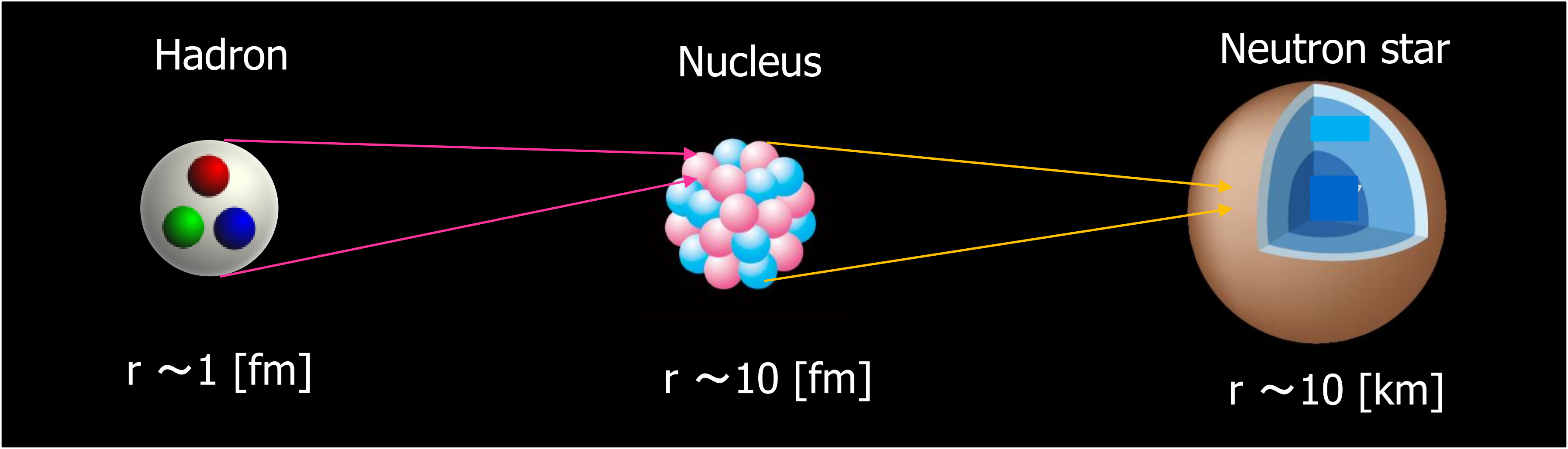
QCD Structure of Hadronic Matter



Hadron

Nucleus

Neutron star

r ∼1 [fm]

r ∼10 [fm]

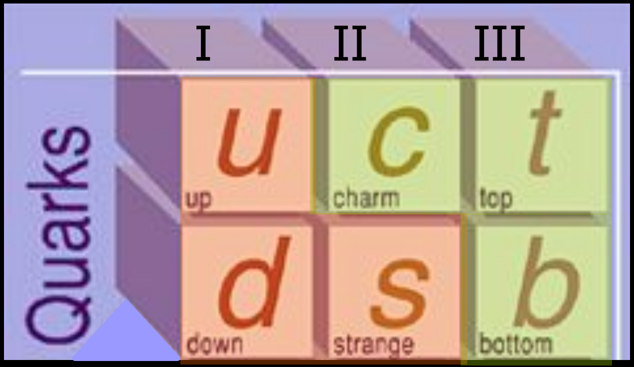
r ∼10 [km]

Light Quarks

$m_u \sim$ 2 MeV

$m_d \sim$ 5 MeV

$m_s \sim$ 90 MeV



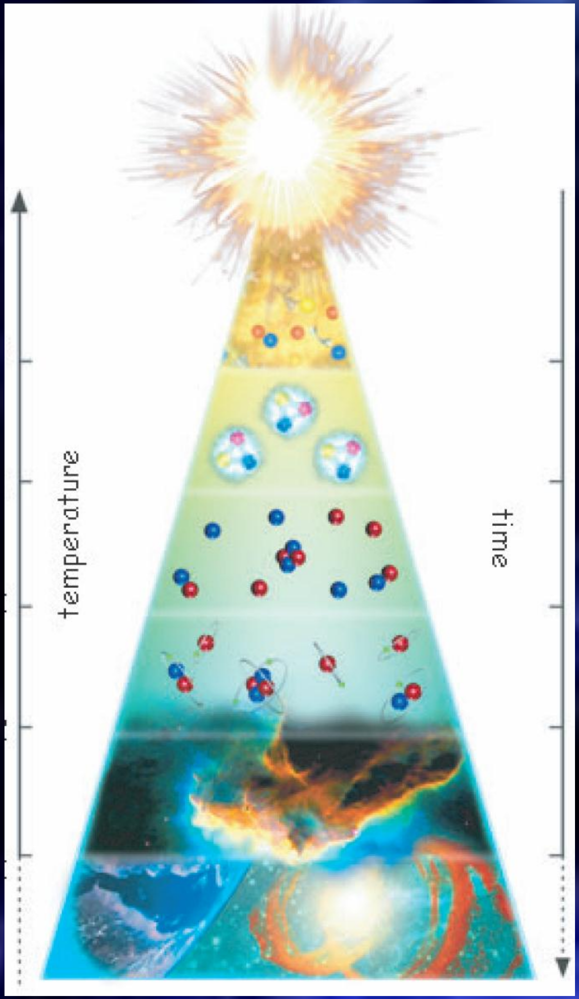
Heavy Quarks

$m_c \sim$ 1.3 GeV

$m_b \sim$ 4.2 GeV

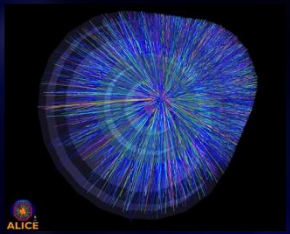
$m_t \sim$ 171 GeV

Modern challenges in Hadron Physics



Primordial form of matter

quark-gluon plasma

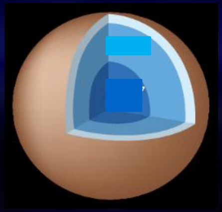


Origin of heavy elements

in explosive astrophysical phenomena

Super dense matter

neutron star, exotic matter, …



"New physics" search

dark matter, …

Contents

[1] Introduction

[2] Precision Lattice QCD

[3] Hot QCD

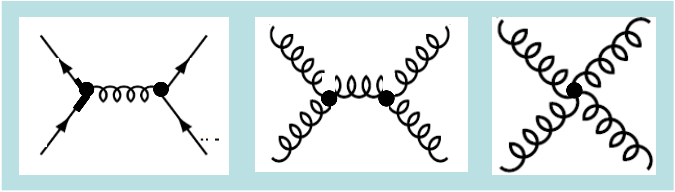
[4] Dense QCD

[5] Nuclear Force from QCD

[6] Dense QCD and Ultra-cold Atoms

[7] Summary

Quantum Chromo Dynamics



$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$$

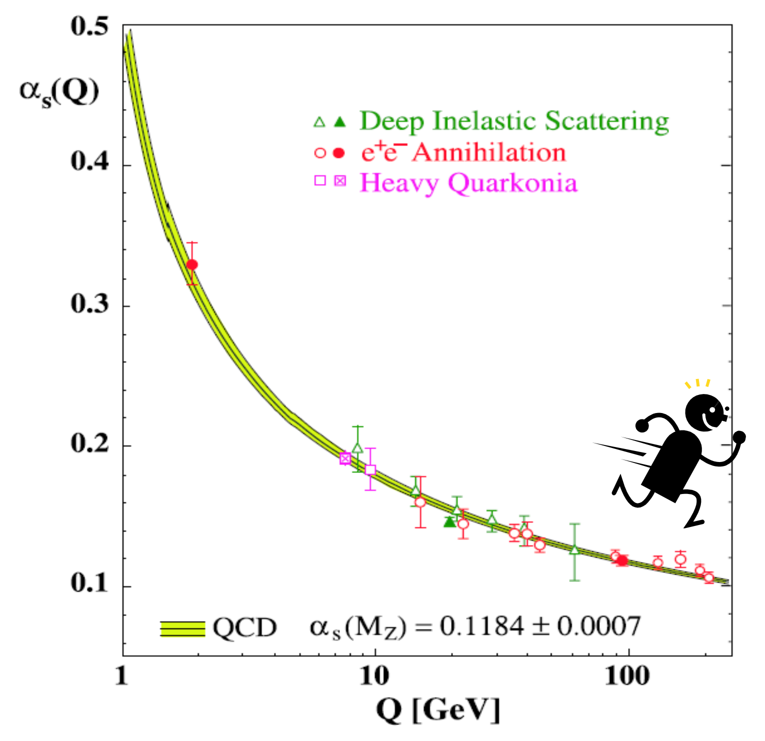
Nambu
(1966)

Running masses: $m_q(Q)$

quark masses **(from lattice QCD)**	**[MeV]** **(MS-bar @ 2GeV)**
m_u	2.19(15)
m_d	4.67(20)
m_s	94(3)

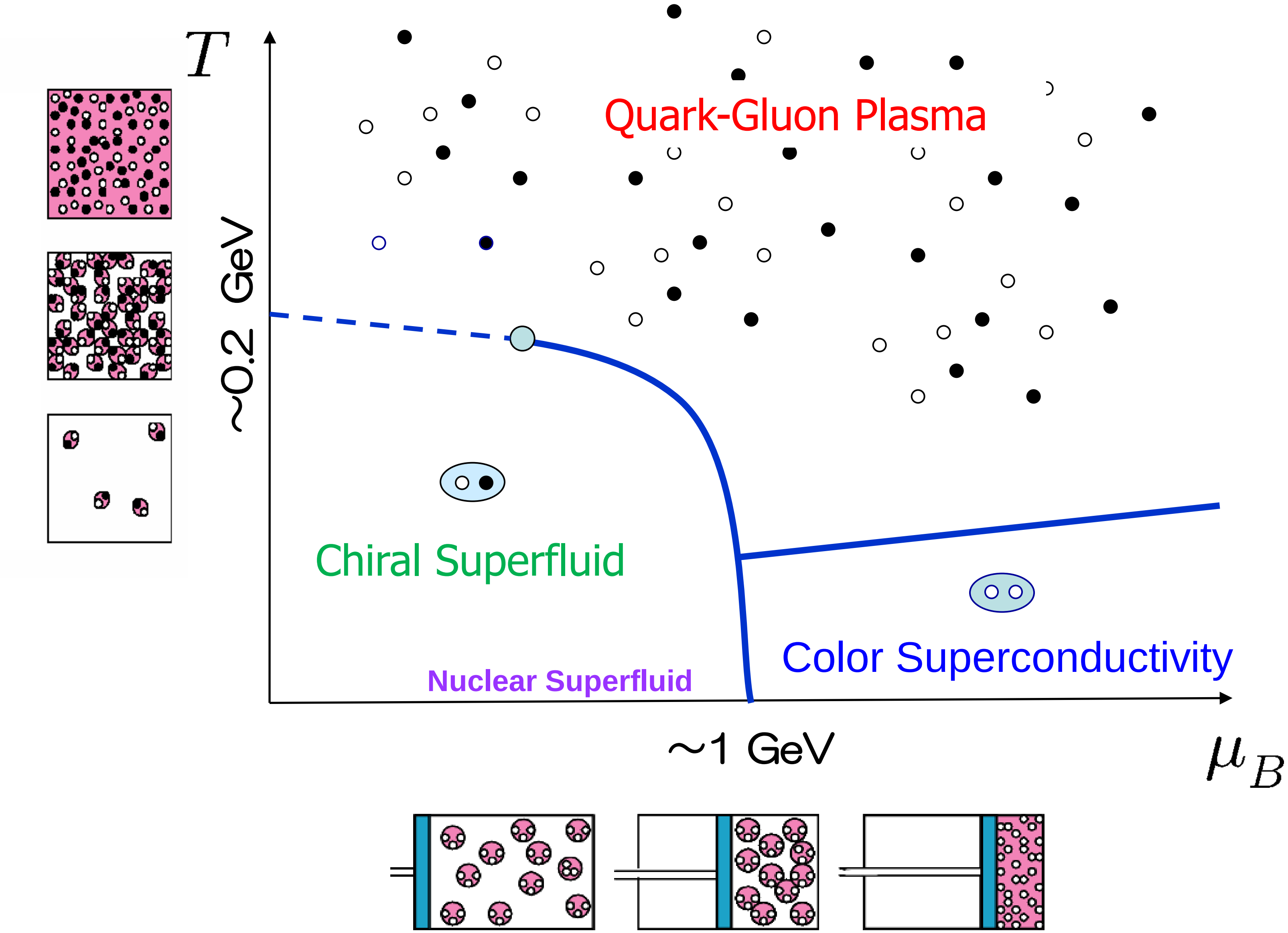
FLAG working group,
arXiv:1011.4408 [hep-lat]

Running coupling: $\alpha_s(Q)=g^2/4\pi$



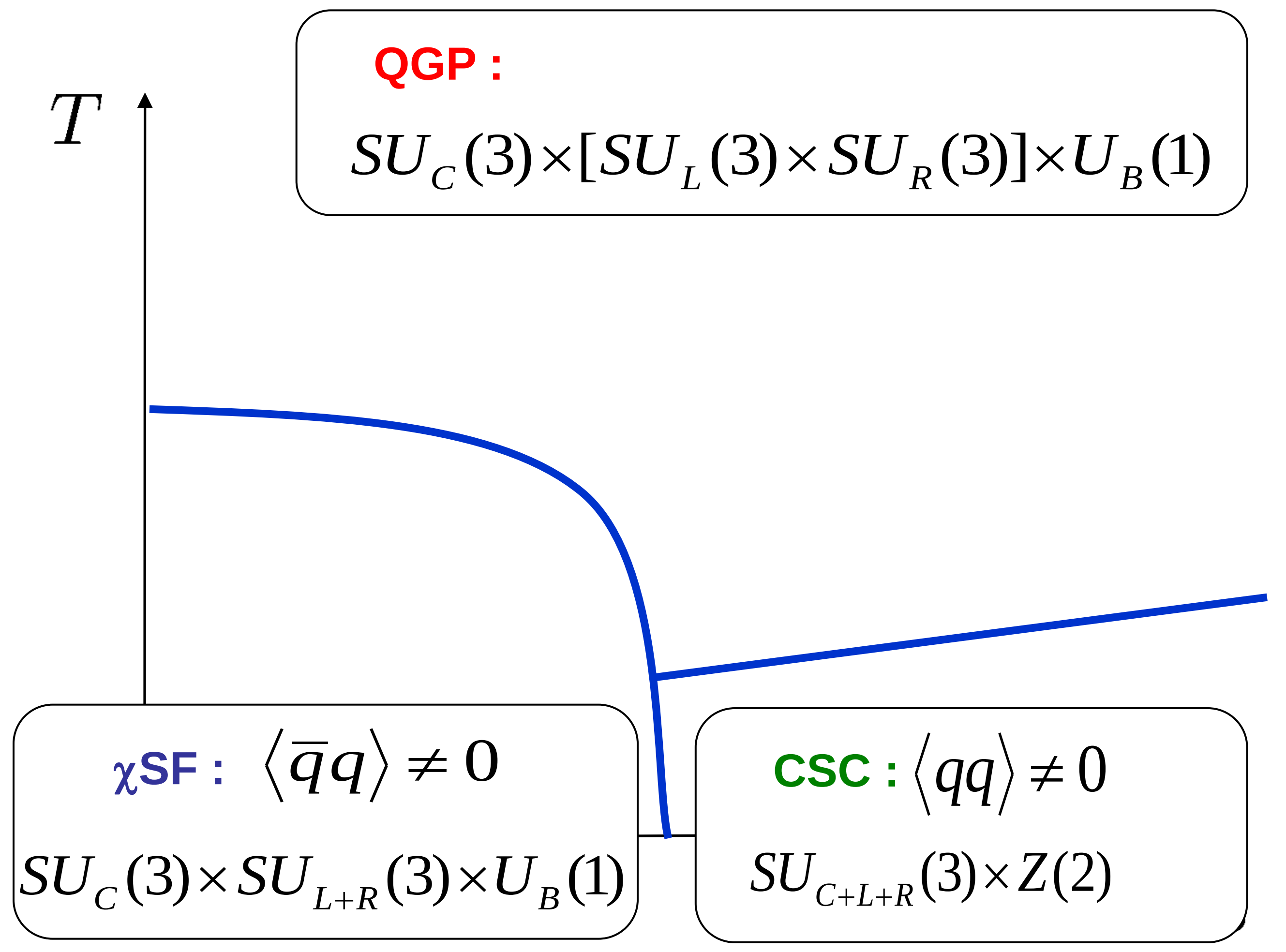
Bethke, Eur. Phys. J C(2009)64:689 →

Schematic QCD phase diagram



Symmetry realization in massless QCD ($N_c=3$, $N_f=3$)

Cabibbo and Parisi, PLB 59 (1975); Collins & Perry, PRL 34 (1975)



QGP :

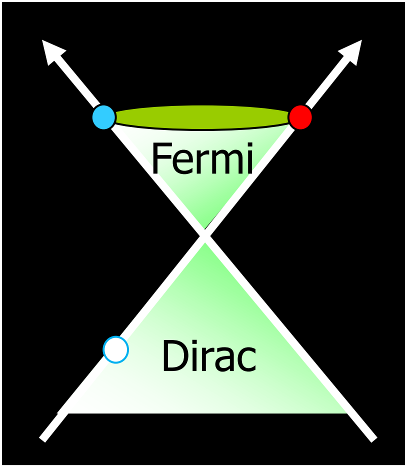
$$SU_C(3)\times[SU_L(3)\times SU_R(3)]\times U_B(1)$$

χSF : $\langle\bar{q}q\rangle\neq 0$

$$SU_C(3)\times SU_{L+R}(3)\times U_B(1)$$

CSC : $\langle qq\rangle\neq 0$

$$SU_{C+L+R}(3)\times Z(2)$$



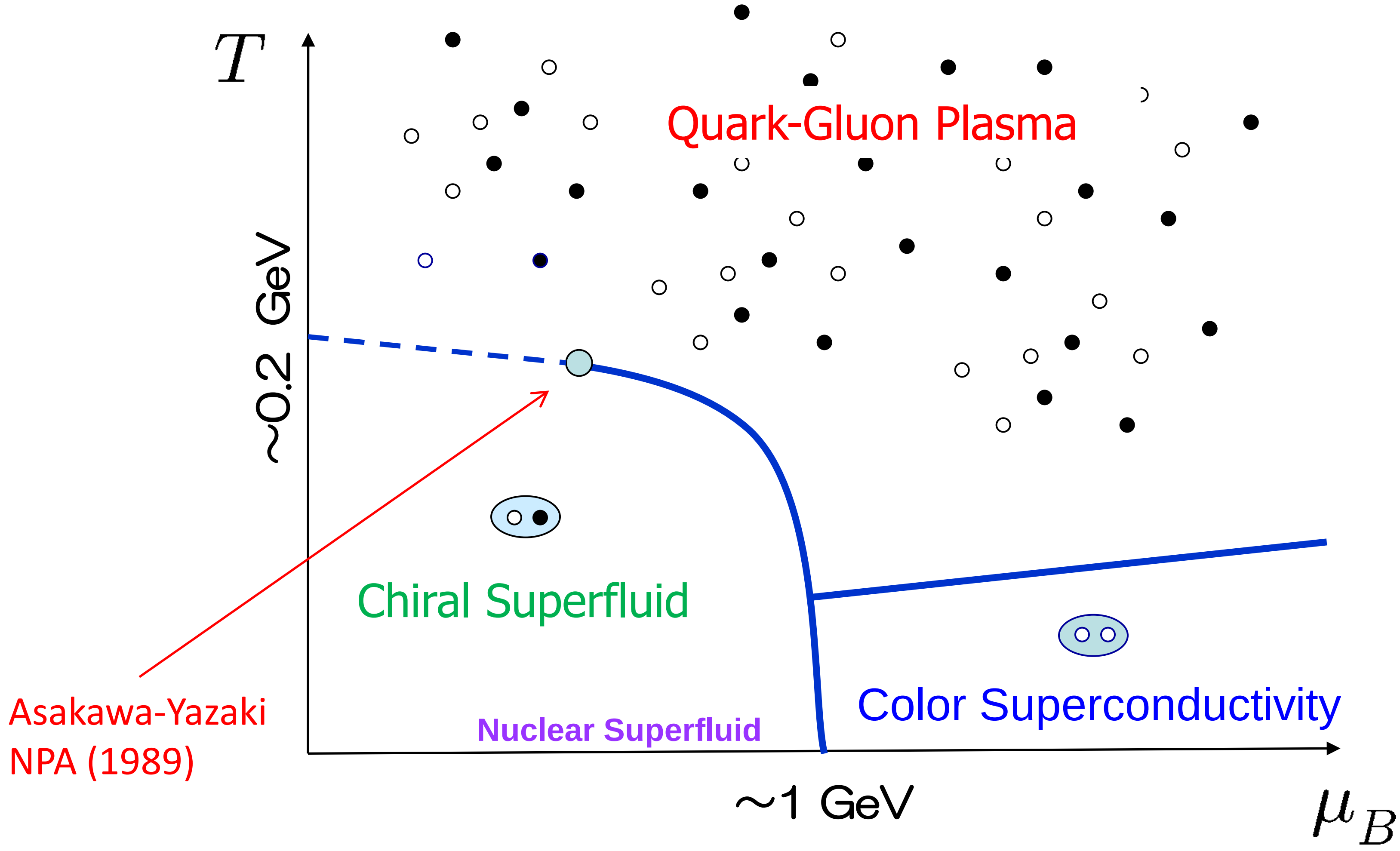
Nambu, PRL 4 (1960)

Alford, Rajagopal & Wilczek, NP B537 (1999)

