QUARKS, GLUONS, AND LATTICES

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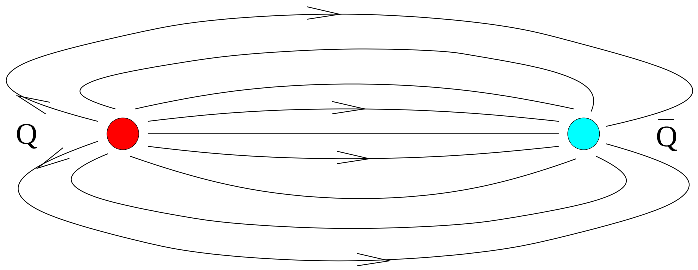
Quarks: fundamental constituents of subnuclear particles

Gluons: what holds them together



Lattices: a mathematical framework for calculation

Quarks

Fundamental constituents feeling the nuclear force

- six known types: u, d, s, c, b, t
- proton (uud); neutron (udd)

Why do we believe in them?

- various combinations give families of observed particles
- high energy scattering suggests pointlike substructure
- heavy quark bound states, i.e. $J = (c\bar{c})$
  - calculable masses
  - “hydrogen atoms” for quarks

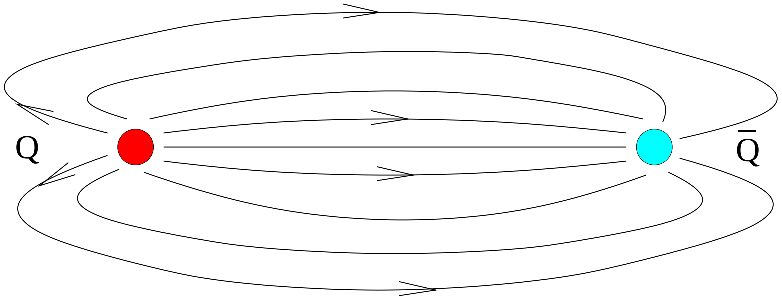
Gluons

Fields that hold the quarks together

- much like electric fields except
  - 8 electric fields, not just one: “non-Abelian” fields
  - charged with respect to each other

Confinement: quarks cannot be isolated

- self interacting gluon flux lines do not spread out



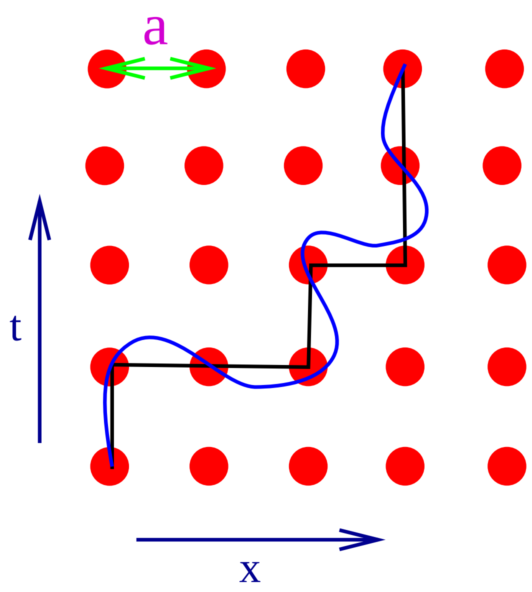
- $1/r^2$ force replaced by a constant at long distances
  - quarks at ends of “strings”

Constant 14 tons of tension pulling the quarks together

Lattices

Quark paths or “world lines” $\longrightarrow$ discrete hops

- four dimensions of space and time



A mathematical trick

- lattice spacing $a \to 0$ for physics
  - $a = \text{minimum length (cutoff)} = \pi/\Lambda$
- allows computations

What led us to the lattice?

