Non-perturbative beta functions and scaling laws in QCD with many quark flavors

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Contents:

Based on Y.Kusafuka, H.T., arXiv:11043606

Y.Kusafuka, E.Ueno, H.T., in preparation

Introduction

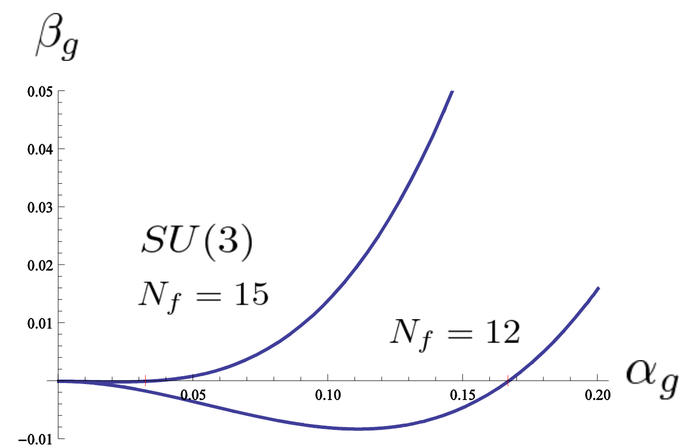
Conformal window of the many flavor QCD

- **SU(Nc) gauge theory with Nf massless flavors**
  - 2-loop gauge beta function $\alpha_g = g^2/(4\pi)^2$

$$\beta_g^{[2]} \equiv \mu \frac{d\alpha_g}{d\mu} = -2b_0\alpha_g^2 - 2b_1\alpha_g^3$$

$$b_0 = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$b_1 = \frac{34}{3}N_c^2 - N_f\left(\frac{N_c^2-1}{N_c} + \frac{10}{3}N_c\right)$$



- **IR (Banks-Zaks) fixed point**

T.Banks, A.Zaks, NP B 196 (1982)
W.E.Caswell, PRL 33 (1974)

  - goes towards strong coupling region as Nf decreases.
- **Spontaneous breaking of the chiral symmetry for Nf < Ncr**
  - Scale invariance is lost there. ⇒ Fixed point cannot exist !
  - Conformal window: $N_{f\text{cr}} < N_f < \frac{11}{2}N_c$

Introduction

Boundary of the conformal window $N_{f\,cr}$

- **Analysis of the Dyson-Schwinger equations**

V.A.Miransky, K.Yamawaki, MPL A4 (1989); PRD 55 (1997)

T.Appelquist et.al. PRL 77 (1996); PRD 58 (1998)

  - DS equations with the fixed point gauge coupling are examined.
  - The ladder approximation is used mostly.
  - Motivation of study: application to New Technicolor models

- **Lattice simulations of the effective gauge coupling**

Y.Iwasaki et. al. PRL 69 (1992)

T. Appelquist et. al. PRL 100 (2008); PRL1 02 (2009);PRD 79 (2009)

A.Hasenfratz, PRD 80 (2009); PRD 82 (2010)

T. DeGrand, et.al. PRD 80 (2009); PRD 82 (2010); PRD 83 (2011)

L. Del Debbio et.al. PRD 81 (2010); PRD 82 (2010)

Z. Fodor et.al.PL B 681 (2009); arXiv:0904.4662

E.Itou et.al arXiv:1011.1964

M.Hyakawa et.al. PRD 83 (2011)

$$\Rightarrow \quad 8 < N_{\rm cr} \leq 12$$

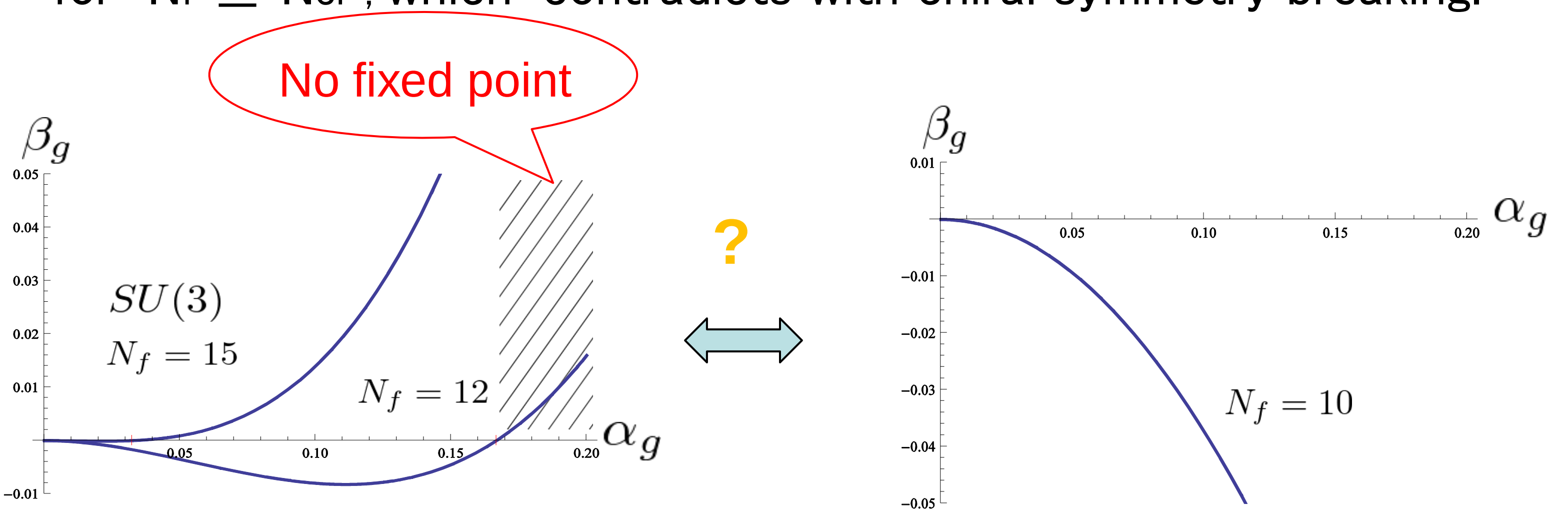
Introduction

Question: How does the beta function change vs. Nf ?

- A fixed point is not allowed at the strong coupling region. Therefore the perturbative beta functions cannot transform into the asymptotically free one smoothly, then what happens?

Decoupling of fermions?

- Chiral symmetry breaking reduces the flavor number effectively. So the beta function may be modified at the strong coupling region.
- But a UV fixed point seems to appear in the strong coupling region for Nf ≧ Ncr , which contradicts with chiral symmetry breaking.



Introduction

⇒ **We need some non-perturbative analyses of the beta functions in the conformal window.**

Use of the Wilson (Exact) Renormalization Group

- Scale invariance:

  Dyson-Schwinger equations treating the chiral order parameters are useless in the conformal window. Also it would be difficult to study almost scale invariant theories by the Lattice MC simulation.

- Non-perturbative calculation:

  In the Wilson RG, renormalized theories can be defined by the renormalized trajectories (RTs), which are given as the continuum limit of the Wilson RG flows. J.Polchinski, N.P. B231 (1984)

- Beta function:

  Then **non-perturbative beta functions** can be given by scale transformation on the RTs.

  ⇒ So the ERG is a quite suitable framework.

