Chiral symmetry breaking in QCD and finite-volume effects

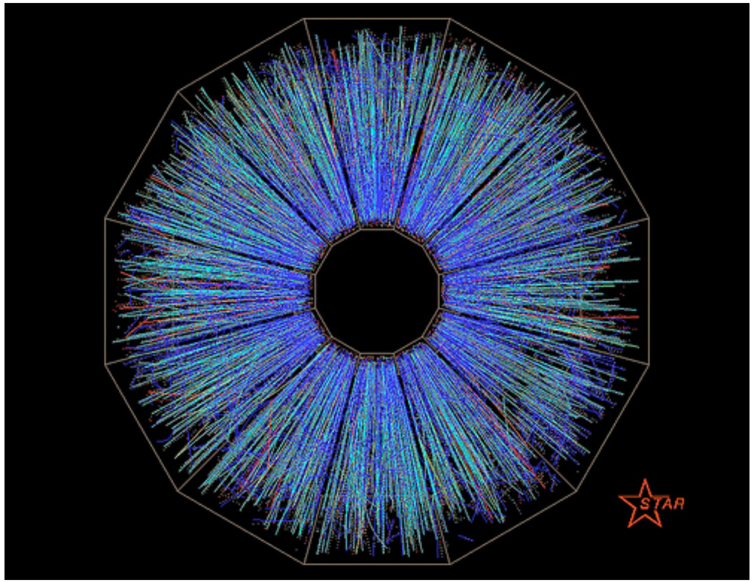
Bertram Klein

Technische Universität München

Renormalization Group Approach from Ultra Cold Atoms to the Hot QGP, Kyoto, August 26 2011

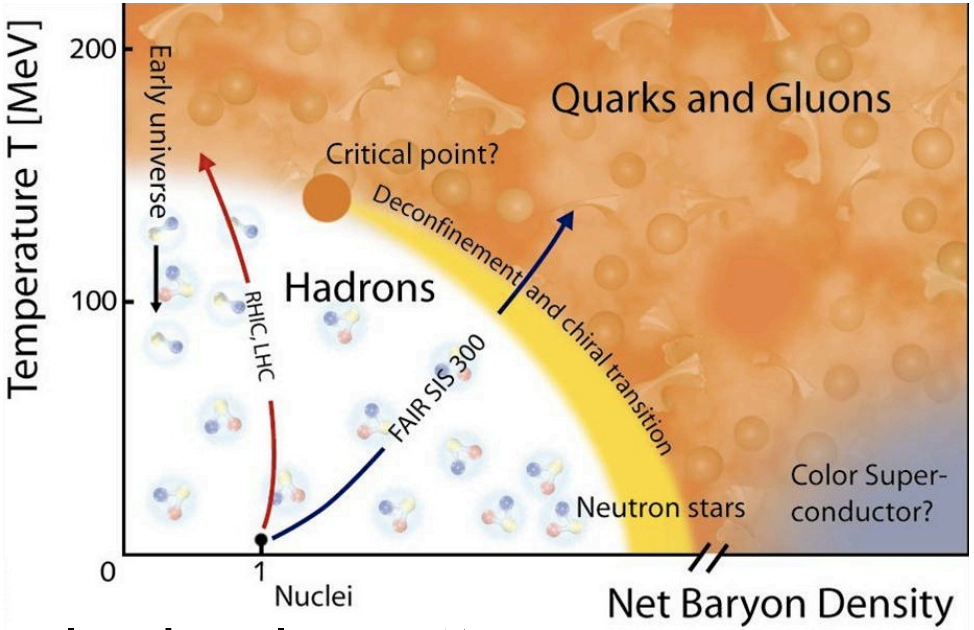
Hadronic matter under extreme conditions

- heavy-ion collisions



[STAR experiment, RHIC, BNL]

[GSI]



- hadronic matter
  at finite temperature and density

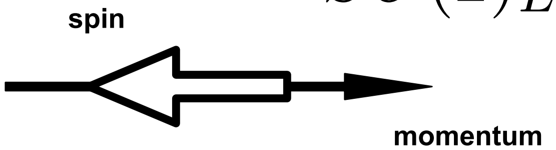
QCD: chiral symmetry and confinement

- SU(3) gauge theory of strongly interacting quarks and gluons

$$\mathcal{S}_{\rm QCD} = \int \mathrm{d}^4x \; \left\{ \bar{\psi}\,(\mathrm{i}\not{D} - m)\,\psi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \right\}$$

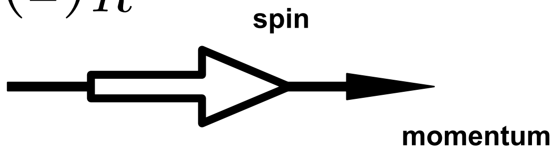
- chiral flavor symmetry: symmetry for 2 massless quarks (flavors (u, d))

$$SU(2)_L \times SU(2)_R$$



left-handed quarks

$$\psi_L \to U_L \psi_L$$

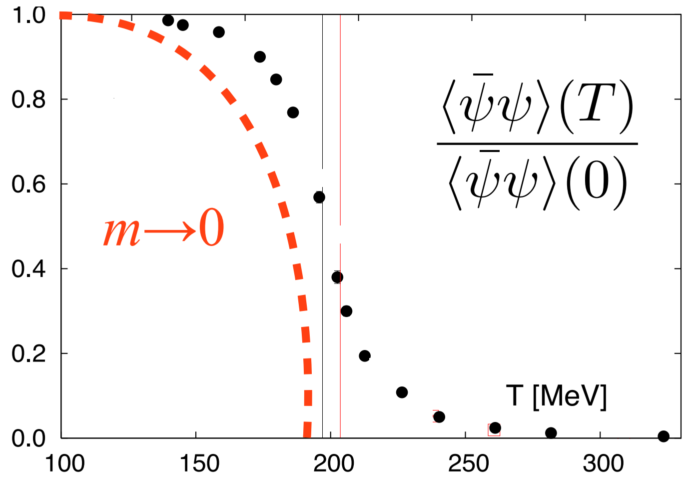


right-handed quarks

$$\psi_R \to U_R \psi_R$$

Spontaneous chiral symmetry breaking

- chiral symmetry is spontaneously broken in the QCD ground state



- order parameter: chiral condensate

$$\langle \bar{\psi}\psi \rangle = \langle \psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L \rangle \neq 0$$

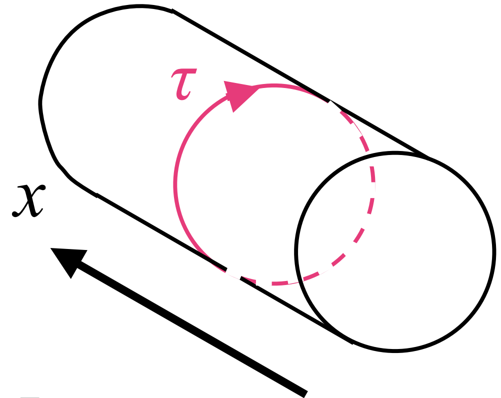
- Goldstone bosons of spontaneous symmetry breaking: **pions** are the (almost massless) low-energy degrees of freedom

- low-momentum QCD: theory of weakly interacting pions

Deconfinement: Z(3) symmetry breaking

- Polyakov loop acts as order parameter

$$\Phi(\vec{x}\,) = \frac{1}{N_c}\mathrm{Tr}\left[\exp \mathrm{i}\int_0^{1/T} \mathrm{d}\tau A_4(\vec{x},\tau)\right]$$



