Holography
and
Strongly Coupled Gauge Theories in 3D

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Perspectives in Theoretical Physics

- From Quark-Hadron Sciences to Unification of Theoretical Physics -

Yukawa Institute for Theoretical Physics

February 8, 2012

Outline

1. "Relativistic materials" in condensed matter
   Graphene
2. D-brane holographic construction of relativistic 3D fermion system.
3. Conclusions

Motivation I

Find an analog in condensed matter of a 2+1D relativistic quantum field theory. (GWS, PRL 53 (26), 2449 (1984)).

Nielsen-Ninomiya Phys.Lett.B130:389,1983.– analog of the 3+1D axial anomaly in a condensed matter system

3D gauge theory has beautiful features – topological mass, parity anomaly, analog of chiral symmetry breaking problem of QCD.

Graphene

Topological insulators

Motivation II

Find a concrete as possible example of AdS/CMT holography

Lattice Dirac equation

$$\sum_{\mu} \gamma^{\mu} [\psi(x+\mu) - \psi(x)] = 0$$

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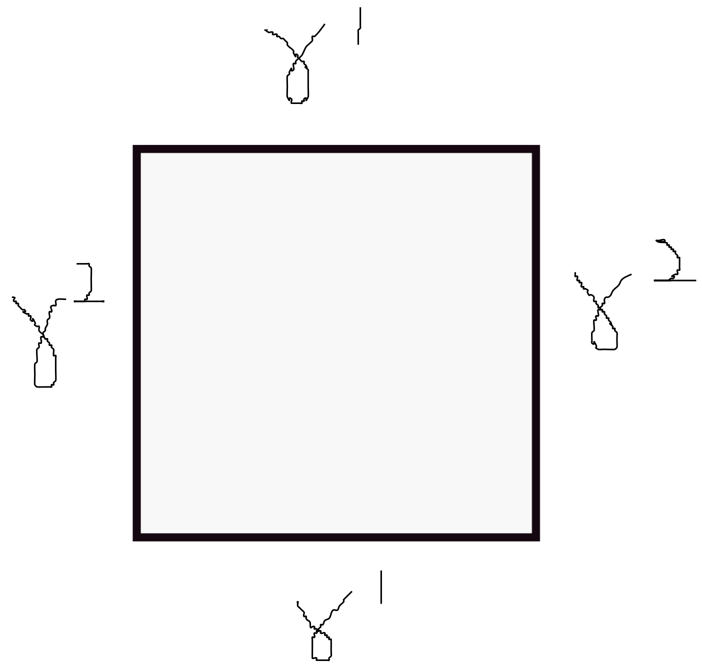
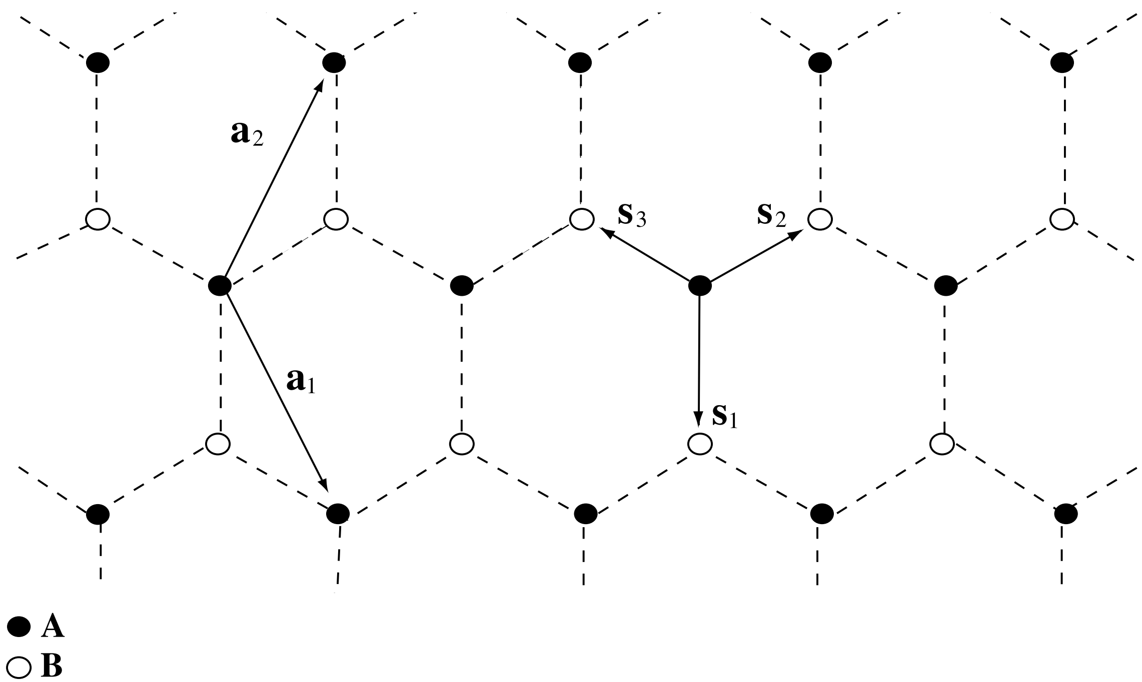