Non-perturbative beta function

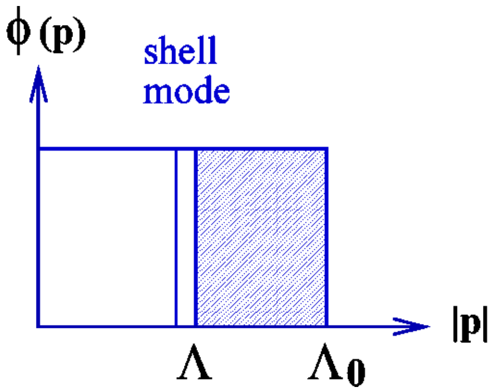
Wilson RG

- **Wilsonian effective action** K.G.Wilson, I.G.Kogut (1974)

Integrating out higher momentum modes

$$
\begin{aligned}
Z &= \int_{|p|<\Lambda_0} \mathcal{D}\phi(p) \exp(-S_0[\phi;\Lambda_0]) \\
&= \int_{|p|<\Lambda} \mathcal{D}\phi(p) \exp(-S_{\rm eff}[\phi;\Lambda])
\end{aligned}
$$

$$S_{\rm eff}[\phi;\Lambda] = \int d^D x \sum_i \frac{g_i}{\Lambda^{d_i}} \mathcal{O}_i[\phi]$$



: Wilsonian effective action contains infinitely many operators

- **Wilson RG**

$$\Lambda \frac{dS_{\rm eff}}{d\Lambda} = \mathcal{F}[S_{\rm eff}], \qquad \text{or} \qquad \Lambda \frac{dg_i}{d\Lambda} = \beta_i(\{g\})$$

- Legendre flow equation (Wetterich eq.) for the cutoff effective action

$$\frac{\partial \Gamma_\Lambda}{\partial \Lambda} = \frac{1}{2} \int_p \mathrm{tr} \left[\frac{\partial R}{\partial \Lambda} \cdot \left(R + \frac{\delta^2 \Gamma_\Lambda}{\delta\phi_p \delta\phi_{-p}} \right)^{-1} \right]$$

Non-perturbative beta function

Scalar field theory as a toy model

- **RG flows in (λ_4, λ_6) space** J.Polchinski, N.P. B231 (1984)
  - Operator truncation

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda_4}{2!}\left(\frac{\phi^2}{2}\right)^2 - \frac{\lambda_6}{3!\Lambda^2}\left(\frac{\phi^2}{2}\right)^3$$

  - Wetterich eqn (sharp cutoff limit)

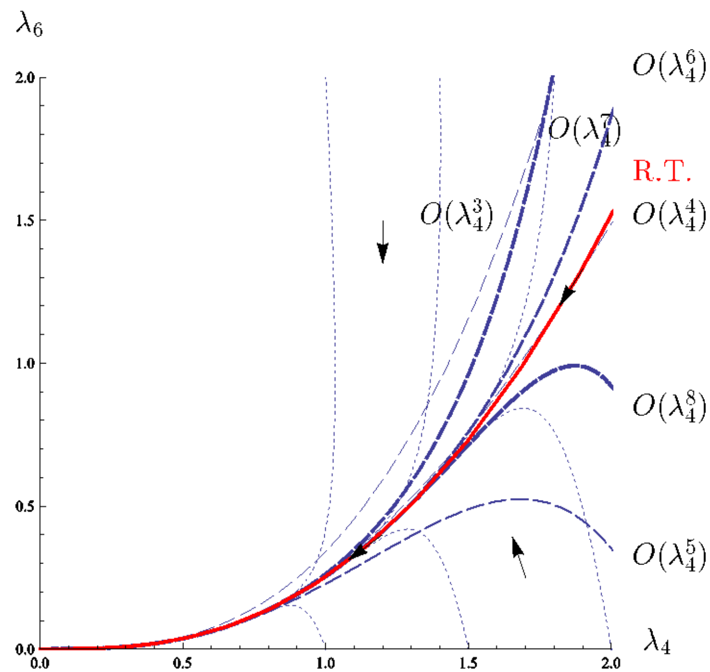
$$\Lambda\frac{d\lambda_4}{d\Lambda} = a\lambda_4^2 - b\lambda_6$$

$$\Lambda\frac{d\lambda_6}{d\Lambda} = 2\lambda_6 - c\lambda_4^3 + 2d\lambda_4\lambda_6$$

$$a = 9A, \quad b = 10A$$

$$c = 27A, \quad d = (45/2)A$$

$$(A = 2/(4\pi)^2)$$



  - Renormalized trajectory

    ⇔ renormalized theory

Non-perturbative beta function

- **Renormalized trajectory**
  - Perturbative analysis

$$\lambda_6^* = \frac{c}{2}\lambda_4^3 + \frac{c}{4}(3a-2d)\lambda_4^4 - \frac{c}{8}(-12a^2+3bc+14ad-4d^2)\lambda_4^5 + \cdots .$$

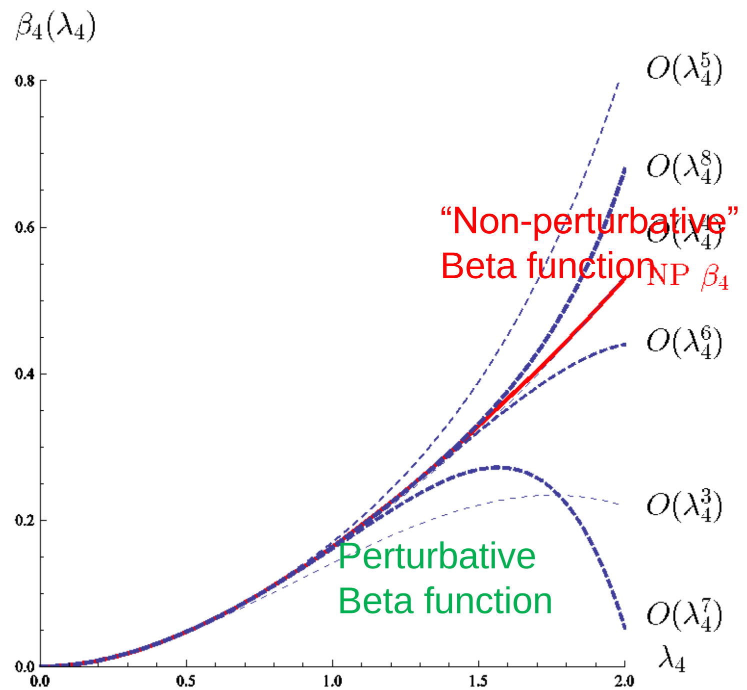
- **Non-perturbative beta function**
  - Renormalized trajectory

    We can find the RT numerically without perturbative expansion.

$$\lambda_6 = \lambda_6^*(\lambda_4)$$

  - "Non-perturbative" beta function

    The beta function of a renormalized parameter is given by the scale transformation on the RT.



$$\begin{aligned}\beta_4(\lambda_4) &= a\lambda_4^2 - b\lambda_6^*(\lambda_4) \\ &= a\lambda_4^2 - \frac{bc}{2}\lambda_4^3 - \frac{bc}{4}(3a-2d)\lambda_4^4 + \cdots\end{aligned}$$

