**QCD Structure of Hadronic Matter**

Hadron: $r \sim 1$ [fm]

Nucleus: $r \sim 10$ [fm]

Neutron star: $r \sim 10$ [km]

**Light Quarks**
- $m_u \sim 2$ MeV
- $m_d \sim 5$ MeV
- $m_s \sim 90$ MeV

**Heavy Quarks**
- $m_c \sim 1.3$ GeV
- $m_b \sim 4.2$ GeV
- $m_t \sim 171$ GeV

T. Hatsuda (U. Tokyo/RIKEN) Feb.6 (2012) at Yukawa Symposium
Modern challenges in Hadron Physics

Primordial form of matter
quark-gluon plasma

Origin of heavy elements
in explosive astrophysical phenomena

Super dense matter
neutron star, exotic matter, ...

“New physics” search
dark matter, ...
Quantum Chromo Dynamics

\[ \mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a}_{\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - gt^a A^a_\mu) q - m\bar{q}q \]

\[ G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf_{abc} A^b_\mu A^c_\nu \]

Nambu (1966)

Running masses: \( m_q(Q) \)

<table>
<thead>
<tr>
<th>Quark Masses (from lattice QCD)</th>
<th>[MeV] (MS-bar @ 2GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_u )</td>
<td>2.19(15)</td>
</tr>
<tr>
<td>( m_d )</td>
<td>4.67(20)</td>
</tr>
<tr>
<td>( m_s )</td>
<td>94(3)</td>
</tr>
</tbody>
</table>

Running coupling: \( \alpha_s(Q) = \frac{g^2}{4\pi} \)

Schematic QCD phase diagram

- Chiral Superfluid
- Nuclear Superfluid
- Color Superconductivity
- Quark-Gluon Plasma

Temperature ($T$) vs. Baryon chemical potential ($\mu_B$)

- $\approx 1 \text{ GeV}$
- $\approx 0.2 \text{ GeV}$
Symmetry realization in massless QCD \((N_c=3, N_f=3)\)

Cabibbo and Parisi, PLB 59 (1975); Collins & Perry, PRL 34 (1975)

QGP:

\[ SU_C(3) \times [SU_L(3) \times SU_R(3)] \times U_B(1) \]

\(\chi_{SF} \): \(\langle \bar{q}q \rangle \neq 0\)

\[ SU_C(3) \times SU_{L+R}(3) \times U_B(1) \]

CSC:

\(\langle qq \rangle \neq 0\)

\[ SU_{C+L+R}(3) \times Z(2) \]

Nambu, PRL 4 (1960)

Alford, Rajagopal & Wilczek, NP B537 (1999)

“Dirac” mass

“Majorana” mass
Color Superconductivity  
Chiral Superfluid

\( \sim 1 \text{ GeV} \)  
\( \sim 0.2 \text{ GeV} \)

Nuclear Superfluid

Quark-Gluon Plasma

Asakawa-Yazaki NPA (1989)

High T critical point: Asakawa & Yazaki, NPA (1989)
Low T critical point: Kitazawa, Koide, Kunihiro & Nemoto, PTP (2002)
Hatsuda, Tachibana, Yamamoto & Baym, PRL (2006)
Lattice QCD

- Color SU(3) gauge theory for strong interaction (Nambu 1966)
- Asymptotic freedom (Gross & Wilczek, Politzer 1973)
- Lattice gauge theory (Wilson 1974)

- Typically $32^4 \approx 10^6$ sites

- Well defined statistical system (finite $a$ and $L$)
- Gauge invariant
- Fully non-perturbative

Monte Carlo simulations (Creutz 1980)
Quark Confinement

Heavy Q-Qbar potential

\[ V(R) = \left( c - \frac{a}{R} + \sigma R \right) \theta(R_0 - R) \]

SESAM Coll., Phys.Rev.D71 ('05)

Bali et al., Nucl.Phys.Proc.Suppl. 153 ('06)
Precision Lattice QCD
Three limits

$L^{-1} \rightarrow 0$ (thermodynamics limit) : finite size scaling
$a \rightarrow 0$ (continuum limit) : asymptotic freedom
$m \rightarrow 0$ (chiral limit) : chiral pert. theory

Techniques

Fermions:
Staggered, Wilson, Domain-wall, Overlap
different ways of handling chiral symmetry

Improved actions:
stout, HEX, asktad, HISQ, clover, ....
different ways of reducing the discretization error

Advanced algorithms:
RHMC, DDHMC, LMA, ....
techniques to make the simulations fast and reliable
QCD running coupling

- Nf=2+1 on the lattice

World average ’10 (w/o lattice)