Late 1960's

- quantum electrodynamics: immensely successful, but "done"
- eightfold way: "quarks" explain particle families
- electroweak theory emerging
- electron-proton scattering: "partons"

Meson-nucleon theory failing

- $\frac{g^2}{4\pi} \sim 15$ vs. $\frac{e^2}{4\pi} \sim \frac{1}{137}$
- no small parameter for expansion

Frustration with quantum field theory

“S-matrix theory”

- particles are bound states of themselves
  - $p + \pi \leftrightarrow \Delta$
  - $\Delta + \pi \leftrightarrow p$
- held together by exchanging themselves
- roots of duality between particles and forces $\longrightarrow$ string theory

What is elementary?

Early 1970’s

- “partons” $\longleftrightarrow$ “quarks”
- renormalizability of non-Abelian gauge theories
  - 1999 Nobel Prize, G. ’t Hooft and M. Veltman
- asymptotic freedom
  - 2004 Nobel prize: D. Gross, D. Politzer, F. Wilczek
- Quark Confining Dynamics (QCD) evolving

Confinement?

- interacting hadrons vs. quarks and gluons
- What is elementary?

Mid 1970’s: a particle theory revolution

- J/ψ discovered, quarks inescapable
- field theory reborn
  - “standard model” evolves

Extended objects in field theory

- “classical lumps” a new way to get particles
- “bosonization” very different formulations can be equivalent
- growing connections with statistical mechanics
- What is elementary?

Field Theory $>>$ Feynman Diagrams

Field theory has infinities

- bare charge, mass divergent
- must “regulate” for calculation
- Pauli Villars, dimensional regularization: perturbative
  - based on Feynman diagrams
  - an expansion in a small parameter, the electric charge

But the expansion misses important “non-perturbative” effects

- confinement
- light pions from chiral symmetry breaking
- no small parameter to expand in

need a “non-perturbative” regulator

Wilson's strong coupling lattice theory (1973)

Strong coupling limit does confine quarks

- only quark bound states (hadrons) can move

space-time lattice $=$ non-perturbative cutoff

Lattice gauge theory

- A mathematical trick
- Minimum wavelength $=$ lattice spacing a
  - Uncertainty principle: a maximum momentum $= \pi/a$
- Allows computations
- Defines a field theory

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Be discrete, do it on the lattice

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Be discrete, do it on the lattice

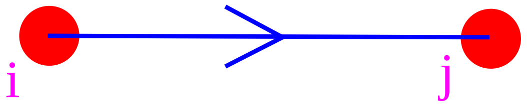
Be indiscreet, do it continuously

Wilson’s formulation

local symmetry + theory of phases

Variables:

- Gauge fields are generalized “phases” $U_{i,j} \sim \exp(i \int_{x_i}^{x_j} A^\mu dx_\mu)$

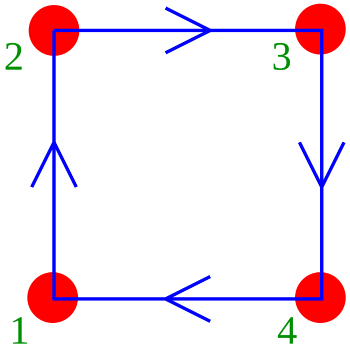


$U_{ij} =$ 3 by 3 unitary ($U^\dagger U = 1$) matrices, i.e. SU(3)

- On links connecting nearest neighbors
- 3 quarks in a proton

Dynamics:

- Sum over elementary squares, ‘‘plaquettes’’



$$U_p = U_{1,2} U_{2,3} U_{3,4} U_{4,1}$$

- like a ‘‘curl’’ $\vec{\nabla} \times \vec{A} = \vec{B}$
- flux through corresponding plaquette.

$$S = \int d^4x\ (E^2 + B^2) \longrightarrow \sum_p \left(1 - \frac{1}{3} \mathrm{ReTr} U_p\right)$$

Quantum mechanics:

- via Feynman's path integrals
- sum over paths $\longrightarrow$ sum over phases
  - $Z = \int (dU) e^{-\beta S}$
  - invariant group measure

Parameter β related to the "bare" charge

  - $\beta = \frac{6}{g_0^2}$
- divergences say we must "renormalize" β as $a \to 0$
  - adjust β to hold some physical quantity constant

