Bose-Einstein Condensation and Thermalization of the Quark-Gluon Plasma

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WORK IN PROGRESS ! and SPECULATIVE....

The sQGP : a puzzle

Where is the apparent strongly coupled character of the quark-gluon plasma coming from ?

The strongly coupled quark-gluon plasma

Empirical evidence from RHIC (and LHC) data

- Strong opacity of matter (jet quenching, energy loss,...)
- Collective behavior (elliptic flow, ...)
- Small ratio of viscosity to entropy density
- Small thermalization time, etc

Why this is puzzling

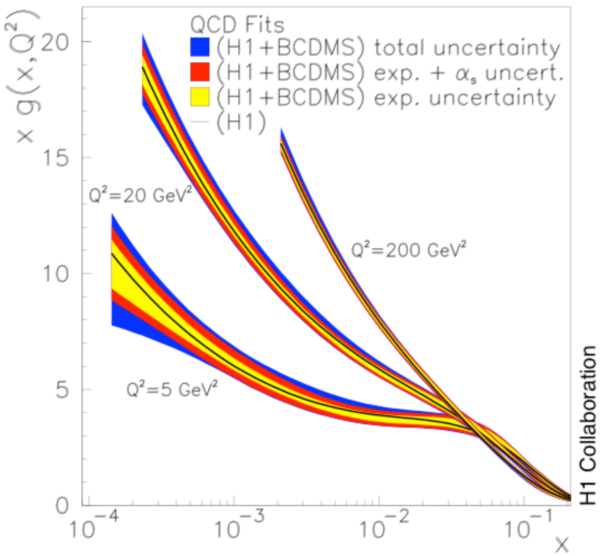
- The coupling constant is not small, but not huge $\alpha_s \sim 0.3 \div 0.4$
- Strict perturbation does not work, but successful resummations exist
- Understanding of early stages of HI collisions relies on weak coupling

Clue

- «Strong coupling» behavior may appear at weak coupling, when many degrees of freedom contribute coherently (e.g. collective phenomena, BCS, CGC, etc)

The over-populated quark-gluon plasma

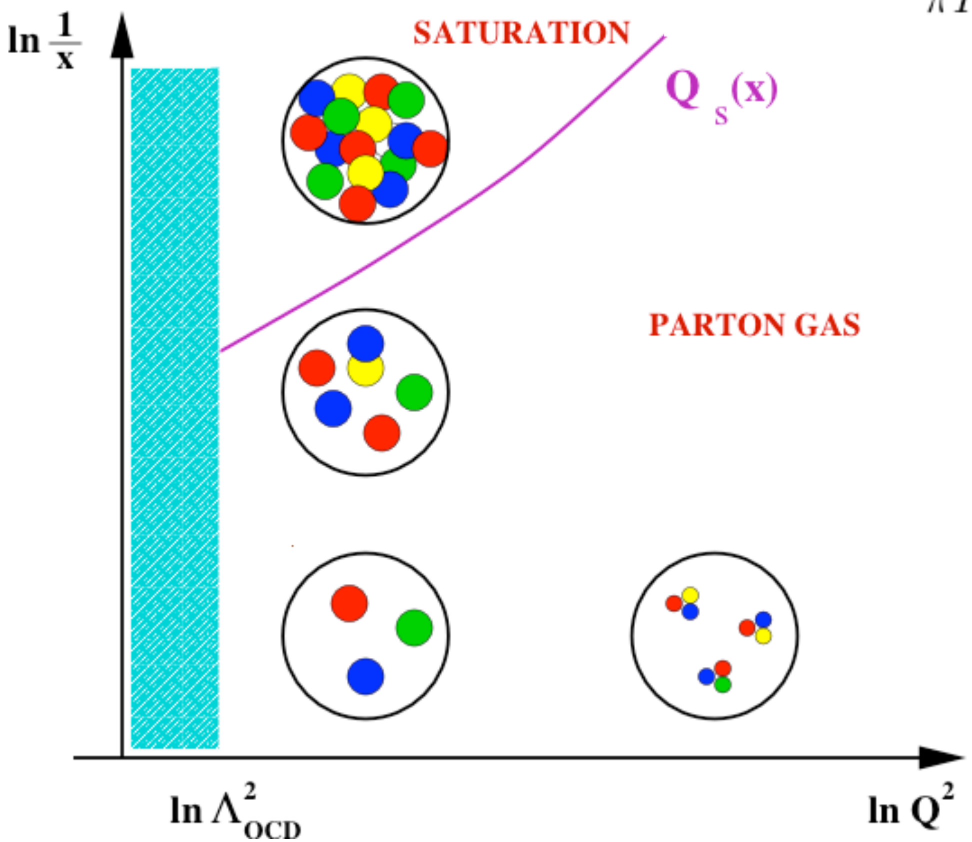
High density partonic systems



Large occupation numbers

$$\frac{xG(x,Q^2)}{\pi R^2 Q_s^2} \sim \frac{1}{\alpha_s}$$

$$Q_s^2 \approx \alpha_s \frac{xG(x,Q^2)}{\pi R^2}$$



Thermodynamical considerations

Initial conditions $(t_0 \sim 1/Q_s)$

$$\epsilon_0 = \epsilon(\tau = Q_s^{-1}) \sim \frac{Q_s^4}{\alpha_s} \qquad n_0 = n(\tau = Q_s^{-1}) \sim \frac{Q_s^3}{\alpha_s} \qquad \epsilon_0 / n_0 \sim Q_s$$