- Z(3) center symmetry

$$\Phi \longrightarrow z\Phi \qquad z = \exp\left[\mathrm{i}\frac{2\pi n}{N_c}\right]$$

de-confined phase: Z(3) symmetry spontaneously broken

$$\langle\Phi\rangle \neq 0$$

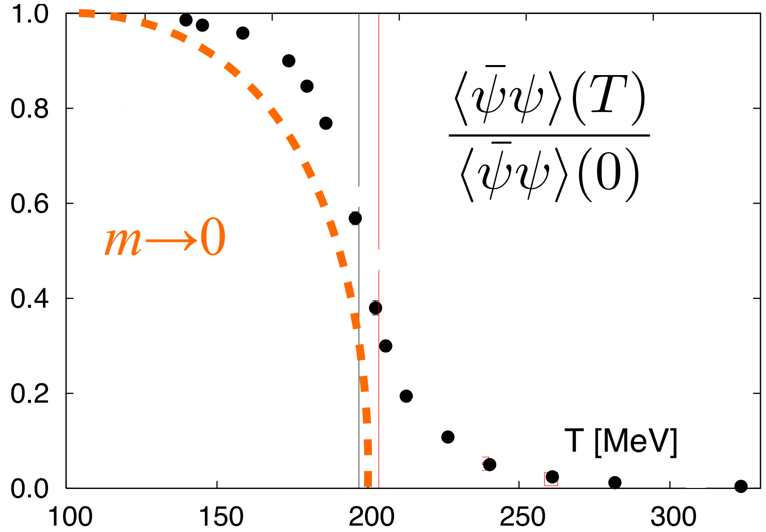
confined phase: Z(3) symmetric

$$\langle\Phi\rangle = 0$$

- Polyakov loop corresponds to free energy of a free static quark: $\Phi \sim \exp\left(-F_q/T\right)$

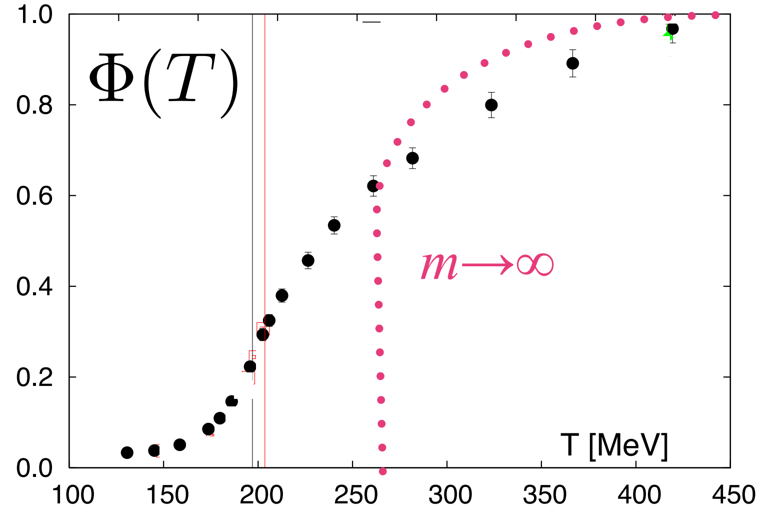
Chiral symmetry restoration and deconfinement

- Chiral quark condensate



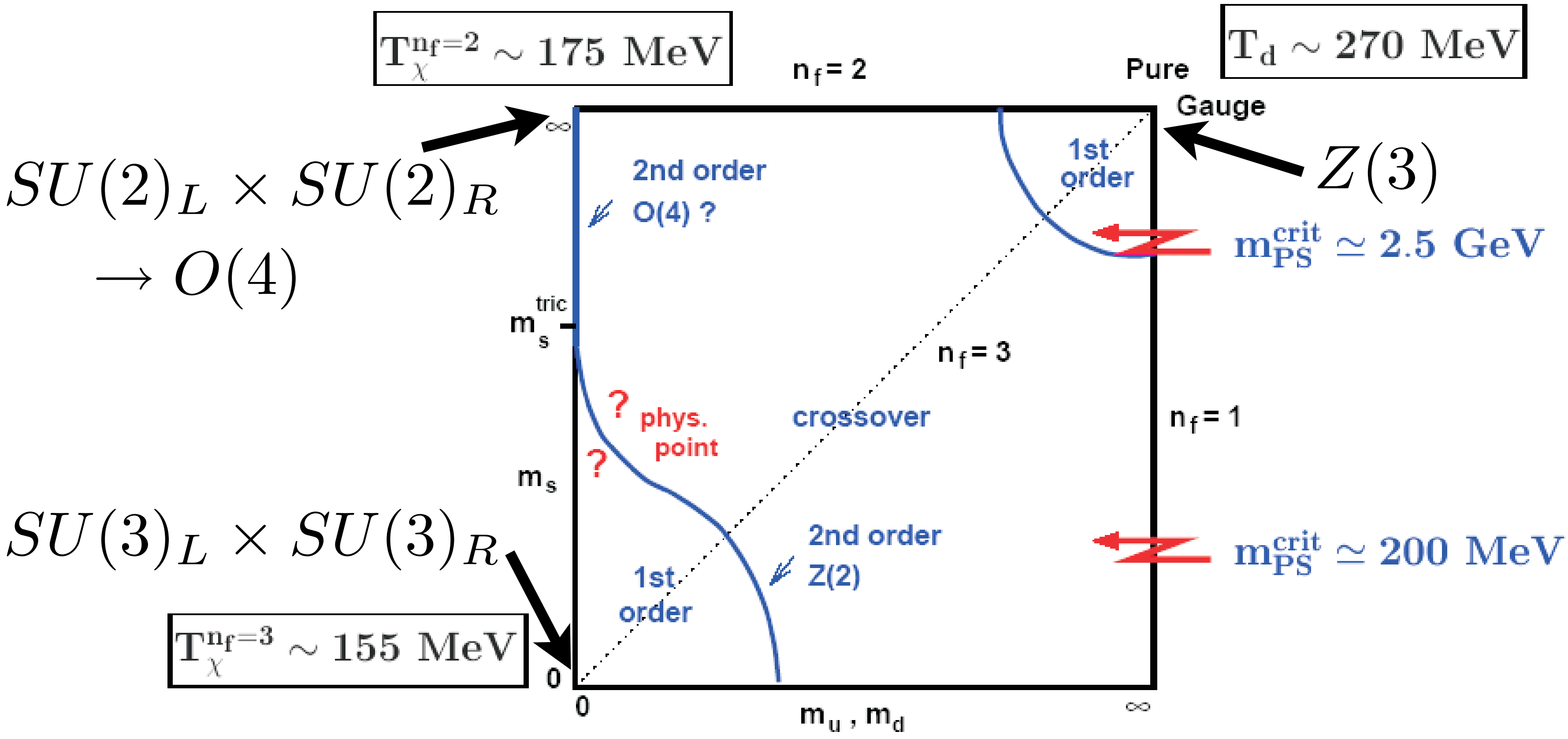
- finite quark mass
- strictly speaking, not a true phase transition
- **crossover**

- Polyakov loop
- crossover in presence of quarks



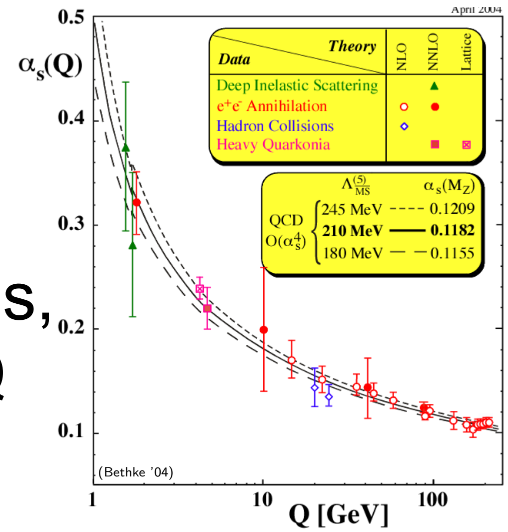
[M. Cheng *et al.*, Phys. Rev. D 77 (2008) 014511.]