"Dirac" mass

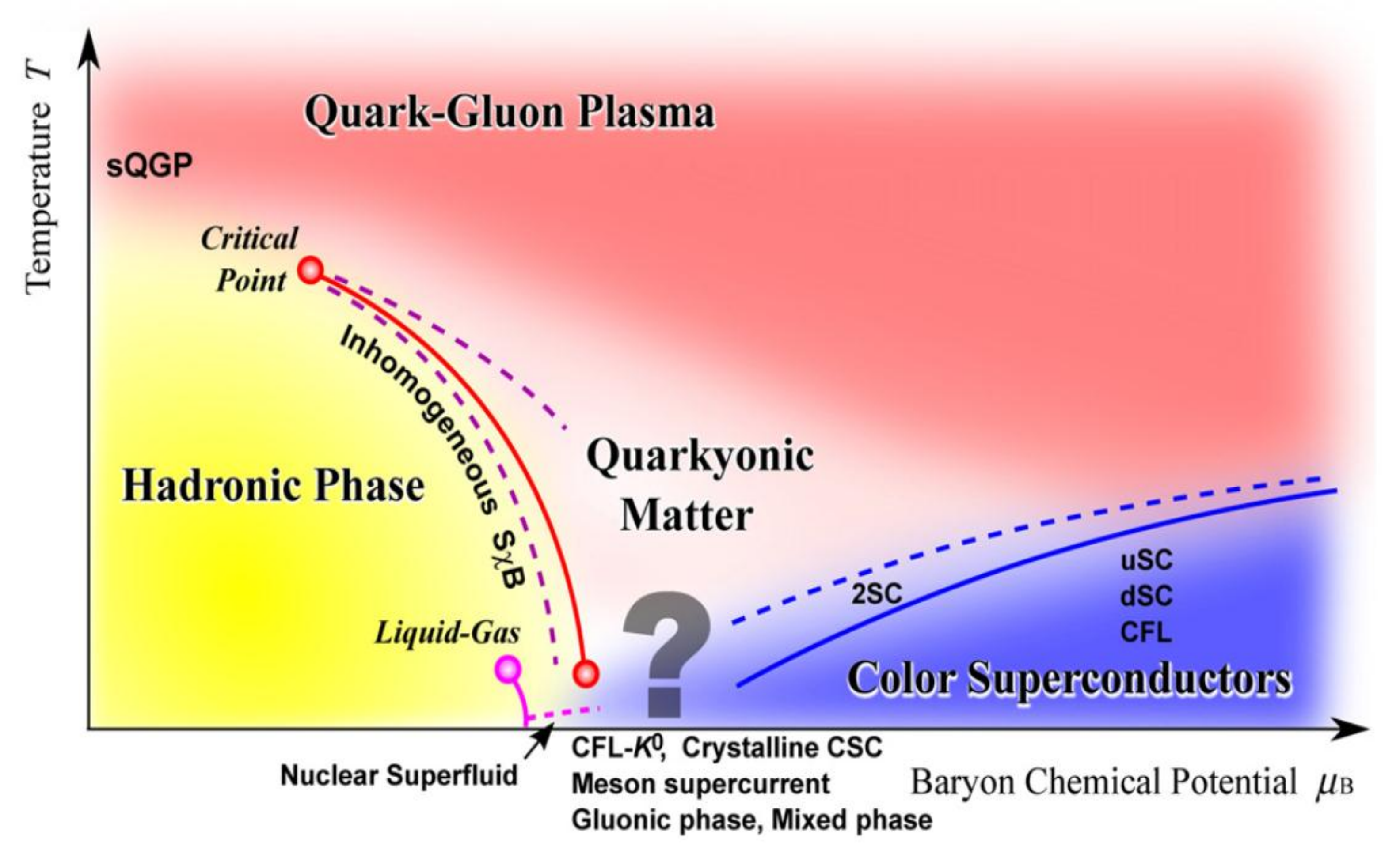
"Majorana" mass

Schematic QCD phase diagram



QCD phase diagram @ 2011

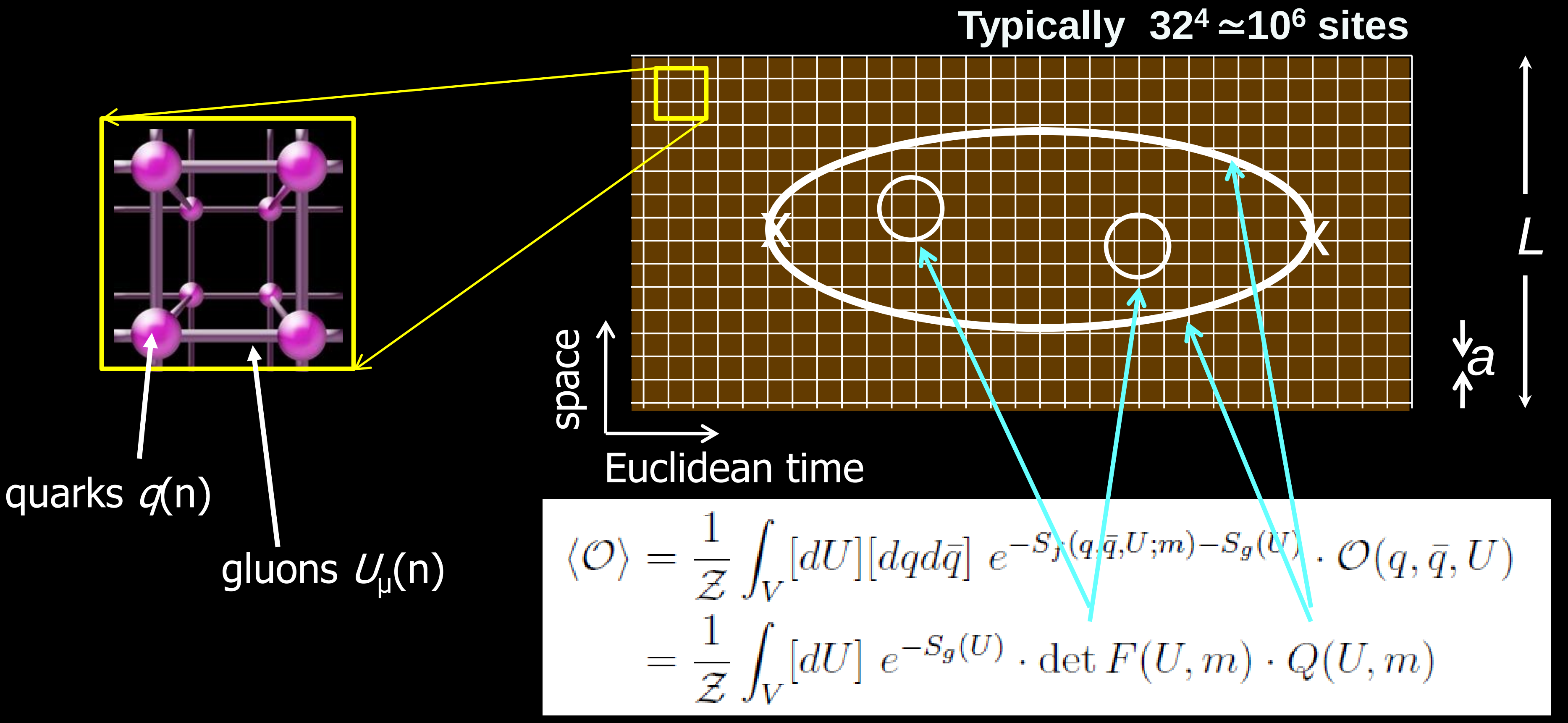
K. Fukushima and T. Hatsuda,
"The Phase Diagram of Dense QCD"
Rep. Prog. Phys. 74 (2011) 014001



High T critical point: Asakawa & Yazaki, NPA (1989)
Low T critical point: Kitazawa, Koide, Kunihiro & Nemoto, PTP (2002)
Hatsuda, Tachibana, Yamamto & Baym, PRL (2006)

Lattice QCD

Color SU(3) gauge theory for strong interaction (Nambu 1966)
Asymptotic freedom (Gross & Wilczek, Politzer 1973)
Lattice gauge theory (Wilson 1974)



$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int_V [dU][dq d\bar{q}]\, e^{-S_f(q,\bar{q},U;m) - S_g(U)} \cdot \mathcal{O}(q,\bar{q},U)$$
$$= \frac{1}{Z} \int_V [dU]\, e^{-S_g(U)} \cdot \det F(U,m) \cdot Q(U,m)$$

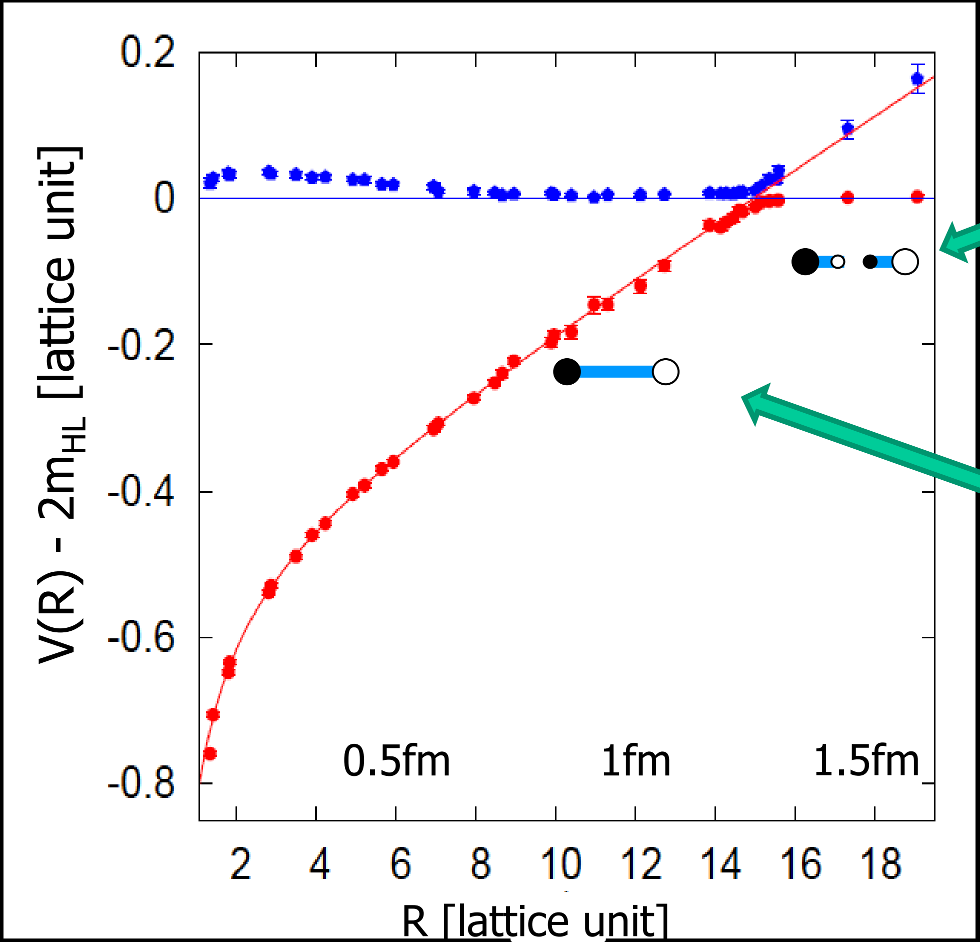
- well defined statistical system (finite a and L)
- gauge invariant
- fully non-perturbative

⇨ Monte Carlo simulations
(Creutz 1980)

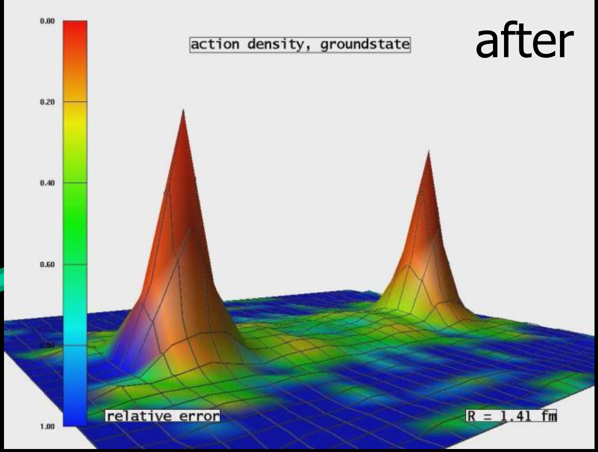
Quark Confinement

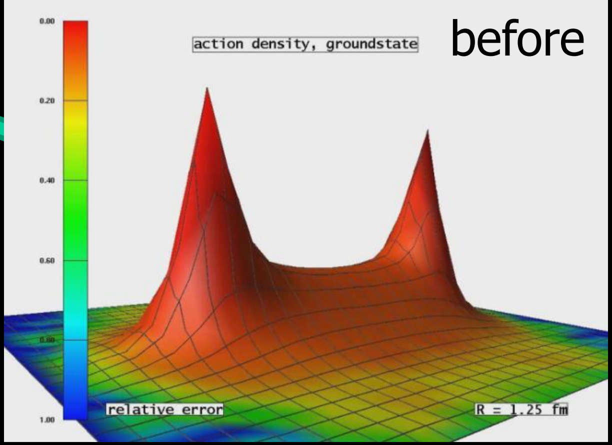
Heavy Q-Qbar potential

$$V(R) = \left(c - \frac{a}{R} + \sigma R\right)\theta(R_0 - R)$$



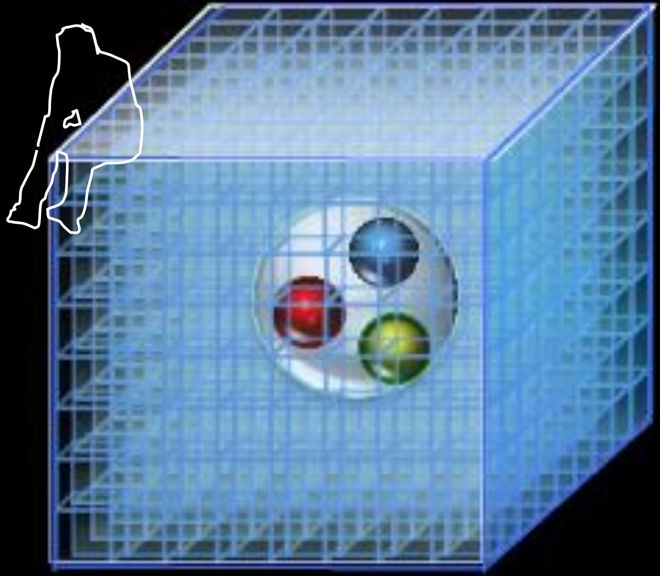
SESAM Coll., Phys.Rev.D71 ('05)





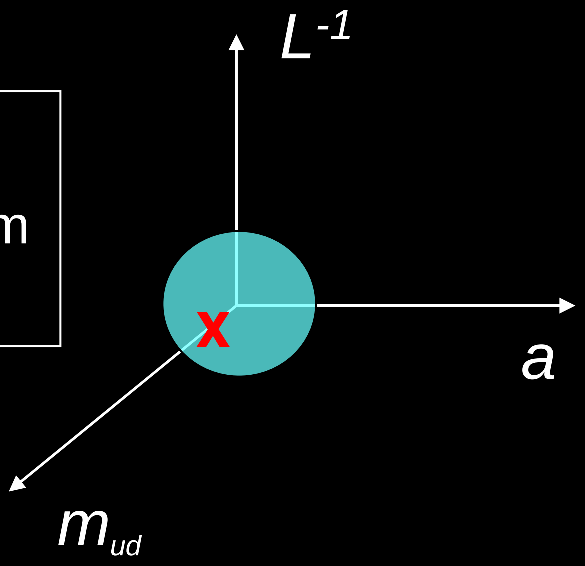
Bali et al., Nucl.Phys.Proc.Suppl. 153 ('06)

Precision Lattice QCD



Three limits

$L^{-1} \rightarrow 0$ (thermodynamics limit) : finite size scaling
$a \rightarrow 0$ (continuum limit) : asymptotic freedom
$m \rightarrow 0$ (chiral limit) : chiral pert. theory



Techniques

Fermions:

Staggered, Wilson, Domain-wall, Overlap

different ways of handling chiral symmetry

Improved actions:

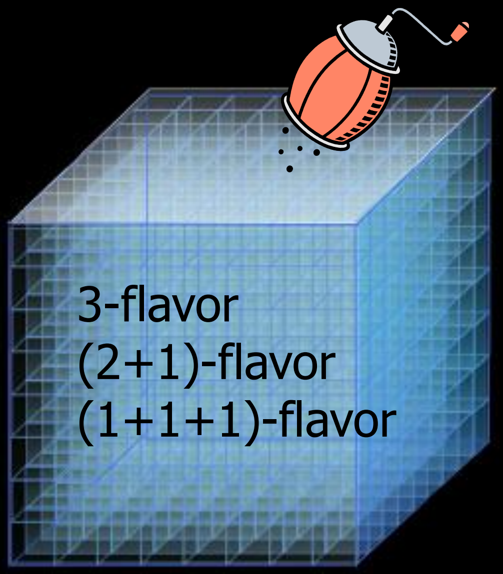
stout, HEX, asktad, HISQ, clover, ...

different ways of reducing the discretization error

Advanced algorithms:

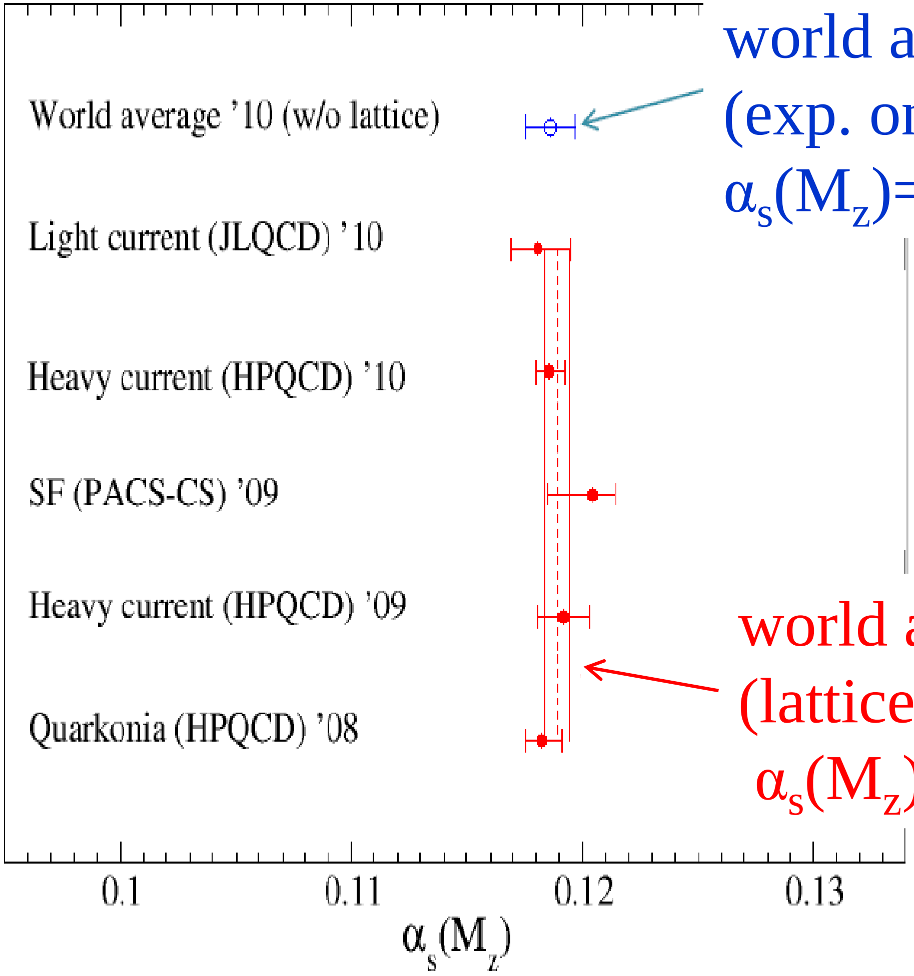
RHMC, DDHMC, LMA, ….

techniques to make the simulations fast and reliable



QCD running coupling

- Nf=2+1 on the lattice

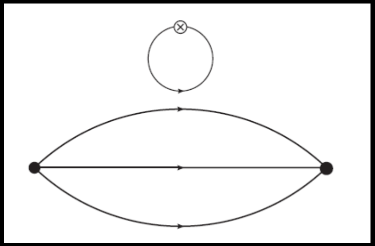


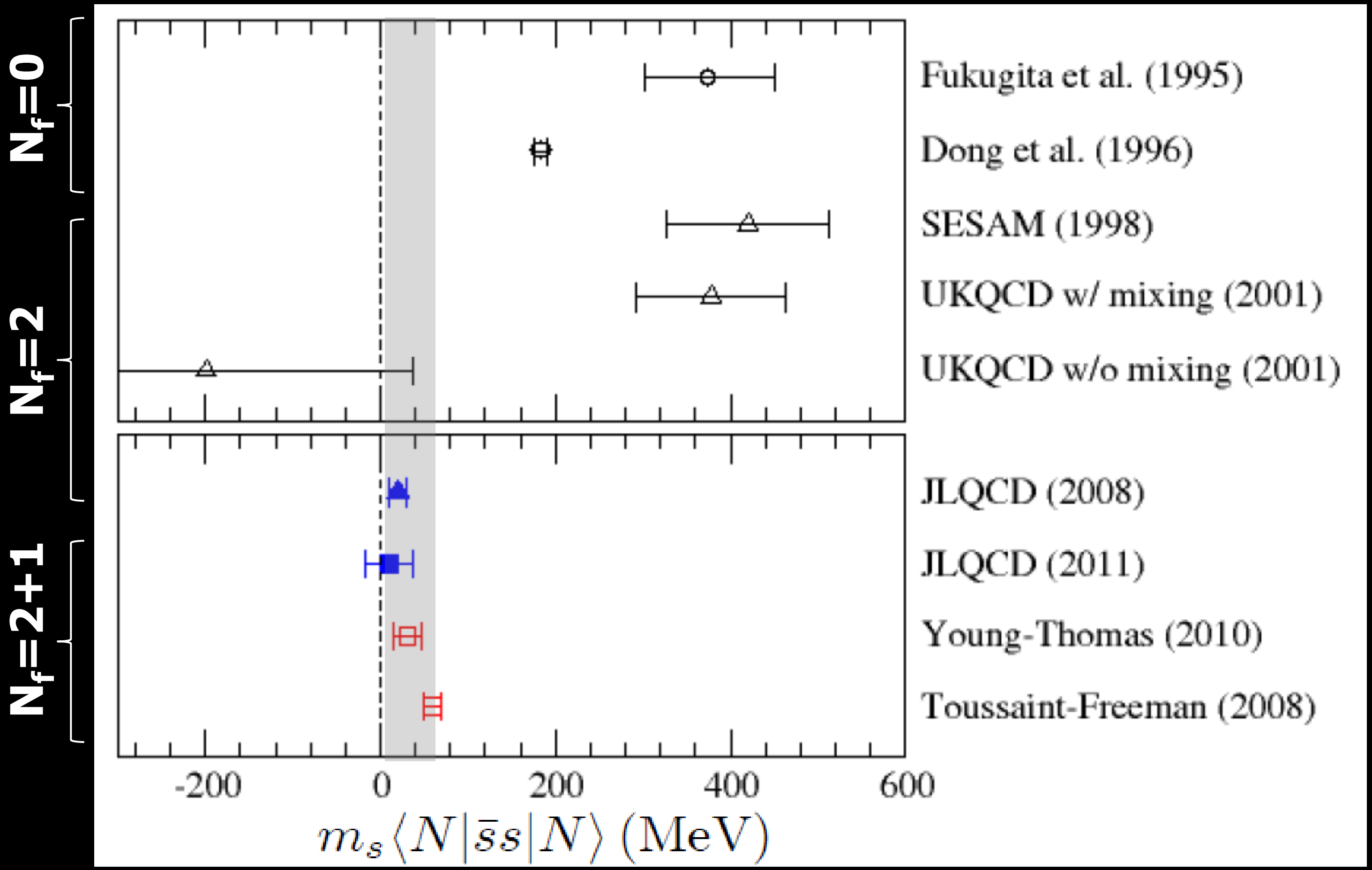
world average
(exp. only)
$\alpha_s(M_z)$=0.1186(11)

world average
(lattice only)
$\alpha_s(M_z)$=0.1189(5)