$$\gamma^1 \gamma^2 \gamma^1 \gamma^2 = -1$$

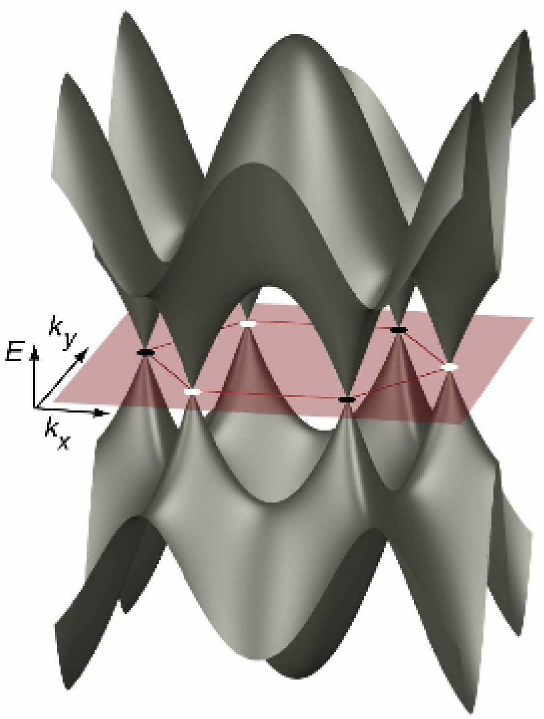
“Theoretical graphene” is the tight-binding model for electrons on a hexagonal lattice



$$H = \sum_{A,i} \left[t b^\dagger_{A+\mathbf{s}_i} a_A + t^* a^\dagger_A b_{A+\mathbf{s}_i} \right]$$

with half-filling (one electron per site)

