r^+ s_{r'}^+ + \mathrm{h.c.}\right)\right]$$

$$J_{\mathrm{n.n.}} = \frac{V_{pf\sigma}^4}{(2U-\Delta)^2}\left(\frac{1}{U}+\frac{1}{2U-\Delta}\right)\tilde{J}(\beta,\gamma,V_{pf\pi}/V_{pf\sigma})$$

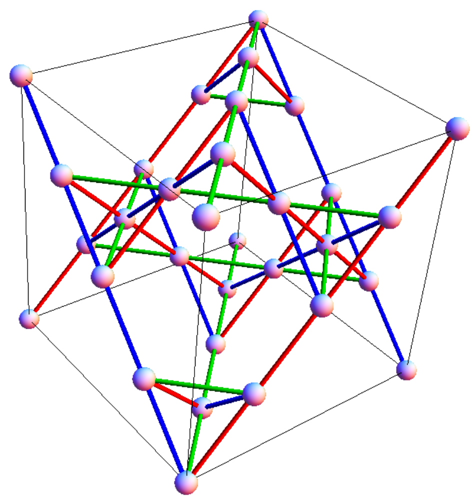
$J_{\mathrm{n.n.}} > 0$ for small $V_{pf\pi}/V_{pf\sigma} < 0$



$\varphi_{r,r'} = 0$

$\varphi_{r,r'} = 2\pi/3$

$\varphi_{r,r'} = -2\pi/3$



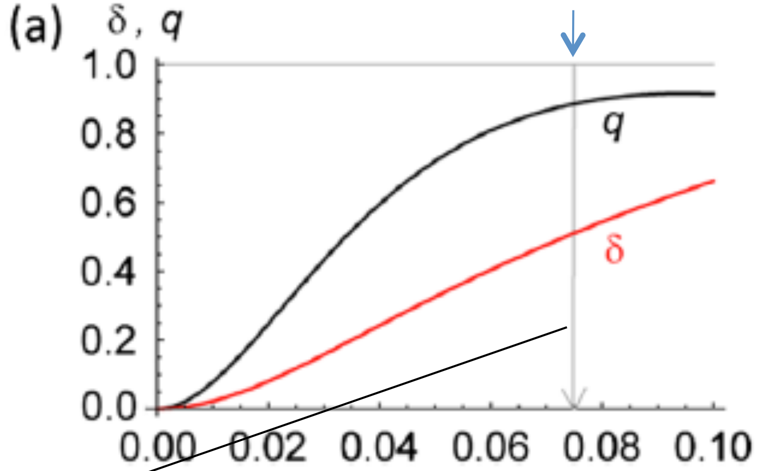
SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).

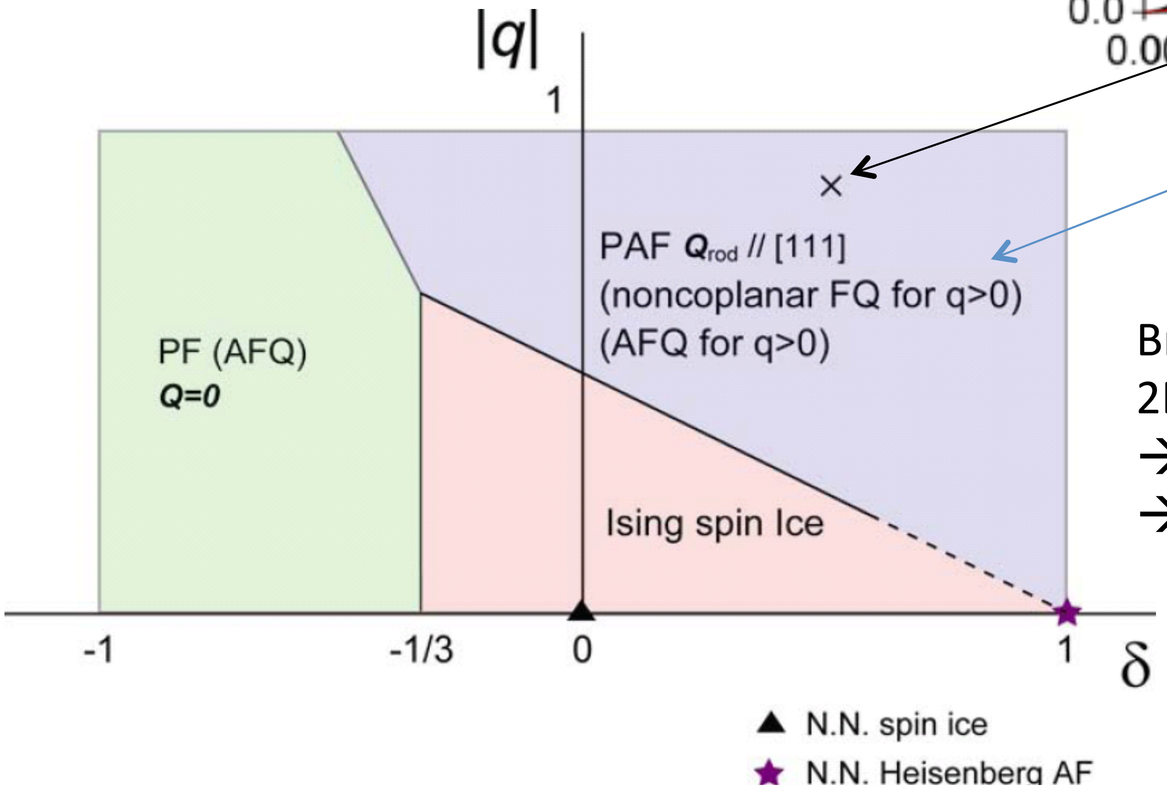
Classical mean-field theory

c.f. Reimers

Instability at finite T.

Pr2Ir2O7





Degeneracy at q=(hhh)

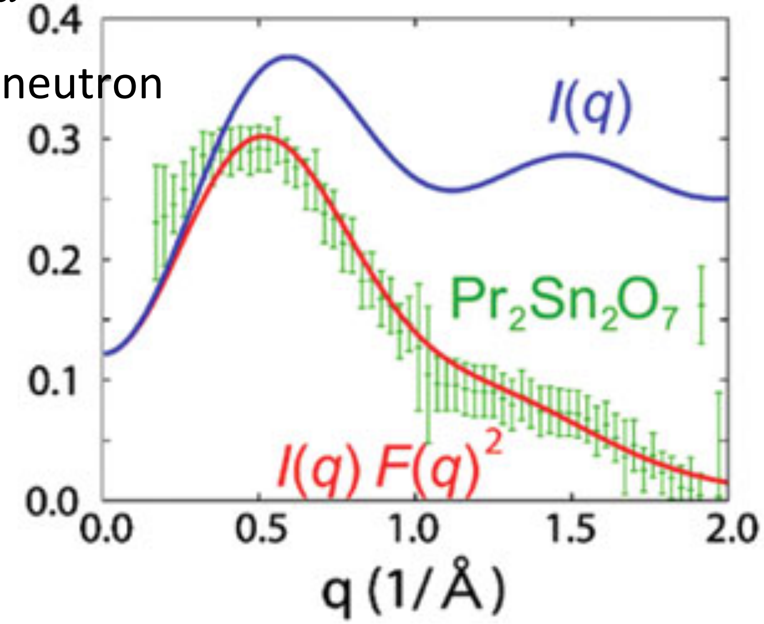
Bragg rod
2D Ferroquadrupole order
→ 3D order by disorder?
→ Possible relevance to Yb2Ti2O7

Neutron scattering profile

$$\frac{S(\vec{q})}{M_0^2} = \frac{1}{N}\sum_{r,r'}\sum_{i,j}\left(\delta_{i,j} - \frac{q_i q_j}{|\vec{q}|^2}\right) n_r^i\, n_{r'}^j \left\langle \sigma_r^z \sigma_{r'}^z \right\rangle_{\text{ave}} e^{i\vec{q}\cdot(r-r')}$$

$$\vec{q} = \frac{2\pi}{a}(hhl),\; M_0 = g_J \mu_B \left(4\alpha^2 + \beta^2 - 2\gamma^2\right)$$

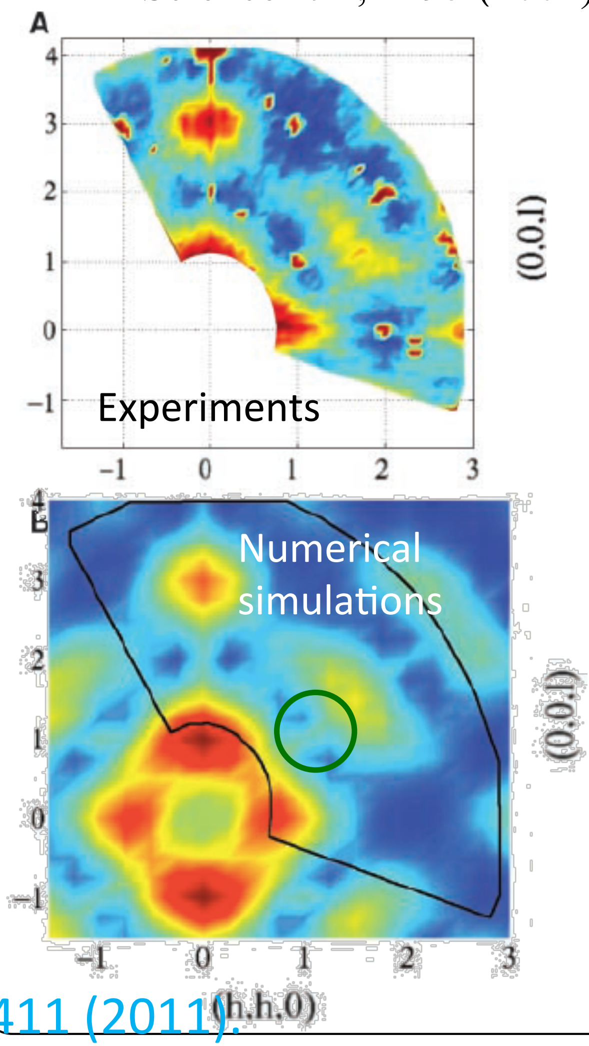
Powder neutron



Dipolar spin ice

S.T. Bramwell and M.J.P. Gingras

Science **294**, 1495 (2001)



Structures around $|q|$~0.5 $Å^{-1}$ and 1.5 $Å^{-1}$

Consistent with exp. on $Pr_2Sn_2O_7$ (Zhou et al.)