Summary by Shintani (Lattice2011)

ssbar content of the proton

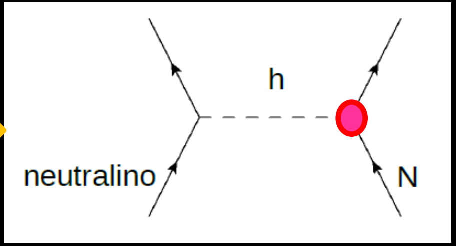




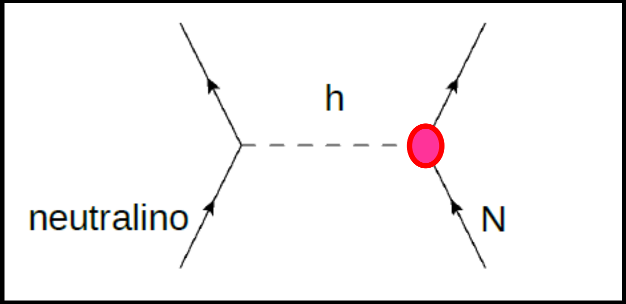
Takeda [JLQCD Coll.], PRD 83 (2011) 114506

Giedt, Thomas, Young PRL103(2009) 201802

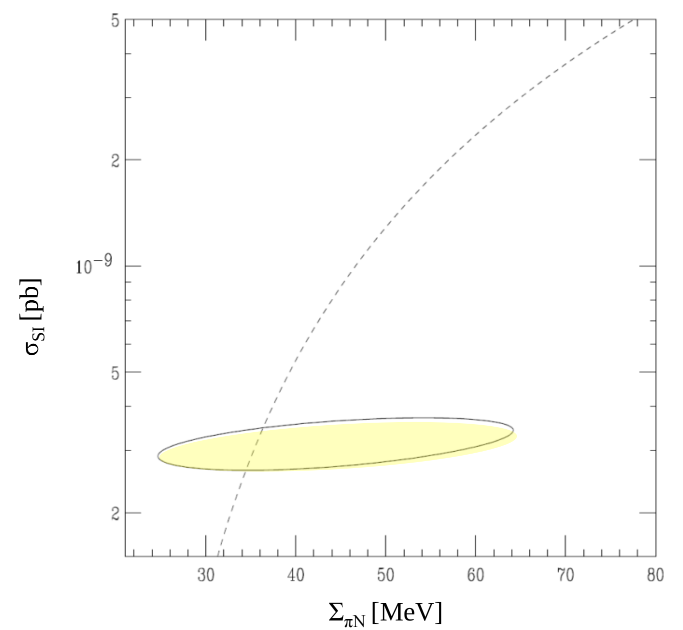
$$m_s\langle N|\bar{s}s|N\rangle < 60\ \text{MeV} \Leftrightarrow y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} < 0.05$$

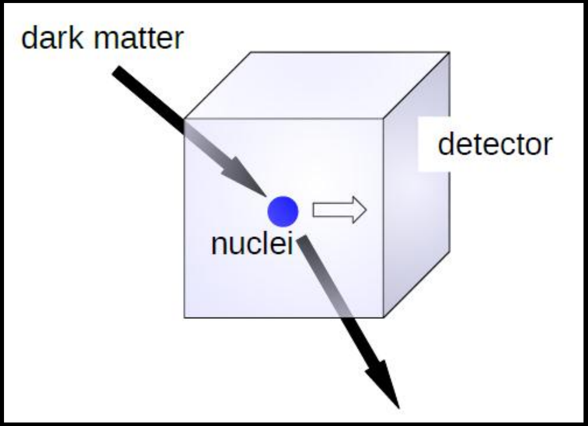


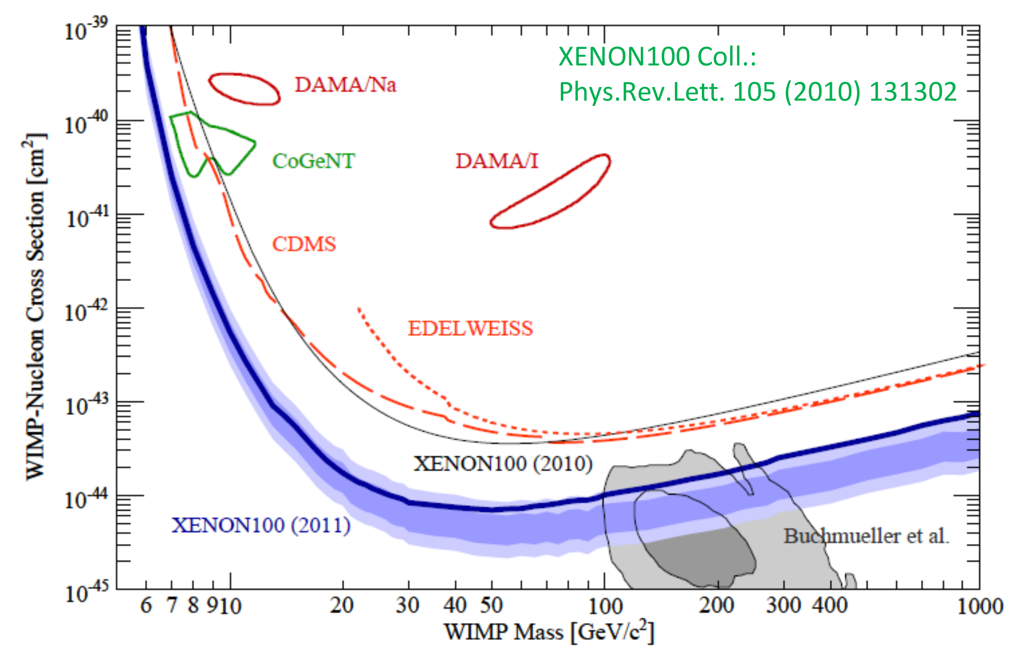
WIMP – Nucleon Interaction



Giedt, Thomas, Young
Phys. Rev. Lett. 103 (2009) 201802

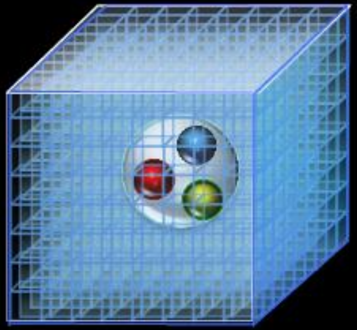






XENON100 Coll.:
Phys.Rev.Lett. 105 (2010) 131302

Hadron masses @ 2009

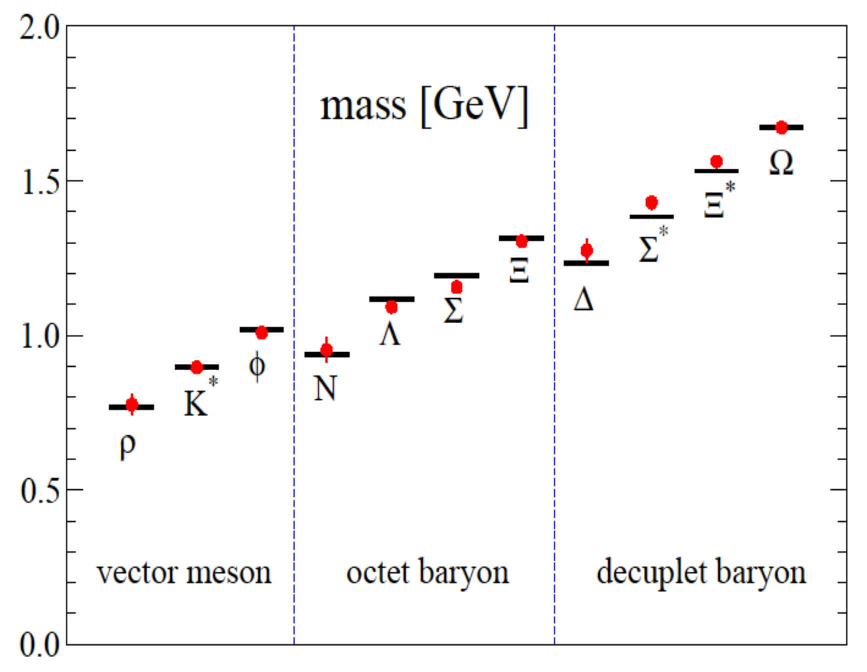


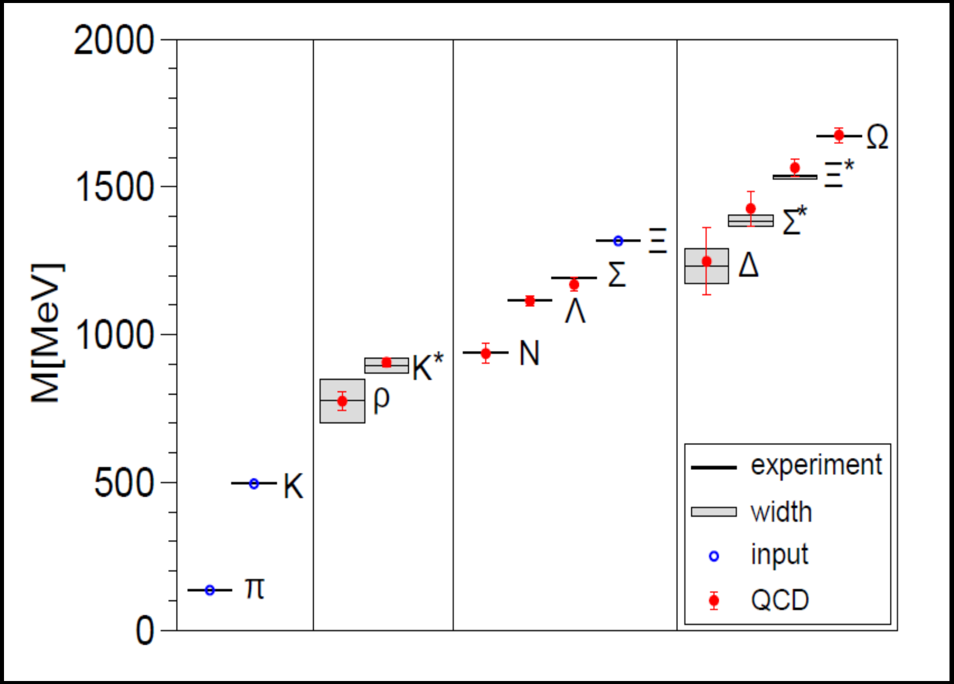
PACS-CS Collaboration,
Phys.Rev.D79(2009)034503

(2+1)-flavor, Wilson
L =2.9 fm, a =0.09 fm
m_π(min) =156 MeV

BMW Collaboration,
Science 322 (2008) 1224

(2+1)-flavor, Wilson
L =(2.0- 4.1) fm, a =0.065, 0.085, 0.125 fm
m_π(min) =190 MeV

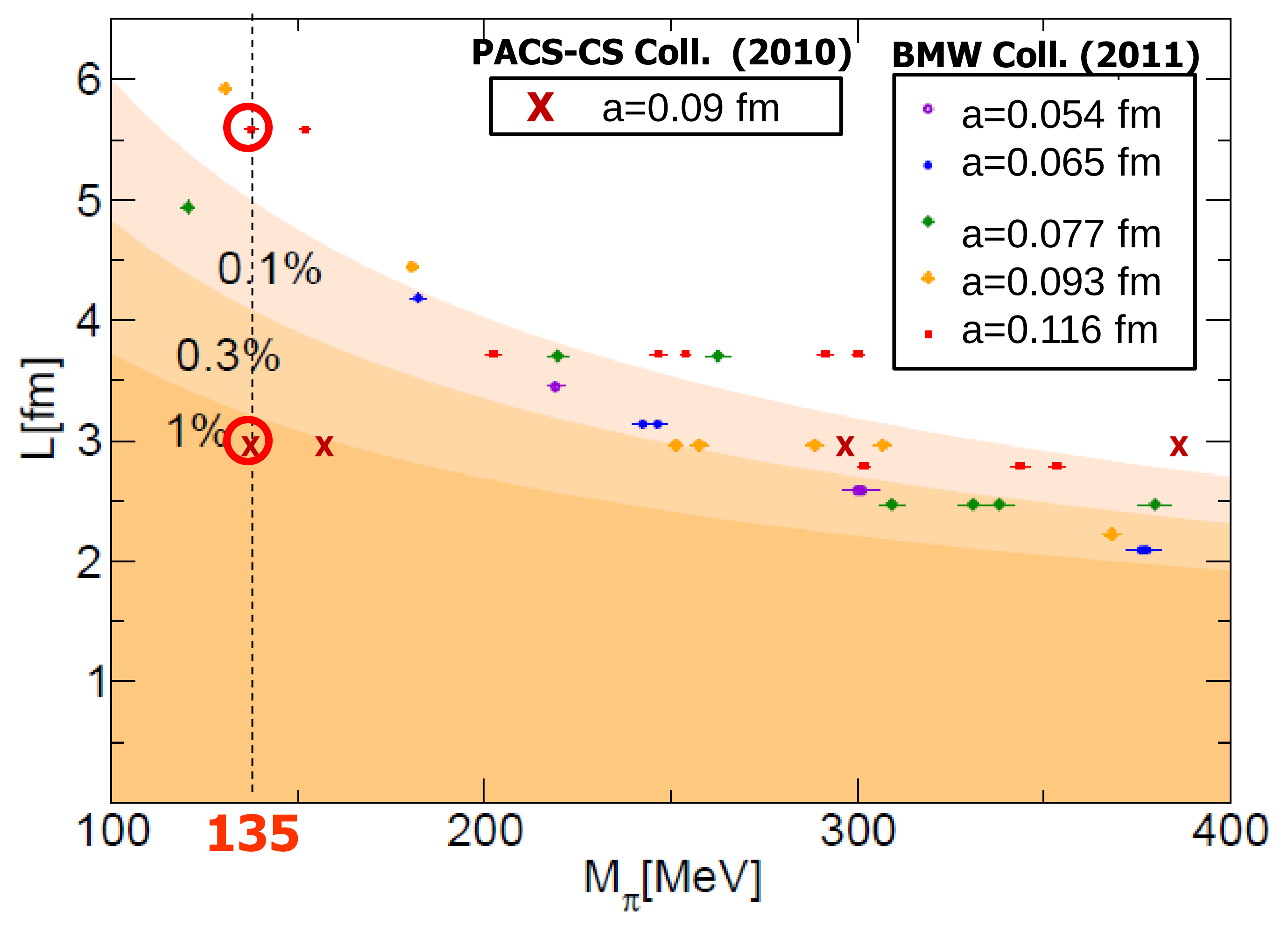




3% accuracy of light hadron masses

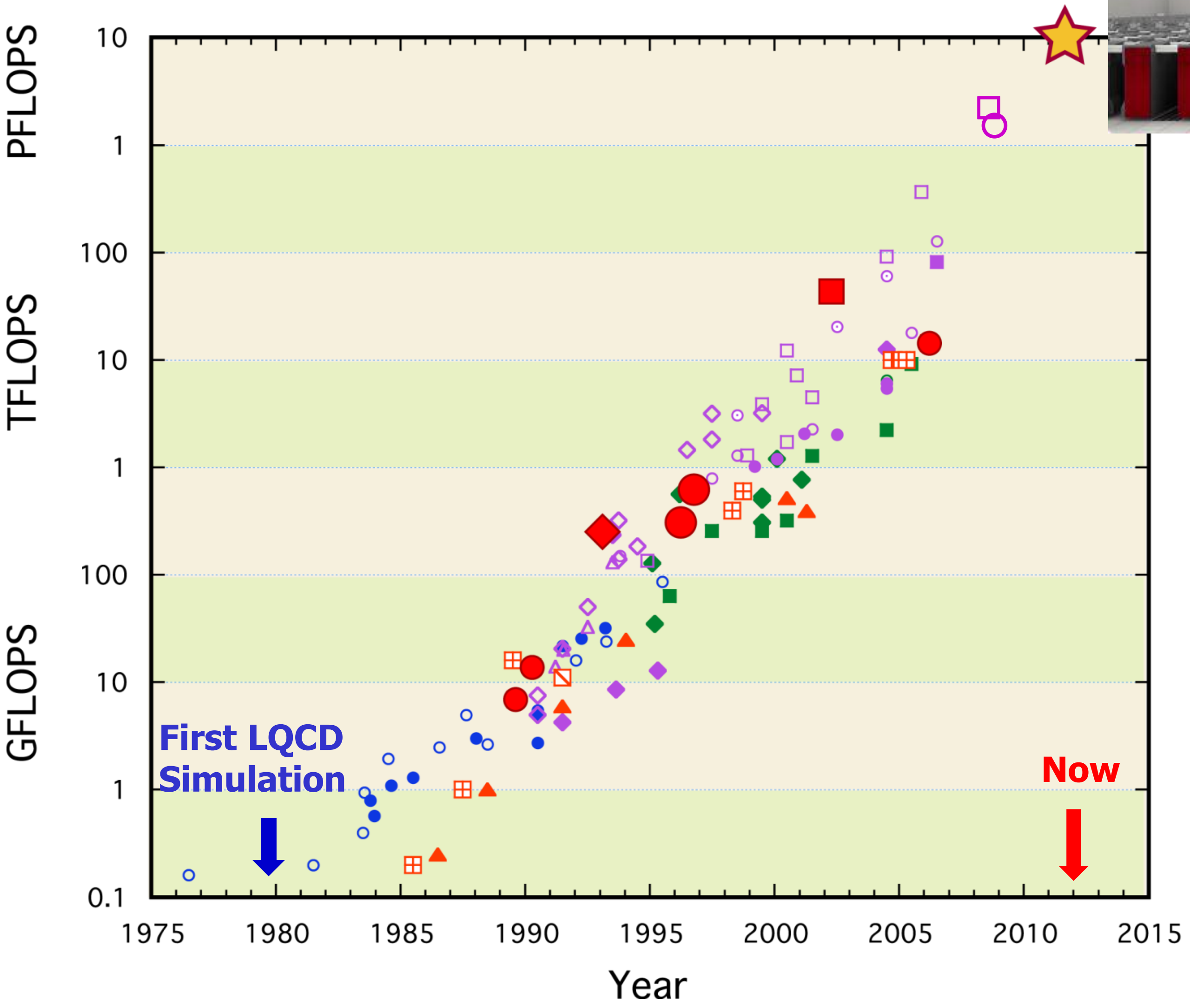
Hadron masses @ 2011

Physical point simulations in (2+1)-flavor QCD



PACS-CS Coll.: Phys. Rev.D81 (2010) 074503
BMW Coll.: Phys. Lett. B701 (2011) 265

Toward large-scale Physical-point simulations



10PFlops K computer (RIKEN)

original plot by A. Ukawa

Advanced Institute for Computational Science (AICS)

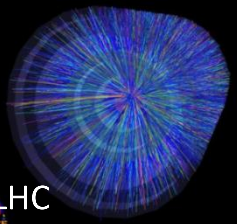
10 Pflops supercomputer KEI “京” (full operation in 2012)

http://www.aics.riken.jp/en/

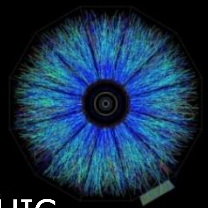
Five “strategic” programs (FY 2010-2015)

