Out-of-equilibrium dynamics of coherent non-abelian fields

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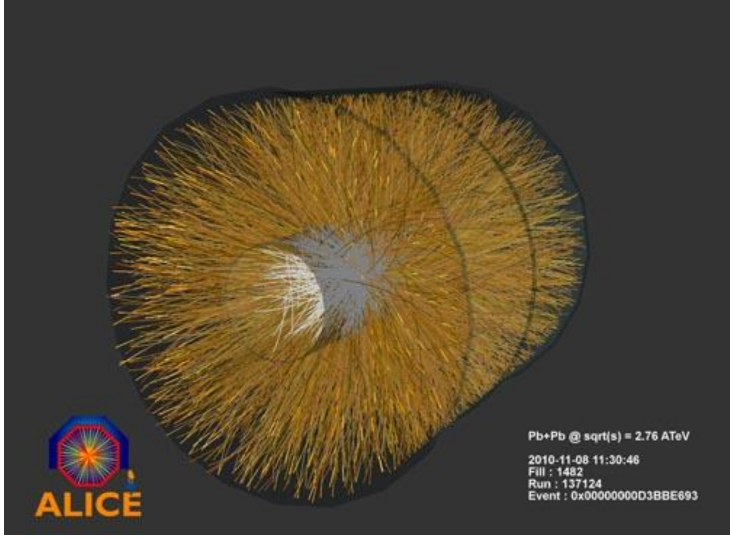
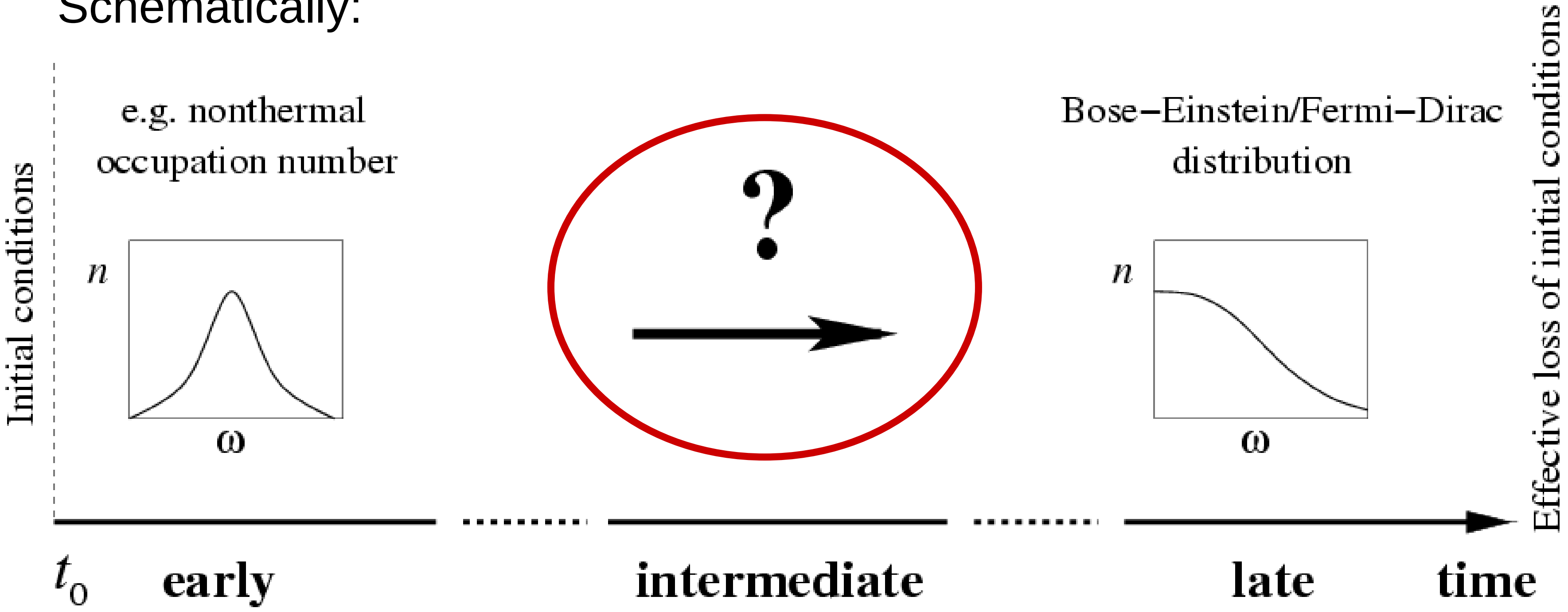
Nonequilibrium QCD

Relativistic heavy-ion collisions explore strong interaction matter starting from a transient ***nonequilibrium*** state

Thermalization process?



Schematically:



- **Characteristic nonequilibrium time scales? Relaxation? Instabilities?**

Nonequilibrium dynamics of coherent fields

Color Glass:

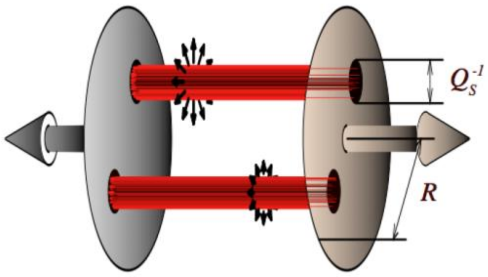


Fig by F Gelis

'color flux tubes'

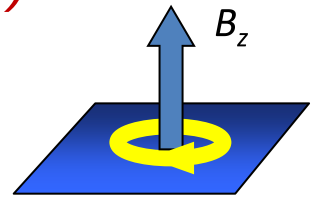
transverse sizes $\Delta R_\perp \sim \dfrac{1}{Q_s}$