RG flow equations for SU(N) gauge theories

Wilsonian effective action

- **Four-fermi operators**
  - Important to describe the chiral symmetry breaking
  - Symmetries
    - Gauge symmetry : $SU(N_c)$
    - Chiral flavor symmetry : $SU(N_f)_L \times SU(N_f)_R$
    - Parity $\psi_L \leftrightarrow \psi_R$

$$\psi_L^{ai} = L^{ai}, \psi_R^{ai} = R^{ai}$$
$$(a = 1, \cdots, N_c)$$
$$(i = 1, \cdots, N_f)$$

  - 4 invariant four-fermi operators

$$\mathcal{L}_{4\mathrm{f}} = \frac{G_S}{\Lambda^2}\mathcal{O}_S + \frac{G_V}{\Lambda^2}\mathcal{O}_V + \frac{G_{V1}}{\Lambda^2}\mathcal{O}_{V1} + \frac{G_{V2}}{\Lambda^2}\mathcal{O}_{V2}$$

$$\begin{aligned}
\mathcal{O}_S &= 2\bar{L}_i R^j \bar{R}_j L^i = \frac{1}{2}\left[\bar{\psi}_i\psi^j\bar{\psi}_j\psi^i - \bar{\psi}_i\gamma_5\psi^j\bar{\psi}_j\gamma_5\psi^i\right] \\
\mathcal{O}_V &= \bar{L}_i\gamma^\mu L^j\bar{L}_j\gamma_\mu L^i + (L \leftrightarrow R) \\
&= \frac{1}{2}\left[\bar{\psi}_i\gamma^\mu\psi^j\bar{\psi}_j\gamma_\mu\psi^i + \bar{\psi}_i\gamma^\mu\gamma_5\psi^j\bar{\psi}_j\gamma_\mu\gamma_5\psi^i\right] \\
\mathcal{O}_{V1} &= 2\bar{L}_i\gamma^\mu L^i\bar{R}_j\gamma_\mu R^j = \frac{1}{2}\left[(\bar{\psi}_i\gamma^\mu\psi^i)^2 - (\bar{\psi}_i\gamma^\mu\gamma_5\psi^i)^2\right] \\
\mathcal{O}_{V2} &= (\bar{L}_i\gamma^\mu L^i)^2 + (L \leftrightarrow R) = \frac{1}{2}\left[(\bar{\psi}_i\gamma^\mu\psi^i)^2 + (\bar{\psi}_i\gamma^\mu\gamma_5\psi^i)^2\right]
\end{aligned}$$

RG flow equations for SU(N) gauge theories

Invariant four-fermi operators

- **Apparent invariants**

$$\psi_L^{ai} = L^{ai}, \psi_R^{ai} = R^{ai} \quad (a = 1, \cdots, N_c) \quad (i = 1, \cdots, N_f)$$

$$2\bar{L}_{ai}\gamma^\mu L^{ai}\bar{R}_{bj}\gamma_\mu R^{bj} = \frac{1}{2}\left[(\bar{\psi}_{ai}\gamma^\mu\psi^{ai})^2 - (\bar{\psi}_{ai}\gamma^\mu\gamma_5\psi^{ai})^2\right]$$

$$2\bar{L}_{ai}\gamma^\mu L^{bi}\bar{R}_{bj}\gamma_\mu R^{aj} = \frac{1}{2}\left[\bar{\psi}_{ai}\gamma^\mu\psi^{bi}\bar{\psi}_{bj}\gamma_\mu\psi^{aj} - \bar{\psi}_{ai}\gamma^\mu\gamma_5\psi^{bi}\bar{\psi}_{bj}\gamma_\mu\gamma_5\psi^{aj}\right]$$

$$\bar{L}_{ai}\gamma^\mu L^{ai}\bar{L}_{bj}\gamma_\mu L^{bj} + (L \leftrightarrow R) = \frac{1}{2}\left[(\bar{\psi}_{ai}\gamma^\mu\psi^{ai})^2 + (\bar{\psi}_{ai}\gamma^\mu\gamma_5\psi^{ai})^2\right]$$

$$\bar{L}_{ai}\gamma^\mu L^{bi}\bar{L}_{bj}\gamma_\mu L^{aj} + (L \leftrightarrow R) = \frac{1}{2}\left[\bar{\psi}_{ai}\gamma^\mu\psi^{bi}\bar{\psi}_{bj}\gamma_\mu\psi^{aj} + \bar{\psi}_{ai}\gamma^\mu\gamma_5\psi^{bi}\bar{\psi}_{bj}\gamma_\mu\gamma_5\psi^{aj}\right]$$

$$\bar{L}_{ai}\gamma^\mu L^{aj}\bar{L}_{bj}\gamma_\mu L^{bi} + (L \leftrightarrow R) = \frac{1}{2}\left[\bar{\psi}_{ai}\gamma^\mu\psi^{aj}\bar{\psi}_{bj}\gamma_\mu\psi^{ai} + \bar{\psi}_{ai}\gamma^\mu\gamma_5\psi^{aj}\bar{\psi}_{bj}\gamma_\mu\gamma_5\psi^{bi}\right]$$

$$\bar{L}_{ai}\gamma^\mu L^{bj}\bar{L}_{bj}\gamma_\mu L^{ai} + (L \leftrightarrow R) = \frac{1}{2}\left[\bar{\psi}_{ai}\gamma^\mu\psi^{bj}\bar{\psi}_{bj}\gamma_\mu\psi^{ai} + \bar{\psi}_{ai}\gamma^\mu\gamma_5\psi^{bj}\bar{\psi}_{bj}\gamma_\mu\gamma_5\psi^{ai}\right]$$

RG flow equations for SU(N) gauge theories

- Fiertz identities
  - $$\bar{\psi}_1\gamma^\mu\psi_2\ \bar{\psi}_3\gamma_\mu\psi_4 + \bar{\psi}_1\gamma^\mu\gamma_5\psi_2\ \bar{\psi}_3\gamma_\mu\gamma_5\psi_4 = \bar{\psi}_1\gamma^\mu\psi_4\ \bar{\psi}_3\gamma_\mu\psi_2 + \bar{\psi}_1\gamma^\mu\gamma_5\psi_4\ \bar{\psi}_3\gamma_\mu\gamma_5\psi_2$$
  - $$\bar{\psi}_1\psi_2\ \bar{\psi}_3\psi_4 - \bar{\psi}_1\gamma_5\psi_2\ \bar{\psi}_3\gamma_5\psi_4 = -\frac{1}{2}\left[\bar{\psi}_1\gamma^\mu\psi_4\ \bar{\psi}_3\gamma_\mu\psi_2 - \bar{\psi}_1\gamma^\mu\gamma_5\psi_4\ \bar{\psi}_3\gamma_\mu\gamma_5\psi_2\right]$$
- Current-current interactions

$$2\sum_{A=1}^{\dim G}(T^A)^a_d\ (T^A)^c_b = \delta^a_b\ \delta^c_d - \frac{1}{N_c}\delta^a_d\ \delta^c_b$$

  - $$2\sum_A \bar{L}_i T^A\gamma^\mu L^i\ \bar{R}_j T^A\gamma_\mu R^j = -\mathcal{O}_S - \frac{1}{2N_c}\mathcal{O}_{V1}$$
  - $$\sum_A \bar{L}_i T^A\gamma^\mu L^i\ \bar{L}_j T^A\gamma_\mu L^j + (L\leftrightarrow R) = \frac{1}{2}\mathcal{O}_V - \frac{1}{2N_c}\mathcal{O}_{V2}$$
  - $$\sum_A \bar{L}_i T^A\gamma^\mu L^j\ \bar{L}_j T^A\gamma_\mu L^i + (L\leftrightarrow R) = \frac{1}{2}\mathcal{O}_{V2} - \frac{1}{2N_c}\mathcal{O}_V$$