1. Life and Medicine
2. New Materials
3. Environment
4. Engineering
5. Particle, Nuclear and Astrophysics

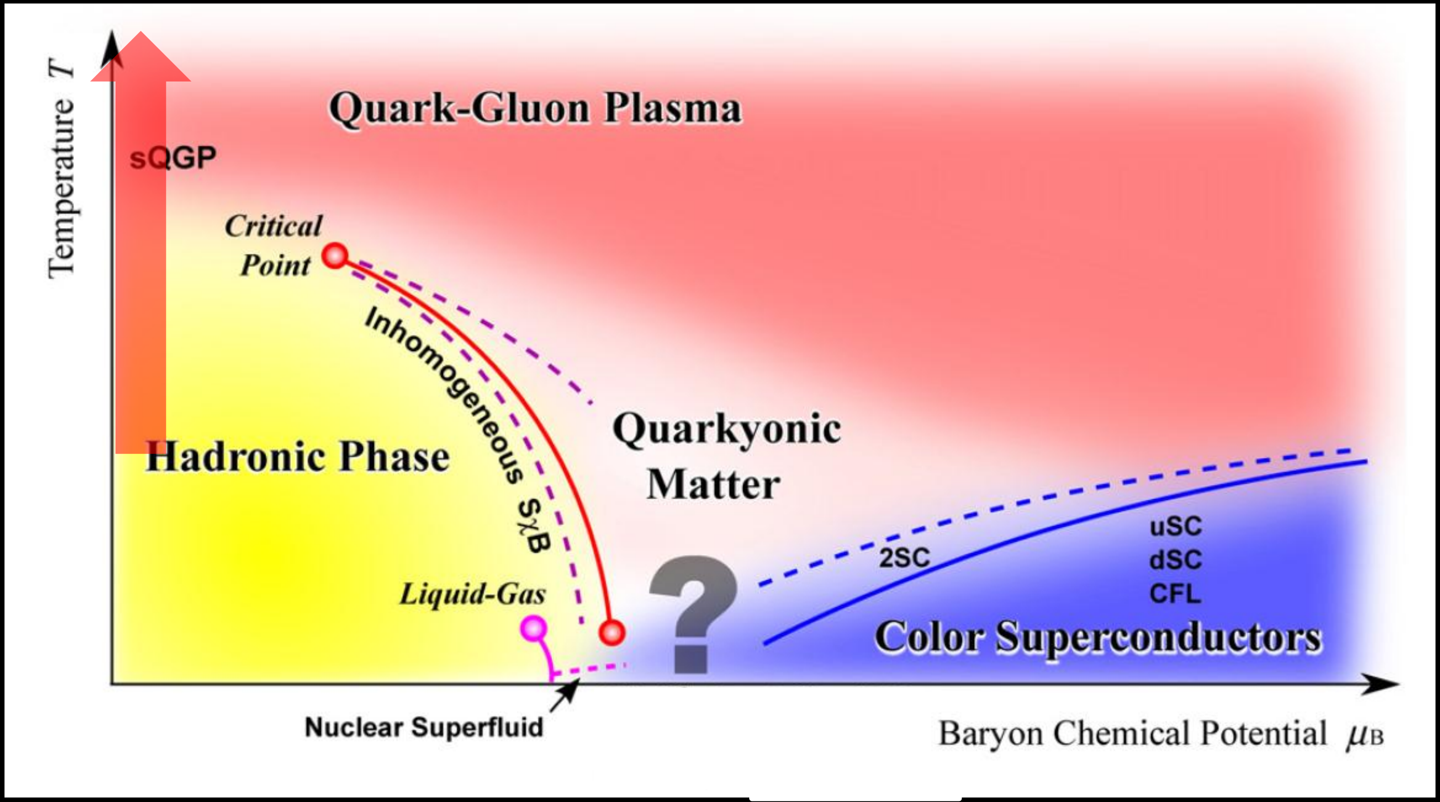
Hot QCD



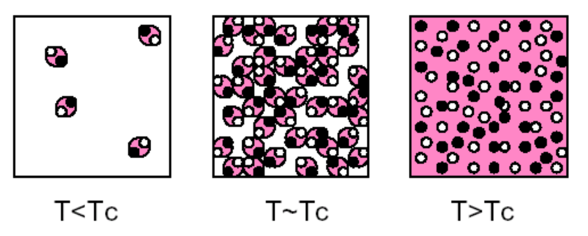
LHC

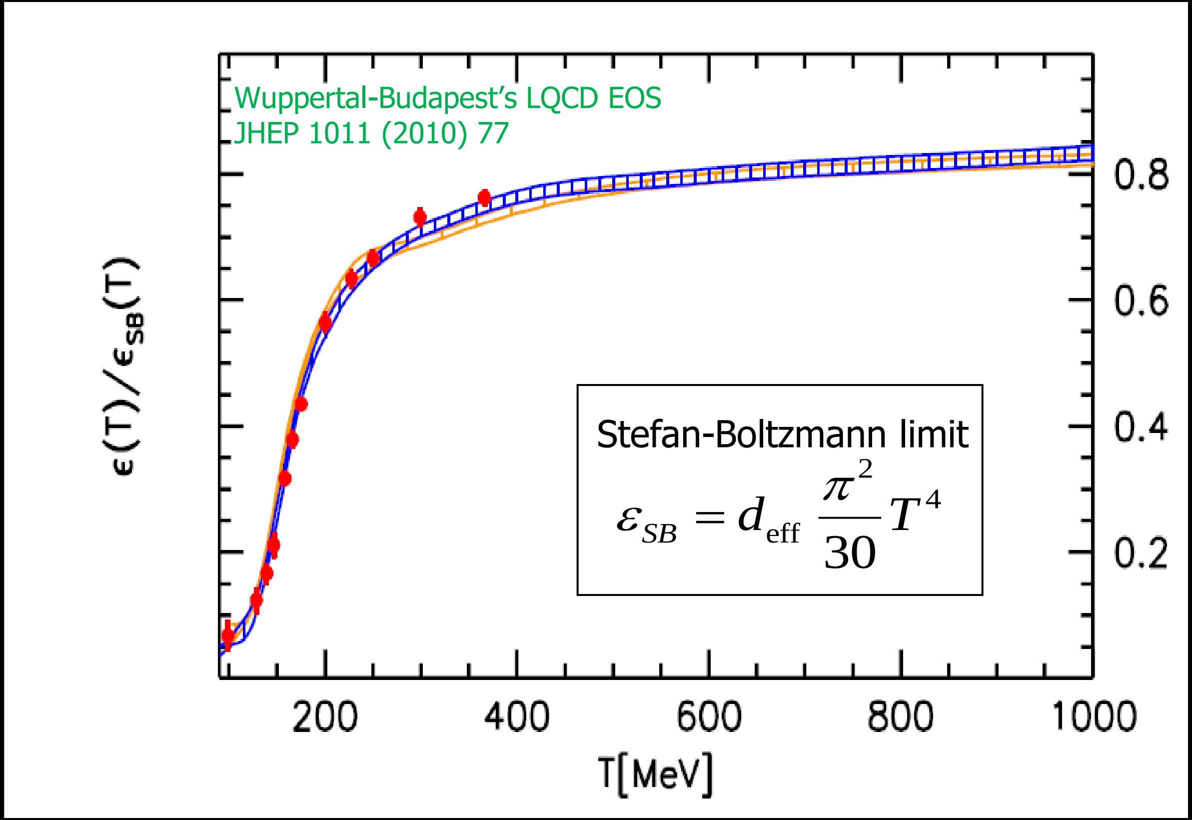


RHIC



Thermal QCD transition





SPS@CERN

RHIC@BNL

LHC@CERN

Order of QCD Transition

2nd order	(u,d; m=0)
1st order	(u,d,s; m=0)
crossover	(real world)

"Critical" Temperature

T_c : ~ 160 MeV

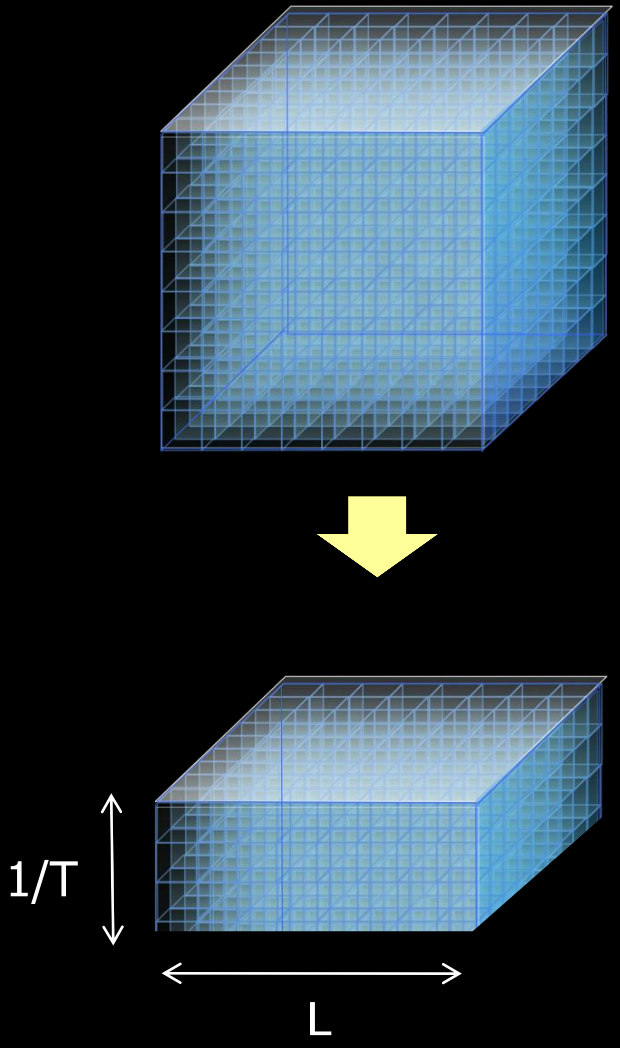
~ 10^{12} [K]

Critical Energy Density

ε_c : ~ 2 GeV/fm^3

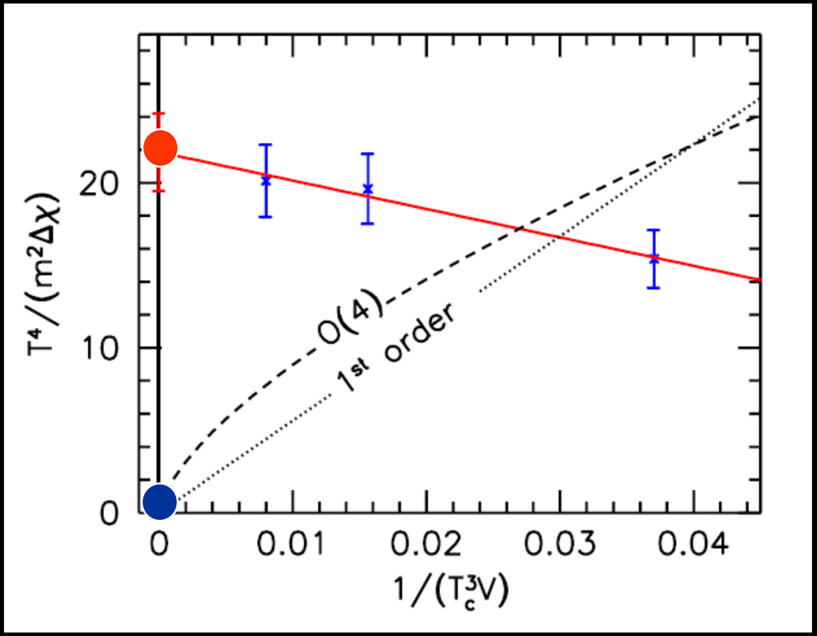
~ 10 ε_{nm}

Order of the QCD transition (T≠0, μ=0)



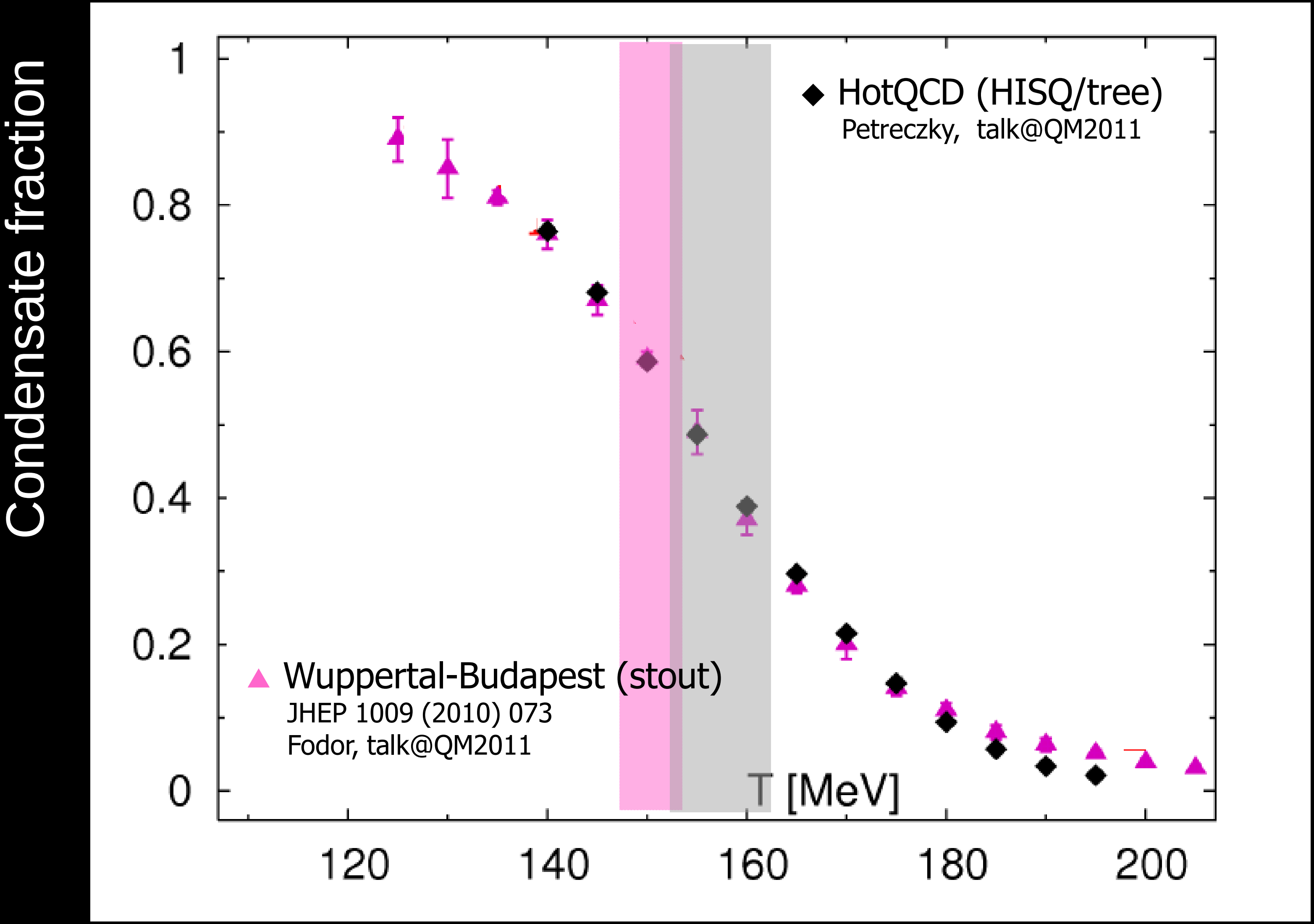
Finite size scaling

$$\chi_m = \frac{\partial^2 P}{\partial m_{\mathrm{ud}}^2} \sim \begin{cases} V & \text{1st order} \\ V^{2/3} & \text{2nd order} \\ V^0 & \text{crossover} \end{cases}$$



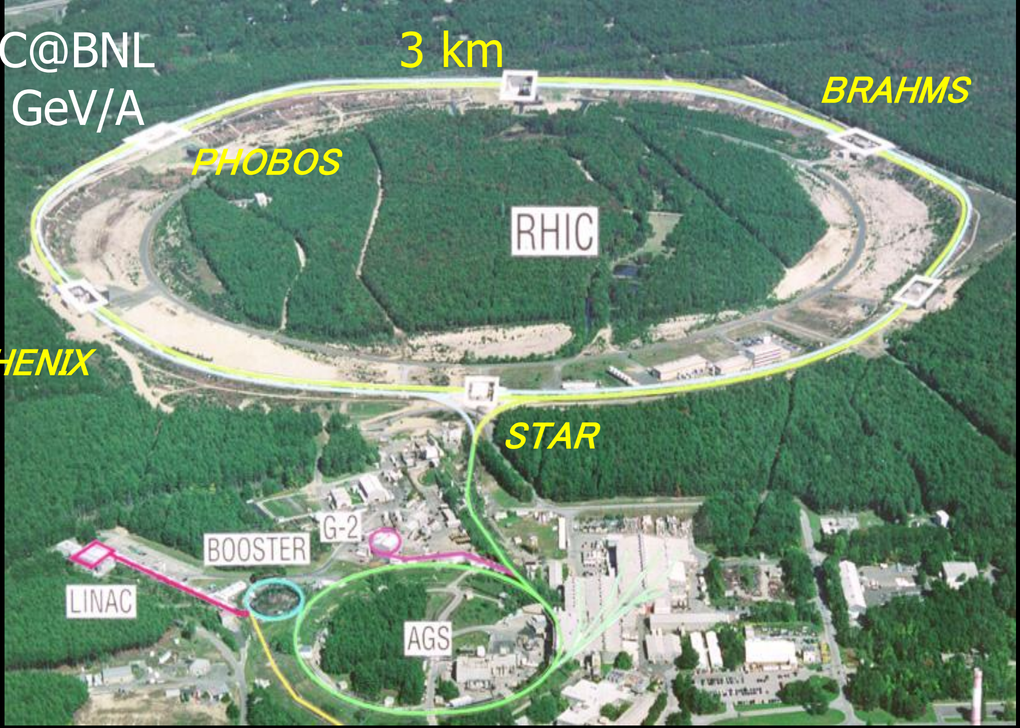
Budapest group, Nature 443 (2006) 675
Staggered, (2+1)-flavor, physical mass

Pseudo-critical temperature



Chiral susceptibility peak ⇒ T_{pc}=150-160MeV

RHIC@BNL
200 GeV/A

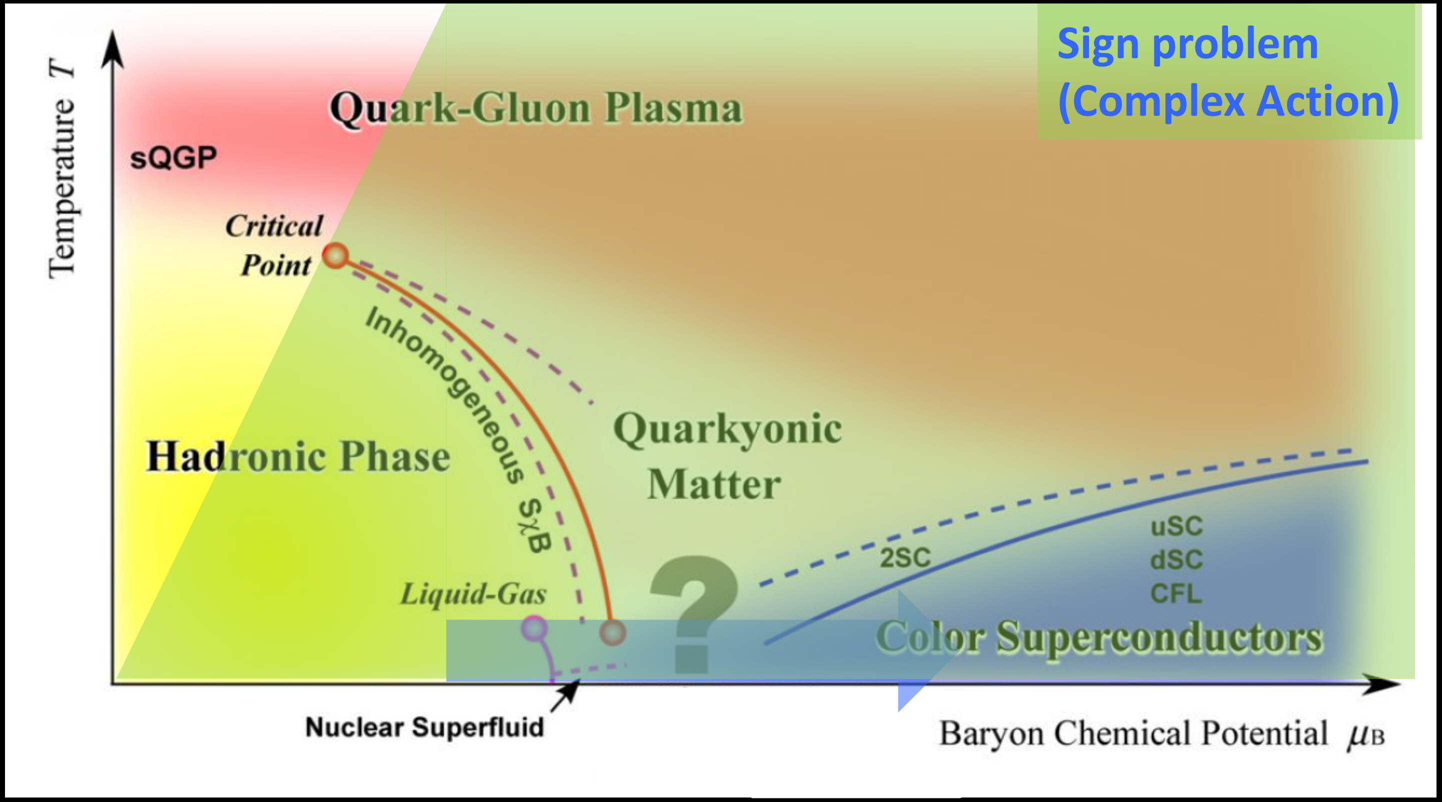


LHC@CERN
5.6 TeV/A

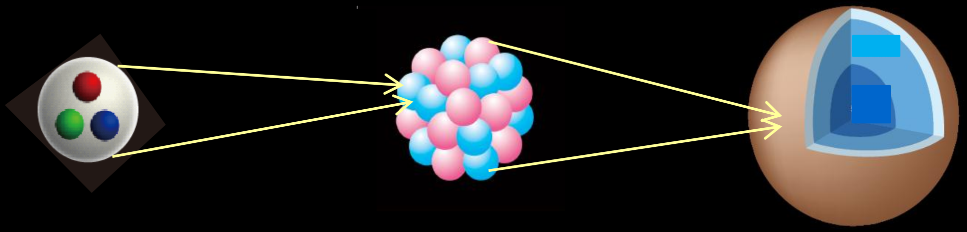
Image courtesy of CERN

→ X. N. Wang's talk

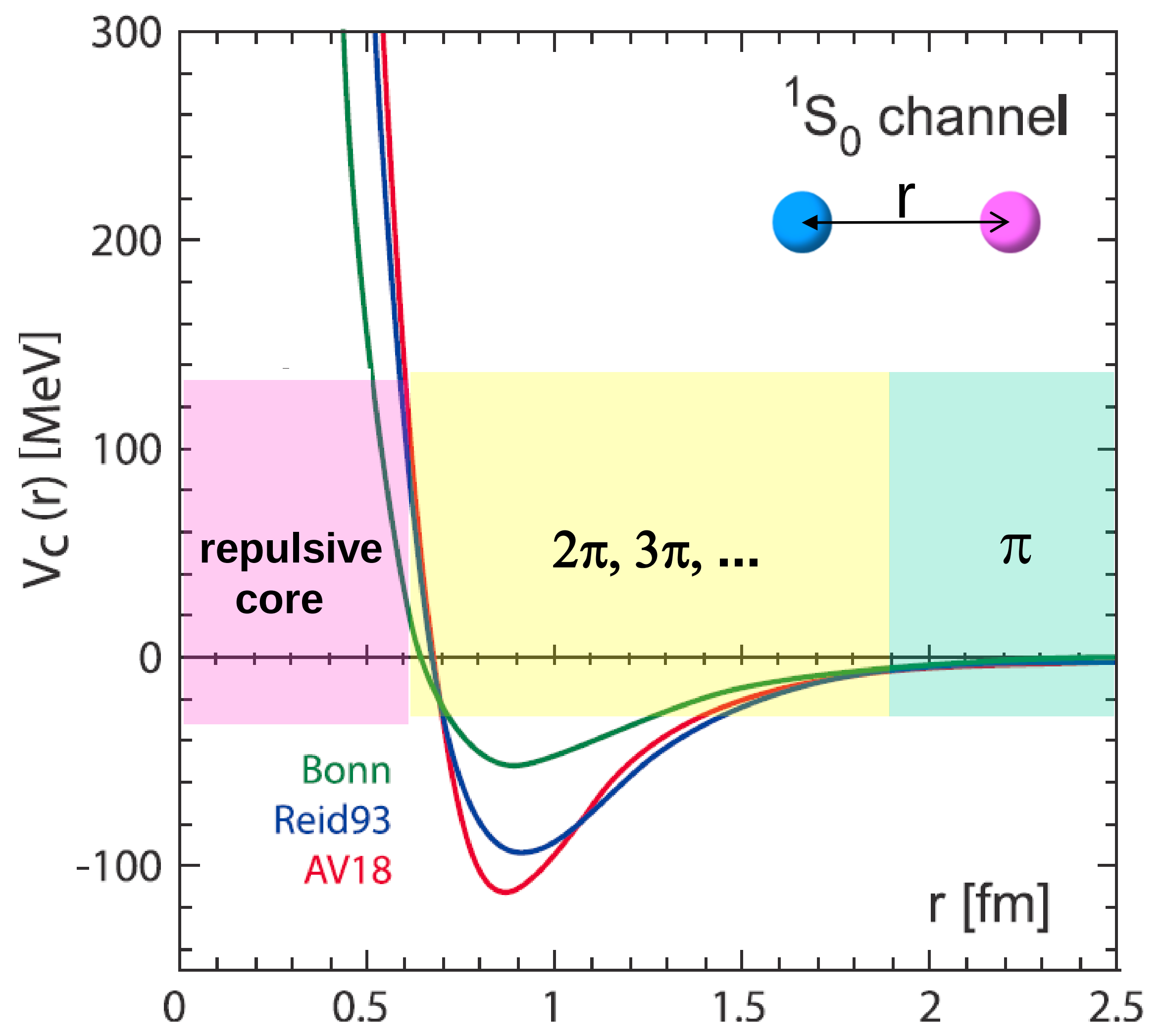
Dense QCD



Sign problem
(Complex Action)