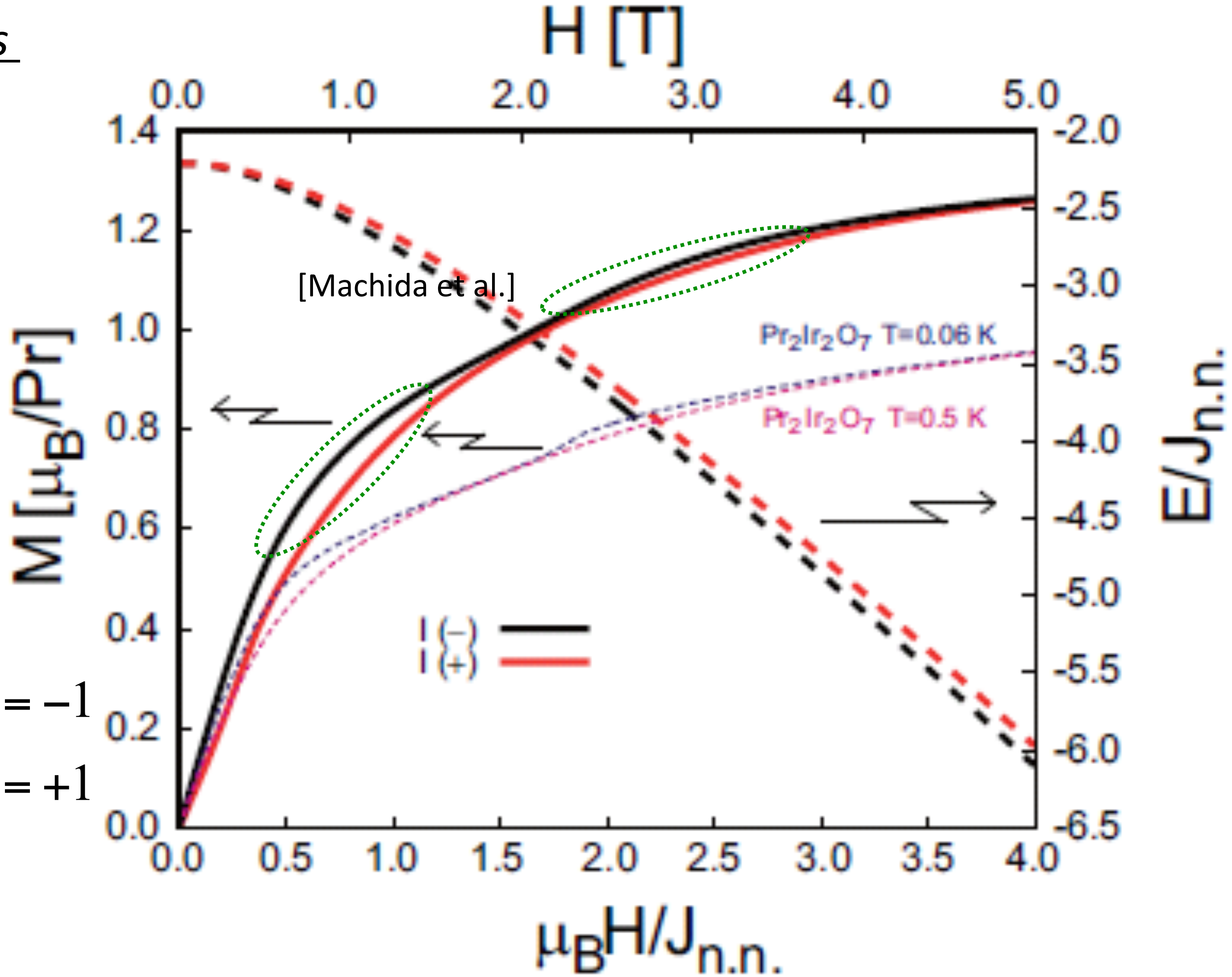
Pinch point singularity should be broadened because of violation of ice rule

SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).

Magnetization curve

Numerical results

$H // \langle 111 \rangle$



Ground state ➡ $\tau_I = -1$

Excited state ➡ $\tau_I = +1$

SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).

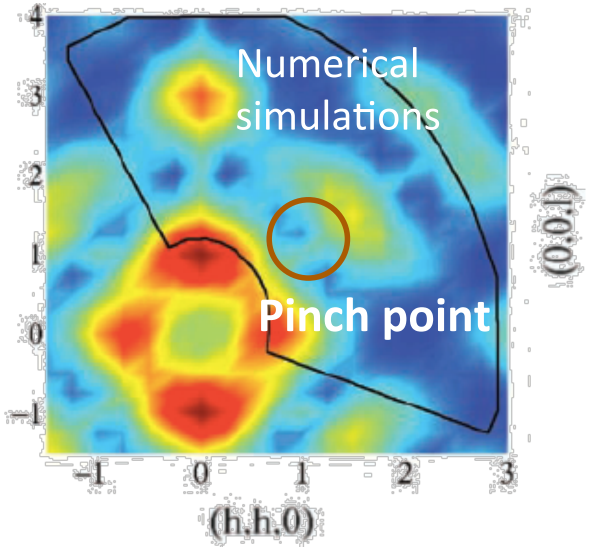
Neutron-scattering profile

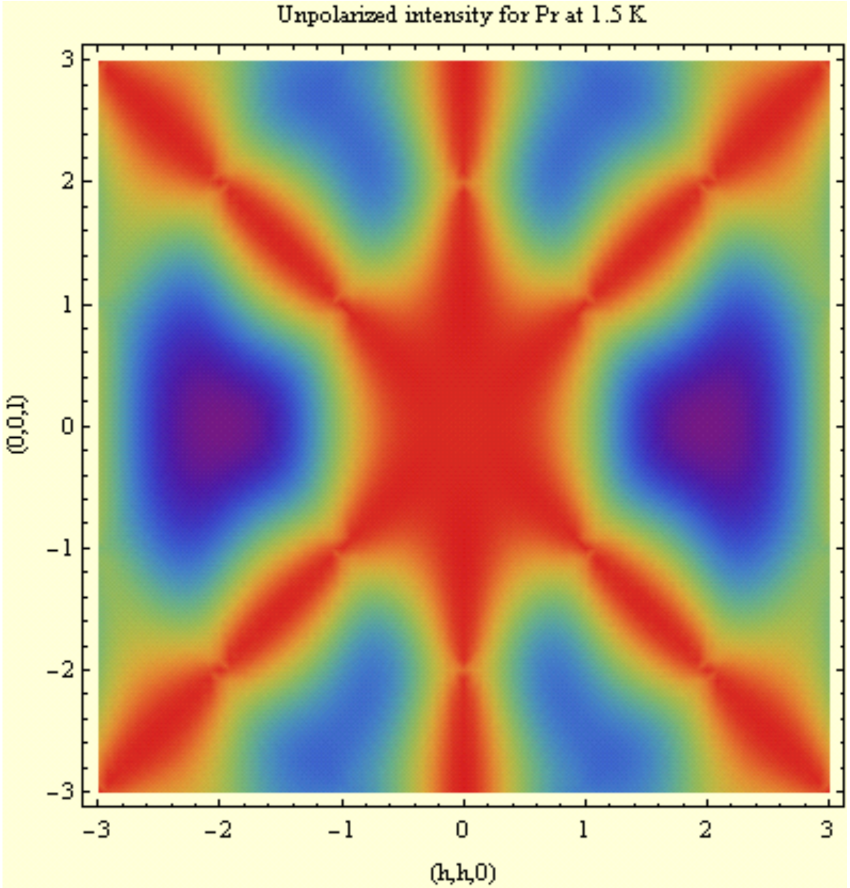
- Pinch points broadened in the energy-integrated profile
- Magnetic coulomb liquid

- For exchange parameters

for $Pr_2Zr_2O_7$

c.f. classical dipolar spin ice





Is Yb2Ti2O7 an XY pyrochlore?

Hodges et al. 2002

Mossbauer and muon spin relaxation spectroscopies:

Local Yb ions → Jz=1/2 doublet

$$|\sigma\rangle_D = -\alpha\sigma|J_z = \frac{7}{2}\sigma\rangle + \beta|J_z = \frac{1}{2}\sigma\rangle + \gamma\sigma|J_z = -\frac{5}{2}\sigma\rangle$$

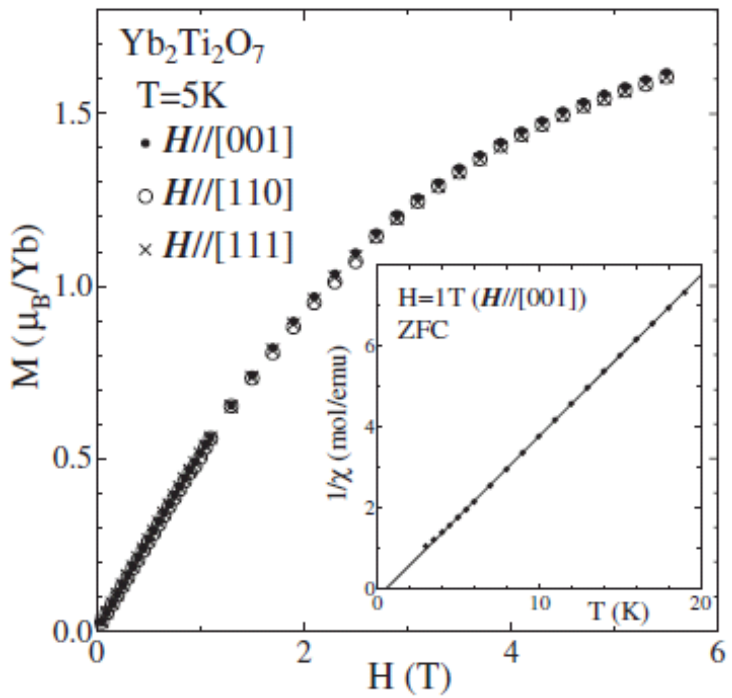
$$\alpha \approx 0.388, \ \beta \approx 0.889, \ \text{and} \ \gamma \approx 0.242$$