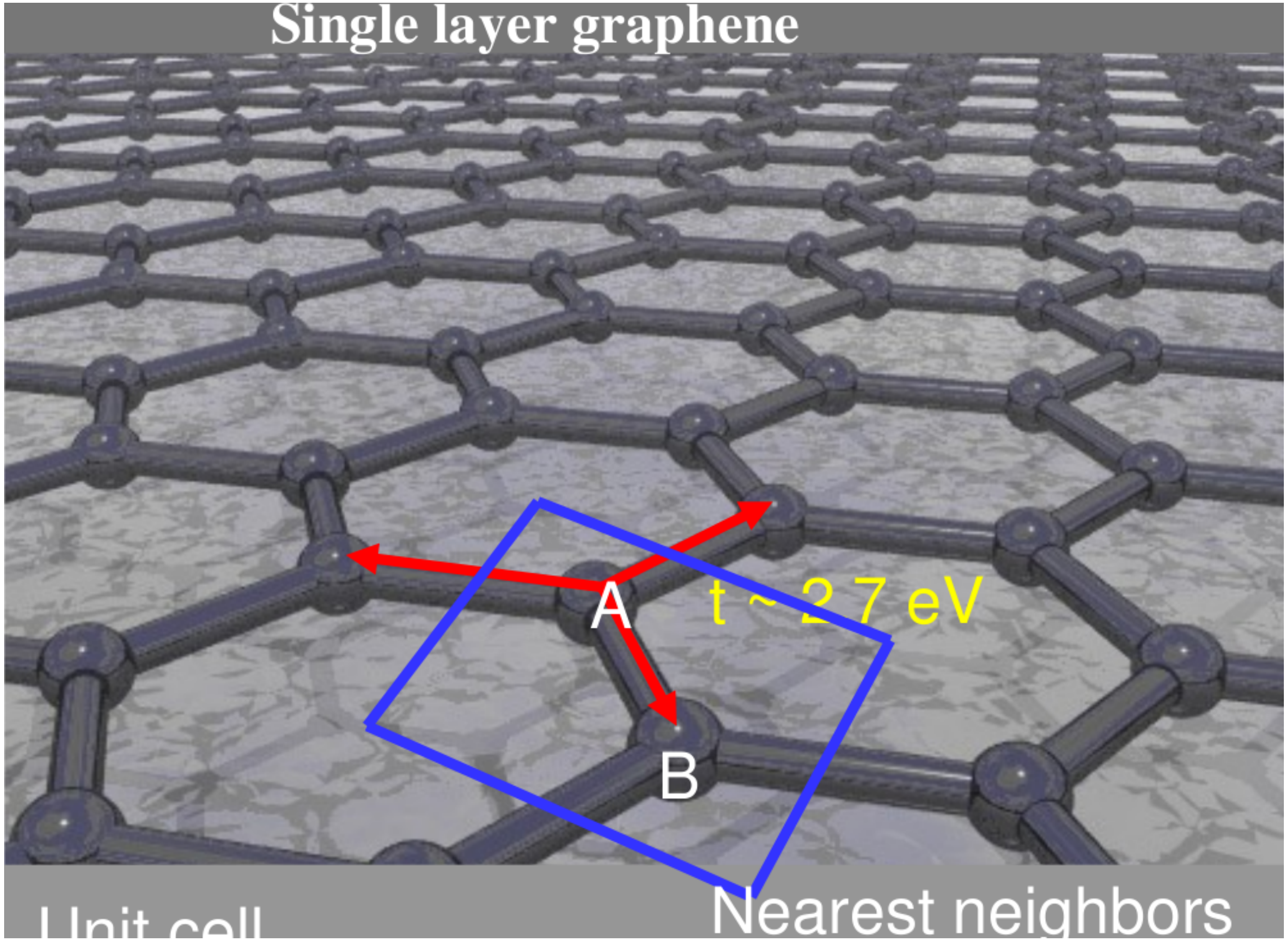
Band structure of graphene



Relativistic fermions, SU(4) symmetry

$$H = \hbar v_F \int d^2x \sum_{a=1}^{4} \left[\, \psi^\dagger_{Aa} \quad \psi^\dagger_{Ba} \,\right] \begin{bmatrix} 0 & \partial_x + i\partial_y \\ -\partial_x + i\partial_y & 0 \end{bmatrix} \begin{bmatrix} \psi_{Aa} \\ \psi_{Ba} \end{bmatrix}$$

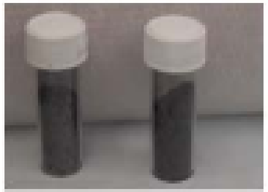
Graphene is a 2 dimensional hexagonal array of carbon atoms



Graphene was produced and identified in the laboratory in 2004

- Micromechanical cleavage of bulk graphite up to 100 micrometer in size via adhesive tapes !

Novoselov et al, Science **306**, 666 (2004)



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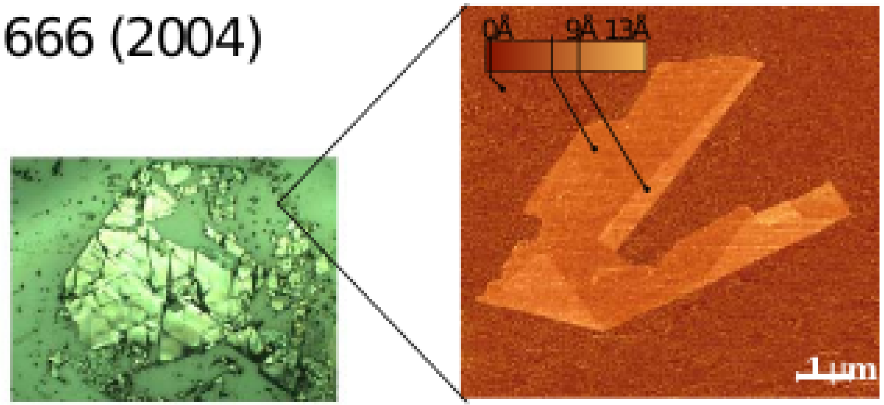
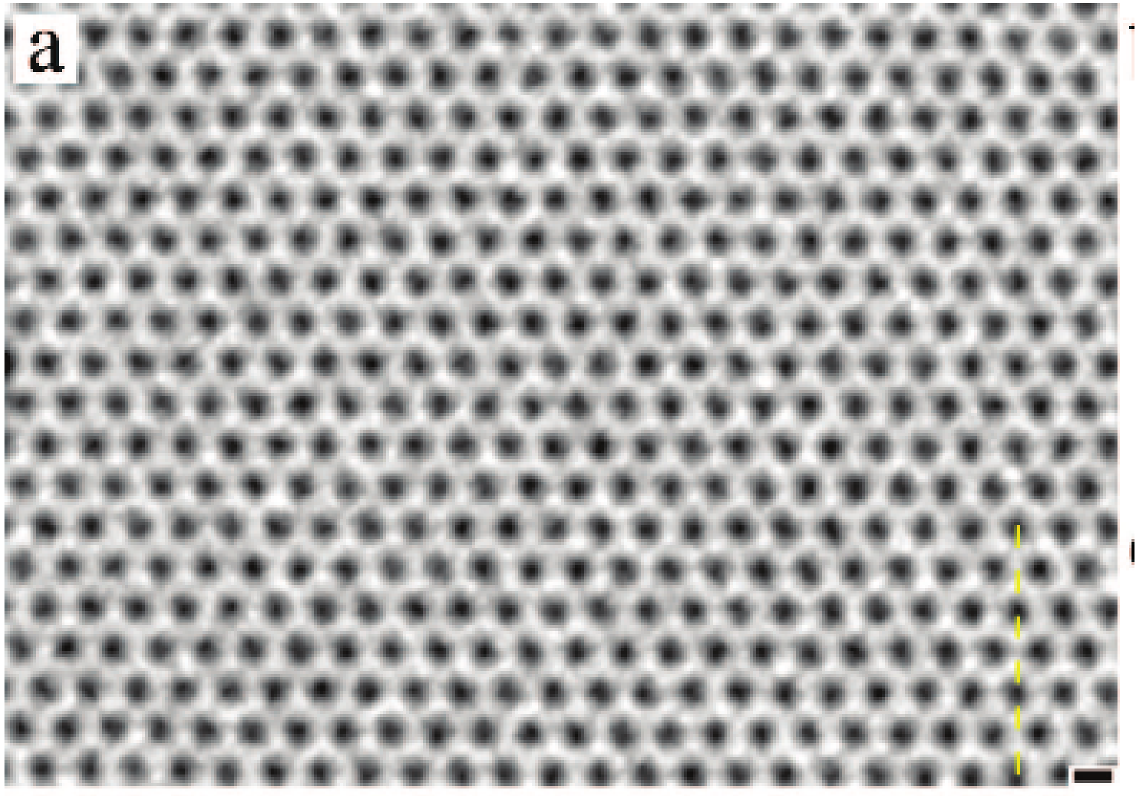


