Infrared fixed point for many flavor SU(N) gauge theory

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To appear arXiv 1108:xxxx

Renormalization Group Approach from Ultra Cold Atoms to the Hot QGP
2011/08/25@ YITP, Kyoto

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Numerical simulation was carried out on
NEC SX-8 and SR16000 in YITP, Kyoto University
NEC SX-8 in RCNP, Osaka University
SR11000 and BlueGene in KEK
100 GPU in Taiwan

Introduction

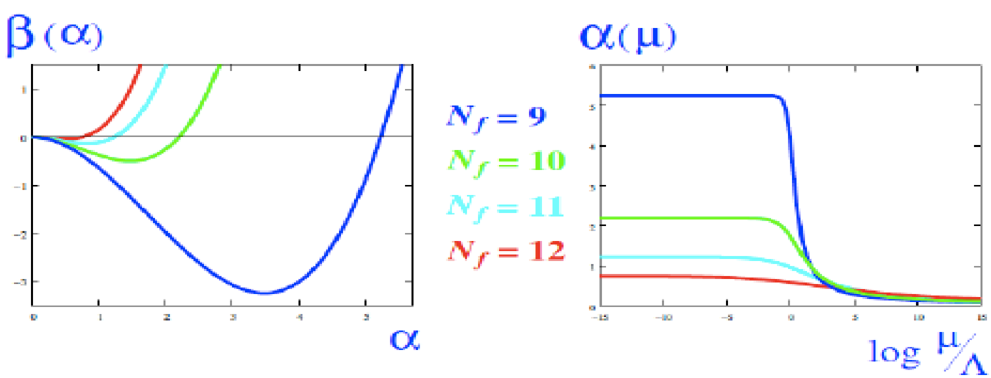
Summary of our work

- We study SU(3) gauge theory coupled with Nf=12 fundamental fermions
- Measure the running coupling constant in this theory and search for an IR fixed point
- Derive the universal quantities around the fixed point

Introduction

In the case of SU(3) coupled with fundamental fermions

- Two-loop running coupling : $\mu\frac{d}{d\mu}\alpha(\mu) \;=\; \beta(\alpha) \;=\; -\,b\;\alpha^2(\mu)\;-c\;\alpha^3(\mu)$



■perturbation (MS bar scheme)

	2-loop	3-loop	4-loop
(alpha)	0.754	0.435	0.470
(g^2)	9.42	5.47	5.90

T.A. Ryttov and R. Shrock, Phys. Rev. D83, 056011 (2011)

■S-D eq. with large Nc

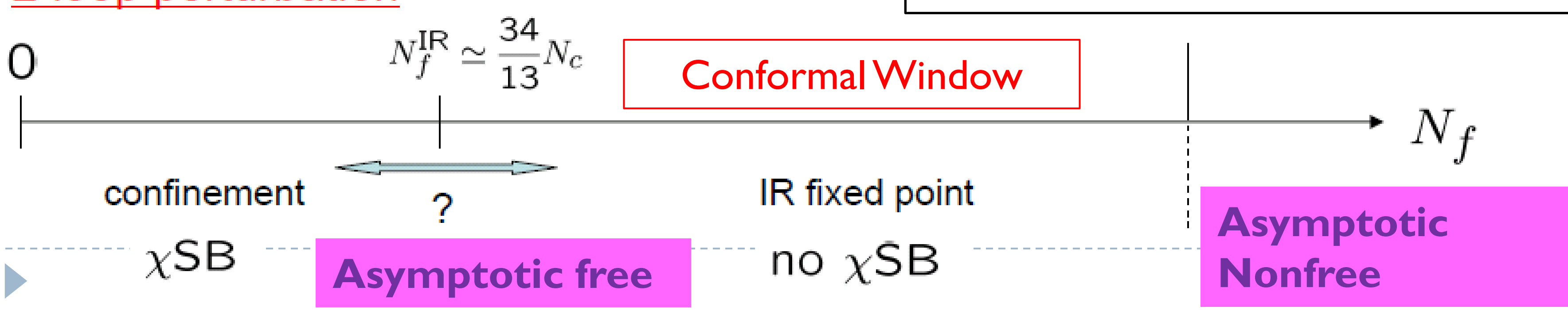
Nf > 11.9, there is IR fixed point

■Exact RG

$$N_f^{cr} = 10.0^{+1.6}_{-0.7}$$

H.Gies and J.Jaeckel, Eur.Phys.J.C46:433-438,2006

2-loop perturbation

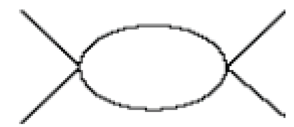


Higgs sector in the Standard Model Lagrangian

$$\mathcal{L}_H \sim \frac{1}{2} D_\mu \phi D^\mu \phi^\dagger + \frac{\lambda}{4}(\phi\phi^\dagger - v^2)^2$$

Problem with a fundamental Higgs boson

- No fundamental scalars observed in nature
- Hierarchy problem (need fine-tuning to cancel a quadratic divergence)
- Triviality problem



$$\beta(\lambda) = \frac{3\lambda^2}{2\pi^2} > 0$$

- No interaction at low energy
- Running coupling constant diverges at a finite energy
- Cuttoff theory?

Candidates for the origin of Higgs sector

- Supersymmetry
- Extra dimension
- Walking techni-color
- Fourth generation

.....

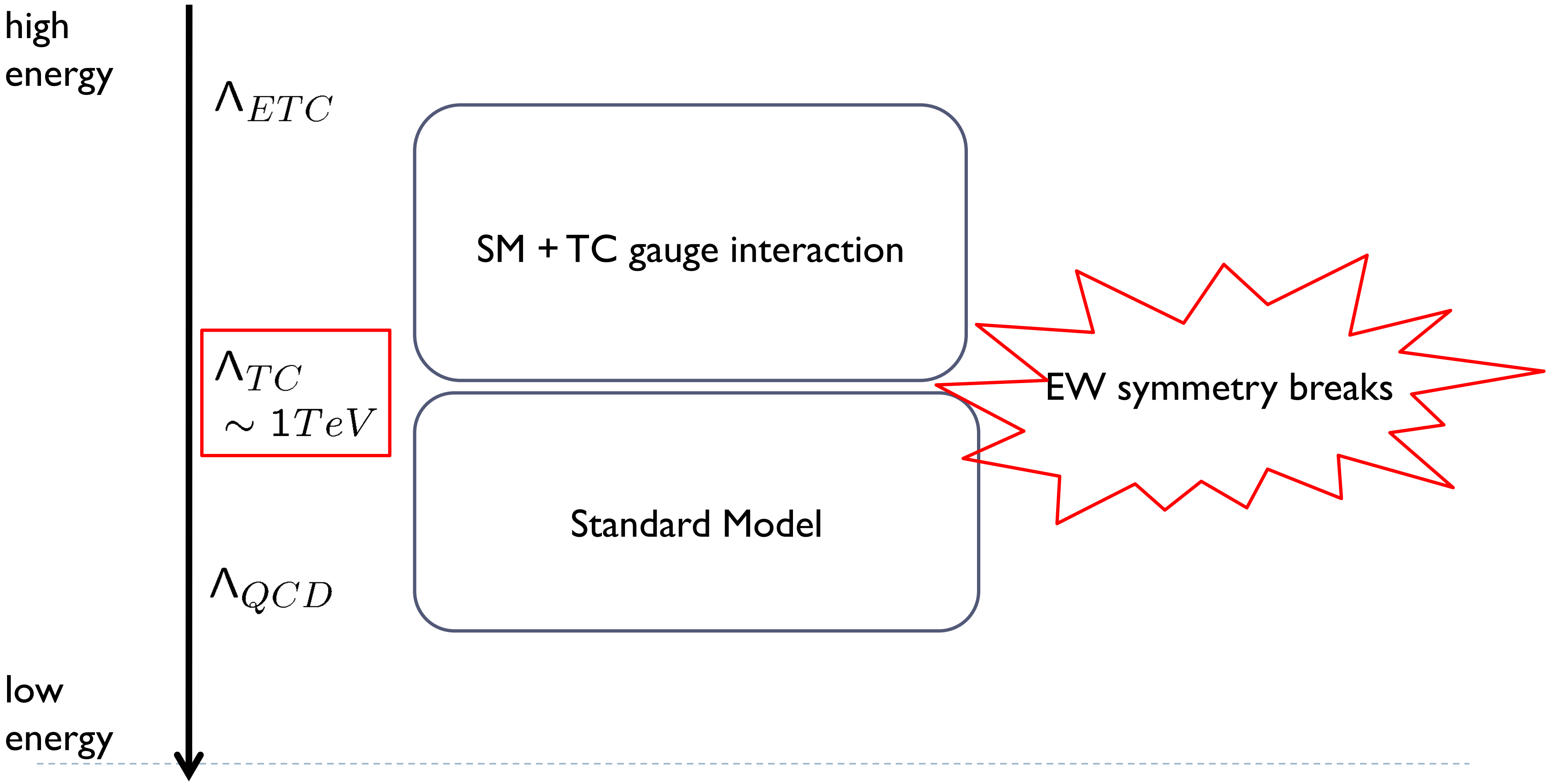
Walking techni-color (TC)

cutoff theory at Λ_{ETC} , (but it is better UV complete theory)

Introduce a new gauge interaction (ex. SU(2), SU(3),SO(N)....)

Add techni-fermions carrying techni-color charge

(adjoint rep, fundamental rep,.....)



Conditions which techni-color gauge interaction should be satisfied

- TC coupling should be large at TC scale
- Λ_{ETC} is larger than 10^3 TeV
- beta fn. has (nearly) zero point at a large TC coupling

Constraints for TC coupling

high energy

Λ_{ETC} $\boxed{g^2_{QCD} \ll 1}$ $\boxed{g^2_{TC} \gg g_{QCD}}$

Λ_{TC}

techni-fermion should be condensed (condensate gives quark mass and seed of EW breaking)

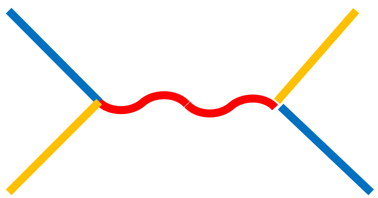
$g^2_{QCD} \leq 1$

Λ_{QCD}

low energy

Techni-color gauge theory is strongly coupled at least TC scale

Constraints for ETC scale



SM+TC theory is Wilsonian effective action.
There are some interaction terms for quark(lepton) (ψ) and techni-fermions (Ψ)

high energy

Λ_{ETC}

4-quark interaction

$$\frac{c_1}{\Lambda_{ETC}^2}(\bar{\psi}\gamma\psi)(\bar{\psi}\gamma\psi)$$

2-quark and 2-techni-fermion

$$\frac{c_2}{\Lambda_{ETC}^2}(\bar{\Psi}\gamma\psi)(\bar{\psi}\gamma\Psi)$$

Λ_{TC}

QCD coupling does not large, then we can neglect the running effects for 4-quark (lepton) interaction

Λ_{QCD}

bound from FCNC ($D^0-\bar{D}^0$) gives a lower limit on ETC scale

$$\left|\frac{c_D}{\Lambda_{ETC}^2}\right| < 7.2\times 10^{-13}\ (\mathrm{GeV}^{-2}) \Rightarrow \boxed{\Lambda_{ETC} \geq 10^3\ (\mathrm{TeV})}$$

low energy

UFit Collaboration: JHEP 0803,049 (2008)

Constraints for quark mass term

high energy

$\Lambda_{ETC} \sim 10^3 TeV$

$\Lambda_{TC} \sim 1TeV$

Λ_{QCD}

low energy

2-quark and 2-techni-fermion

$$\frac{c_2}{\Lambda_{ETC}^2}\langle\bar{\Psi}\Psi\rangle_{ETC}(\bar{\psi}\psi)$$

$$\frac{c_2}{\Lambda_{ETC}^2}\langle\bar{\Psi}\Psi\rangle_{TC} exp[\int_{\Lambda_{TC}}^{\Lambda_{ETC}}\frac{d\mu}{\mu}\gamma(\mu)](\bar{\psi}\psi)$$

