Quantum Criticality, high Tc superconductivity and the AdS/CFT correspondence.

Jan Zaanen
String theory: what is it really good for?

- Hadron (nuclear) physics: quark-gluon plasma in RIHC.
- Quantum matter: quantum criticality in heavy fermion systems, high Tc superconductors, …

Quantum critical matter

Quark gluon plasma

High Tc superconductors

Heavy fermions

Iron superconductors (?)
High-Tc Has Changed Landscape of Condensed Matter Physics

- High-resolution ARPES
- Magneto-optics
- Transport-Nernst effect
- Spin-polarized Neutron
- STM
- High Tc Superconductivity
- Angle-resolved MR/Heat Capacity
- Inelastic X-Ray Scattering
QuickTime™ and a decompressor are needed to see this picture.

Photoemission spectrum
Holography and quantum matter

“Planckian dissipation”: quantum critical matter at high temperature, perfect fluids and the linear resistivity (Son, Policastro, …, Sachdev).

Reissner Nordstrom black hole: “critical Fermi-liquids”, like high Tc’s normal state (Hong Liu, John McGreevy).

Dirac hair/electron star: Fermi-liquids emerging from a non Fermi liquid (critical) ultraviolet, like overdoped high Tc (Schalm, Cubrovic, Hartnoll).

Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Gubser, Hartnoll …).
Plan


2. Crash course: the AdS/CFT correspondence.

Twenty five years ago ...

Mueller  Bednorz

Ceramic CuO’s, like YBa2Cu3O7

Superconductivity jumps to ‘high’ temperatures
Graveyard of Theories
The quantum in the kitchen: Landau’s miracle

Electrons are waves

Pauli exclusion principle: every state occupied by one electron

Unreasonable: electrons strongly interact!!

Landau’s Fermi-liquid: the highly collective low energy quantum excitations are like electrons that do not interact.
**BCS theory: fermions turning into bosons**

Fermi-liquid fundamentally unstable to attractive interactions.

*Quasiparticles pair and Bose condense:*

Ground state \( \Psi_{BCS} = \prod_k \left( u_k + \nu_k c^+_k c^+_{-k\downarrow} \right) |\text{vac.}\> \)

*Conventional superconductors (Tc < 40K): “pairing glue”= exchange of quantized lattice vibrations (phonons)*
Fermion sign problem

Imaginary time path-integral formulation

\[
\mathcal{Z} = \text{Tr} \exp(-\beta \hat{H}) = \int dR \rho(R, R; \beta)
\]

\[
R = (r_1, \ldots, r_N) \in \mathbb{R}^{Nd}
\]

\[
\rho_{B/F}(R, R; \beta) = \frac{1}{N!} \sum_{\mathcal{P}} (\pm 1)^{\mathcal{P}} \rho_{D}(R, \mathcal{P}R; \beta)
\]

\[
= \frac{1}{N!} \sum_{\mathcal{P}} (\pm 1)^{\mathcal{P}} \int_{R \rightarrow \mathcal{P}R} dR(\tau) \exp \left\{ -\frac{1}{\hbar} \int_0^{\hbar/T} d\tau \left( \frac{m}{2} \dot{R}^2(\tau) + V(R(\tau)) \right) \right\}
\]

Boltzmannons or Bosons:
- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:
- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!
Phase diagram high Tc superconductors

The clash: the quantum critical metal

… which is good for superconductivity!

The quantized traffic jam

The quantum fog (Fermi gas) returns
Divine resistivity

- Ioffe–Regel limit
- Saturation
- Electron-phonon
- Electron-electron

(resistivity vs. temperature graph)
Fractal Cauliflower (romanesco)
Quantum critical cauliflower
Quantum critical cauliflower
Quantum critical cauliflower
Quantum critical cauliflower
Quantum criticality or ‘conformal fields’
Quantum critical hydrodynamics: Planckian relaxation time

Relaxation time $\tau$: time it takes to convert work in entropy.

Viscosity: $\eta = (\varepsilon + p)\tau$

Entropy density: $s = \frac{\varepsilon + p}{T}$

“Planckian viscosity” $\Rightarrow \frac{\eta}{s} = T\tau \approx \frac{\hbar}{k_B}$ ??