Phase transitions in QCD

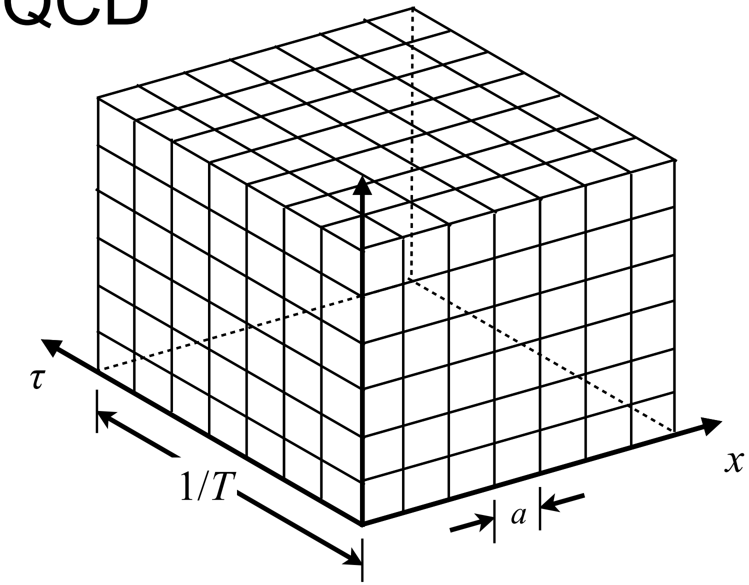


[F. .Karsch and E. Laermann, arXiv:hep-lat/0305025.]

Methods for studying QCD



- QCD: perturbative at large momentum scales, non-perturbative at low momentum scales Q
- chiral perturbation theory: Effective Field Theory systematic expansion of low-energy QCD
- lattice QCD: simulation of the gauge theory in finite Euclidean space-time volume



Methods for studying QCD

- Dyson-Schwinger Equations for QCD
  [C. Fischer and R. Alkofer, J. Lücker, ...]

- functional Renormalization Group methods
  - for models, including confinement [B.-J. Schaefer, T. Herbst, A. Tripolt, V. Skokov, B. Friman, K. Redlich ...]
  - for QCD [J. Braun, H. Gies, L. Haas, L. Fister, Pawlowski, ...]

- (chiral) model calculations for transition regions
  → this talk

QCD in a finite volume

- finite volume introduces an additional scale 1/L:
  probes different momentum regions, like temperature T

- no real phase transitions, critical long-range behavior affected by the volume size

- quark boundary conditions: what to choose in a finite spatial volume? periodic or anti-periodic?

→ finite-volume effects in the analysis?

Topics

- Critical scaling for the chiral phase transition in infinite and finite volume

- Phenomenological implications of finite-volume effects for the QCD phase diagram

long-range fluctuations: Renormalization Group (RG) methods

The quark-meson model

- a model for chiral symmetry breaking
- no gauge degrees of freedom

$$\Gamma_\Lambda[\bar{\psi},\psi,\sigma,\vec{\pi}] = \int \mathrm{d}^4x \Big\{ \bar{\psi}(\mathrm{i}\,\partial\!\!\!/\,)\psi + g\bar{\psi}(\sigma + \mathrm{i}\gamma_5\vec{\tau}\cdot\vec{\pi})\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi}\,)^2 + U_\Lambda(\sigma,\sigma^2+\vec{\pi}^2) \Big\}$$

- chiral symmetry breaking: SU(2) × SU(2) → SU(2)
  as O(4) → O(3) (meson sector) $\langle\sigma\rangle \neq 0$
- specify effective action for the model at initial scale Λ
- use functional Renormalization Group
  (Wetterich equation) to obtain effective action
  [C.Wetterich, Phys. Lett. B 301 (1993) 90.]

Renormalization Group calculation

- RG flow equation with 3*d* optimized cutoff function: change of effective potential with change of RG scale *k*

$$k\partial_k U_k(\phi^2) = k^5 \left[\frac{3}{E_\pi} \left(\frac{1}{2} + n_B(E_\pi) \right) \mathcal{B}_{\mathrm{p}}(kL) + \frac{1}{E_\sigma} \left(\frac{1}{2} + n_B(E_\sigma) \right) \mathcal{B}_{\mathrm{p}}(kL) - \frac{2N_c N_f}{E_q} \left(1 - n_F(E_q,\mu) - n_F(E_q,-\mu) \right) \mathcal{B}_{\mathrm{p}}(kL) \right]$$

→ see also talk by M. Scherer

- volume dependence encoded in mode-counting functions

- depends on choice of boundary conditions (for the spatial directions)

Critical behavior, scaling and universality

- Critical point (e.g. second-order phase transition)
- diverging *correlation length* $\xi \sim 1/M_\sigma \rightarrow \infty$
- critical *long-range* fluctuations dominate the physics of a system close to the critical point
- universality classes: the same critical behavior is observed in different systems with the same *symmetries* and *dimensionality*
- critical behavior is *characterized* by a few *critical exponents* $\beta, \gamma, \delta, \nu$ and *scaling functions*

→ finite volume cuts off long-range fluctuation and impacts critical scaling behavior!

Scaling behavior in infinite volume

- two relevant couplings: temperature T and symmetry breaking (quark mass m) H

$$t = (T - T_c)/T_0, \qquad h = H/H_0$$

- Scaling function for the order parameter *M* (f_π , $\langle\bar{\psi}\psi\rangle$):

$$M(t, h = 0) = (-t)^\beta \qquad M(t = 0, h) = h^{1/\delta}$$

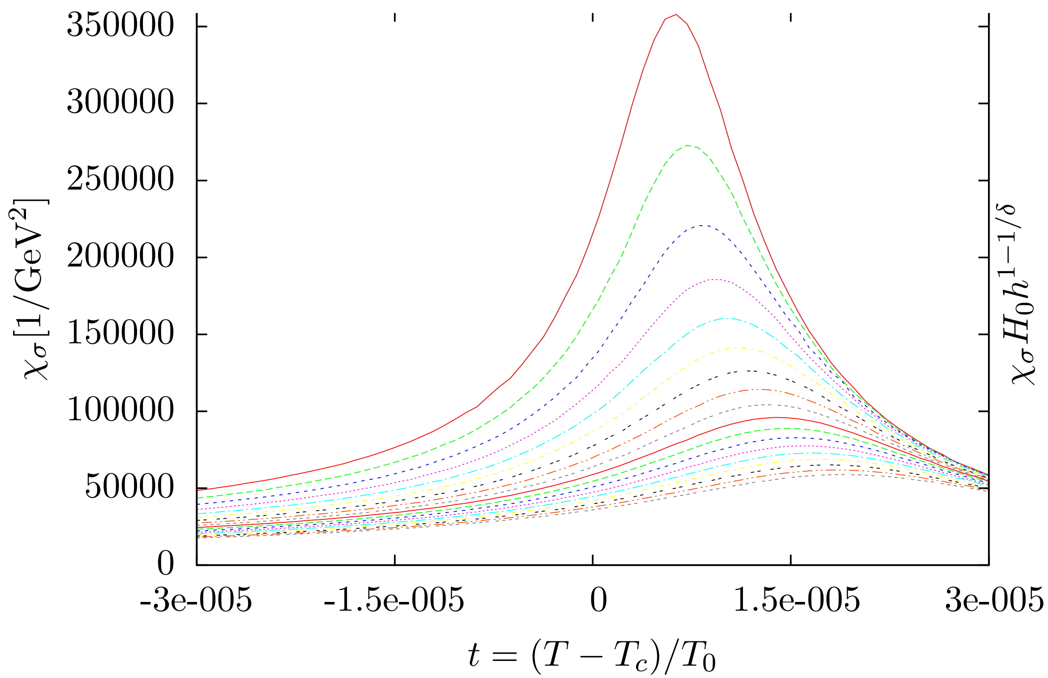
$$M(t, h) = h^{1/\delta} f_M(z) \qquad z = t/h^{1/(\beta\delta)}$$