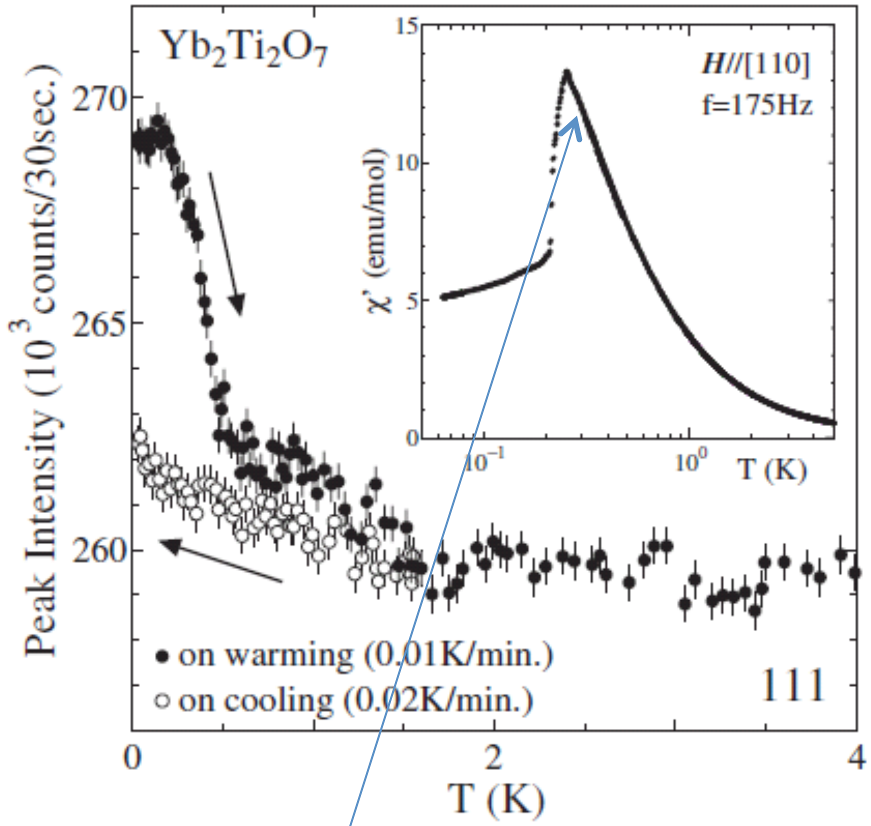
Blotte et al. 1969

1st-order phase transition at T ~ 0.24 K (specific heat)

Evidence of the 1st–order phase transition at ~0.24 K (Kramers case of Yb2Ti2O7)

Yasui et al. JPSJ (2003)





Anomaly in the specific heat [Blote et al. 1969]

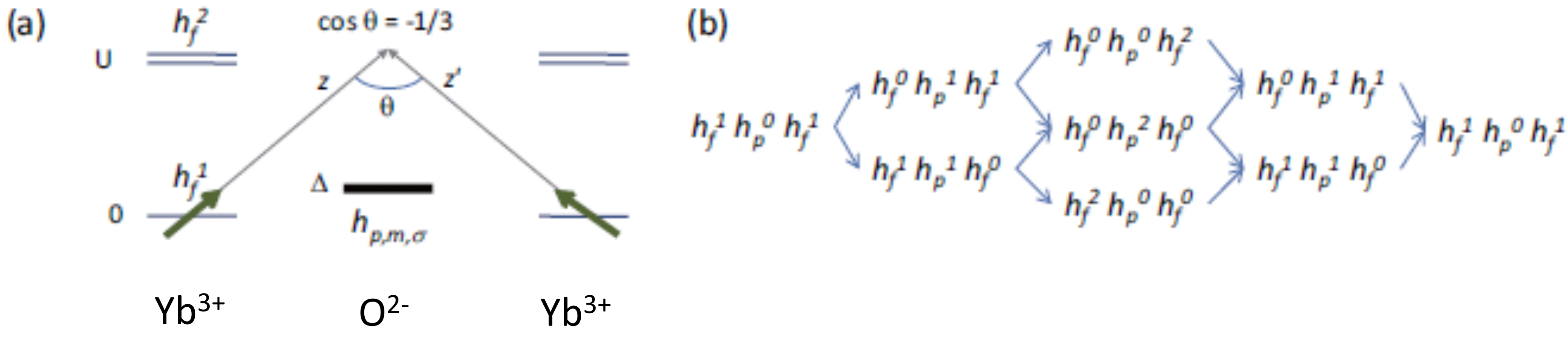
Effective model for Yb pyrochlore

Yb^{3+} 13 4f-electorns or a single 4f-hole

Crystal-field ground-state Kramers doublet:

$$|\sigma\rangle_D = -\alpha\sigma|J_z = \frac{7}{2}\sigma\rangle + \beta|J_z = \frac{1}{2}\sigma\rangle + \gamma\sigma|J_z = -\frac{5}{2}\sigma\rangle$$

Superexchange interaction must be seriously included!



SO, arXiv: 1101.1230

Enhanced quantum fluctuations in otherwise classical spin ice (Kramers case of Yb3+)

$$\hat{\boldsymbol{m}}_{\boldsymbol{r}} = g_J \mu_B \hat{\boldsymbol{J}}_{\boldsymbol{r}} = \frac{1}{2}\mu_B \left[g_\perp \left(\hat{\sigma}^x_{\boldsymbol{r}} \boldsymbol{x}_{\boldsymbol{r}} + \hat{\sigma}^y_{\boldsymbol{r}} \boldsymbol{y}_{\boldsymbol{r}} \right) + g_\parallel \hat{\sigma}^z_{\boldsymbol{r}} \boldsymbol{z}_{\boldsymbol{r}} \right]$$

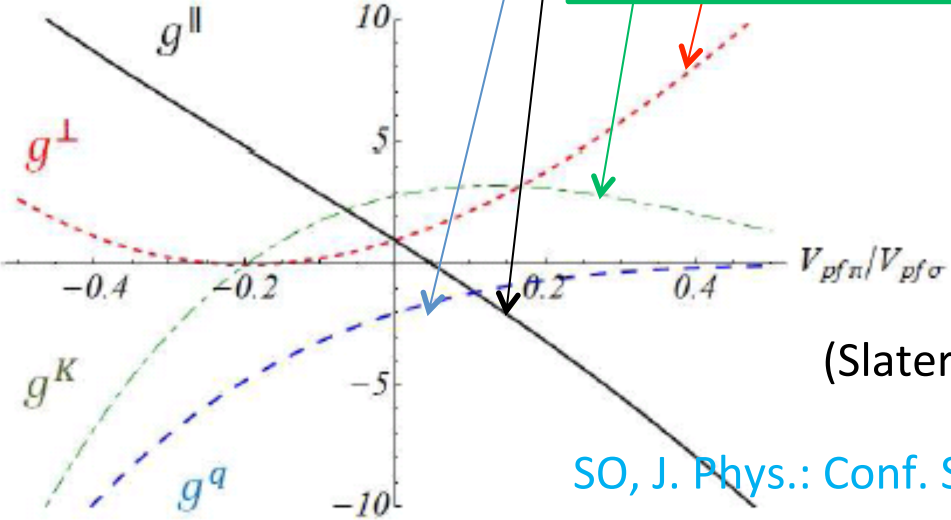
Hodges et al.

$g_\perp$ ~4.2 $g_\parallel$ ~1.8

$$\hat{H}_{ex} = -J_{n.n.} \sum_{\langle \boldsymbol{r},\boldsymbol{r}' \rangle}^{n.n.} \Big[g^\parallel \hat{\sigma}^z_{\boldsymbol{r}} \hat{\sigma}^z_{\boldsymbol{r}'} + g^\perp \left(\hat{\sigma}^x_{\boldsymbol{r}} \hat{\sigma}^x_{\boldsymbol{r}'} + \hat{\sigma}^y_{\boldsymbol{r}} \hat{\sigma}^y_{\boldsymbol{r}'} \right)$$
$$+ g^q \left(\left(\hat{\vec{\sigma}}_{\boldsymbol{r}} \cdot \vec{n}_{\boldsymbol{r}} \right) \left(\hat{\vec{\sigma}}_{\boldsymbol{r}'} \cdot \vec{n}_{\boldsymbol{r}'} \right) - \left(\hat{\vec{\sigma}}_{\boldsymbol{r}} \cdot \vec{n}'_{\boldsymbol{r},\boldsymbol{r}'} \right) \left(\hat{\vec{\sigma}}_{\boldsymbol{r}'} \cdot \vec{n}'_{\boldsymbol{r},\boldsymbol{r}'} \right) \right)$$
$$+ g^K \left(\hat{\sigma}^z_{\boldsymbol{r}} \left(\hat{\vec{\sigma}}_{\boldsymbol{r}'} \cdot \vec{n}_{\boldsymbol{r},\boldsymbol{r}'} \right) + \left(\hat{\vec{\sigma}}_{\boldsymbol{r}} \cdot \vec{n}_{\boldsymbol{r},\boldsymbol{r}'} \right) \hat{\sigma}^z_{\boldsymbol{r}'} \right) \Big],$$

$$2(e^{2i\phi_{\mathbf{r},\mathbf{r}'}} \hat{\sigma}^+_{\mathbf{r}} \hat{\sigma}^+_{\mathbf{r}'} + h.c.)$$

$$2(e^{i\phi_{\mathbf{r},\mathbf{r}'}} \hat{\sigma}^+_{\mathbf{r}} \hat{\sigma}^z_{\mathbf{r}'} + h.c.)$$



xy plane $n'_{r,r'}$

$n_{r,r'}$

(Slater-Koster parameters)

120°rotation depending on the bond

SO, J. Phys.: Conf. Series 320, 012065

RPA calculation (Gingras group)

