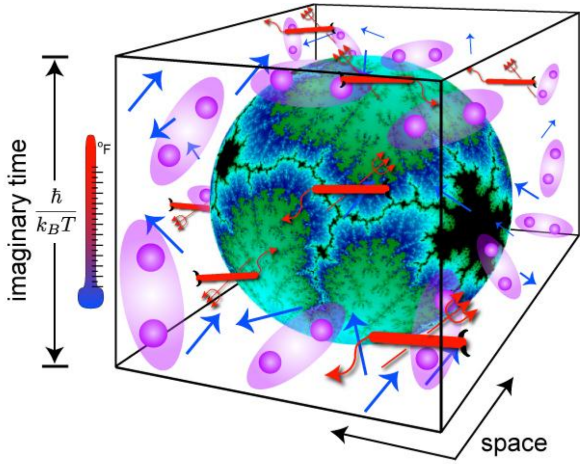
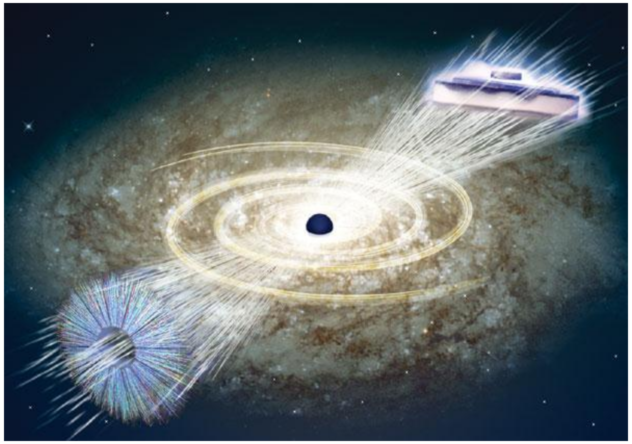
Quantum Criticality, high Tc superconductivity and the AdS/CFT correspondence.

Jan Zaanen





Nederlandse Organisatie voor Wetenschappelijk Onderzoek

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

FOM

String theory: what is it really good for?

- Hadron (nuclear) physics: quark-gluon plasma in RIHC.

- Quantum matter: quantum criticality in heavy fermion systems, high Tc superconductors, …

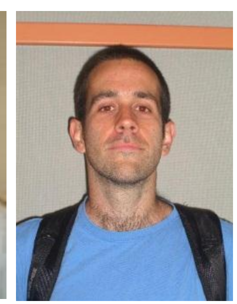
Started in 2001, got on steam in 2007.


Son


Hartnoll


Herzog


Kovtun


McGreevy

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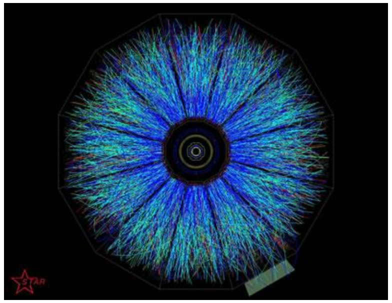
Liu

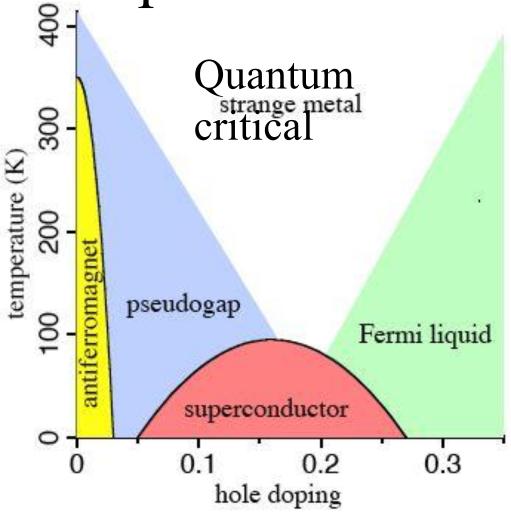

Schalm

Quantum critical matter

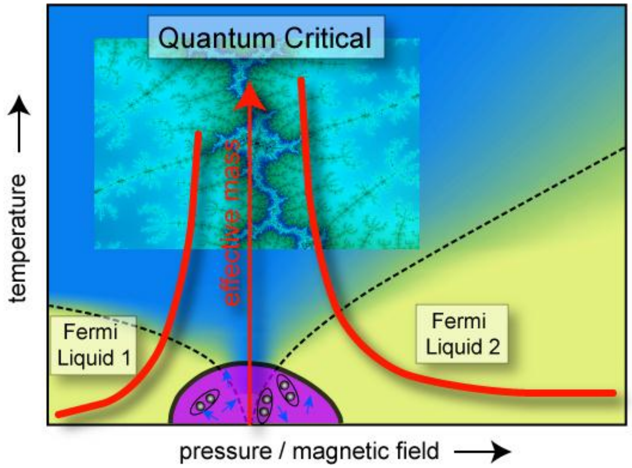
Quark gluon plasma



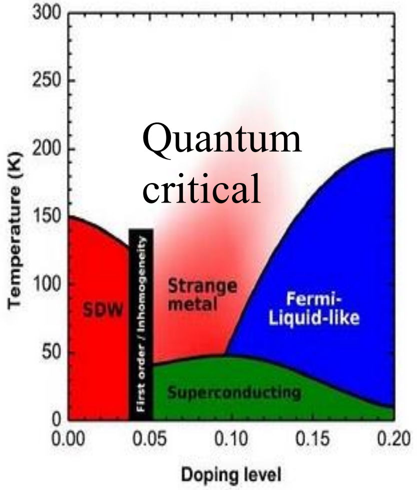
High Tc superconductors



Heavy fermions

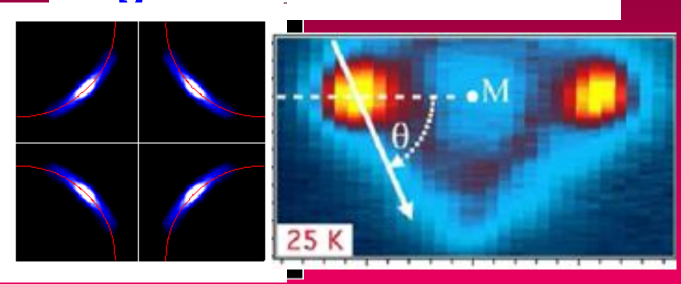


Iron superconductors (?)

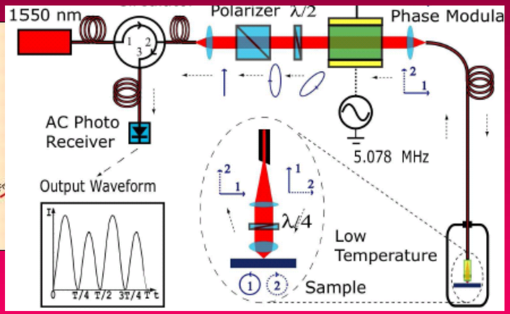


High-Tc Has Changed Landscape of Condensed Matter Physics

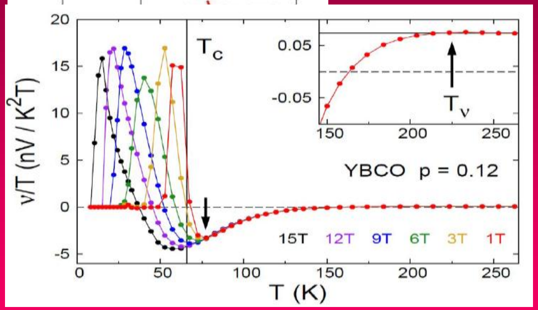
High-resolution ARPES



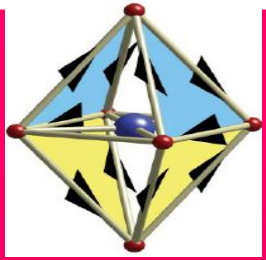
Magneto-optics



Transport-Nernst effect

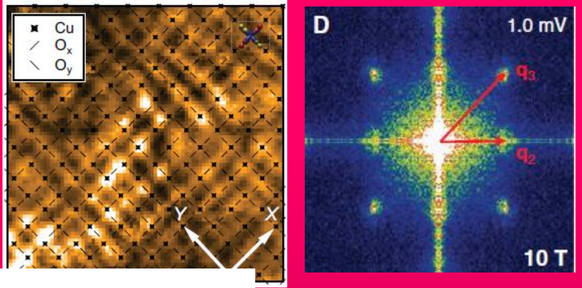


Spin-polarized Neutron

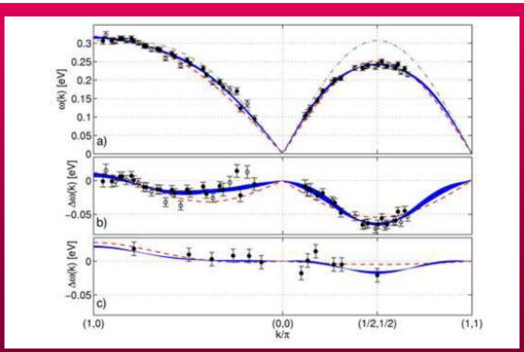


High Tc Superconductivity

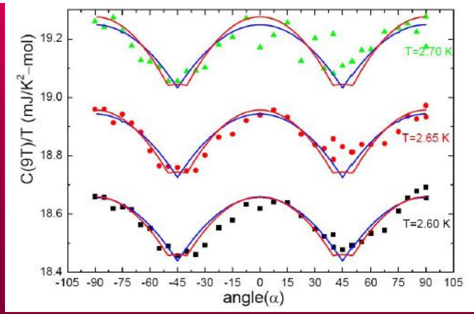
STM

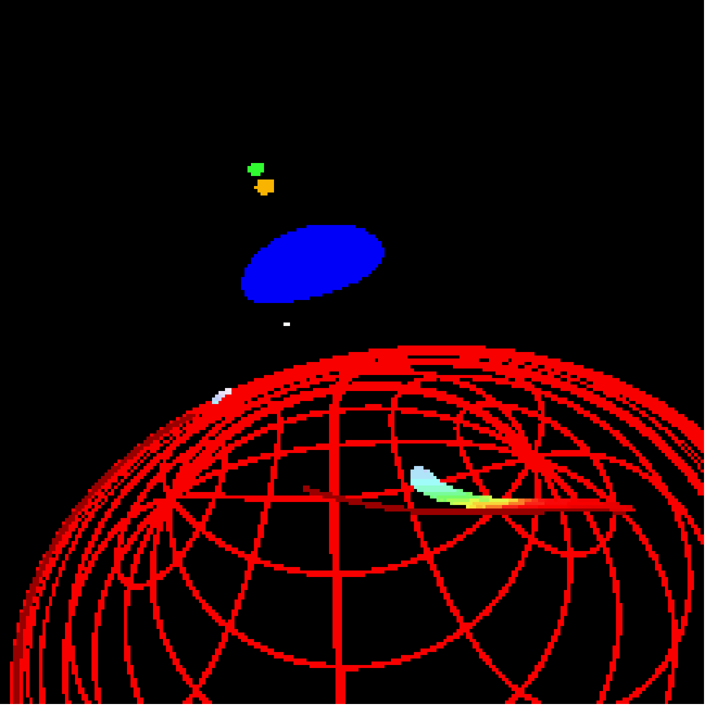


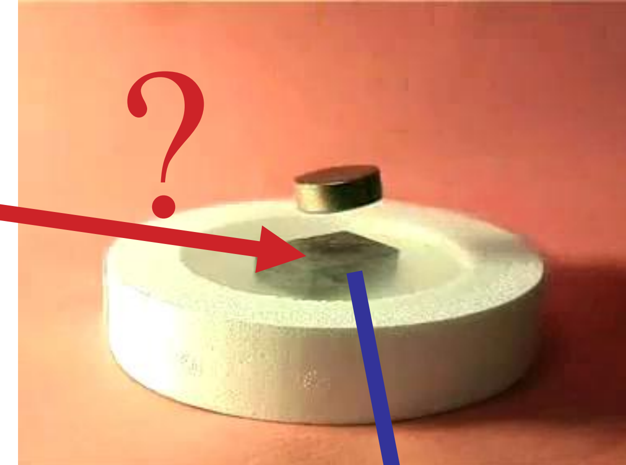
Inelastic X-Ray Scattering



Angle-resolved MR/Heat Capacity







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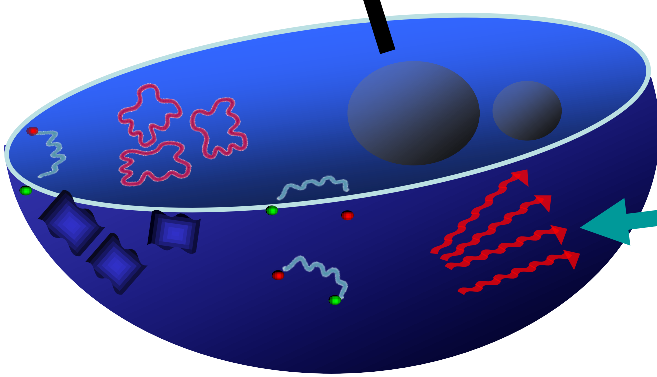
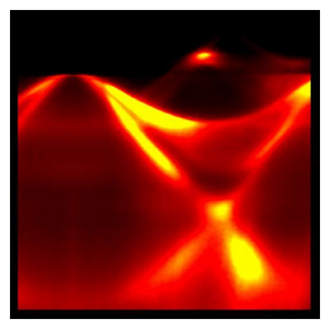
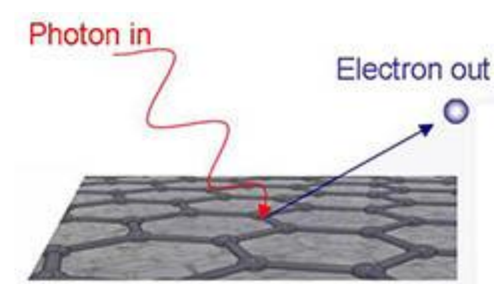
decompressor
are needed to see this picture.





Photoemission spectrum



Holography and quantum matter

"Planckian dissipation": quantum critical matter at high temperature, perfect fluids and the linear resistivity (Son, Policastro, …, Sachdev)**.**

Reissner Nordstrom black hole: "critical Fermi-liquids", like high Tc's normal state (Hong Liu, John McGreevy)**.**

Dirac hair/electron star: Fermi-liquids emerging from a non Fermi liquid (critical) ultraviolet, like overdoped high Tc (Schalm, Cubrovic, Hartnoll)**.**

Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Gubser, Hartnoll …)**.**

Plan

1. Crash course: quantum critical electron matter in solids.

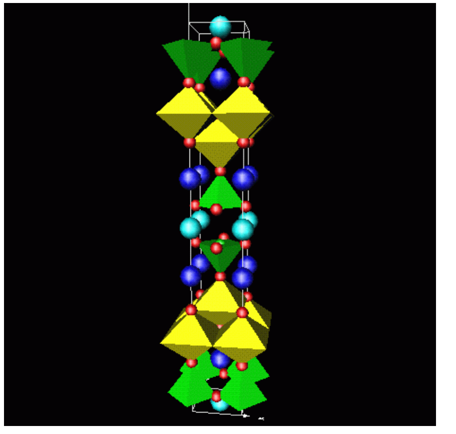
2. Crash course: the AdS/CFT correspondence.

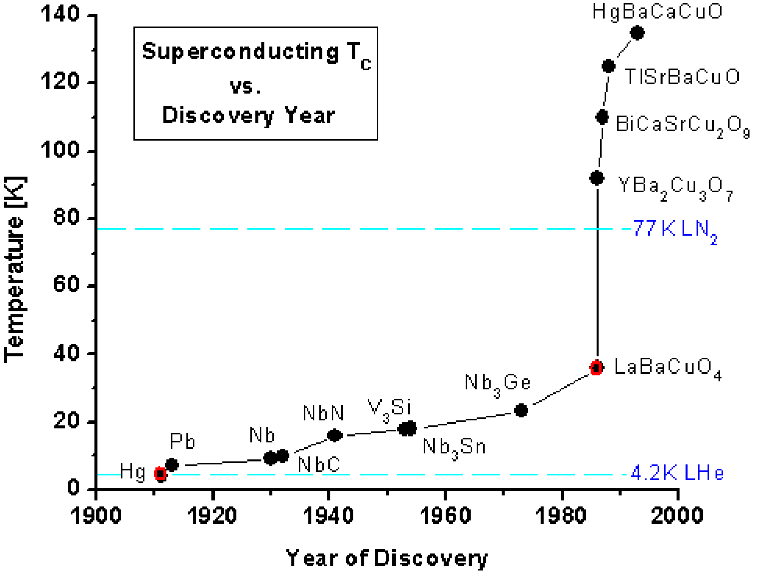
3. Holographic quantum matter: Planckian dissipation, marginal/critical Fermi-liquids, Fermi liquids and superconductors.

Twenty five years ago …

Mueller **Bednorz**

Ceramic CuO's,
likeYBa2Cu3O7





Superconductivity jumps to 'high' temperatures

Graveyard of Theories

Mott

Laughlin

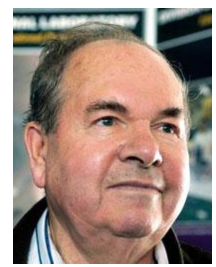
Mueller

Schrieffer

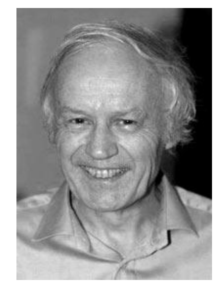
De Gennes

Bednorz

Anderson



Abrikosov



Leggett

QuickTime™ and a
decompressor
are needed to see this picture.

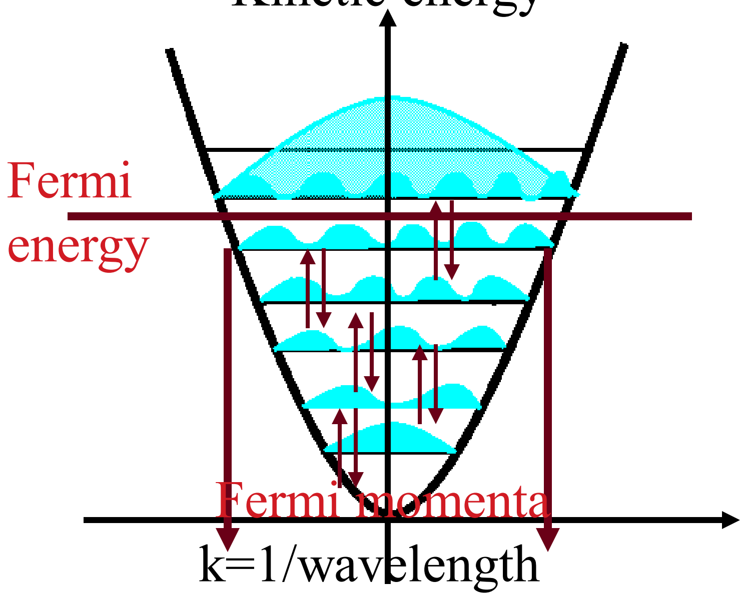
Lee

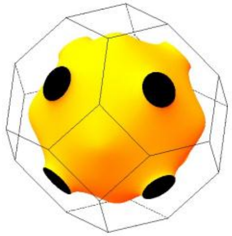
Wilczek

Ginzburg

Yang

The quantum in the kitchen: Landau's miracle





Fermi surface of copper

Electrons are waves

Pauli exclusion principle: every state occupied by one electron

Unreasonable: electrons strongly interact !!

Landau's Fermi-liquid: the highly collective low energy quantum excitations are like electrons that do not interact.

BCS theory: fermions turning into bosons

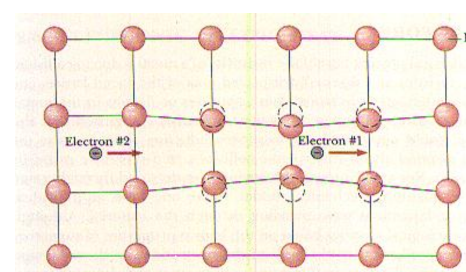
Bardeen Cooper Schrieffer

Fermi-liquid fundamentally unstable to attractive interactions.

Quasiparticles pair and Bose condense:

Ground state $$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) |vac.\rangle$$

Conventional superconductors (Tc < 40K): "pairing glue"= exchange of quantized lattice vibrations (phonons)

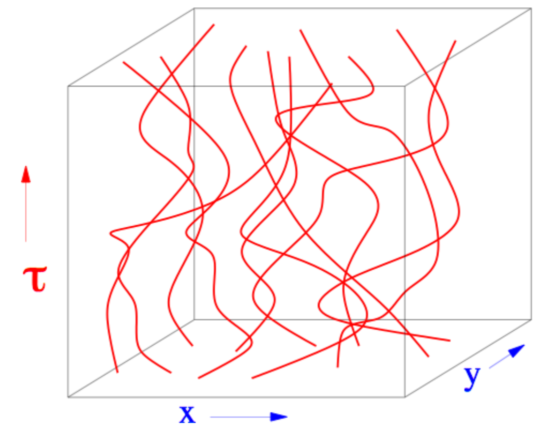


Fermion sign problem

Imaginary time path-integral formulation

$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int d\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$
$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}d\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Boltzmannons or Bosons:

- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:

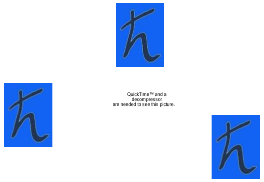
- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!

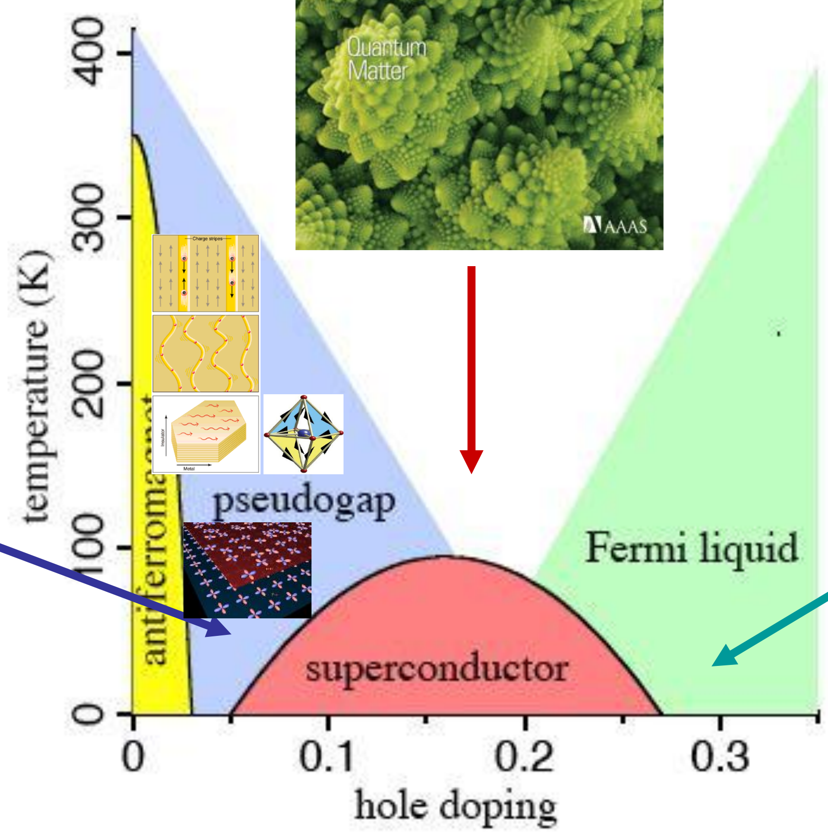
Phase diagram high Tc superconductors

The clash: the quantum critical metal

… which is good for superconductivity!