γ: anomalous dim. of $\langle\bar{\Psi}\Psi\rangle$

$$\begin{aligned} M_q &\sim \frac{\langle\bar{\Psi}\Psi\rangle_{ETC}}{\Lambda_{ETC}^2} \\ &= \frac{\langle\bar{\Psi}\Psi\rangle_{TC}}{\Lambda_{ETC}^2} exp[\int dg_{TC}^2 \frac{\gamma(g_{TC})}{\beta(g_{TC})}] \end{aligned}$$

quark mass should be enhanced → large coupling but beta fn~0

$$\begin{aligned} M_q &\sim \frac{1}{\Lambda_{ETC}^2}\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma*}\langle\bar{\Psi}\Psi\rangle_{ETC} \\ &\sim 1\text{GeV}(\gamma* = 1) \end{aligned}$$

Let's find such theory which has an fixed point with large coupling

Recent Study of Lattice groups

Program at Lattice 2011

There are two plenary talks, more than18 parallel session talks and several poster session talks concerning with ``conformal window"

J Kuti	Twelve flavors with three colors and two flavors with six colors below the conformal window
G Fleming	Infrared conformality and lattice simulations
X-Y Jin	Lattice QCD with 12 Degenerate Quark Flavors
K Ogawa	The Infrared behavior of SU(3) Nf=12 gauge theory -about the existence of conformal fixed point-

E Pallante	On the spectrum of many-flavor QCD
Y Aoki	KMI project on many flavor QCD with N_f=12 and 16
K-I Nagai	KMI (Nagoya) project; Many flavor QCD as exploration of the walking behavior with approximate IR fixed point
M Lin	Lattice simulations of SU(3) gauge theory with ten flavors of Dirac fermions
C Schroeder	The Running Coupling and Finite Temperature for Twelve Flavors and Three Colors

D Schaich	S parameter and parity doubling below the conformal window
H Ohki	Study of the infrared behavior in SU(2) gauge theory with eight flavors
T Karavirta	Exploring the conformal window: SU(2) gauge theory on the lattice
G Voronov	Lattice Study of the Extent of the Conformal Window in Two-Color Yang-Mills Theory
M Buchoff	Pion scattering in QCD-like theories below conformality

T DeGrand	Gauge theories with fermions in the two-index symmetric representation
Y Shamir	Renormalized coupling from gluon exchange in the Schrodinger functional
D Sinclair	The chiral phase transition for QCD with sextet quarks
S Sint	Perturbative lattice artefacts in the SF coupling for technicolor-inspired models

SU(3) gauge theory coupled with fundamental fermion

The other gauge theory

The other representation of fermion

The Lattice study for IR behavior of many flavor QCD theory

- Study the running coupling constant
- Measure the universal quantities
- Investigate the phase structure (chiral symmetry)

Running coupling

A definition of nonperturbative renormalized coupling

$$\begin{aligned} \langle O \rangle_{NP} &\equiv Z_O \langle O \rangle_{tree} \\ &= \boxed{Z_O g_0^2} \cdot k \\ &\equiv g_R^2 k \end{aligned}$$

Lattice simulation can calculate the vev of $\langle O \rangle_{NP}$

$$g_R^2 \equiv \langle O \rangle_{NP} / k$$

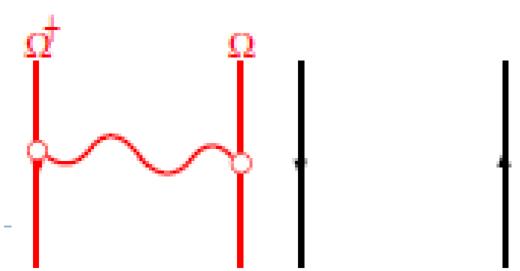
Examples of scheme

- Schrodinger functional scheme
- Wilson loop scheme : Phys.Rev.D80:034507(2009)
- Twisted Polyakov Loop scheme :

no O(a/L) error scheme

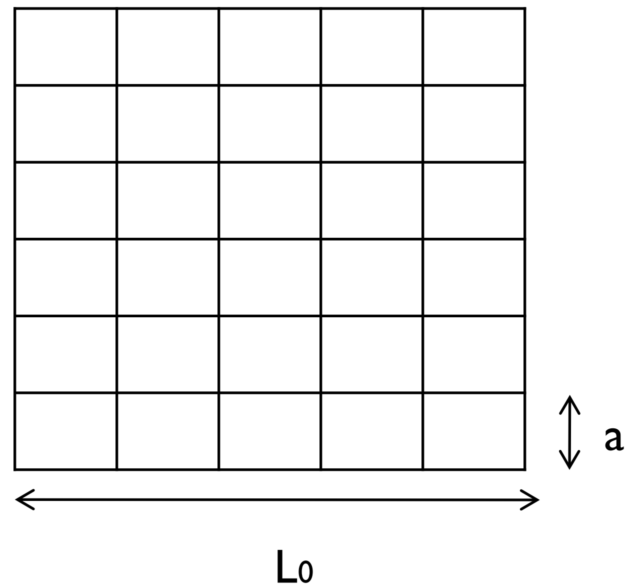
Nonperturbative definition of renormalized coupling in TPL scheme

$$g_{TP}^2 = \frac{1}{k} \frac{\langle \sum_{y,z} P_1(y,z,L/2a) P_1(0,0,0)^* \rangle}{\langle \sum_{x,y} P_3(x,y,L/2a) P_3(0,0,0)^* \rangle}$$



In SU(3) lattice gauge theory,
the VEV of operator *O* depends on

- Lattice size (L_0)
- Lattice spacing (a)
- bare coupling constant $\boxed{\beta = 2N/g_0^2}$



"renormalized coupling" on lattice $g_R^2(a, L_0, g_0)$

How to take the continuum limit

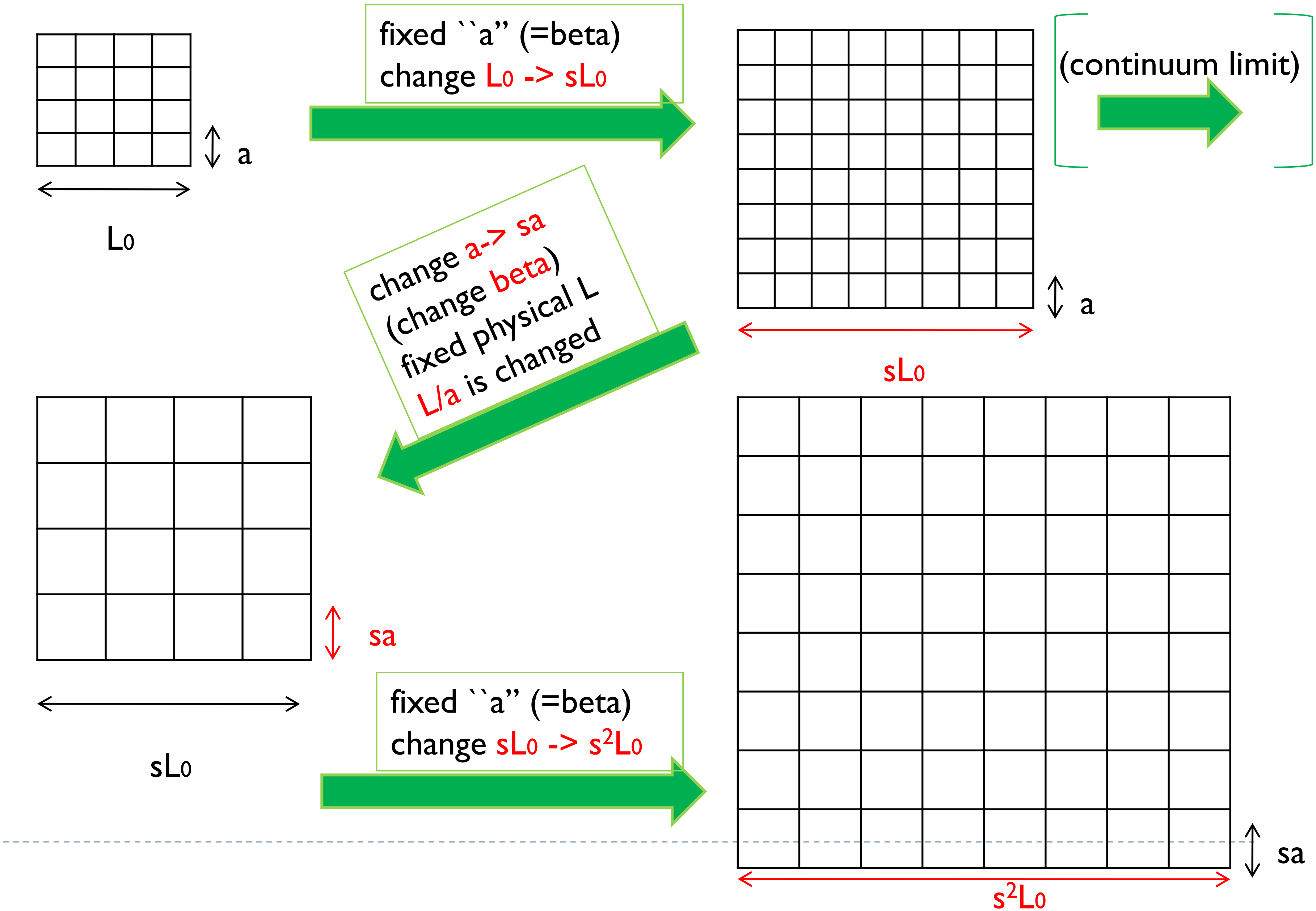
$$g_R^2\left(\frac{1}{L_0}\right) \equiv \lim_{a \to 0} Z_R\left(\frac{a}{L_0}, g_0^2\right)\Big|_{L_0} g_0^2$$

To take the continuum limit, we have to set the scale "a ".
It corresponds to tuning g_0^2 to keep a certain input physical parameter constant.

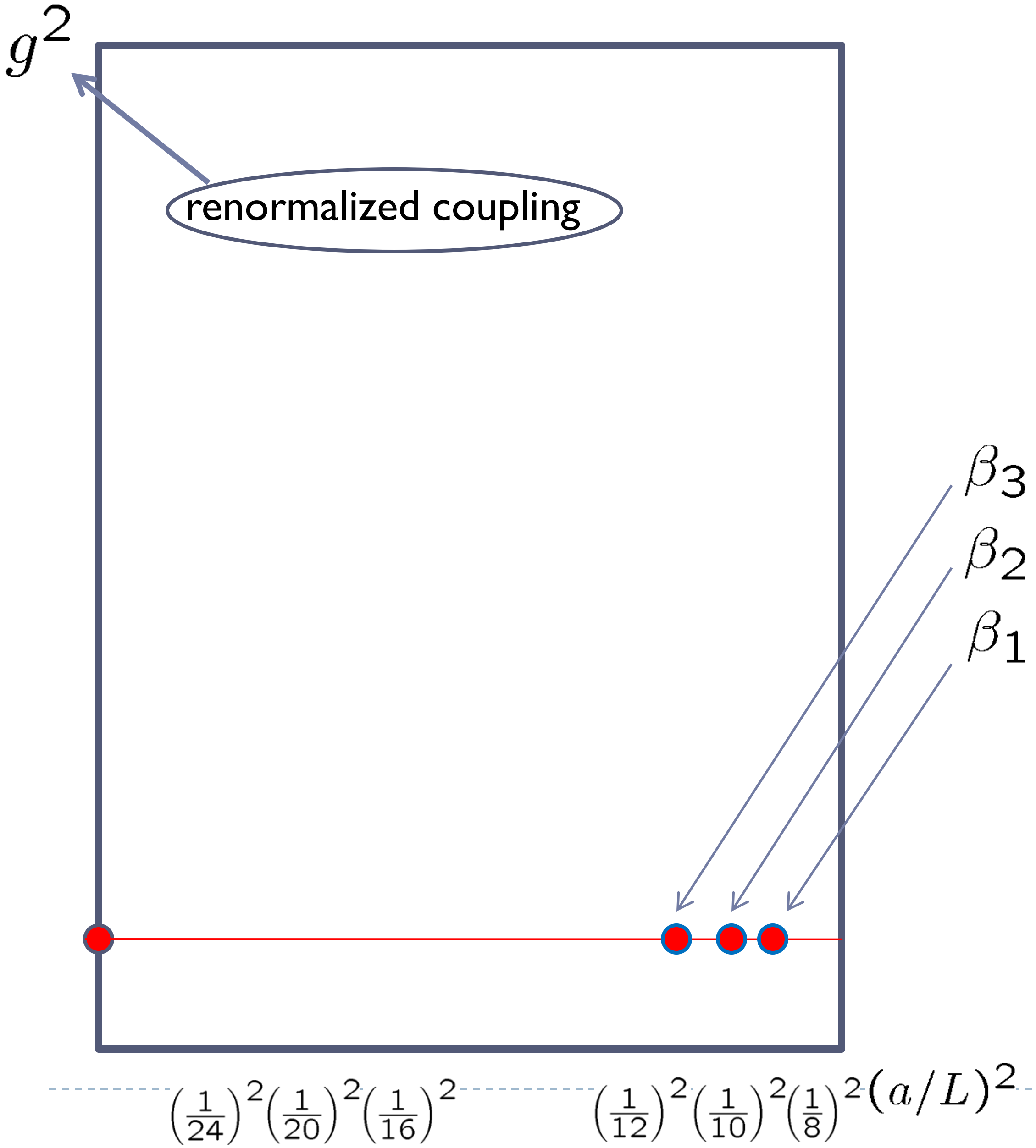
input $g_R^2(1/L_0)$ $\Rightarrow$ output $g_R^2(1/sL_0)$ IR cutoff is moving