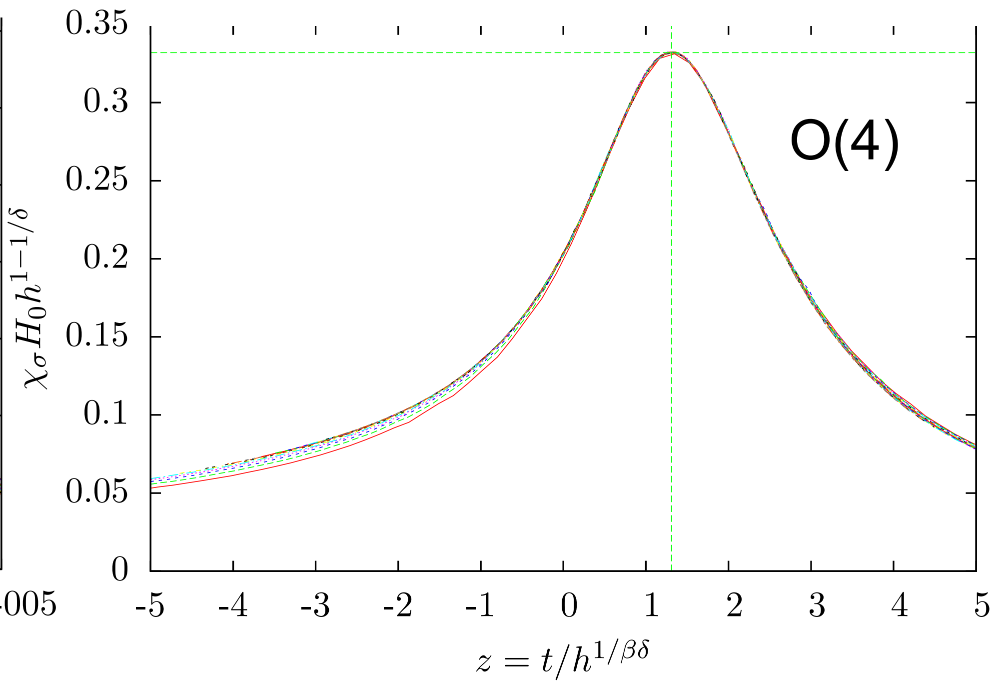
- Scaling function for the susceptibility χ_σ :

$$\chi_\sigma(t, h) = \frac{\partial M}{\partial H}(t, h) = \frac{1}{H_0} h^{1/\delta - 1} f_\chi(z)$$

Susceptibility χ_σ from the model: O(4)

- susceptibility χ_σ for small values of $m_\pi < 0.9$ MeV



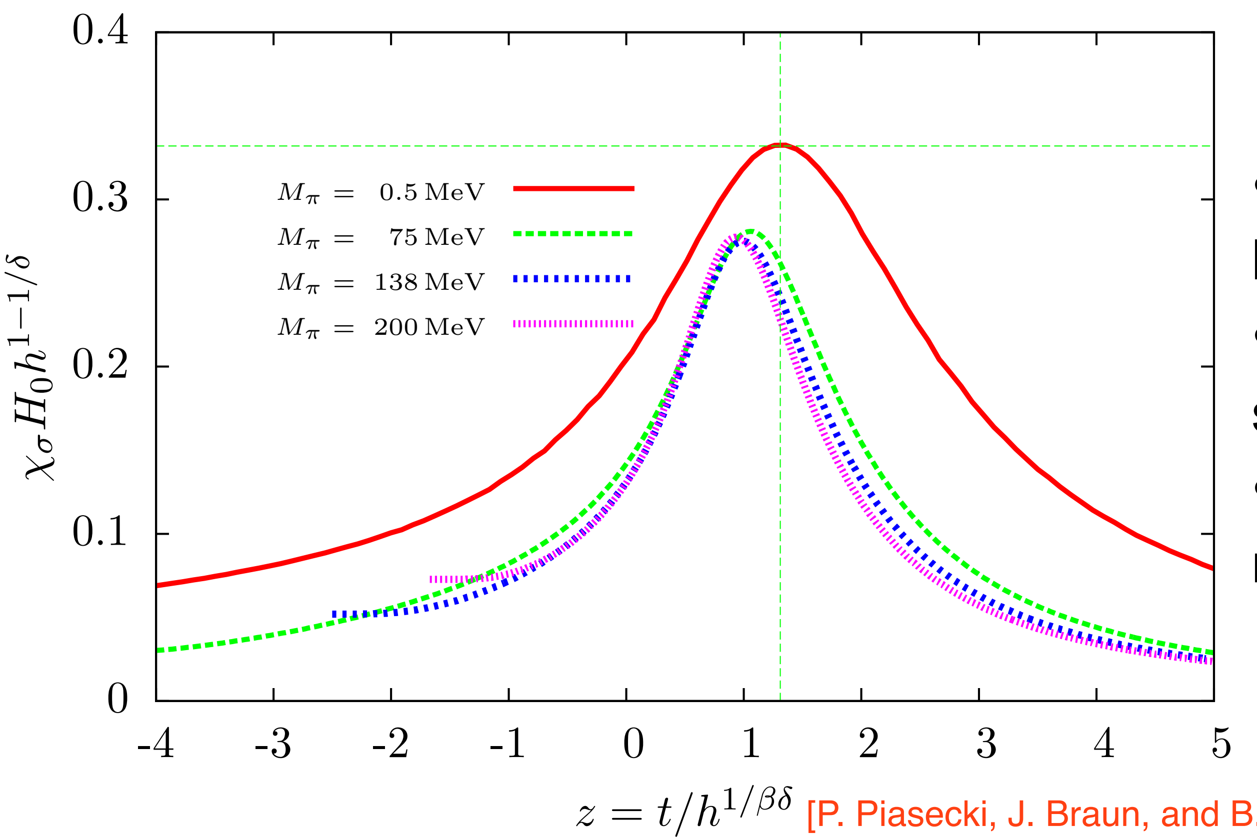


- rescaled susceptibility $\chi_\sigma H_0 h^{1-1/\delta}$

[P. Piasecki, J. Braun, and B. Klein, Eur. Phys. J. C71, 1576 (2011)]

Susceptibility χ_σ from the model: O(4)

- rescaled susceptibility $\chi_\sigma H_0 h^{1-1/\delta}$ for realistic values of m_π