Kostya Novoselov

Andre Geim



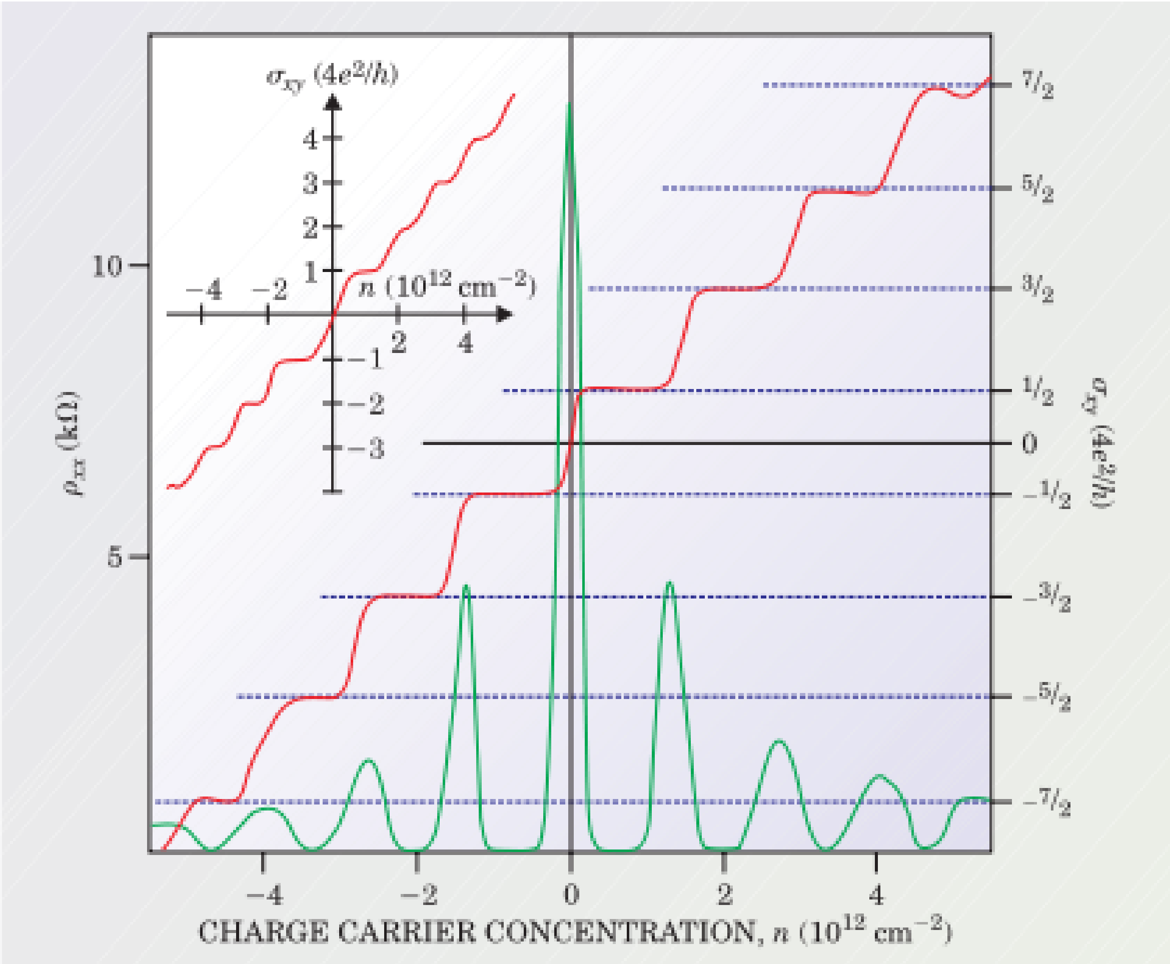
Jannik C. Meyer, C. Kisielowski, R. Erni, Marta D. Rossell, M. F. Crommie, and A. Zettl, Nano Letters 8, 3582 (2008).