1) Consider extreme case: constant color magnetic field pointing in *z*-direction

$$B_j^a = \delta^{1a}\delta_{3j}B \qquad \text{from} \qquad A_x^1 = -\frac{1}{2}yB\,,\; A_y^1 = \frac{1}{2}xB \qquad \text{(all other zero)}$$

→ *exponential growth of fluctuations (Nielsen-Olesen instability) with maximum rate*

$$\sqrt{gB} \sim Q_s$$



Nielsen, Olesen '78; Chang, Weiss '79; … Iwasaki '08; Fujii, Itakura '08 …

2) Consider less extreme case: *temporal* modulations on scales $\gtrsim 1/\sqrt{gB}$

$B_j^a = \delta^{1a}\delta_{3j}B$ from $A_x^2 = A_y^3 = \sqrt{\dfrac{B}{g}}$ (all other zero)

$\rightarrow$ non-linear part of field strength tensor

Classical equation of motion:

$$(D_\mu[A]F^{\mu\nu}[A])^a = 0$$

Time-dependent background field $\bar{A}_\mu^a(x^0)$:

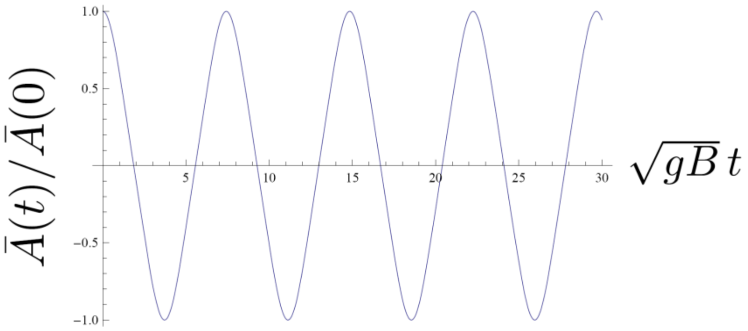
$$A_\mu^a(x) = \bar{A}_\mu^a(x^0) + \delta A_\mu^a(x)$$

temporal (Weyl) gauge with $A_0^a = 0$ and

$$\bar{A}_i^a(t) = \bar{A}(t)\left(\delta^{a2}\delta_{i1} + \delta^{a3}\delta_{i2}\right) \;, \quad \bar{A}(t=0) = \sqrt{B/g}$$

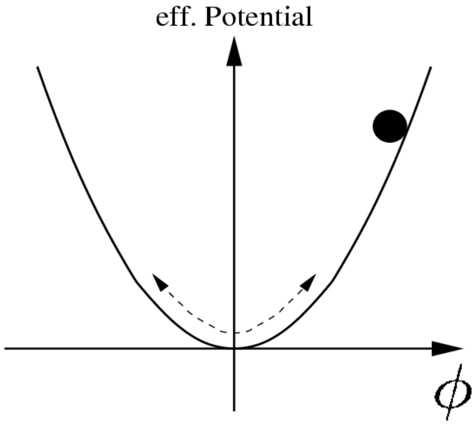
• Background-field equation: $\left(D_\mu[\bar{A}]F^{\mu\nu}[\bar{A}]\right)^a = 0$

$$\Rightarrow \quad \boxed{\partial_t^2 \bar{A}(t) + g^2 \bar{A}(t)^3 = 0}$$

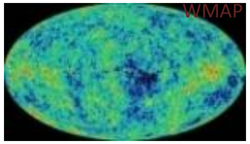


Oscillating solution: $\bar{A}(t) = \sqrt{\frac{B}{g}}\,\mathrm{cn}\left(\sqrt{gB}\,t\,,\frac{1}{2}\right)$ with period $\Delta t_B = \frac{4K(1/2)}{\sqrt{gB}} \simeq \frac{7.42}{\sqrt{gB}}$

Compare e.g. scalar $\lambda\Phi^4$ theory:

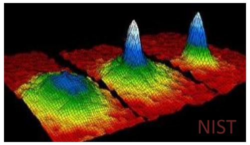


- early universe inflaton dynamics (preheating)



- non-rel. gas of ultracold atoms (Gross-Pitaevski), λ~a (s-wave scattering length)

  B. Novak, RG-conference



$\rightarrow$ talks next week

- Linearized fluctuation equation, *SU*(2):

$$\left(D_\mu[\bar{A}]D^\mu[\bar{A}]\delta A^\nu\right)^a - \left(D_\mu[\bar{A}]D^\nu[\bar{A}]\delta A^\mu\right)^a + g\epsilon^{abc}\delta A^b_\mu F^{c\mu\nu}[\bar{A}] = 0$$

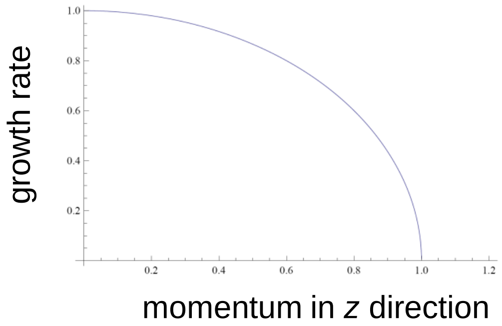
maximally amplified modes: $\delta A_- = \delta A^3_2 - \delta A^2_1$ or $\delta A^3_1 + \delta A^2_2$

$$\Rightarrow \quad \boxed{\partial_t^2 \delta A_-(t,p_z) = \left(g^2 \bar{A}(t)^2 - p_z^2\right)\delta A_-(t,p_z)} \qquad (p_x = p_y = 0)$$