Planckian relaxation time $= \tau_{\hbar} \approx \frac{\hbar}{k_B T}$

Planckian relaxation time = the shortest possible relaxation time under equilibrium conditions that can only be reached when the quantum dynamics is scale invariant !!
Critical Cuprates are Planckian Dissipators

van der Marel, JZ, … Nature 2003:

Optical conductivity QC cuprates

Frequency less than temperature:

\[ \sigma_1(\omega, T) = \frac{1}{4\pi} \frac{\omega^2_{pr} \tau_r}{1 + \omega^2 \tau_r^2}, \quad \tau_r = A \frac{\hbar}{k_B T} \]

\[ \Rightarrow \left[ \frac{\hbar}{k_B T \sigma_1} \right] = \text{const.} \left( 1 + A^2 \left[ \frac{\hbar \omega}{k_B T} \right]^2 \right) \]

A = 0.7: the normal state of optimally doped cuprates is a Planckian dissipator!
Divine resistivity

![](image)

- Ioffe–Regel limit
- saturation
- electron–phonon
- electron–electron

**resistivity** vs **temperature**
Quantum Phase transitions

Quantum scale invariance emerges naturally at a zero temperature continuous phase transition driven by quantum fluctuations:

JZ, Science 319, 1205 (2008)
Phase diagram high Tc superconductors

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... which is good for superconductivity!

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Fermionic quantum phase transitions in the heavy fermion metals


JZ, Science 319, 1205 (2008)

\[ m^* = \frac{1}{E_F} \]
\[ E_F \rightarrow 0 \Rightarrow m^* \rightarrow \infty \]


Coleman
Rutgers
Critical Fermi surfaces in heavy fermion systems

Antiferromagnetic order

Blue = Fermi liquid

Yellow = quantum critical regime

\[ \rho(T) \sim T^\varepsilon \]

FL Fermi surface

Coexisting critical Fermi surfaces?

FL Fermi surface
Hertz-Millis and Chubukov’s “critical glue”

Bosonic (magnetic, etc.) order parameter drives the quantum phase transition

Electrons: fermion gas = heat bath damping bosonic critical fluctuations

Bosonic critical fluctuations ‘back react’ as pairing glue on the electrons

\[ \lambda(i\Omega) = \left( \frac{\Omega_0}{|\Omega|} \right)^\gamma \]

“Strong coupling” Migdal-Eliashberg theory

Attractive interaction due to “glue boson”, two parameters:

Coupling strength: \( \lambda = V / E_F \)

Migdal parameter: \( \frac{\hbar \omega_{boson}}{E_F} \)

Migdal-Eliashberg: dress boson and fermion propagators up to all orders ignoring vertex corrections which are \( O(\hbar \omega_B / E_F) \).
Computing the pair susceptibility: full Eliashberg

\[ \chi(k, k'; q) = \chi_0(k, k'; q) + u^2 \sum_{k_1, k_2} \chi_0(k, k_1; q) D(k_2 - k_1) \chi(k_2, k'; q) \]

\[ \Gamma(k; q) = \sum_{k'} \chi(k, k'; q) \]

\[ \Gamma(i\nu; i\Omega) = \Gamma_0(i\nu; i\Omega) + A \Gamma_0(i\nu; i\Omega) \sum_{\nu'} \lambda(i\nu' - i\nu) \Gamma(i\nu'; i\Omega) \]

\[ \chi_{\text{pair}}(i\Omega, q = 0) = \sum_{\nu} \Gamma(i\nu; i\Omega) \quad \text{as} \quad i\Omega \to \omega + i\delta \]

\[ \chi_{\text{pair}}(\omega, q = 0) \]
Watching electrons: photoemission

Electron spectral function: probability to create or **annihilate** an electron at a given momentum and energy.