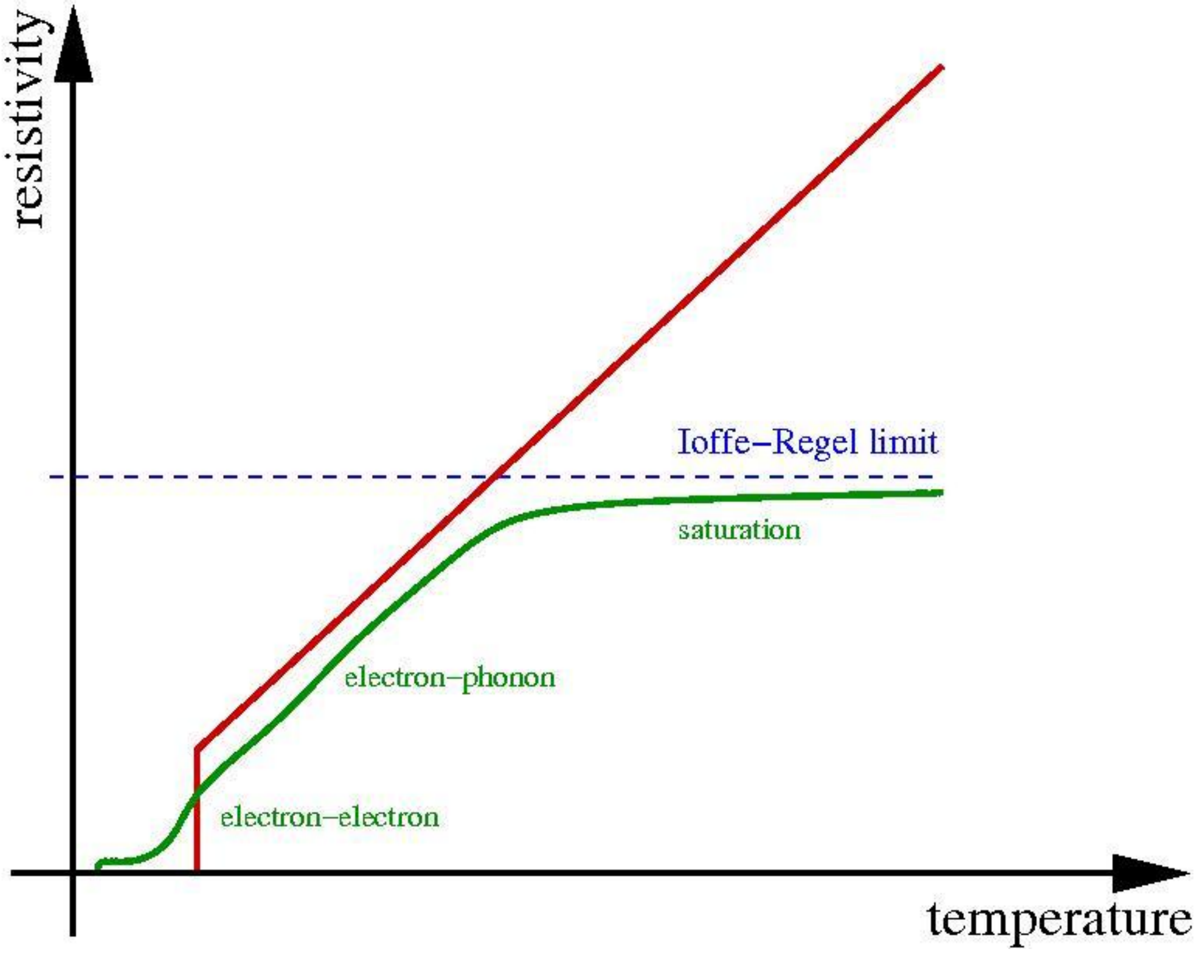
The quantized traffic jam





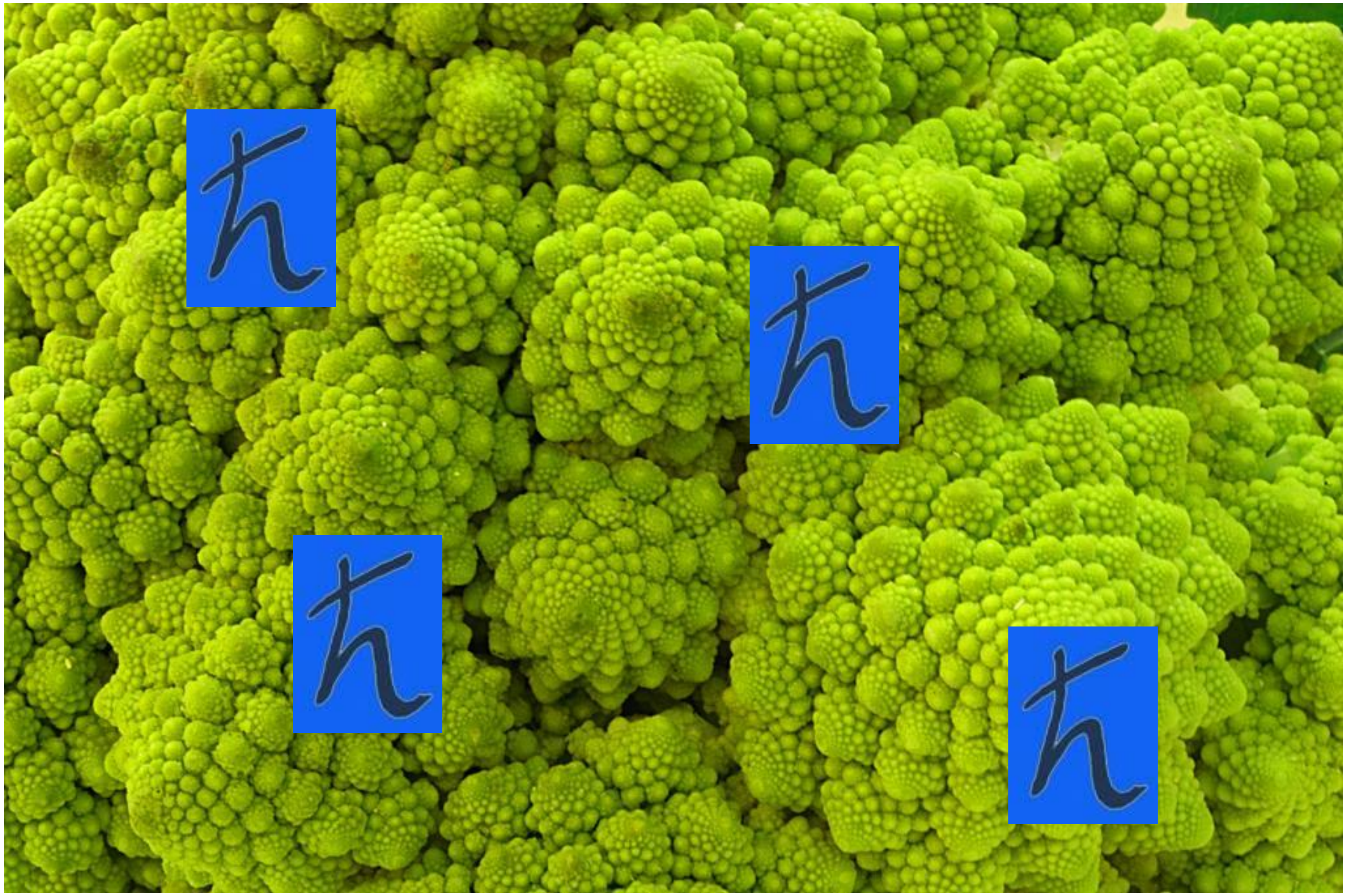
The quantum fog (Fermi gas) returns

Divine resistivity

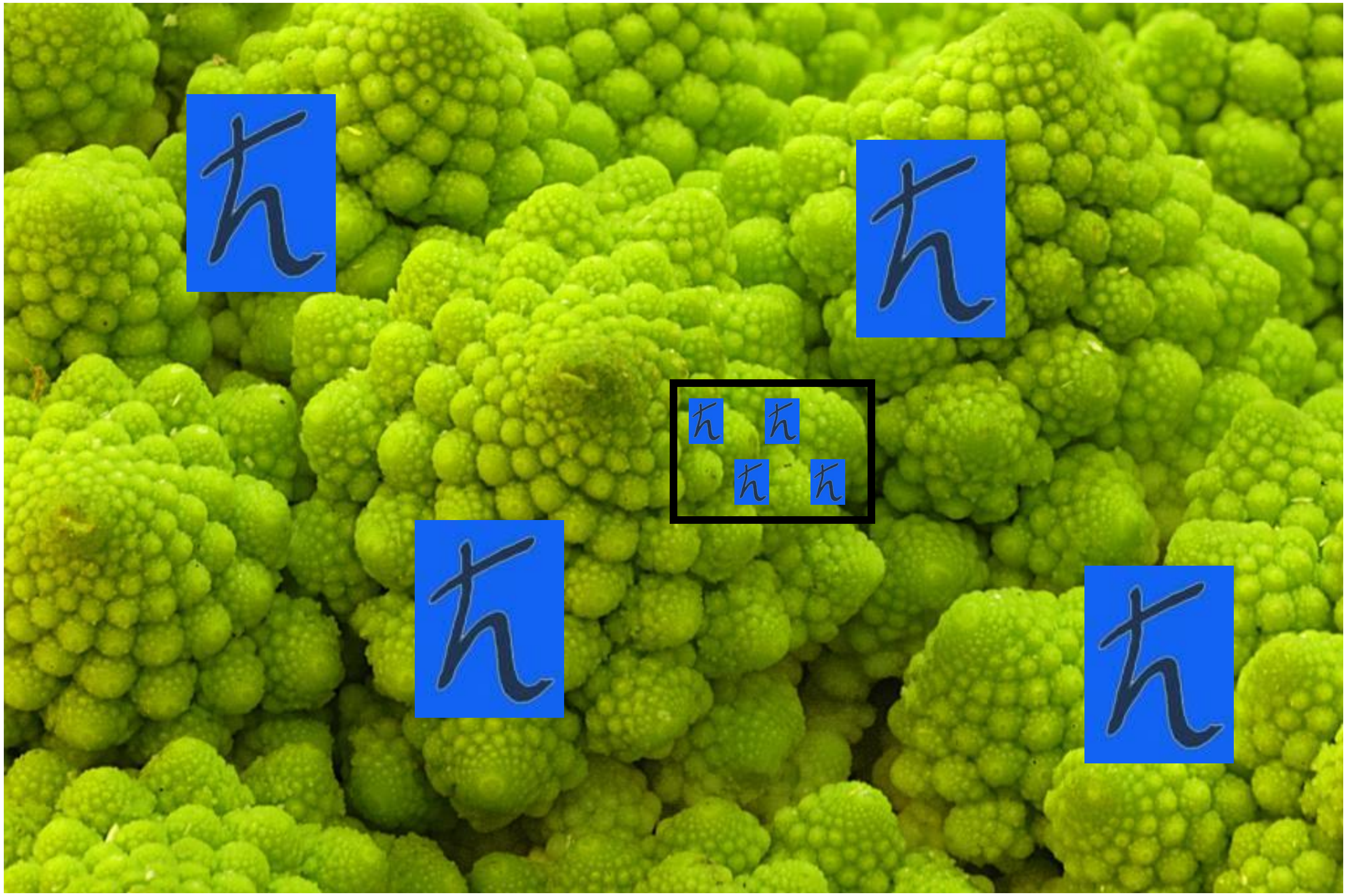


Fractal Cauliflower (romanesco)

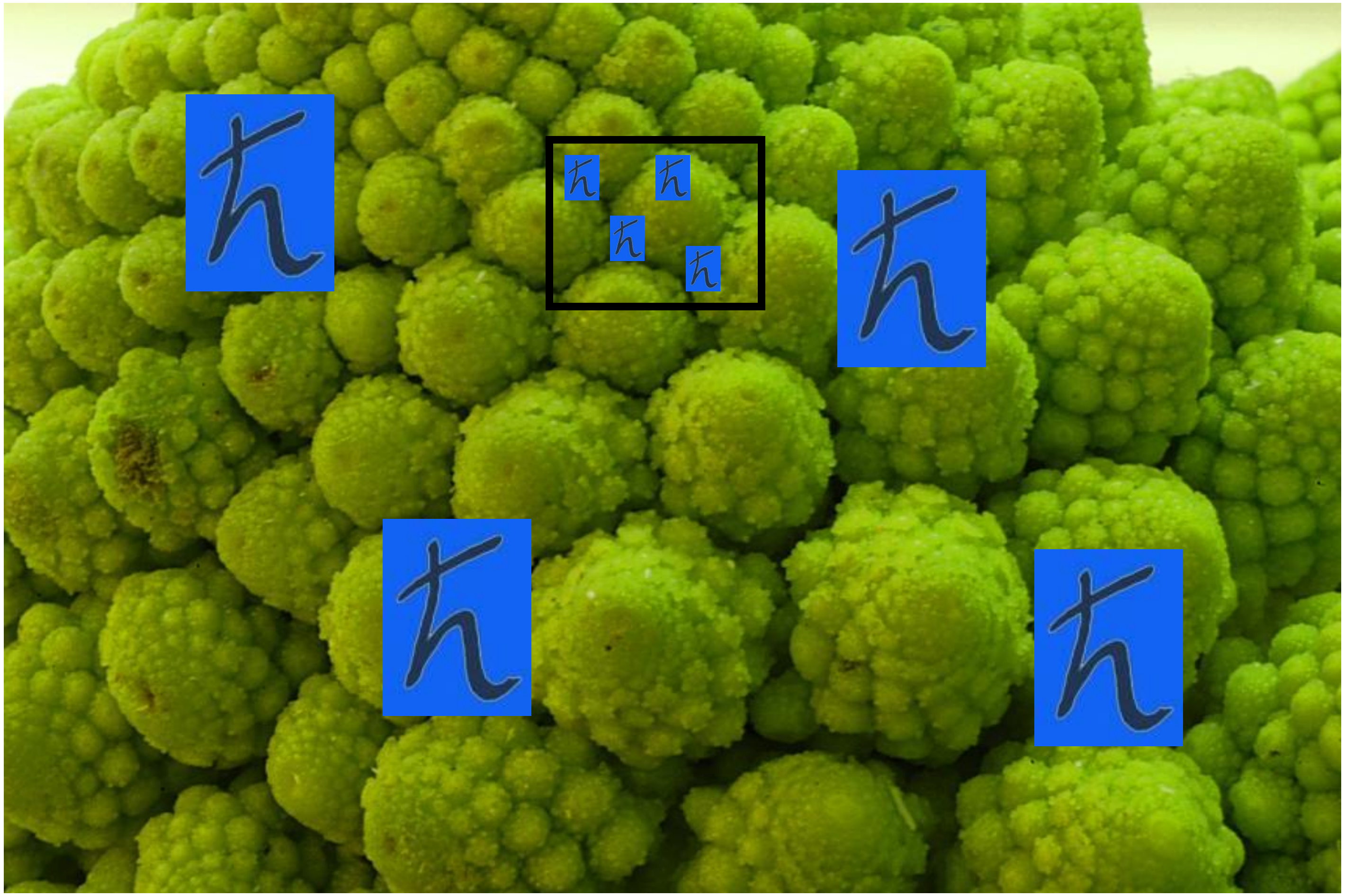
Quantum critical cauliflower



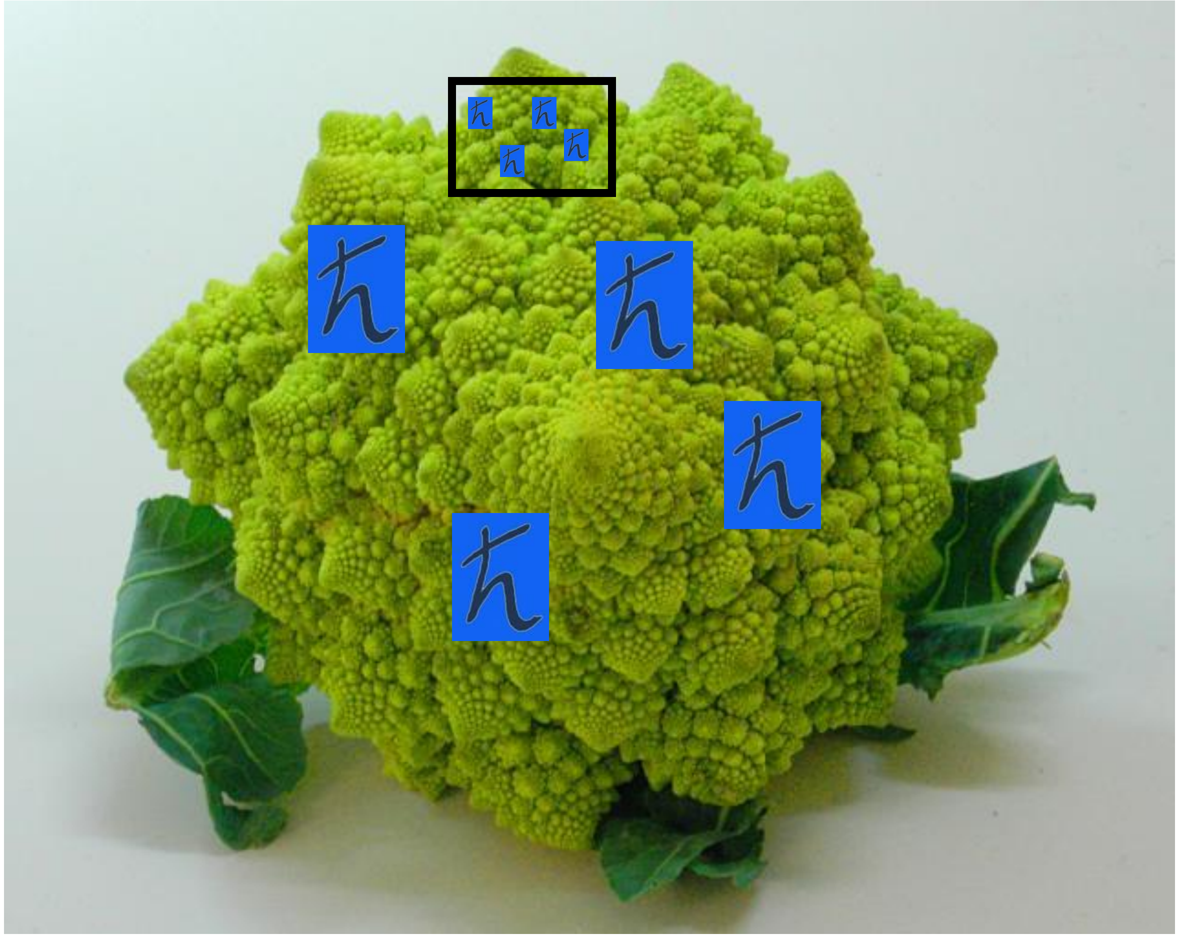
Quantum critical cauliflower



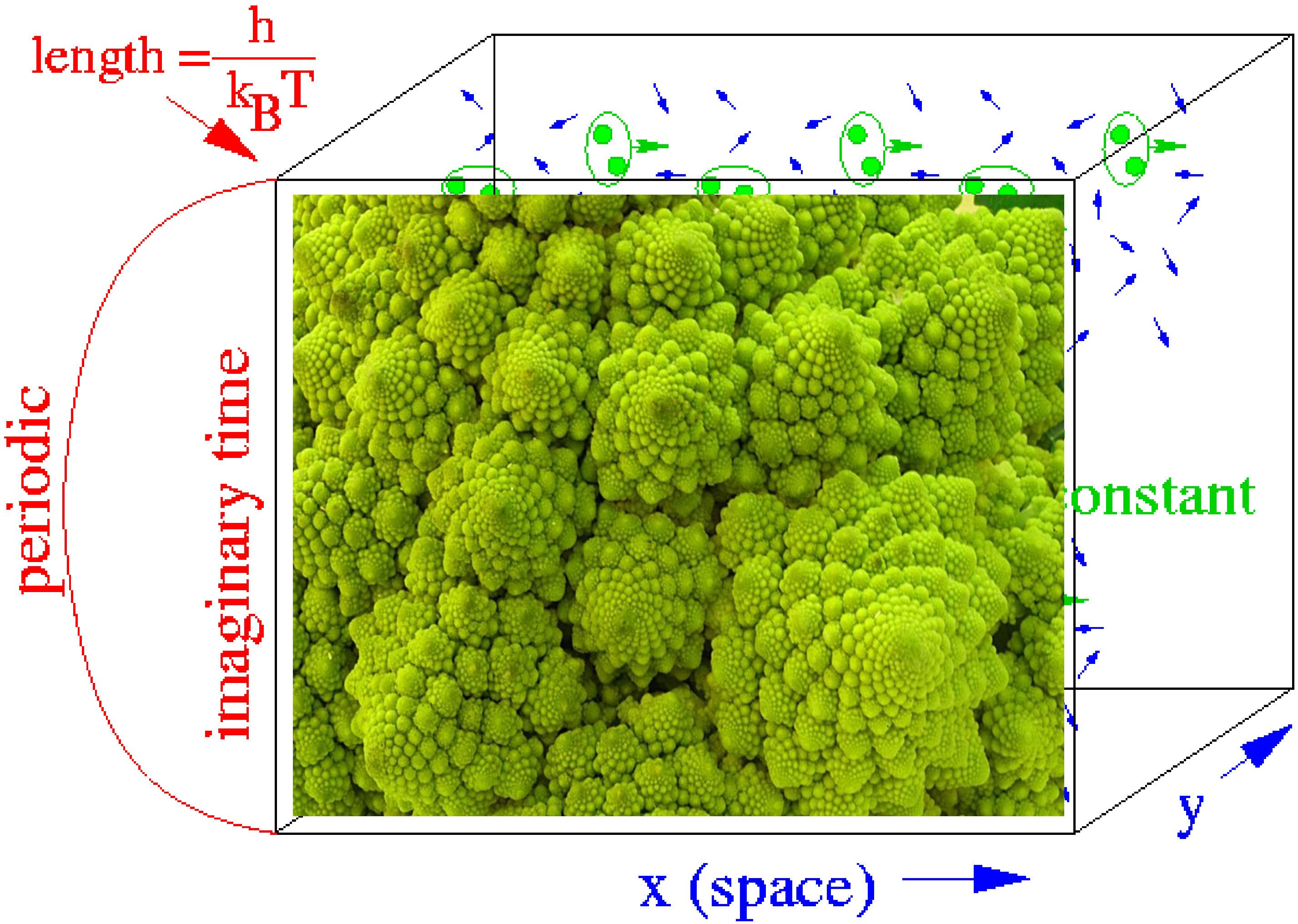
Quantum critical cauliflower



Quantum critical cauliflower



Quantum criticality or 'conformal fields'

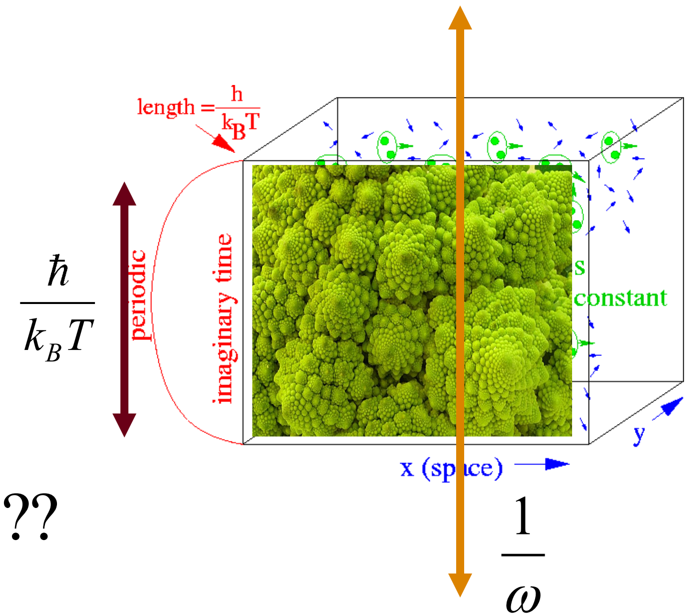


Quantum critical hydrodynamics: Planckian relaxation time

Relaxation time τ : time it takes to convert work in entropy.

Viscosity: $\eta = (\varepsilon + p)\tau$

Entropy density: $s = \dfrac{\varepsilon + p}{T}$



"Planckian viscosity" $\Rightarrow \dfrac{\eta}{s} = T\tau \approx \dfrac{\hbar}{k_B}$??