Oscillator with time-dependent frequency with ‘wrong sign‘ for $p_z^2 < g^2\bar{A}(t)^2$

approximate solution: ($\bar{A}(t=0) = \sqrt{B/g}$)

$$\delta A_-(t,p_z) \sim e^{\sqrt{g\overline{B}-p_z^2}\,t}$$



→ similar to *Nielsen-Olesen instability with time-averaged magnetic field*

$$g\overline{B} \equiv \frac{gB(t=0)}{2K(1/2)}\int_0^{2K(1/2)} \mathrm{d}x\, \mathrm{cn}^2\left(x, \frac{1}{2}\right) \approx 0.457\, gB(t=0)$$

Non-linear evolution: Classical-statistical lattice gauge theory

Wilson action:

(real time!)

$$S[U] = -\beta_0 \sum_x \sum_i \left\{ \frac{1}{2\mathrm{Tr}\mathbf{1}} \left(\mathrm{Tr}\, U_{x,0i} + \mathrm{Tr}\, U_{x,0i}^{-1} \right) - 1 \right\} + \beta_s \sum_x \sum_{\substack{i,j \\ i<j}} \left\{ \frac{1}{2\mathrm{Tr}\mathbf{1}} \left(\mathrm{Tr}\, U_{x,ij} + \mathrm{Tr}\, U_{x,ij}^{-1} \right) - 1 \right\}$$

Plaquette variables $U_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu} \approx \exp\left[-iga^2 F_{\mu\nu}(x)\right]$

Here: $\beta = \beta_0 / \gamma = \beta_s \gamma = 4$, temporal gauge, $SU(2)$, *no expansion*

Sampling introduces classical-statistical fluctuations (‘loops‘) → non-linear evolution, accurate for sufficiently ’large fields/high occupation’ numbers:

anti-commutator $\langle\{A, A\}\rangle \gg \langle[A, A]\rangle$ commutator

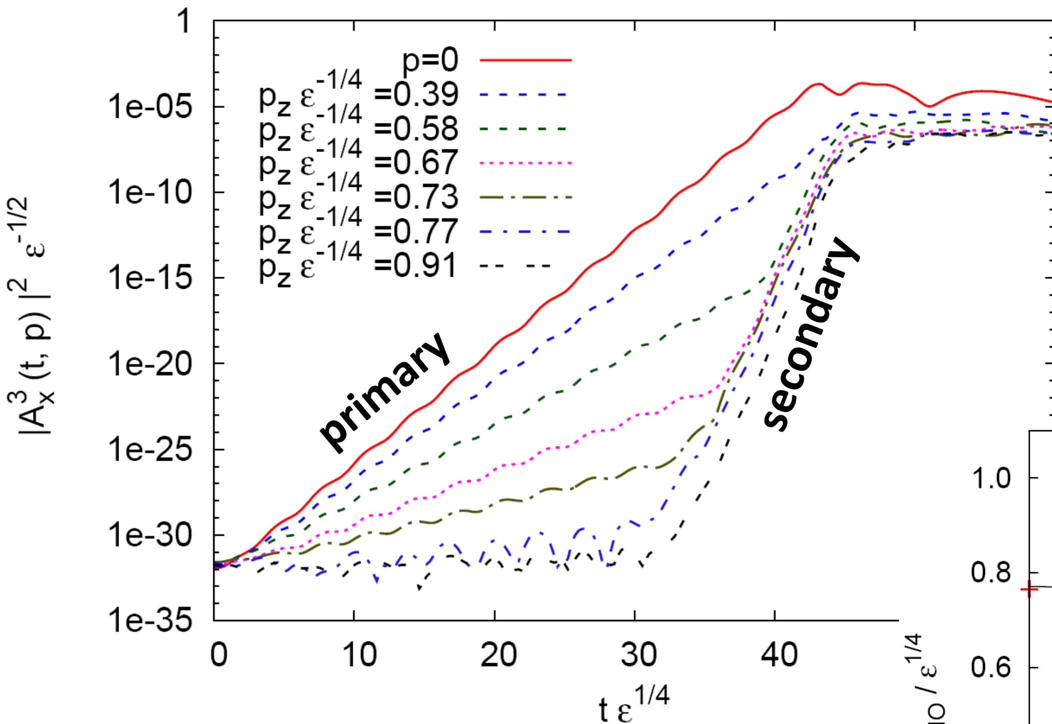
→ ‘working horse‘ for instability dynamics

Romatschke, Venugopalan; Berges, Gelfand, Sexty, Scheffler, Schlichting;
Kunihiro, Müller, Ohnishi, Schäfer, Takahashi, Yamamoto; Fukushima, Gelis; …

Nonequilibrium coherent fields on the lattice

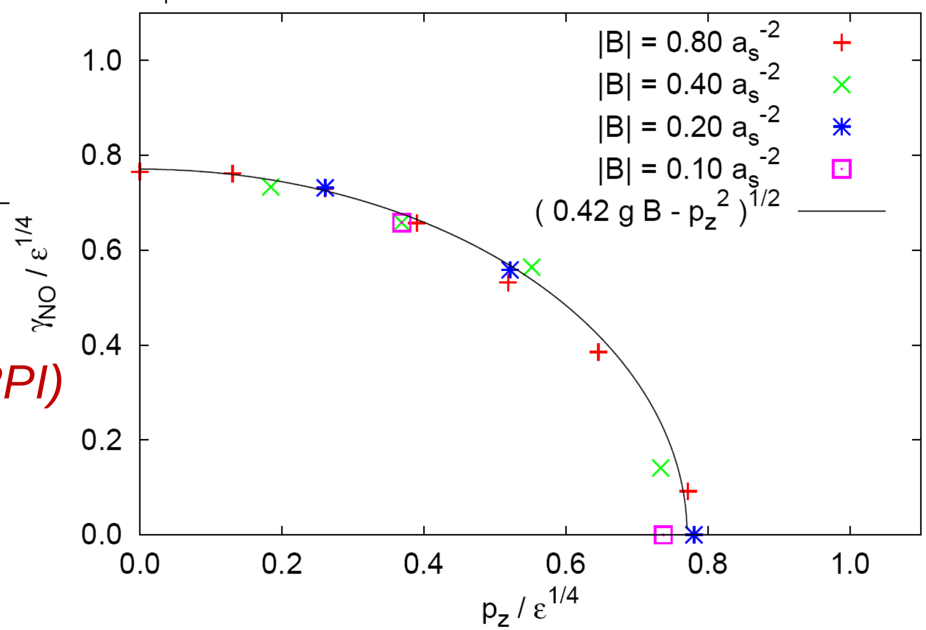
Exponential growth of fluctuations:

with Sexty, Scheffler, in preparation

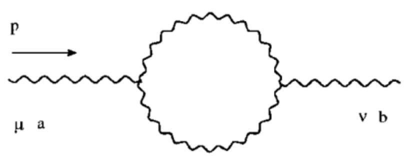


- *good agreement of primary growth with linear analysis!*

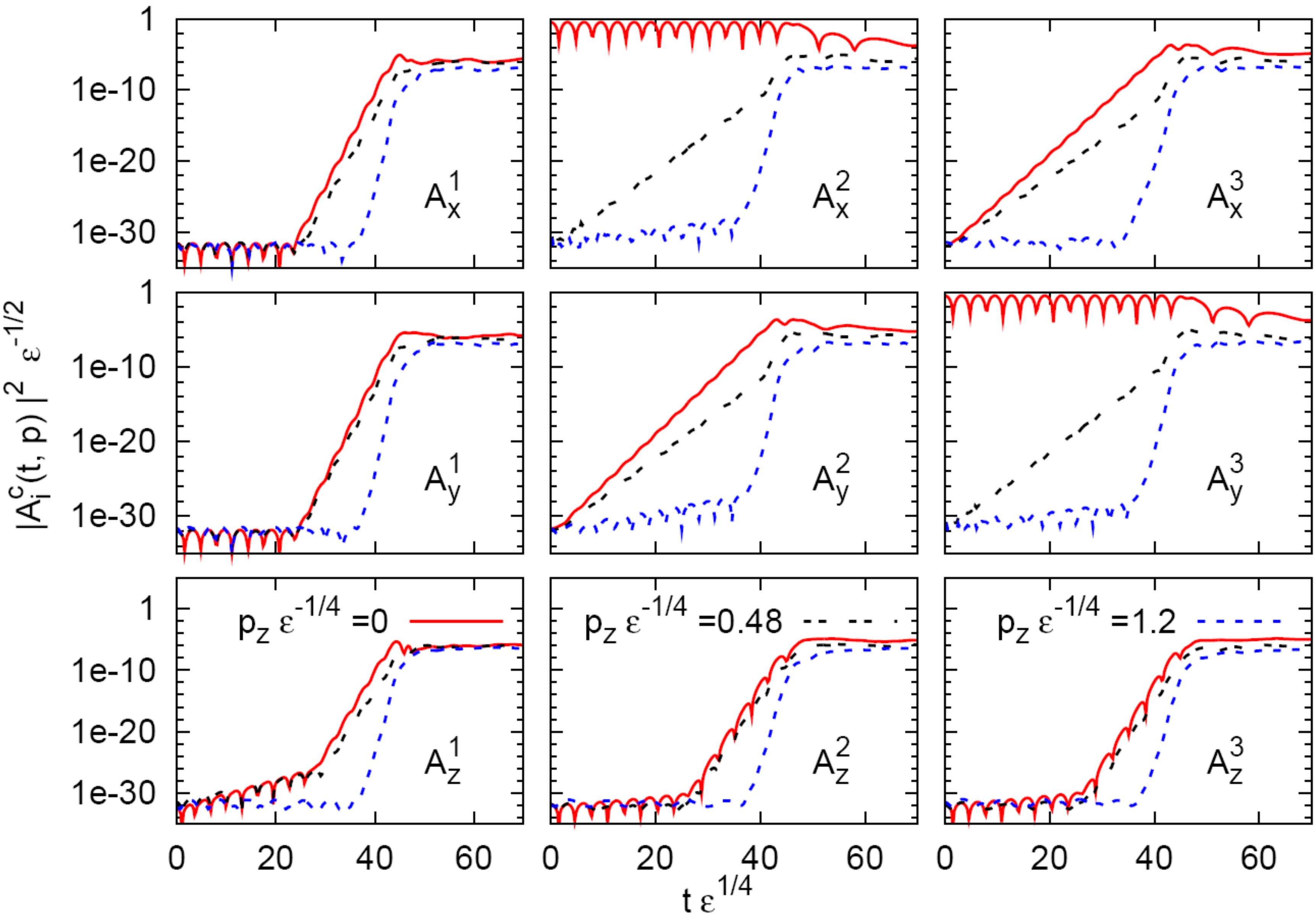
Primary growth rates:



- *secondary rate ~2γ_{NO} from non-linear (2PI)*



cf. also Berges, Scheffler, Sexty, PRD 77 (2008) 034504



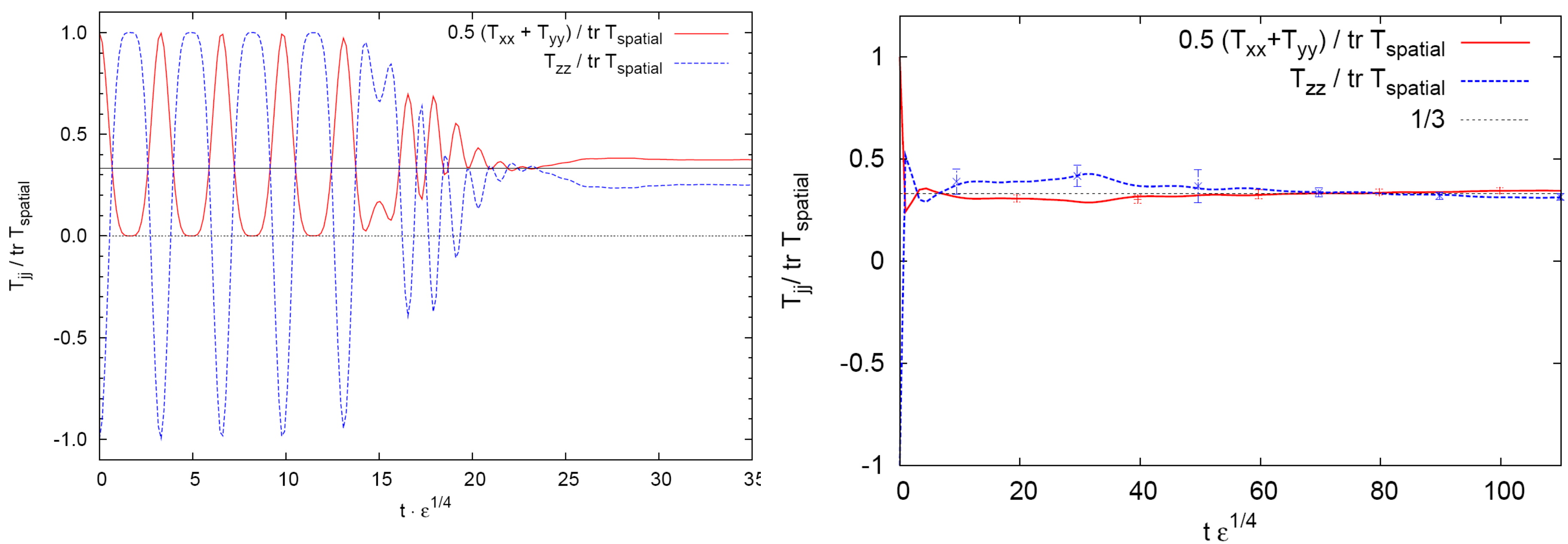
with Sexty, Scheffler, in preparation

Isotropization

3) Choose initial homogeneous fields randomly (ensemble)

with zero mean and width $< \sqrt{gB} \sim Q_s$ with Sexty, Scheffler, in preparation

Compare: **original** **vs.** **ensemble average**



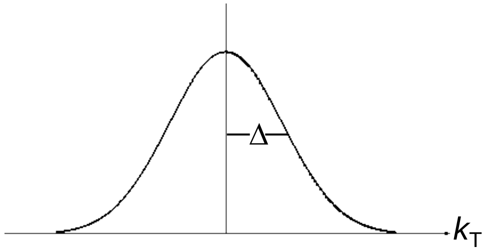
• *very efficient isotropization for ensemble averaged homogeneous fields!*

$\rightarrow$ *early equation of state for hydrodynamics*

Comparison to previous ensembles

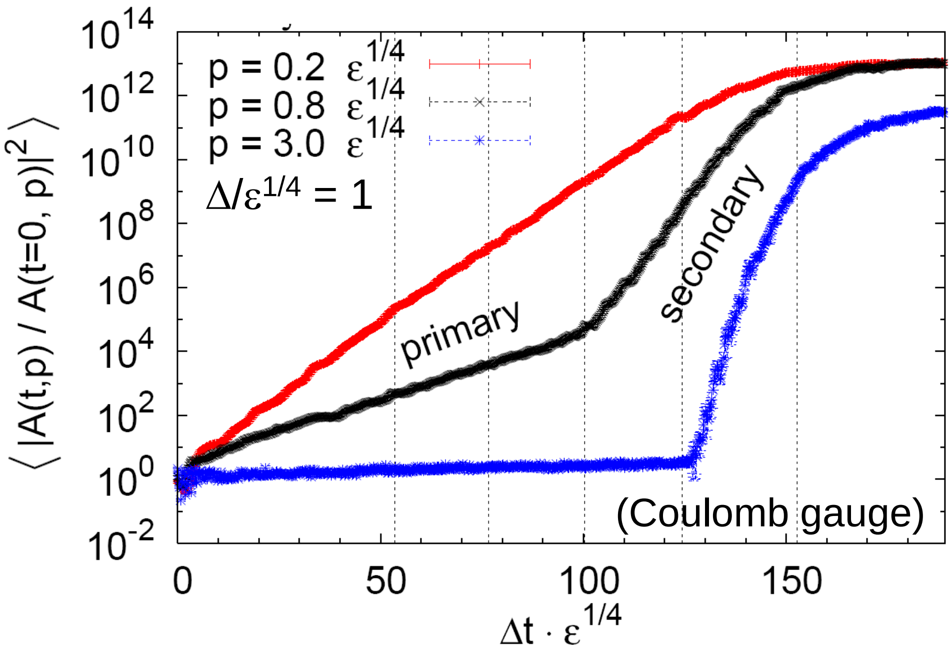
Initial conditions: *stochastically generated inhomogeneous fields with*

$$\langle\, |A^a_j(t=0,\vec{k})|^2\,\rangle \sim C\,\exp\{-\frac{k_x^2+k_y^2}{2\Delta^2}-\frac{k_z^2}{2\Delta_z^2}\}$$