Fermi energy

Fermi surface of copper

Fermi momenta

K = 1/wavelength

Kinetic energy
Fermi-liquid phenomenology

Bare single fermion propagator ‘enumerates the fixed point’:

\[ G(\omega, k) = \frac{1}{\omega - \mu_0 - k^2 / 2m - (\Sigma' + i\Sigma'')} = \frac{Z}{(\omega - E_F) - \nu_R (k - k_F) + \ldots} \]

Spectral function:

\[ \text{Im} G(\omega, k) = A(\omega, k) = \frac{\Sigma''(\omega, k)}{|\omega + \mu + (k - k_F)^2 / 2m + \Sigma'(\omega, k)|^2 + |\Sigma''(\omega, k)|^2} \]

The Fermi liquid ‘lawyer list’:

- At \( T = 0 \) the spectral weight is zero at the Fermi-energy except for the quasiparticle peak at the Fermi surface:
  \[ A(E_F, k) = Z \delta(k - k_F) \]

- Analytical structure of the self-energy:
  \[ \Sigma''(\omega, k) \propto (\omega - E_F)^2 + \ldots \]
  \[ \Sigma'(\omega, k) = \Sigma'(E_F, k_F) + \frac{\partial \Sigma'}{\partial \omega} \bigg|_{\omega = E_F} (\omega - E_F) + \frac{\partial \Sigma'}{\partial k} \bigg|_{k = k_F} (k - k_F) + \ldots \]

- Temperature dependence:
  \[ \Sigma''(E_F, k_F, T) \propto T^2 + \ldots \]
ARPES: Observing Fermi liquids

‘MDC’ at $E_F$ in conventional 2D metal (NbSe$_2$)

Fermi-liquids: sharp Quasiparticle ‘poles’
Cuprates: “Marginal” or “Critical” Fermi liquids

Fermi ‘arcs’ (underdoped) closing to Fermi-surfaces (optimally-, overdoped).

EDC lineshape: ‘branch cut’ (conformal), width proportional to energy
Varma’s Marginal Fermi liquid phenomenology.

**Fermi-gas** interacting by **second order** perturbation theory with ‘singular heat bath’:

\[
\text{Im} P(q, \omega) \propto -N(0) \frac{\omega}{T}, \quad \text{for } |\omega| < T
\]

\[
\propto -N(0) \text{sign}(\omega), \quad \text{for } |\omega| > T
\]

Directly observed in e.g. Raman ??

Single electron response (photoemission):

\[
G(k, \omega) = \frac{1}{\omega - v_F(k - k_F) - \Sigma(k, \omega)}
\]

\[
\Sigma(k, \omega) \propto g \left( \frac{\omega}{\omega_c} \right)^2 \left[ \omega \ln \left( \max \left( |\omega|, T \right)/\omega_c \right) - i \frac{\pi}{2} \max \left( |\omega|, T \right) \right]
\]

**Single particle** life time \( \frac{1}{\tau} \propto \max \left( |\omega|, T \right) \) is coincident (?!?) with the **transport life time** \( \Rightarrow \) linear resistivity.
The fermionic criticality conundrum

Pauli exclusion principle generates the Fermi-energy, Fermi surface.

How to reconcile the quantum statistical scales with scale invariance?

Why is this quantum scale invariance of a local, purely temporal kind?

How can a (heavy) Fermi-liquid emerge from a ‘microscopic’ quantum critical state?

Why is this state good for high Tc superconductivity, and a phletora of exotic “competing orders”?

AdS/CFT gives an answer!
Plan


2. Crash course: the AdS/CFT correspondence.

General relativity “=“ quantum field theory

Gravity

Quantum fields

Maldacena 1997
Holography with lasers

Components of a Hologram:
- Laser
- Beam Splitter
- Object
- Reference Beam
- Mirror
- Film Plate

Three dimensional image

Encoded on a two dimensional photographic plate
Gravity - quantum field holography

Einstein world “AdS” = Anti de Sitter universe

Quantum fields in flat space “CFT” = quantum critical
‘t Hooft’s holographic principle

Hawking Temperature: \[ T = \frac{\hbar g}{2\pi k c} \]

\( g \) = acceleration at horizon

BH entropy: \[ S = \frac{k c^3 A}{4\hbar G} \]

\( A \) = area of horizon

Number of degrees of freedom (field theory) scales with the area and not with the volume (gravity)
The bulk: Anti-de Sitter space

Extra radial dimension of the bulk $\iff$ scaling “dimension” in the field theory

Bulk AdS geometry = scale invariance of the field theory

$$dr^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

$$F(r) = -\Lambda r^2 + 1, \quad \Lambda < 0$$
Weak-Strong Duality

Bulk: weakly coupled gravity

Boundary: strongly coupled Quantum Field theory

Einstein-Maxwell

Large N Yang-Mills at large ‘t Hooft coupling

Kramers-Wannier
Quantum critical dynamics: classical waves in AdS

\[ W_{CFT}(J) = S_{AdS}(\phi)_{\phi x_0 \to 0 = J} \]

\[ g_{YM}^2 N = \frac{R^4}{\alpha} \]

\[ g_{YM}^2 = g_s \]
Fermionic renormalization group

The Magic of AdS/CFT!