Dipole-dipole interactions originating from

i) the magnetic dipolar interaction

ii) the nearest-neighbor Heisenberg exchange interaction

Fitting with diffuse elastic neutron scattering

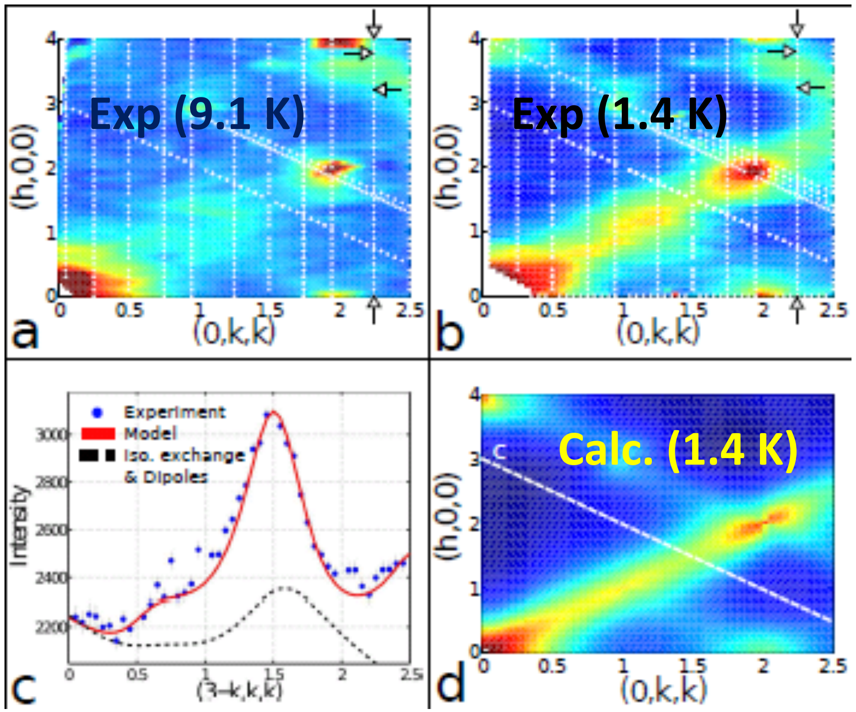
Thompson et al.

Estimated values of coupling constants

c.f. Spin-wave at high field

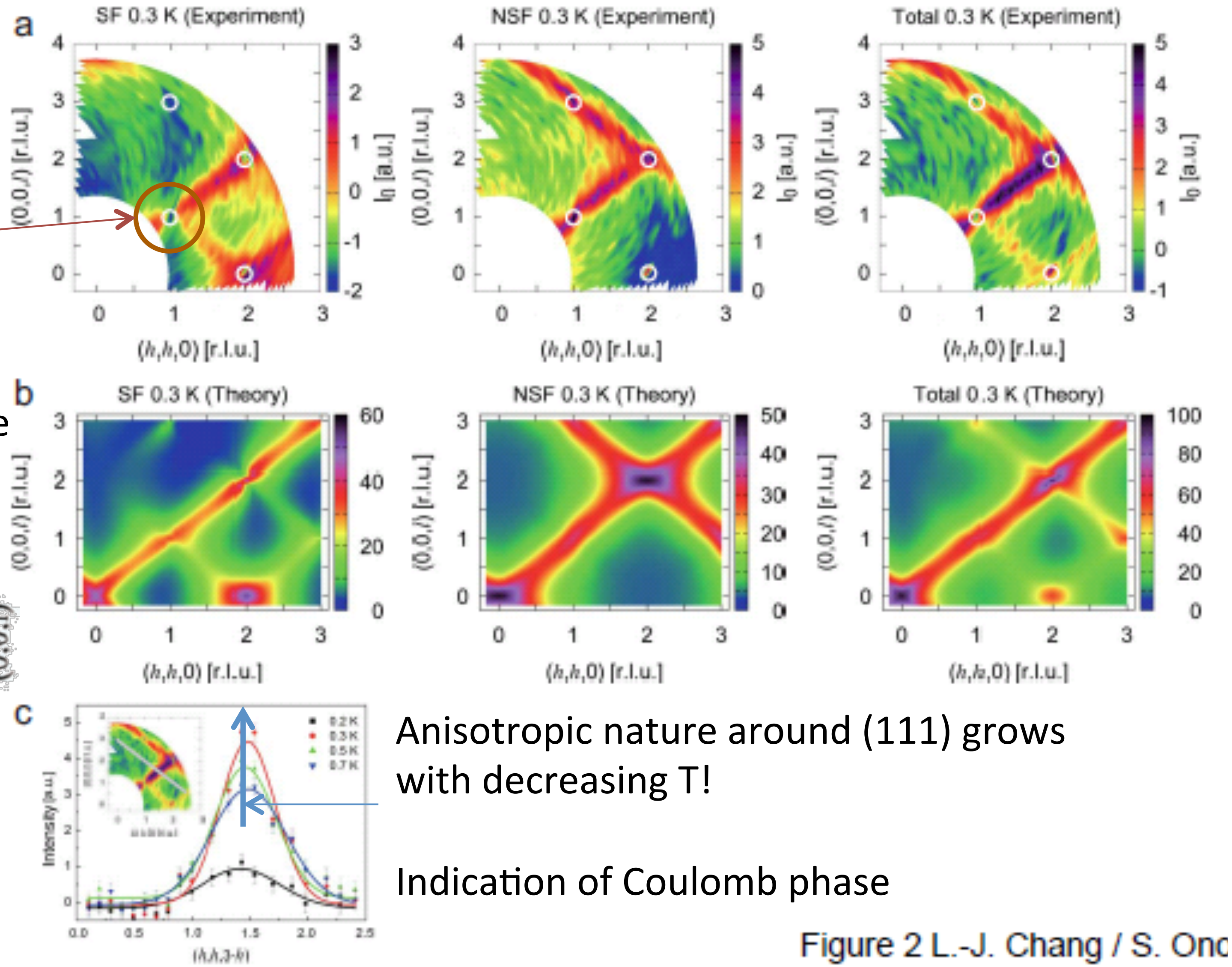
$J_{n.n} \sim 0.064\text{K}$	$J_{\text{n.n.}} \sim 0.04\text{ K}$
$g^{\parallel} = -1$	$g^{\parallel} = -1$
$g^{\perp} \sim 0.73$	$g^{\perp} \sim 0.3$
$g^{q} \sim -0.14$	$g^{q} \sim -0.3$
$g^{K} \sim -2.42$	$g^{K} \sim -0.8$
Thompson et al.	Ross-Savary-Gaulin-Gardner-Balents

Discrepancy

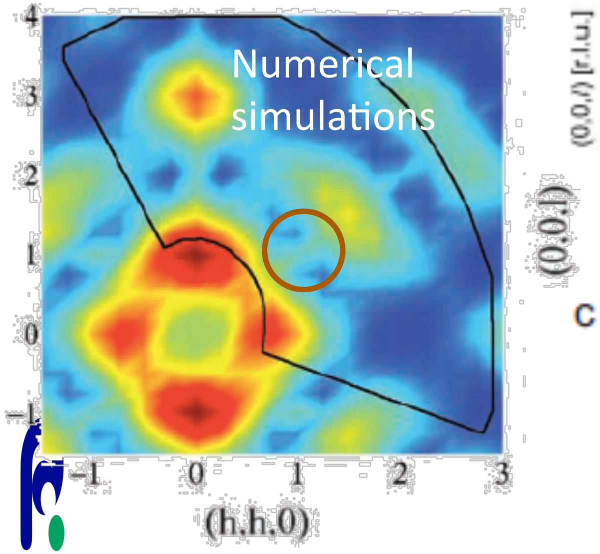


New results from polarized neutron scattering and RPA

a remnant of
pinch-point singularity



c.f. classical dipolar spin ice



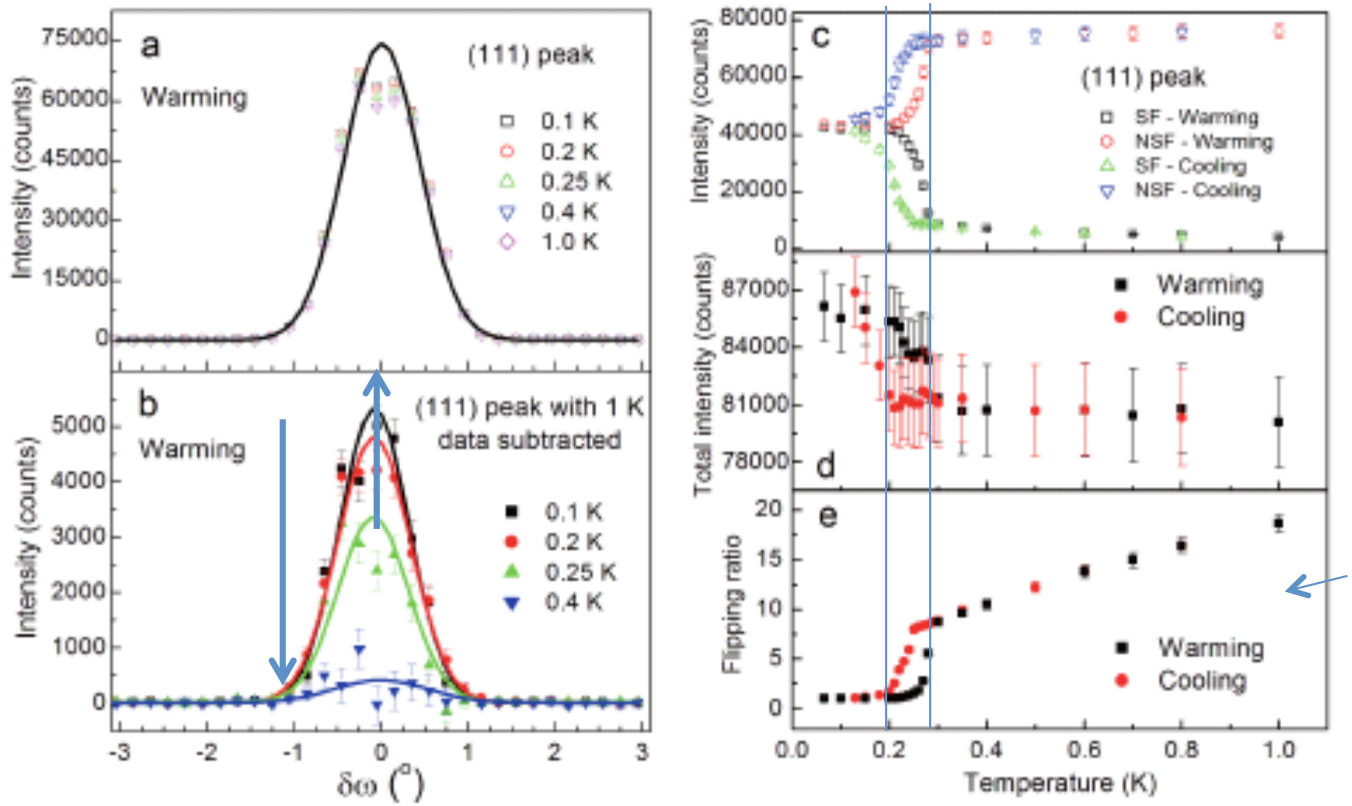
Anisotropic nature around (111) grows with decreasing T!