Graphene superlatives (from Andre Geim)

- Thinnest imaginable material
- Strongest material "ever measured" (theoretical limit)
- Stiffest known material (stiffer than diamond)
- Most stretchable crystal (up to 20 percent)
- Record thermal conductivity (outperforming diamond)
- Highest current density at room temperature (million times higher than Copper)
- Highest intrinsic mobility (100 times more than Silicon)
- Conducts electricity even with no electrons.
- Lightest charge carriers (massless).
- Longest mean free path at room temperature (microns)
- Most impermeable (even Helium atoms can't squeeze through).

K. Novoselov et. al. *Nature 438, 197 (2005)*

Y. Zhang et. al. *Nature 438, 201 (2005)*



$$\sigma_{xy} = 4\frac{e^2}{h}\left(n + \frac{1}{2}\right)$$

Graphene with Coulomb interaction

$$S = \int d^3x \, \sum_{k=1}^{4} \bar{\psi}_k \left[\gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A}) \right] \psi_k$$

$$+ \frac{\epsilon}{2e^2} \int d^3x \; F_{0i} \frac{1}{2\sqrt{-\partial^2}} F_{0i} - \frac{1}{4e^2} \int d^3x \; F_{ij} \frac{1}{2\sqrt{-\partial^2}} F_{ij}$$

Kinetic terms have U(4)×SO(3,2) symmetry, $v_F \sim c/300$ $(c = 1)$

Interaction is non-relativistic with U(4) symmetry

Graphene fine structure constant

$$\alpha_{\text{graphene}} = \frac{e^2}{4\pi\hbar\epsilon v_F} = \frac{e^2}{4\pi\hbar c} \frac{c}{v_F} \frac{1}{\epsilon} \approx \frac{300}{137} \frac{1}{\epsilon}$$

AC Conductivity

$\omega >> k_B T$ RG improved one-loop correction

$$\sigma(\omega) = \frac{4e^2}{h}\left(1 + \mathcal{C}\frac{\frac{e^2}{2h}}{\left(v_F + \frac{e^2}{2h}\frac{1}{4}\ln(\Lambda/\omega)\right)}\right)$$

$$\sigma(\omega) = \frac{4e^2}{h}\left(1 + \mathcal{C}\frac{\frac{e^2}{4\pi\hbar v_F}}{1 + \frac{e^2}{4\pi\hbar v_F}\frac{1}{4}\ln(\Lambda/\omega)}\right)$$

V. Juricic et.al. Phys. Rev. B 82, 235402 (2010)

R. Nair et.al., Science 320, 1308 2008.

Experiments $\mathcal{C} = 0\pm ?$

Theory $\mathcal{C} \sim .2 - .5$

Gauge theory – String theory Duality

- $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory: gauge fields, adjoint representation scalar and spinor quarks
  conformal field theory with tuneable coupling constant g_{YM}
  and SU(N) gauge group

is exactly dual to

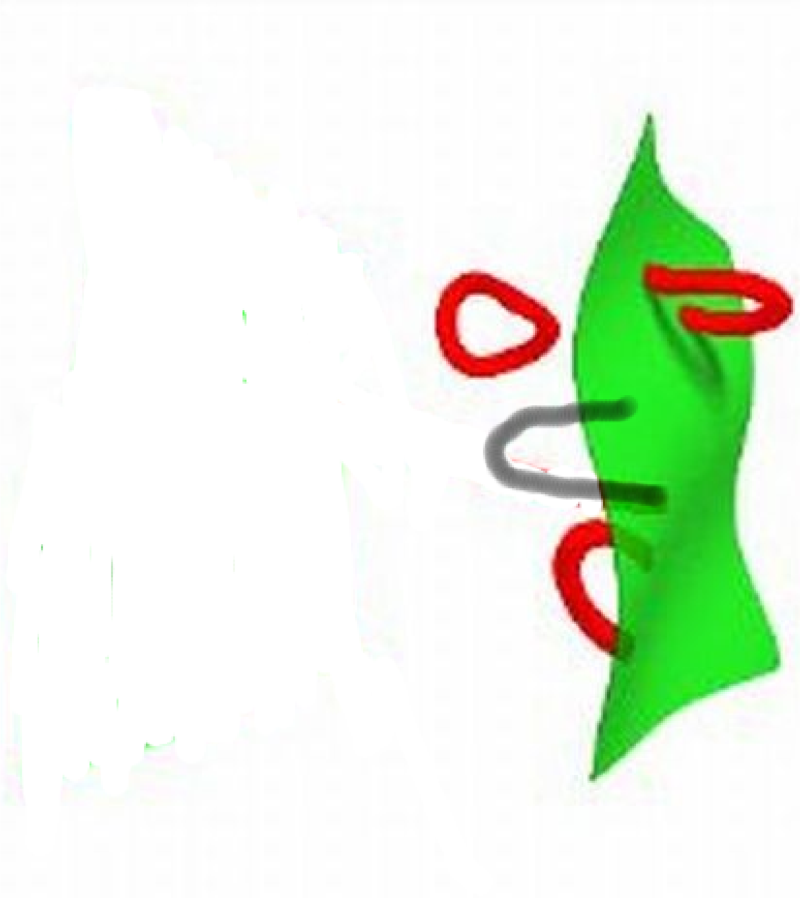
- IIB superstring theory on $AdS_5 \times S^5$ background
  N units of RR 4-form flux
  radius of curvature $R = \left(g_{YM}^2 N\right)^{\frac{1}{4}} \sqrt{\alpha'}$