Parameters

Asymptotic freedom

- 2004 Nobel prize: D. Gross, D. Politzer, F. Wilczek

$$g_0^2 \sim \frac{1}{\log(1/a\Lambda)} \to 0$$

Λ sets the overall scale via “dimensional transmutation”

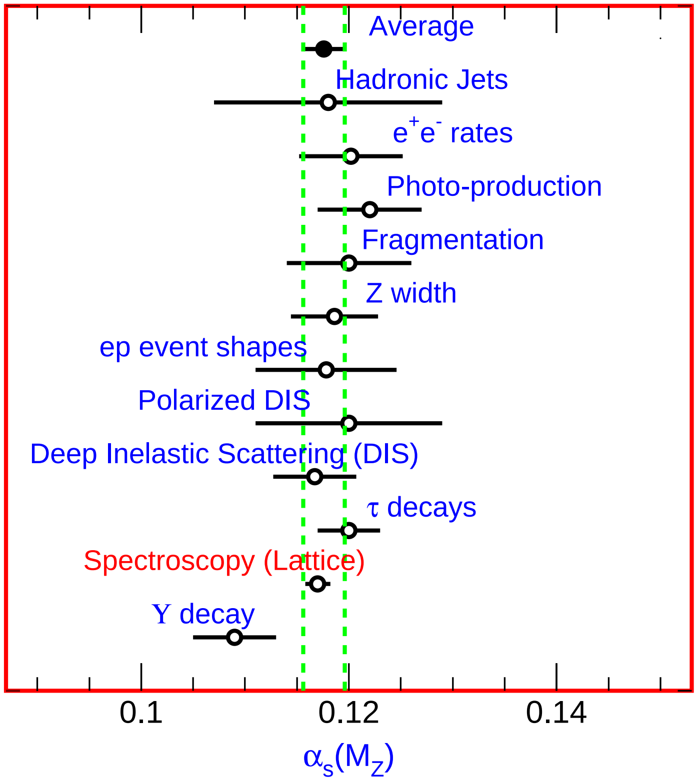
- Sidney Coleman and Erick Weinberg
- Λ depends on units: not a real parameter

Only the quark masses!

$m_q = 0$: parameter free theory

- $m_\pi = 0$
- m_ρ / m_p determined
- close to reality

Example: strong coupling determined



(PDG, 2008)

(charmonium spectrum for input, fermion dynamics treated approximately)

Numerical Simulation

$$Z = \int dU e^{-\beta S}$$

10^4 lattice ⇒

- $10^4 \times 4 \times 8 = 320,000$ dimensional integral
- 2 points/dimension ⇒

$$2^{320,000} = 3.8 \times 10^{96,329} \qquad \text{terms}$$

- age of universe $\sim 10^{27}$ nanoseconds

Use statistical methods

- $Z \longleftrightarrow$ partition function
- $\frac{1}{\beta} \longleftrightarrow$ temperature