Nuclear force: a brief history



- One-pion exchange
  Yukawa (1935)

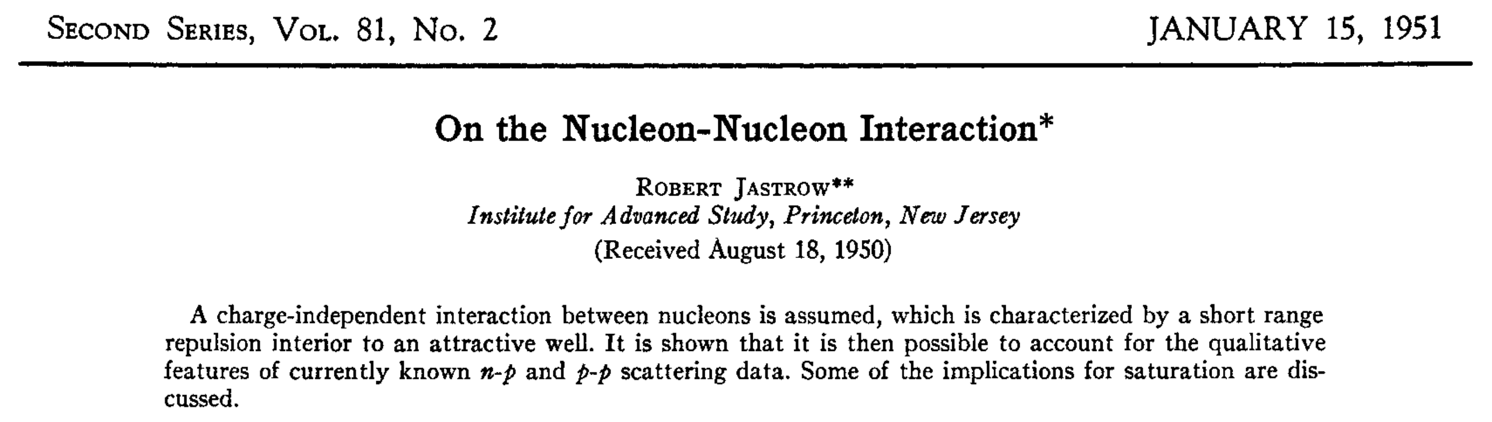
- Multi-pion
  Taketani et al. (1951)

- Repulsive core

Jastrow (1951) Nambu (1957)

- EFT
  Weinberg (1990)

high precision NN force (90's-)
30-40 parameters
5000 phase shift data



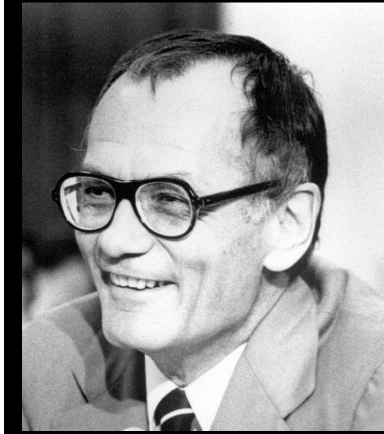
SECOND SERIES, VOL. 81, NO. 2 JANUARY 15, 1951

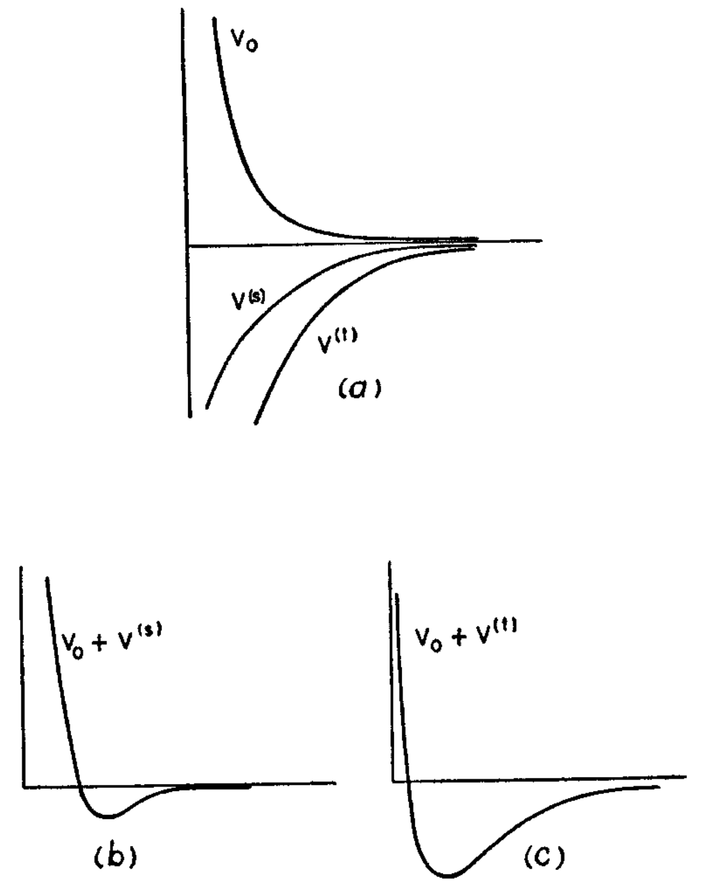
On the Nucleon-Nucleon Interaction*

ROBERT JASTROW**
Institute for Advanced Study, Princeton, New Jersey

(Received August 18, 1950)

A charge-independent interaction between nucleons is assumed, which is characterized by a short range repulsion interior to an attractive well. It is shown that it is then possible to account for the qualitative features of currently known *n-p* and *p-p* scattering data. Some of the implications for saturation are discussed.





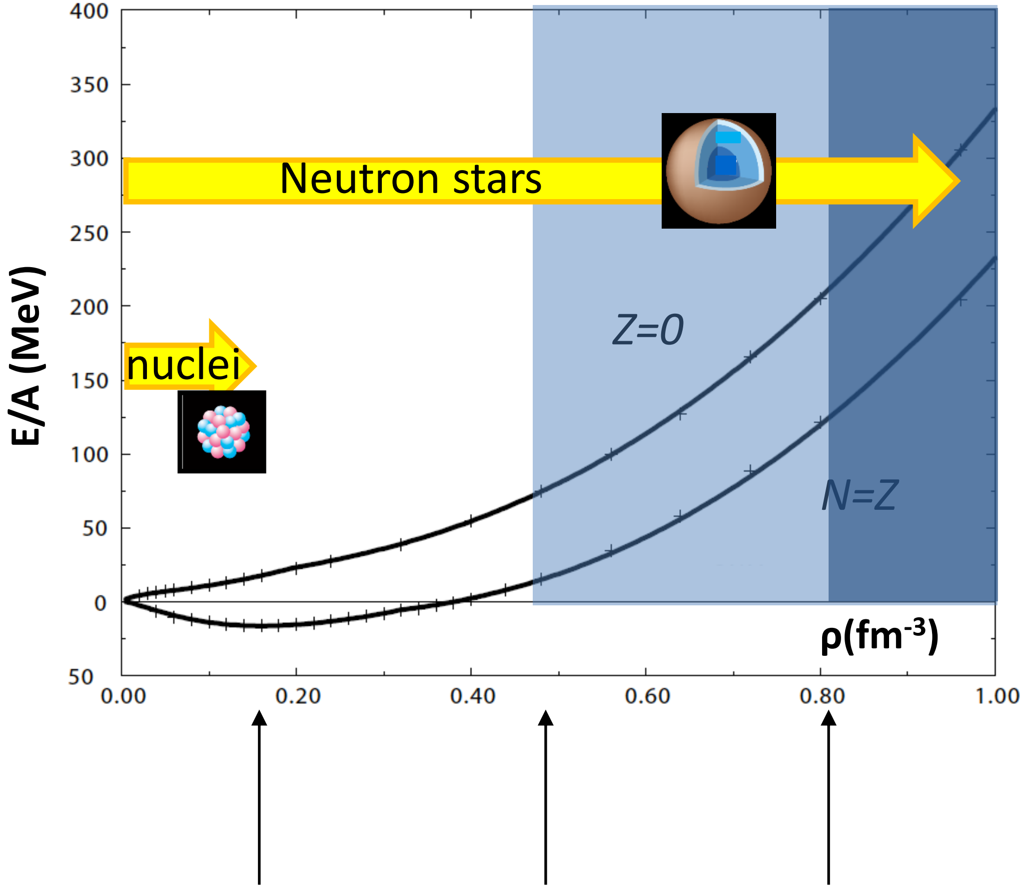
So I got up in the question period and I said, "Maybe the reason is that inside the nuclear force of attraction, which holds nuclei together, there's a very strong short-range force of repulsion, like a little hard sphere inside this attractive Jell-O."

I'll never forget, Oppenheimer got up, he liked to needle the young fellows and he said, very dryly, "Thank you so much for, we are grateful for every tiny scrap of help we can get." But I ignored his needle and pursued my idea, and actually calculated the scattering of neutrons by protons. I showed that it fit the data very well. Oppenheimer read my paper for the Physical Review and took back his criticisms. This work became a permanent element of the literature of physics.

http://www.marshall.org/article.php?id=30

Nuclear Force and EoS of Dense Matter

Akmal, Pandharipande & Ravenhall, PRC58 ('98)

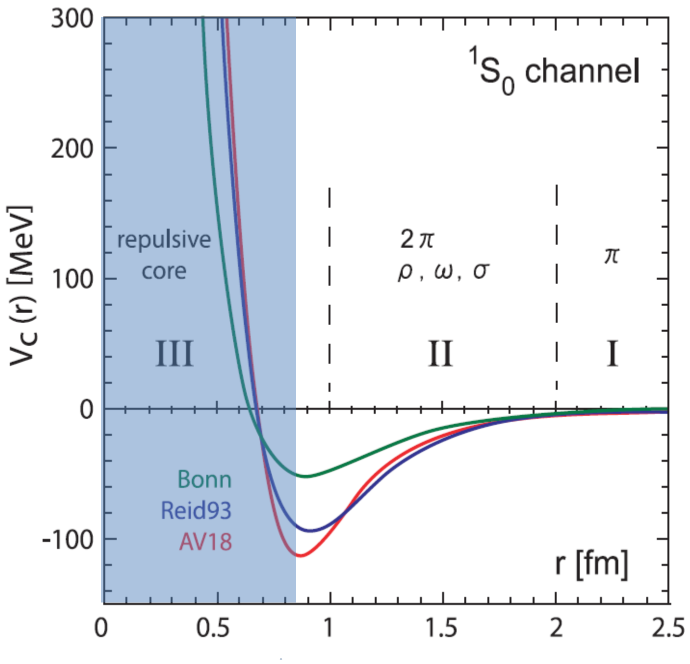


ρ_0
= 0.16 fm^{-3}

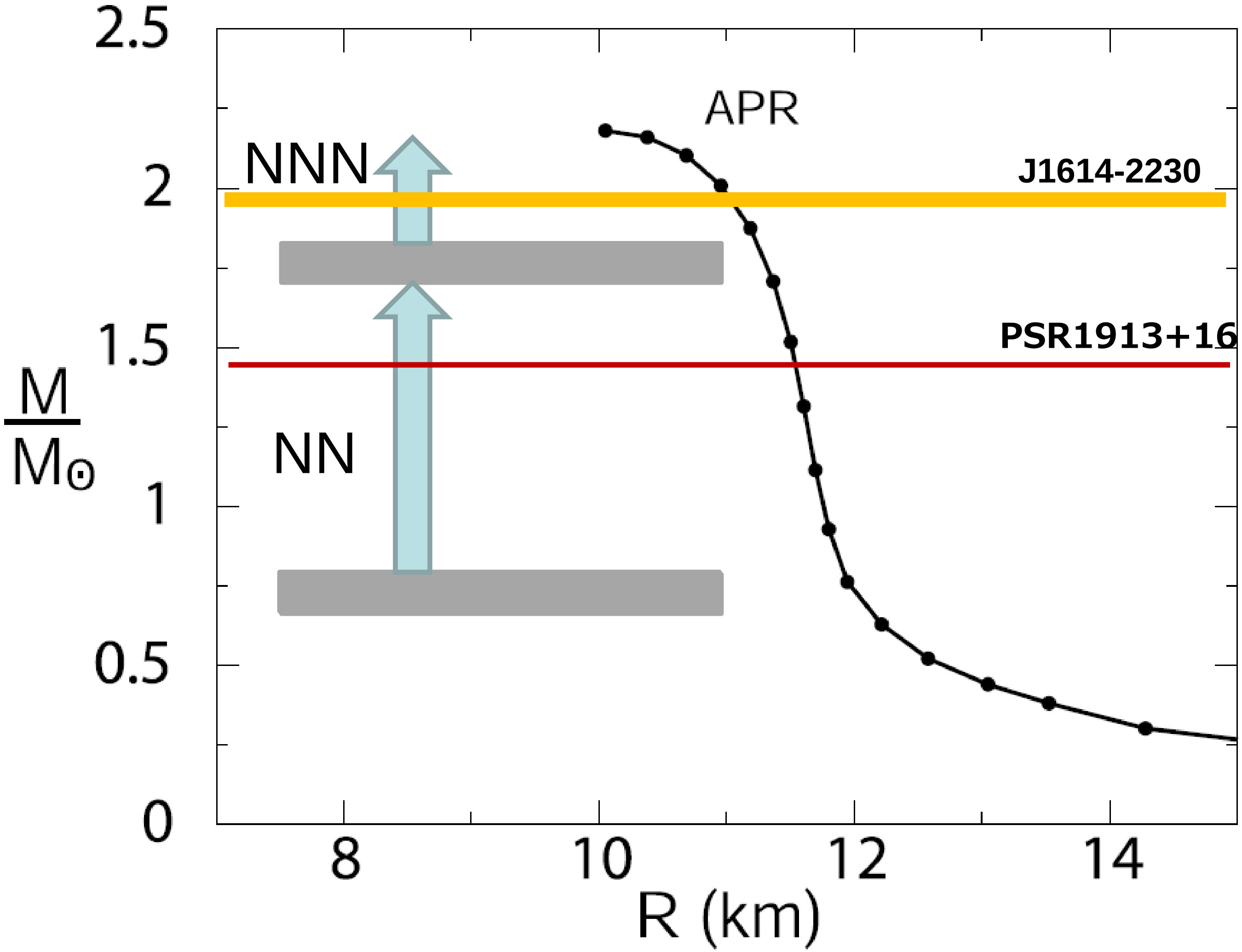
$3\rho_0$

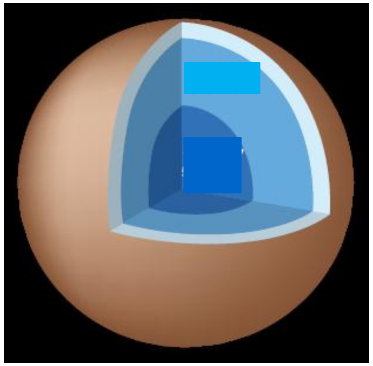
$5\rho_0$

Phenomenological NN forces



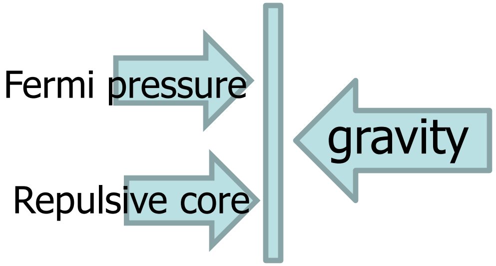
Nuclear Force and Neutron Star



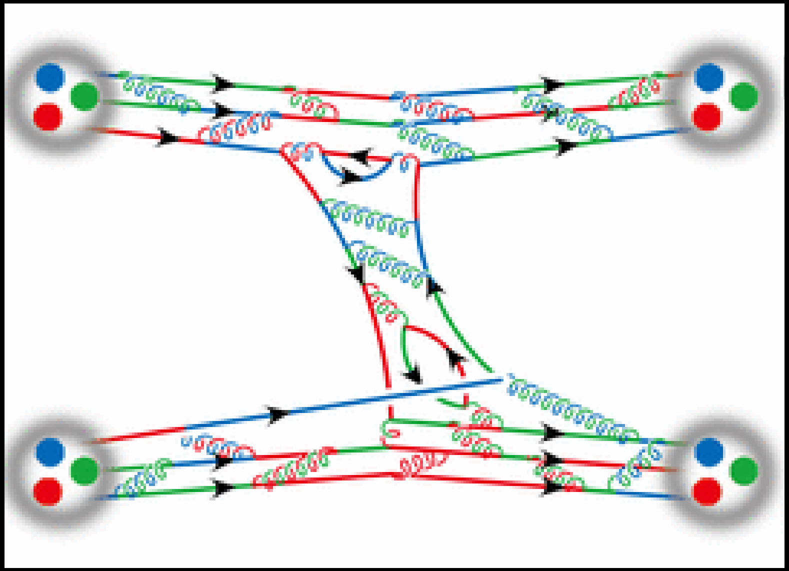


(ρ_{max} ~ $6\rho_0$)

Pressure balance



Nuclear Force from Lattice QCD



Hadrons to Atomic nuclei
HAL
from Lattice QCD

Univ. Tsukuba	S. Aoki, N. Ishii, H. Nemura, K. Sasaki
RIKEN	K. Murano, T. Doi, T. Hatsuda
Nihon Univ.	T. Inoue
Tokyo Inst. Tech.	Y. Ikeda
Univ. Tokyo	B. Charron

How to define the NN potential from QCD ?

1. NN wave function from lattice QCD

$$\phi_n(\vec{r},t) = \langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)|E_n\rangle$$

$$\phi(\vec{r},t) = \sum_n c_n \phi_n(\vec{r},t)$$

1. NN potential from the NN wave function

$$\left(-\frac{\partial}{\partial t} - H_0\right)\phi(\vec{r},t) = \int U(\vec{r},\vec{r}')\phi(\vec{r}',t)d^3r'$$

3. Derivative expansion

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta^3(\vec{r}-\vec{r}')$$

$$V(\vec{r},\nabla) = V_{\mathrm{C}}(r) + S_{12}V_{\mathrm{T}}(r) + \vec{\mathrm{L}}\cdot\vec{\mathrm{S}}\ V_{\mathrm{LS}}(r) + \{V_{\mathrm{D}}(r),\nabla^2\} + \cdots$$

LO LO NLO NNLO

Ishii, Aoki, Hatsuda, Phys.Rev.Lett. 99 (2007) 022001
\+ Ishii et al. (HAL QCD Coll.)