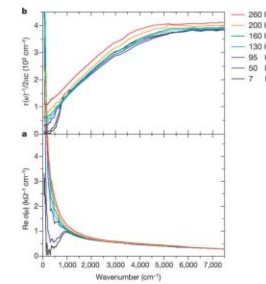
$$\tau = \tau_{\hbar} \approx \frac{\hbar}{k_B T}$$

Planckian relaxation time = the shortest possible relaxation time under equilibrium conditions that can only be reached when the quantum dynamics is scale invariant !!

Critical Cuprates are Planckian Dissipators

van der Marel, JZ, … Nature 2003:

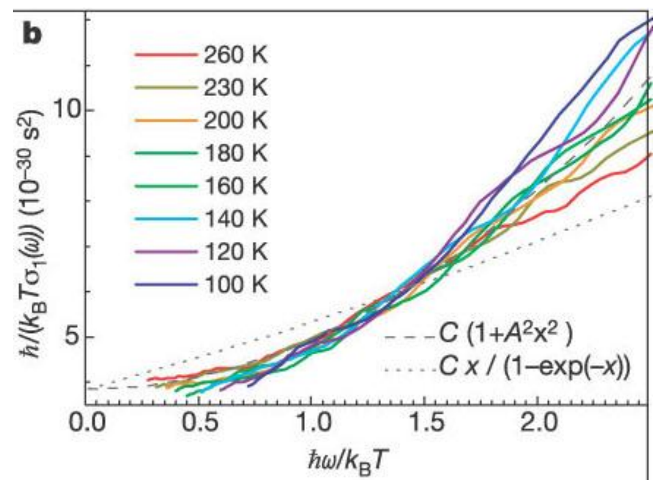
Optical conductivity QC cuprates



Frequency less than temperature:

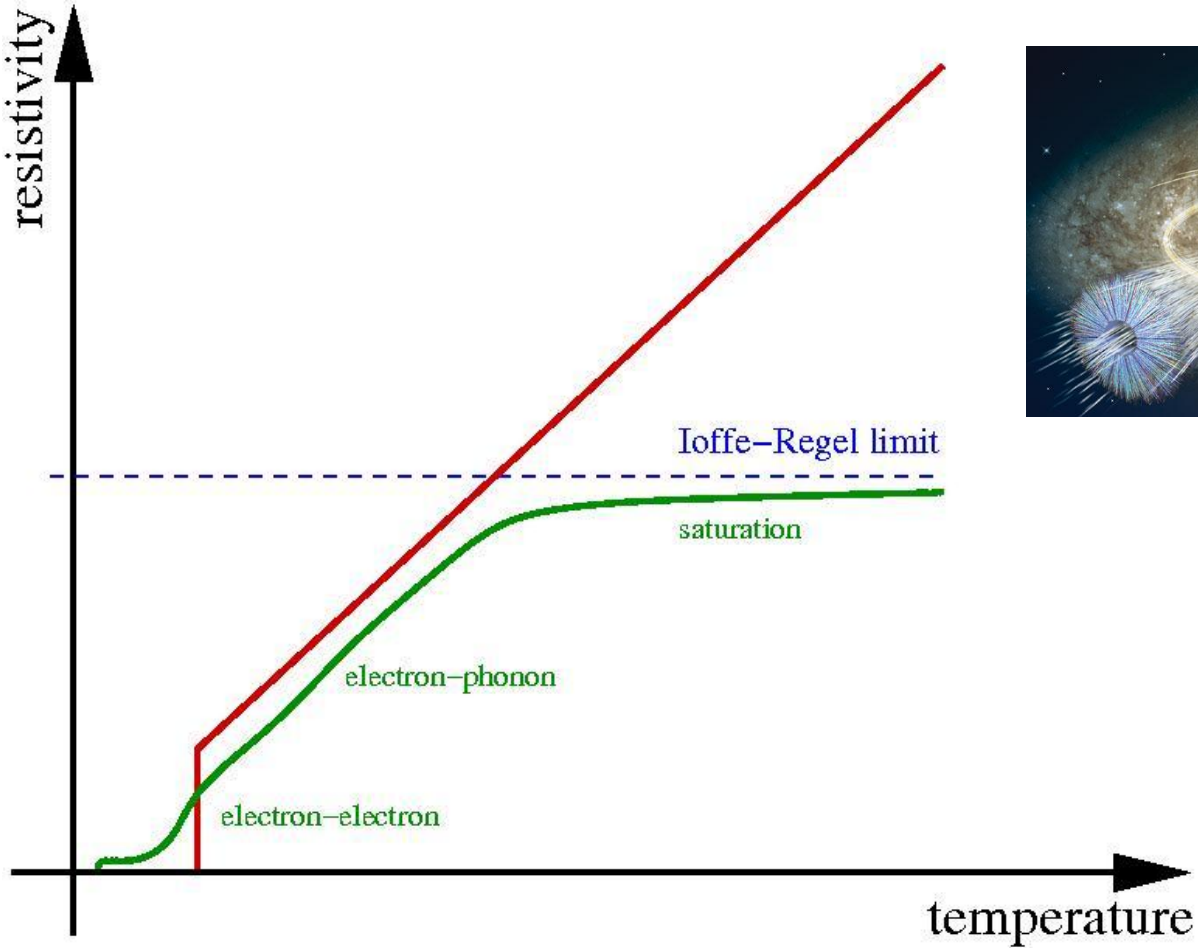
$$\sigma_1(\omega,T) = \frac{1}{4\pi}\frac{\omega_{pr}^2 \tau_r}{1+\omega^2\tau_r^2}, \qquad \tau_r = A\frac{\hbar}{k_B T}$$

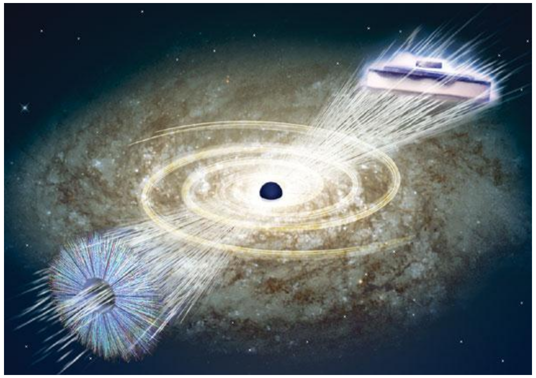
$$\Rightarrow \quad [\frac{\hbar}{k_B T \sigma_1}] = const.(1 + A^2[\frac{\hbar\omega}{k_B T}]^2)$$



A= 0.7: the normal state of optimallly doped cuprates is a Planckian dissipator!

Divine resistivity

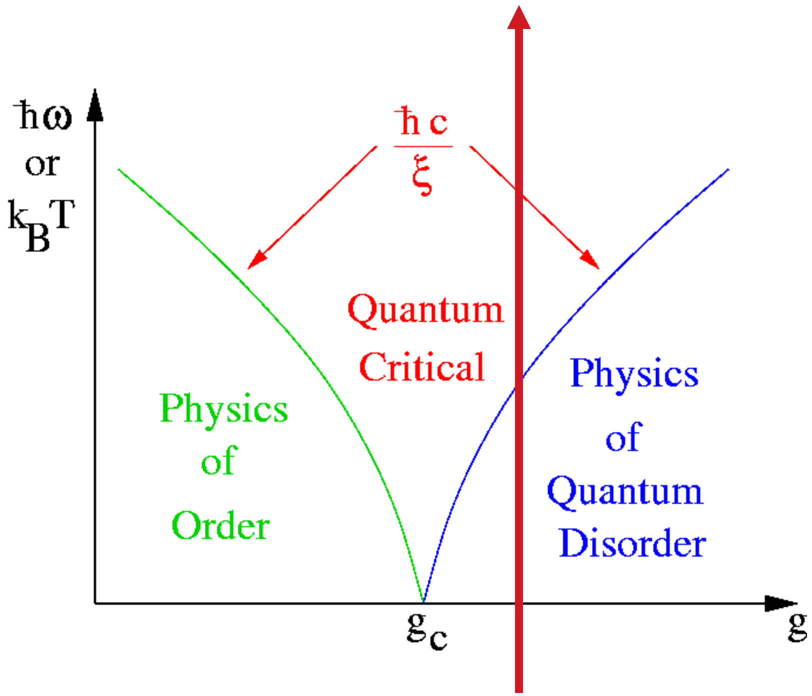


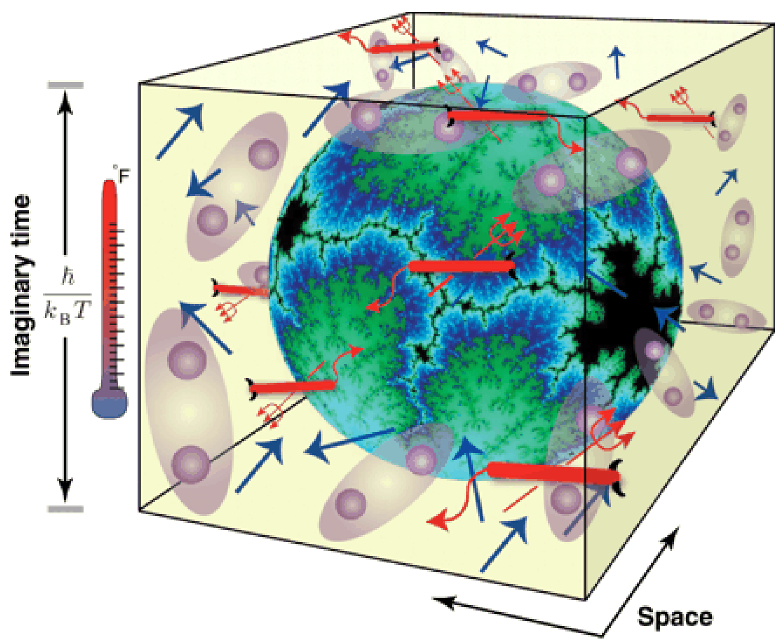


?!

Quantum Phase transitions

Quantum scale invariance emerges naturally at a zero temperature continuous phase transition driven by quantum fluctuations:





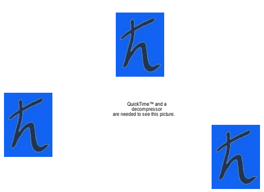
JZ, Science 319, 1205 (2008)

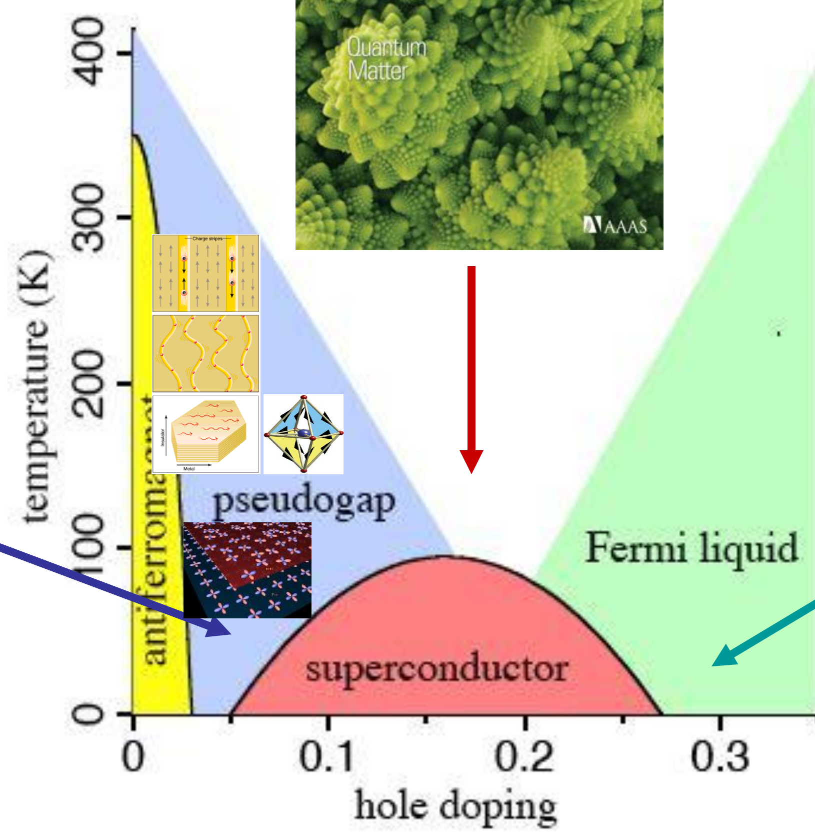
Phase diagram high Tc superconductors

The clash: the quantum critical metal

… which is good for superconductivity!

The quantized traffic jam

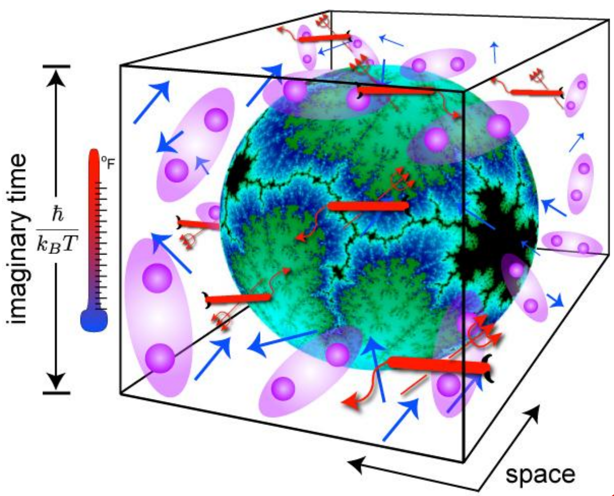


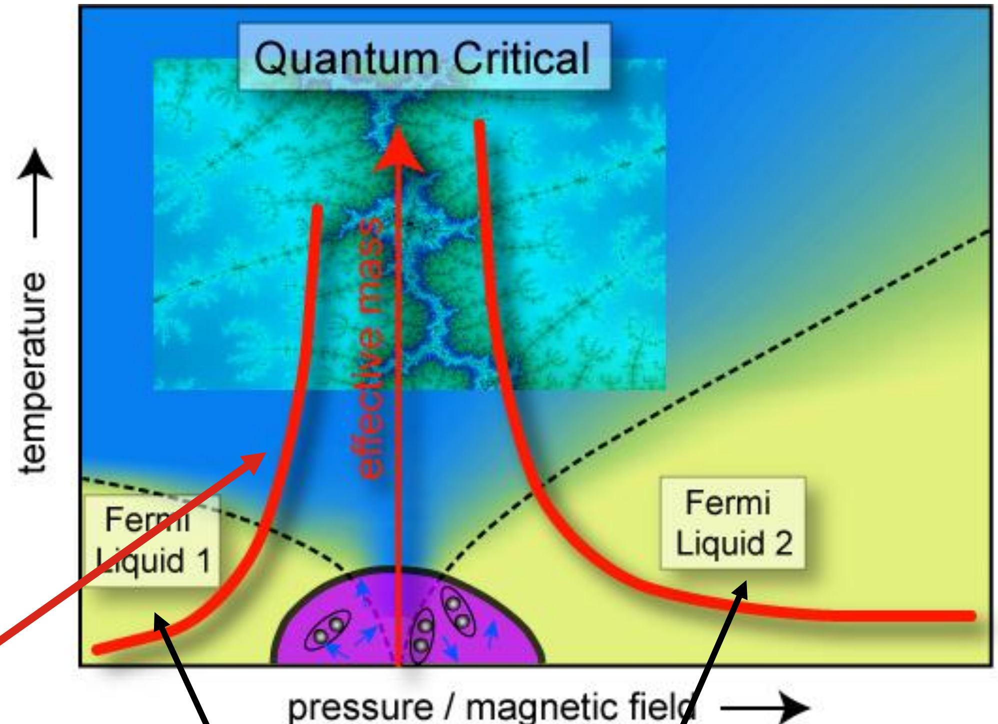


The quantum fog (Fermi gas) returns

Fermionic quantum phase transitions in the heavy fermion metals

JZ, Science 319, 1205 (2008)

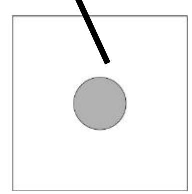




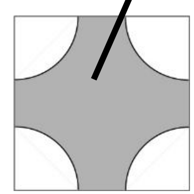
QP effective mass

$$m^* = \frac{1}{E_F}$$

$$E_F \to 0 \Rightarrow m^* \to \infty$$



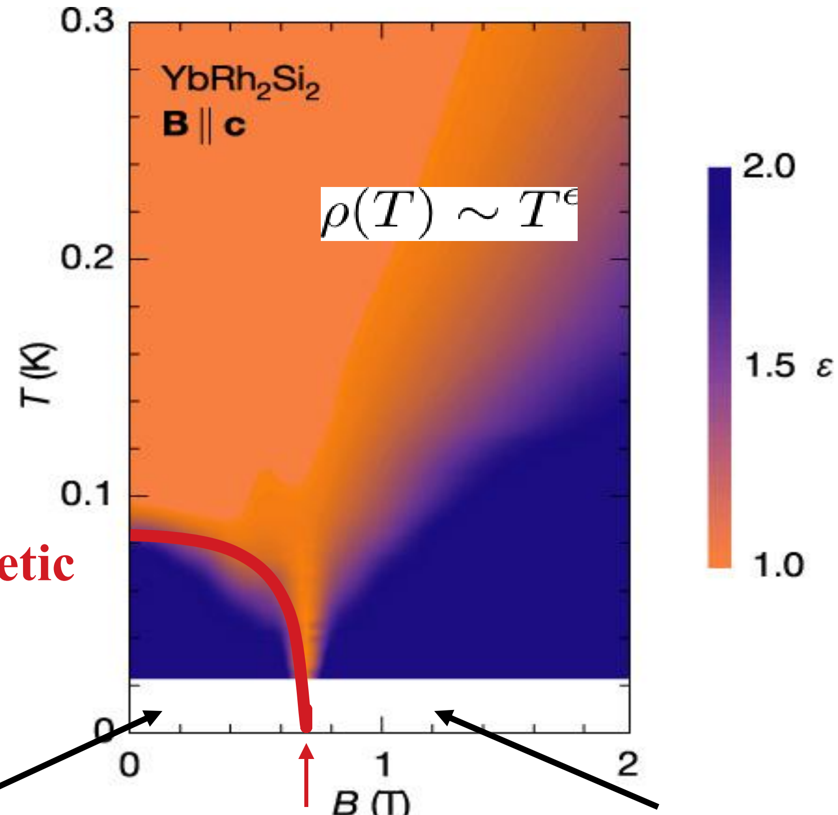
'bad actors'



Paschen et al., Nature (2004)

Coleman Rutgers

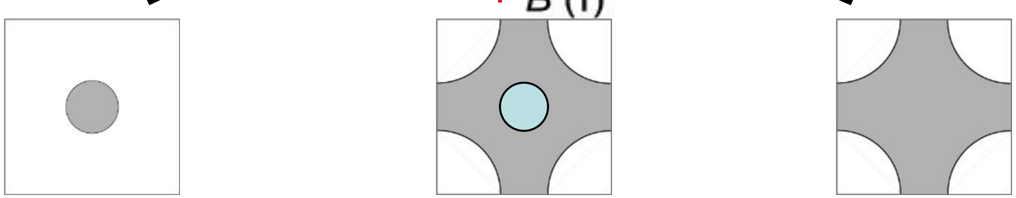
Critical Fermi surfaces in heavy fermion systems



Blue = Fermi liquid

Yellow= quantum critical regime

Antiferromagnetic order

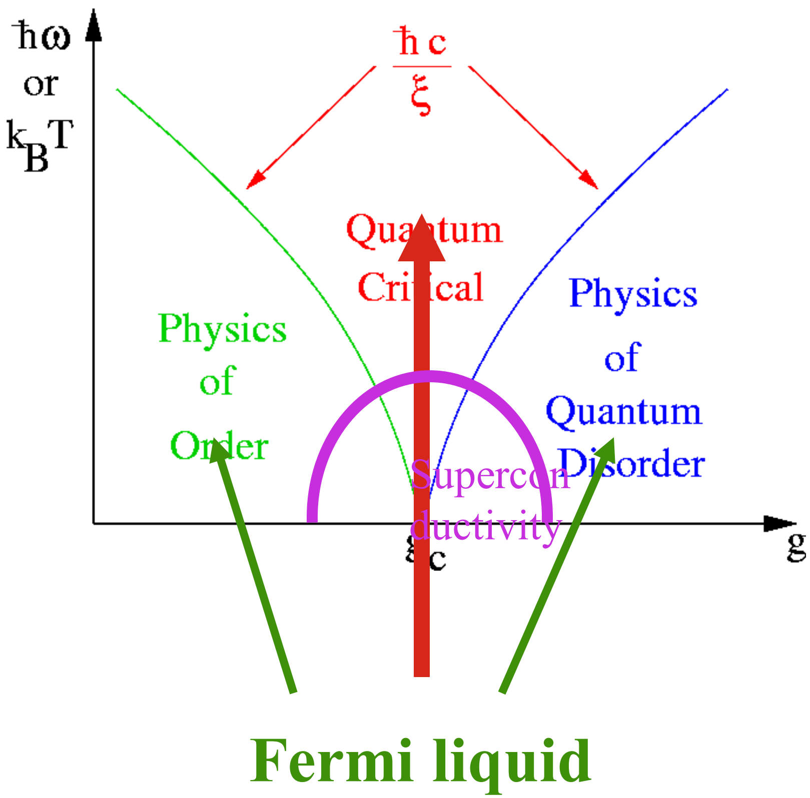


FL Fermi surface

Coexisting critical Fermi surfaces ?

FL Fermi surface

Hertz-Millis and Chubukov's "critical glue"



Bosonic (magnetic, etc.) order parameter drives the quantum phase transition

Electrons: fermion gas = heat bath damping bosonic critical fluctuations

Bosonic critical fluctuations 'back react' as pairing glue on the electrons

$$\lambda(i\Omega) = \left(\frac{\Omega_0}{|\Omega|}\right)^{\gamma}$$

E.g.: Moon, Chubukov, J. Low Temp. Phys. 161, 263 (2010)

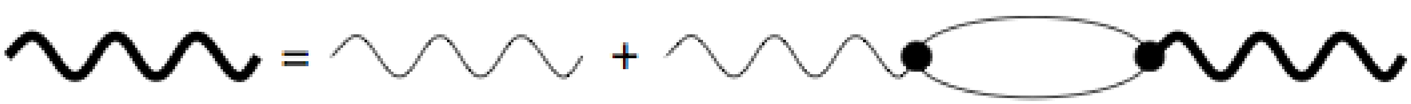
“Strong coupling” Migdal-Eliashberg theory

Attractive interaction due to “glue boson”, two parameters:

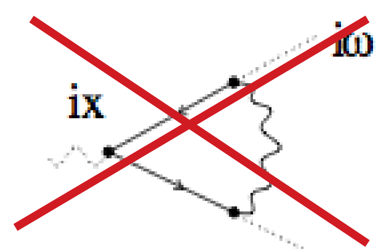
Coupling strength: $\lambda = V / E_F$

Migdal parameter: $\dfrac{\hbar\omega_{boson}}{E_F}$

Migdal-Eliashberg: dress boson and fermion propagators up to all orders ignoring vertex corrections which are O($\hbar\omega_B / E_F$).