Light current (JLQCD) ’10

Heavy current (HPQCD) ’10

SF (PACS-CS) ’09

Heavy current (HPQCD) ’09

Quarkonia (HPQCD) ’08

world average (exp. only)

$\alpha_s(M_Z) = 0.1186(11)$

world average (lattice only)

$\alpha_s(M_Z) = 0.1189(5)$

Summary by Shintani (Lattice2011)
ssbar content of the proton

$m_s \langle N|\bar{s}s|N\rangle < 60 \text{ MeV} \iff y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} < 0.05$

Takeda [JLQCD Coll.], PRD 83 (2011) 114506
Giedt, Thomas, Young
PRL103(2009) 201802
WIMP – Nucleon Interaction

Giedt, Thomas, Young

Hadron masses @ 2009

PACS-CS Collaboration,

(2+1)-flavor, Wilson
$L = 2.9$ fm, $a = 0.09$ fm
$m_{\pi}(\text{min}) = 156$ MeV

BMW Collaboration,
Science 322 (2008) 1224

(2+1)-flavor, Wilson
$L = (2.0-4.1)$ fm, $a = 0.065, 0.085, 0.125$ fm
$m_{\pi}(\text{min}) = 190$ MeV

3% accuracy of light hadron masses
Hadron masses @ 2011
Physical point simulations in (2+1)-flavor QCD

PACS-CS Coll. (2010)

BMW Coll. (2011)

Toward large-scale Physical-point simulations

10PFlops K computer (RIKEN)

First LQCD Simulation

Now
Advanced Institute for Computational Science (AICS)
10 Pflops supercomputer KEI “京” (full operation in 2012)


Five “strategic” programs (FY 2010-2015)
1. Life and Medicine   2. New Materials   3. Environment
4. Engineering   5. Particle, Nuclear and Astrophysics
Hot QCD
Thermal QCD transition

**Order of QCD Transition**
- 2\(^{nd}\) order \((u,d; m=0)\)
- 1\(^{st}\) order \((u,d,s; m=0)\)
- crossover \((\text{real world})\)

**“Critical” Temperature**
- \(T_c : \sim 160\text{ MeV}\)
- \(\sim 10^{12} \text{ [K]}\)

**Critical Energy Density**
- \(\varepsilon_c : \sim 2\text{ GeV/fm}^3\)
- \(\sim 10 \varepsilon_{\text{fm}}\)

\[\varepsilon_{SB} = d_{\text{eff}} \frac{\pi^2}{30} T^4\]

Stefan-Boltzmann limit

Wuppertal-Budapest’s LQCD EOS
JHEP 1011 (2010) 77
Order of the QCD transition ($T \neq 0, \mu = 0$)

Staggered, (2+1)-flavor, physical mass

Finite size scaling

$$\chi_m = \frac{\partial^2 P}{\partial m_{ud}^2} \sim \begin{cases} V & 1\text{st order} \\ V^{2/3} & 2\text{nd order} \\ V^0 & \text{crossover} \end{cases}$$

Staggered, (2+1)-flavor, physical mass
Chiral susceptibility peak $\Rightarrow T_{pc}=150$-160 MeV
Dense QCD

Sign problem (Complex Action)
Nuclear force: a brief history

- One-pion exchange
  Yukawa (1935)

- Multi-pion
  Taketani et al. (1951)

- Repulsive core
  Jastrow (1951) Nambu (1957)

- EFT
  Weinberg (1990)

- high precision NN force (90’s-)
  30-40 parameters
  5000 phase shift data
So I got up in the question period and I said, "Maybe the reason is that inside the nuclear force of attraction, which holds nuclei together, there's a very strong short-range force of repulsion, like a little hard sphere inside this attractive Jell-O."

I'll never forget, Oppenheimer got up, he liked to needle the young fellows and he said, very dryly, "Thank you so much for, we are grateful for every tiny scrap of help we can get." But I ignored his needle and pursued my idea, and actually calculated the scattering of neutrons by protons. I showed that it fit the data very well. Oppenheimer read my paper for the Physical Review and took back his criticisms. This work became a permanent element of the literature of physics.