RG flow equations for SU(N) gauge theories

- **Spontaneous breaking of the chiral symmetry**

K.–I.Aoki, K.Morikawa, W.Souma, J.–I.Sumi, H.T.,M.Tomoyose,
PTP97 (1997), PTP102 (1999), PRD61 (2000)

  - $G_S \to \infty$: Chiral symmetry breaking

$$\langle \bar{\psi}_i \psi^j \rangle = M^3 \delta_i^j \quad \Rightarrow \quad SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

- **Approximation scheme**
  - Operator truncation

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4g^2} F^A_{\mu\nu} F^{A\mu\nu} + \bar{\psi}_f i \not{D} \psi^f + \mathcal{L}_{4\mathrm{f}}$$

  - We discard all gauge non–invariant corrections.

    Note: Cutoff breaks gauge invariance. Gauge non–invariant corrections may be controlled by the modified WT identities.

RG flow equations for SU(N) gauge theories

RG flow equations (sharp cutoff limit)

- **Four-fermi couplings** $(g_i = G_i/4\pi^2,\ \alpha_g = g^2/(4\pi)^2)$

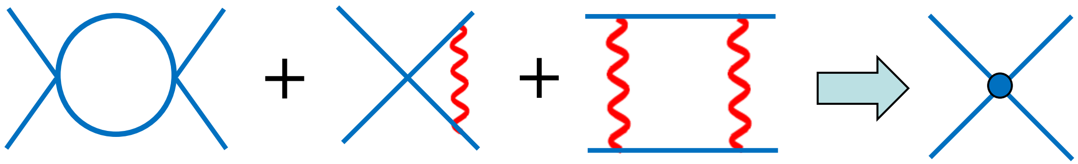
H.Gies, J.Jackel, C.Wetterich, PRD 69 (2004)

H.Gies, J.Jackel, EPJC 46 (2006)

$$\begin{aligned}
\Lambda\frac{dg_S}{d\Lambda} &= 2g_S - 2N_c g_S^2 + 2N_f g_S g_V + 6 g_S g_{V1} + 2 g_S g_{V2} \\
&\quad -12C_2(F) g_S \alpha_g + 12 g_{V1}\alpha_g - \frac{3}{2}\left(3N_c - \frac{4}{N_c} - \frac{1}{N_c^2}\right)\alpha_g^2 \\
\Lambda\frac{dg_V}{d\Lambda} &= 2g_V + (N_f/4) g_S^2 + (N_c + N_f) g_V^2 - 6 g_V g_{V2} \\
&\quad -\frac{6}{N_c}(g_V + g_{V2})\alpha_g - \frac{3}{4}\left(N_c - \frac{8}{N_c} + \frac{3}{N_c^2}\right)\alpha_g^2 \\
\Lambda\frac{dg_{V1}}{d\Lambda} &= 2g_{V1} - (1/4) g_S^2 - g_S g_V - 3 g_{V1}^2 - N_f g_S g_{V2} + 2(N_c + N_f) g_V g_{V1} \\
&\quad +2(N_c N_f + 1) g_{V1} g_{V2} + \frac{6}{N_c} g_{V1}\alpha_g + \frac{3}{4}\left(1 + \frac{3}{N_c^2}\right)\alpha_g^2 \\
\Lambda\frac{dg_{V2}}{d\Lambda} &= 2g_{V2} - 3g_V^2 - N_c N_f g_{V1}^2 + (N_c N_f - 2) g_{V2}^2 - N_f g_S g_{V1} \\
&\quad +2(N_c N_f + 1) g_V g_{V2} + 6(g_V + g_{V2})\alpha_g - \frac{3}{4}\left(3 + \frac{1}{N_c^2}\right)\alpha_g^2
\end{aligned}$$

RG flow equations for SU(N) gauge theories

- **Loop corrections for the four-fermi operators**



- **Large N_c, N_f limit** $(r = N_f/N_c \ : \ \text{fixed})$
  - rescale as $N_c g_{S(V)} \rightarrow g_{S(V)}, \ N_c^2 g_{V1(V2)} \rightarrow g_{V1(V2)},$
    $N_c \alpha_g \rightarrow \alpha_g$

$$\Lambda \frac{dg_S}{d\Lambda} = 2g_S - 2g_S^2 + 2r g_S g_V - 6 g_S \alpha_g - \frac{9}{2}\alpha_g^2$$
$$\Lambda \frac{dg_V}{d\Lambda} = 2g_V + \frac{r}{4} g_S^2 + (1+r) g_V^2 - \frac{3}{4}\alpha_g^2$$

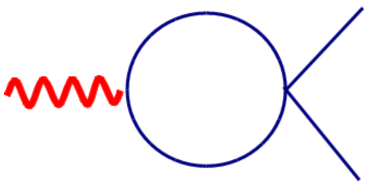
  - Note: Four-fermi couplings g_{V1}, g_{V2} do not involve in the large N_c and N_f limit.
  - Note: The large N_c corrections contain only the ladder diagrams. But the non-ladder ones come through the large N_f part.

RG flow equations for SU(N) gauge theories

- **Gauge coupling**
  - We use the perturbative beta functions in the large N_c, N_f limit and **add a part of higher order corrections via the four−fermi effective couplings.**

1. Vertex correction : H.Gies, J.Jackel, C.Wetterich, PRD 69 (2004)

   We discard all vertex corrections with the four−fermi couplings, since the gauge symmetry should forbid them.

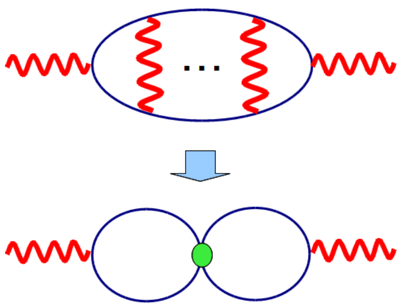


2. Vacuum polarization :

   The higher order corrections via four−fermi effective operators should be incorporated into the vacuum polarization.

$$\beta_g^{[2]} = -\frac{2}{3}(11-2r)\alpha_g^2 - \frac{2}{3}(34-13r)\alpha_g^3$$

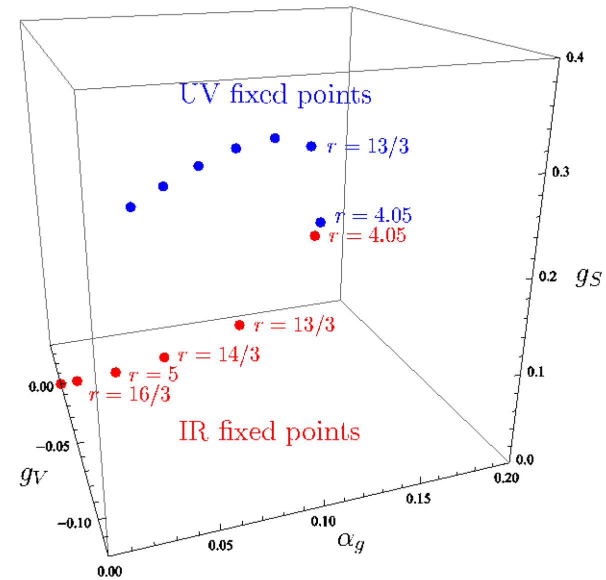
$$\boxed{\Lambda\frac{d\alpha_g}{d\Lambda} = \beta_g^{[2]} + 2rg_V\alpha_g^2}$$