overpopulation parameter $\qquad n_0\, \epsilon_0^{-3/4} \sim 1/\alpha_s^{1/4}$

In equilibrated quark-gluon plasma

$$\epsilon_{eq} \sim T^4 \qquad n_{eq} \sim T^3 \qquad n_{eq}\, \epsilon_{eq}^{-3/4} \sim 1$$

mismatch by a large factor (at weak coupling) $\qquad \alpha_s^{-1/4}$

Chemical potential does not help

Assume that the nmber of gluons is conserved

$$f_{\rm eq}(\boldsymbol{k}) \equiv \frac{1}{e^{\beta(\omega_{\boldsymbol{k}}-\mu)}-1}$$

growing function of μ

Maximum value of μ

$$\mu \leq \omega_{\boldsymbol{p}=0} = m \neq 0$$

Screening mass

$$m_0^2 \sim \alpha_s \int_{\boldsymbol{p}} \frac{df_0}{d\omega_p} \sim Q_s^2$$

$$m_{eq} \sim \alpha^{1/2} T \sim \alpha^{1/4} Q_s$$

Maximum number of gluons that can be accommodated by a BE distribution

$$n_{\rm max} = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{e^{\beta(\omega_{\boldsymbol{k}}-m)}-1} \sim T^3$$

Formation of a Bose-Einstein condensate

(when elastic processes dominate)

Most particles are in the BEC

$$n_{\rm c} \sim \frac{Q_{\rm s}^3}{\alpha}\left(1-\alpha^{1/4}\right) \qquad n_c = n - n_g$$

BEC contributes little to the energy density

$$n_{\rm c}\, m \sim \frac{Q_{\rm s}^3}{\alpha_{\rm s}}\,\alpha_{\rm s}^{1/4} Q_{\rm s} \sim \alpha^{1/4} T^4 \ll \epsilon_0$$

Entropy considerations

$$s \sim \int_{\boldsymbol{p}} \ln f_{\boldsymbol{p}} \qquad s_0 \sim Q_s^3 \qquad s_{eq} \sim T^3 \sim Q_{\rm s}^3/\alpha^{3/4}$$

Note: overpopulation disappears at equilibrium

$$n_g \sim T^3 \qquad n_{\rm g}\,\epsilon^{-3/4} \sim 1$$

Note: inelastic processes inhibate the formation of a condensate

Kinetic evolution dominated by elastic collisions

Simple kinetic equation and two important scales

Non expanding plasma

$$\partial_t f(\boldsymbol{k}, X) = C_{\boldsymbol{k}}[f]$$

A very schematic distribution function

$$f(p) \sim \frac{1}{\alpha_s} \text{ for } p < \Lambda_s, \qquad f(p) \sim \frac{1}{\alpha_s}\frac{\Lambda_s}{\omega_p} \text{ for } \Lambda_s < p < \Lambda, \qquad f(p) \sim 0 \text{ for } \Lambda < p$$

Initially, $\quad t \sim 1/Q_s \qquad \Lambda_s \sim \Lambda \sim Q_s$

In small angle approximation

$$\left.\frac{\partial f}{\partial t}\right|_{\text{coll}} \sim \frac{\Lambda_s^2 \Lambda}{p^2}\partial_p \left\{p^2\left[\frac{df}{dp} + \frac{\alpha_s}{\Lambda_s} f(p)(1+f(p))\right]\right\}$$

$$\frac{\Lambda\Lambda_s}{\alpha_s} \equiv -\int_0^\infty dp\, p^2 \frac{df}{dp}$$

$$\frac{\Lambda\Lambda_s^2}{\alpha_s^2} \equiv \int_0^\infty dp\, p^2 f(1+f)$$

Note: when $f \sim 1/\alpha_s$ all dependence on coupling disappears

Note fixed point for BE distribution with $T = \Lambda_s/\alpha_s$

Then, $T \sim \Lambda \sim \Lambda_s/\alpha_s$ (thermalization condition)

Simple estimates

Momentum integrals dominated by hard scale

$$n_{\mathrm{g}} \sim \frac{1}{\alpha_{\mathrm{s}}}\Lambda^2\Lambda_s \qquad \epsilon_{\mathrm{g}} \sim \frac{1}{\alpha_{\mathrm{s}}}\Lambda_{\mathrm{s}}\Lambda^3 \qquad \frac{\epsilon_g}{n_g} \sim \Lambda$$

$$n = n_{\mathrm{c}} + n_{\mathrm{g}}$$

$$\epsilon_{\mathrm{c}} \sim n_{\mathrm{c}}\, m \sim n_c \sqrt{\Lambda\Lambda_s}$$

$$m^2 \sim \alpha_{\mathrm{s}} \int dp\, p^2 \frac{df(p)}{d\omega_p} \sim \Lambda\Lambda_{\mathrm{s}}$$