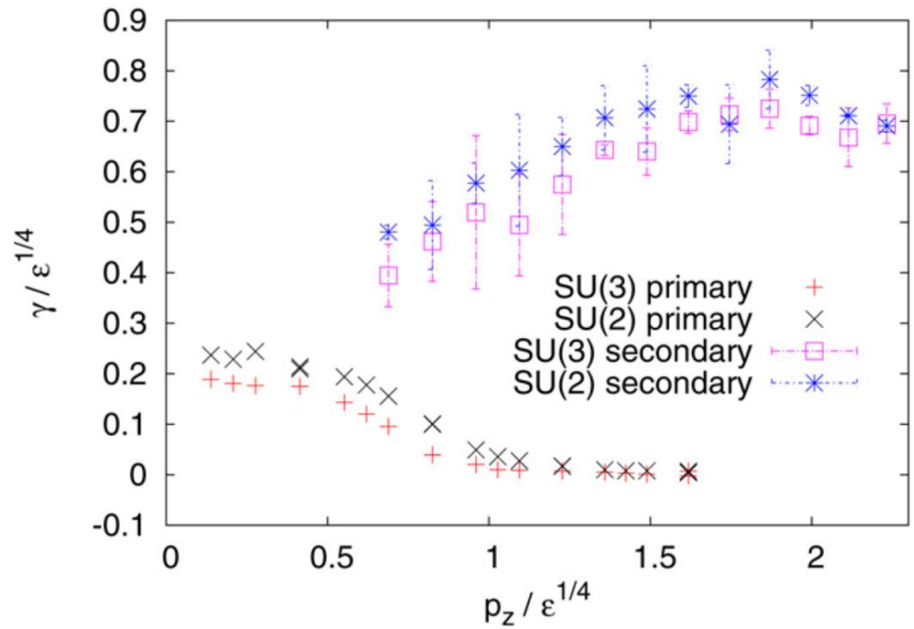


and $Q_s \sim \Delta \gg \Delta_z$ (extreme oblate anisotropy)

Exponential growth of fluctuations:



Primary/secondary growth rates:

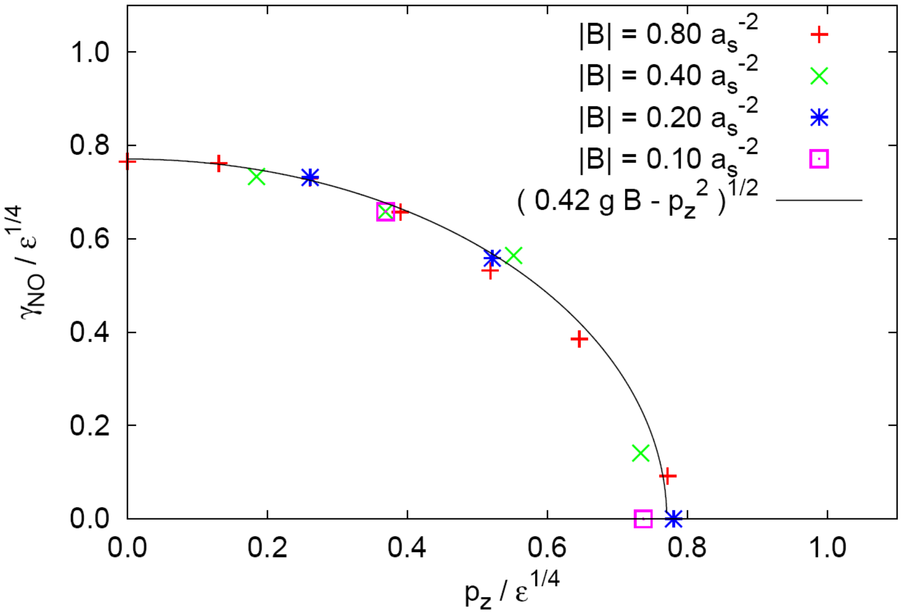


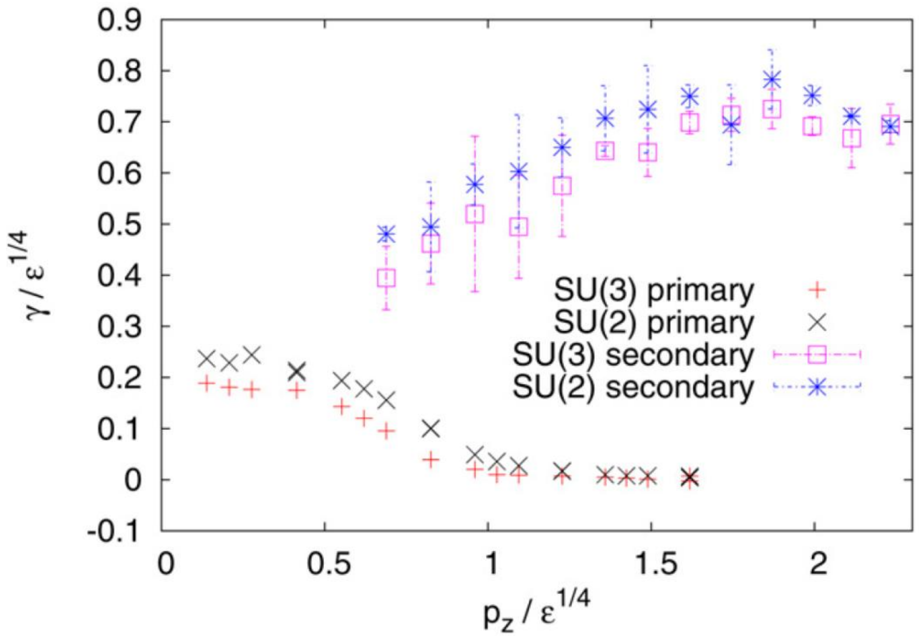
Berges, Scheffler, Sexty, PRD 77 (2008) 034504 (**SU(2)**); + Gelfand, PLB 677 (2009) 210 (**SU(3)**)