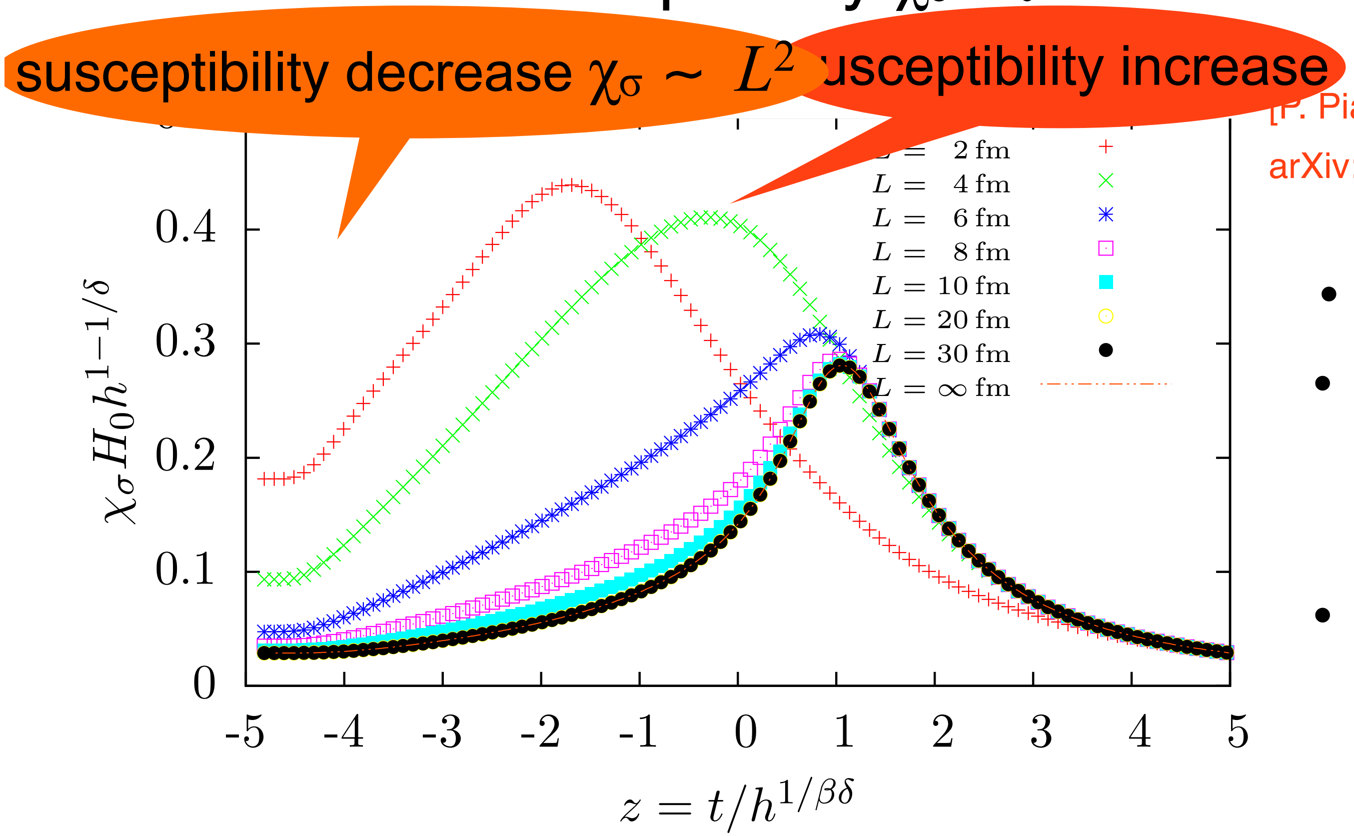


- apparent scaling for large quark masses
- differs from actual scaling function!
- sensitivity to quark mass larger than in QCD

[P. Piasecki, J. Braun, and B. Klein, Eur. Phys. J. C71, 1576 (2011)]

Infinite volume scaling in finite volume?

- rescaled susceptibility $\chi_\sigma H_0 h^{1-1/\delta}$ in finite volume



[P. Piasecki, J. Braun, and B. Klein (2010), arXiv:1008.2155]

- m_π = 75 MeV
- deviations from infinite-volume scaling for $L < 6$ fm
- effects probably weaker in lattice QCD

Scaling behavior in finite volume

- correlation length is cut off by finite volume size L
- Volume size L appears as additional relevant coupling
- Finite-size scaling functions with additional scaling variable

- Finite-size scaling function for the order parameter:

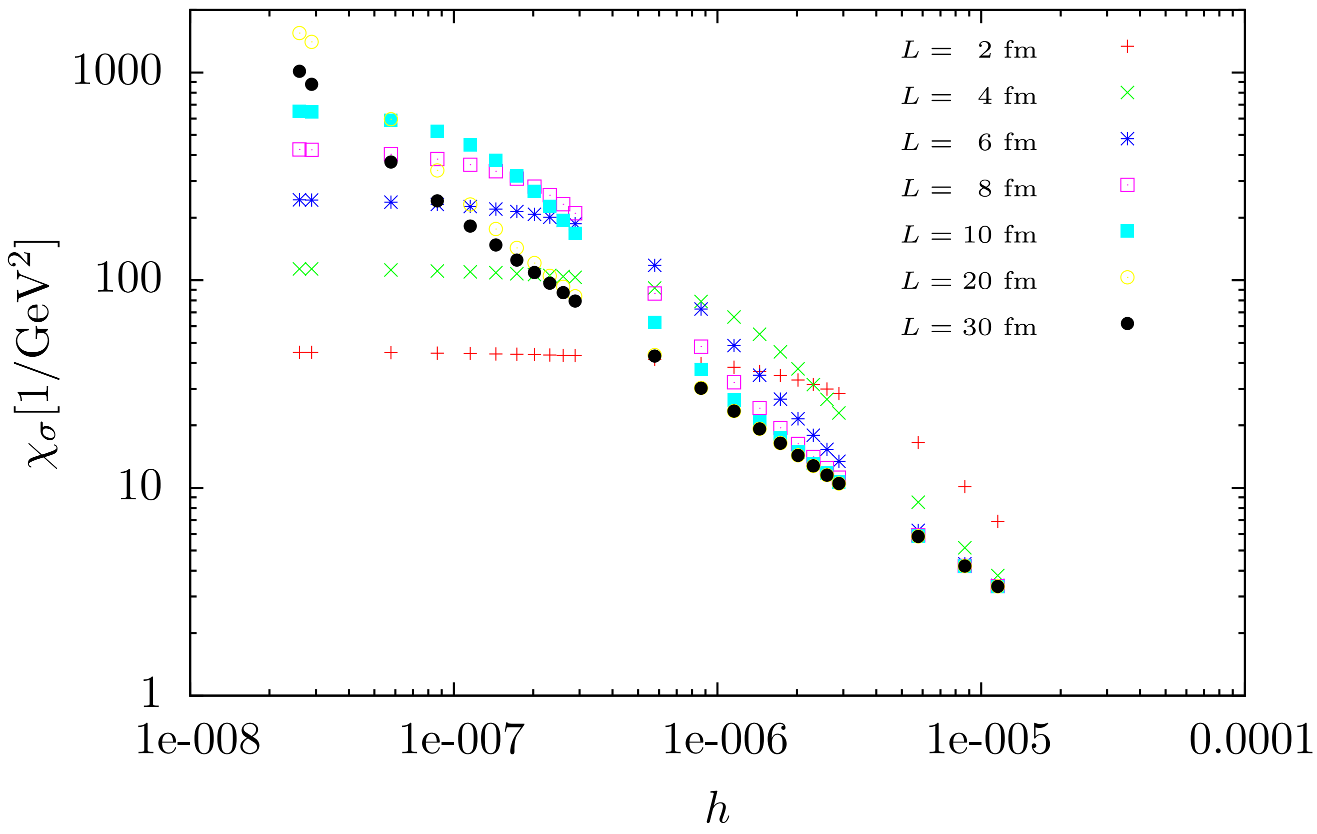
$$M(t, h, L) = L^{-\beta/\nu} Q_M(z, \bar{h}), \quad \bar{h} = hL^{\beta\delta/\nu}$$