Indication of Coulomb phase

Figure 2 L.-J. Chang / S. Onc

Evidence of first-order ferromagnetic transition

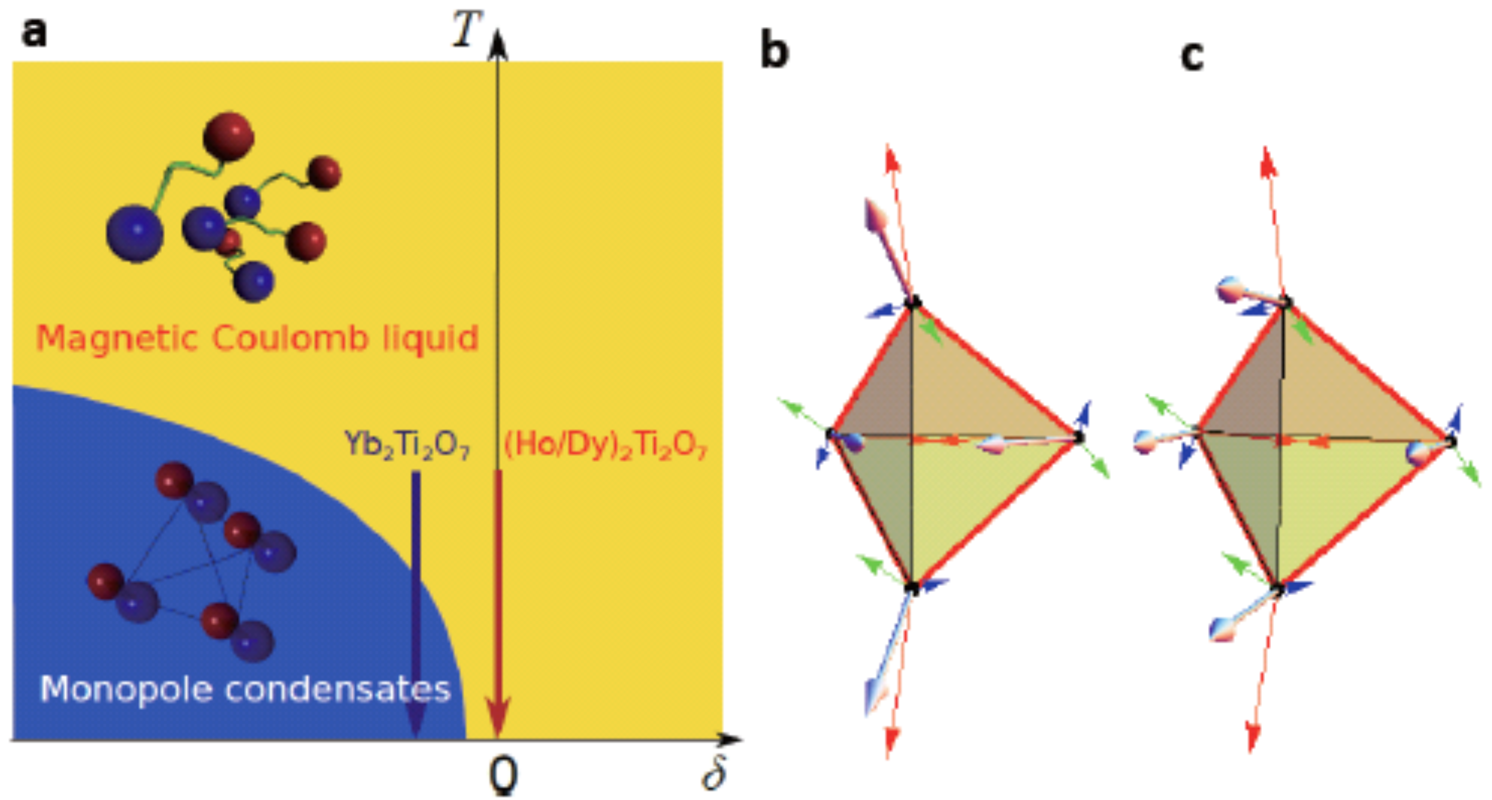
Polarized neutron-scattering intensity and neutron-spin flipping ratio showing thermal hysteresis



Spin flipping ratio: NSF/SF at (111)

Phase diagram and the hypothetical magnetic structure

Ground state in the mean-field approximation



Pseudospin structure

Magnetic structure
Nearly collinear ferromagnet
M//[100]

Towards compact U(1) gauge theory

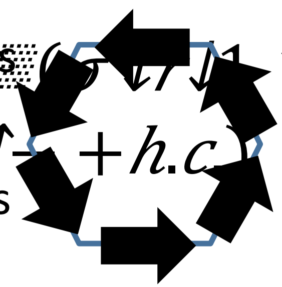
Quantum pseudospin-1/2 Hamiltonian
(PRL105, 047201; arXiv:1011.4981, arXiv:1101.1230)

$$H = J_{n.n.} \sum_{\langle r,r' \rangle}^{n.n.} \left[g^{\parallel} \sigma_r^z \sigma_{r'}^z + g^{\perp} (\sigma_r^x \sigma_{r'}^x + \sigma_r^y \sigma_{r'}^y) + g^q \{ (\sigma_r \cdot n_{r,r'})(\sigma_{r'} \cdot n_{r,r'}) - (\sigma_r \cdot n_{r,r'})(\sigma_{r'} \cdot n_{r,r'}) \} + g^K \{ \sigma_r^z (\sigma_{r'} \cdot n_{r,r'}) + (\sigma_r \cdot n_{r,r'}) \sigma_{r'}^z \} \right]$$

1. Assume $J_{n.n.} > 0$, $g^{\parallel} > 0$.
2. Start from degenerate spin-ice ground states
3. 3rd-order perturbation in $g^{\perp}$ →

$$H_{Ring} = 12 J_{n.n.} (2g^{\perp})^3 / (4g^{\parallel})^2 \sum_{hex} (\sigma_{r_1}^{+} \sigma_{r_2}^{-} \sigma_{r_3}^{+} \sigma_{r_4}^{-} \sigma_{r_5}^{+} \sigma_{r_6}^{-} + h.c.)$$

$$\sigma_r^{\pm} = (\sigma_r^x \pm i \sigma_r^y)/2$$



(Hermele-Fisher-Balents, PRB 69, 64404)

Rotor representation of Pauli matrices

$\sigma_{r_i}^{z} = 2(n_{r_i} - 1/2)$

$\sigma_{r_i}^{\pm} = (\sigma_{r_i}^{x} \pm i\sigma_{r_i}^{y})/2 = e^{\pm i\phi_{r_i}}$

Constraint of hard-core bosons : $n_{r_i} = 0,1$

Commutation relation:

$[\phi_{r_i}, n_{r_j}] = i\delta_{r_i, r_j}$

(Hermele-Fisher-Balents, PRB 69, 64404)

Electromagnetic charge (monopole) / fields

$Q_{R_\pm} = \boldsymbol{divE} = \pm\sum_{i=0}^{3} (n_{R_\pm \pm a_i} - 1/2)$

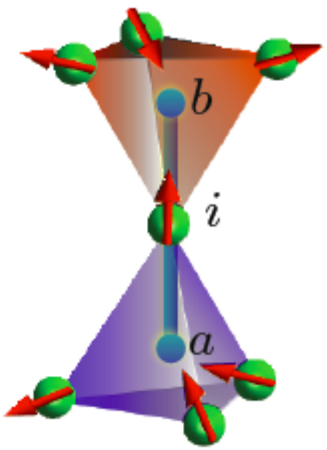
$\boldsymbol{E}_{R_\pm \pm a_i} = \pm \boldsymbol{a}_i/|\boldsymbol{a}_i| \, (n_{R_\pm \pm a_i} - 1/2)$

or $E_{R_\pm, R_\pm \pm 2a_i} = \pm (n_{R_\pm \pm a_i} - 1/2)$

$A_{R_\pm, R_\pm \pm 2a_i} = \pm \phi_{R_\pm \pm a_i}$

$R_\pm$: +, - FCC sublattice of the diamond lattice

Compact U(1) gauge theory coupled to charged bosons



$$S_i^z = \eta_a E_{ab}$$

$$\eta_a = \pm 1[a \in A(B)]$$

$$S_i^+ = \Phi_a^\dagger e^{iA_{ab}} \Phi_b \qquad \Phi_a = e^{-i\varphi_a}$$

$$Q_a = (divE)_a \qquad \Phi_a^\dagger \Phi_a = 1$$

$$[A_{ab}, E_{ab}] = i$$

$$[\Phi_a, Q_a] = \Phi_a$$

Monopolar spinons

$$H_{QED} = \frac{J_{zz}}{2}\sum_{\mathbf{r}} Q_{\mathbf{r}}^2 - J_{\pm}\sum_{\mathbf{r}}\sum_{\mu\neq\nu} \Phi^\dagger_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu}\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} \mathrm{s}^{-\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu} \mathrm{s}^{+\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu}$$

$$+\frac{J_{\pm\pm}}{2}\sum_{\mathbf{r}}\sum_{\mu\neq\nu}(\gamma_{\mu\nu}^{-2\eta_{\mathbf{r}}}\Phi_{\mathbf{r}}^\dagger\Phi_{\mathbf{r}}^\dagger\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu}\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} \mathrm{s}^{\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu} \mathrm{s}^{\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} + h.c)$$

$$+J_{z\pm}\sum_{\mathbf{r}}\sum_{\mu\neq\nu} \mathrm{s}^z_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\mu}\left(\gamma_{\mu\nu}^{-\eta_{\mathbf{r}}}\Phi_{\mathbf{r}}^\dagger\Phi_{\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} \mathrm{s}^{\eta_{\mathbf{r}}}_{\mathbf{r},\mathbf{r}+\eta_{\mathbf{r}}\mathbf{e}_\nu} + h.c.\right) + \mathrm{const..}$$