Find “typical equilibrium” configurations C

$$P(C) \sim e^{-\beta S(C)}$$

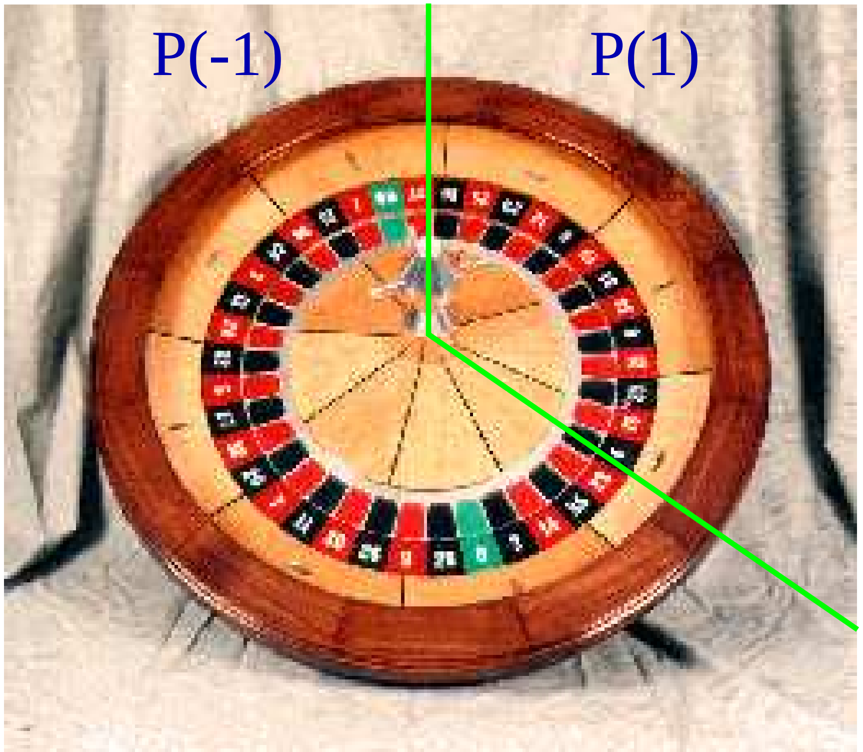
Use a Markov process

$$C \to C' \to \ldots$$

Z_2 example: (L. Jacobs, C. Rebbi, MC)

$$U = \pm 1$$

$$P(1) = \frac{e^{-\beta S(1)}}{e^{-\beta S(1)} + e^{-\beta S(-1)}}$$

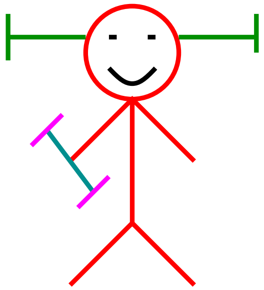


Random field changes biased by Boltzmann weight.

- converge towards "thermal equilibrium."
    - $P(C) \sim e^{-\beta S}$

In principle can measure anything

Fluctuations → theorists have error bars!