- ➢ Potential is a nice tool to calculate observables
- ➢ Potential is volume insensitive (=Lattice Friendly)

Key channels in NN scattering ($(^{2s+1}L_J)$)

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\;V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

LO LO NLO NNLO

$^1S_0$ Central force ↔ nuclear BCS pairing

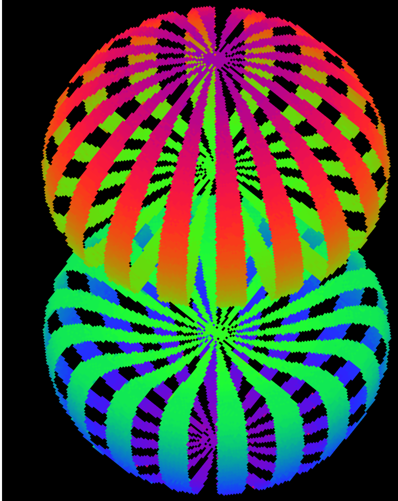
Bohr, Mottelson & Pines, Phys. Rev. 110 (1958)

$^3S_1$-$^3D_1$ Tensor force ↔ deuteron binding

Schwinger, Phys. Rev. 55 (1939), Bethe, ibid.57 (1940)
Rarita & Schwinger, ibid. 59 (1941)

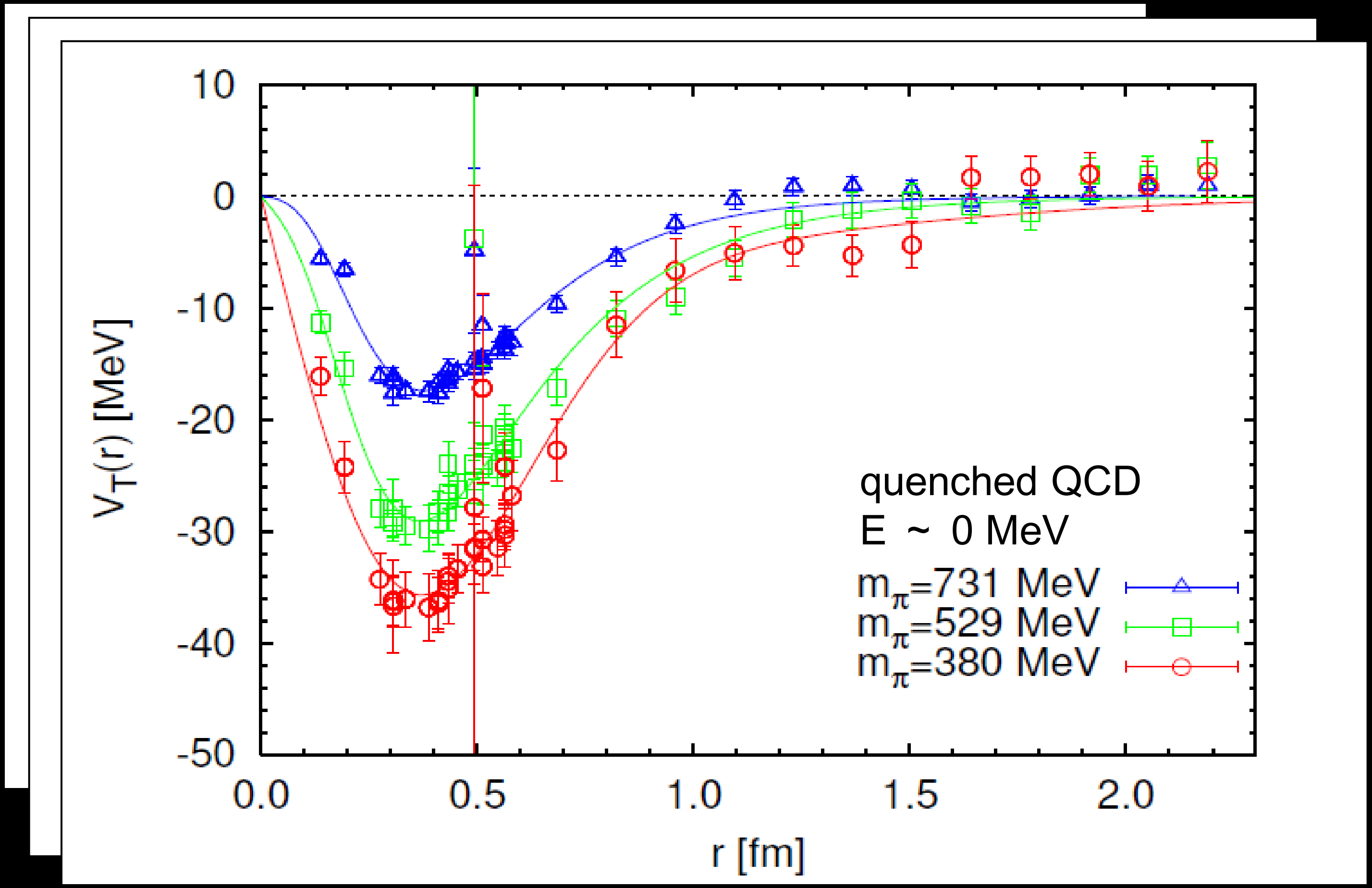
$^3P_2$-$^3F_2$ LS force ↔ neutron superfluidity in neutron stars

Tamagaki, Prog. Theor. Phys. 44 (1970)
Hoffberg et al., Phys. Rev. Lett. 24 (1970)



Density profile of the deuteron with $S_z=\pm1$
Pandharipande et al., (1998)

[Exercise 1] LO potentials : $V_C(r)$ & $V_T(r)$



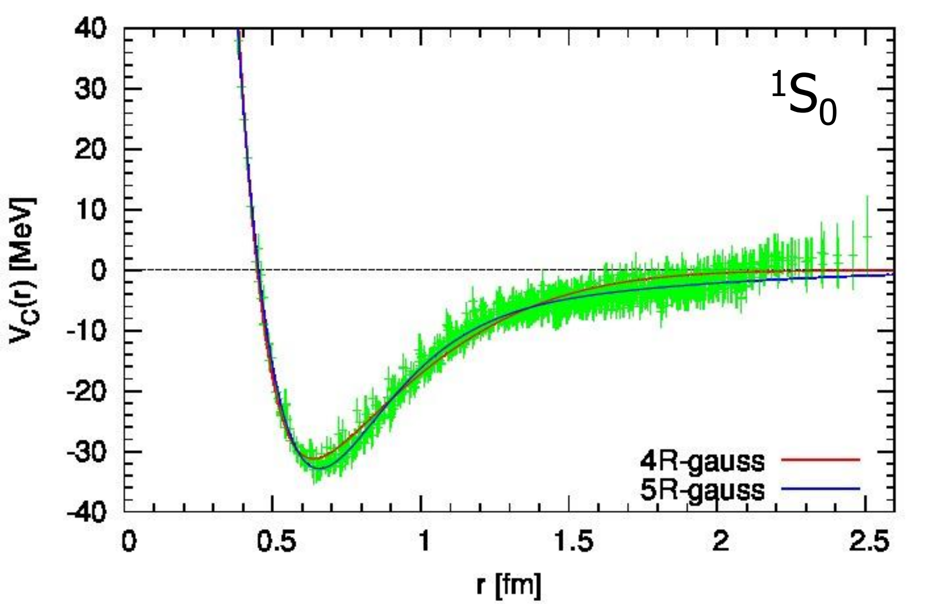
Aoki, Ishii & Hatsuda,
Prog. Theor. Phys. 123 (2010) 89

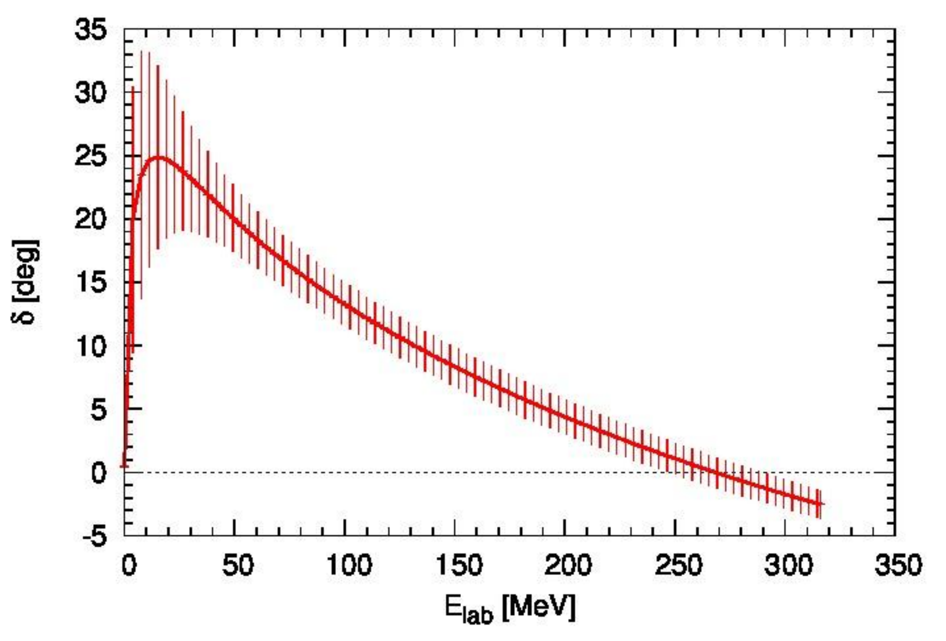
- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

Central potential in (2+1)-flavor QCD

HAL QCD Coll., in preparation

PACS-CS gauge config.
(Clover + Iwasaki)
a= 0.09 fm, L=2.9 fm
m_π= 700 MeV

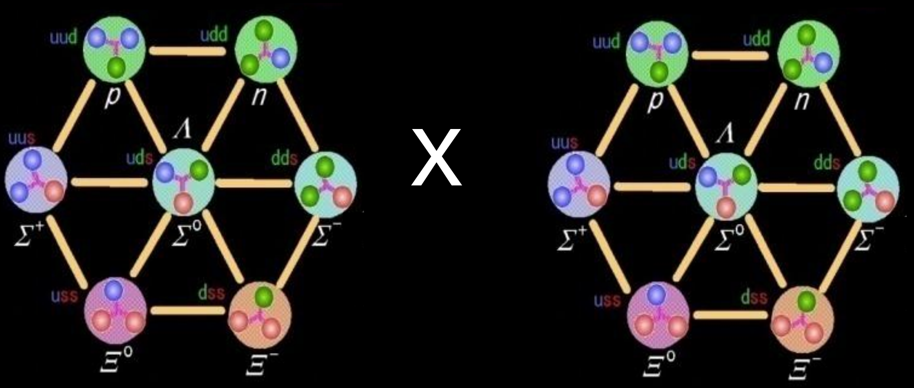




Physical point simulations (m_π=135MeV with L=6fm & 9fm)
will be carried out at KEI computer

Origin of the "short range NN repulsion" ?

⇒ Baryon-baryon force in flavor SU(3)



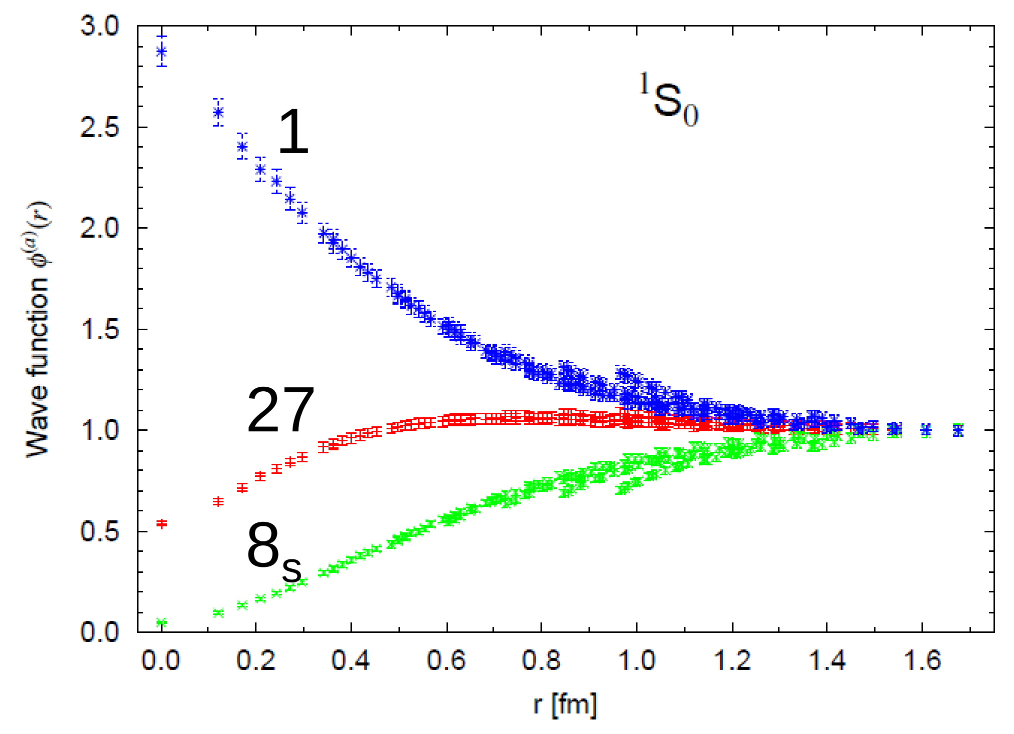
$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{Symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{Anti-symmetric}}$$

<u>Six</u> independent potentials in the flavor-basis

Byproducts

- Hyperon forces : important for hyper-nuclei & neutron stars
- Fate of H-dibaryon: exotic 6-quark state (uuddss) Jaffe, PRL 38 ('77)

Lattice BB wave functions



Iwasaki + clover
(CP-PACS/JLQCD config.)
L=1.9 fm, a=0.12 fm, 16^3x32
m_π=835 MeV, m_B=1752 MeV

Inoue et al. (HAL QCD Coll.)
Prog. Theor. Phys. 124 (2010) 591

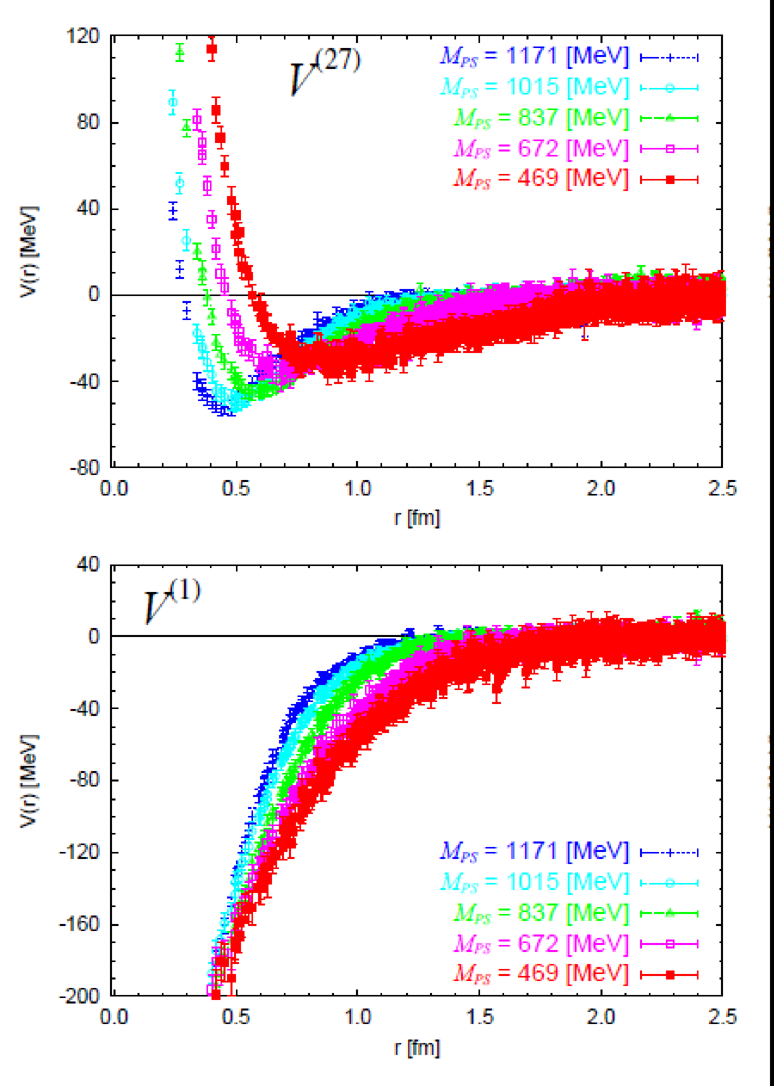
Short range BB int. ⇔ Quark Pauli principle

1 : allowed, 27 : partially blocked, 8_s : blocked

c.f. Urbaryon models (Otsuku-Yasuno-Tamagaki 1965, Machida & Namiki 1965)
Constituent quark model (Oka, Yazaki, Shimizu 1986)

BB potentials in flavor-basis

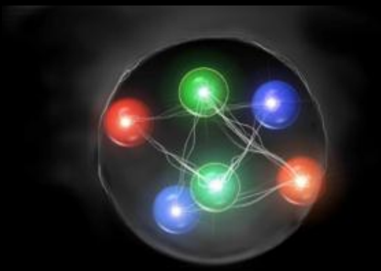
Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002
+ NPA (2012) to appear

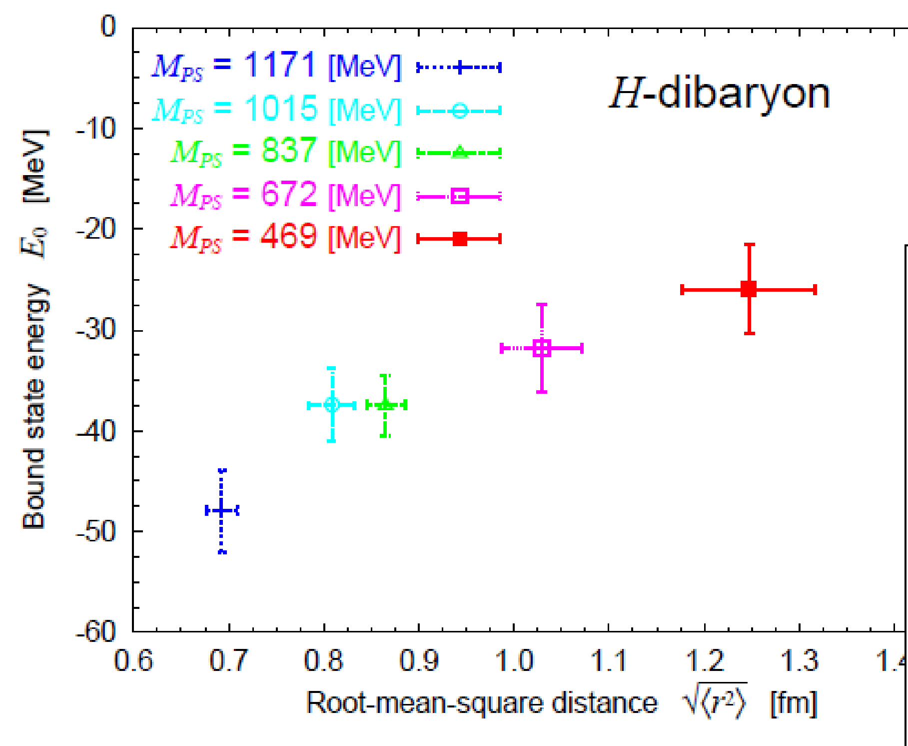


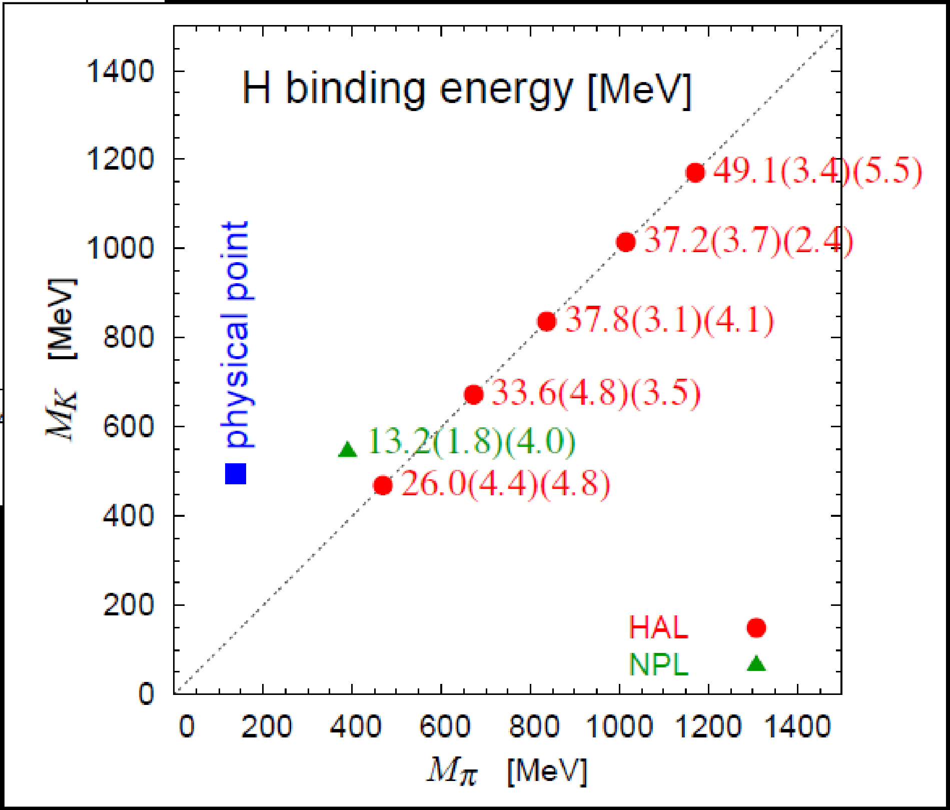
Repulsive core in NN channel

Attractive core in H channel

H-dibaryon from LQCD







At physical point:

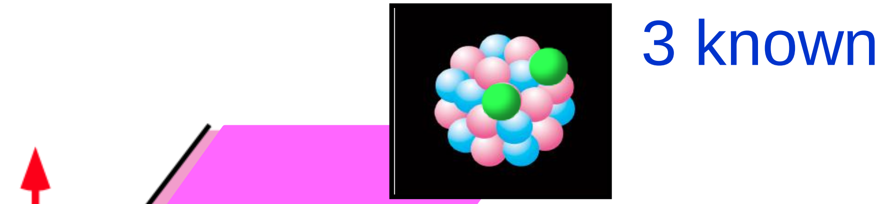
$M_{\Lambda\Lambda} < M_H < M_{\Xi N}$?