http://www.marshall.org/article.php?id=30
Nuclear Force and EoS of Dense Matter

**Akmal, Pandharipande & Ravenhall, PRC58 (‘98)**

- **E/A (MeV)**
- **ρ(fm⁻³)**
- **Z=0**
- **N=Z**

#### Neutron stars

- **ρ₀ = 0.16 fm⁻³**
- **3ρ₀**
- **5ρ₀**

**Phenomenological NN forces**

- **1S₀ channel**
- **Vc(r) [MeV]**
- **r [fm]**
- **III**
- **II**
- **I**

- **repulsive core**
- **2π, ρ, ω, σ**
- **π**

- **Bonn Reid93 AV18**
Nuclear Force and Neutron Star

Pressure balance

Fermi pressure
Repulsive core
gravity

$\rho_{\text{max}} \sim 6\rho_0$
Nuclear Force from Lattice QCD

Univ. Tsukuba
RIKEN
Nihon Univ.
Tokyo Inst. Tech.
Univ. Tokyo

S. Aoki, N. Ishii, H. Nemura, K. Sasaki
K. Murano, T. Doi, T. Hatsuda
T. Inoue
Y. Ikeda
B. Charron
How to define the NN potential from QCD?

1. NN wave function from lattice QCD

\[ \phi_n(\vec{r}, t) = \langle 0 | N(\vec{x} + \vec{r}, t) N(\vec{x}, t) | E_n \rangle \]

\[ \phi(\vec{r}, t) = \sum_n c_n \phi_n(\vec{r}, t) \]

1. NN potential from the NN wave function

\[ \left( -\frac{\partial}{\partial t} - H_0 \right) \phi(\vec{r}, t) = \int U(\vec{r}, \vec{r}') \phi(\vec{r}', t) d^3r' \]

3. Derivative expansion

\[ U(\vec{r}, \vec{r}') = V(\vec{r}, \nabla) \delta^3(\vec{r} - \vec{r}') \]

\[ V(\vec{r}, \nabla) = V_C(r) + S_{12} V_{T}(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{ V_D(r), \nabla^2 \} + \cdots \]

LO          LO               NLO                NNLO

+ Ishii et al. (HAL QCD Coll.)

- Potential is a nice tool to calculate observables
- Potential is volume insensitive (=Lattice Friendly)
Key channels in NN scattering \((^{2s+1}L_J)\)

\[ V(\vec{r}, \nabla) = V_C(r) + S_{12} V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{V_D(r), \nabla^2\} + \ldots \]

<table>
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<tr>
<th>LO</th>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
</tr>
</thead>
</table>

\( ^1S_0 \)

Central force ↔ nuclear BCS pairing

Bohr, Mottelson & Pines, Phys. Rev. 110 (1958)

\( ^3S_1-^3D_1 \)

Tensor force ↔ deuteron binding

Schwinger, Phys. Rev. 55 (1939), Bethe, ibid. 57 (1940)
Rarita & Schwinger, ibid. 59 (1941)

\( ^3P_2-^3F_2 \)

LS force ↔ neutron superfluidity in neutron stars


Density profile of the deuteron with \( S_z = \pm 1 \)
Pandharipande et al., (1998)
[Exercise 1] LO potentials: $V_C(r)$ & $V_T(r)$

- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

Central potential in \((2+1)\)-flavor QCD

Physical point simulations \((m_{\pi}=135\text{MeV} \text{ with } L=6\text{fm} \text{ & } 9\text{fm})\) will be carried out at KEI computer

HAL QCD Coll., in preparation

PACS-CS gauge config. (Clover + Iwasaki)
\(a = 0.09 \text{ fm}, L=2.9 \text{ fm}

\(m_{\pi} = 700 \text{ MeV}\)
Origin of the “short range NN repulsion”?

⇒ Baryon-baryon force in flavor SU(3)

Byproducts
• Hyperon forces : important for hyper-nuclei & neutron stars
• Fate of H-dibaryon: exotic 6-quark state (uuddss)  Jaffe, PRL 38 (’77)

\[ 8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a \]
Symmetric  Anti-symmetric

Six independent potentials in the flavor-basis
Lattice BB wave functions

Inoue et al. (HAL QCD Coll.)

Iwasaki + clover
(CP-PACS/JLQCD config.)
L = 1.9 fm, a = 0.12 fm, 16^3 x 32
m_n = 835 MeV, m_B = 1752 MeV

Short range BB int. ⇔ Quark Pauli principle

1 : allowed, 27 : partially blocked, 8_s : blocked

c.f. Urbaryon models (Otsuku-Yasuno-Tamagaki 1965, Machida & Namiki 1965)
Constituent quark model (Oka, Yazaki, Shimizu 1986)
BB potentials in flavor-basis

Inoue et al. [HAL QCD Coll.]
+ NPA (2012) to appear

Repulsive core in NN channel

Attractive core in H channel
H-dibaryon from LQCD

At physical point:
\[ M_{\Lambda\Lambda} < M_H < M_{\Xi N} ? \]