U(1) spin liquid
with deconfined spinons

Mean-field phases

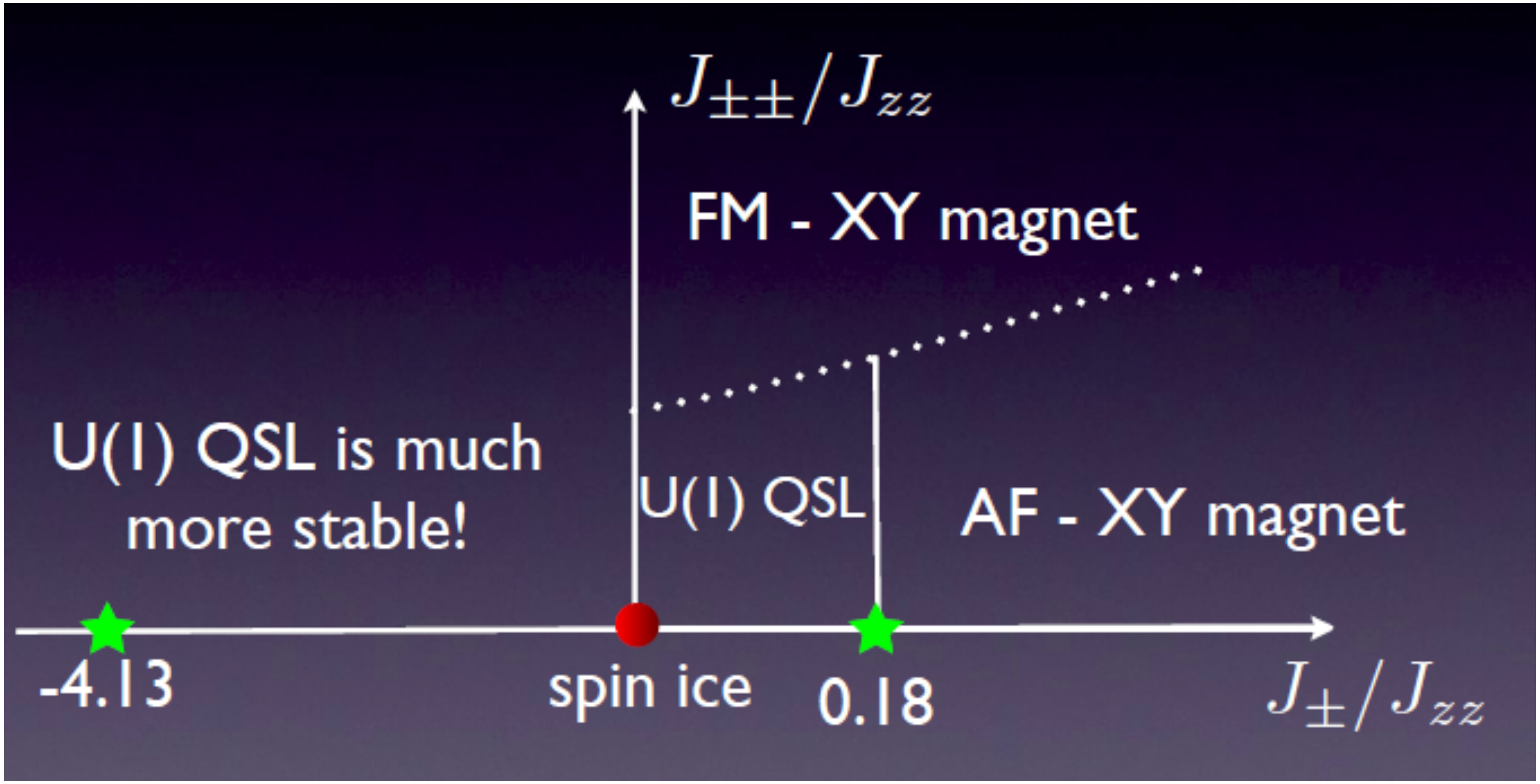
From U(1) QSL to ???

quartic spinon hopping $\Phi_r^\dagger \Phi_r^\dagger \Phi_{r+e_\mu} \Phi_{r+e_\nu}$

(Q) Which one is energetically favored ?

	$\langle \Phi \rangle$	$\langle \Phi_r \Phi_{r'} \rangle$	$\langle \Phi_{r_A}^\dagger \Phi_{r_B} \rangle$	characteristics
XY magnet	$\neq 0$	$\neq 0$	$\neq 0$	ordering on XY
Z_2	0	$\neq 0$	0	no ordering gapped excitation
U(1)-XY*	0	0	$\neq 0$	ordering on XY gapless photon
Z_2-XY*	0	$\neq 0$	$\neq 0$	ordering on XY gapped excitation

Mean-field phase diagram



Summary 1

- Effective quantum pseudospin-1/2 model for Pr,$Yb_2 TM_2 O_7$
  - AF/F anisotropic superexchange interaction
  - → Emergent U(1) spin liquid
  - → Quantum phase transitions from U(1) spin liquid to others
  - → Higgs transitions of monopolar spinons

Consistent with
- Magnetization curve
- Neutron scattering profile

→ Already observed in Yb2Ti2O7

- Possible Z(2) spin-liquid phase ??? Require further studies
  - Monopole-monopole pair condensates → charge-2 Higgs phase (but not a monopole-antimonopole pair)

Skyrmion motion

Thanks to

For a work on skyrmion Hal l effect: Kim-SO, arXiv:1012.0631v2

K.S.Kim (APCTP)

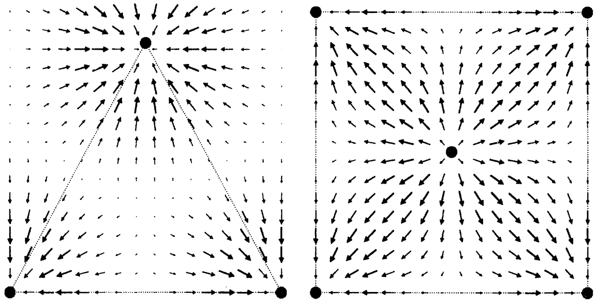
O. Tchernyshyov (JHU), H. Kohno (Osaka U), M. Mostovoy(U. Groningen)

Skyrmions: chiral spin states

Skyrme (1961, 1962) originally aimed at $\pi_3(SU(N)) = Z$

3D nonlinear continuum field theory describing

nuclear particles as localized states $\pi_2(SU(2)) = Z$



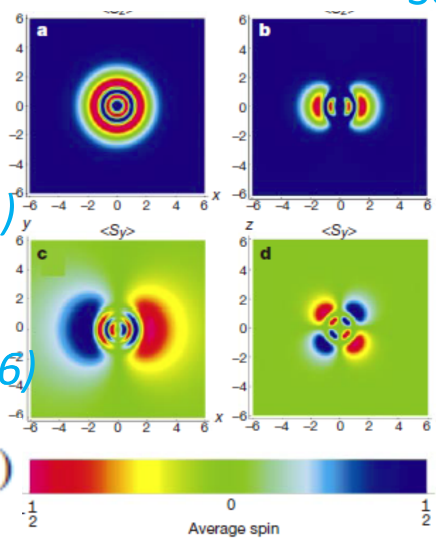
- *Liquid-crystal blue phases* *Wright-Mermin (1989)*
- *Quantum-Hall ferromagnet*

  Sondhi-Karlhede-Kivelson-Rezayi (1993)

$$\mathcal{L}_{\rm eff} = \alpha \mathcal{A}(\mathbf{n}(\mathbf{r})) \cdot \partial_t \mathbf{n}(\mathbf{r}) + \alpha' (\nabla \mathbf{n}(\mathbf{r}))^2 + g\bar{\rho}\mu_B \mathbf{n}(\mathbf{r}) \cdot \mathbf{B} - \tfrac{1}{2} \int d^2r' \, V(\mathbf{r}-\mathbf{r}') q(\mathbf{r}) q(\mathbf{r}') \ , \qquad (5)$$

- *Skyrme crystal in 2DEG*

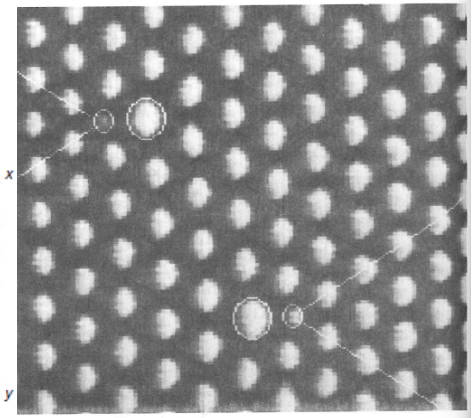
  Brey-Fertig-Cote-MacDonald (1996)
- *Cold atom (ferromagnetic spin-1/2 BEC condensate of ^{87}Rb)* *Khawaja-Stoof (2001)*
- *Noncentrosymmetric ferromagnets (MnSi, etc):* *Rossler-Bogdanov-Pfleiderer (2006)*



Garnet film

Seul-Murray, Science('93)

518 9 Topological defects



9.2.16. Magnetic bubble domain pattern showing a dislocation pair in otherwise ideal hexagonal lattice. The size of the magnetic bubbles can ust to minimize energy. The bubbles at five-fold sites at dislocation cores tract whereas those at seven-fold sites expand. This pattern is on a gnetic garnet film of composition $(YGdTm)_3(FeGa)_6O_{12}$ grown to a thickness of approximately 13μm on a single crystal substrate of gadolinium gallium garnet in the (111) orientation. It was produced by cooling the film from the paramagnetic state in a small normal field ($H \sim 1$ oersted). [M. S. Seul and C.A. Murray, *Science* **262**, 558 (1993).]

$$f = Am^2 \sum_{i,j} (\partial_i n_j)^2 + \eta A (\nabla m)^2 + f_{\rm D}(\mathbf{m}) + f_0(m)$$

Dzyaloshinskii-Moriya interaction is crucial.