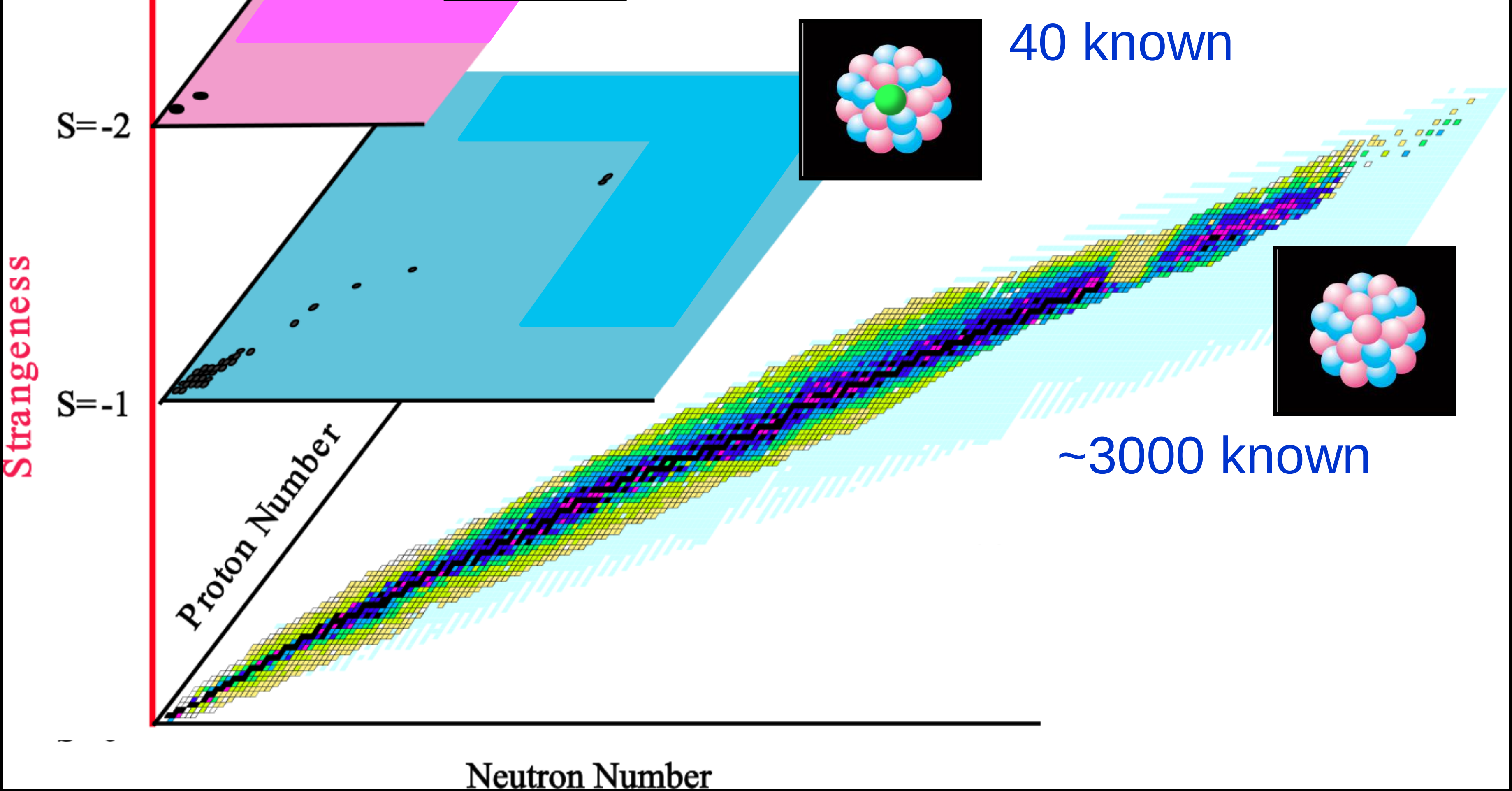
Inoue et al. [HAL QCD Coll.],
NPA (2012) to appear

⇒ exp. search at RHIC & J-PARC

Hypernuclei at J-PARC


J-PARC@KEK, Japan（2009-）


3 known

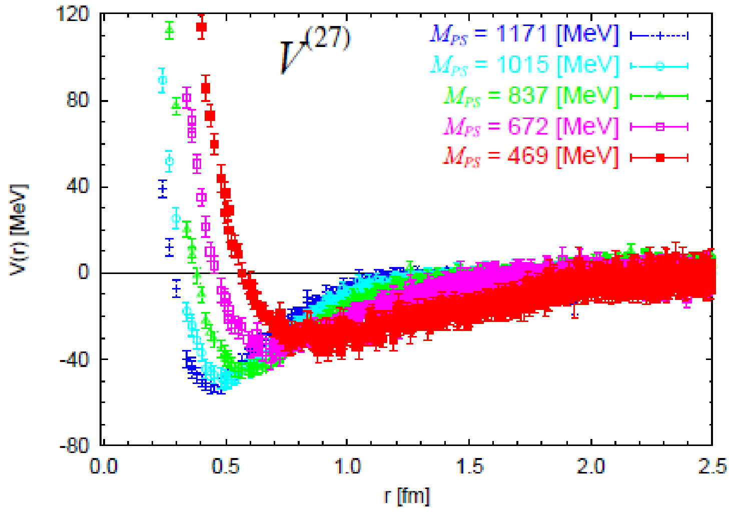

40 known

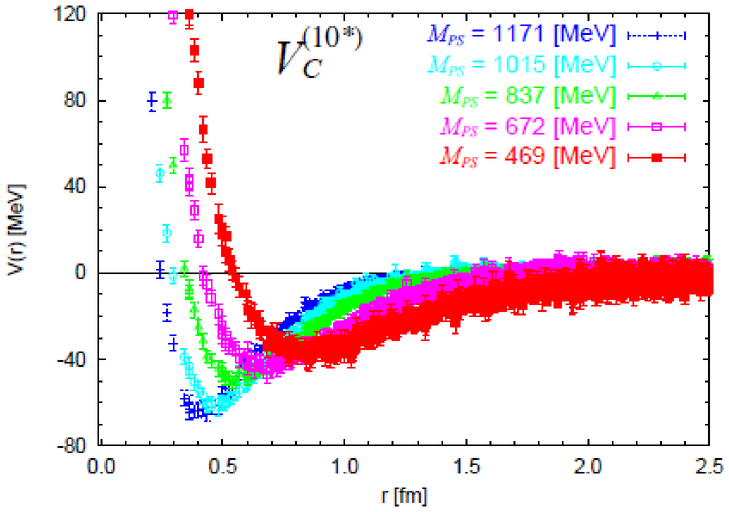
~3000 known

BB potentials in flavor-basis

Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002
+ NPA (2012) to appear

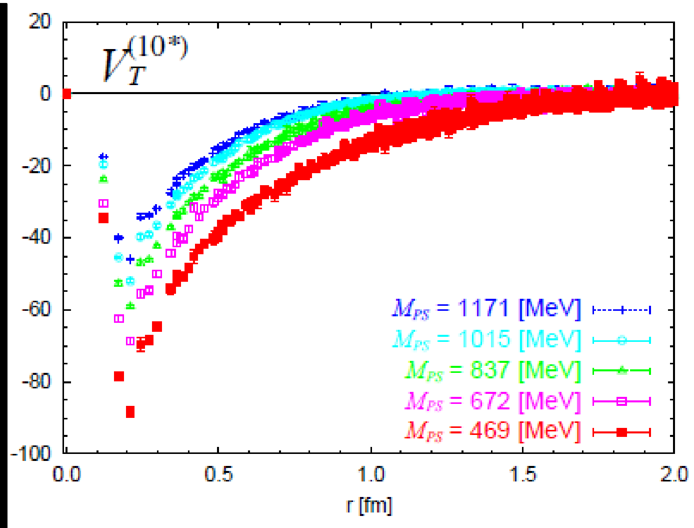
Repulsive core in NN channel





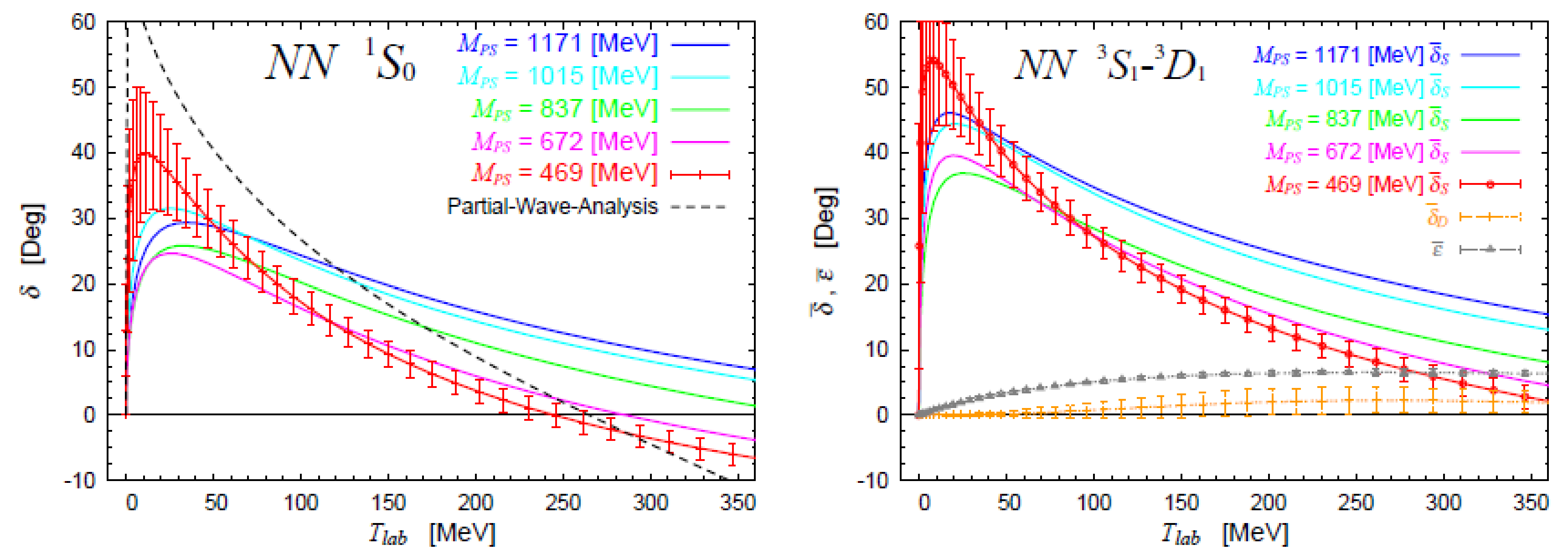
$^1S_0$ channel

$^3S_1 - {}^3D_1$ channel



Growing NN tensor force

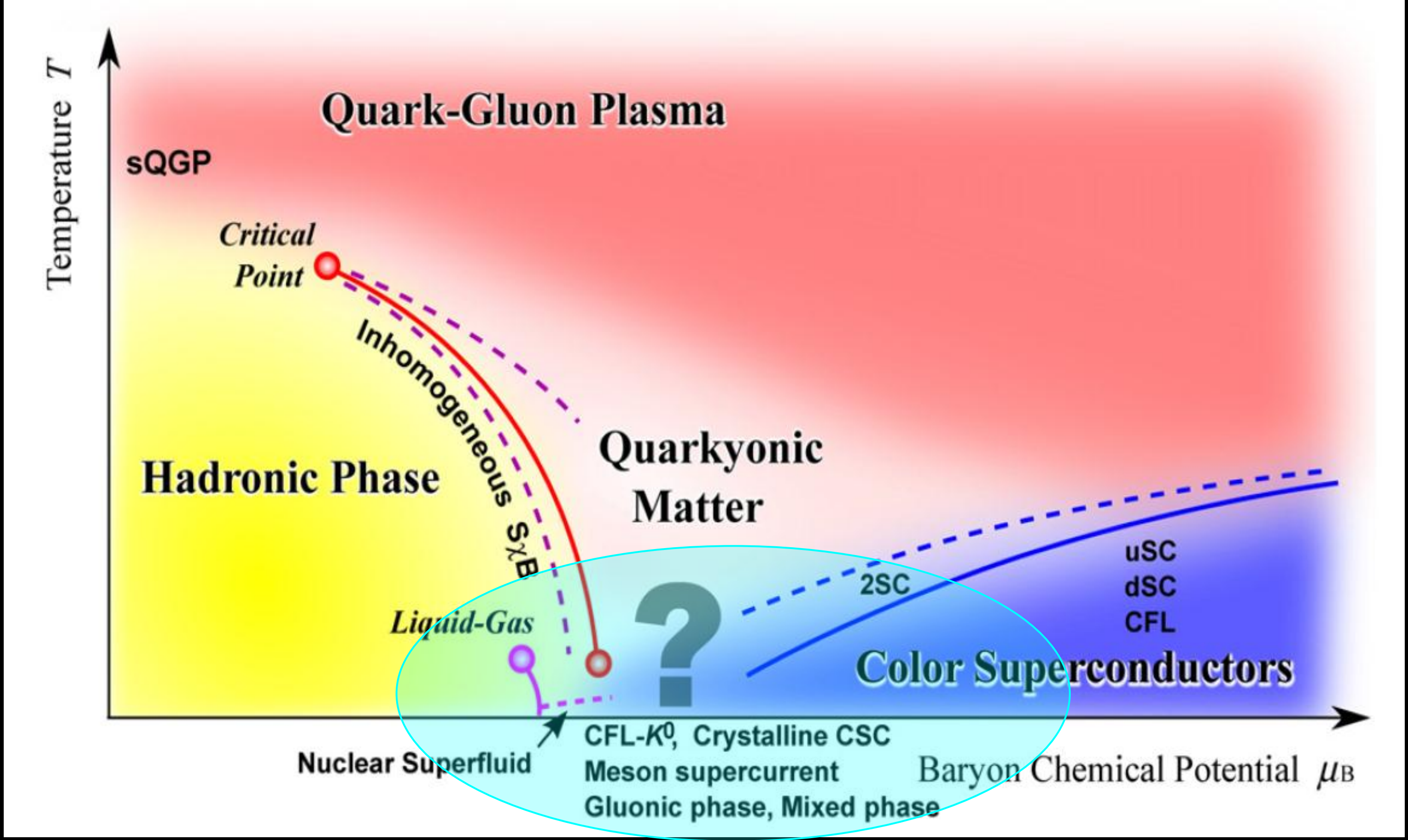
NN phase shifts in the SU(3) symmetric world



Stronger attraction in the deuteron channel

Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002 + NPA (2012) to appear

Simulating dense QCD by ultra-cold atoms



Ultra-cold atomic Gasses

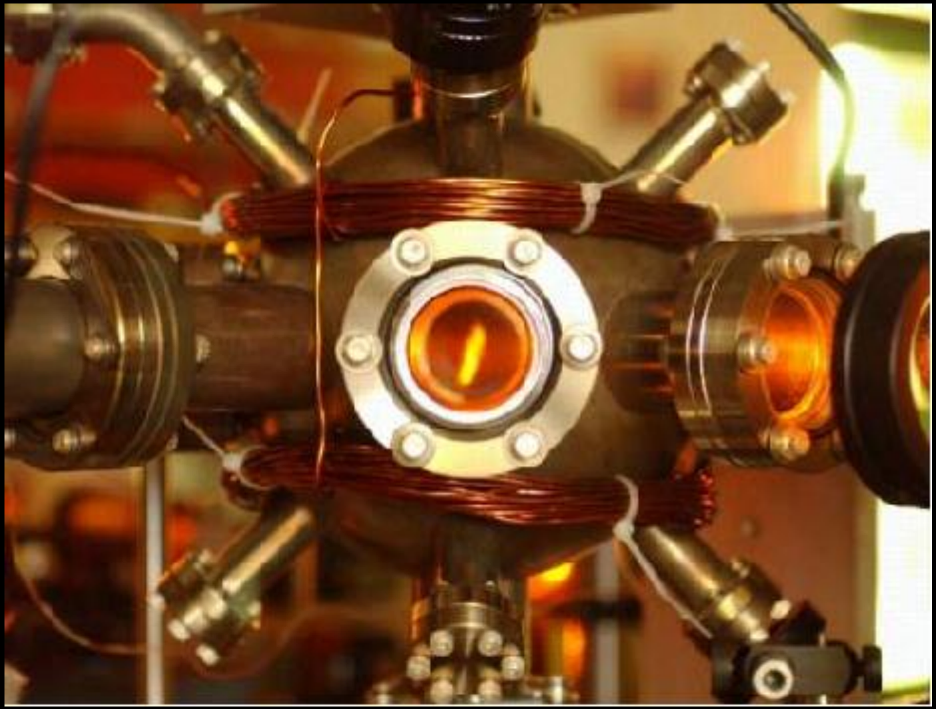
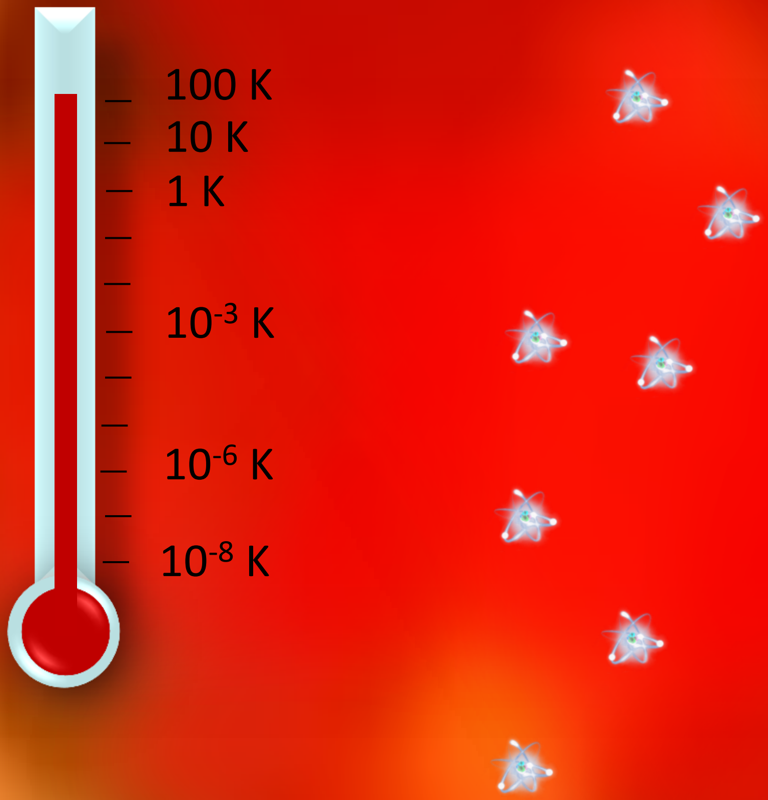
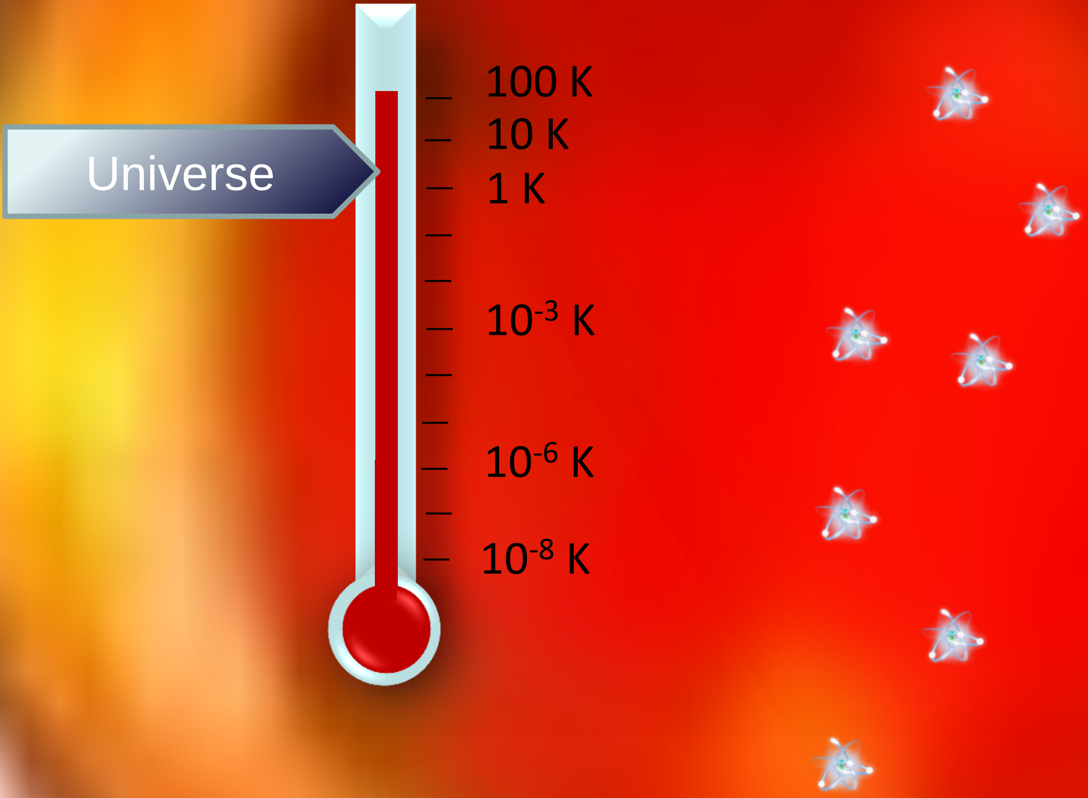


Figure from Pascal Naidon (RIKEN)


100 K
10 K
1 K
10^{-3} K
10^{-6} K
10^{-8} K



Universe

100 K

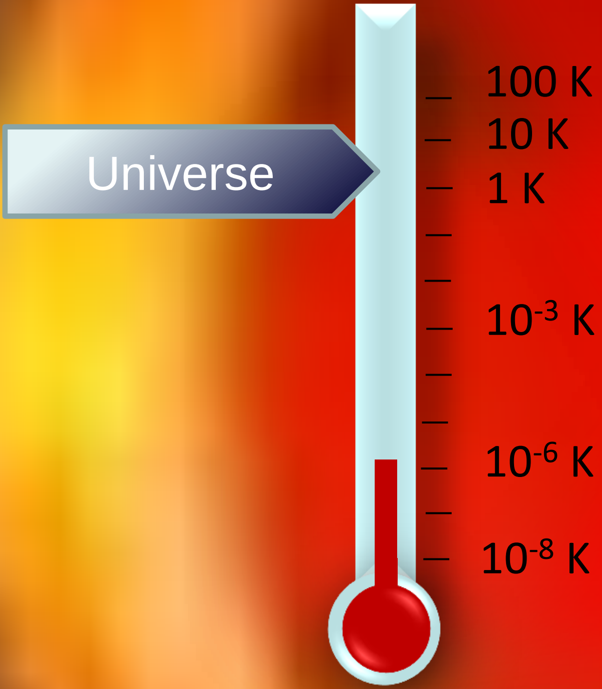
10 K

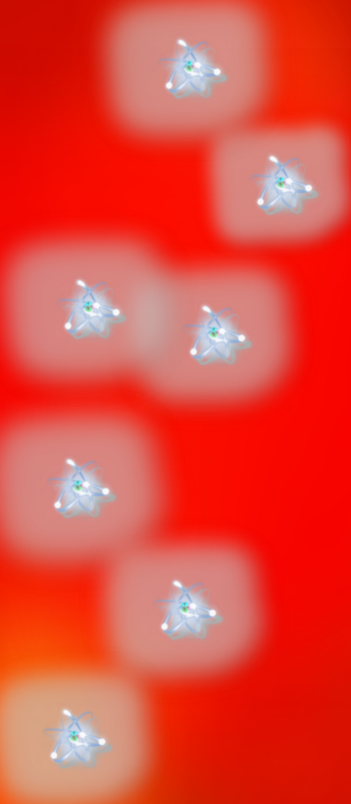
1 K

10^{-3} K

10^{-6} K

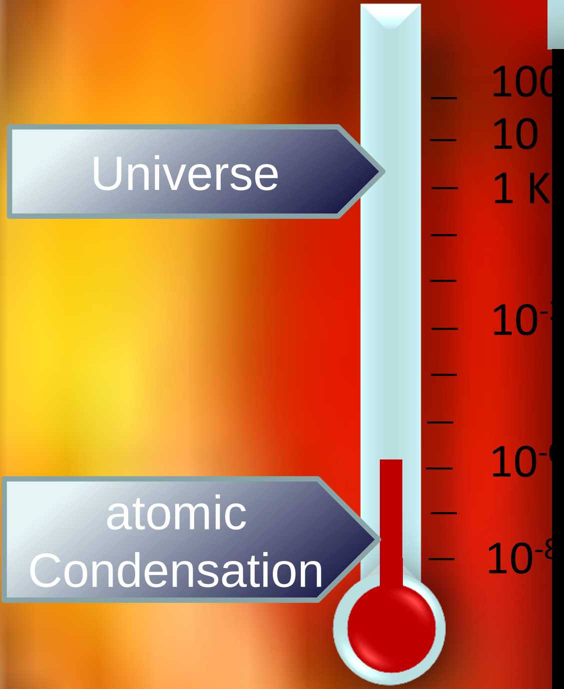
10^{-8} K



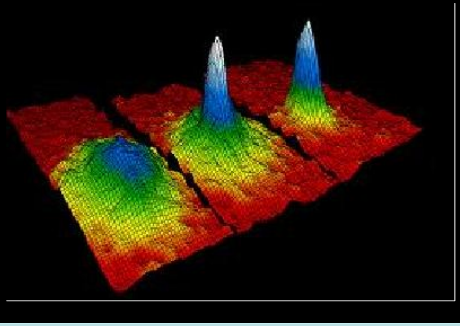


At these temperatures, *quantum effects* appear.