(a) 

(b) 

(c) 

Computing the pair susceptibility: full Eliashberg

$$\chi(k,k';q) = \chi_0(k,k';q) + u^2 \sum_{k_1,k_2} \chi_0(k,k_1;q) D(k_2-k_1) \chi(k_2,k';q)$$

$$\Gamma(k;q) = \sum_{k'} \chi(k,k';q)$$

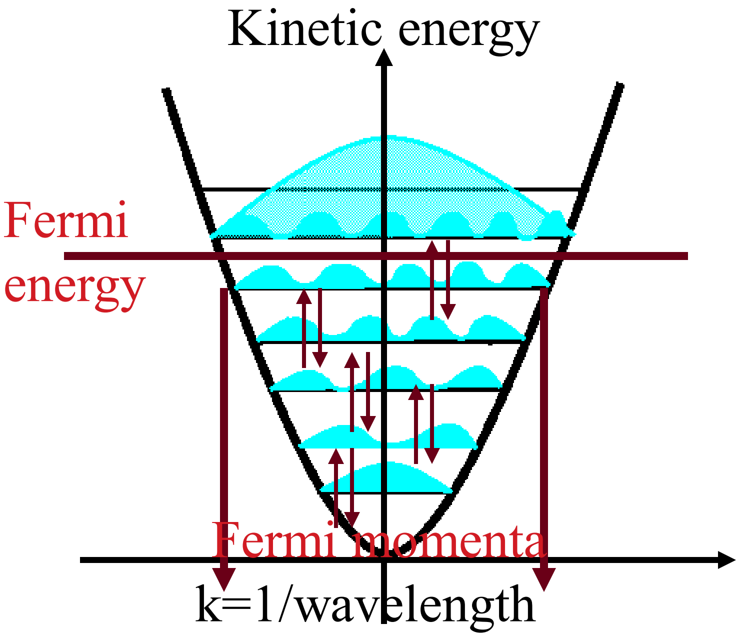
$$\Gamma(i\nu;i\Omega) = \Gamma_0(i\nu;i\Omega) + \mathcal{A}\,\Gamma_0(i\nu;i\Omega) \sum_{\nu'} \lambda(i\nu'-i\nu)\Gamma(i\nu';i\Omega)$$

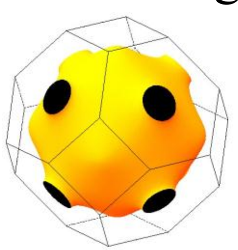
$$\chi_{\text{pair}}(i\Omega, \mathbf{q}=0) = \sum_{\nu} \Gamma(i\nu;i\Omega) \qquad i\Omega \to \omega + i\delta$$

$$\chi_{\text{pair}}(\omega, \mathbf{q}=0)$$

Watching electrons: photoemission

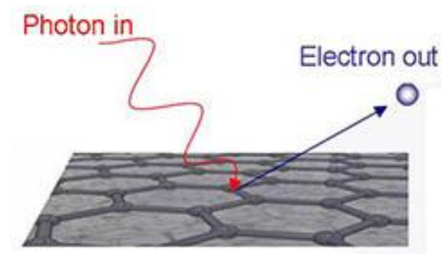
QuickTime™ and a decompressor are needed to see this picture.

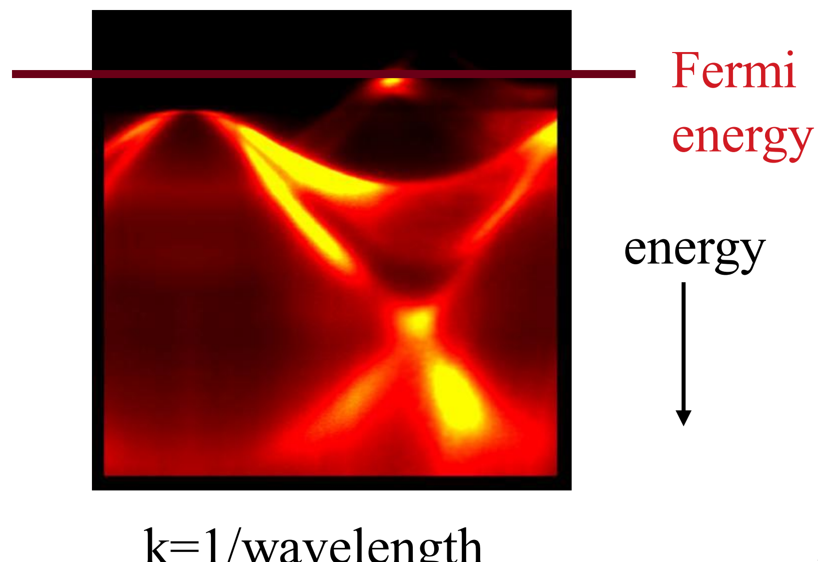




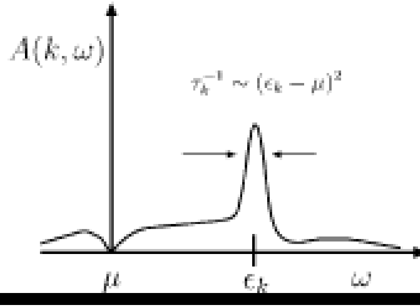
Fermi surface of copper

Electron spectral function: probability to create or annihilate an electron at a given momentum and energy.





Fermi-liquid phenomenology



Bare **single fermion propagator** 'enumerates the fixed point':

$$G(\omega,k)=\frac{1}{\omega-\mu_0-k^2/2m-(\Sigma'+i\Sigma'')}=\frac{Z}{(\omega-E_F)-v_R(k-k_F)+\ldots}$$

Spectral function: $$\boxed{\mathrm{Im}\,G(\omega,k)=A(\omega,k)=\frac{\Sigma''(\omega,k)}{\left|\omega+\mu+(k-k_F)^2/2m+\Sigma'(\omega,k)\right|^2+\left|\Sigma''(\omega,k)\right|^2}}$$

The Fermi liquid 'lawyer list':

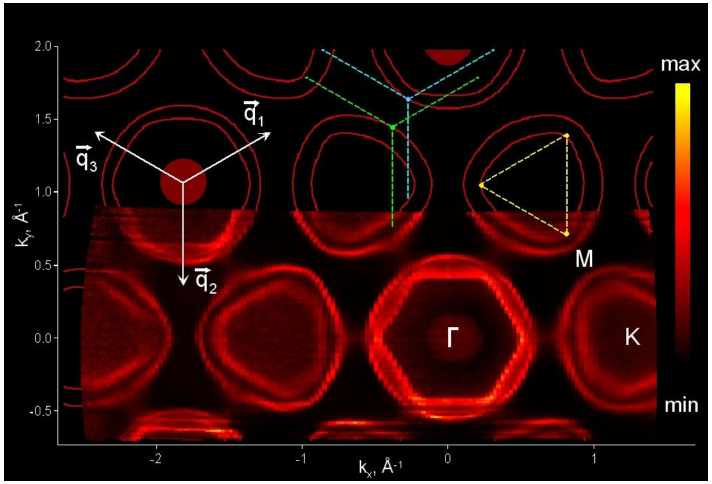
- At T= 0 the spectral weight is zero at the Fermi-energy except for the quasiparticle peak at the Fermi surface: $A(E_F,k)=Z\,\delta(k-k_F)$

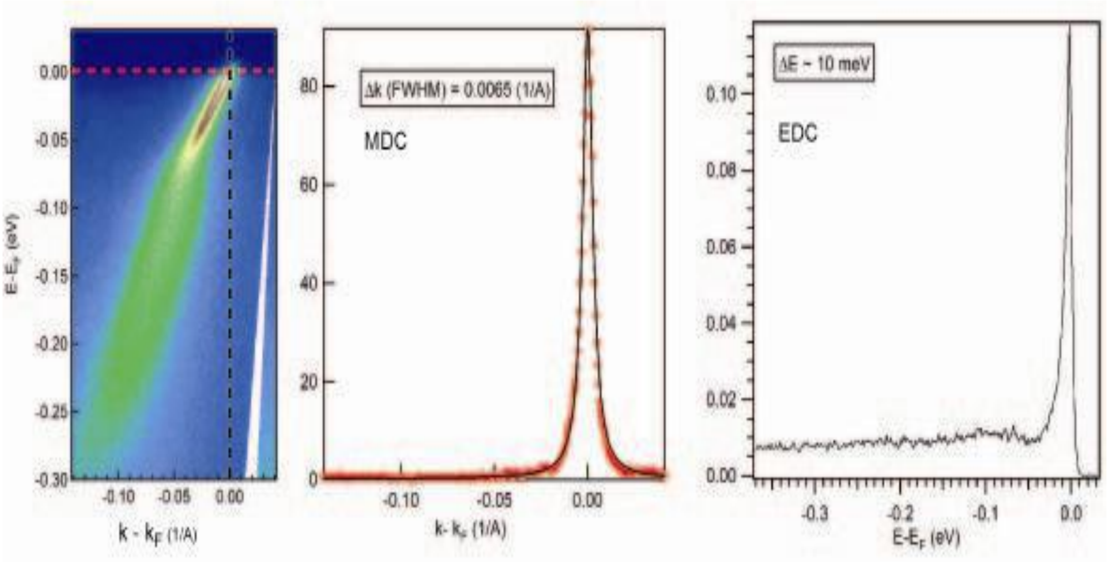
- Analytical structure of the self-energy: $\Sigma''(\omega,k)\propto(\omega-E_F)^2+\ldots$

$$\Sigma'(\omega,k)=\Sigma'(E_F,k_F)+\left.\frac{\partial\Sigma'}{\partial\omega}\right|_{\omega=E_F}(\omega-E_F)+\left.\frac{\partial\Sigma'}{\partial k}\right|_{k=k_F}(k-k_F)+\ldots$$

- Temperature dependence: $\Sigma''(E_F,k_F,T)\propto T^2+\ldots$

ARPES: Observing Fermi liquids

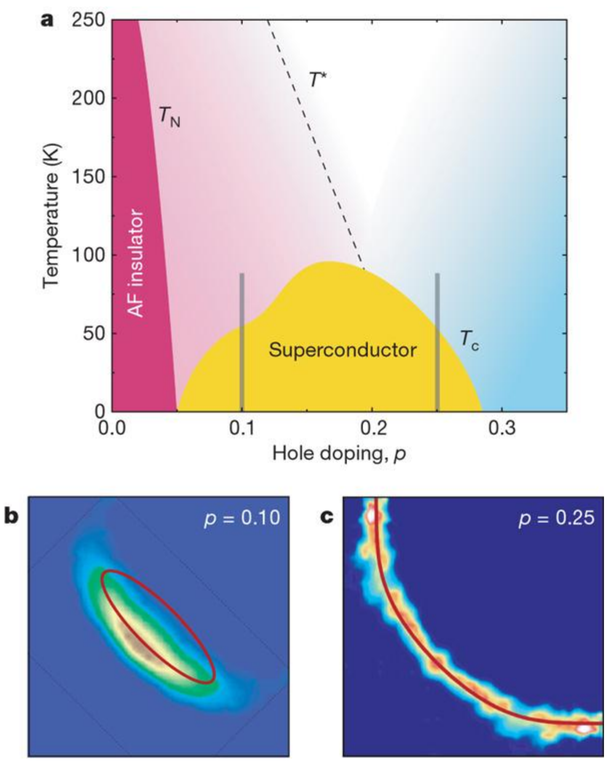




'MDC' at E_F in conventional 2D metal ($NbSe_2$)

Fermi-liquids: sharp Quasiparticle 'poles'

Cuprates: “Marginal” or “Critical” Fermi liquids



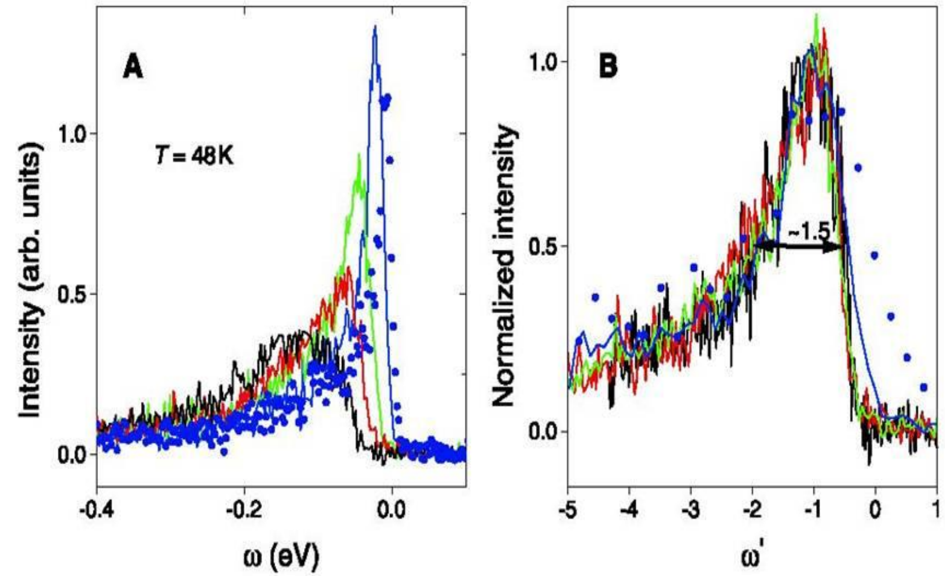


Fig. 3. (A) EDCs obtained in the (0,0) → (π,π) direction after background subtraction. (B) EDCs scaled to the same peak position, showing that the overall shape scales linearly with binding energy. The peak width is approximately 1.5 times the binding energy in all spectra (double-headed arrow).

Fermi ‘arcs’ (underdoped) closing to Fermi-surfaces (optimally-, overdoped).

EDC lineshape: ‘branch cut’ (conformal), width propotional to energy

Varma's Marginal Fermi liquid phenomenology.

Fermi-gas interacting by **second order** perturbation theory with 'singular heat bath':

QuickTime™ and a
decompressor
are needed to see this picture.

$$\mathrm{Im}P(q,\omega) \propto -N(0)\frac{\omega}{T}, \quad for\ |\omega| < T$$

$$\propto -N(0)\,sign(\omega),\ for\ |\omega| > T$$

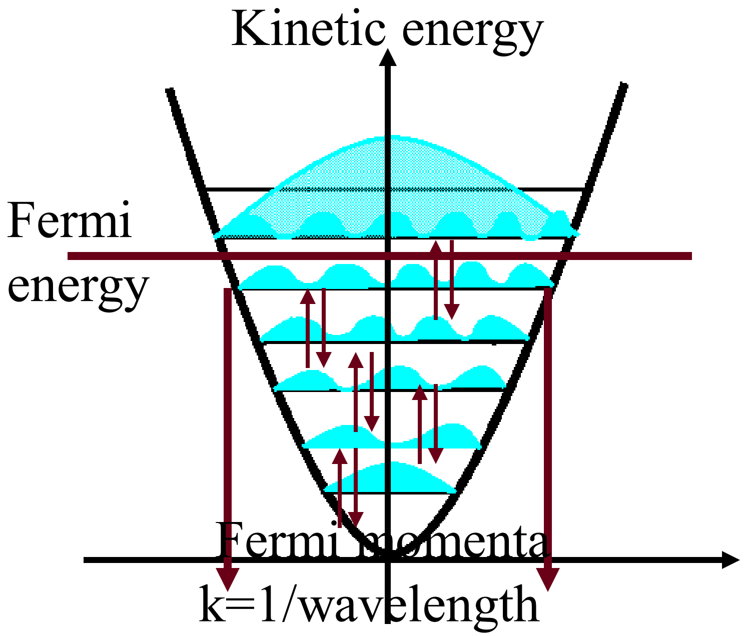
Directly observed in e.g. Raman ??

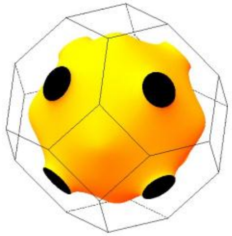
Single electron response (photoemission): $$G(k,\omega) = \frac{1}{\omega - v_F(k-k_F) - \Sigma(k,\omega)}$$

$$\Sigma(k,\omega) \propto \left(\frac{g}{\omega_c}\right)^2 \left[\omega \ln\left(\max(|\omega|,T)/\omega_c\right) - i\frac{\pi}{2}\max(|\omega|,T)\right]$$

Single particle life time $\frac{1}{\tau} \propto \max(|\omega|,T)$ is coincident (?!) with the **transport life time** => linear resistivity.

The fermionic criticality conundrum





Fermi surface of copper

Pauli exclusion principle generates the Fermi-energy, Fermi surface.

How to reconcile the quantum statistical scales with scale invariance?

Why is this quantum scale invariance of a local, purely temporal kind?

How can a (heavy) Fermi-liquid emerge from a 'microscopic' quantum critical state?

Why is this state good for high Tc superconductivity, and a phletora of exotic "competing orders" ?

AdS/CFT gives an answer!

Plan

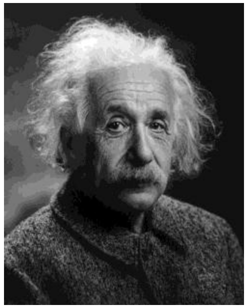
1. Crash course: quantum critical electron matter in solids.

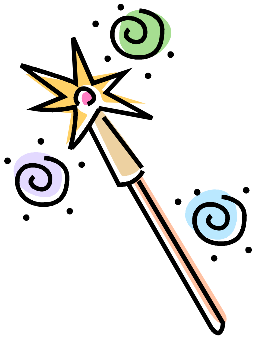
2. Crash course: the AdS/CFT correspondence.

3. Holographic quantum matter: marginal/critical Fermi-liquids, Fermi liquids and superconductors.

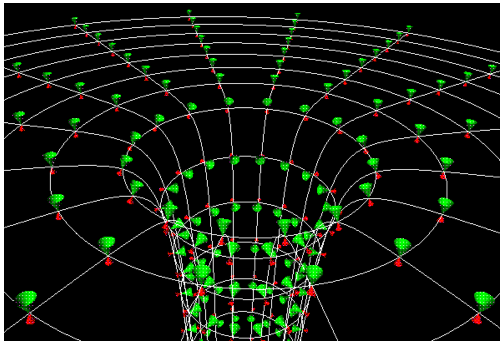
General relativity "=" quantum field theory

Gravity





Quantum fields



Maldacena 1997

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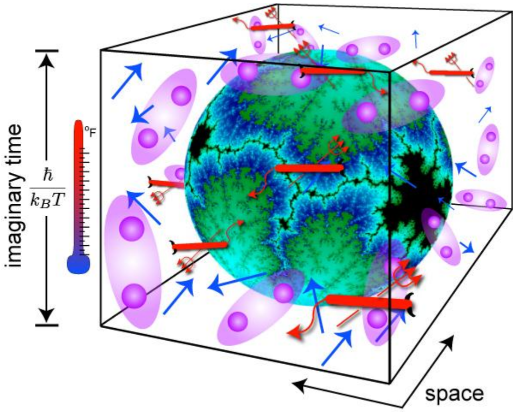
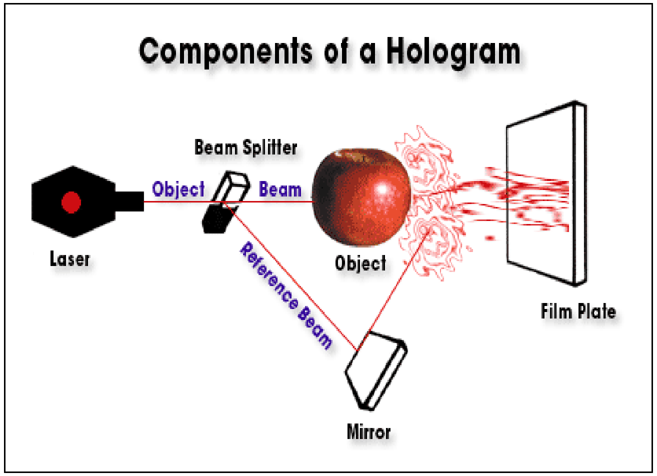
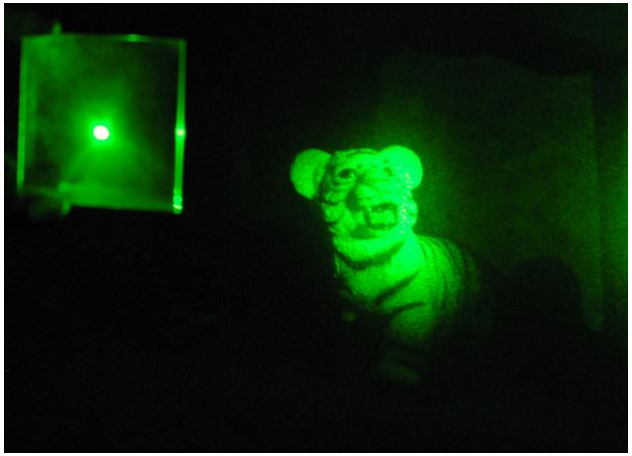
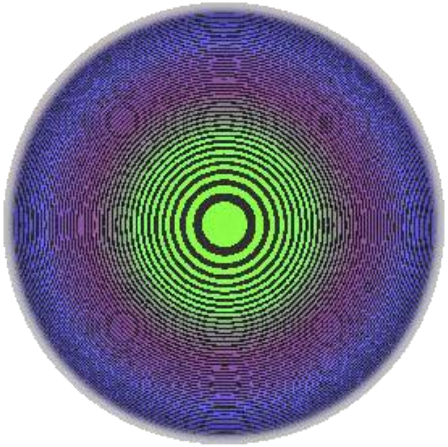


Holography with lasers



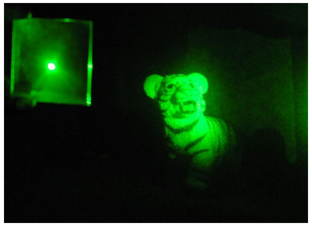


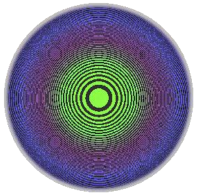
Three dimensional image



Encoded on a two dimensional photographic plate

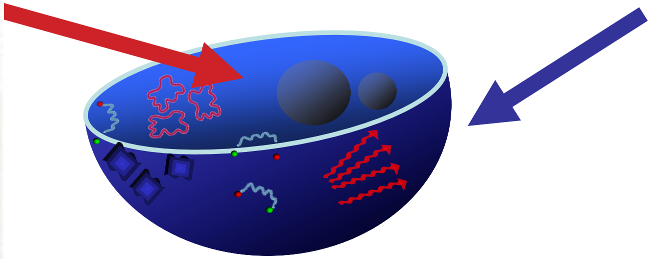
Gravity - quantum field holography



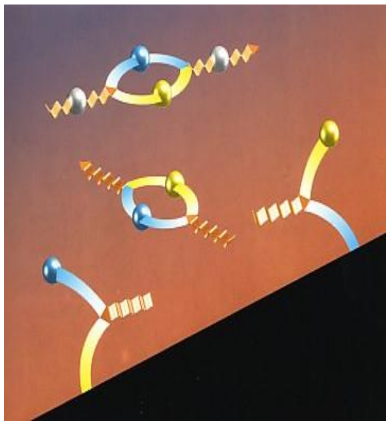


Einstein world "AdS" = Anti de Sitter universe

Quantum fields in flat space "CFT"= quantum critical



‘t Hooft’s holographic principle

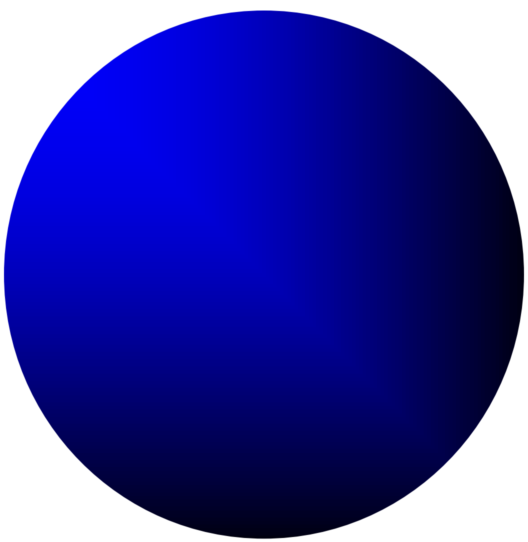


Hawking Temperature: $$T = \frac{\hbar g}{2\pi k c}$$

g = acceleration at horizon

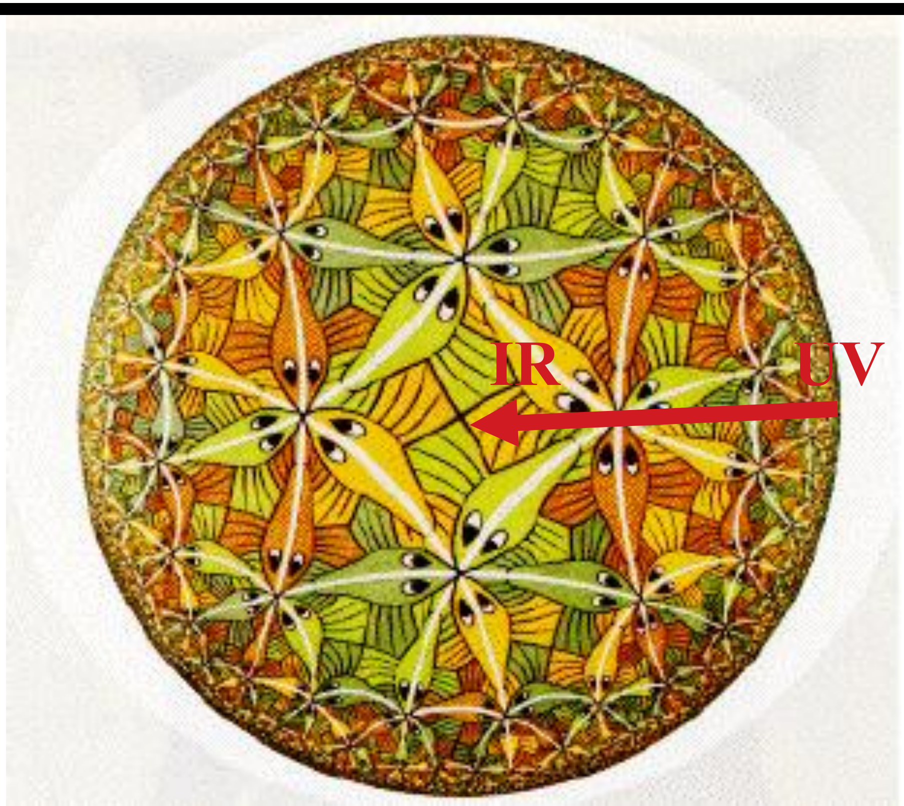
BH entropy: $$S = \frac{k c^3 A}{4\hbar G}$$