From transport equation

$$t_{\mathrm{scat}} = \frac{\Lambda}{\Lambda_{\mathrm{s}}^2},$$

Note: the collision time is independent of α_s

Thermalization

Time dependence fixed by 2 conditions

$$\Lambda_s \Lambda^3 \sim \text{constant} \qquad t_{\text{scat}} \sim \frac{\Lambda}{\Lambda_s^2} \sim t$$

Then

$$\Lambda_{\text{s}} \sim Q_s \left(\frac{t_0}{t}\right)^{\frac{3}{7}} \qquad \Lambda \sim Q_s \left(\frac{t}{t_0}\right)^{\frac{1}{7}}$$

$$n_g \sim n_0 \left(\frac{t_o}{t}\right)^{1/7} \qquad m \sim Q_{\text{s}}(t_0/t)^{1/7} \qquad \frac{\epsilon_{\text{c}}}{\epsilon_{\text{g}}} \sim \left(\frac{t_0}{t}\right)^{1/7}$$

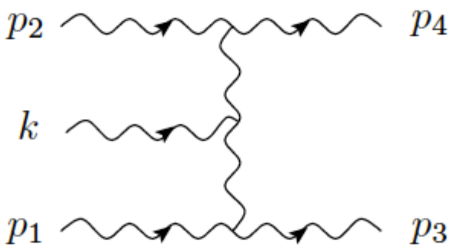
$$s \sim \Lambda^3 \sim Q_{\text{s}}^3 (t/t_0)^{3/7}$$

At thermalization $(\Lambda_s \sim \alpha_s \Lambda)$

$$t_{\text{th}} \sim \frac{1}{Q_{\text{s}}} \left(\frac{1}{\alpha_{\text{s}}}\right)^{\frac{7}{4}} \qquad s \sim Q_{\text{s}}^3/\alpha_{\text{s}}^{3/4} \sim T^3$$

Inelastic processes

The rate of inelastic and elastic processes are comparable



$$\frac{1}{t_{scat}} \sim \alpha_s^{n+m-2} \left(\frac{\Lambda_s}{\alpha_s}\right)^{n+m-2} \left(\frac{1}{m^2}\right)^{n+m-4} \Lambda^{n+m-5}$$

$$m^2 \sim \Lambda_s \Lambda$$

$$t_{\rm scat} = \frac{\Lambda}{\Lambda_{\rm s}^2},$$

Recall that there is no 'equilibrium' condensate with inelastic processes

However the formation of a transient condensate is possible. Whether it occurs or not is an interesting, difficult, question.

Effect of the longitudinal expansion

Longitudinal expansion

Simple (boost invariant) expansion

$$\partial_t f - \frac{p_z}{t}\partial_{p_z} f = \left.\frac{df}{dt}\right|_{p_z t} = C[f] \qquad \partial_t \epsilon + \frac{\epsilon + P_L}{t} = 0$$

Note: mean field terms have been dropped. But they are important. E.g. instabilities may lead/maintain momentum isotropy. Instead, ASSUME

$$P_L = \delta\,\epsilon \qquad 0 < \delta < 1/3$$

Then

$$\epsilon_g(t) \sim \epsilon(t_0)\left(\frac{t_0}{t}\right)^{1+\delta} \qquad \Lambda_s \sim Q_s\left(\frac{t_0}{t}\right)^{(4+\delta)/7}, \qquad \Lambda \sim Q_s\left(\frac{t_0}{t}\right)^{(1+2\delta)/7}$$

and

$$\left(\frac{t_{th}}{t_0}\right) \sim \left(\frac{1}{\alpha_s}\right)^{\frac{7}{3-\delta}}$$

Bose-Einstein condensation....

or not ??

With particle number conserved,

$$\partial_t n + \frac{n}{t} = 0\,, \qquad n = n_0\left(\frac{t_0}{t}\right) \sim \frac{Q_s^2}{\alpha_s}\frac{1}{t}.$$

$$n\epsilon^{-3/4} \sim \left(\frac{t_0}{t}\right)^{1/4}\left(\frac{t_0}{t}\right)^{-3\delta/4}$$

$$n_c \sim \frac{Q_s^3}{\alpha_s}\left(\frac{t_0}{t}\right)\left[1 - \left(\frac{t_0}{t}\right)^{(-1+5\delta)/7}\right]$$

If inelastic processes important, situation unclear....

Summary

The Quark-Gluon plasma formed in the early stages of heavy ion collisions is strongly interacting with itself up to parametrically late times when the system thermalizes.

A transient BEC may form on the way to thermalization