- gauge theory is perturbative for small $\lambda = g_{YM}^2 N$
  string theory is perturbative for small $4\pi g_s = g_{YM}^2$
  and large R equivalent to $\lambda \to \infty$

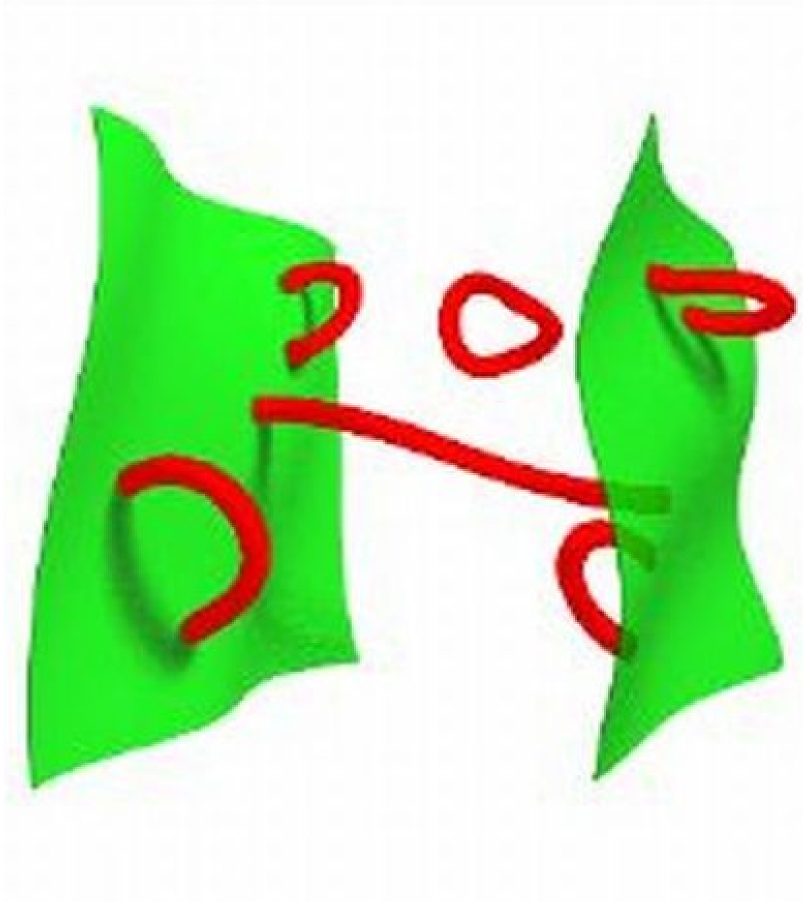
Symmetry $SO(2,4) \times SO(6) \subset SU(2,2|4)$

Additional degrees of freedom with probe branes

$AdS_5 \times S^5$ is sourced by a stack of D3 branes



Fundamental representation matter is introduced by including probe Dbrane



and taking the decoupling limit.

- D-brane construction of graphene using (unstable) D3-D7
  S-J.Rey, Strings 2007 (Madrid) and YITP; Prog.Theor.Phys.Suppl.177, 128 (2009) arXiv:0911.5295
- chiral symmetry breaking
  D.Kutasov, J.Lin, A.Parnachev, arXiv:1107.2324
- stabilize with instanton bundle on S^4.
  R.Myers, M.Wapler, JHEP 0812, 115 (2008) arXiv:0811.0480 [hepth].
- can use abelian flux
  O.Bergman, N.Jokela, G.Lifschytz, M.Lippert, JHEP 1010 (2010) 063 arXiv:1003.4965[hep-th].
- $C\ P\ T$ and D7-brane boundary conditions
  J.Davis, H.Omid, G.S., arXiv:1107.4397[hep-th]
- bilayers **J.Davis, N.Kim, arXiv:1109.4952[hep-th]**