s : scaling parameter

Step scaling method



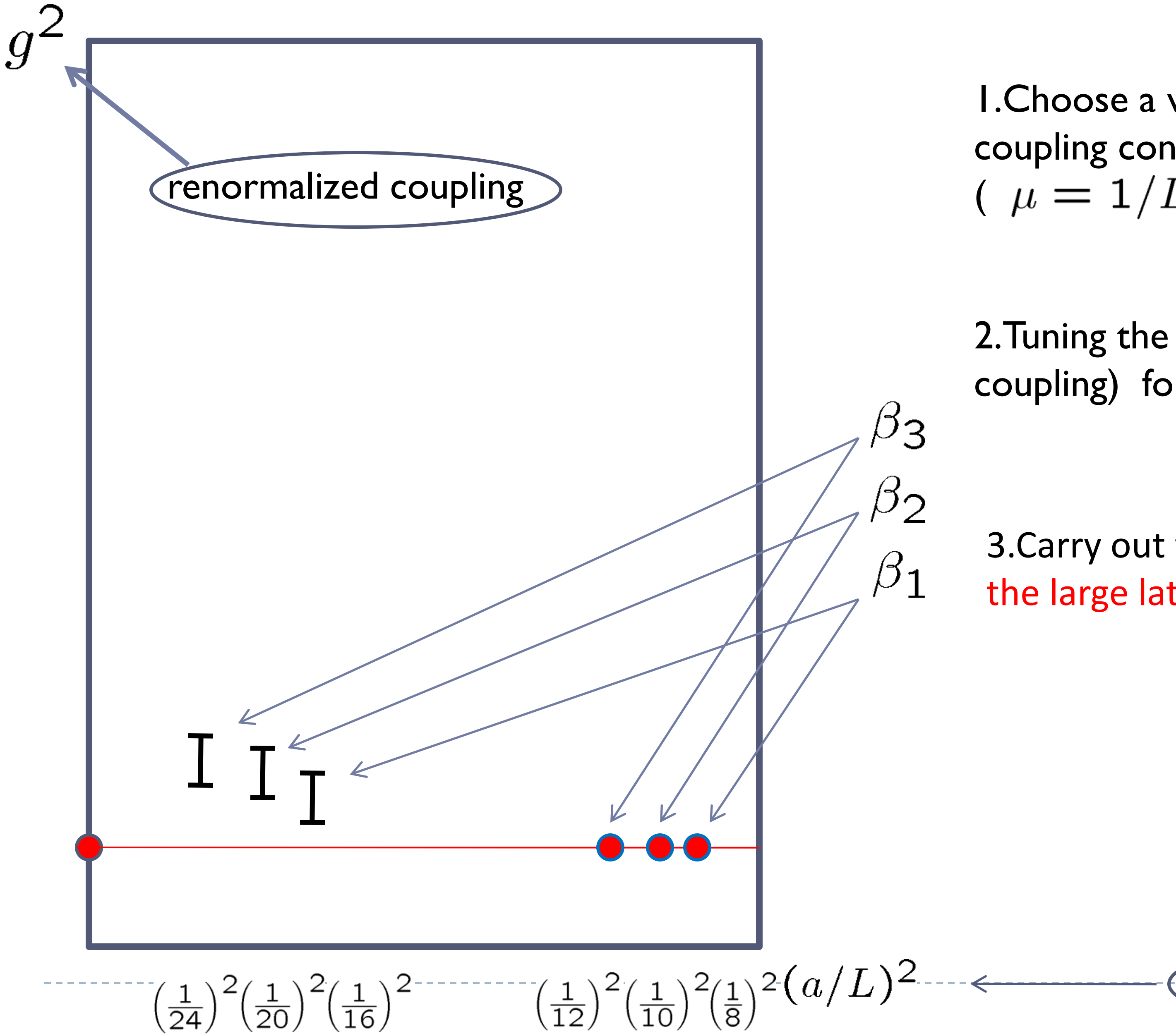
Step scaling method



1.Choose a value of renormalized coupling constant at energy scale ($\mu = 1/L_0$)

2.Tuning the beta (bare coupling) for small lattice size

Step scaling method

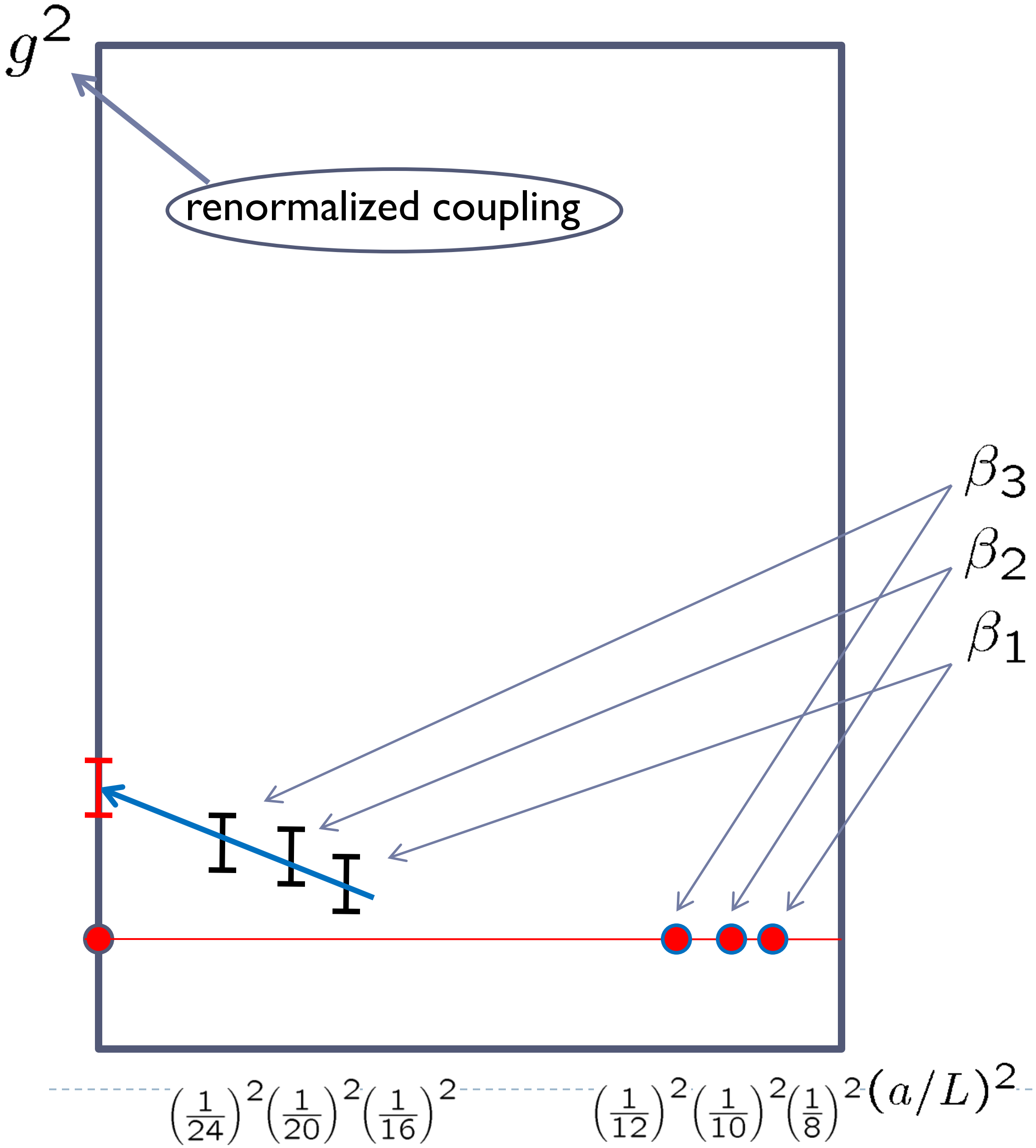


1.Choose a value of renormalized coupling constant at energy scale ($\mu = 1/L_0$)

2.Tuning the beta (bare coupling) for small lattice size

3.Carry out the simulation for the large lattice size

Step scaling method



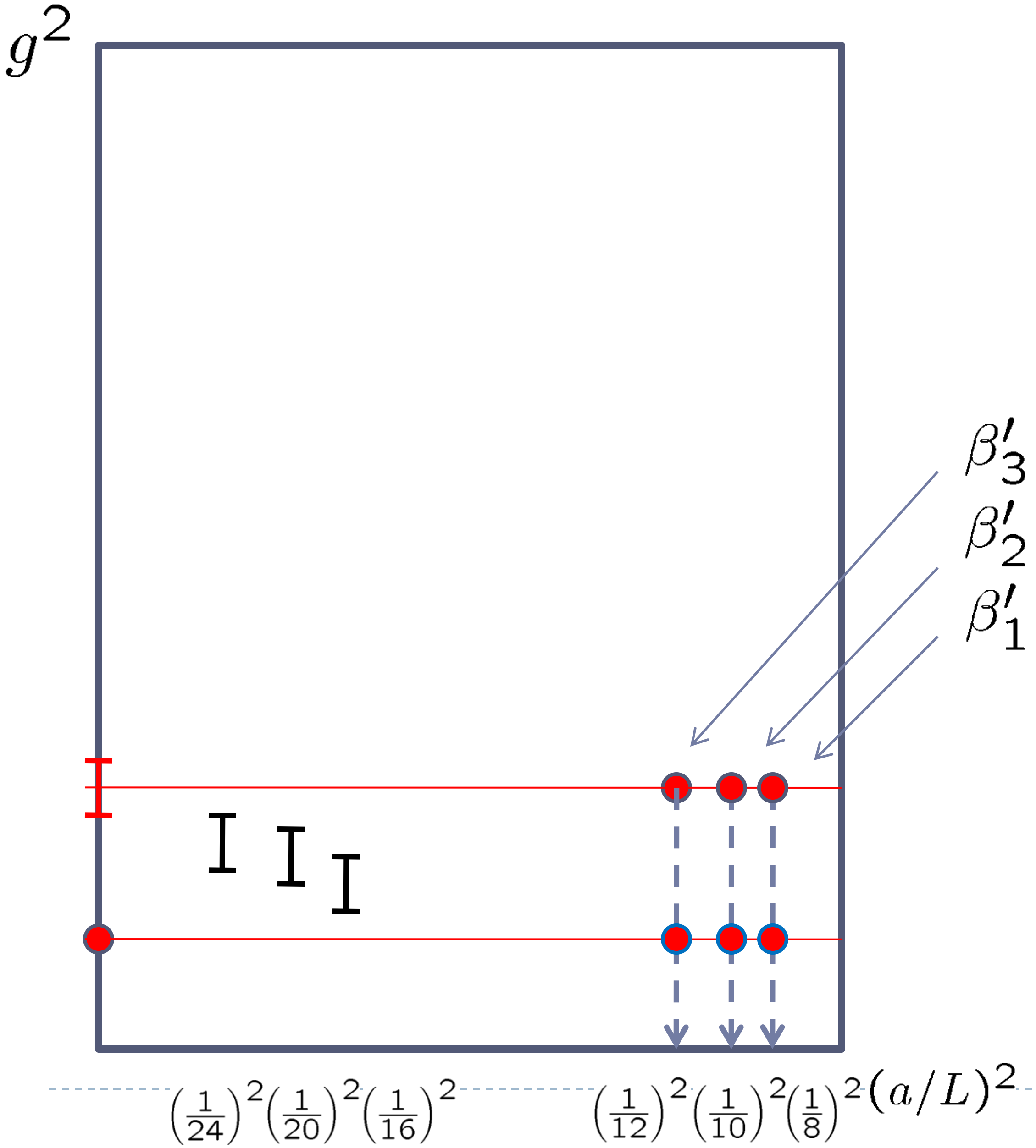
1.Choose a value of renormalized coupling constant at energy scale ($\mu = 1/L_0$)