Coherence speed-up

Compare:

spatially homogeneous fields vs. stochastic inhomogeneous fields



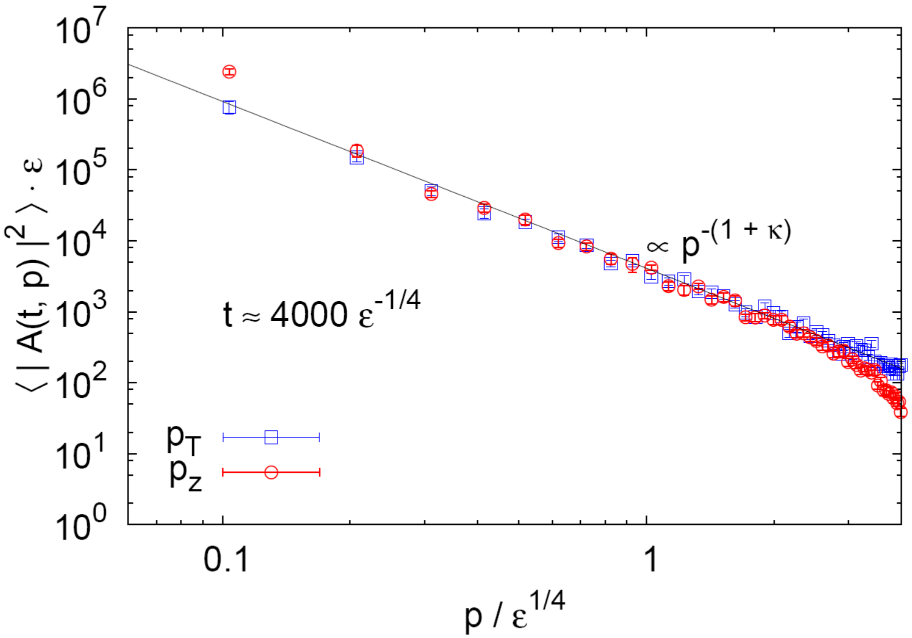


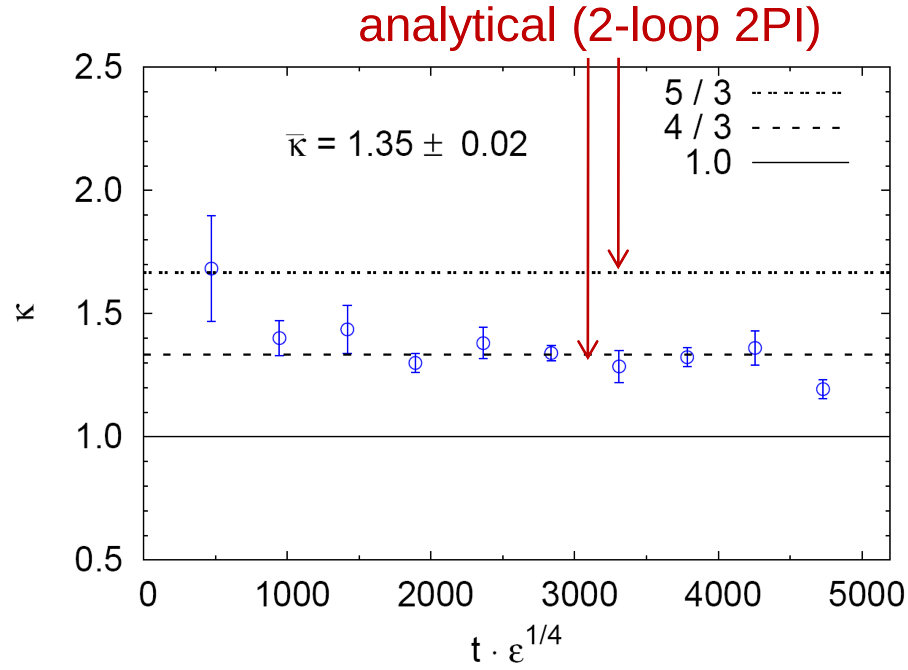
Inverse primary growth rates: e.g. $\varepsilon_{RHIC} \sim$ 5-25 GeV/fm^3, $\varepsilon_{LHC} \sim 2 \times \varepsilon_{RHIC}$

$\boxed{1/\gamma_{NO} \simeq 0.3 - 0.6 \text{ fm/c}}$

$\boxed{1/\gamma \simeq 1.0 - 1.8 \text{ fm/c}}$

Non-linear dynamics leading to turbulence





- Scaling exponent close to perturbative Kolmogorov value at high *p*: $\kappa = 4/3$

Berges, Scheffler, Sexty, PLB 681 (2009) 362; see also Fukushima, Gelis, arXiv:1106.1396 [hep-ph]

- Nonperturbative infrared scaling behavior with $\kappa = 4$ ($\kappa = 5$) expected

Infrared "occupation number" ~ 1/g² → strongly correlated! Universal!