Inoue et al. [HAL QCD Coll.], NPA (2012) to appear

⇒ exp. search at RHIC & J-PARC
Hypernuclei at J-PARC

3 known

40 known

~3000 known
BB potentials in flavor-basis

Inoue et al. [HAL QCD Coll.]
+ NPA (2012) to appear

Repulsive core in NN channel

Growing NN tensor force

$^1S_0$ channel

$^3S_1 - ^3D_1$ channel
NN phase shifts in the SU(3) symmetric world

Stronger attraction in the deuteron channel

Inoue et al. [HAL QCD Coll.]
Simulating dense QCD by ultra-cold atoms
Ultra-cold atomic Gasses

Figure from Pascal Naidon (RIKEN)
At these temperatures, quantum effects appear.

Quantum motion is described by waves.
Quantum fluids (Superfluids)

- Superfluid helium
- Superconducting electrons
- Superfluid nucleons
- Superconducting quarks

At temperatures below $10^{-7}$ K, quantum effects appear.

- Bose-Einstein Condensate 1995
- Fermi superfluid 2003

Universe

atomic Condensation
Neutron Star Structure

The structure of a neutron star can be divided into several layers, each with different properties and compositions. From the outer to the inner regions, these layers include:

1. **Crust**: The outermost layer, consisting of a mixture of neutrons, protons, electrons, and muons. It is about 10 km thick.
2. **Hyperon Core**: This layer contains hyperons (Σ, Λ, Ξ) along with neutrons and protons. The thickness of this core is also approximately 10 km.
3. **Quark Core**: This innermost layer is composed of quarks, which are the fundamental particles of matter. The quark core is believed to be the densest part of a neutron star.
4. **Strange Quark Star**: A theoretical model where a substantial portion of the star's mass consists of strange quarks.

Each of these layers is characterized by different pressures and temperatures due to the immense gravitational forces inside the neutron star. The overall size of a neutron star, determined by these layers, is typically around 10 km in radius.
“Quark-Hadron transition”
in boson-fermion mixture of ultracold atoms

\[ \langle qq \rangle \]

Nuclear superfluid ⇔ Fermion+Diquark ⇔ Quark superfluid

Induced superfluid ⇔ Fermi-Bose mixture

\[ a_{\text{Born}} = -\frac{m_N}{2m_R}a_{\text{bf}} \]

Maeda, Baym & Hatsuda, Phys. Rev. Lett. 103 ('09)
\( \pi^0 \) and/or \( \rho^0 \) condensation in neutron matter

\[
(- \nabla^2 + m_{\pi}^2) \varphi_c(r) = \left( \frac{f}{m_{\pi}} \right) \nabla \cdot \left< \psi^\dagger \sigma \psi \right>
\]

\[
(- \nabla^2 + m_{\rho}^2) \rho_c(r) = \left( \frac{f_{\rho}}{m_{\rho}} \right) \nabla \times \left< \psi^\dagger \sigma \psi \right>
\]

A. B. Migdal, NPA (1972)
Takatsuka, Tamagaki & Tatsumi,
Prog. Theor. Phys. Suppl. 112 (‘83) 67

Kunihiro, Prog. Theor. Phys. 60 (‘78) 1229
“Meson condensation” in ultracold dipolar atoms

\[ U = \frac{\mu^2}{r^3} \left\{ \vec{\sigma}_1 \cdot \vec{\sigma}_2 - 3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) \right\} + g\delta(\vec{r}), \]


Sogo et al., NJP (2009)

Fregoso & Fradkin, PRL 103 (2009)
“Summary”

1. New era of Lattice QCD arrived
   - massive physical point simulations will start from 2012
     (u,d,s-flavor, L ~ 6fm, a ~ 0.05fm, m_π = 135MeV)

2. LQCD started to provide 1st principle input for the physics of quark-gluon plasma
   - EoS (P(T), ε(T)), spectral functions, etc ↔ RHIC, LHC

3. LQCD provides qualitative pictures on the NN, YN, YY & NNN forces
   - better constraints of the EoS of dense matter
   - physical point simulations will start from 2012

4. Ultracold atomic experiments may provide
   - better understanding of the hadron-quark transition in dense matter
“Future”

In a few years, we will hear more on

1. Physical point LQCD results for many observables

2. Simulations with better fermions
   staggered, Wilson $\rightarrow$ domain wall, overlap

3. BB and BBB interactions
   $\rightarrow$ better understanding of nuclei and neutron stars from QCD

4. UCA/QCD correspondence

![Diagram showing RHIC/LHC, Lattice QCD, N_{\star} obs., AdS/CFT, Ultracold atoms]
END