- Finite-size scaling function for the susceptibility:

$$\chi_\sigma(t, h, L) = L^{\gamma/\nu} Q_\chi(z, \bar{h})$$

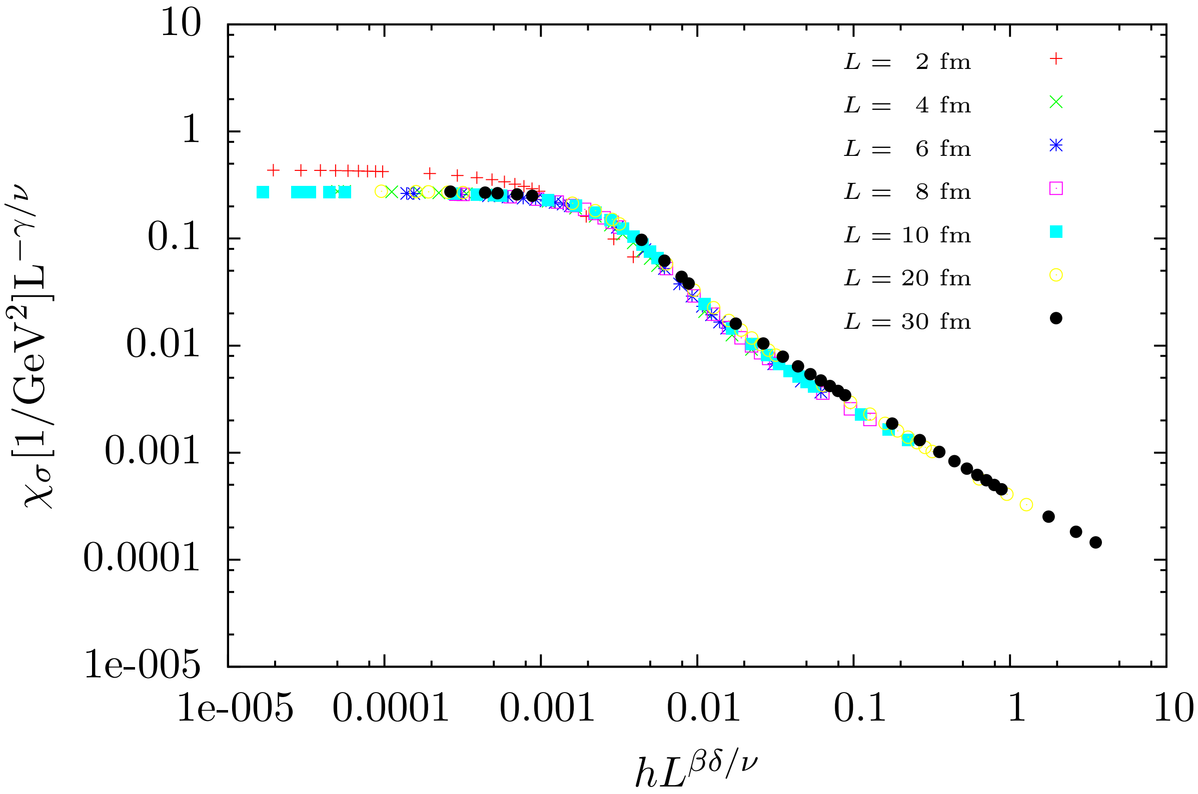
Finite size scaling: Susceptibility χ_σ

- susceptibility in finite volume



Finite size scaling: Susceptibility χ_σ

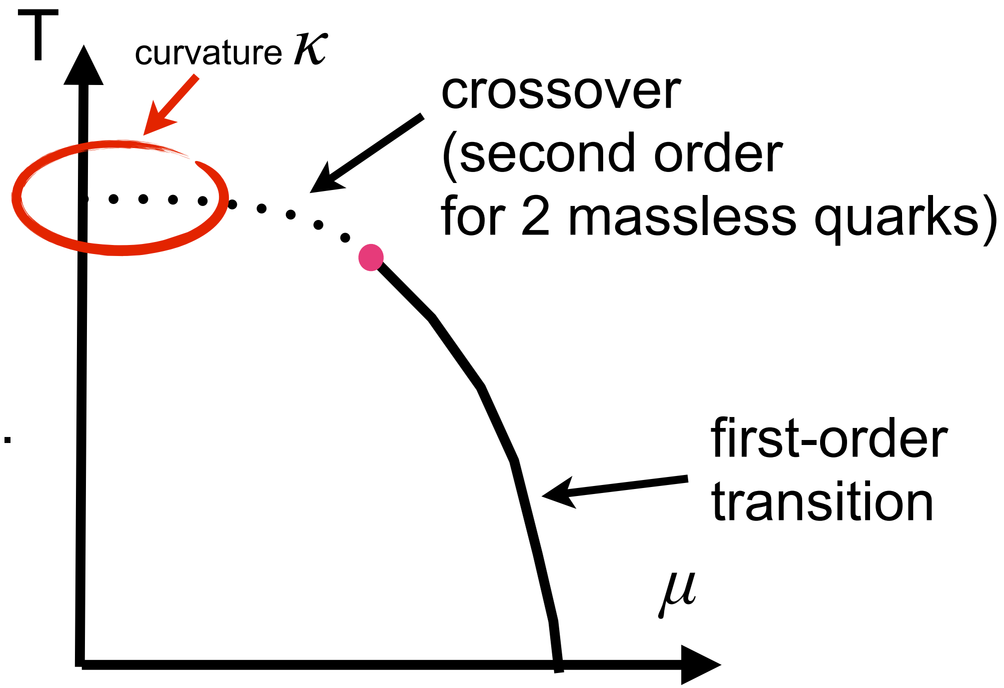
- finite-size scaled susceptibility



L [fm]	M_π [MeV]	$M_\pi L$
4	139	2.82
6	85	2.59
8	60	2.43
10	45	2.30
20	19	1.94

QCD phase diagram

at finite chemical potential
...very schematically...



- second-order phase transition for two flavors in the chiral limit
  [R. D. Pisarski and F. Wilczek, Phys. Rev. D 29 (1984) 338]
- crossover at finite quark masses for finite temperature at $\mu = 0$
- conventional expectation:
  first-order phase transition with critical end point

Curvature of the transition line

- at small baryon chemical potential μ, the phase transition line is characterized by the curvature κ

$$\frac{T_\chi(L, m_\pi, \mu)}{T_\chi(L, m_\pi, \mu = 0)} = 1 - \kappa \left(\frac{\mu}{(\pi T_\chi(L, m_\pi, 0))} \right)^2 + \ldots$$

- "sign problem" in lattice QCD: simulations are difficult at finite μ

RG methods: [J. Braun, Eur. Phys. J. C64, 459 (2009)]

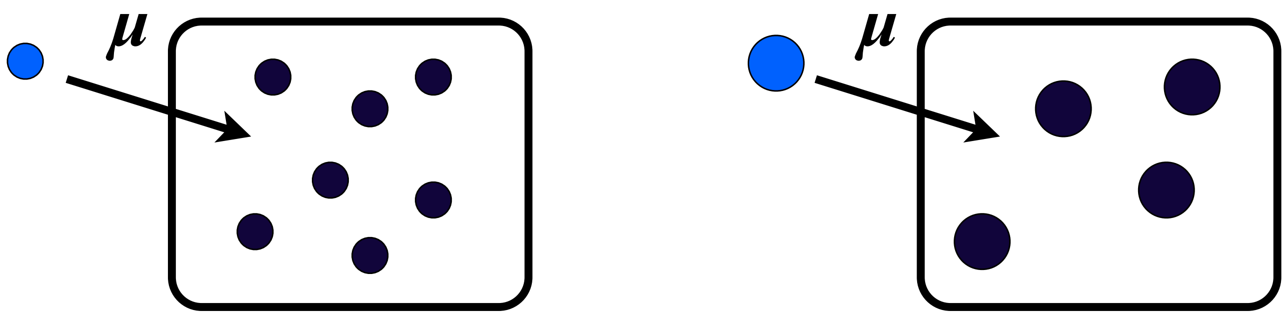
- curvature can be calculated in lattice QCD (imaginary chemical potential, Taylor expansion) [P. de Forcrand and O. Philipsen, Nucl. Phys. B 642 (2002) 290, JHEP 01 (2007) 077; F. Karsch et al., Nucl. Phys. Proc. Suppl. 129, 614 (2004).]