Wilson-Fisher RG: based on Boltzmannian statistical physics
Plan


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Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Gubser, Hartnoll …).
The black hole is the heater

GR in Anti de Sitter space

\[ dr^2 = -F(r) dt^2 + \frac{dr^2}{F(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ F(r) = -\Lambda r^2 + 1 - \frac{GM}{r} \]

Quantum-critical fields on the boundary:
- at the Hawking temperature
- entropy = black hole entropy
Planckian dissipation

Schwarzschild Black Hole: encodes for the finite temperature dissipative quantum critical fluid.

Universal entropy production time:

$$\tau = \tau_{\hbar} \approx \frac{\hbar}{k_B T}$$

Minimal viscosity: quark gluon plasma, unitary cold atom fermion gas

$$\frac{\eta}{\hbar} = \frac{\sigma^A}{J} \frac{k_B}{T}$$

Linear resistivity high Tc metals:

$$\rho \propto \frac{1}{\tau_{\hbar}} \propto k_B T$$
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“AdS-to-ARPES”: Fermi-liquid (?) emerging from a quantum critical state.

QuickTime™ and a decompressor are needed to see this picture.

String Theory, Quantum Phase Transitions, and the Emergent Fermi Liquid

Mihailo Ćubrović, Jan Zaane, Koenraad Schalm*

A central problem in quantum condensed matter physics is the critical theory governing the zero-temperature quantum phase transition between strongly renormalized Fermi liquids as found in heavy fermion intermetallics and possibly in high-temperature superconductors. We found that the mathematics of string theory is capable of describing such fermionic quantum critical states. Using the anti–de Sitter/conformal field theory correspondence to relate fermionic quantum critical fields to a gravitational problem, we computed the spectral functions of fermions in the field theory. By increasing the fermion density away from the relativistic quantum critical point, a state emerges with all the features of the Fermi liquid.

Quantum many-particle physics lacks a general mathematical theory to deal with fermions at finite density. This is known as the “fermion sign problem”.

There is no recourse to brute-force lattice models because the statistical path-integral methods that work for any bosonic quantum field theory do not work for finite-density Fermi systems.
Breaking fermionic criticality with a chemical potential

‘Dirac waves’

Electrical monopole

Fermi-surface??
AdS/ARPES for the Reissner-Nordstrom non-Fermi liquids

Fermi surfaces but no quasiparticles!

A

Critical FL

B

Marginal FL

C

Non Landau FL
Holographic quantum critical fermion state

Strange Metal Transport Realized by Gauge/Gravity Duality

Thomas Faulkner, Habil Iqbal, Hong Liu, John McGreevy, and David Vegh

Fermi liquid theory explains the thermodynamic and transport properties of most metals. The so-called non-Fermi liquids deviate from these expectations and include exotic systems such as the strong-coupling phase of cuprate superconductors and heavy fermion materials near a quantum phase transition. We used the anti-de-Sitter/conformal field theory correspondence to identify a class of non-Fermi liquids: their low-energy behavior is found to be governed by a non-trivial infrared fixed point, which exhibits nanoscale scaling behavior only in the time direction. For some representatives of this class, the resistivity has a linear temperature dependence, as is the case for strange metals.

During the past decade, developments in string theory have revealed surprising and profound connections between gravity and many-body systems, resulting in the emergence of a new description for strongly coupled many-body systems. The anti-de-Sitter/conformal field theory (AdS/CFT) correspondence ([1,5]) relates a gravity theory in a weakly curved \((d + 1)\)-dimensional anti-de Sitter (AdS, \(d = 4\)) spacetime to a strongly coupled \(d\)-dimensional quantum field theory defined on its boundary. This correspondence maps questions about complicated many-body phenomena at strong coupling to solvable single- or few-body classical problems in a curved geometry. Black holes in this geometry play a surprising and universal role in characterizing the dynamics of the boundary theory at finite temperature and density: a development anticipated by the discovery of Hawking and Bekenstein in the 1970s (4, 7) that black holes are intrinsically thermodynamic objects. Important dynamical insights into the thermodynamics (6) and transport behavior (7) of strongly correlated systems have been obtained from simple geometric aspects of black hole spacetimes.