2.Tuning the beta (bare coupling) for small lattice size

3.Carry out the simulation for the large lattice size

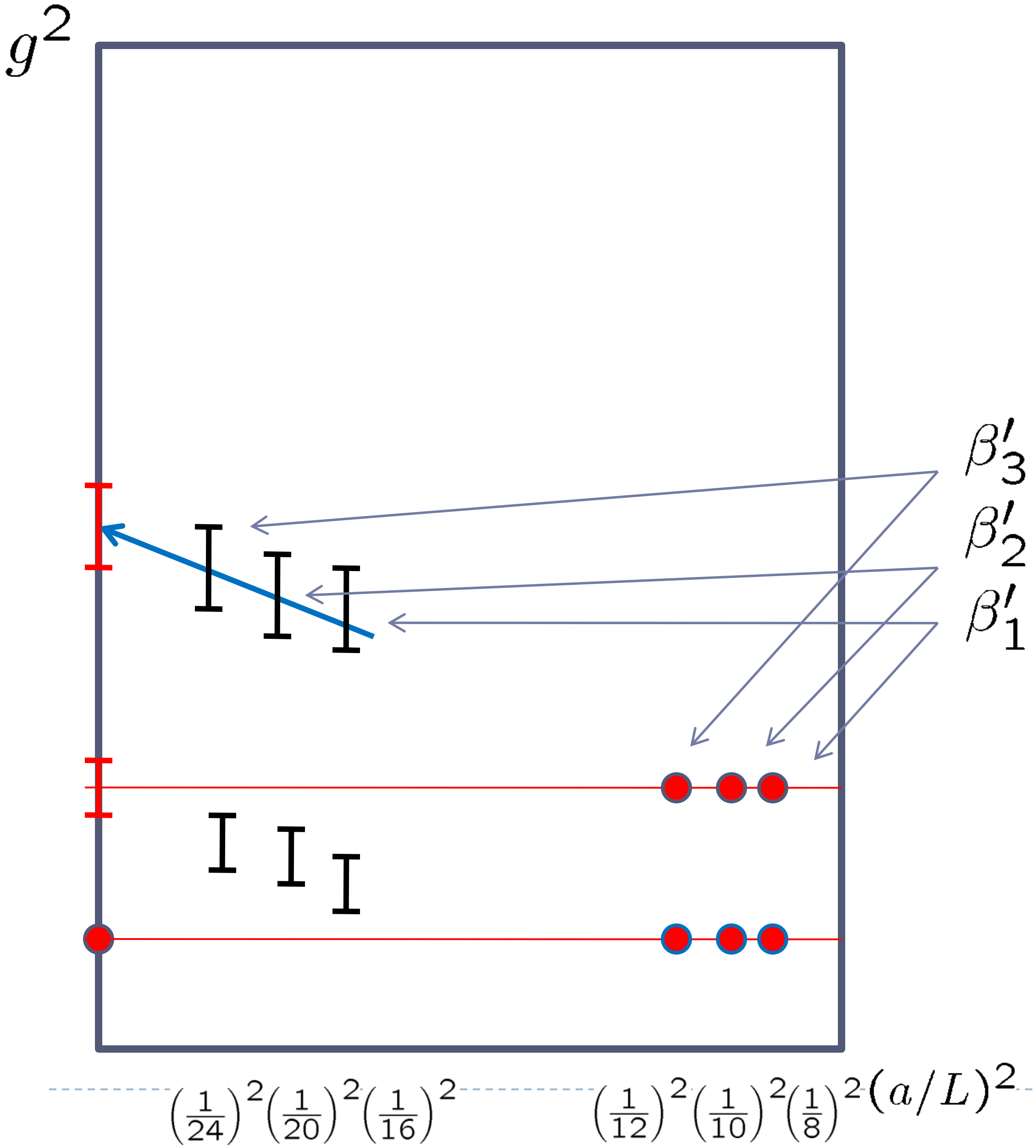
4.Take the continuum limit (energy scale $\mu = 1/2L_0$)

Step scaling method



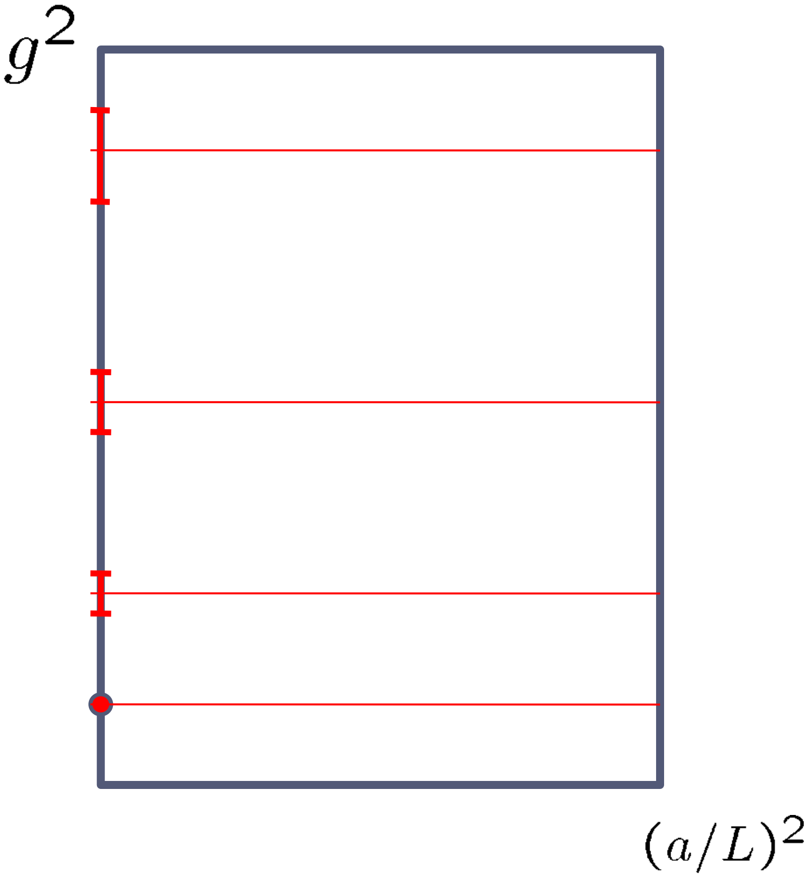
Tune the beta at small lattice size

Step scaling method

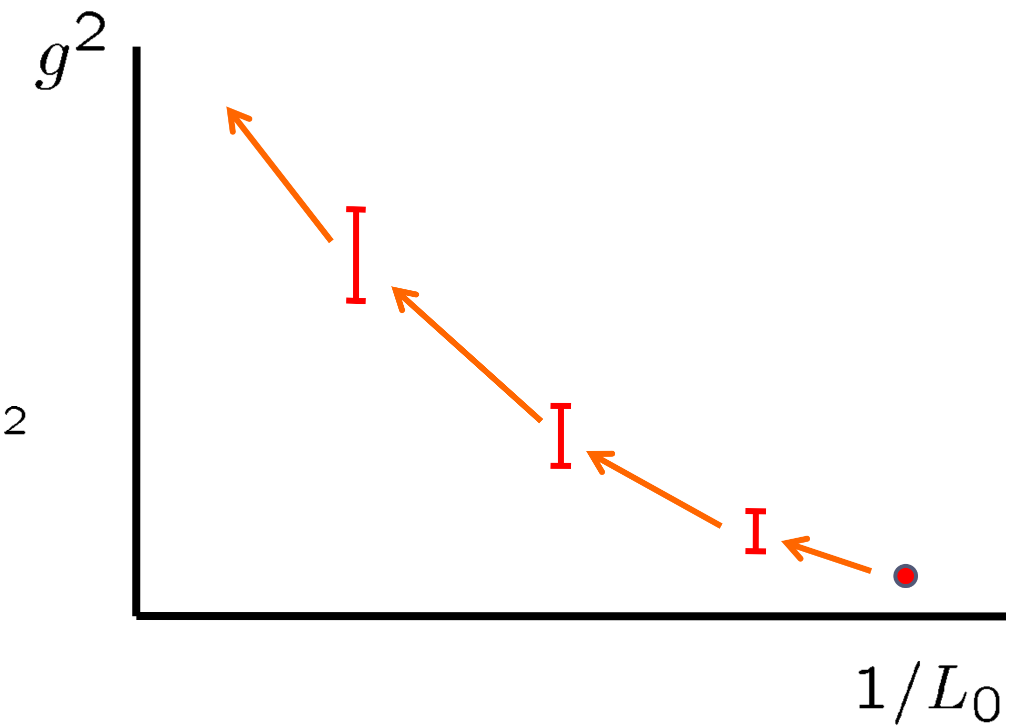


Take the continuum limit
(energy scale $\mu = 1/4L_0$)

Step scaling method

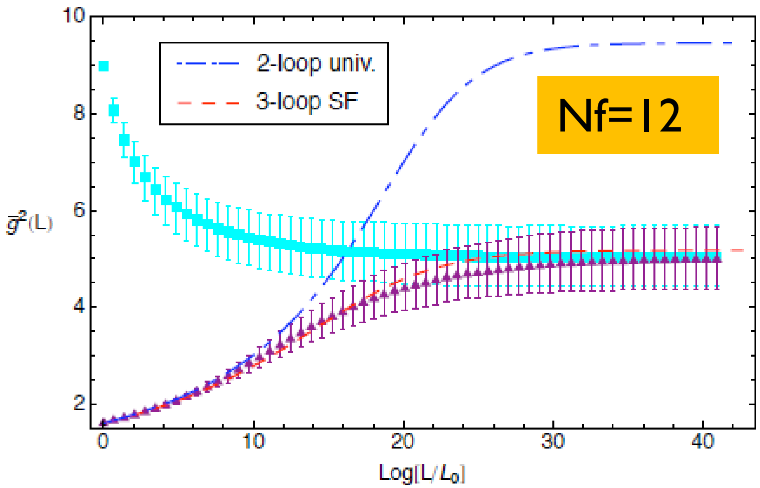


We obtain the scaling of the running coupling.



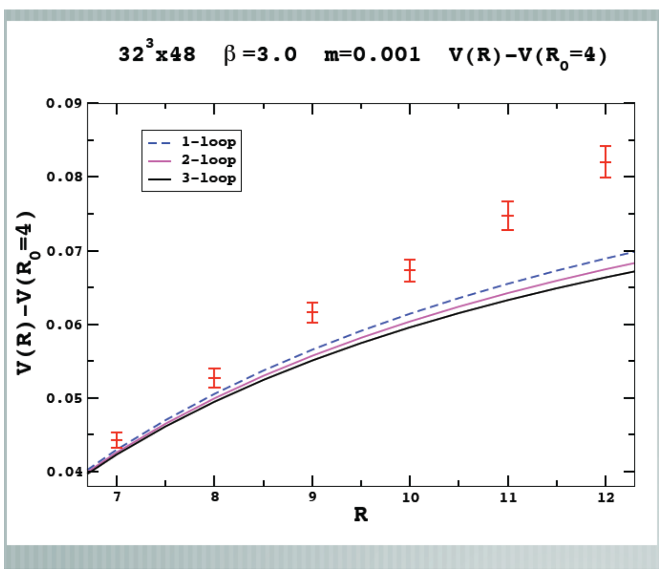
Status of SU(3) Nf=12 theory

Running coupling constant on Lattice



Appelquist' group (SF scheme) ,
Phys.Rev.D79:076010,2009

The running coupling shows a flat region
The value of fixed point coupling g^2~5.
It consists with 3-loop lattice perturbative analysis.

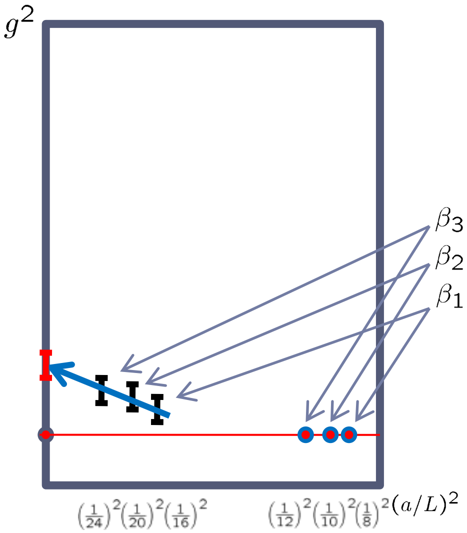


Fodor's group: (potential scheme)
arXiv:0911:2463 [hep-lat], talk at Lattice2010

The renormalized coupling is larger than 1,2,3-loop perturbartive result.