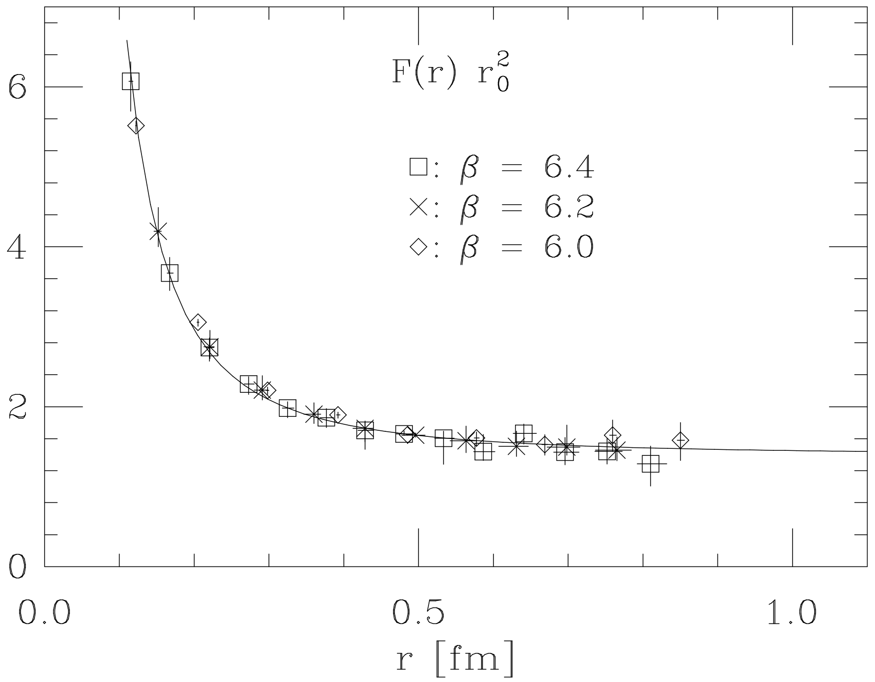


Also have systematic errors

- finite volume
- finite lattice spacing
- quark mass extrapolations

Interquark force

- constant at large distance
- confinement



UKQCD Collaboration, hep-lat/9411075

Extracting particle masses

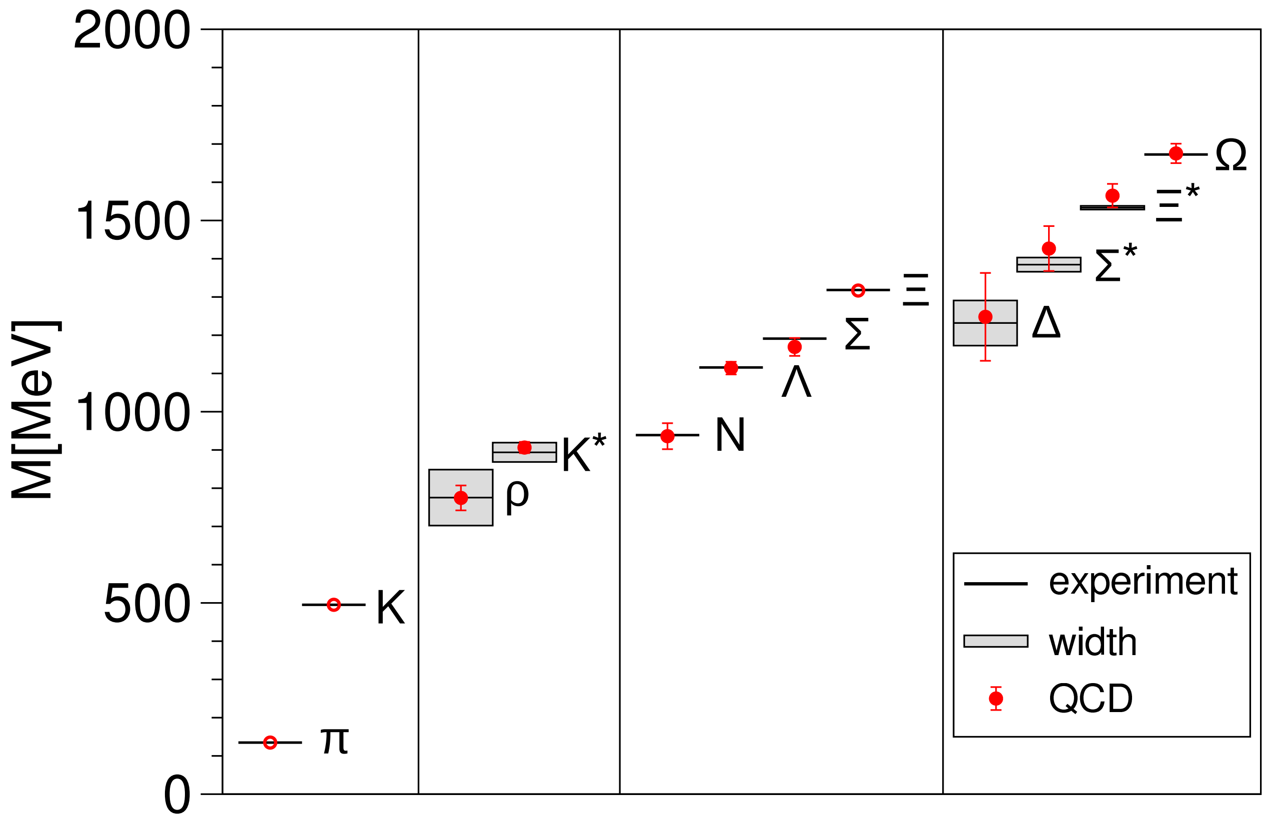
- let $\phi(t)$ be some operator that can create a particle at time t
- As $t \to \infty$
  - $\langle \phi(t)\phi(0) \rangle \longrightarrow e^{-mt}$
- $m =$ mass of lightest hadron created by ϕ
- Bare quark mass is a parameter