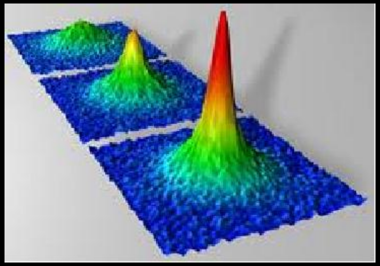
Quantum motion is described by waves



Bose-Einstein Condensate 1995



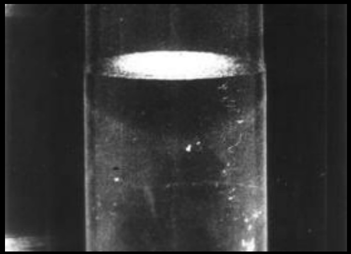
Fermi superfluid 2003



10-7 K

QUANTUM FLUIDS (SUPERFLUIDS)

Superfluid helium



10^{-1} K
1 K

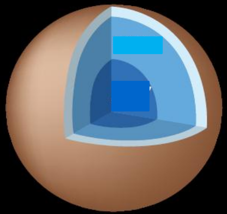
r.

Superconducting electrons

10 K

Superfluid nucleons

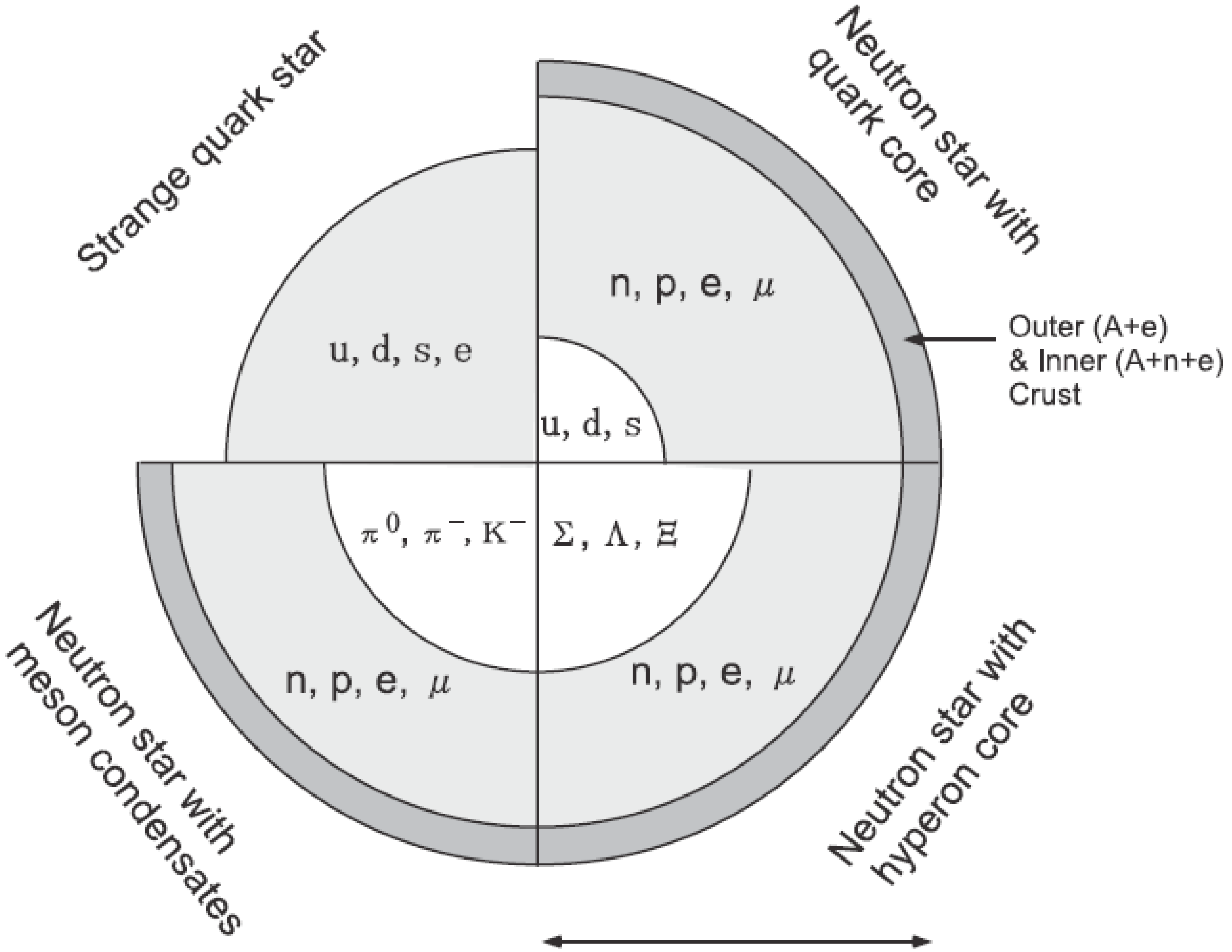
Superconducting quarks



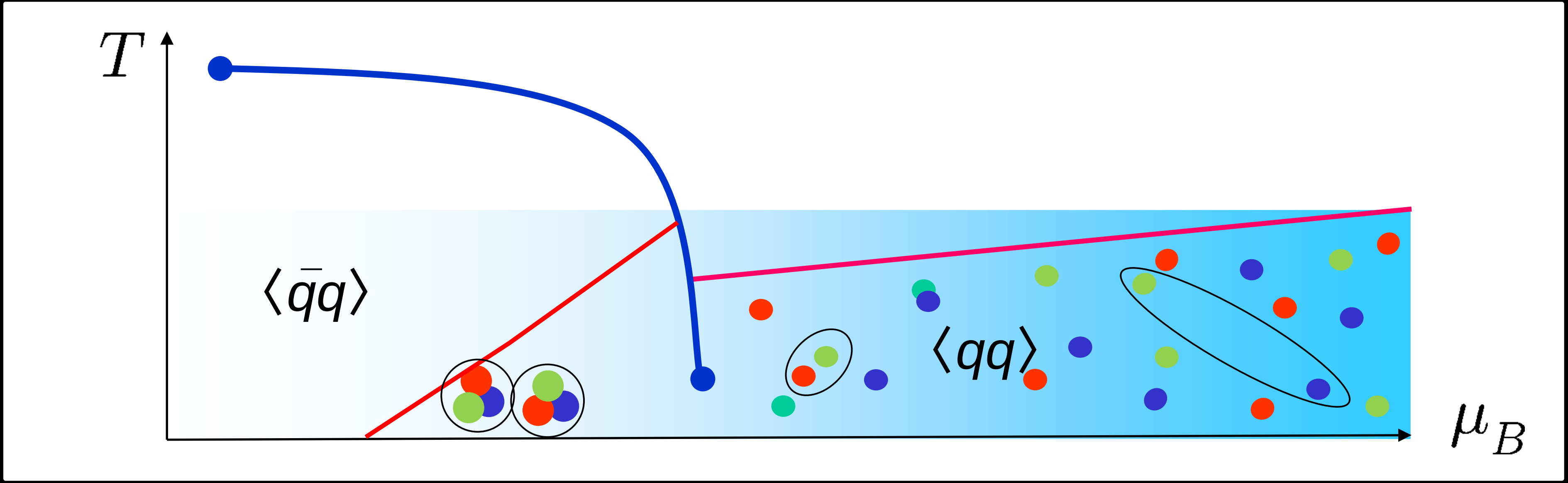
10^{9} K

10^{10}K

Neutron Star Structure

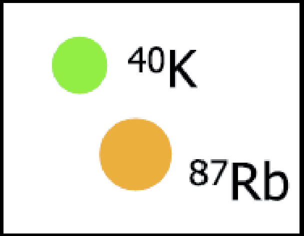


“Quark-Hadron transition” in boson-fermion mixture of ultracold atoms



Nuclear superfluid ⇔ Fermion+Diquark ⇔ Quark superfluid

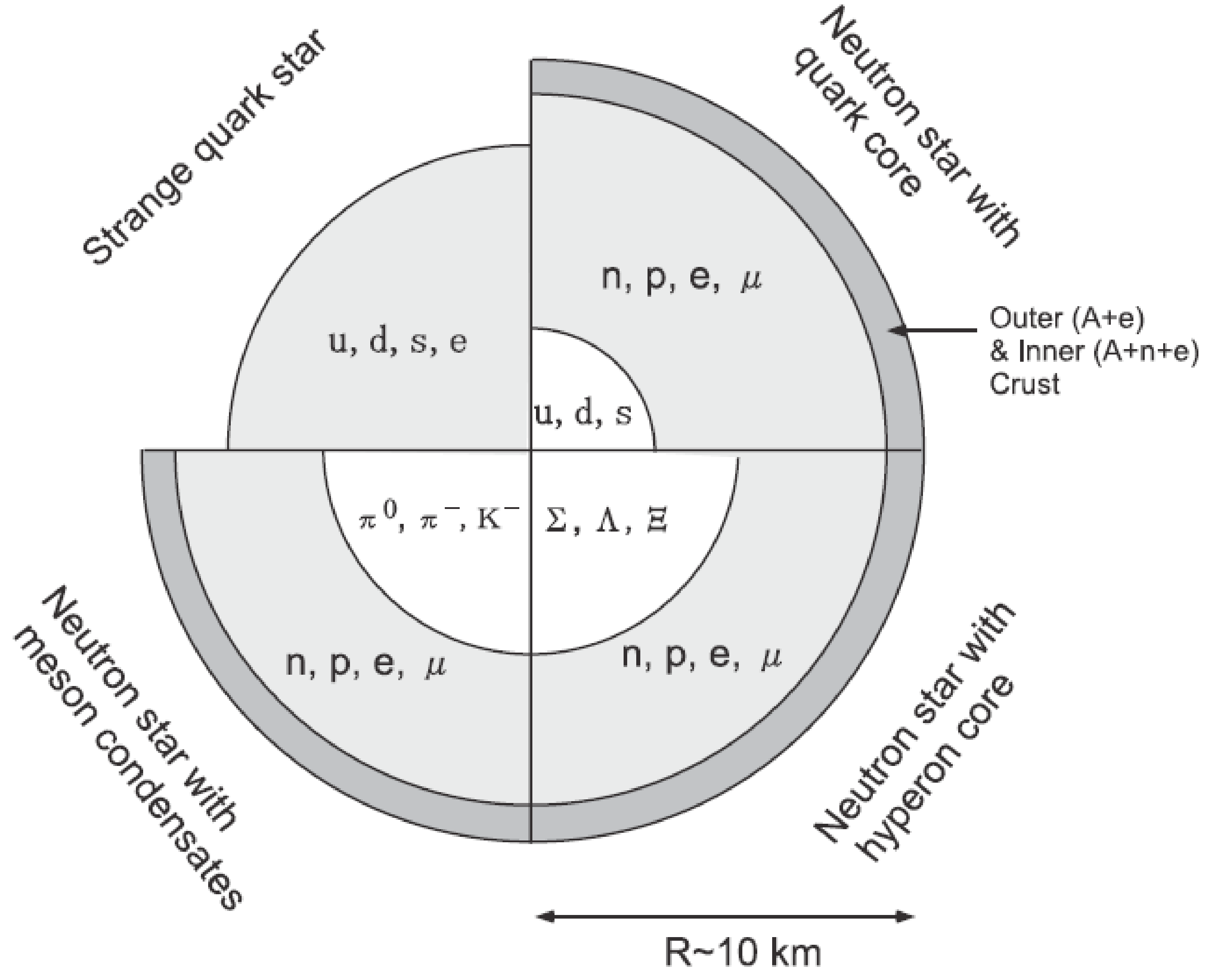
Induced superfluid ⇔ Fermi-Bose mixture



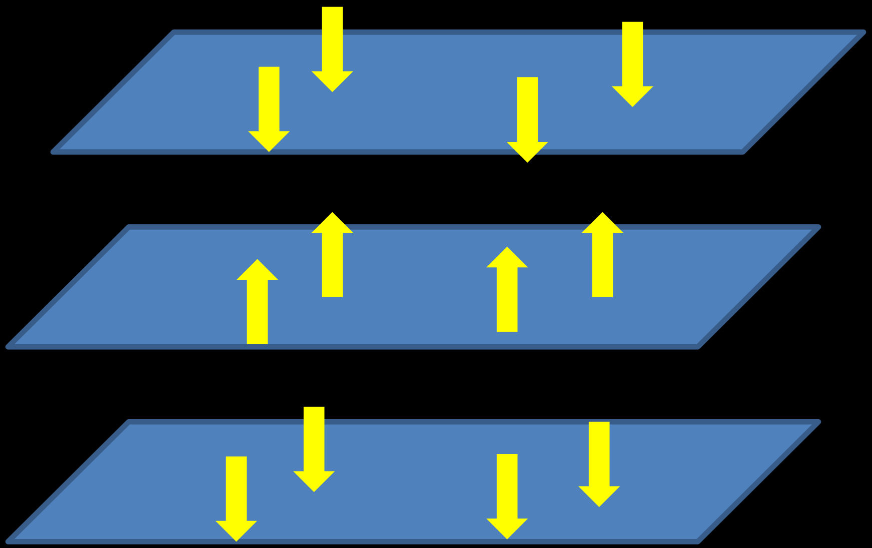
$$a_{\rm NN}^{\rm Born} = -\frac{m_{\rm N}}{2m_{\rm R}} a_{\rm bf}$$

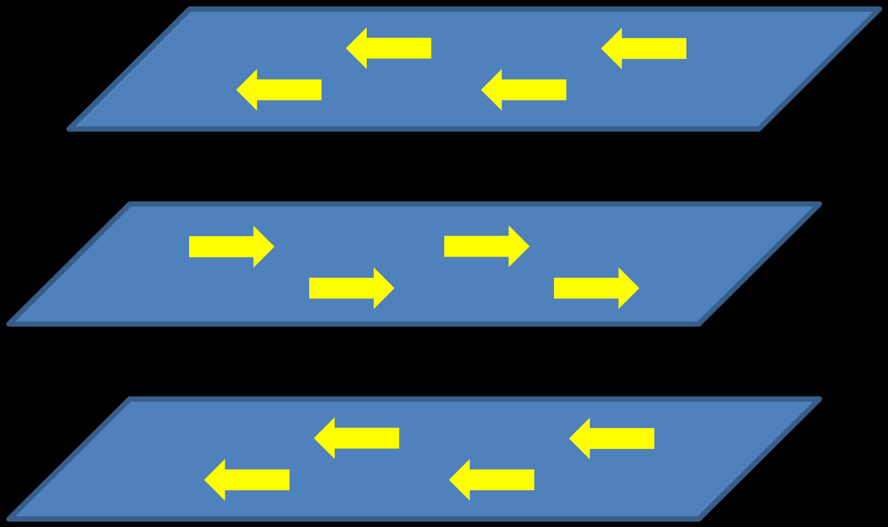
Maeda, Baym & Hatsuda, Phys. Rev. Lett. 103 (’09)

Neutron Star Structure



π^0 and/or ρ^0 condensation in neutron matter





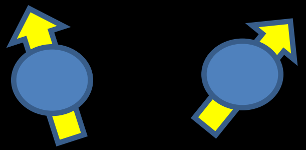
$$(-\nabla^2 + m_\pi{}^2)\,\varphi_c(\mathbf{r}) = (f/m_\pi)\,\boldsymbol{\nabla}\cdot\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

$$(-\nabla^2 + m_\rho{}^2)\,\boldsymbol{\rho}_c(\mathbf{r}) = (f_\rho/m_\rho)\,\boldsymbol{\nabla}\times\langle\psi^\dagger\boldsymbol{\sigma}\psi\rangle$$

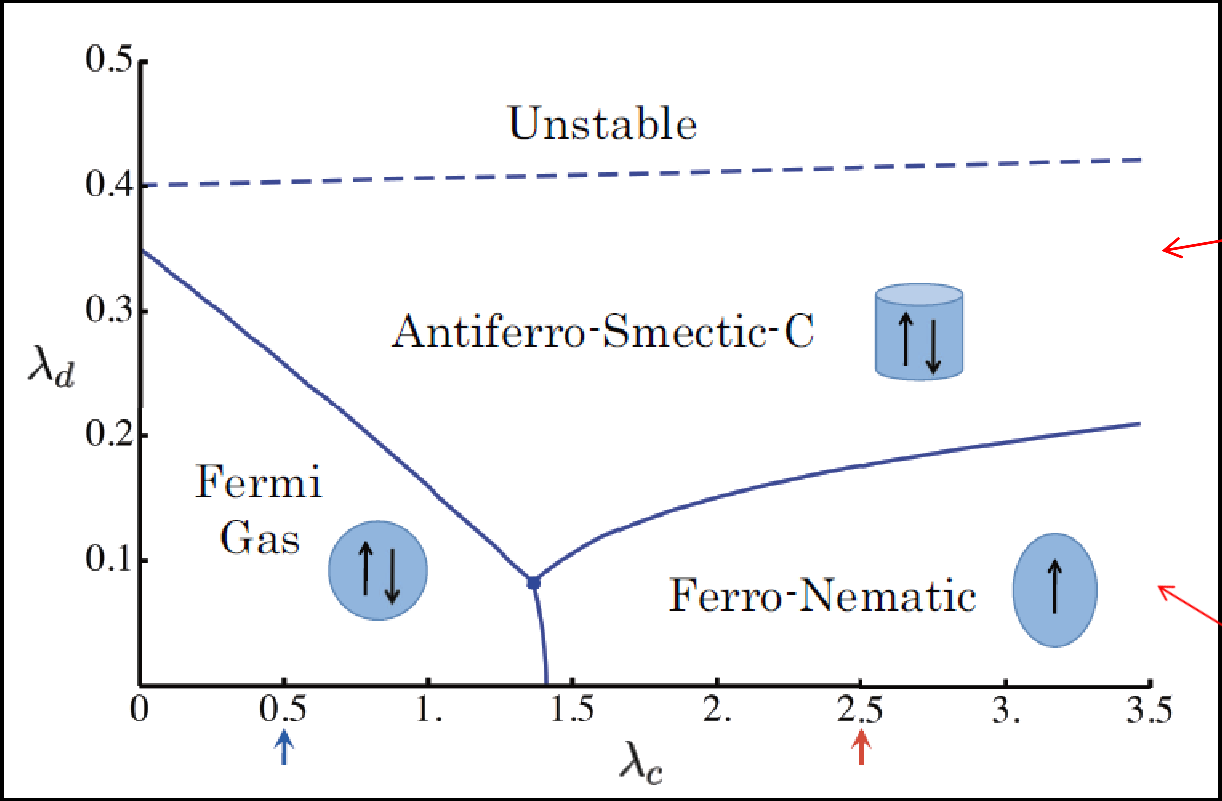
A. B. Migdal, NPA (1972)
Takatsuka, Tamagaki & Tatsumi,
Prog. Theor. Phys. Suppl. 112 ('83) 67

Kunihiro, Prog. Theor. Phys. 60 ('78) 1229

“Meson condensation” in ultracold dipolar atoms



$$U = \frac{\mu^2}{r^3}\{\vec{\sigma}_1\cdot\vec{\sigma}_2 - 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})\} + g\delta(\vec{r}),$$



Maeda, Hatsuda & Baym, in preparation.

Sogo et al., NJP (2009)
Fregoso & Fradkin, PRL 103 (2009)

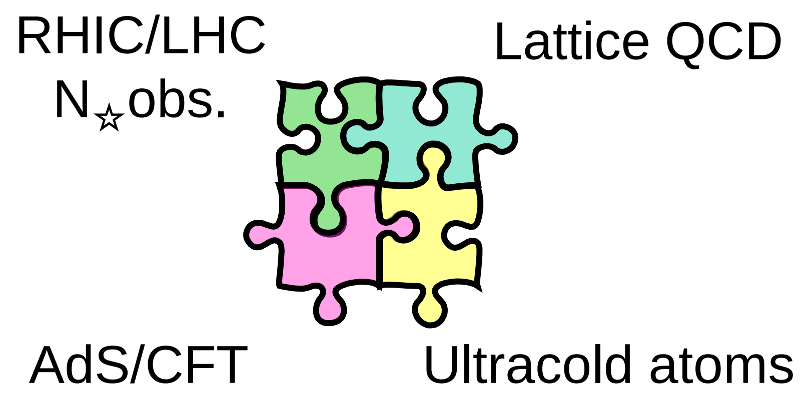
“Summary”

1. New era of Latice QCD arrived
   - massive physical point simulations will start from 2012 (u,d,s-flavor, L ~ 6fm, a ~ 0.05fm, m_π=135MeV)
2. LQCD started to provide 1st principle input for the physics of quark-gluon plasma
   - EoS (P(T), ε(T)), spectral functions, etc ⇔ RHIC, LHC
3. LQCD provides qualitative pictures on the NN, YN, YY & NNN forces
   - better constraints of the EoS of dense matter
   - physical point simulations will start from 2012
4. Ultracold atomic experiments may provide
   - better understanding of the hadron-quark transition in dense matter

“Future”

In a few years, we will hear more on

1. Physical point LQCD results for many observables
2. Simulations with better fermions
   staggered, Wilson → domain wall, overlap
3. BB and BBB interactions
   → better understanding of nuclei and neutron stars from QCD
4. UCA/QCD correspondence



END