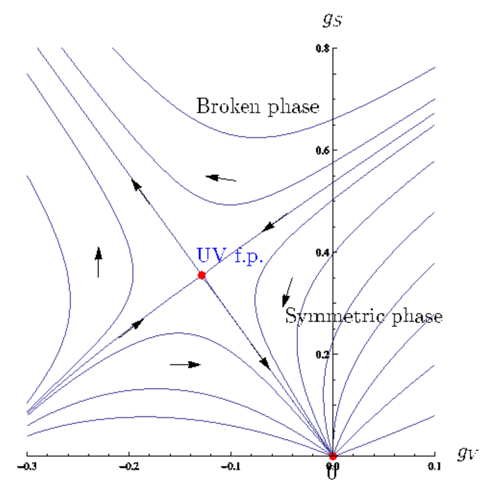


Aspect of RG flows

Numerical analysis of the flow equations

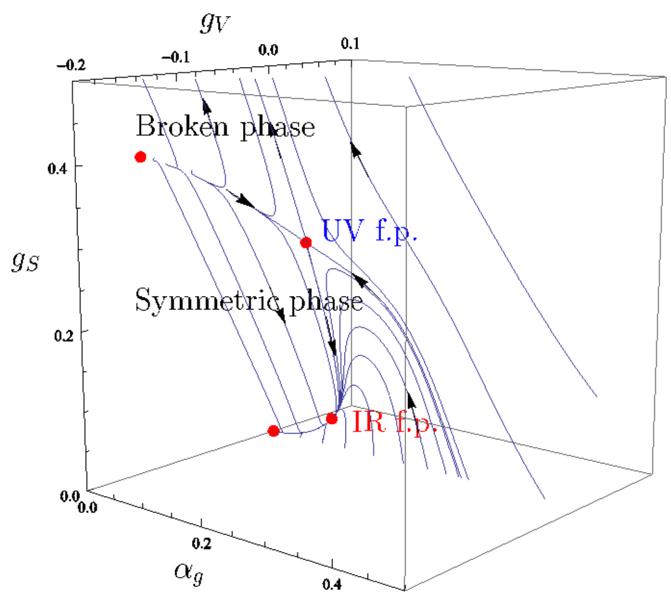
- **RG flows in large Nc and Nf**
  - RG flows are given in 3 dimensional coupling space of (α_g, g_V, g_S).
- **Fixed points in the conformal window**
  - A UV fixed point exists as well as the IR fixed point.
  - The UV fixed point and the IR fixed point merge with each other at r = 4.05.
- **RG flows in (gs, gv) space**
  - One linear combination of gs and gv gives the relevant operator, which induces the chiral phase transition.



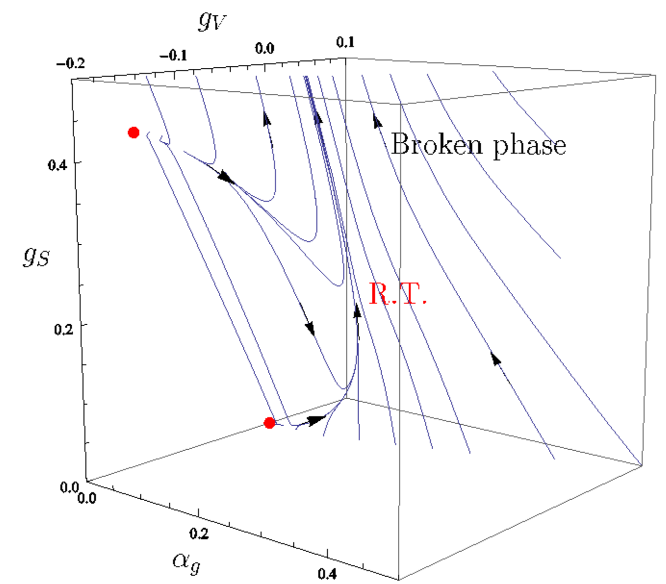


Aspect of RG flows

- **RG flows in the 3D space**
  - There is the phase boundary of chiral symmetry and the UV fixed point lies on the boundary.
  - Flows in the unbroken phase approach towards the IR fixed point.
  - The phase boundary disappears for r < 4.05 and the entire region becomes the broken phase.



$N_f = 13$



$N_f = 12$

“Non-perturbative” gauge beta functions

RT in the conformal window

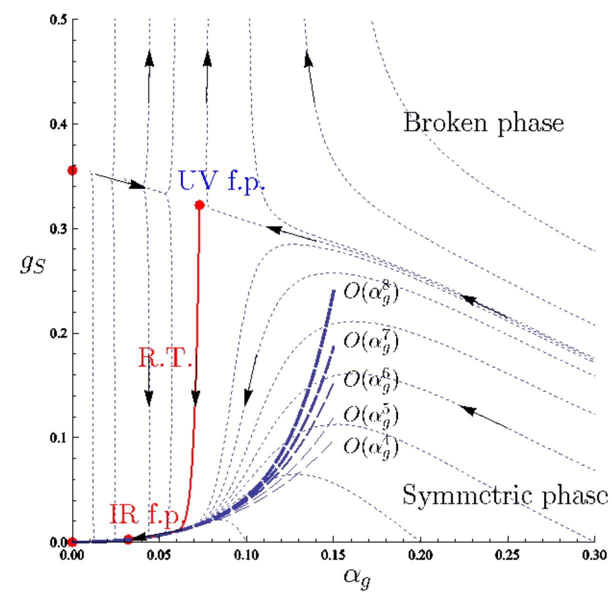
- **Perturbative RT**
  - We may extract the RT by solving the truncated RG flow equations in perturbative expansion.

$$g_S^* = \frac{9}{4}\alpha_g^2 - \frac{9}{4}(-3+2b_0)\alpha_g^3 + \frac{9}{32}(90-120b_0+48b_0^2-16b_1-3r)\alpha_g^4 + \cdots ,$$
$$g_V^* = \frac{3}{8}\alpha_g^2 - \frac{3}{4}b_0\alpha_g^3 + \frac{3}{128}(-3+96b_0^2-32b_1-30r)\alpha_g^4 + \cdots .$$

Note: These equations give continuum limit of the truncated ERG equation, not the full QCD.

  - This “RT” does not seem to give a continuum limit.
  - However this “RT” seems to survive for Nf > (11/2)Nc.

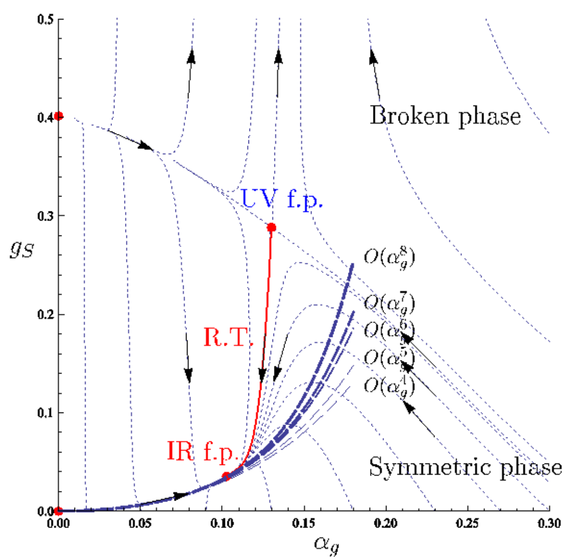
Note: How about the RT of QED?