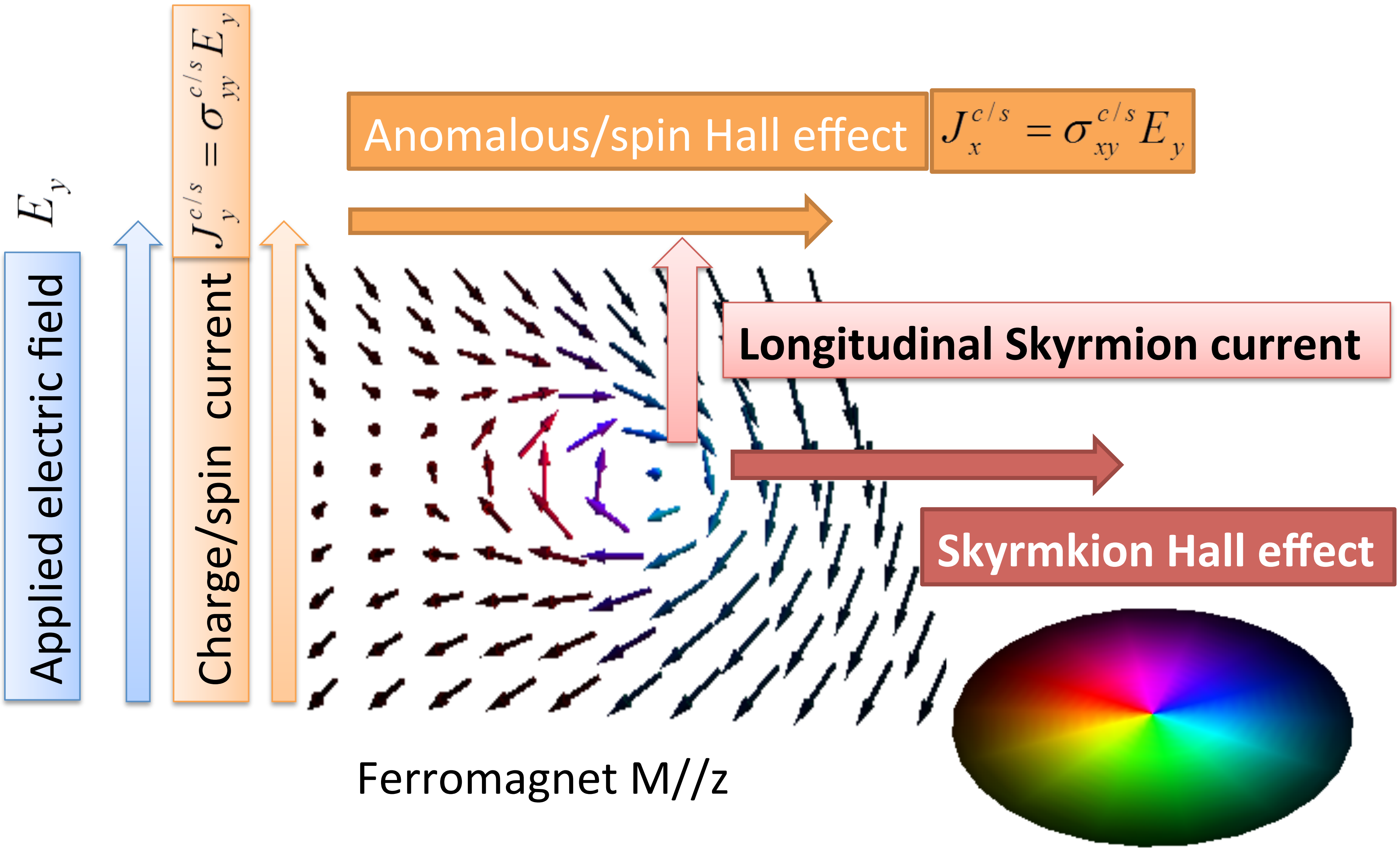
DM vectors rotates depending on the bond direction→Multiple(3)-q spiral

Microscopy: stereograph of spins on $Fe_{0.5}Co_{0.5}Si$ and FeGe Tokura

Rosch 2010

RIKEN

“Zoo” of Hall effects



Single skyrmion transport in the bulk?

Double-exchange ferromagnet with the Rashba-spn-orbit coupling

$$H = -\sum_{\langle \mathbf{r},\mathbf{r}'\rangle}^{N.N.} \left[c_{\mathbf{r}}^{\dagger} h(\vec{S}_{\mathbf{r}}, \vec{S}_{\mathbf{r}'}) c_{\mathbf{r}'} + J \vec{S}_{\mathbf{r}} \cdot \vec{S}_{\mathbf{r}'} \right]$$

S: localized spins

$$h(\vec{S}_{\mathbf{r}}, \vec{S}_{\mathbf{r}'}) = \hat{U}_{\mathbf{r}}^{\dagger}(\vec{S}_{\mathbf{r}}) \left[t_{\mathbf{r},\mathbf{r}'} + \lambda \mathbf{e}_z \cdot (\mathbf{r} - \mathbf{r}') \times \hat{\boldsymbol{\sigma}} + J_H \delta_{\mathbf{r},\mathbf{r}'} \vec{S}_{\mathbf{r}} \cdot \hat{\boldsymbol{\sigma}} \right] \hat{U}_{\mathbf{r}'}(\vec{S}_{\mathbf{r}'})$$

→ Continuum model

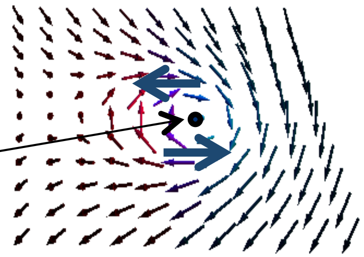
Applied electric field → $(\partial_t, \nabla) \to (\partial_t, \nabla) + ie(A_t, \mathbf{A})$

Non-collinear spin configuration of localized spins

$$\to (\partial_t, \nabla) + ie(A_t, \mathbf{A}) + i(a_t, \mathbf{a})$$

Berry's phase curvature

ξ



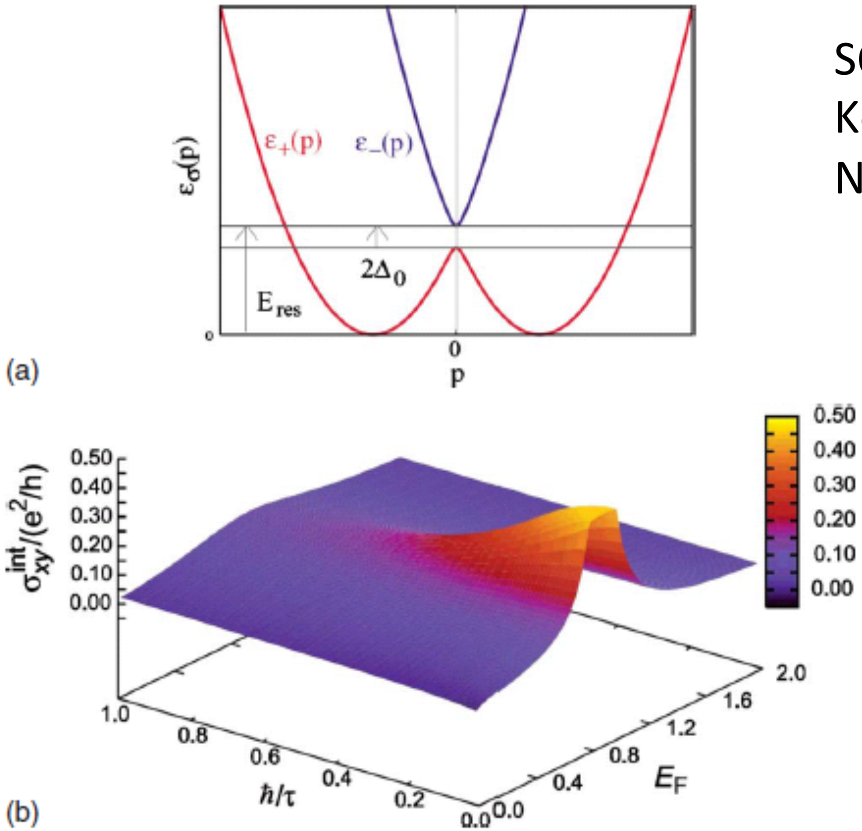
A simple single-skyrmion case: $\mathbf{a} = \dfrac{a^2 - |\mathbf{r} - \xi|^2}{a^2 + |\mathbf{r} - \xi|^2} \dfrac{n_\xi}{|\mathbf{r} - \xi|} \mathbf{e}_\theta$

Of course, coupled to current
(Spin-transfer torque)

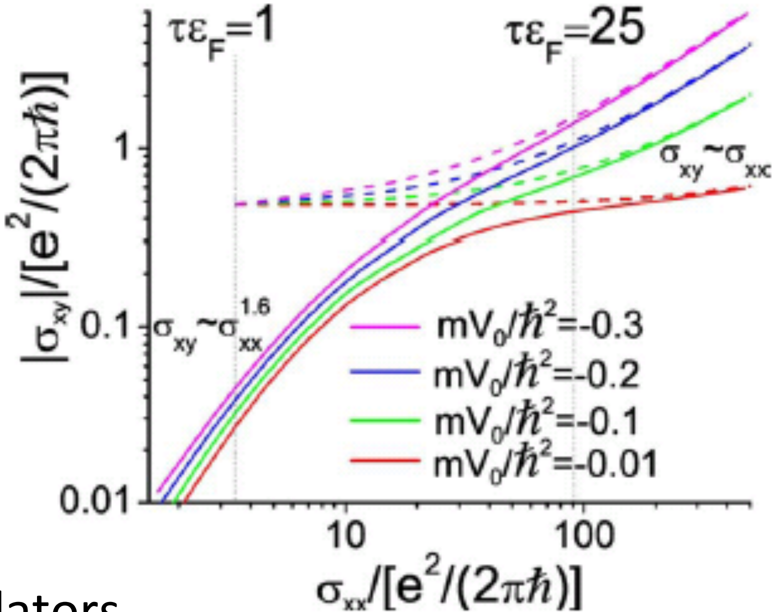
First, assume that localized spins are fixed

→Spin-polarized ferromagnetic Rashba model

→ Intrinsic and extrinsic anomalous Hall effects



SO-Sugimoto-Nagaosa (PRL 2006)
Kovalev-Tserkovnyak-Sinova (PRB 2009)
Nagaosa-Sinova-SO-MacDonald-Ong (RMP 2010)



Berry-phase contribution that survives in insulators

Now let's allow spin dynamics!