Chiral symmetry:

$$m_\pi^2 \sim m_q$$

Adjust m_q to get m_π / m_ρ (m_s for the kaon)

all other mass ratios determined

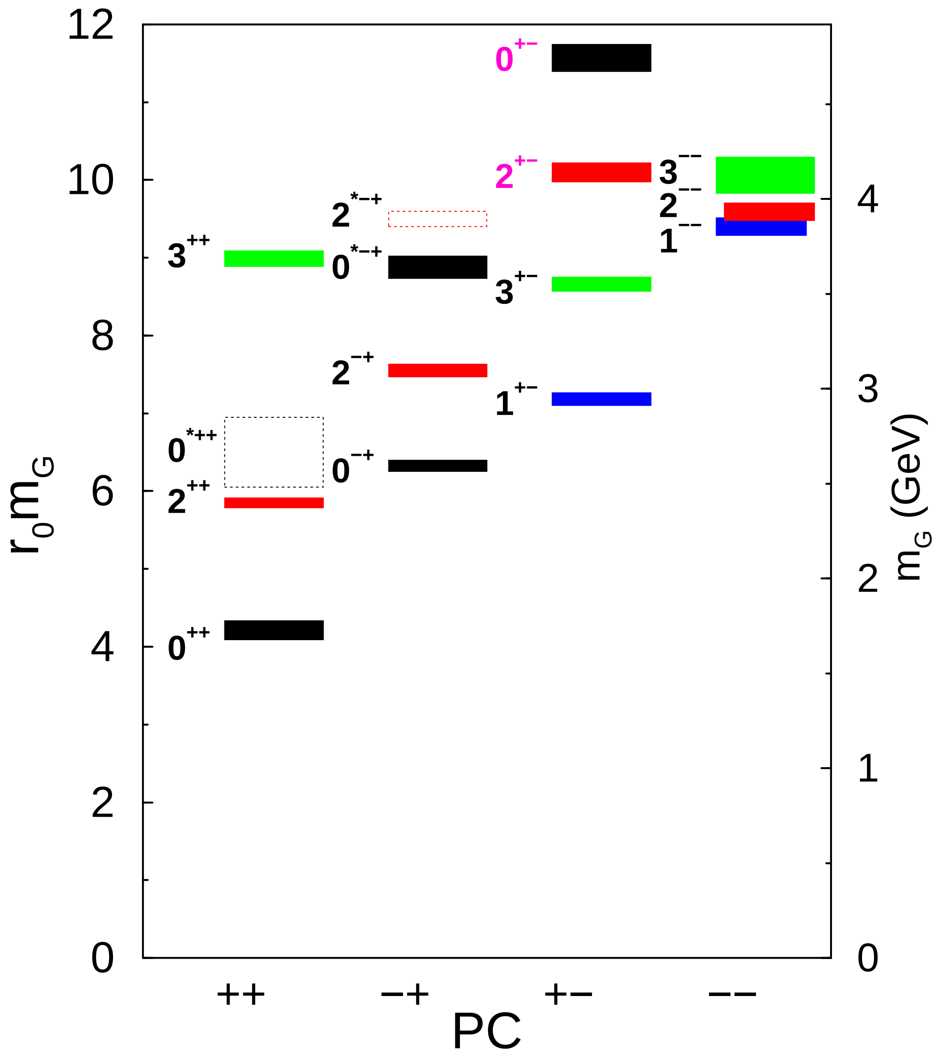


Budapest-Marseille-Wuppertal collaboration

- Lattice 2008 conference
- Science 322:1224-1227,2008
- improved Wilson fermions