***A* = area of horizon**



Number of degrees of freedom (field theory) scales with the area and not with the volume (gravity)

The bulk: Anti-de Sitter space



Extra radial dimension of the bulk <=> scaling "dimension" in the field theory

Bulk AdS geometry = **scale invariance of the field theory**

$$dr^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$F(r) = -\Lambda r^2 + 1, \qquad \Lambda < 0$$

Weak-Strong Duality

Bulk: weakly coupled gravity

Boundary: strongly coupled Quantum Field theory

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Einstein-Maxwell

Large N Yang-Mills at large ‘t Hooft coupling

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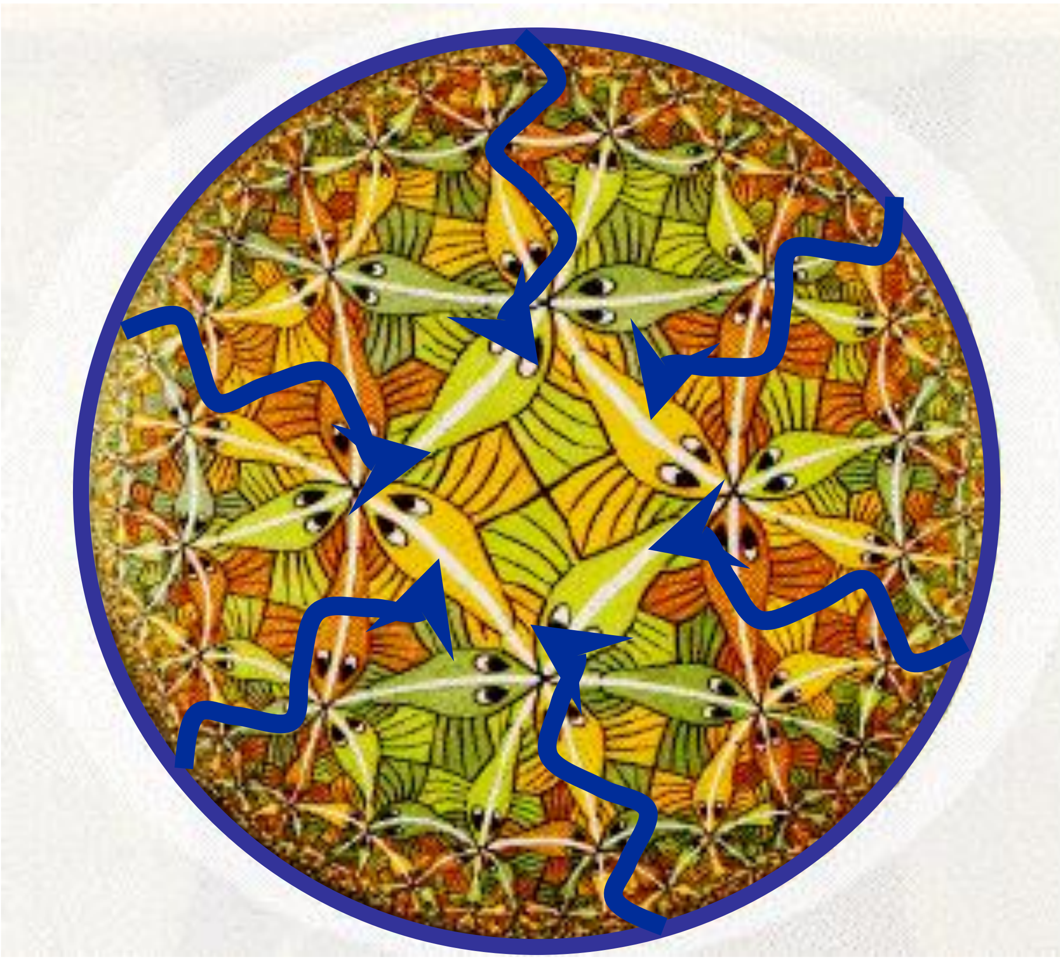
Kramers-Wannier

Quantum critical dynamics: classical waves in AdS

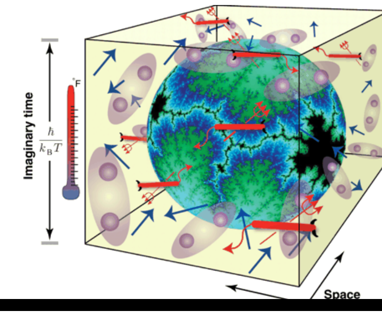


$$W_{CFT}(J) = S_{AdS}(\phi)_{\phi_{x0} \to 0 = J}$$

$$g_{YM}^2 N = \frac{R^4}{\alpha}$$

$$g_{YM}^2 = g_s$$

Fermionic renormalization group



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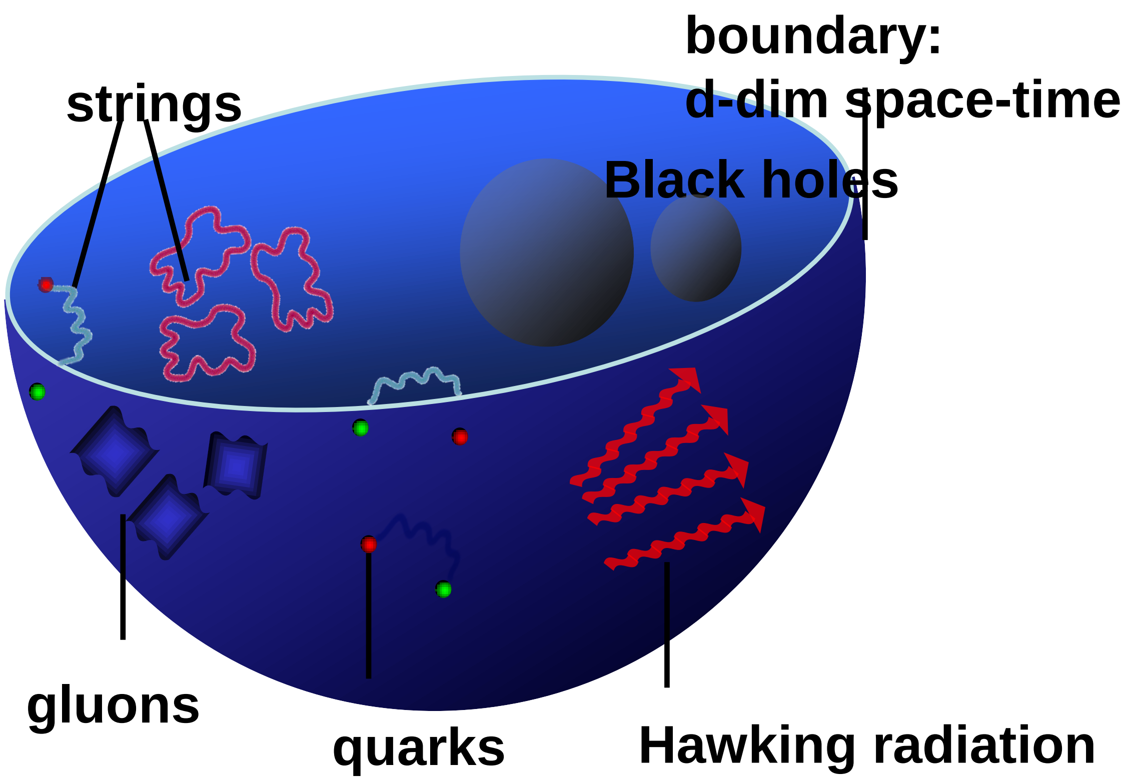
Wilson-Fisher RG:
based on Boltzmannian statistical physics

The Magic of AdS/CFT!



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Holography and quantum matter

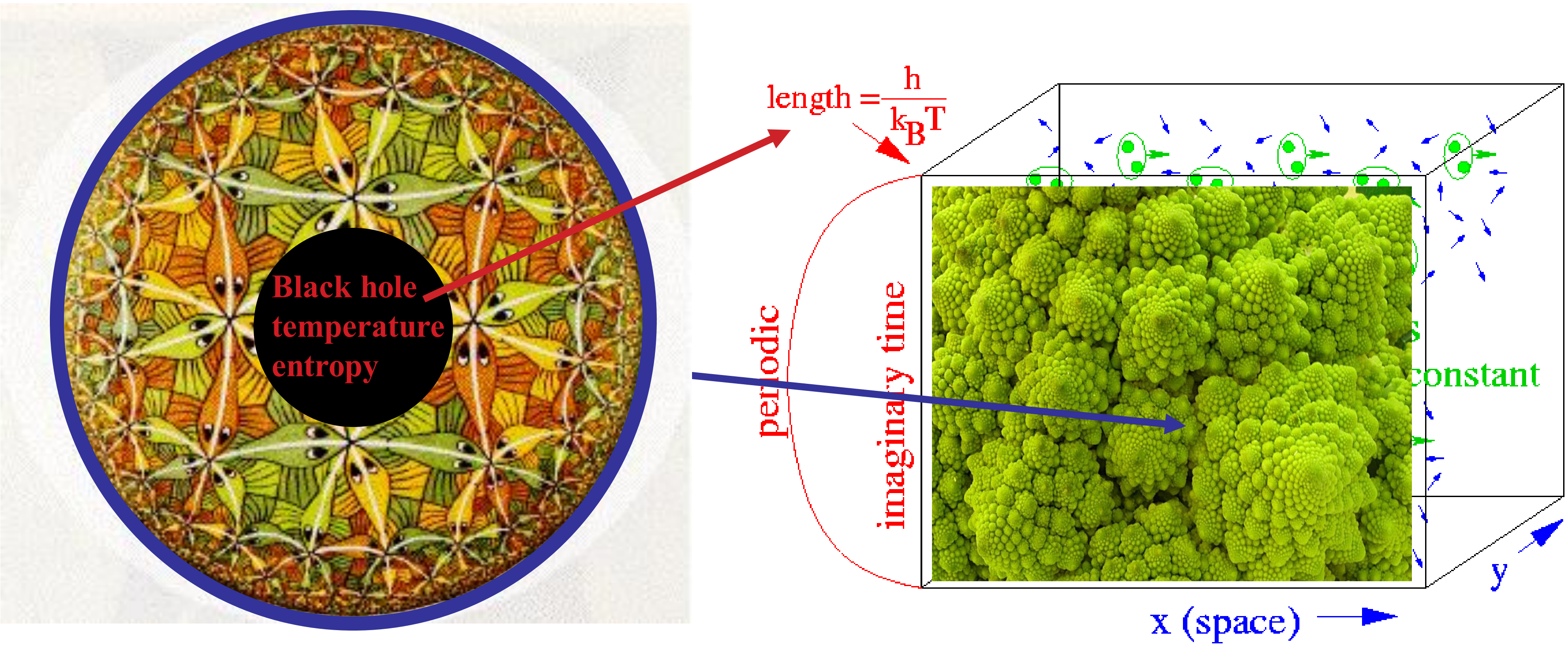
"Planckian dissipation": quantum critical matter at high temperature, perfect fluids and the linear resistivity (Son, Policastro, …, Sachdev)**.**

Reissner Nordstrom black hole: "critical Fermi-liquids", like high Tc's normal state (Hong Liu, John McGreevy)**.**

Dirac hair/electron star: Fermi-liquids emerging from a non Fermi liquid (critical) ultraviolet, like overdoped high Tc (Schalm, Cubrovic, Hartnoll)**.**

Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Gubser, Hartnoll …)**.**

The black hole is the heater



GR in Anti de Sitter space

$$dr^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

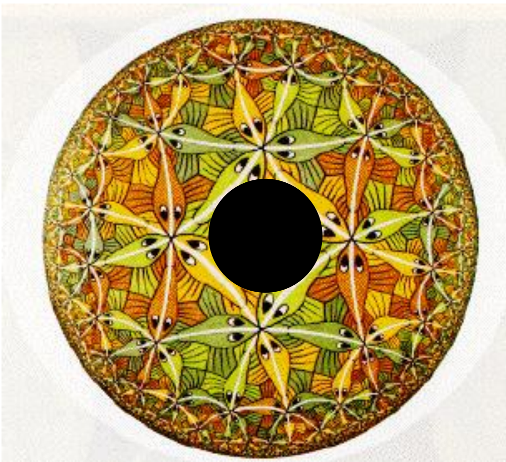
$$F(r) = -\Lambda r^2 + 1 - \frac{GM}{r}$$

Quantum-critical fields on the boundary:

- at the Hawking temperature

- entropy = black hole entropy

Planckian dissipation

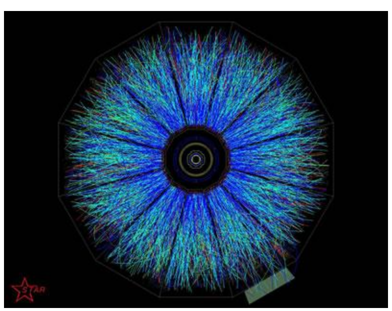


Schwarzschild Black Hole: encodes for the finite temperature dissipative quantum critical fluid.

Universal entropy production time:

$$\tau = \tau_\hbar \approx \frac{\hbar}{k_B T}$$

Minimal viscosity: quark gluon plasma, unitary cold atom fermion gas $\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$



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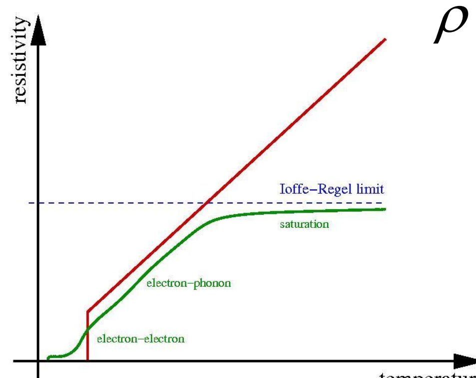
GIF decompressor
are needed to see this picture.

Linear resistivity high Tc metals:

$$\rho \propto \frac{1}{\tau_\hbar} \propto k_B T$$



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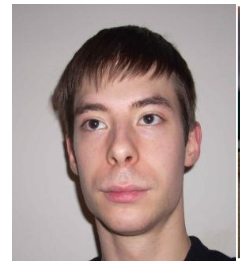
"AdS-to-ARPES": Fermi-liquid (?) emerging from a quantum critical state.



Cubrovic Schalm

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analysis of TDH transcripts expressed in human fetal liver tissue showed complete skipping of exon 4 and either complete skipping or aberrant splicing of exon 6 (fig. S8). Given that exons 4 and 6 encode segments of the enzyme critical to its function and that truncation via the nonsense codon at amino acid 214 would also be predicted to yield an inactive variant, it appears that the human gene is incapable of producing an active TDH enzyme. Remarkably, all metazoans whose genomes have been sequenced to date, including chimpanzees, appear to contain an intact TDH gene (*14*). Unless humans evolved adaptive capabilities sufficient to overcome three mutational lesions, it would appear they are TDH deficient.

Human ES cells grow at a far slower rate than mouse ES cells, with a doubling time of 35 hours (*15*). Whether the slower growth rate of human ES cells reflects the absence of a functional TDH enzyme can perhaps be tested by introducing, into human ES cells, either a repaired human TDH gene or the intact TDH gene of a closely related mammal. That this strategy might work is supported by the expression in human cells of a functional form of the 2-amino-3-ketobutyrate-

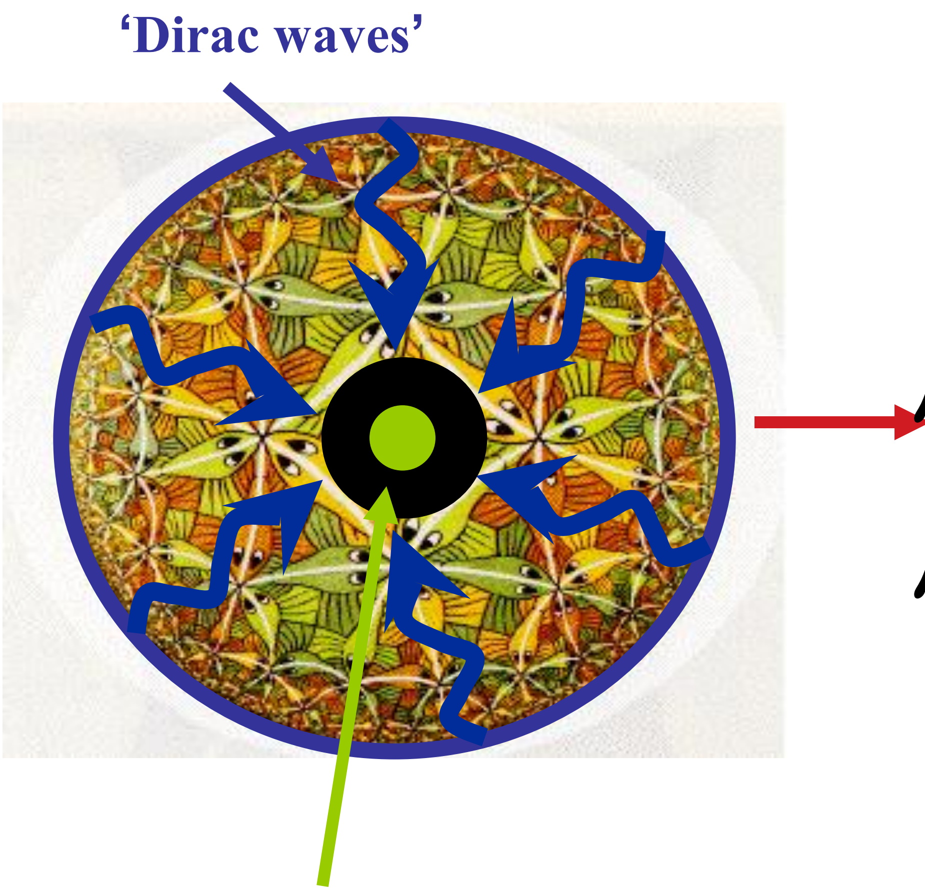
String Theory, Quantum Phase Transitions, and the Emergent Fermi Liquid

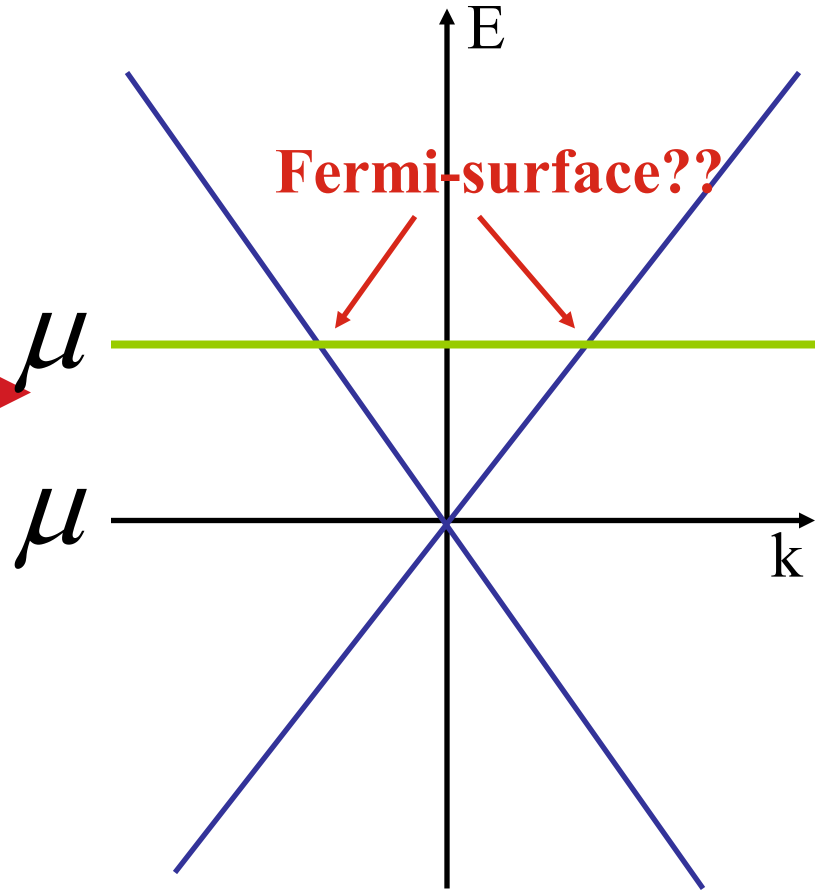
Mihailo Čubrović, Jan Zaanen, Koenraad Schalm*

A central problem in quantum condensed matter physics is the critical theory governing the zero-temperature quantum phase transition between strongly renormalized Fermi liquids as found in heavy fermion intermetallics and possibly in high–critical temperature superconductors. We found that the mathematics of string theory is capable of describing such fermionic quantum critical states. Using the anti–de Sitter/conformal field theory correspondence to relate fermionic quantum critical fields to a gravitational problem, we computed the spectral functions of fermions in the field theory. By increasing the fermion density away from the relativistic quantum critical point, a state emerges with all the features of the Fermi liquid.

Quantum many-particle physics lacks a general mathematical theory to deal with fermions at finite density. This is known as the "fermion sign problem": There is no recourse to brute-force lattice models because the statistical path-integral methods that work for any bosonic quantum field theory do not work for finite-density Fermi systems.