→ differences partially due to finite-volume effects?

Why Finite-volume effects?

- curvature depends on the sensitivity of the system on the chemical potential

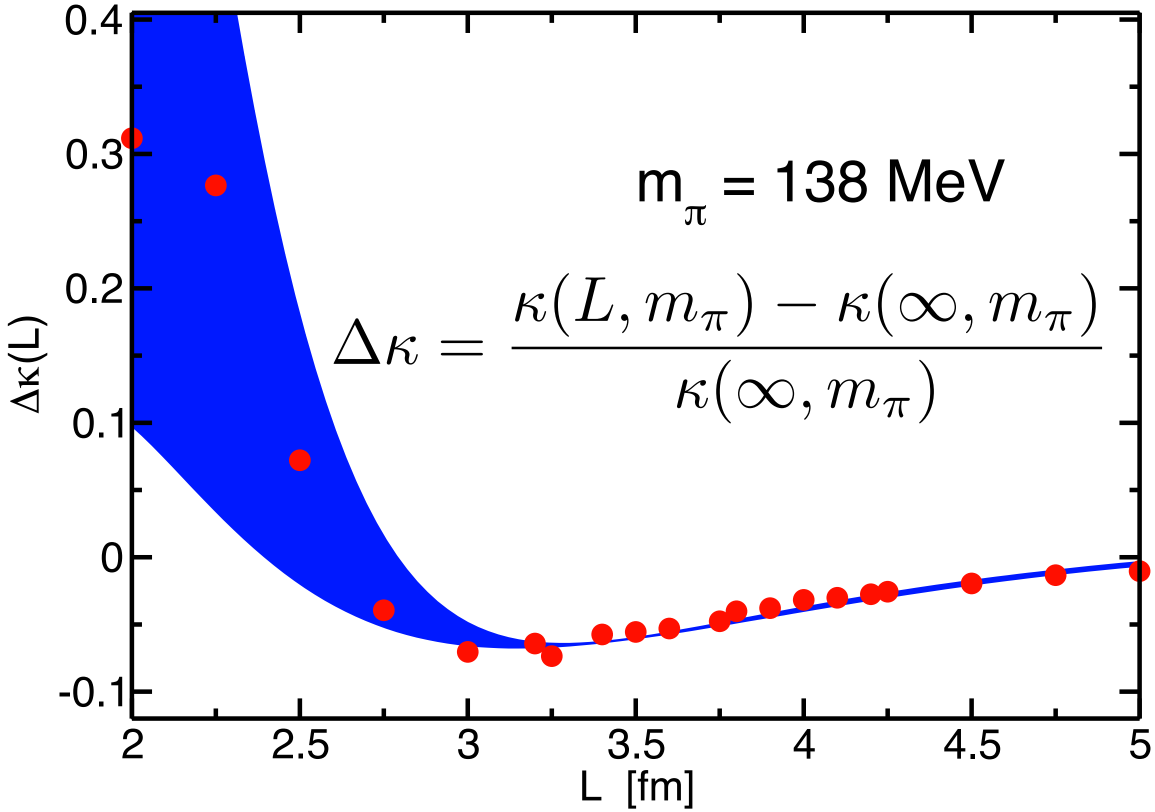
$$\mu = \frac{\partial F}{\partial N_q}\bigg|_{T,V}$$



- sensitivity in turn depends on the "constituent quark mass"
- constituent quark mass affected by volume!

Change of curvature in finite volume

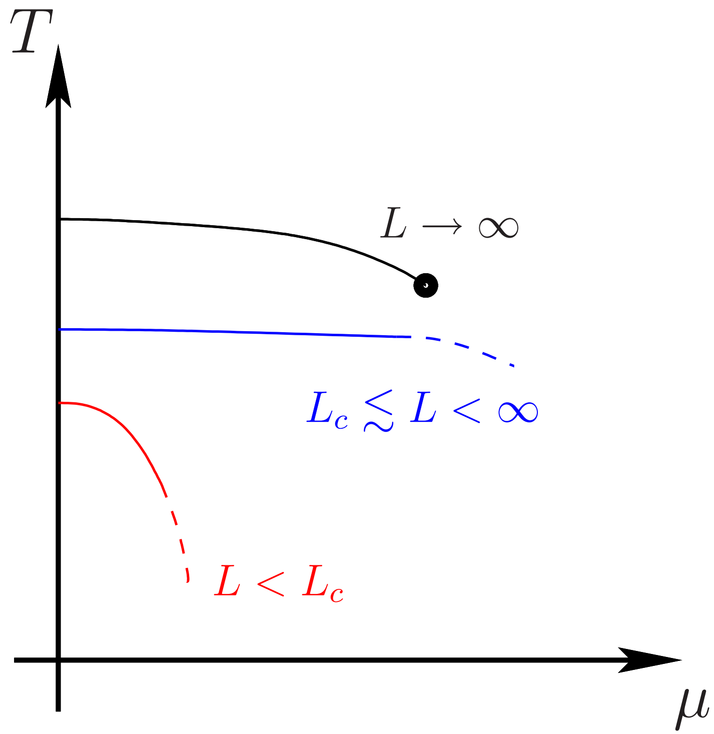
- **periodic** boundary conditions for quarks
- decreasing curvature in intermediate volume
- corresponds to decreasing pion mass/increasing constituent quark mass
- decreased sensitivity to chemical potential



[B.-J. Schaefer, J.Braun, B. Klein]

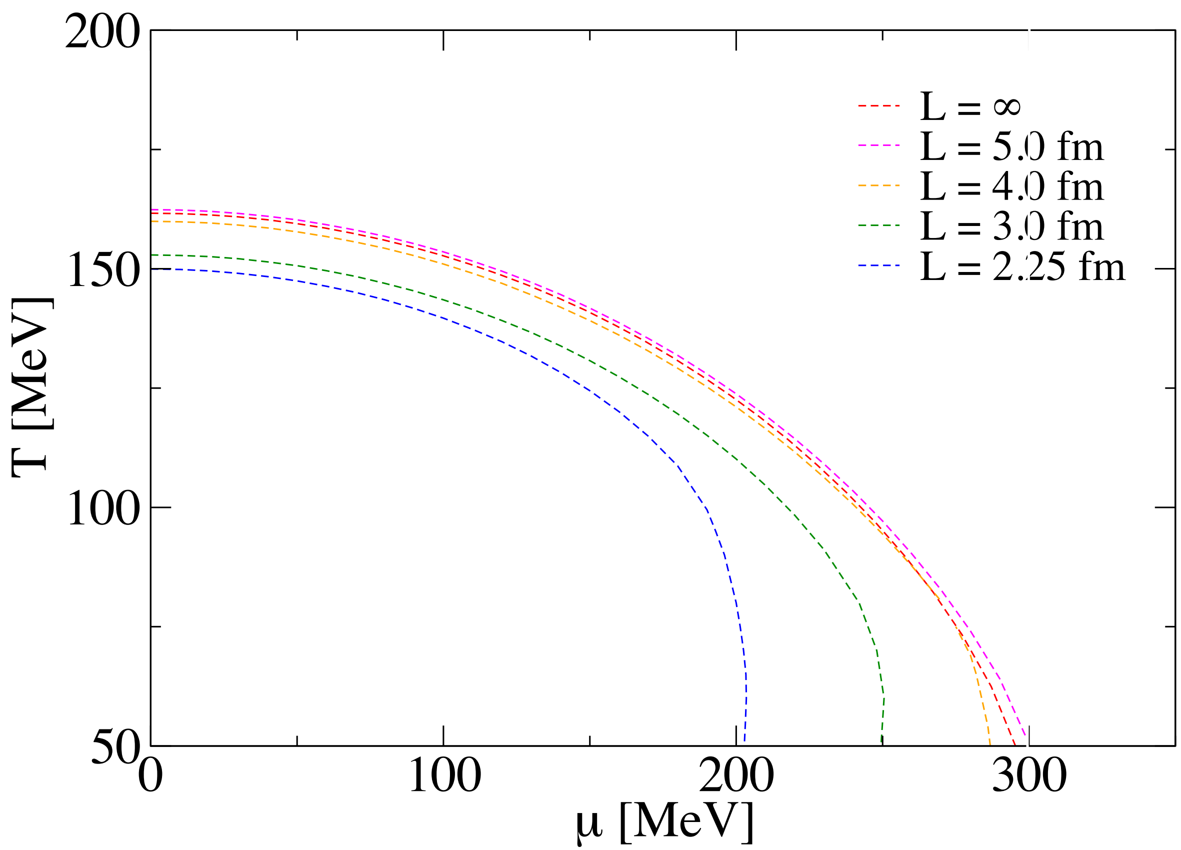
Phase diagram for QCD models in finite volume - qualitative results