The coupling at low energy region is growing.
(No fixed point.)

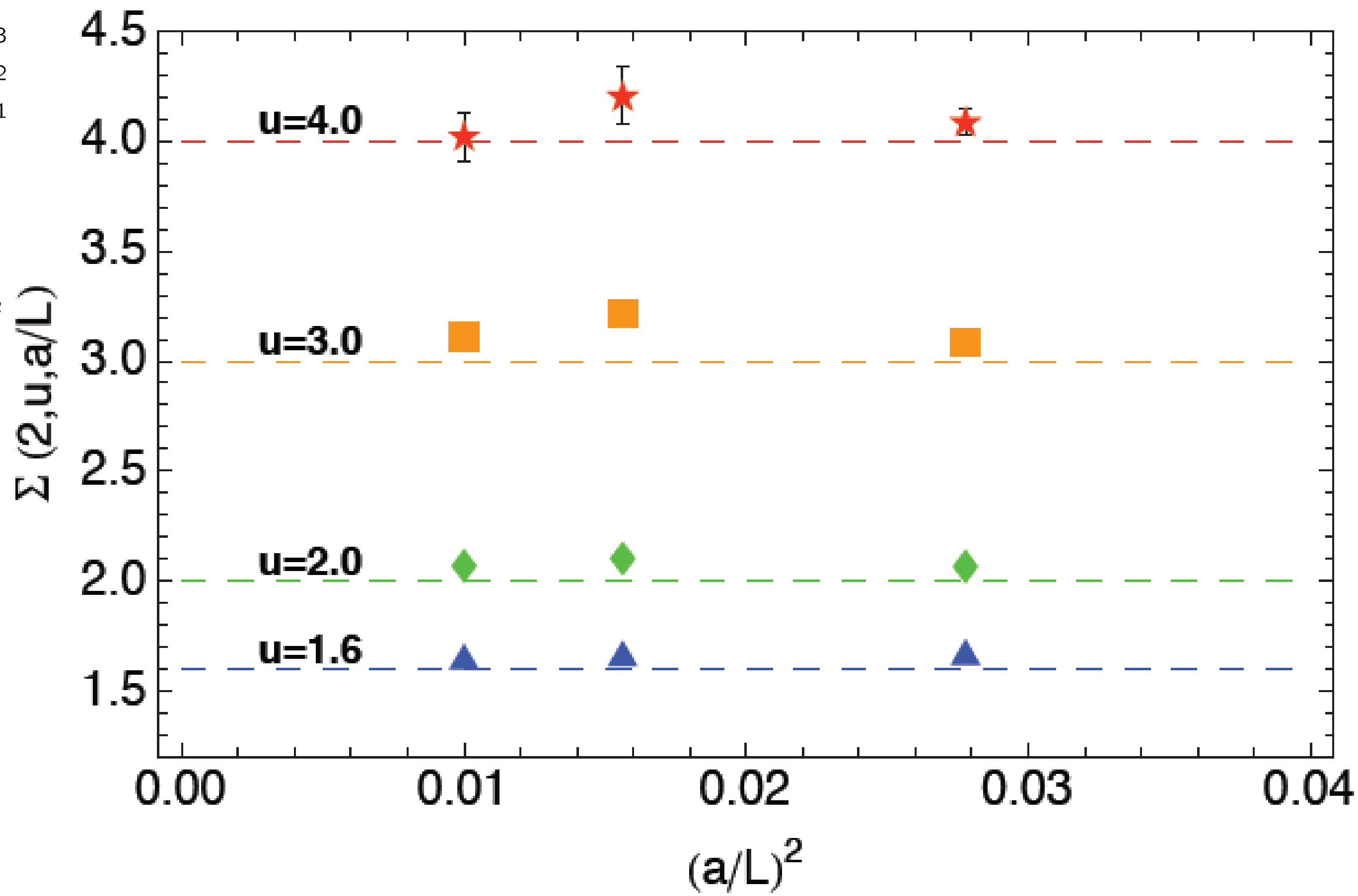
Continuum extrapolation in SF scheme



s=2 step scaling

L=6 -> L=12
L=8 -> L=16
L=10 -> L=20

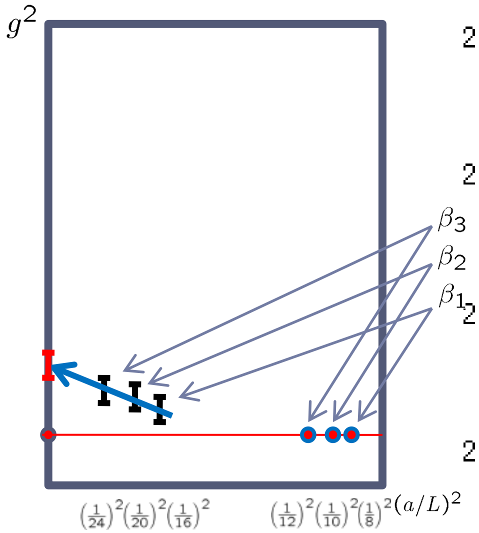
constant extrapolation?



Our result

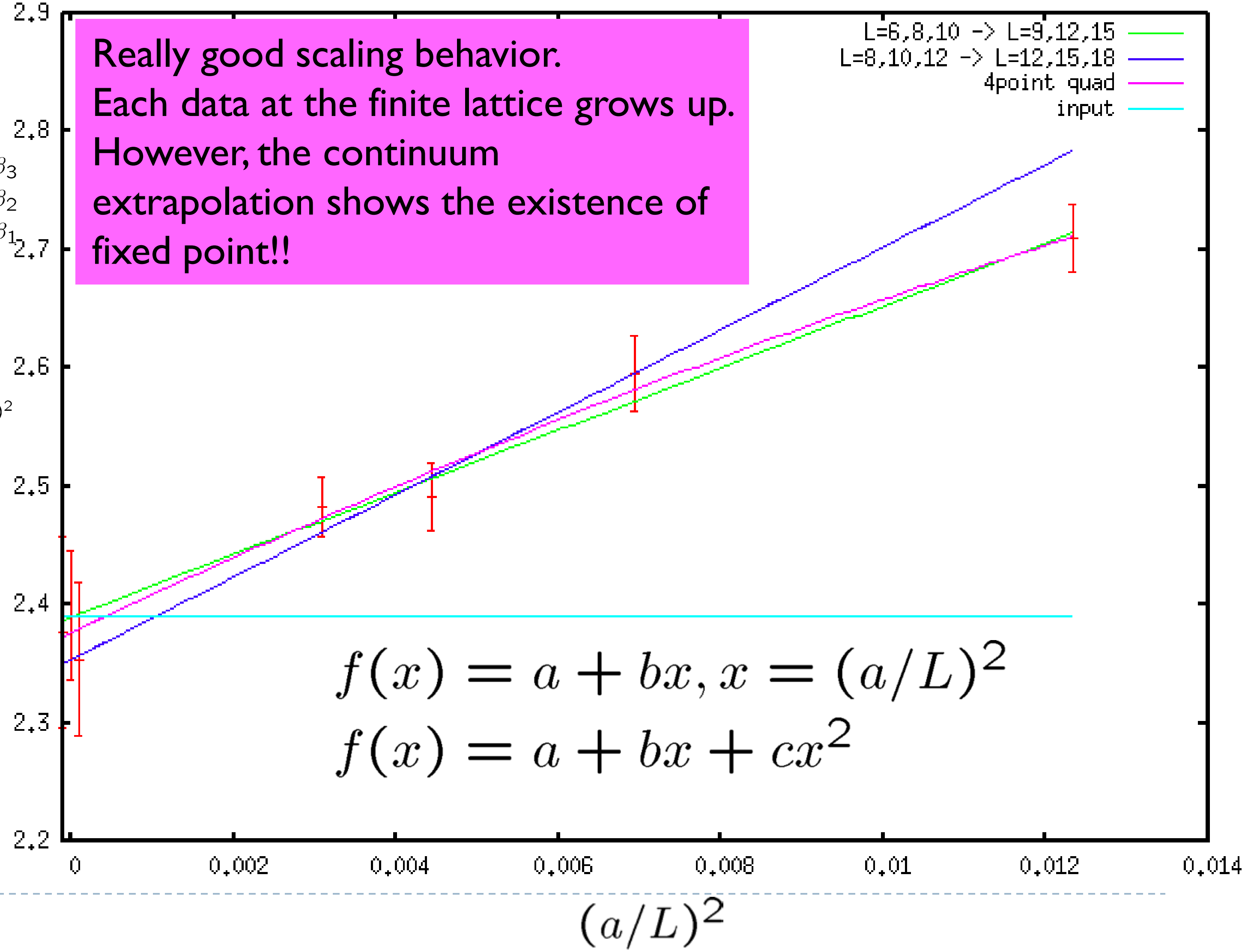
Continuum Extrapolation

(TPL scheme)



s=1.5 step scaling

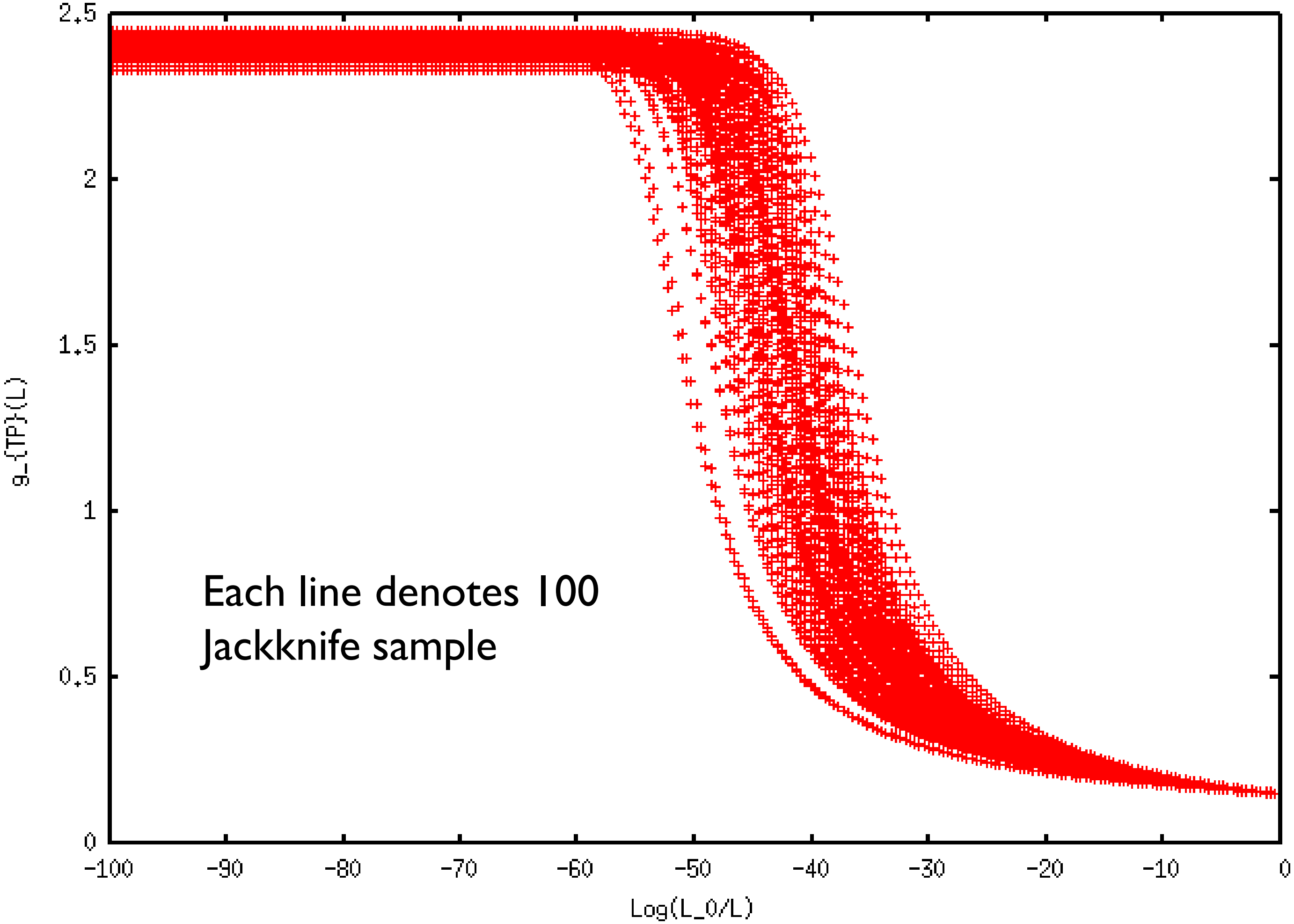
L=6 -> L=9
L=8 -> L=12
L=10 -> L=15
L=12 -> L=18