- Still assume a reasonably long lifetime for the skyrmion configuration
- Stationary but nonequilibrium problem!
- ***V***: the steady velocity of the skyrmion core

$$\xi = \boldsymbol{V}t \qquad (\partial_t, a_t) = (\partial_t - \mathbf{V}\cdot\nabla, a_t - \mathbf{V}\cdot\mathbf{a})$$

- ***V*** *can have the* ***E****-linear term!*
- Coupled self-consistent equations:

  transport equations for electrons and skyrmions

semi-classical EQM of spins

$$i\hbar\dot{\vec{S}} = -i\vec{S}\times\frac{\delta\mathcal{H}_T}{\delta\vec{S}} \quad \Longrightarrow \quad 4\pi n_\xi \hbar\epsilon_{ij}\dot{\xi}_j = -\int_A d^2r\,\frac{\langle\delta\mathcal{H}_T\rangle}{\delta\xi_i}$$

Skyrmion number   Skyrmion core coordinates

Another approach: U(1) gauge theory

Integrate out the fermions around the skyrmion configuration
Expansion of the Lagrangian in fluctuations of Berry phases $\delta a = a - a\uparrow Skyrmion$
Ignore spinon flucutations

$$\mathcal{L}_{eff} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{CS},$$

$$\mathcal{L}_M = \frac{1}{2}\begin{pmatrix} \delta a_i & A_i \end{pmatrix}\begin{pmatrix} \sigma_{ss}|\partial_\tau| + \chi_{ss}(-\partial^2) & \sigma_{sc}|\partial_\tau| + \chi_{sc}(-\partial^2) \\ \sigma_{sc}|\partial_\tau| + \chi_{sc}(-\partial^2) & \sigma_{cc}|\partial_\tau| + \chi_{cc}(-\partial^2) \end{pmatrix} P^T_{ij}\begin{pmatrix} \delta a_j \\ A_j \end{pmatrix},$$

$$\mathcal{L}_{CS} = i\frac{\Theta_{ss}(\mathbf{x}-\mathbf{X})}{2\pi}\epsilon_{\mu\nu\lambda}\delta a_\mu \partial_\nu \delta a_\lambda + i\frac{\Theta_{sc}(\mathbf{x}-\mathbf{X})}{\pi}\epsilon_{\mu\nu\lambda}\delta a_\mu \partial_\nu A_\lambda + i\frac{\Theta_{cc}(\mathbf{x}-\mathbf{X})}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu \partial_\nu A_\lambda,$$

$$P^T_{ij} = \delta_{ij} + \partial_i\partial_j/(-\partial^2)$$

$$v_x = \frac{\left((S-M)\sigma^H_{sc} + \rho_{el}\sigma^H_{ss}\right)\left(\sigma_{ss}\sigma^H_{cc} + \sigma_{cc}\sigma^H_{ss}\right) + \rho_{el}\sigma_{ss}\left(\pi^2 L^4 \sigma_{ss}\sigma_{cc} - \sigma^H_{ss}\sigma^H_{cc} + \frac{\sigma^{H2}_{sc}}{4}\right)}{\left((S-M)\sigma^H_{sc} + \rho_{el}\sigma^H_{ss}\right)^2 + \pi^2 L^4 (\rho_{el}\sigma_{ss})^2}E_x,$$

$$v_y = \frac{-\frac{1}{\pi L^2}\left((S-M)\sigma^H_{sc} + \rho_{el}\sigma^H_{ss}\right)\left(\pi^2 L^4 \sigma_{ss}\sigma_{cc} - \sigma^H_{ss}\sigma^H_{cc} + \frac{\sigma^{H2}_{sc}}{4}\right) + \rho_{el}\sigma_{ss}\left(\pi^2 L^4 \sigma_{ss}\sigma_{cc} - \sigma^H_{ss}\sigma^H_{cc}\right)}{\left((S-M)\sigma^H_{sc} + \rho_{el}\sigma^H_{ss}\right)^2 + \pi^2 L^4 (\rho_{el}\sigma_{ss})^2}E_x.$$

Consider a stationary motion of skyrmion
Ignore relative change of spin configurations

$$\partial_\tau \longrightarrow \partial_\tau - \mathbf{v}_r \cdot \boldsymbol{\partial}_r, \qquad \delta a_\tau \longrightarrow \delta a_\tau - \mathbf{v}_r \cdot \delta \mathbf{a}_r$$

Skyrmion velocity

In particular,

Insulator

$$v_x = 0,$$

$$v_y = \frac{1}{\pi L^2}\frac{\sigma^H_{ss}\sigma^H_{cc} - \frac{\sigma^{H2}_{sc}}{4}}{(S-M)\sigma^H_{sc} + \rho_{el}\sigma^H_{ss}}E_x.$$

Dissipationless current

- In the thermdynamic limit, a single skyrmion can not affect the bulk transport. Nevertheless, thanks to the Berry phase of the spin-polarized Rashba model, there appears the intrinsic anomalous Hall effect and the associated skyrmion Hall current.

In a particular limit of the Dirac-fermion case
i.e., a case of 2D quantized anomalous Hall effect in insulators
(Haldane 1988)
e.g., a surface state of a 3DTI under magnetic field (S. C. Zhang …)

$$\mathcal{S} = \mathcal{S}_B + \int d^3\boldsymbol{x}\Big\{\bar{\psi}\Big(i\hat{\boldsymbol{D}} - m\vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{\tau}}\Big)\psi + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2\Big\}.$$

$$\begin{aligned}\mathcal{S}_{eff} &= \mathcal{S}_B + \int d^3\boldsymbol{x}\Big(\frac{1}{2g^2}|(\partial_\mu - ia_\mu)z_\sigma|^2 + \frac{i}{2\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu a_\lambda \\ &\quad + \frac{i}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \frac{i}{4\pi}\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2\Big)\end{aligned}$$

Chern-Simons term
Edge currents

$$\sigma_{xx} \to 0, \sigma_{xy} = \frac{e^2}{2h} \Rightarrow \mathbf{V} \perp \mathbf{E}$$

$$V_x / E_y \propto \sigma_H$$ X Skyrmion number

Relation to Tatara-Kohno theory for the motion of magnetic vortex in a ferromagnetic nanodisk

Shibata-Nakatani-Tatara-Kohno-Ohtani 2006

The LLG equation

$$\frac{\partial \boldsymbol{M}}{\partial t} = -\gamma_0 \boldsymbol{M} \times \boldsymbol{H}_{\text{eff}} + \frac{\alpha}{M_s} \boldsymbol{M} \times \frac{\partial \boldsymbol{M}}{\partial t} - \boldsymbol{v}_s \cdot \nabla \boldsymbol{M}$$

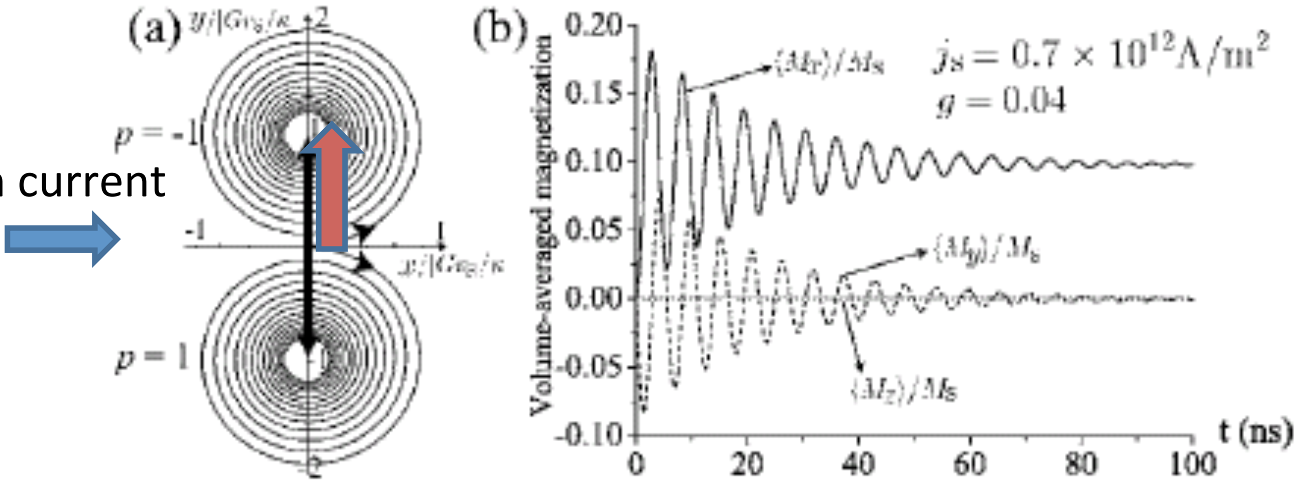
$$G = e_z \hbar S \int \frac{d^3x}{a^3} \boldsymbol{n} \cdot (\partial_x \boldsymbol{n} \times \partial_y \boldsymbol{n})$$

∝ skyrmion number

Magnetic vortex

Landau-Gilbert damping (Not topological, extrinsic)

Spin current



Topological effect is not included here.!

c.f. Dynamically induced skyrmion current
Mostovoy-Nomura-Nagaosa

Summary

- Skyrmion Hall effect
- We have shown that it appears in relativistic insulators with spin-orbit coupling.
- Skrymion current produces a Hall voltage drop.