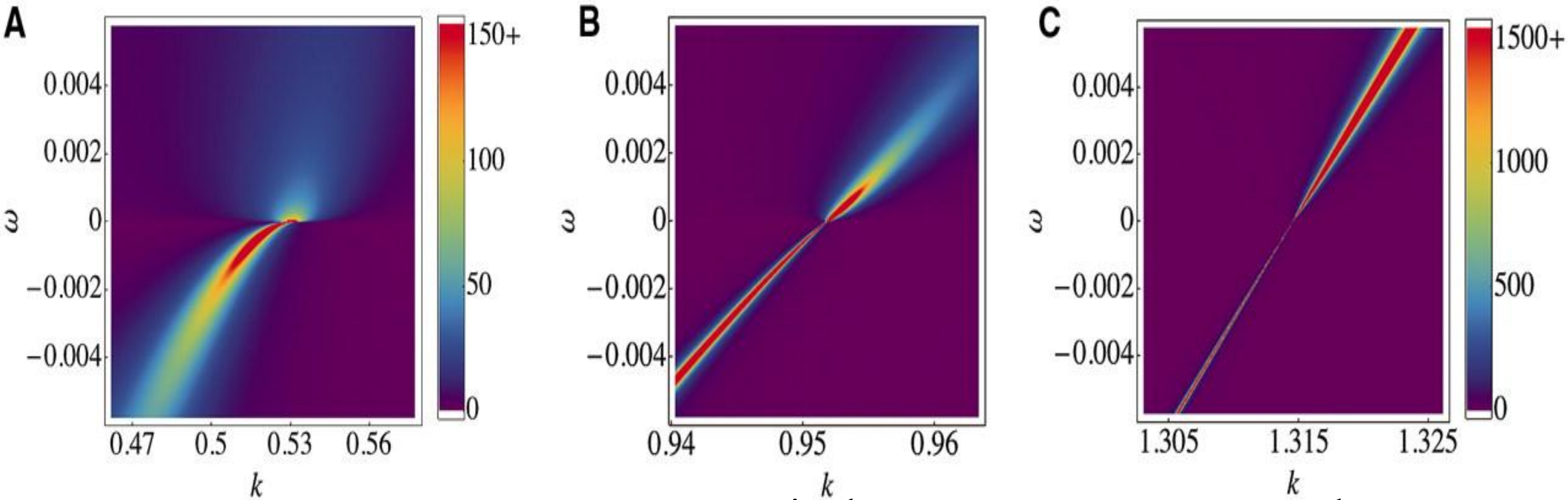
Breaking fermionic criticality with a chemical potential





AdS/ARPES for the Reissner-Nordstrom non-Fermi liquids

Fermi surfaces but no quasiparticles!



Critical FL

Marginal FL

Non Landau FL

Holographic quantum critical fermion state

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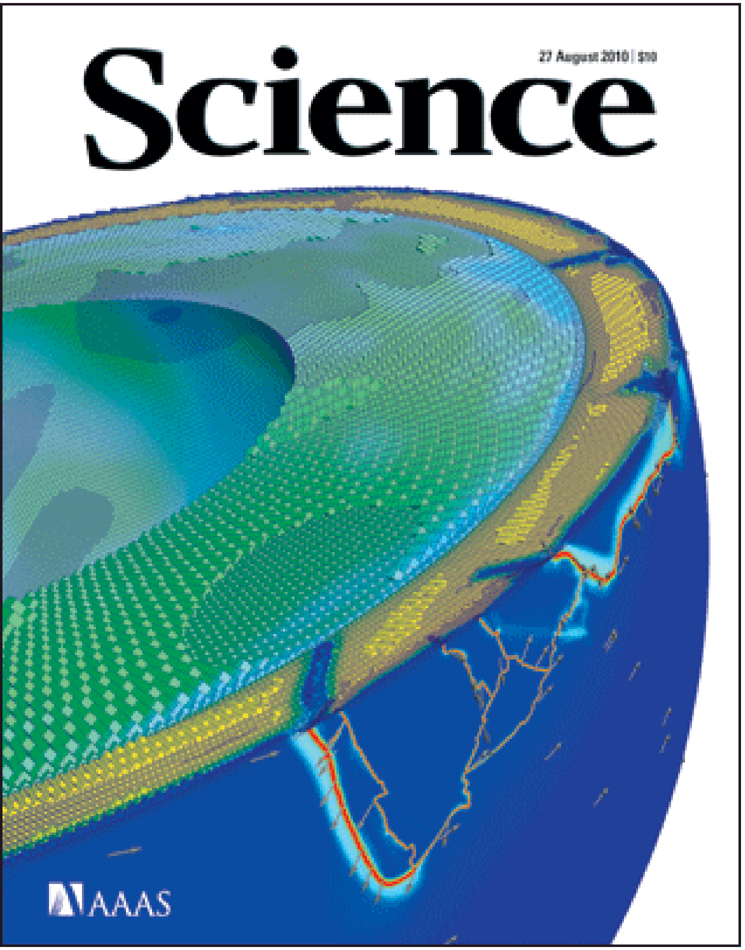
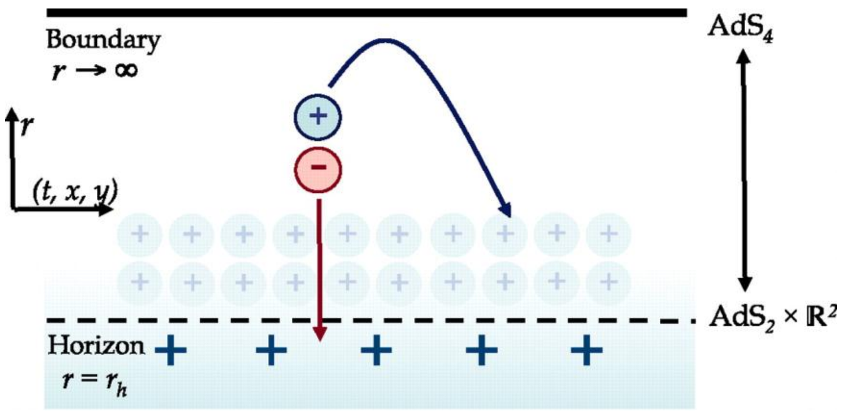
Liu McGreevy





Strange Metal Transport Realized by Gauge/Gravity Duality

Thomas Faulkner,[1] Nabil Iqbal,[2] Hong Liu,[2]* John McGreevy,[2] David Vegh[3]

Fermi liquid theory explains the thermodynamic and transport properties of most metals. The so-called non-Fermi liquids deviate from these expectations and include exotic systems such as the strange metal phase of cuprate superconductors and heavy fermion materials near a quantum phase transition. We used the anti–de-Sitter/conformal field theory correspondence to identify a class of non-Fermi liquids; their low-energy behavior is found to be governed by a nontrivial infrared fixed point, which exhibits nonanalytic scaling behavior only in the time direction. For some representatives of this class, the resistivity has a linear temperature dependence, as is the case for strange metals.

During the past decade, developments in string theory have revealed surprising and profound connections between gravity and many-body systems, resulting in the emergence of a new description for strongly coupled many-body systems. The anti–de-Sitter/conformal field theory (AdS/CFT) correspondence (*1–3*) relates a gravity theory in a weakly curved $(d + 1)$-dimensional anti–de Sitter (AdS_{d+1}) spacetime to a strongly coupled d-dimensional quantum field theory defined on its boundary. This correspondence maps questions about complicated many-body phenomena at strong coupling to solvable single- or few-body classical problems in a curved geometry. Black holes in this geometry play a surprising and universal role in characterizing the dynamics of the boundary theory at finite temperature and density, a development anticipated by the discovery of Hawking and Bekenstein in the 1970s (*4*, *5*) that black holes are intrinsically thermodynamic objects. Important dynamical insight into the thermodynamics (*6*) and transport behavior (*7*) of strongly correlated systems has been obtained from simple geometric aspects of black hole spacetimes.

Very recently, this apparatus has been brought to bear on the problem of fermions near quantum criticality (*8–11*). The basic strategy is to perform angle-resolved photoemission (ARPES) thought experiments on a charged black hole, which describes the ground state of a class of strongly coupled many-body systems. The fermionic response, which is proportional to the ARPES intensity, may be computed by studying the scattering of Dirac particles off this black hole. By exploring different regions in parameter space, both Fermi liquid–like (*10*) and non–Fermi-liquid behavior (*9*, *11*) were discovered, establishing the black hole as a new tool for addressing outstanding questions related to interacting fermions at finite density.

A prime example of a theoretical challenge to which such a tool may be applied is the strange metal phase of the cuprate high-temperature superconductors. The metallic state above the superconducting transition temperature T_c near optimal doping has unusual transport properties different from those of a normal metal, and was thus dubbed a strange metal; understanding this phase is believed to be essential for deciphering the mechanism for high-T_c superconductivity. The anomalous behavior of the strange metal (perhaps most prominently the simple and robust linear temperature dependence of the resistivity) implies that the low-

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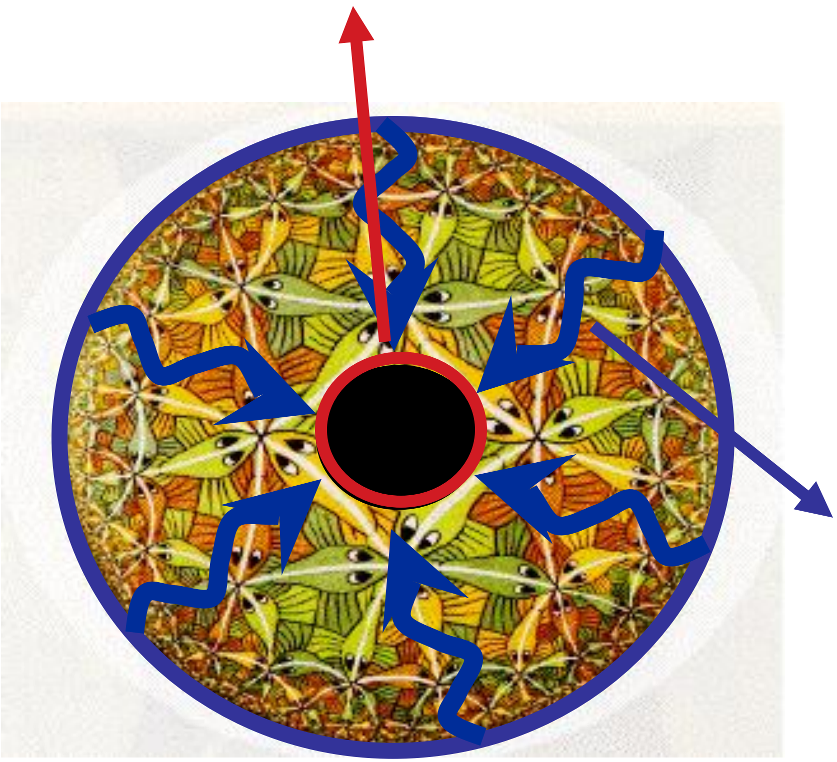
*To whom correspondence should be addressed. E-mail: hong_liu@mit.edu

Gravitationally coding the fermion propagators (Faulkner et al. Science 329, 1043, 2010)

Horizon geometry of the extremal black hole: ‘emergent’ AdS^2 =>
IR of boundary theory controlled by emergent temporal criticality



Gravitational ‘mechanism’ for marginal (critical) Fermi-liquids:

$$G^{-1} = \omega - v_F\left(k - k_F\right) - \Sigma(k,\omega)$$

QuickTime™ and a
decompressor
are needed to see this picture.

Temporal scale invariance IR “lands” in probing fermion self energy!

$$\Sigma'' \propto \omega^{2\nu_{k_F}}$$

Fermi-surface “discovered” by matching UV-IR: like Mandelstam “fermion insertion” for Luttinger liquid!

Gravitationally coding the fermion propagators (Faulkner et al. Science Aug 27. 2010)

T=0 extremal black hole, **near horizon geometry 'emergent scale invariant'**:

$$AdS_2 \otimes R_2 \Rightarrow g_k(\omega) = c(k)\omega^{2\nu_k}$$

Matching with the UV infalling Dirac waves:

$$G_R(\omega,k) = F_0(k) + F_1(k)\omega + F_2(k) g_k(\omega)$$

Special momentum shell: $|k| \equiv k_F$

$$G_R(\omega,k) = \frac{h_1}{k - k_F - \omega/v_F - \Sigma(\omega,k)}; \quad \Sigma(\omega,k) = h g_{k_F}(\omega) = h_2 e^{i\gamma_{k_F}} \omega^{2\nu_{k_F}}$$

Space-like: IR-UV matching 'organizes' Fermi-surface.

Time-like: IR scale invariance picked up via AdS2 self energy

Miracle, this is like critical/marginal Fermi-liquids!!

Marginal Fermi liquid phenomenology.

Fermi-gas interacting by **second order** perturbation theory with 'singular heat bath':

QuickTime™ and a decompressor are needed to see this picture.

$$\mathrm{Im}P(q,\omega) \propto -N(0)\frac{\omega}{T}, \quad \mathit{for}\ |\,\omega\,| < T$$

$$\propto -N(0)\,sign(\omega), \mathit{for}\ |\,\omega\,| > T$$

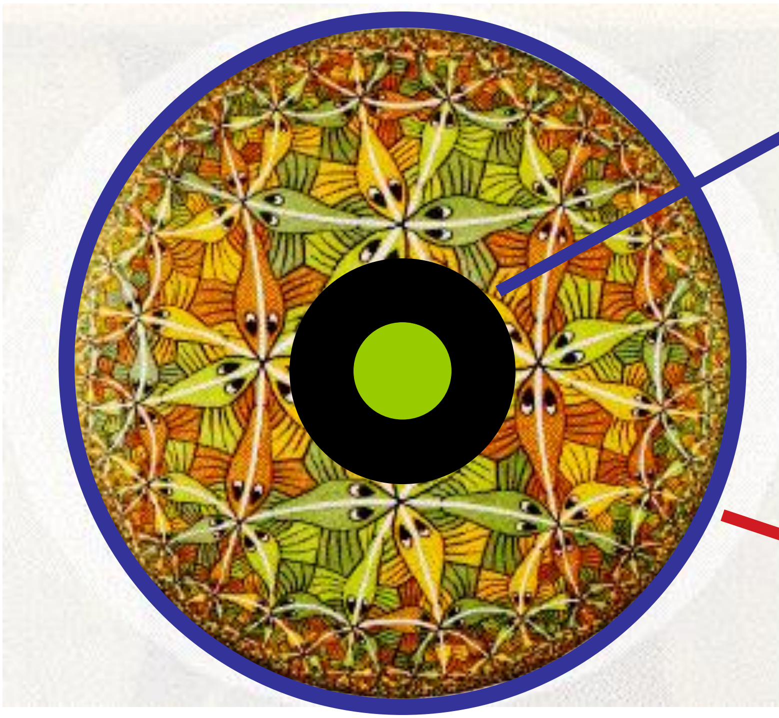
Directly observed in e.g. Raman ??

Single electron response (photoemission): $G(k,\omega) = \dfrac{1}{\omega - v_F(k-k_F) - \Sigma(k,\omega)}$

$$\Sigma(k,\omega) \propto \left(\frac{g}{\omega_c}\right)^2 \left[\omega \ln\left(\max(|\,\omega\,|,T)/\omega_c\right) - i\frac{\pi}{2}\max(|\,\omega\,|,T)\right]$$

Single particle life time $\dfrac{1}{\tau} \propto \max(|\,\omega\,|,T)$ is coincident (?!) with the **transport life time** => linear resistivity.

The zero temperature extensive entropy 'disaster'



The 'extremal' charged black hole with AdS^2 horizon geometry has zero Hawking temperature but a finite horizon area.

AdS-CFT

The 'seriously entangled' quantum critical matter at zero temperature should have an extensive ground state entropy (?*##!!)

Holography and quantum matter

"Planckian dissipation": quantum critical matter at high temperature, perfect fluids and the linear resistivity (Son, Policastro, …, Sachdev)**.**

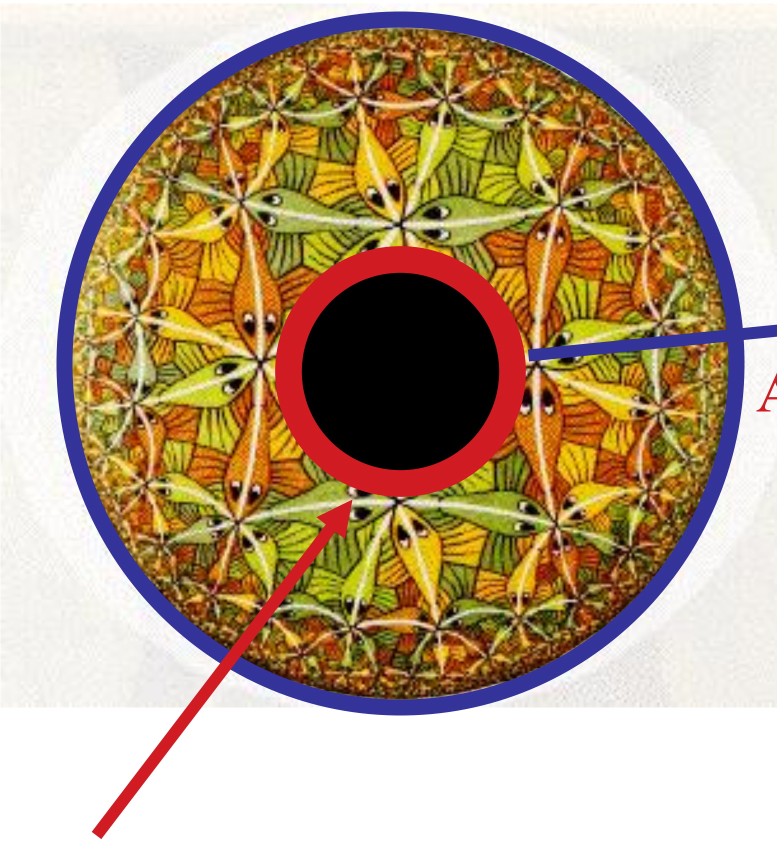
Reissner Nordstrom black hole: "critical Fermi-liquids", like high Tc's normal state (Hong Liu, John McGreevy)**.**

Dirac hair/electron star: Fermi-liquids emerging from a non Fermi liquid (critical) ultraviolet, like overdoped high Tc (Schalm, Cubrovic, Hartnoll)**.**

Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Gubser, Hartnoll …)**.**

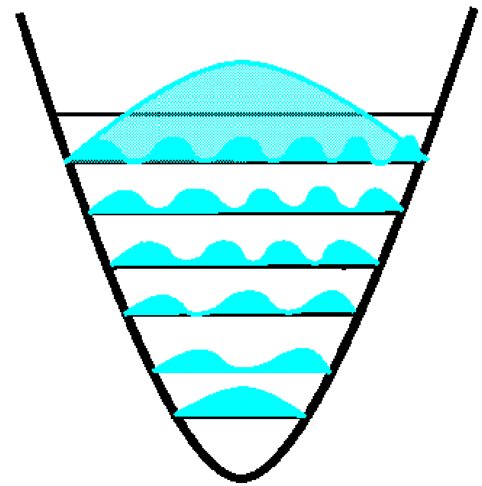
Black hole hair can be fermionic!

Schalm, Cubrovic, JZ (arXiv:1012.5681)

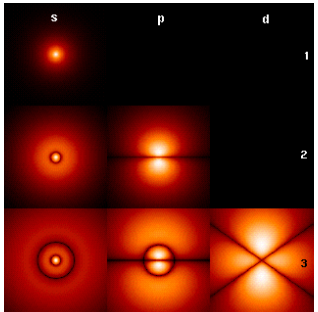


Stable Fermi liquid on the boundary!

AdS-CFT

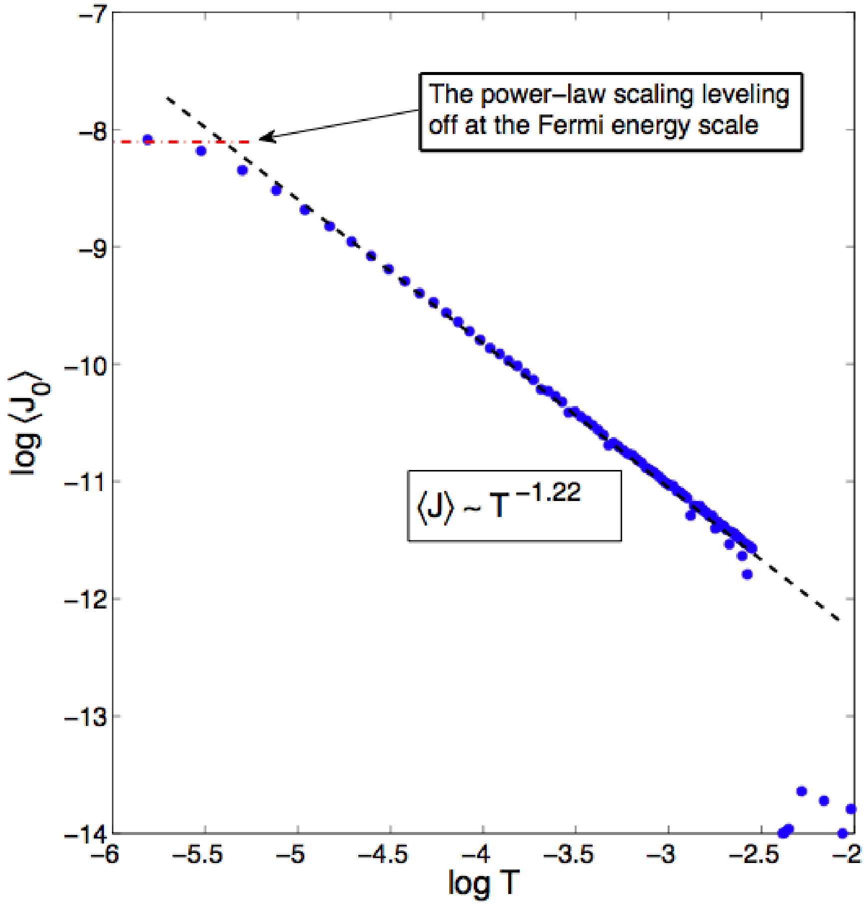


'Hydrogen atom': one Fermion quantum mechanical probability density.



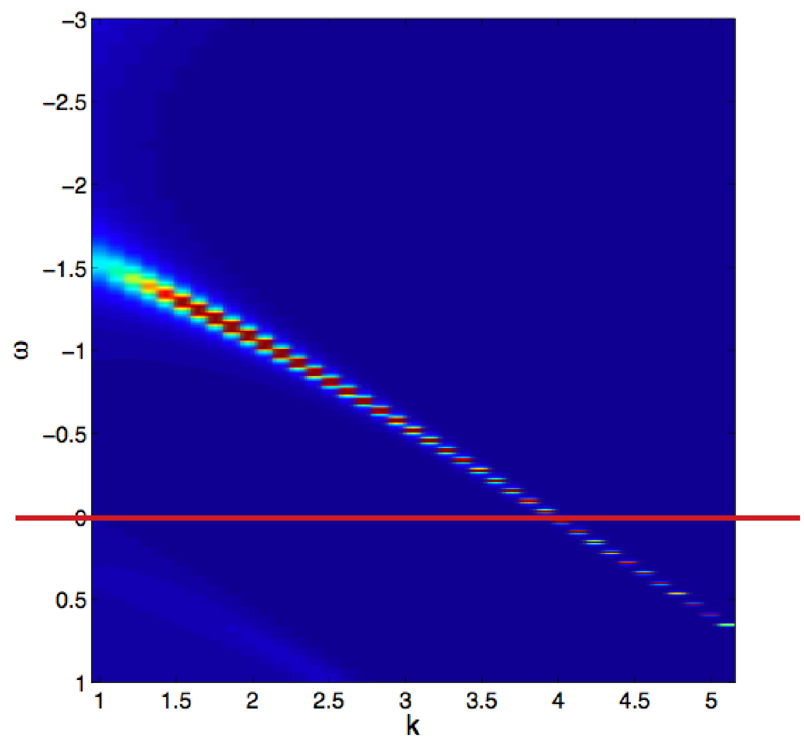
Fermionic hair: stability and equation of state.


Strongly renormalized E_F



Single Fermion spectral function:

non Fermi-liquid Fermi surfaces have disappeared!