Really good scaling behavior.
Each data at the finite lattice grows up.
However, the continuum extrapolation shows the existence of fixed point!!

$$f(x) = a + bx, \; x = (a/L)^2$$
$$f(x) = a + bx + cx^2$$

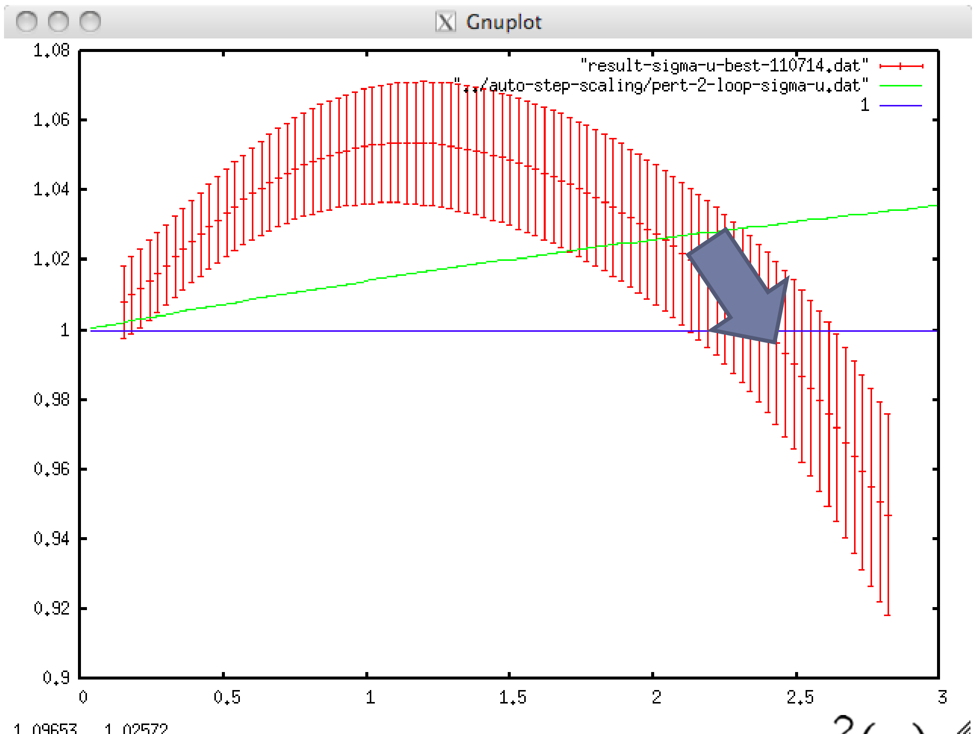
Running coupling constant in TPL scheme



$$g^{2*} = 2.39 \pm 0.20(stat.)$$

Exponent of the beta function

$g^2(\mu/s)/g^2(\mu)$



$g^2(\mu)$

Around the fixed point, the beta function can be approximated by linear function:

$$\beta(g^2) \sim \gamma_g^*(g^{2*} - g^2)$$

Our result:

$$s^{-\gamma_g^*} = 0.82 \pm 0.07$$

$$\gamma_g^* = 0.27 - 0.71$$

SF scheme (Appelquist)

$$\gamma_g^* = 0.13 \pm 0.03$$

2 loop

$$\gamma_g^* = 0.36$$

Mass anomalous dimension
-preliminary results-

New definition of the anomalous dimension using psuedo-scalar correlator

Measure the psuedo-scalar correlator

$$P(t,x) = \bar{\psi}(t,x)\gamma_5\psi(t,x)$$

$$C_P(t) = \sum_x \langle P(t,x)P(0,0)\rangle$$

Z factor

$$P_R(g_0, a/L) \equiv Z_P(g_0, a/L)P(t)$$

$$C_P^{tree}(t) \equiv Z_P(g_0, a/L)^2 C_P(t)$$ at fixed t

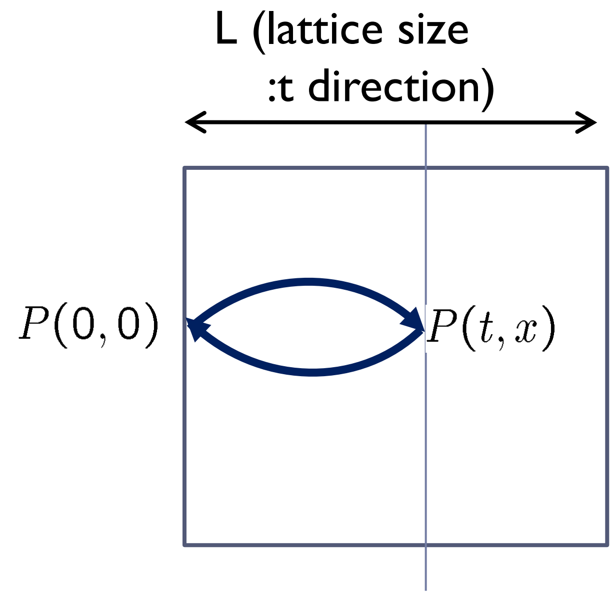
$$\boxed{Z_P(g_0, a/L) \equiv \sqrt{\frac{C_P^{tree}(t)}{C_P(t)}}}$$

scaling function

$$\sigma_P(u,s) = \lim_{a \to 0} \frac{Z_P(g_0, a/sL)}{Z_P(g_0, a/L)}\bigg|_{g^2=u}$$

anomalous dimension at the fixed point

$$\gamma^* = -\frac{log(\sigma_R(u,s))}{log(s)}$$

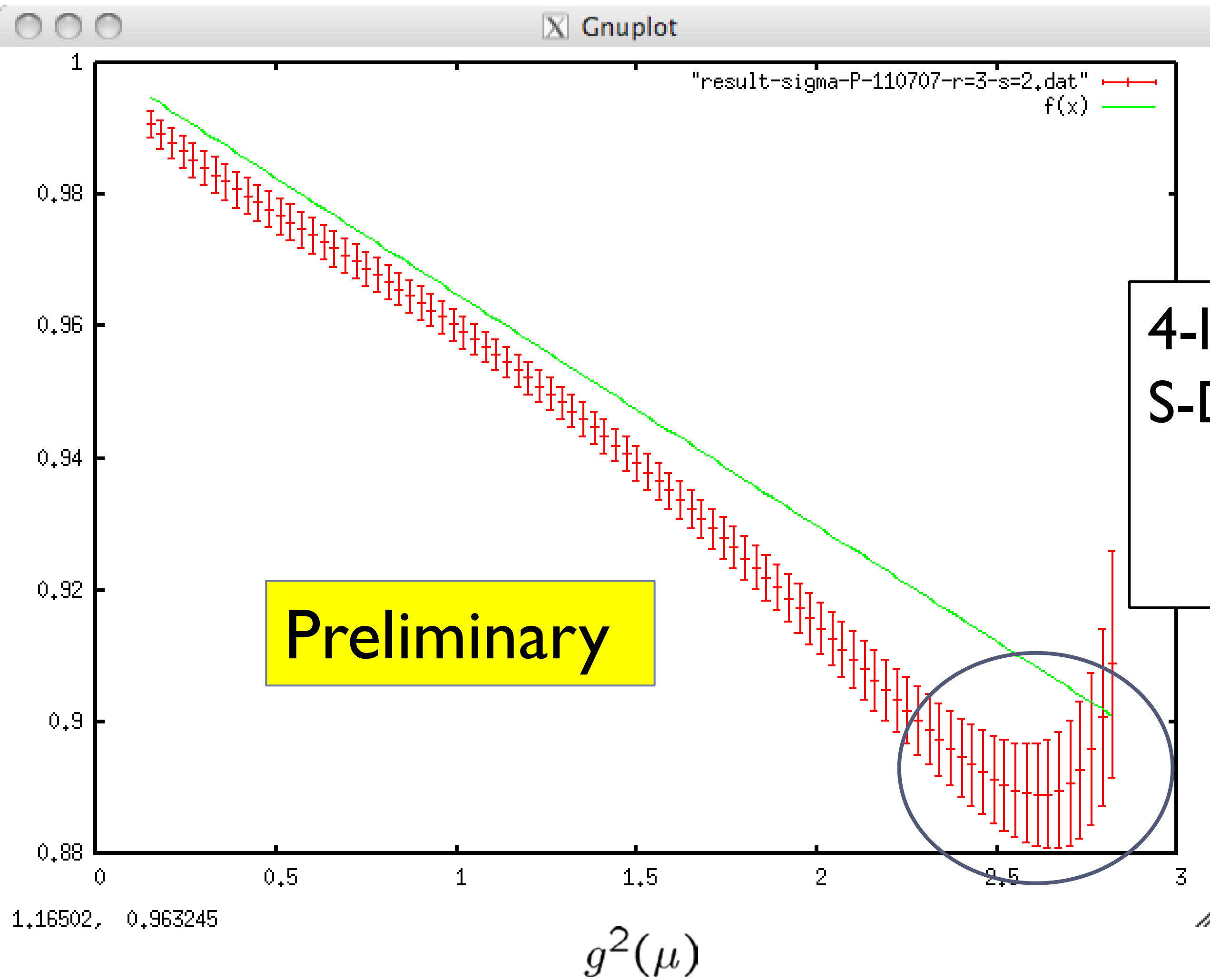


Here, we fix t/L for each lattice size.
Choice of t/L corresponds scheme.

PCAC relation in QCD

$$\partial_\mu (A_R)_\mu = 2mP_R$$

Our result (Mass step scaling fn. ~ Zm factor)



¥gamma*~0.29

4-loop ¥gamma*~0.253
S-D eq. ¥gamma*~1

T.A. Ryttov and R. Shrock, Phys. Rev. D83, 056011 (2011)

Comments:
We introduce m=1$0^{-5}$ to measure the correlator.
We checked the data does not change if we use m=1$0^{-7}$.

The other works

Study IR behavior of Masses for composite operators and chiral condensate

They measured lattice data include a fermion mass .Then, they extrapolated to massless limit using small mass expansion.

- Fodor group (arXiv:1104.3124)

Chiral symmetry broken hypothesis works well rather than conformal hypothesis!

$$\boxed{< \bar{\Psi}\Psi >= (c_0 + c_1 m + c_2 m^2)}$$

- Appelquist group (arXiv:1106.2148)

Conformal hypothesis also works well!