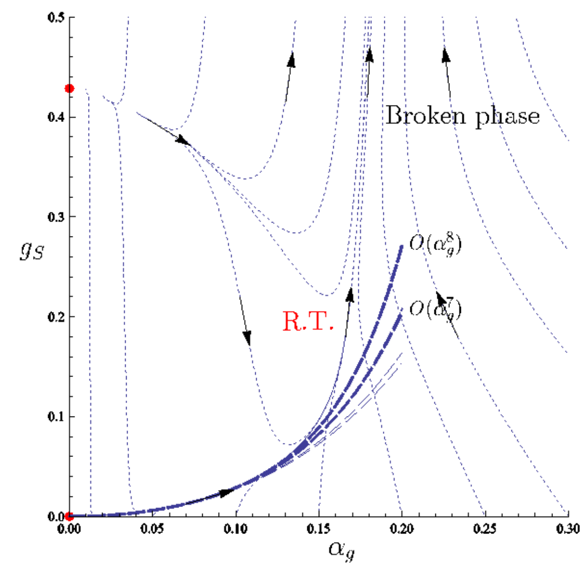
$N_f = 15$

"Non-perturbative" gauge beta functions

- **RTs near boundary of the conformal window**
  - The perturbative continuum limit lines approaches towards the non-perturbative RT as the flavor number is lowered.
  - Out of the window the fixed points disappears. However the non-perturbative RT survives as the RT of the asymptotically free QCD.
  - The perturbative continuum limit seems to converge towards the RT.



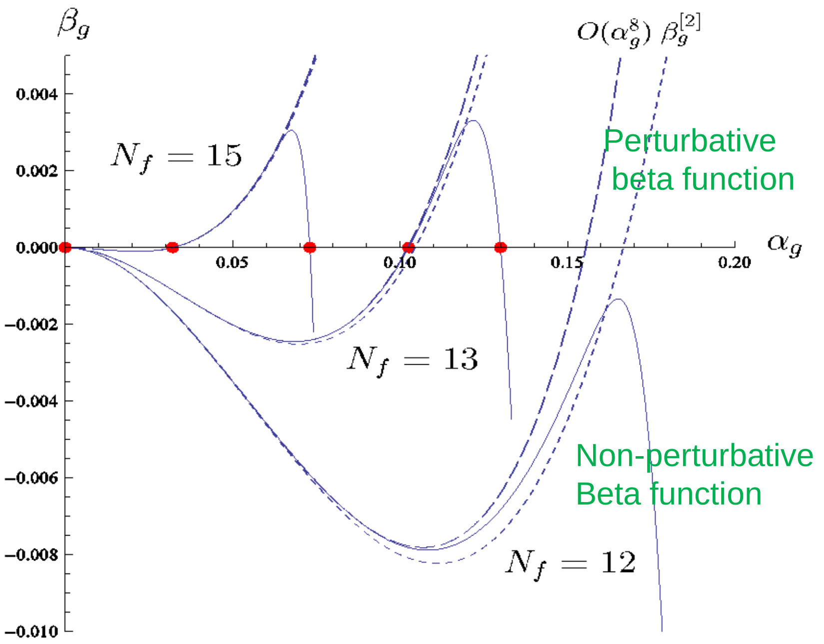
$N_f = 13$



$N_f = 12$

“Non-perturbative” gauge beta functions

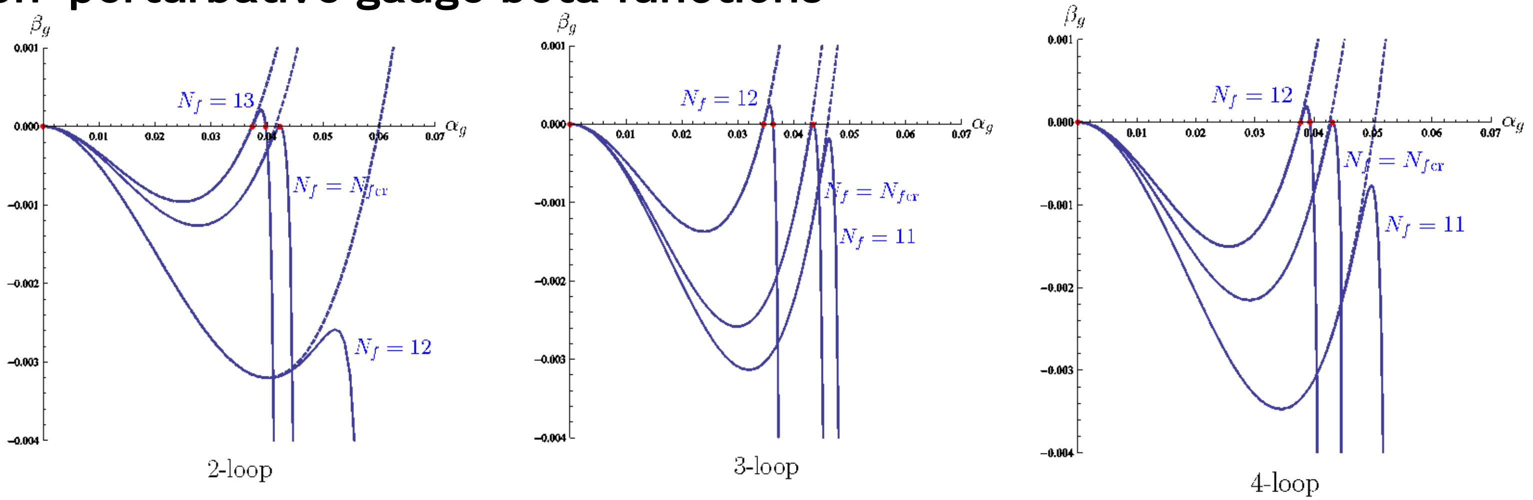
- **Non-perturbative gauge beta functions**
  - We define the non-perturbative gauge beta function by scale transformation of the gauge coupling on the RTs.
  - A UV fixed point appears in the gauge beta function due to higher order corrections generated through the four-fermi operators.
  - This behavior is not due to chiral symmetry breaking.
  - The UV fixed point does not appear in the strong coupling region.
- **“Conformality lost”**
  - The IR fixed point merges with the UV fixed point at the edge of conformal window.



Anomalous dimensions in many flavor QCD

Critical flavor number

- **Analysis for the many flavor QCD for comparison with the lattice MC.**
  - Solve the RG equations with a finite N_f, (N_c=3).
  - Use 2–, 3,– and 4–loop perturbative beta functions
- **Non–perturbative gauge beta functions**



  - Critical flavor numbers

$N_{f\,\mathrm{cr}} \simeq 12.78$ (2-loop) $\quad N_{f\,\mathrm{cr}} \simeq 11.24$ (3-loop) $\quad N_{f\,\mathrm{cr}} \simeq 11.58$ (4-loop)

Note: Lattice analyses indicate that QCD with 12 flavors is conformal.