Very recently, this apparatus has been brought to bear on the problem of fermions near quantum criticality (6–13). The basic strategy is to perform angle-resolved photoemission spectroscopy (ARPES) on a charged black hole, which describes the ground state of a class of strongly coupled many-body systems. The fermionic response, which is proportional to the ARPES intensity, may be computed by studying the scattering of Dirac particles off this black hole. By exploring different regions in parameter space, both Fermi liquid-like (13) and non-Fermi liquid behavior (9, 14) were discovered, establishing the black hole as a new tool for addressing outstanding questions related to intersting fermions at finite density.

A prime example of a theoretical challenge to which such a tool may be applied is the strange metal phase of the cuprate high-temperature superconductors. The metallic state above the superconducting transition temperature \(T_c\), near optimal doping has unusual transport properties different from those of a normal metal, and was thus dubbed a strange metal. Understanding this phase is believed to be essential for deciphering the mechanisms for high-\(T_c\) superconductivity. The anomalous behavior of the strange metal (perhaps most prominently the simple and robust linear temperature dependence of the resistivity) implies that the low-
Horizon geometry of the extremal black hole: ‘emergent’ AdS$^2$ \Rightarrow IR of boundary theory controlled by emergent temporal criticality

Gravitational ‘mechanism’ for marginal (critical) Fermi-liquids:

$$G^{-1} = \omega - \nu_F (k - k_F) - \Sigma(k, \omega)$$

Temporal scale invariance IR “lands” in probing fermion self energy!

$$\Sigma'' \sim \omega^{2\nu k_F}$$

Fermi-surface “discovered” by matching UV-IR: like Mandelstam “fermion insertion” for Luttinger liquid!
Gravitationally coding the fermion propagators (Faulkner et al. Science Aug 27. 2010)

T=0 extremal black hole, **near horizon geometry ‘emergent scale invariant’**:

\[ AdS_2 \otimes R_2 \Rightarrow g_k(\omega) = c(k)\omega^{2\nu_k} \]

Matching with the **UV** infalling Dirac waves:

\[ G_R(\omega,k) = F_0(k) + F_1(k)\omega + F_2(k)g_k(\omega) \]

**Special momentum shell:** \( |k| \equiv k_F \)

\[ G_R(\omega,k) = \frac{h_1}{k - k_F - \omega/\nu_F - \Sigma(\omega,k)}; \quad \Sigma(\omega,k) = hg_{k_F}(\omega) = h_2e^{i\gamma_{kF}}\omega^{2\nu_{kF}} \]

**Space-like:** IR-UV matching ‘organizes’ Fermi-surface.

**Time-like:** IR scale invariance picked up via AdS2 self energy

**Miracle, this is like critical/marginal Fermi-liquids!!**
Marginal Fermi liquid phenomenology.

Fermi-gas interacting by **second order** perturbation theory with ‘singular heat bath’:

\[
\text{Im} P(q, \omega) \propto -N(0) \frac{\omega}{T}, \quad \text{for } |\omega| < T
\]

\[
\propto -N(0) \text{sign}(\omega), \quad \text{for } |\omega| > T
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Directly observed in e.g. Raman ??

Single electron response (photoemission):

\[
G(k, \omega) = \frac{1}{\omega - v_F(k - k_F) - \Sigma(k, \omega)}
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\[
\Sigma(k, \omega) \propto \left( \frac{g}{\omega_c} \right)^2 [\omega \ln(\max(|\omega|, T)/\omega_c) - i \frac{\pi}{2} \max(|\omega|, T)]
\]

**Single particle** life time \( \frac{1}{\tau} \propto \max(|\omega|, T) \) is coincident (?!?) with the **transport life time** => linear resistivity.
The zero temperature extensive entropy ‘disaster’

The ‘extremal’ charged black hole with AdS$^2$ horizon geometry has zero Hawking temperature but a finite horizon area.

AdS-CFT

The ‘seriously entangled’ quantum critical matter at zero temperature should have an extensive ground state entropy (??!)
“Planckian dissipation”: quantum critical matter at high temperature, perfect fluids and the linear resistivity (Son, Policastro, …, Sachdev).