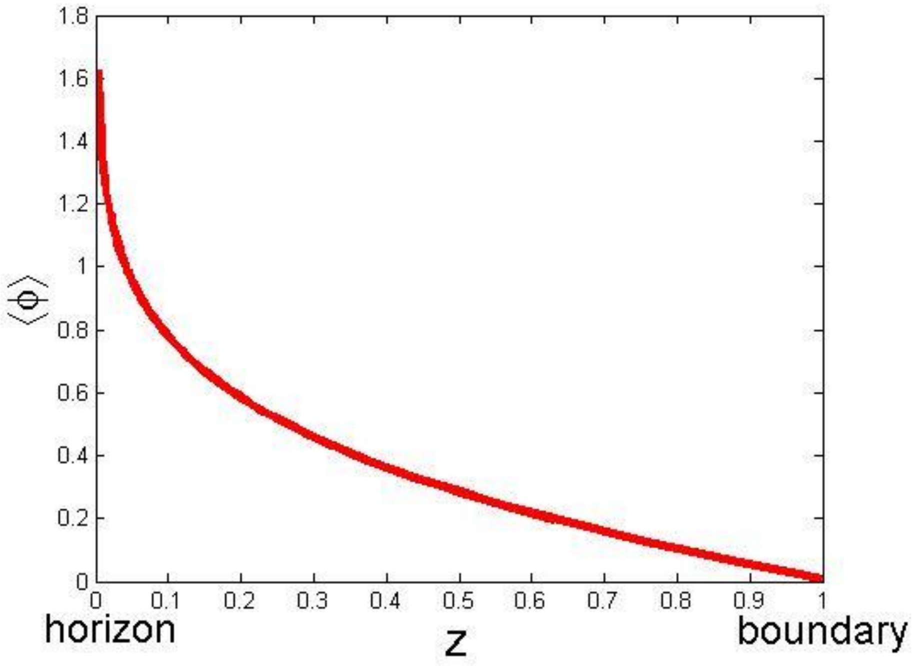


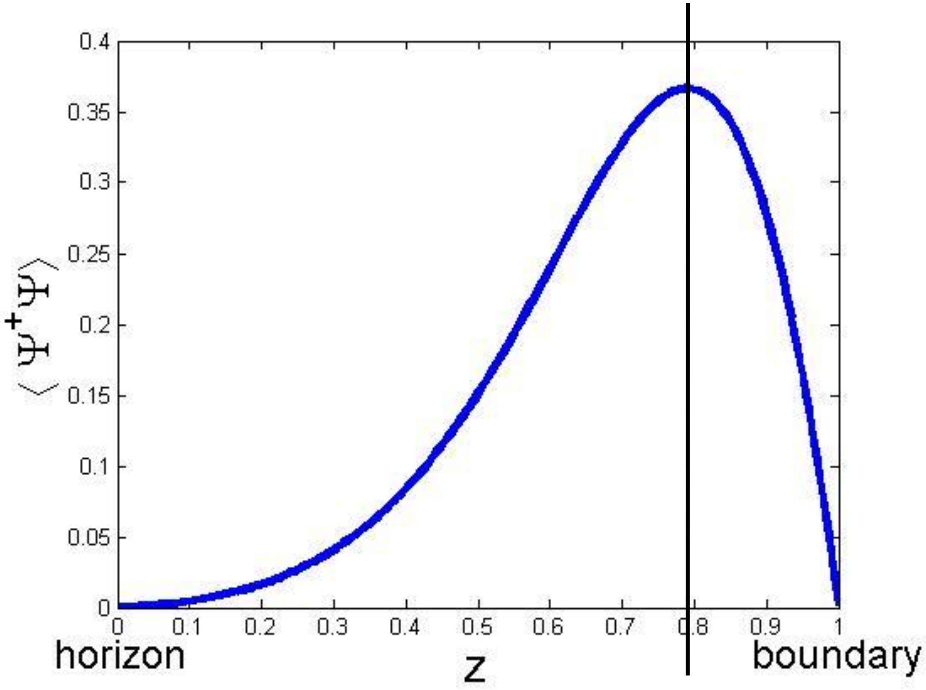
The Fermi-liquid VEV: Hair profile vs. statistics

- Scalar vs. fermionic hair: scale-free vs. scale-ful profile

Bosons accumulate at the horizon

Position of the maximum determines the Fermi energy





Holography and quantum matter

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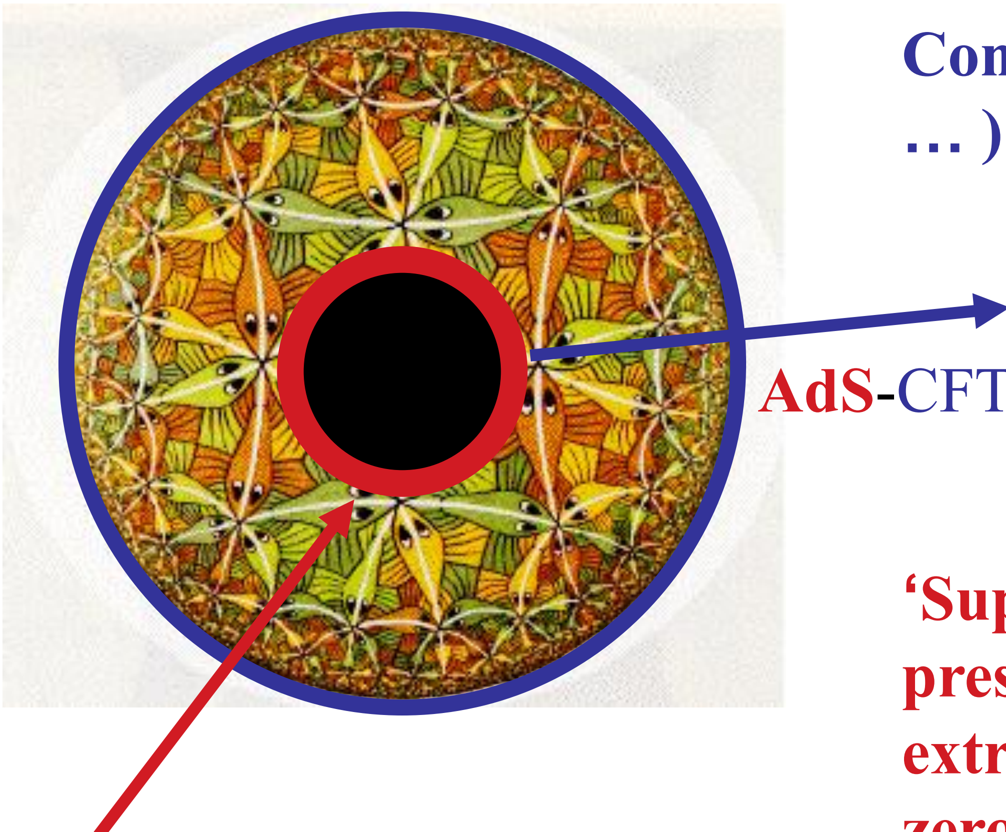
The holographic superconductor

Hartnoll, Herzog, Horowitz, arXiv:0803.3295



Condensate (superconductor, …) on the boundary!

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

(Scalar) matter ‘atmosphere’

‘Super radiance’: in the presence of matter the extremal BH is unstable => zero T entropy always avoided by low T order!!!

"Bottom-up" : Minimal holographic superconductivity (H^3)

What are the minimal bulk ingredients to capture the boundary superconductor?

- Continuum theory $\Rightarrow T_{\mu\nu} \Rightarrow g_{\mu\nu}$ in bulk.
- Conserved charge $\Rightarrow J_{\mu} \Rightarrow A_{\mu}$ in bulk.
- Fermion pair operator $\langle \psi\psi \rangle \Rightarrow \psi$ in bulk.

Write a minimal phenomenological bulk Lagrangian

$$L = R + \frac{6}{L^2} - \frac{1}{4} F^{ab} F_{ab} - V(|\psi|) - |\nabla \psi - iqA\psi|^2$$

Bulk geometry: AdS Reissner-Nordstrom black hole

Finite temperature and finite charge density: AdS RN black hole

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right)$$

where $$g(r) = r^2 - \frac{1}{r}\left(r_+^3 + \frac{\rho^2}{4r_+}\right) + \frac{\rho^2}{4r^2}$$

Scalar potential:

$$A^0 = \rho\left(\frac{1}{r_+} - \frac{1}{r}\right)$$

Hawking temperature:

$$T = \frac{12r_+^4 - \rho^2}{16\pi r_+^3}$$

The hairy black hole …

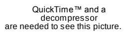
Minimal model: $V(|\psi|) = -2\psi^2$, the dual operator Ψ can have conformal dimensions $\Delta = 1, 2$

The Reissner-Nordstrom BH describes the normal state, but it goes unstable at a $T < T_c \approx \sqrt{\rho}$ because $m_{eff}^2 \approx m^2 - q^2 A_0^2$ turns negative.

Below T_c the black hole gets hair in the form of a "scalar atmosphere": via the dictionary, a VEV emerges in the field theory in the absence of a source.

The global U(1) symmetry of the CFT is spontaneously broken into a superfluid!

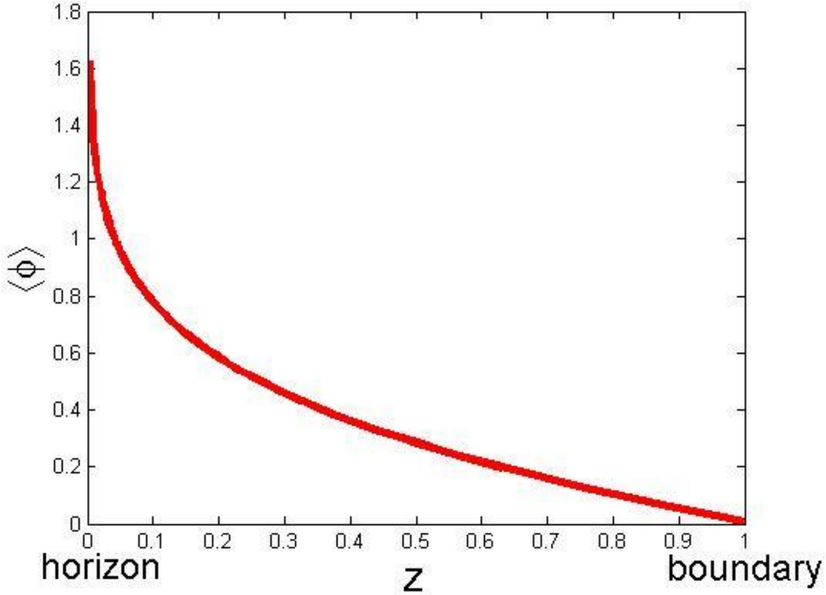
The Bose-Einstein Black hole hair

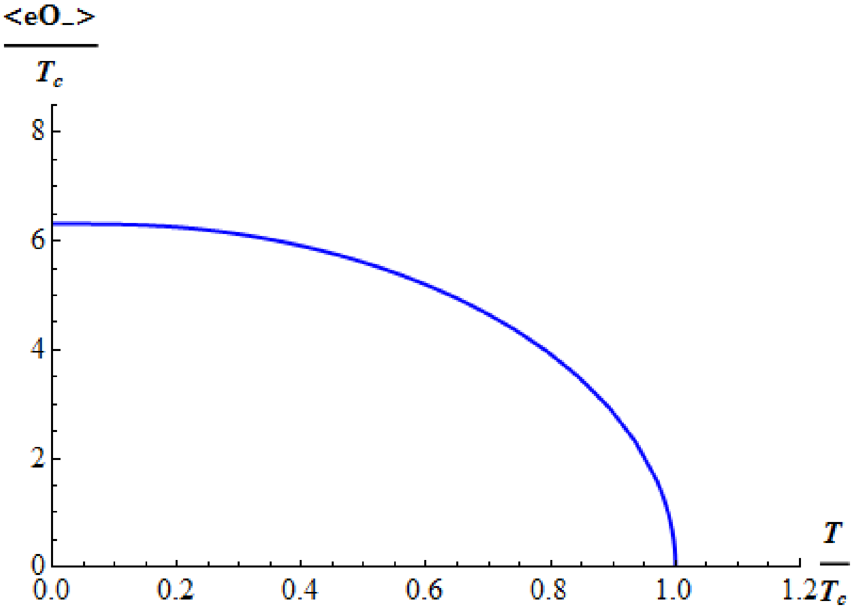


Hartnoll Herzog Horowitz

Scalar hair accumulates at the horizon

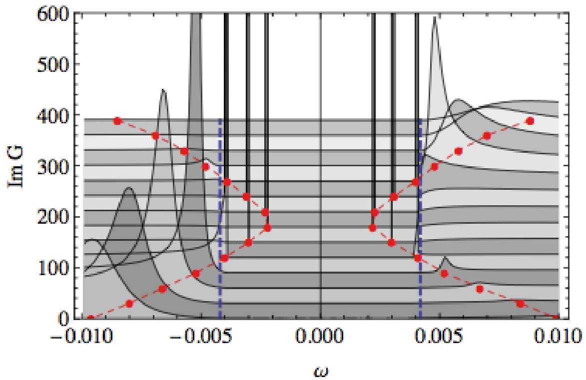
Mean field thermal transition.

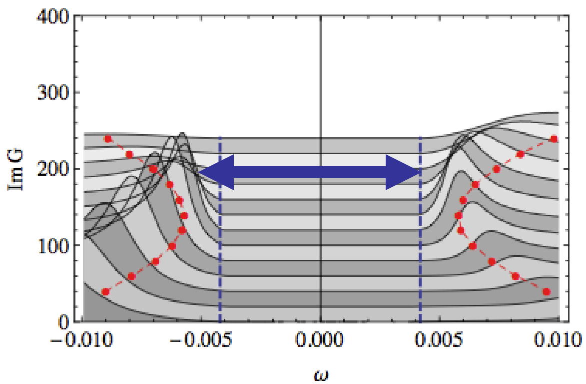




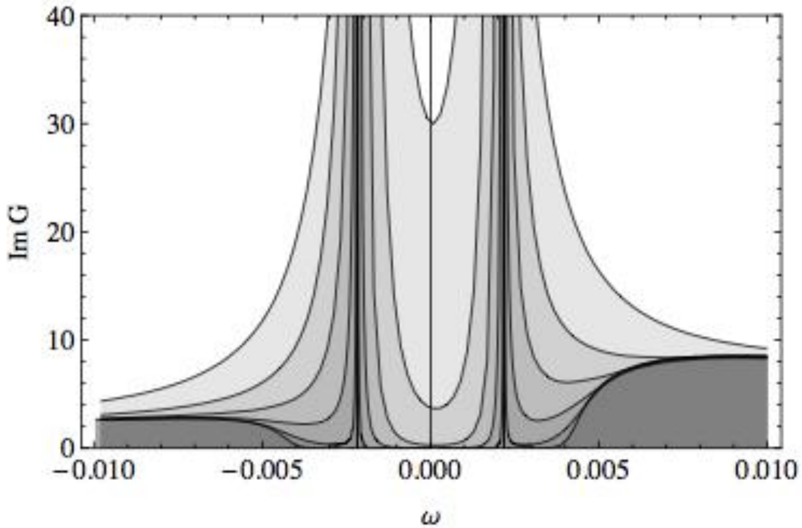
Holographic superconductivity: stabilizing the fermions.

Fermion spectrum for scalar-hair back hole (Faulkner et al., 911.340; Chen et al., 0911.282):





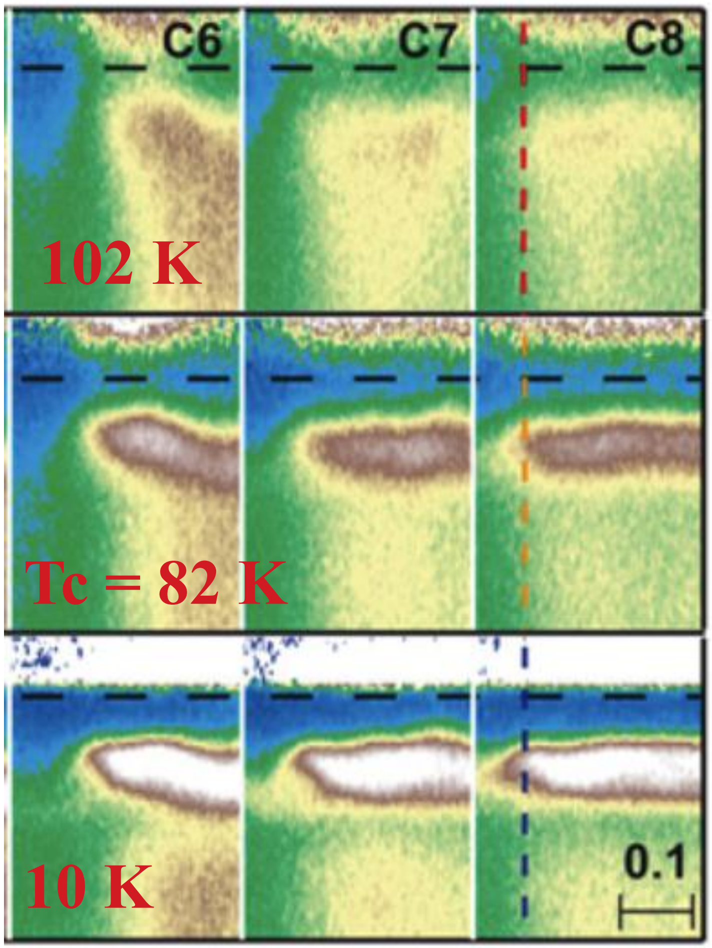
'BCS' Gap in fermion spectrum !!



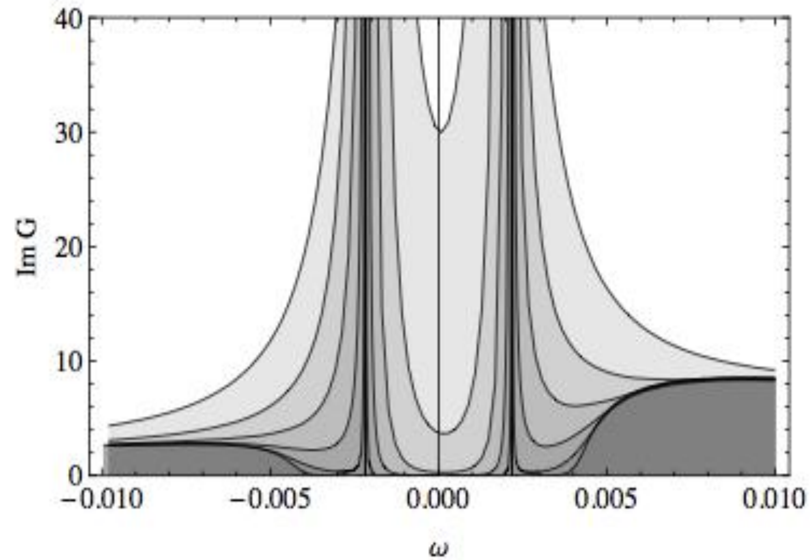
Temperature dependence as expected for 'quantum-critical' superconductivity (She, JZ, 0905.1225)

Excessive temperature dependence 'pacified' !

‘Pseudogap’ fermions in high Tc superconductors



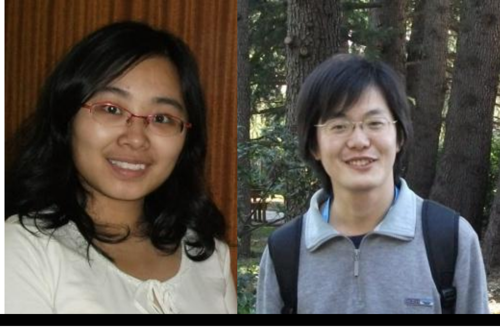
Shen group, Nature 450, 81 (2007)

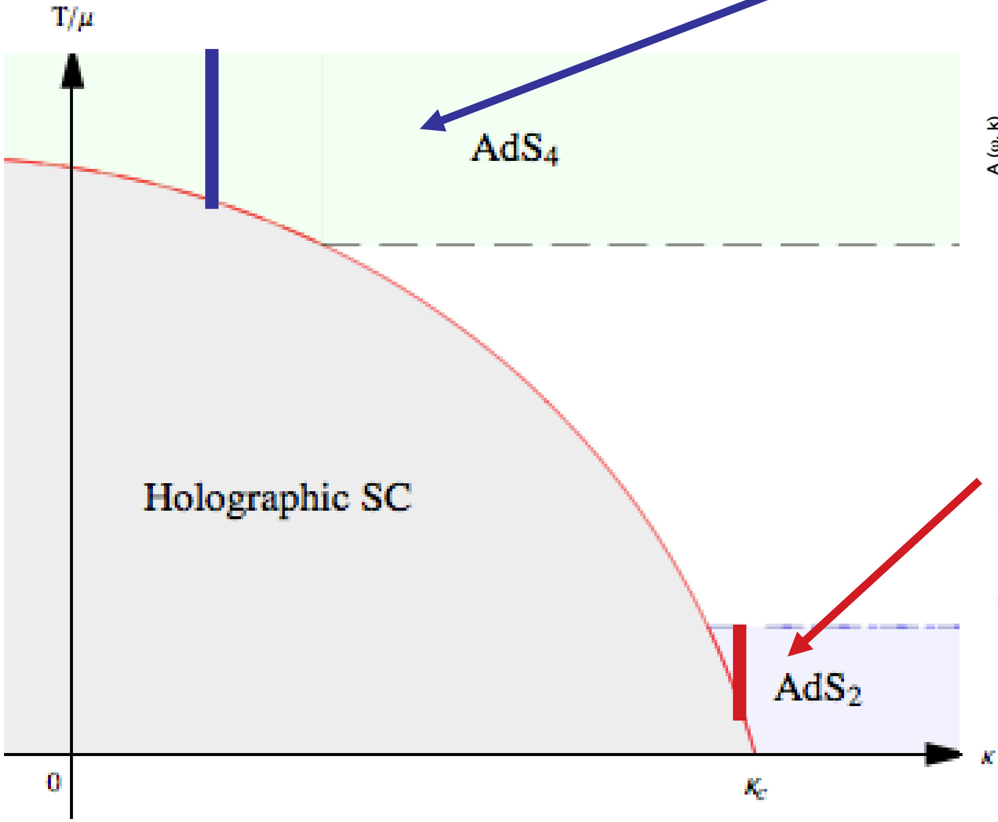


Gap stays open above Tc

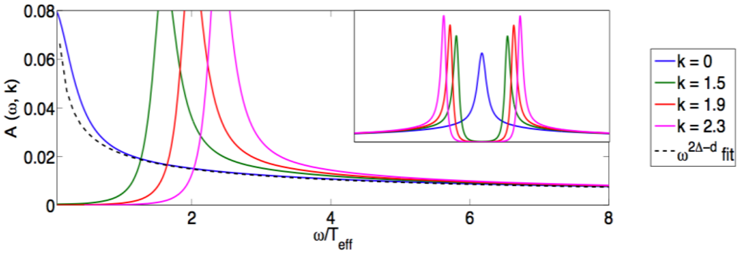
But sharp quasiparticles disappear in incoherent ‘spectral smears’ in the metal

“Double trace” Phase Diagram

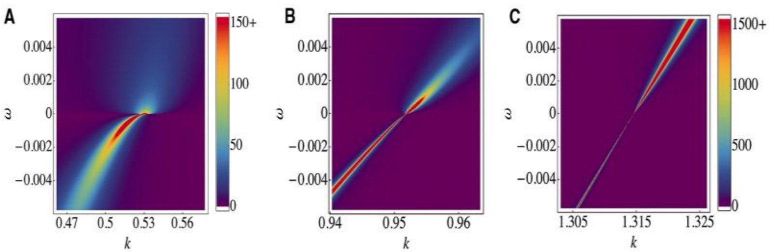




This looks like “quantum critical graphene” at zero density



This is the “marginal Fermi-liquid” Liu style



More fanciful: Quantum phase transitions

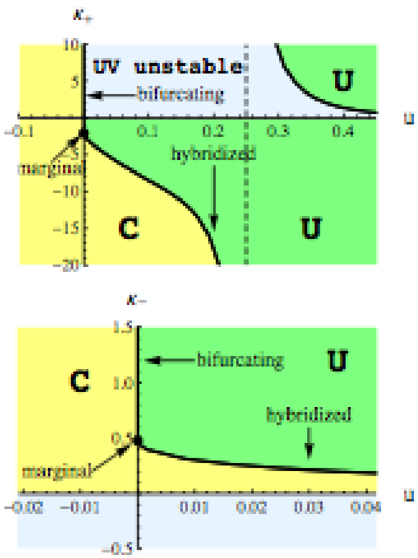


FIG. 7. The full phase diagram of the system for a neutral scalar. C (U) denotes regions with (without) IR instabilities. *Top plot*: standard quantization. For $u < 0$, i.e. $m^2R^2 < -\frac{3}{2}$ the system is always unstable in the IR with $u = 0$ the critical line for a bifurcating QCP. For $-\frac{3}{2} < m^2R^2 < 0$, i.e. $0 < u < \frac{1}{4}$, the system develops an IR instability for $\kappa_+ < \kappa_c(m^2) < 0$ with $\kappa_c(m^2)$ giving the critical line for hybridized QCP. The marginal critical point lies at the intersection for the critical lines for bifurcating and hybridized instabilities. The system has a vacuum UV instability for $\kappa_+ > 0$. For $m^2 > 0$, i.e. $u > \frac{1}{4}$, as discussed in the caption of Fig. 6, the vacuum instability is cured by finite density effect for sufficiently large κ_+. *Bottom plot*: phase diagram for the alternative quantization (for $\nu_U \in (0, 1)$, hence the limited range in u compared to the top plot, $u < \frac{1}{24}$), which can be obtained from that of the standard quantization by using the relation (3.4). In the vacuum, the system has an IR instability for $\kappa_- < 0$, i.e. with $\kappa_- = 0$ the critical line. At a finite density the critical line is pushed into the region $\kappa_- > 0$.