D3-D7 system

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

brane extends in directions X

brane sits at single point in directions O

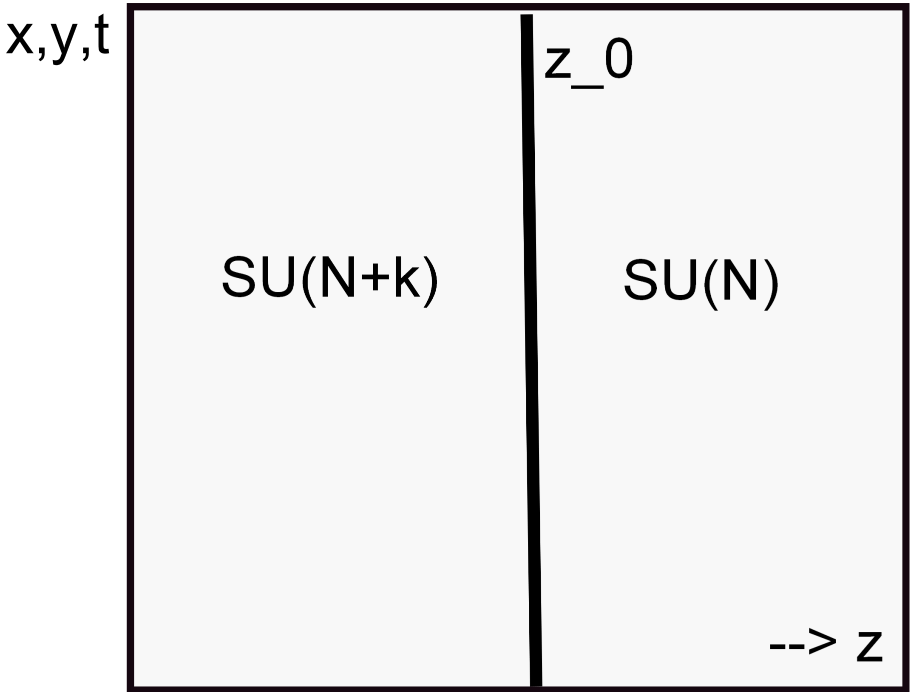
$\#ND = 6$ system – no supersymmetry – no tachyon – only zero modes of 3-7 strings are in R-sector and are 2-component fermions (N_7 flavors and N_3 colors).

Mass = separation in x_9-direction.

$$S = \int d^3x \sum_{\sigma=1}^{N_7} \sum_{\alpha=1}^{N_3} \bar{\psi}_\alpha^\sigma [i\gamma^\mu \partial_\mu - m] \psi_\alpha^\sigma + \text{interactions}$$

$N_3 \to \infty$, $\lambda = 4\pi g_s N_3$ fixed $\to$ replace D3's by $AdS_5 \times S^5$, large λ

Defect conformal field theory



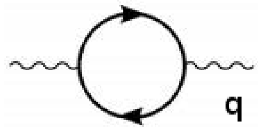
2+1-dimensional defect separates two regions where $\mathcal{N} = 4$ SYM has different gauge groups. $k = n_D^2 = \lambda f^2$.

Conformally invariant solution

D7-brane $(AdS_4 \subset AdS_5) \times (S^2 \times S^2 \subset S^5)$ with flux
$F = f d\Omega_2 + f d\tilde{\Omega}_2$

Current-current correlation

$$\left\langle e\bar{\Psi}\gamma_\mu \Psi \; e\bar{\Psi}\gamma_\nu \Psi \right\rangle = N_7 \frac{\lambda(f^2+1)}{2\pi^2} \frac{q^2 g_{\mu\nu} - q_\mu q_\nu}{q}$$

compare with $N_7 \frac{\lambda}{16} \frac{q^2 g_{\mu\nu} - q_\mu q_\nu}{q}$ at weak coupling 

Dangerously relevant operator

$$\left\langle \bar{\Psi}\Psi(x) \; \bar{\Psi}\Psi(0) \right\rangle = \frac{\text{const.}}{x^{2\Delta}} \; , \; \Delta = \frac{3}{2} + \frac{3}{2}\sqrt{1 - \frac{32}{9}\frac{1-f^2}{1+2f^2}}$$

compare with $\Delta = 2$ (free field theory), $\Delta = 1/2$ unitarity bound, $\Delta = 3/2$ stability bound.

Turn on Mass operator

Flows to parity violating CFT in IR with gapless matter

$< \bar{\Psi}\Psi >= \chi(f^2) m^{\Delta_+/\Delta_-}$

$$L = -\frac{N}{4\lambda} F \frac{1}{\sqrt{-D^2}} F + i\frac{k}{4\pi}(AdA + \frac{2}{3}A^3) + \bar{\psi}\gamma^\mu D_\mu \psi$$

S.Giombi et.al. arXiv:1110.4386

$$\text{one loop}: \quad < j_\mu j_\nu >= N_7 \frac{\lambda}{16} \frac{q^2 \delta_{\mu\nu} - q_\mu q_\nu}{q}$$

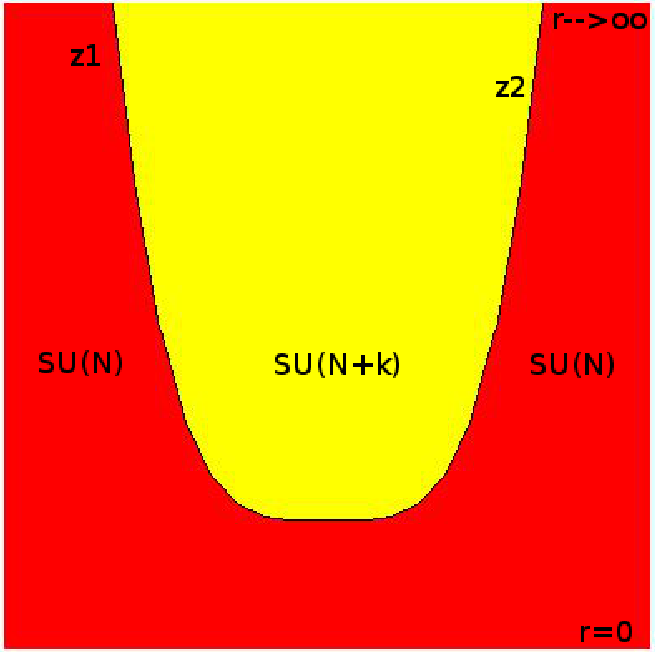
$$\text{large } q: \quad < j_\mu j_\nu >= N_7 \frac{\lambda(f^2+1)}{2\pi^2} \frac{q^2 \delta_{\mu\nu} - q_\mu q_\nu}{q}$$

small q:

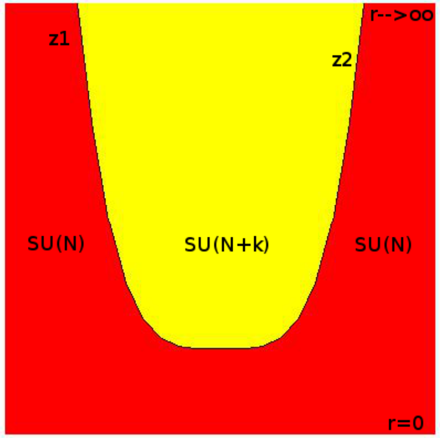
$$< j_\mu j_\nu >= N_7 \frac{2\lambda f}{2\pi^2} \frac{q^2 \delta_{\mu\nu} - q_\mu q_\nu}{q} + \frac{N_7 \lambda}{2\pi^2}(f\sqrt{1-f^2} - \cos^{-1} f) i\epsilon_{\mu\nu\lambda} q^\lambda$$

What about solutions with a charge gap?

Suspended brane solutions D7-D5 brane join



$\longleftarrow z \longrightarrow$



$$\langle j_{+a} j_{+b} \rangle = N_7 \frac{\lambda}{4\pi} \epsilon_{acb} q_c + \mathcal{O}(q^2)$$

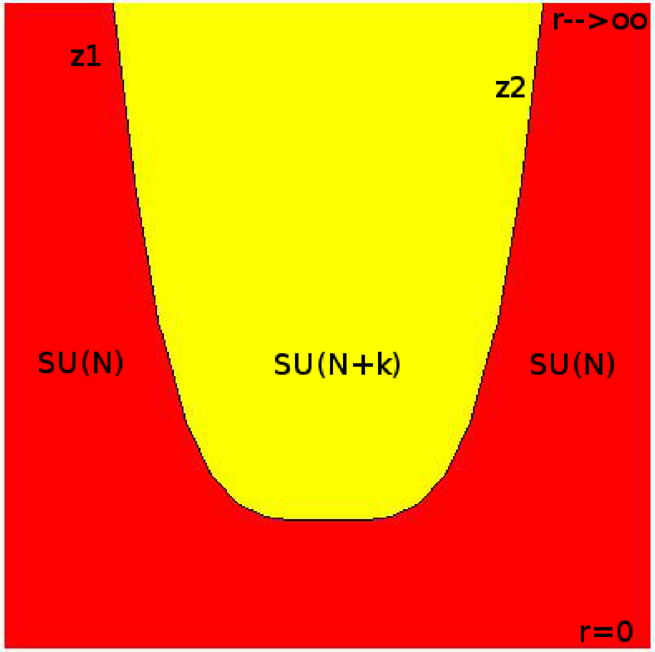
$$\langle j_{-a} j_{-b} \rangle = N_7 \frac{\lambda}{\pi^2 \rho_m} \frac{q^2 \delta_{ab} - q_a q_b}{q^2} + \epsilon_{acb} q_c \Delta^{(-)}_{\rm CS}(0) + \ldots$$

where

$$\rho_m = \int_{r_{\rm min}}^{\infty} \frac{d\tilde{r}}{\tilde{r}^2} \frac{\sqrt{(f^2 + 4\sin^4\psi)(f^2 + 4\cos^4\psi)}}{\sqrt{1 + \tilde{r}^2 {\psi'}^2 + \tilde{r}^4 {z'}^2}}$$

$$\Delta^{(-)}_{\rm CS}(0) = N_7 \frac{\lambda}{\pi^2} \int_0^{\pi/4} d\psi (1 - \cos 4\psi) \left(1 - \frac{\rho(\psi)}{\rho_m}\right)^2$$

Suspended brane solutions: D7-$\bar{\mathrm{D}}7$



J.Davis and N.Kim, arxiv...

Conclusions

- An attempt at holographic graphene.
- D7-D3 system as strongly coupled 2+1-dimensional relativistic fermions
- Conformal field theory at strong coupling
- gapless state with explicitly broken P and T symmetry
- only gapped states are joined branes D7-D5 and D7-D7 with $U(N_7) \times U(N_7) \to U(N_7)$ symmetry breaking pattern
- evidence for no renormalization of Chern-Simons at strong coupling