$$\boxed{M_X \sim C_X m^{1/(1+\gamma*)}}$$

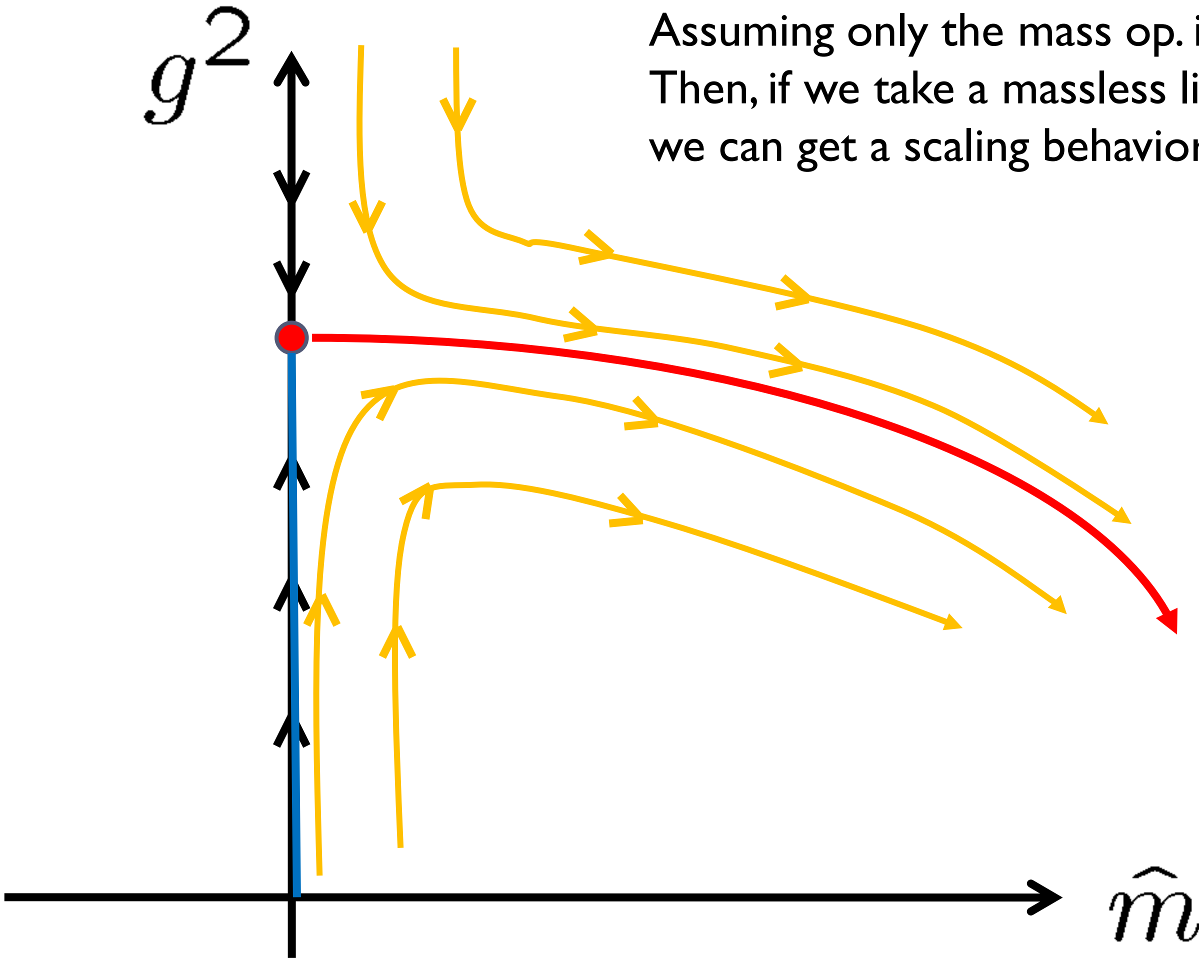
- X-Y. Jin (Columbia U group)

Chiral symmetry is broken in IR simulation

- KMI group

Measure the psedoscalar mass

Mass deformation



Assuming only the mass op. is relevant in the IR.
Then, if we take a massless limit in the low energy region, we can get a scaling behavior around IR fixed point.

Raw Data (arXiv:1104.3124, Fodor et al.)

The simulation input parameters are lattice size and fermion mass.(beta=2.2 =enough low energy)

Appelquist's group used only largest volume data for each m value .

Lattice size	Fermion mass
$24^3 \times 48$	0.035
$32^3 \times 64$	0.020
$40^3 \times 80$	....
$48^3 \times 96$	0.015
	0.010
nearly cont.lim.	massless lim.

Measured masses
"Pion"
"Pion" decay constant
"axial vector"
"scalar"
….
Chiral condensate

Table 1: Measured masses and F_π with the three largest volumes in the $m = 0.01 - 0.02$ range and the largest volume for $m > 0.02$. Asterisks indicate $L_s = 32$ when different from the spatial volume of the second column. M_{pnuc} is the mass of the nucleon's parity partner.

mass	lattice	M_π	F_π	M_{i5}	M_{sc}	M_{ij}	M_{nuc}	M_{pnuc}	M_{Higgs}	M_{rho}	M_{A1}
0.0100	$48^3 \times 96$	0.1647(23)	0.02474(49)	0.1650(13)	0.16437(95)	0.1657(10)	0.3066(69)	0.3051(81)	0.247(13)	0.1992(28)	0.2569(83)
0.0100	$40^3 \times 80$	0.1819(28)	0.02382(39)	0.1842(29)	0.1835(35)	0.1844(44)	0.3553(93)	0.352(16)	0.2143(81)	0.2166(73)	0.237(12)
0.0100	$32^3 \times 64$	0.2195(35)	0.02234(46)	0.2171(31)	0.194(10)	0.195(11)	0.386(16)	0.387(22)	0.2162(53)	0.239(19)	0.246(21)
0.0150	$48^3 \times 96$	0.2140(14)	0.03153(51)	0.2167(16)	0.2165(17)	0.2185(18)	0.3902(67)	0.3881(84)	0.296(13)	0.2506(33)	0.3245(87)
0.0150	$40^3 \times 80$	0.2200(23)	0.03167(53)	0.2210(21)	0.2218(30)	0.2239(34)	0.4095(84)	0.411(10)	0.291(11)	0.2574(36)	0.327(14)
0.0150	$32^3 \times 64$	0.2322(34)	0.03168(64)	0.2319(11)	0.2318(17)	0.2341(16)	0.4387(60)	0.4333(84)	0.2847(33)	0.2699(41)	0.324(16)
0.0200	$40^3 \times 80$	0.2615(17)	0.03934(56)	0.2736(22)*	0.2651(8)	0.2766(42)*	0.4673(62)	0.4699(66)	0.330(17)	0.3049(28)	0.361(32)
0.0250	$32^3 \times 64$	0.3098(18)	0.04762(53)	0.3179(17)	0.3183(18)	0.3231(20)	0.563(12)	0.563(14)	0.4137(88)	0.3683(19)	0.469(14)
0.0275	$24^3 \times 48$	0.3348(29)	0.05218(85)	0.3430(18)	0.3425(25)	0.3471(26)	0.609(21)	0.628(23)	0.460(16)	0.4050(69)	0.523(34)
0.0300	$24^3 \times 48$	0.3576(15)	0.0561(11)	0.3578(15)*	0.3726(29)	0.3790(40)	0.640(12)*	0.633(16)*	0.470(15)	0.4160(26)*	0.5222(90)*
0.0325	$24^3 \times 48$	0.3699(66)	0.0588(15)	0.3790(34)	0.3814(62)	0.3879(62)	0.680(18)	0.686(26)	0.500(21)	0.4481(39)	0.548(31)
0.0350	$24^3 \times 48$	0.3927(17)	0.06422(57)	0.4065(18)	0.4074(19)	0.4149(26)	0.703(28)	0.741(20)	0.538(30)	0.4725(64)	0.669(65)

Conformal hypothesis with mass deformation

The running mass

$$m(\mu) = m(\Lambda)\,\mathrm{exp}(\int_{\mu}^{\Lambda} \frac{\gamma(\mu)}{\mu} d\mu)$$

(if there is a fixed point)

$$= m(\Lambda/\mu)^{\gamma *}$$

We assume the running mass satisfies at some IR scale M $m(M) = M$

Mass of physical state X is set by the scale M. $M_X = m(\Lambda/M_X)^* \simeq C_X m^{1/1+\gamma *}$

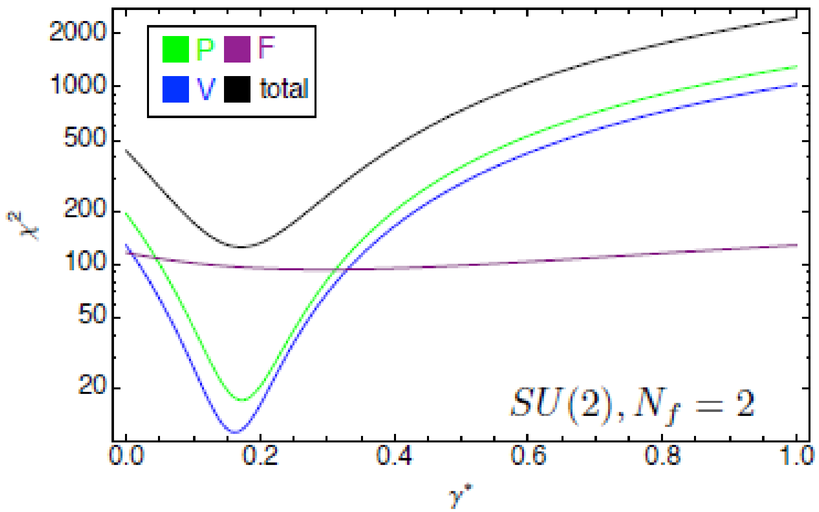
There is a correction term in a small-m expansion: $\boxed{M_X = C_X m^{1/(1+\gamma *)} + D_X m}$

The chiral condensate can be written by $\langle\bar{\psi}\psi\rangle_M \simeq M^3(\Lambda/M)^{\gamma_*} \simeq B_c m^{(3-\gamma *)/(1+\gamma *)}$

The leading term should be added in small-m expansion:

$$\boxed{\langle\bar{\psi}\psi\rangle_M = A_c m + B_c m^{(3-\gamma *)/(1+\gamma *)}}$$

Conformal and broken examples



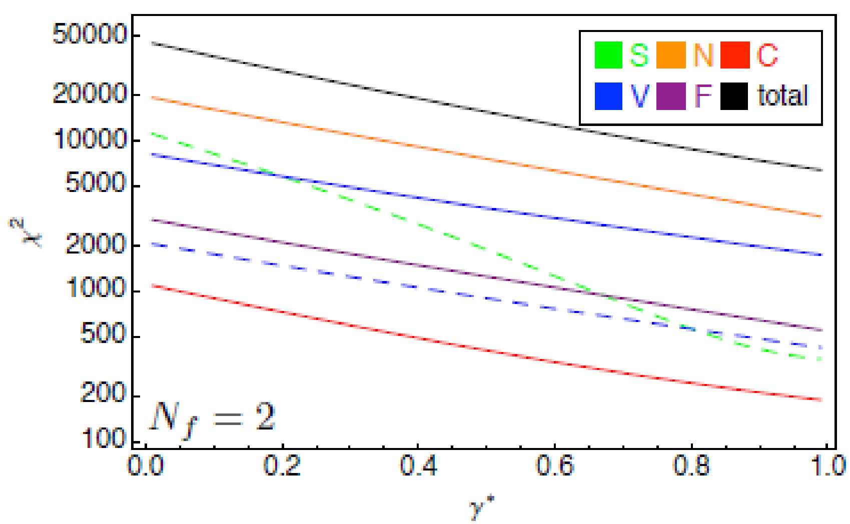
SU(2) Nf=2 ADOJOINT fermions

(widely believed to be conformal in IR.)

Gamma* is free parameter for each fit.
Plot the χ^2 as a function of gamma*.
The best fit value: gamma*=0.17(5).
This value is consistent with a direct measurement.

Bursa et al. Phys.Rev.D81,014505 (2010)

$$0.05 < \gamma* < 0.56$$



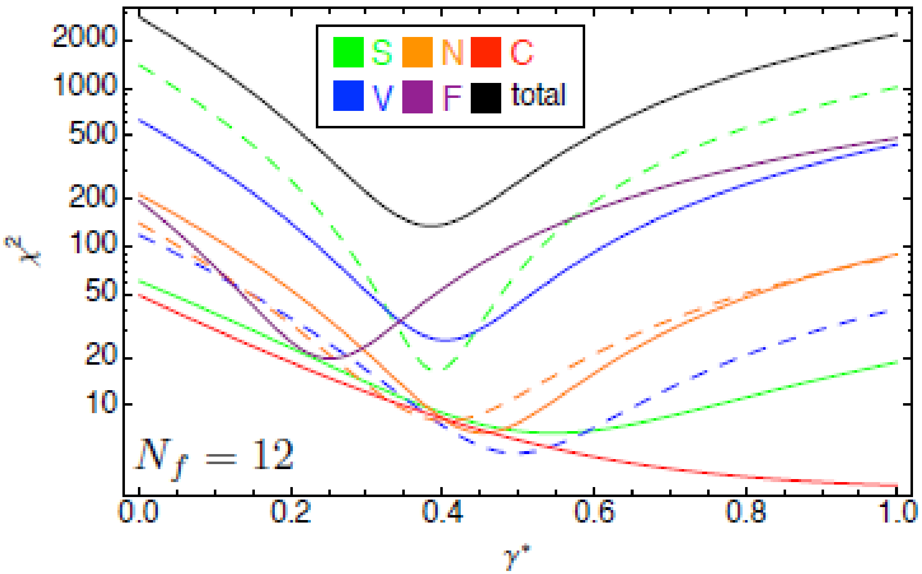
SU(3) Nf=2 fundamental fermions

(known to be in broken phase,
but use the conformal hypothesis fit.)

No clear minimum for any channel.