Quite different behaviors of the holographic quantum phase transitions by tuning the holographic SC down by mass or double trace deformation

Iqbal, Liu, Mezeiar, arXiv: 1108.0425

K.Jensen arXiv:1108.0421

Why is Tc high?

"Because there is superglue binding the electrons in pairs"

Wrong!

The superfluid density is tiny, it is very easy to 'bend and twist' a high Tc superconductor. **Its cohesive energy sucks.**

Tc's are set by the competition between the two sides …

The theory of the mechanism should explain why the free energy of the metal is seriously BAD.

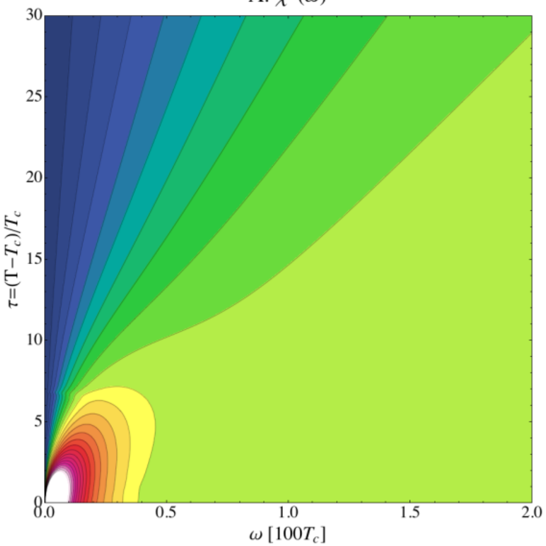
Observing the pairing mechanism …

Claim: the maximal knowledge on the pairing mechanism is encoded in the temperature evolution of the normal state dynamical pair susceptibility,

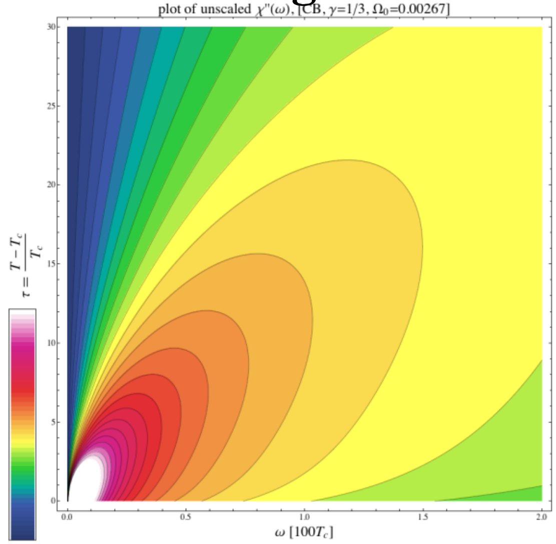
$$\chi_p(q,\omega) = -i\int_0^{\infty} dt\, e^{i\omega t - 0^+ t}\left\langle \left[b^+(q,0), b(q,t)\right]\right\rangle$$

$$b^+(q,t) = \sum_k c^+_{k+q/2,\uparrow}(t)\, c^+_{-k+q/2,\downarrow}(t)$$

Standard BCS



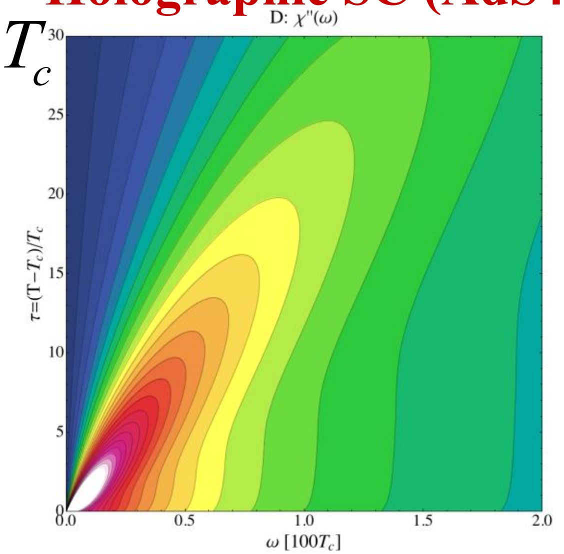
"Critical glue"



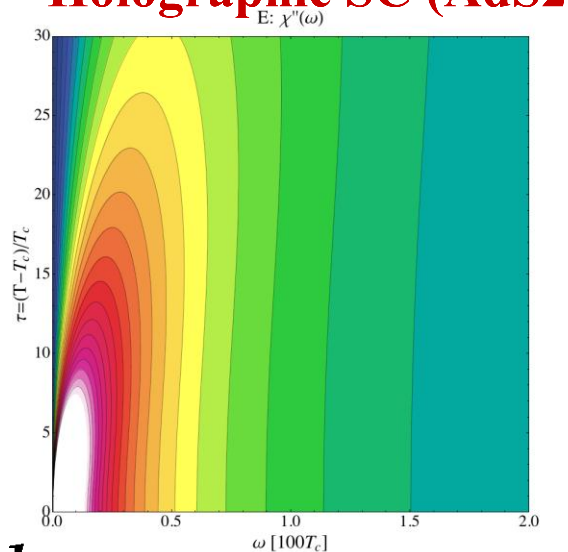
$$\chi_p''(\hbar\omega)$$

Holographic SC (AdS4)

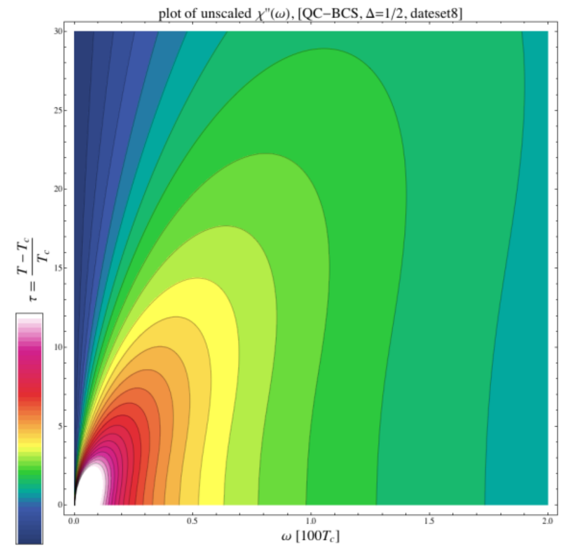
$T - T_c$



Holographic SC (AdS2)

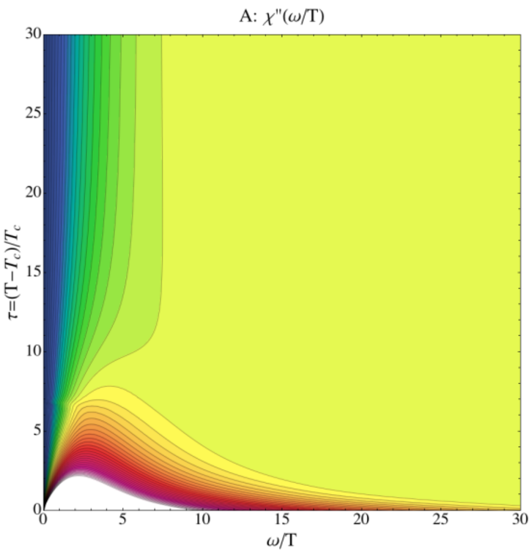


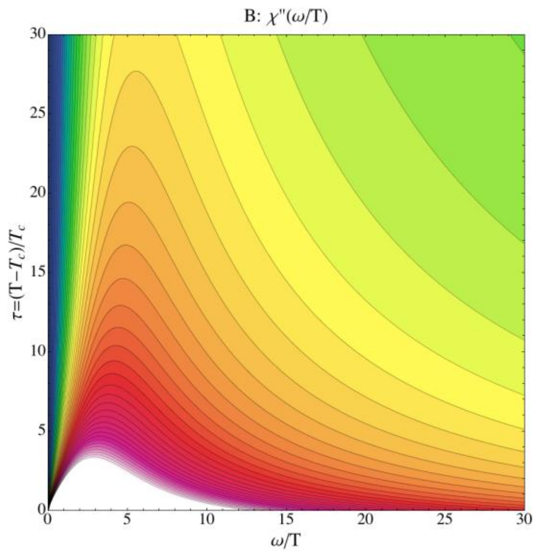
QC-BCS



$\hbar\omega$

Standard BCS **“Critical glue”**

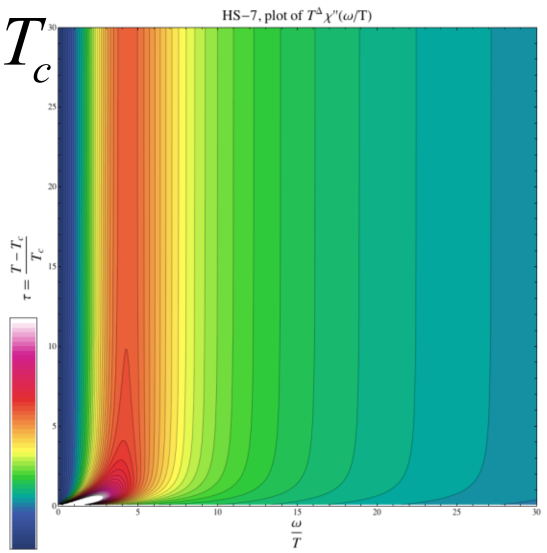


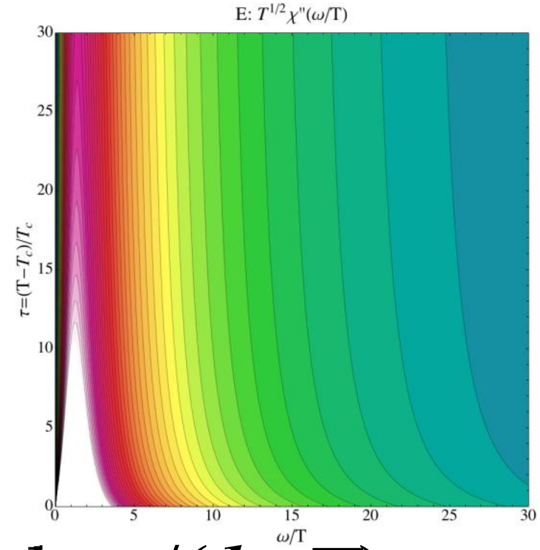


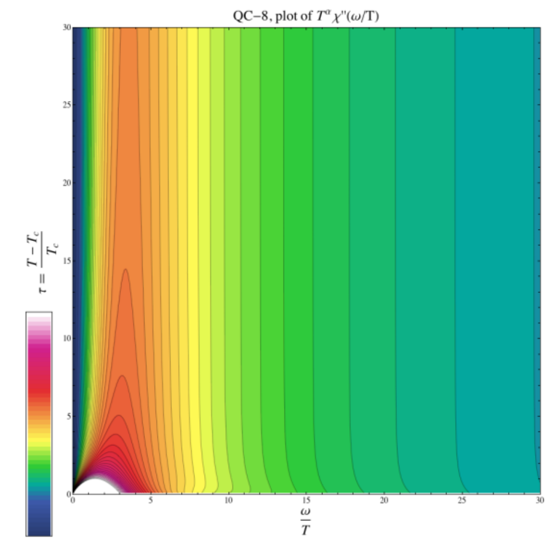
$$T^{\Delta}\chi_p^{''}\left(\frac{\hbar\omega}{k_B T}\right)$$

Holographic SC (AdS4) **Holographic SC (AdS2)** **QC-BCS**

$T - T_c$

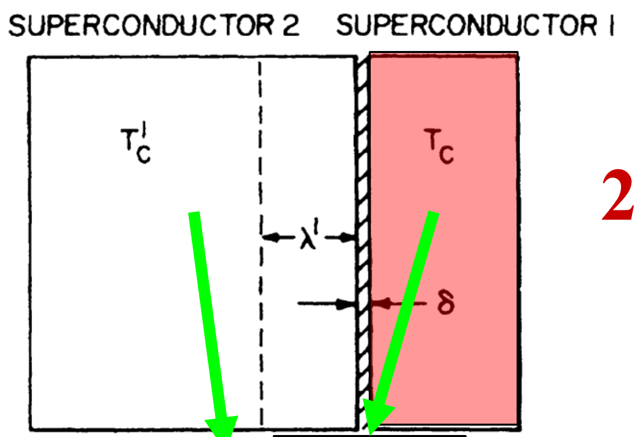






$\hbar\omega/(k_B T)$

Observing the origin of the pairing mechanism



$$\Delta \boxed{c^\dagger c^\dagger}$$

$$T_c' > T > T_c$$

2nd order Josephson effect

Ferrell Scalapino

1969 1970

$$I_s(\mathbf{H}, V) \sim \frac{1}{R_N^2} \boxed{\mathrm{Im}\chi_{\mathrm{pair}}(\mathbf{k}, \omega)} \qquad \omega = 2eV$$

Why Webb Pairing telescope?

QuickTime™ and a decompressor are needed to see this picture.

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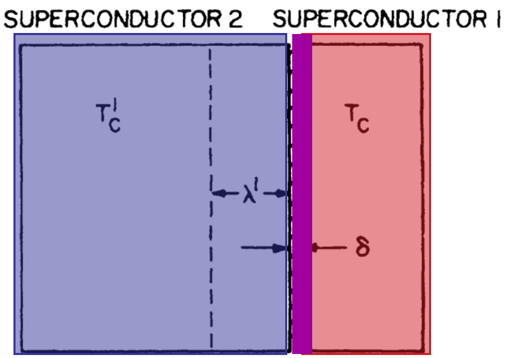

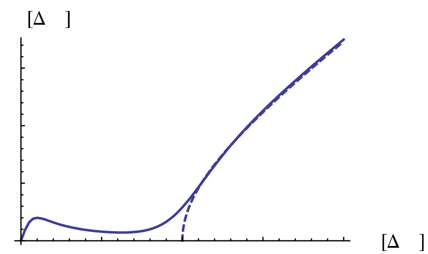

$$I_{tun}(V) = I_{qp}(V) + I_{pair}(V)$$

QC metal:

Need large dynamical range:

$$T, \omega \propto 10 - 100\, T_c$$

QC superconductor at ambient conditions with low Tc:

$CeIrIn_5$, $T_c = 0.4K$

Probe superconductor:

High T_c

Tunneling into d(?)-wave channel

Cuprate ?

Full gap to suppress QP current (?)

MgB_2 (Tc=40K)?

Barrier is the challenge!

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Dirac hair/electron star: Fermi-liquids emerging from a non Fermi liquid (critical) ultraviolet, like heavy fermions (Schalm, Cubrovic, Hartnoll).

Scalar hair: holographic superconductivity, a new mechanism for superconductivity at a high temperature (Hartnoll, Herzog,Horowitz) **.**

Further reading

Condensed matter tutorials:
J. Zaanen, Science 319, 1205; arXiv:1012.5461

AdS/CMT tutorials:
J. Mc Greevy, arXiv:0909.0518; S. Hartnoll, arXiv:0909.3553,1106.4324

Non fermi-liquids:
M. Cubrovic et al. Science 325,429 (2009); T. Faulkner et al., Science 329, 1043 (2010); N. Iqbal et al., arXiv:1105.4621

Fermi-liquids:
M. Cubrovic et al. arXiv:1012.5681,1106.1798; S. Hartnoll et al., arXiv:1105.3197

Holographic superconductors:
J.-H. She et al., arXiv:1105.5377

Thermodynamics: where are the fermions?

Hartnoll et al.: arXiv:0908.2657,0912.0008

Large N limit: thermodynamics entirely determined by AdS geometry.

Fermi surface dependent thermodynamics, e.g. Haas van Alphen oscillations?

Leading 1/N corrections: "Fermionic one-loop dark energy"

Quantum corrections: one loop using Dirac quasinormal modes: 'generalized Lifshitz-Kosevich formula' for HvA oscillations.

$$\chi_{osc.} = -\frac{\partial^2 \Omega_{osc.}}{\partial B^2} = \frac{\pi A T c k_F^4}{e B^3} \cos \frac{\pi c k_F^2}{e B} \sum_{n=0}^{\infty} e^{-\frac{c T k_F^2}{e b \mu}\left(\frac{T}{\mu}\right)^{2\nu-1}} F_n(\mu)$$

Collective transport: fermion currents

Hong Liu (MIT)

QuickTime™ and a decompressor are needed to see this picture.

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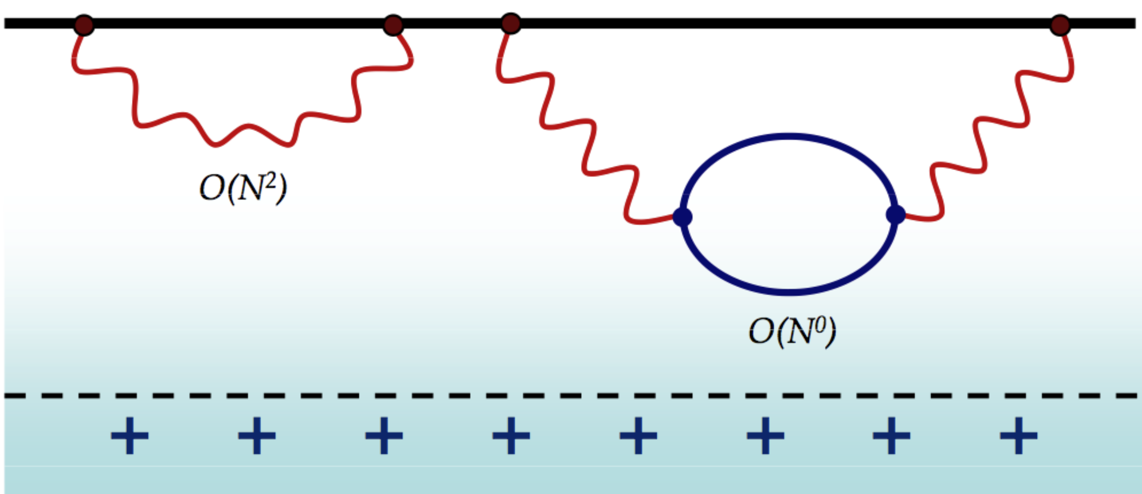
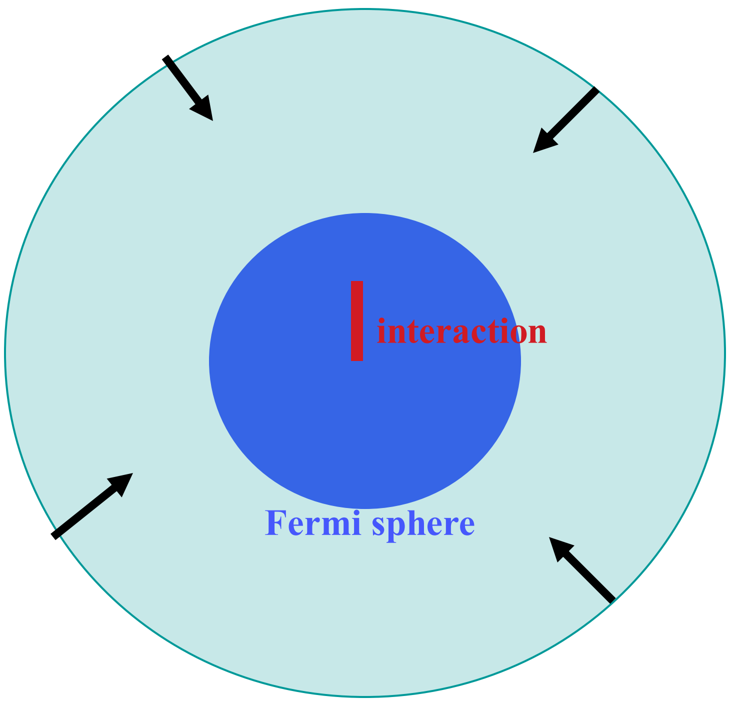
Conductivity



Tedious one loop calculation, 'accidental' cancellations: $\rho_{FS} \propto \Sigma''_{1-fermion} \propto T^{2\nu}$

'Strange coincidence' of one electron and transport lifetime of marginal fermi liquid finds gravitational explanation!

'Shankar/Polchinski' functional renormalization group

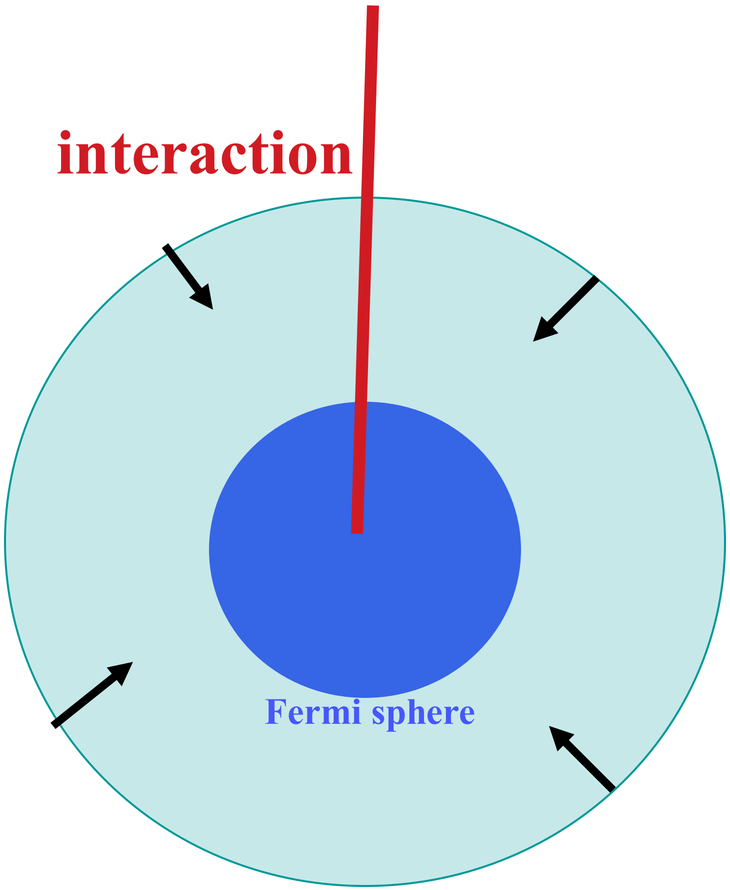


UV: weakly interacting Fermi gas

Integrate momentum shells: functions of running coupling constants

All interactions (except marginal Hartree) irrelevant => **Scaling limit might be perfectly ideal Fermi-gas**

The end of weak coupling



Strong interactings:

Fermi gas as UV starting point does not make sense!

=> 'emergent' Fermi liquid fixed point remarkably resilient (e.g. 3He)

=> Non Fermi-liquid/non 'Hartree-Fock' (BCS etc) states of fermion matter?

Empty

Empty