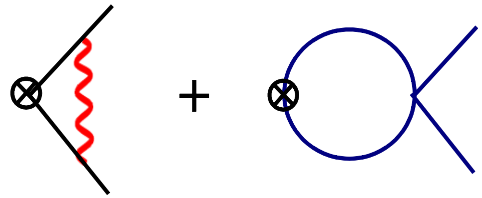
e.g. E.Itou et.al. arXiv:10110516

Anomalous dimensions in many flavor QCD

Anomalous dimensions of fermion mass

- **Anomalous dimension of $\bar{\psi}\psi$ in the ERG approach**

$\gamma_{\bar{\psi}\psi} =$

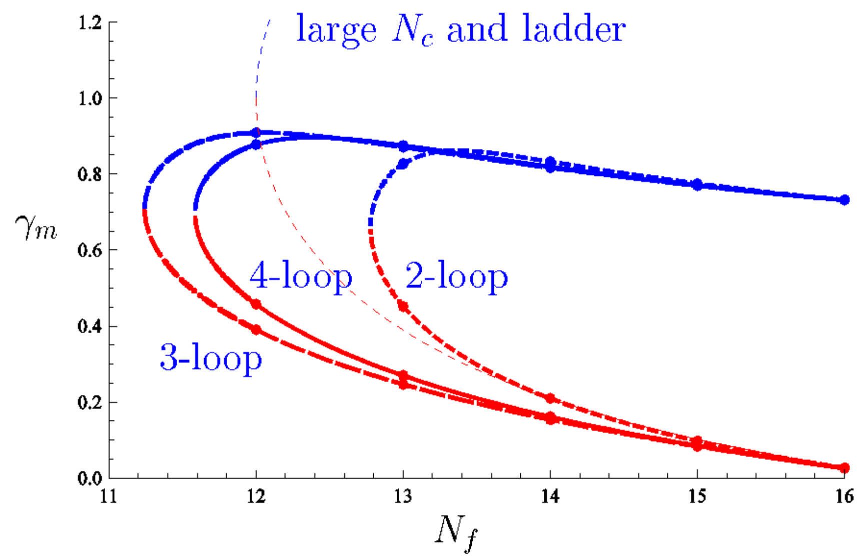


$$= -6C_2(F)\alpha_g - 2N_c g_S + 4g_{V1}$$
$$\equiv -\gamma_m$$

  - RG scheme and gauge independent at the fixed points

- **Results by the RG equations**

  Note: the anomalous dimension is fairly suppressed compared with the conventional value in the large N and ladder approx.



- **Lattice MC results**

  T.Appelquist et.al. (2011)

$$\gamma_{m*} \simeq 0.386 \pm 0.010 \quad (N_f = 12)$$

  - The 3− and 4−loop results are close to the Lattice estimations.

Scaling laws in nearly conformal theories

Scaling of the dynamical scale in the broken phase

- "Conformality lost" and the Miransky scaling

V.A.Miransky, K.Yamawaki MPL A4 (1989); PRD 55 (1997)

D.B.Kaplan, J-W.Lee, D.T.Son, M.A.Stephanov, PRD 80 (2009)

- Suppose that a UV fixed point and an IR fixed point merge.

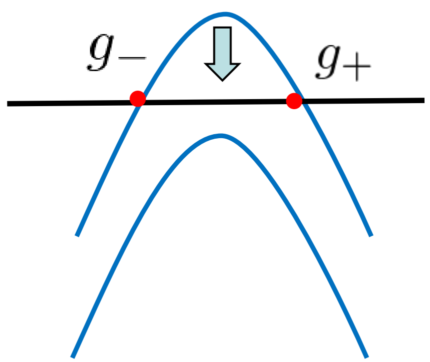
  The beta function with an external parameter c is given as

$$\beta(g;c) = \Lambda \frac{dg}{d\Lambda} = (c - c_{\rm cr}) - (g - g_*)^2$$

  Then the fixed point couplings are $g_\pm = g_* \pm \sqrt{c - c_{\rm cr}}$

- BKT type phase transition for $c = c_{\rm cr} - \epsilon$

$$\frac{\Lambda_{\rm IR}}{\Lambda_{\rm UV}} = \exp\left(\int_{g_{\rm UV}}^{g_{\rm IR}} \frac{dg}{\beta(g;c)}\right) \simeq \exp\left(-\frac{\pi}{\sqrt{c_{\rm cr} - c}}\right)$$



- Fermion mass generation $N_f \leq N_{\rm cr}$

  Miransky scaling $m_f \sim M e^{-\frac{C}{\sqrt{N_{\rm cr} - N_f}}}$

Scaling laws in nearly conformal theories

- **Approximation by a parabolic function**
  - We may approximate the RT as a parabolic function as follows;

1. Expand the RG flow equations around the critical fixed point.

   $x^i = x^i_* + \tilde{x}^i$: effective couplings near a fixed point x^i_*

$$\Lambda \frac{d\tilde{x}^k}{d\Lambda} = M[x_*]^k_i \tilde{x}^i + \frac{1}{2}\frac{\partial^2 \beta^k}{\partial x^i \partial x^j}[x_*]\tilde{x}^i \tilde{x}^j + \cdots \qquad M[x_*]^k_i = \frac{\partial \beta^k}{\partial x^i}[x_*]$$

   Note: An exactly marginal operator appears at fixed point merger.

   The RT passes along the exactly marginal direction

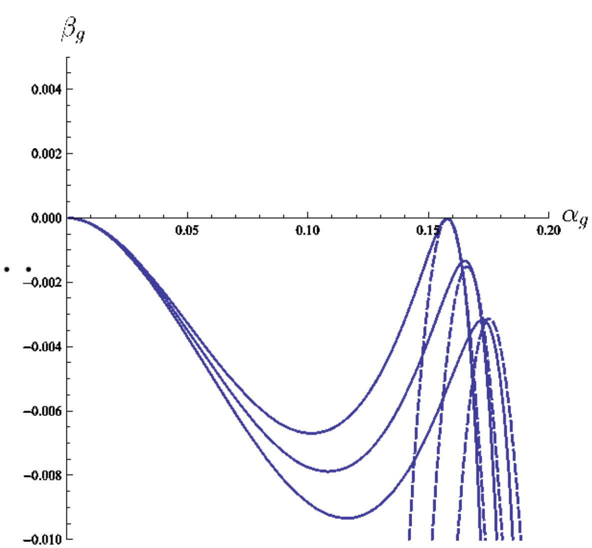
   . $M[x_{\rm cr}]^k_i u^i = 0$

2. Extract the beta function along the exactly marginal direction.

$$\tilde{x}^i = \tilde{\alpha}_g u^i \Rightarrow \Lambda \frac{d\alpha_g}{d\Lambda} = -A(\alpha_g - \alpha_{g{\rm cr}})^2 + \cdots$$