In the case of SU(3) Nf=12

They fit Fodor's data at largest volume for each m value.
The small volume data may be suffered from a finite volume effect.



$$M_X \sim C_X m^{1/(1+\gamma*)}$$

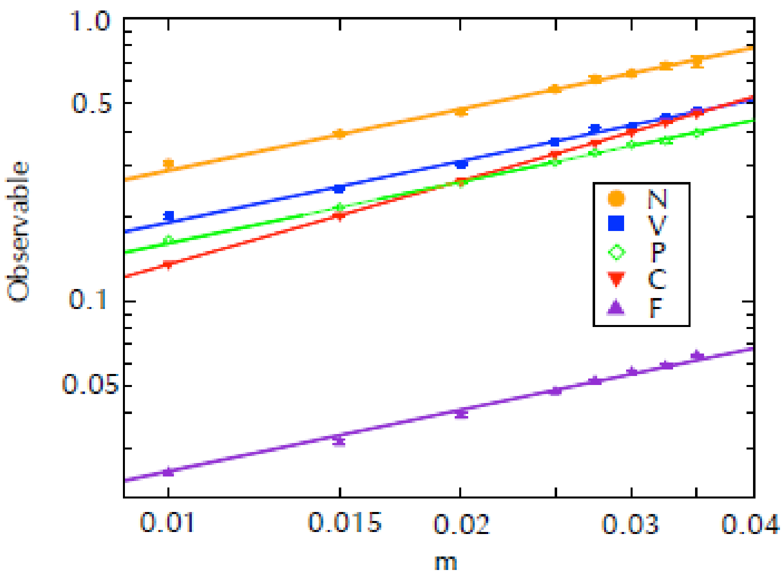
X=pseudoscalar state
vector state
"nucleon" state
"pion" decay constant

$$\langle \bar{\psi}\psi \rangle \sim A_c m + B_c m^{(3-\gamma*)/(1+\gamma*)} + C_c m^{3/(1+\gamma*)}$$

The minimum χ^2 for each mass occur at a similar value of gamma*.
Except for chiral condensate…
B_c and C_c is strongly correlated and A_c is dominated.
(Need more simulation data.)

Fitting the data universal anomalous dim.

They global fit the data using universal anomalous dimension.



$$M_X \sim C_X m^{1/(1+\gamma*)}$$

X=pseudoscalar state
vector state
"nucleon" state
"pion" decay constant

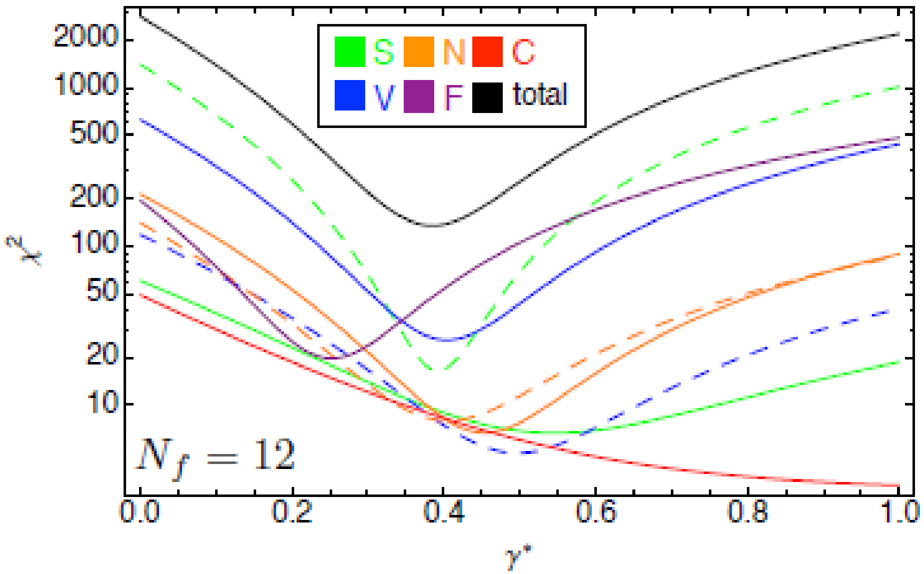
$$\langle\bar{\psi}\psi\rangle \sim A_c m + B_c m^{(3-\gamma*)/(1+\gamma*)} + C_c m^{3/(1+\gamma*)}$$

The data can be fitted by a function using conformal hypothesis.
X^2/dof~2.508 (dof=53).
The mass anomalous dimension at the fixed point: gamma*= 0.403(13)

Conclusion of Appelquist's papers

Fodor's data can be described with similar fit quality between
chiral broken hypothesis (x^2/dof~0.81)
with
conformal hypothesis (x^2/dof~2.5).
In these hypotheses, m should be small to control massless extrapolation.

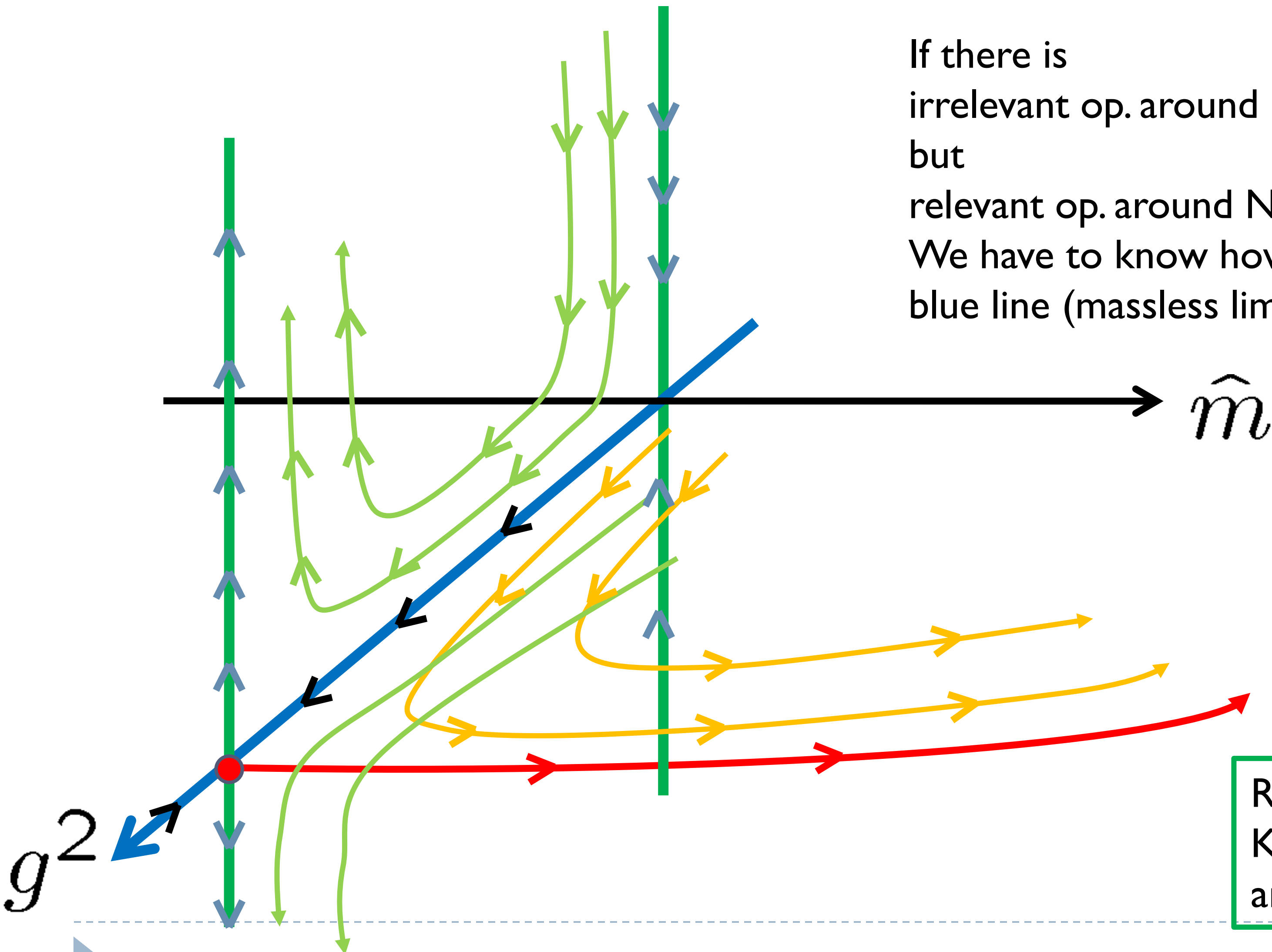
Further simulations at additional m will help to distinguish the two scenario.



The other question comes from this plot in the case of Nf=12.

Why does the minimum of each channel appear different gamma?

Mass deformation with additional relevant op.



If there is
irrelevant op. around Gaussian FP
but
relevant op. around Non-trivial FP
We have to know how to get the
blue line (massless limit)

Related works:
Kusafuka and Terao
arXiv:1104.3606

Summary and Discussion

Is there IR fixed point in SU(3) Nf=12 theory?

Appelquist group:YES (SF scheme g^2*~5, constant continuum extrapolation)
Fodor group : NO (Potential scheme, before taking continuum limit)
A. Hasenfratz :YES (MonteCalro RG, bare step scaling)
Our group :YES! (TPL scheme g^2*~2.4, a^2 quadratic extrapolation)

Chiral condensate

Fodor group : chiral symmetry is broken (huge lattice size and low beta region)
Columbia U group : chiral symmetry is broken
Appelquist group : the same data can be fitted by conformal hypothesis

Anomalous dimension for psuedo-scalar operator

Appelquist group : gamma*= 0.403(13) using conformal hypothesis with mass deformation
KMI group : gamma*~0.47-0.58
Our group : gamma*~0.29 by directly measurement

Discussion

The other theories

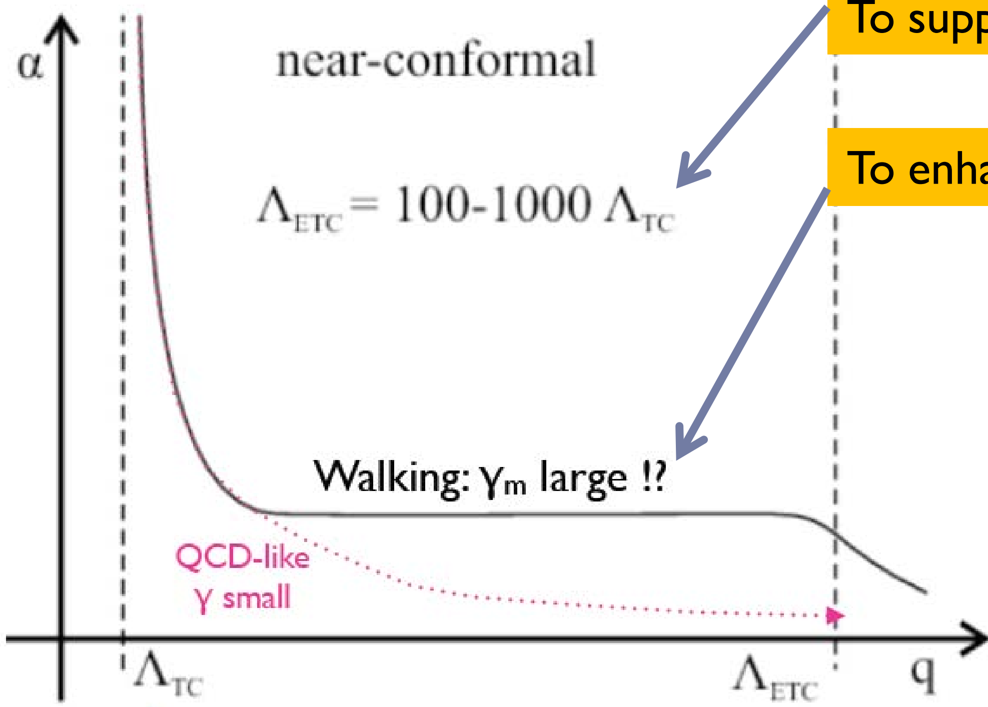
SU(3) Nf=10 fundamental fermion by N.Yamada's group using Wilson fermion
SU(2) Nf=2 adjoint fermion by del Debbio's group
SU(2) Nf=8 fundamental fermion by H.Ohki (Our group)
....

Lattice data will give phenomenological and theoretical information around nontrivial fixed point in near future.

BACK UP SLIDES

Introduction

To explain the origin of electro-weak symmetry breaking, we introduce an additional strong gauge interaction, and the composite state will be a seed of SSB.
There is a scenario based on a conformal theory in IR region.



To suppress FCNC process

To enhance quark mass

Asymptotic free (UV complete)

IR fixed point (or walking behavior)

Large anomalous dimension for composite state