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Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Gubser, Hartnoll …).
Black hole hair can be fermionic!

Schalm, Cubrovic, JZ (arXiv:1012.5681)

Stable Fermi liquid on the boundary!

‘Hydrogen atom’: one Fermion quantum mechanical probability density.
Fermionic hair: stability and equation of state.

Strongly renormalized $E_F$

Single Fermion spectral function: non Fermi-liquid Fermi surfaces have disappeared!
The Fermi-liquid VEV: Hair profile vs. statistics

- Scalar vs. fermionic hair: scale-free vs. scale-ful profile

Bosons accumulate at the horizon

Position of the maximum determines the Fermi energy
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The holographic superconductor
Hartnoll, Herzog, Horowitz, arXiv:0803.3295

Condensate (superconductor, … ) on the boundary!

(Scalar) matter ‘atmosphere’

‘Super radiance’: in the presence of matter the extremal BH is unstable => zero T entropy always avoided by low T order!!!
“Bottom-up” : Minimal holographic superconductivity (H³)

What are the minimal bulk ingredients to capture the boundary superconductor?

- Continuum theory $\Rightarrow T_{\mu\nu} \Rightarrow g_{\mu\nu}$ in bulk.
- Conserved charge $\Rightarrow J_\mu \Rightarrow A_\mu$ in bulk.
- Fermion pair operator $\langle \hat{\mathcal{H}} \rangle \Rightarrow \hat{\chi}$ in bulk.

Write a minimal phenomenological bulk Lagrangian

$$L = R + \frac{6}{L^2} - \frac{1}{4} F^{ab} F_{ab} - V(|\psi|) - |\nabla \psi - iqA \psi|^2$$
Bulk geometry: AdS Reissner-Nordstrom black hole

Finite temperature and finite charge density: AdS RN black hole

\[ ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2) \]

where \[ g(r) = r^2 - \frac{1}{r} \left( r_+^3 + \frac{\rho^2}{4r_+^2} \right) + \frac{\rho^2}{4r^2} \]

Scalar potential:

\[ \mathcal{A}^0 = \mathcal{A} \left( \frac{\rho}{r} - \frac{\rho}{r} \right) \]

Hawking temperature:

\[ T = \frac{12r_+^4 - \rho^2}{16\pi r_+^3} \]
Minimal model: $V(\mathcal{O}) = -2\nu^2$, the dual operator $\Psi$ can have conformal dimensions $\Delta = 1, 2$

The Reissner-Nordstrom BH describes the normal state, but it goes unstable at a $T < T_c \approx \lambda$ because $m_{\text{eff}}^2 \approx m^2 - q^2 A_0^2$ turns negative.

Below $T_c$ the black hole gets hair in the form of a “scalar atmosphere”: via the dictionary, a VEV emerges in the field theory in the absence of a source.

The global U(1) symmetry of the CFT is spontaneously broken into a superfluid!
The Bose-Einstein Black hole hair

Scalar hair accumulates at the horizon

Mean field thermal transition.

Graphs showing the accumulation of scalar hair and the mean field thermal transition.
Holographic superconductivity: stabilizing the fermions.

**Fermion spectrum for scalar-hair back hole** (Faulkner et al., 911.340; Chen et al., 0911.282):

- Temperature dependence as expected for ‘quantum-critical’ superconductivity (She, JZ, 0905.1225)

- Excessive temperature dependence ‘pacified’!

‘BCS’ Gap in fermion spectrum !!
‘Pseudogap’ fermions in high Tc superconductors

Gap stays open above Tc

But sharp quasiparticles disappear in incoherent ‘spectral smears’ in the metal

Shen group, Nature 450, 81 (2007)
“Double trace” Phase Diagram

This looks like “quantum critical graphene” at zero density

This is the “marginal Fermi-liquid” Liu style
More fanciful: Quantum phase transitions

Quite different behaviors of the holographic quantum phase transitions by tuning the holographic SC down by mass or double trace deformation

Iqbal, Liu, Mezeiar, arXiv: 1108.0425
K.Jensen arXiv:1108.0421
Why is Tc high?

“Because there is superglue binding the electrons in pairs”

Wrong!