Glueballs

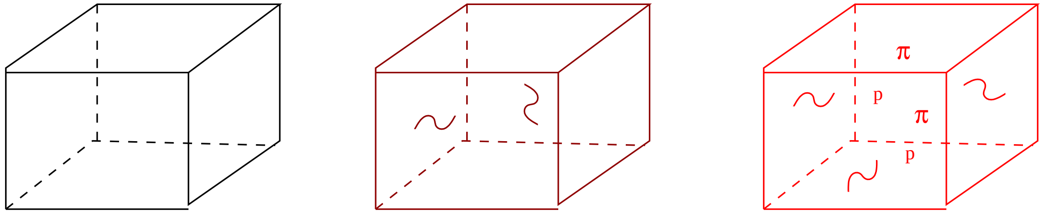
- closed loops of gluon flux
- no quarks



Morningstar and Peardon, Phys. Rev. D **60**, 034509 (1999)

- used an anisotropic lattice, ignored virtual quark-antiquark pairs

Quark Gluon Plasma

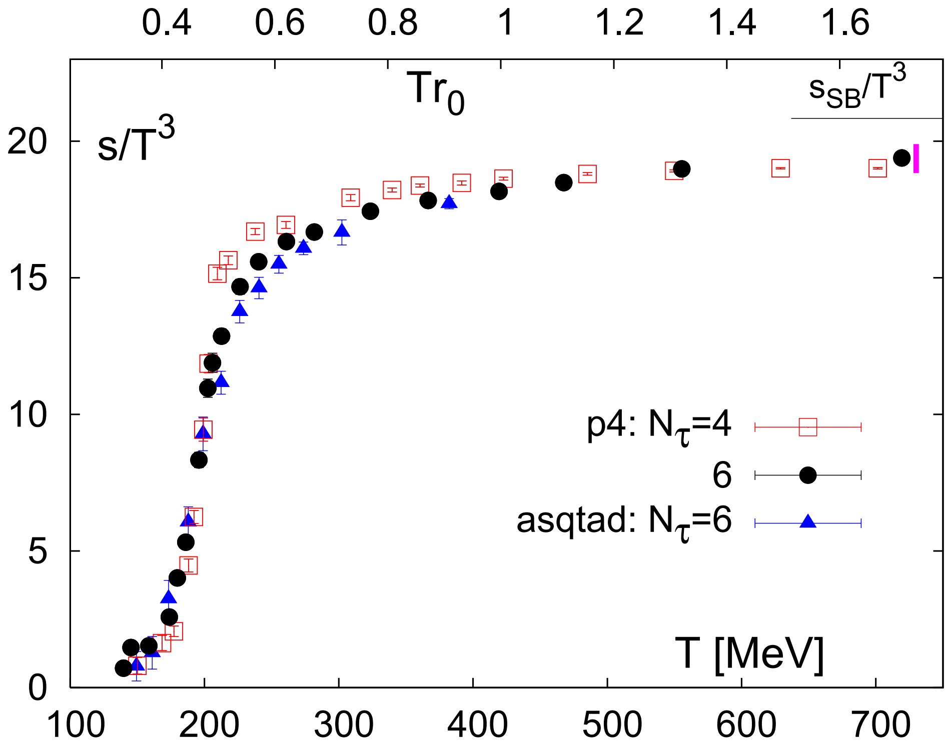


Finite temporal box of length t

  - $Z \sim \mathrm{Tr}\ e^{-Ht}$

- $1/t \leftrightarrow$ temperature
- confinement lost at high temperature
- chiral symmetry restored
- $T_c \sim 170 - 190$ MeV
  - not a true transition, but a rapid “crossover”

Big jump in entropy versus temperature



M. Cheng et al., Phys.Rev.D77:014511,2008.

- uses a non-rigorous approximation to QCD

Unsolved Problems

Chiral gauge theories

- parity conserving theories in good shape
- chiral theories (neutrinos) remain enigmatic
    - non-perturbative definition of the weak interactions?

Sign problems

- finite baryon density: nuclear matter
    - color superconductivity at high density
- $\theta \neq 0$
    - spontaneous CP violation at $\theta = \pi$

Fermion algorithms (quarks)

- remain very awkward
- why treat fermions and bosons so differently?

Lots of room for new ideas!