3. Find the (imaginary) fixed points $\alpha_{g*1,2}$ for a off-critical flavor number Nf.

$$\beta_g = -A(\alpha_g - \alpha_{g*1})(\alpha_g - \alpha_{g*2}) + \cdots$$



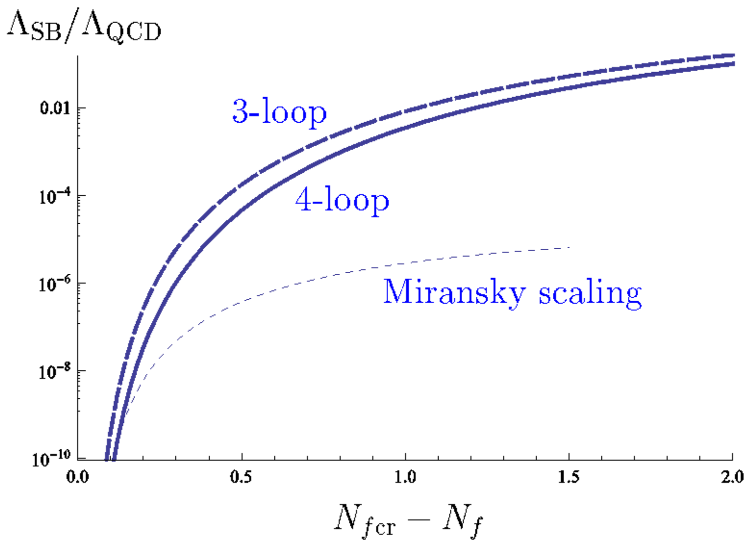
Scaling laws in nearly conformal theories

- **Dynamical scale of the chiral symmetry breaking**
  - Running effect must be taken into account in the broken phase.

    J.Braun,C.S.Fischer,H.Gies arXiv: 10124279
  - The four-fermi coupling diverges at the dynamical scale $\Lambda_{\rm SB}$.

    Note: Breakdown of the description in terms of the local fermi fields indicates the spontaneous chiral symmetry breaking.

    K-I.Aoki et al.PTP 97 (1997); PTP 102 (1999); PRD 61(2000)
  - Take difference with $\Lambda_{\rm QCD}$ obtained by the 1-loop beta function.
- **Approximation for the scaling law**
  - Large deviation from the Miransky scaling.
  - Fit with perturbative beta functions + the parabolic function is good.



Scaling laws in nearly conformal theories

Scaling of the explicit fermion mass

- **Recent lattice analyses**
  - MC simulations of mass deformed QCD (adding a bare fermion mass)

L.Del Debbio, R.Zwicky, PRD82 014502 (2010); arXiv:1009.2894

Z.Foder et. al. arXiv:1104.3124

T.Applequist et.al. arXiv:1106.2148

- **Scaling law in the conformal window**
  - The RG eqn for a fermion mass and IR enhancement

$$\Lambda \frac{dm}{d\Lambda} = -\gamma_{m*} m \qquad m(\Lambda) = \left(\frac{\Lambda_0}{\Lambda}\right)^{\gamma_{m*}} m_0$$

  - Decoupling at the scale of the fermion mass:

$$m(\Lambda = m_f) = m_f$$

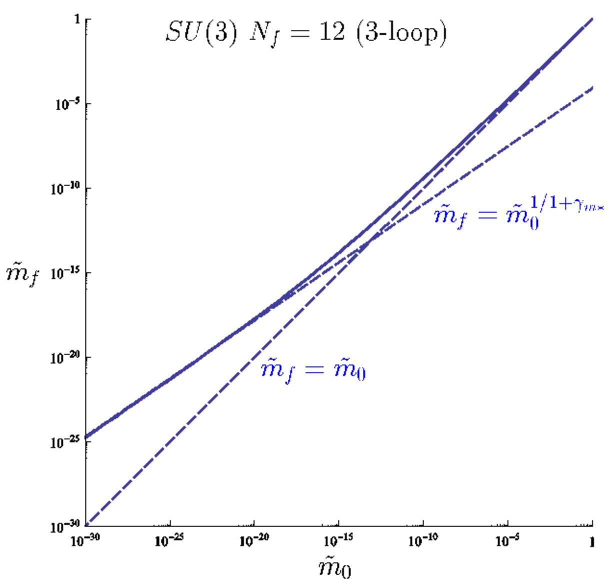
  - The scaling law of the dimensionless mass parameter $\tilde{m}_f = m_f/\Lambda_0$

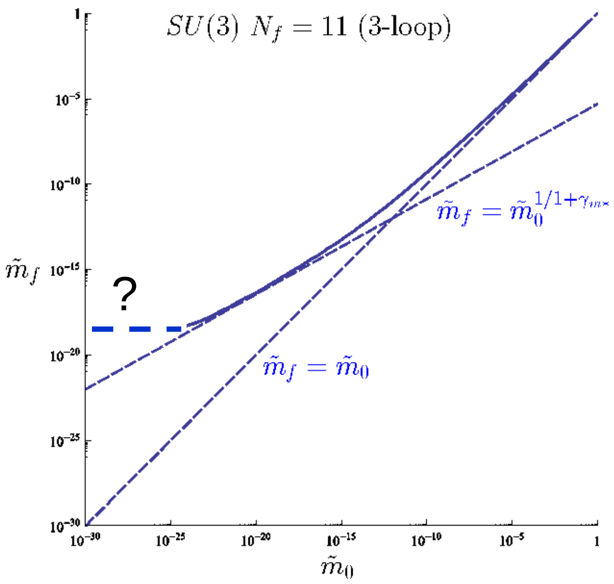
$$\tilde{m}_f = \tilde{m}_0^{\frac{1}{1+\gamma_{m*}}}$$

Scaling laws in nearly conformal theories

Scaling of the explicit fermion mass

- **Scaling laws in the slightly broken case**
  - We may solve the RG equations for the effective couplings including the fermion mass m on the RT numerically.
  - It seems to be difficult to distinguish whether the theory is conformal or chirally broken.
  - It is necessary to see dynamical mass generation to future work .





Scaling laws in nearly conformal theories

Scaling of the chiral condensate

- **Hyperscaling relation in the conformal window**
  - We can also deduce the hyper scaling relation by the RG flow equations as

$$\frac{\langle\bar{\psi}\psi\rangle}{\Lambda_0^3} = \frac{N_c N_f}{4\pi^2(1-\gamma_{m*})}\left(\tilde{m}_0 - \frac{2}{1+\gamma_{m*}}\tilde{m}_0^{\eta_*}\right) \qquad (\tilde{m}_0 = m_0/\Lambda_0)$$

$$\eta_* = \frac{3-\gamma_{m*}}{1+\gamma_{m*}}$$

  - The linear term (contact term) is dominant for $\gamma_{m*} < 1$.

    ⇒ Therefore, it seems to be difficult to see the hyperscaling relation.

Summary and discussions

- **We extended the RG flow equations for the gauge couplings so as to include the "non-perturbative" corrections through the effective four-fermi operators.**
- **We gave the non-perturbative gauge beta functions by scale transformation on the RT, which shows merge of the UV and the IR fixed points. ⇒ manifestation of the "Conformality Lost" picture.**
- **The anomalous dimension of the fermion mass and the critical flavor number were evaluated for QCD by using the 3-, 4-loop beta functions.**
- **Scaling of the dynamical scale was evaluated by using the beta functions.**
- **The scaling relations of the fermion mass for the mass deformed QCD were also examined near the boundary of the conformal window.**

Future issues

- Derivation of the RG flow equation for the gauge coupling including the four-fermi couplings by the ERG formalism.
- Evaluation of the chiral order parameters near the conformal boundary.

Many Thanks