The superfluid density is tiny, it is very easy to ‘bend and twist’ a high Tc superconductor. Its cohesive energy sucks.

Tc’s are set by the competition between the two sides …

The theory of the mechanism should explain why the free energy of the metal is seriously BAD.
Observing the pairing mechanism ...

Claim: the maximal knowledge on the pairing mechanism is encoded in the temperature evolution of the normal state dynamical pair susceptibility,

\[
\chi_p(q,\omega) = -i \int_0^\infty dt e^{i\omega t - 0^+ t} \left\langle \left[ b^+ (q,0), b(q,t) \right] \right\rangle
\]

\[
b^+ (q,t) = \sum_k \left[ c^+_{k+q/2,\uparrow}(t) c^+_{-k+q/2,\downarrow}(t) \right]
\]
Standard BCS

“Critical glue”

\( \chi_p(\hbar \omega) \)

Holographic SC (AdS4)

Holographic SC (AdS2)

QC-BCS
Standard BCS

"Critical glue"

Holographic SC (AdS4)

Holographic SC (AdS2)

QC-BCS

\[ T^\Delta \chi''_p \left( \frac{\hbar \omega}{k_B T} \right) \]
Observing the origin of the pairing mechanism

\[ T'_c > T > T_c \]

2\textsuperscript{nd} order Josephson effect

\[ I_s (H, V) \sim \frac{1}{R_N^2} \text{Im} \chi_{\text{pair}} (k, \omega) \]

\[ \omega = 2eV \]
Why Webb Pairing telescope?

QC metal:
Need large dynamical range: $T, \omega \approx 10 - 100 T_c$
QC superconductor at ambient conditions with low $T_c$:

CeIrIn$_5$, $T_c = 0.4$K

Probe superconductor:
High $T_c$
Tunneling into d(?)-wave channel
Cuprate ?
Full gap to suppress QP current (?)

MgB$_2$ ($T_c=40$K)?

Barrier is the challenge!
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Dirac hair/electron star: Fermi-liquids emerging from a non Fermi liquid (critical) ultraviolet, like heavy fermions (Schalm, Cubrovic, Hartnoll).

Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Hartnoll, Herzog, Horowitz).
Further reading

**Condensed matter tutorials:**
J. Zaanen, Science 319, 1205; arXiv:1012.5461

**AdS/CMT tutorials:**

**Non fermi-liquids:**

**Fermi-liquids:**

**Holographic superconductors:**
J.-H. She et al., arXiv:1105.5377
Thermodynamics: where are the fermions?

Hartnoll et al.: arXiv:0908.2657,0912.0008

**Large N limit**: thermodynamics entirely determined by AdS geometry.

**Fermi surface** dependent thermodynamics, e.g. Haas van Alphen oscillations?

**Leading 1/N corrections**: “Fermionic one-loop dark energy”

Quantum corrections: one loop using Dirac quasinormal modes:
‘generalized Lifshitz-Kosevich formula’ for HvA oscillations.

\[
\chi_{osc.} = -\frac{\partial^2 \Omega_{osc.}}{\partial B^2} = \frac{\pi A T c k_F^4}{eB^3} \cos \frac{\pi c k_F^2}{eB} \sum_{n=0}^{\infty} e^{-\frac{cT_k^2}{eB\mu} \left(\frac{T}{\mu}\right)^{2\nu-1}} F_n(\mu)
\]
Collective transport: fermion currents

Conductivity

O(N^2)

O(N^0)

Tedious one loop calculation, ‘accidental’ cancellations: \( \rho_{FS} \propto \Sigma''_{1-\text{fermion}} \propto T^{2v} \)

‘Strange coincidence’ of one electron and transport lifetime of marginal fermi liquid finds gravitational explanation!
‘Shankar/Polchinski’ functional renormalization group

UV: weakly interacting Fermi gas

Integrate momentum shells: functions of running coupling constants

All interactions (except marginal Hartree) irrelevant $\Rightarrow$ Scaling limit might be perfectly ideal Fermi-gas
The end of weak coupling

Strong interactings:

Fermi gas as UV starting point does not make sense!

=> ‘emergent’ Fermi liquid fixed point remarkably resilient (e.g. 3He)

=> Non Fermi-liquid/non ‘Hartree-Fock’ (BCS etc) states of fermion matter?
Empty
Empty