Berges, Rothkopf, Schmidt '08; Berges, Hoffmeister '09; Scheppach, Berges, Gasenzer '10; Carrington, Rebhan '10; Nowak, Sexty, Gasenzer '10; …

→ see also talks next week

Quantum corrections and fermions

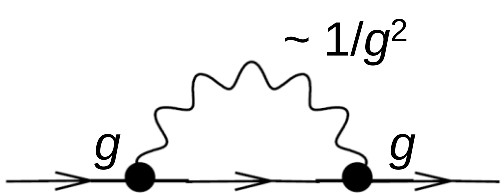
- Classical-statistical gauge field description accurate for

  → sufficiently large field expectation values/highly occupied modes
  → but quantum corrections at low occupied higher momenta
  inclusion into simulations using *inhomogeneous 2PI effective action*

cf. Berges, Roth, NPB 847 (2011) 197

- Fermions: $n_\psi(p) \leq 1$ (Pauli principle)

  → *no* classical-statistical approximation
  → enhancement of quantum corrections from highly occupied bosons!



$\sim \; O(1)$

Requires real-time lattice simulations with dynamical fermions!

cf. Berges, Gelfand, Pruschke, PRL 107 (2011) 061301

Conclusions & Outlook

- coherent fields can lead to ultra-fast dynamics:

$$1/\gamma_{NO} \simeq 0.3 - 0.6 \text{ fm/c}$$

for typical LHC/RHIC energies

- very efficient isotropization for ensemble averaged homogeneous fields!

$\rightarrow$ early equation of state for hydrodynamics

- non-linear dynamics crucial for efficient development of turbulence

$\rightarrow$ perturbative Kolmogorov scaling exponent at high p: $\kappa = 4/3$

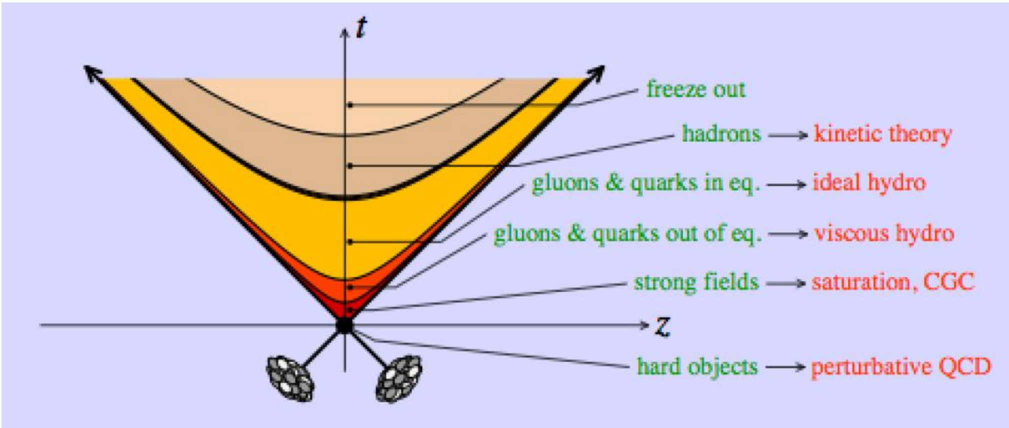
$\rightarrow$ non-perturbative scaling exponent at low p? shown to be true for scalars PRL 101 (2008) 041603

- enhancement of quark corrections to $O(1)$ from highly occupied bosons?

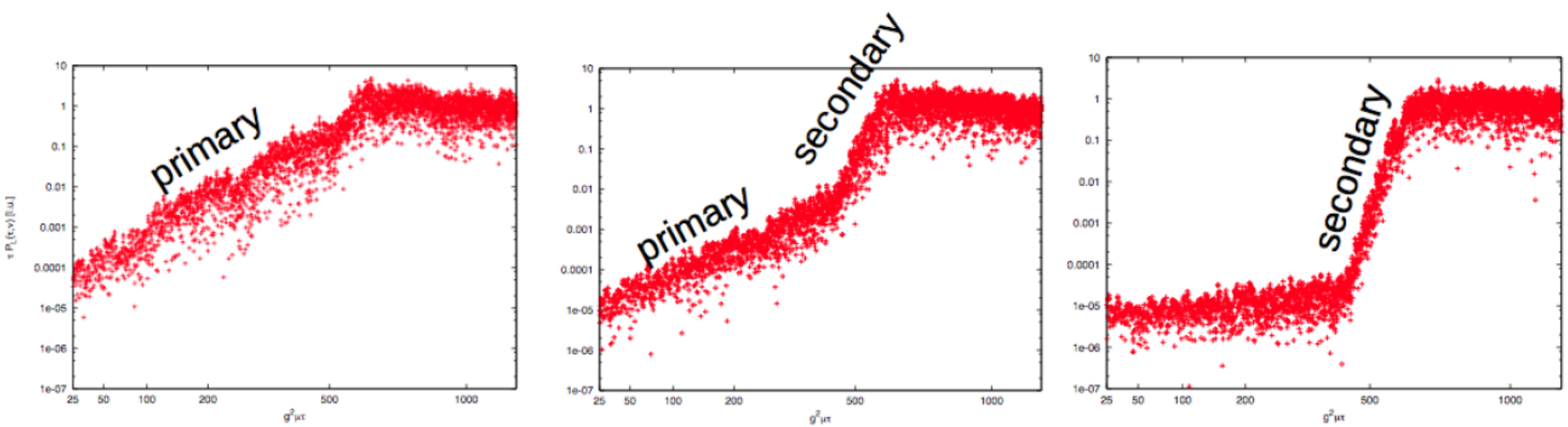
shown to be true for quark-meson model PRL 107 (2011) 061301

$\rightarrow$ real-time dynamical fermions on the lattice in 3+1 dimensions

Non-linear dynamics with expansion



see also Venugopalan, Romatschke ’06; Fukushima, Gelis ’11



soft ⟶ hard

Sören Schlichting