- qualitatively clear effects of finite volume on curvature
- phase transition line tends to *flatten* in an intermediate volume range
- curvature increases dramatically for very small volumes



Phase diagram for QCD models in finite volume - first results

- potential discretized on a mesh grid
- first-order phase transition can be determined
- effects on critical point can be determined



[A. Tripolt, B.-J. Schaefer, J.Braun, B. Klein]

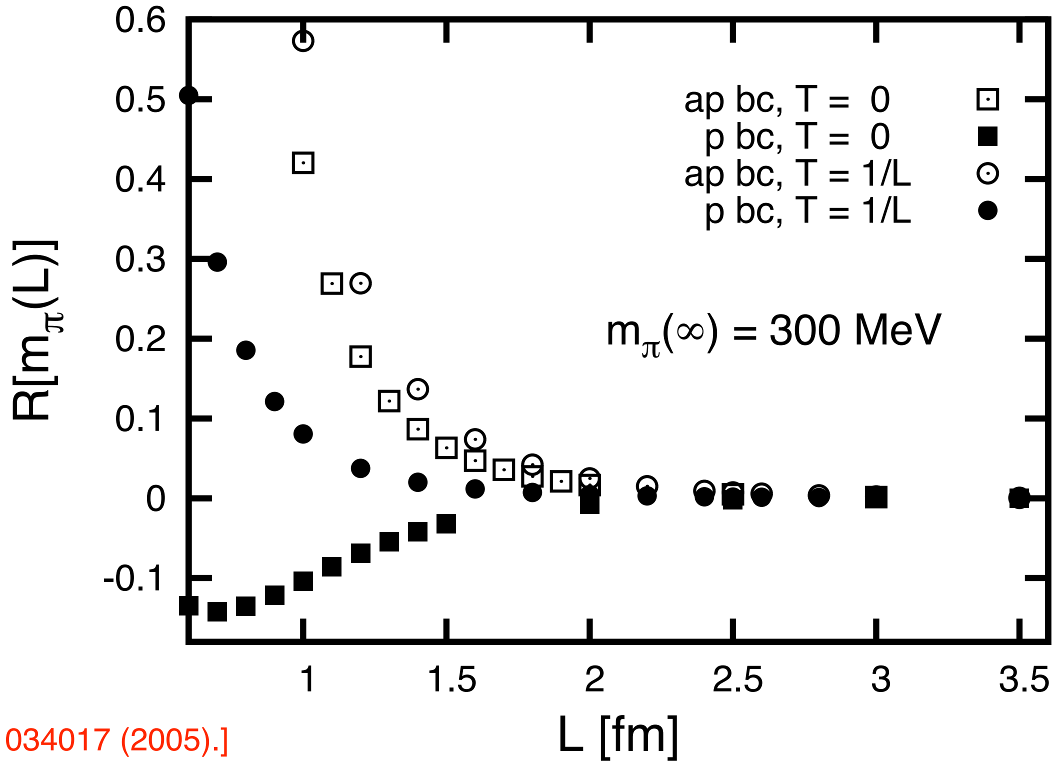
Effects of the quark boundary conditions

- Pion mass shift in V = $L^3 \times 1/T$ in quark-meson model
- periodic vs. anti-periodic quark boundary conditions (b.c.)

$$f_\pi \sim \langle \sigma \rangle$$

$$\langle \bar{\psi}\psi \rangle \sim \langle \sigma \rangle$$

$$m_\pi^2 = m \frac{\langle \bar{\psi}\psi \rangle}{f_\pi^2} \sim \frac{m}{\langle \sigma \rangle}$$

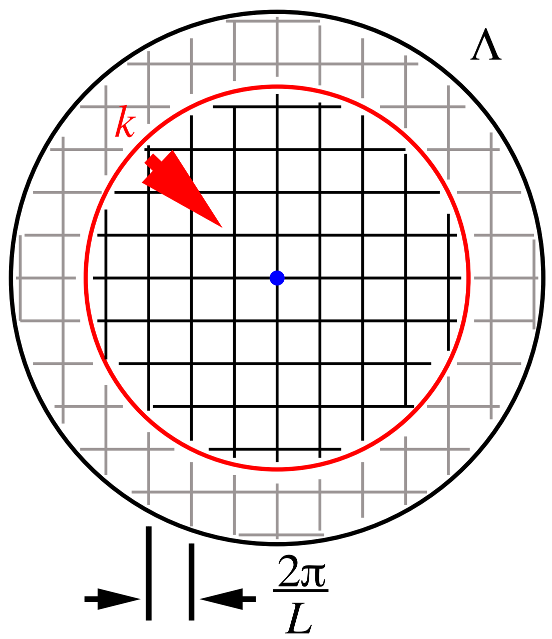


[J. Braun, B. Klein, H.-J. Pirner, Phys. Rev. D72, 034017 (2005).]

[J. Luecker et al., Phys. Rev. D81, 094005 (2010); D.B. Carpenter, C.F. Baillie, Nucl. Phys. B 260, 103 (1985).]

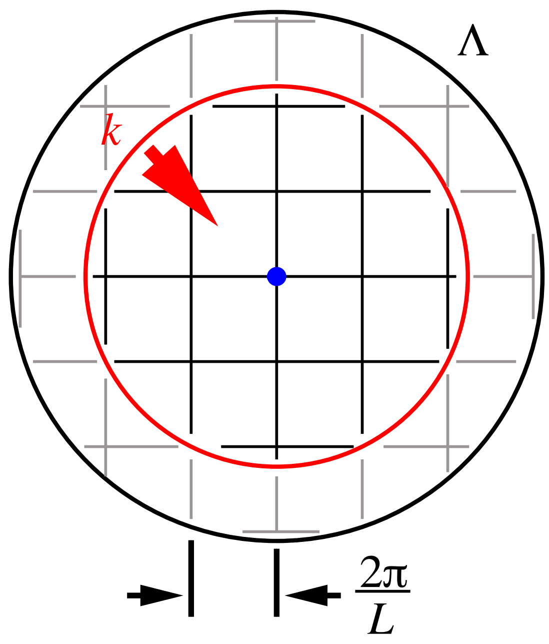
Quark contributions for a finite volume

- quark momentum modes contributing to the condensate (and the constituent quark mass) in a large finite volume
- zero-mode
  for **periodic** b.c.
- no zero mode
  for **anti-periodic** b.c.



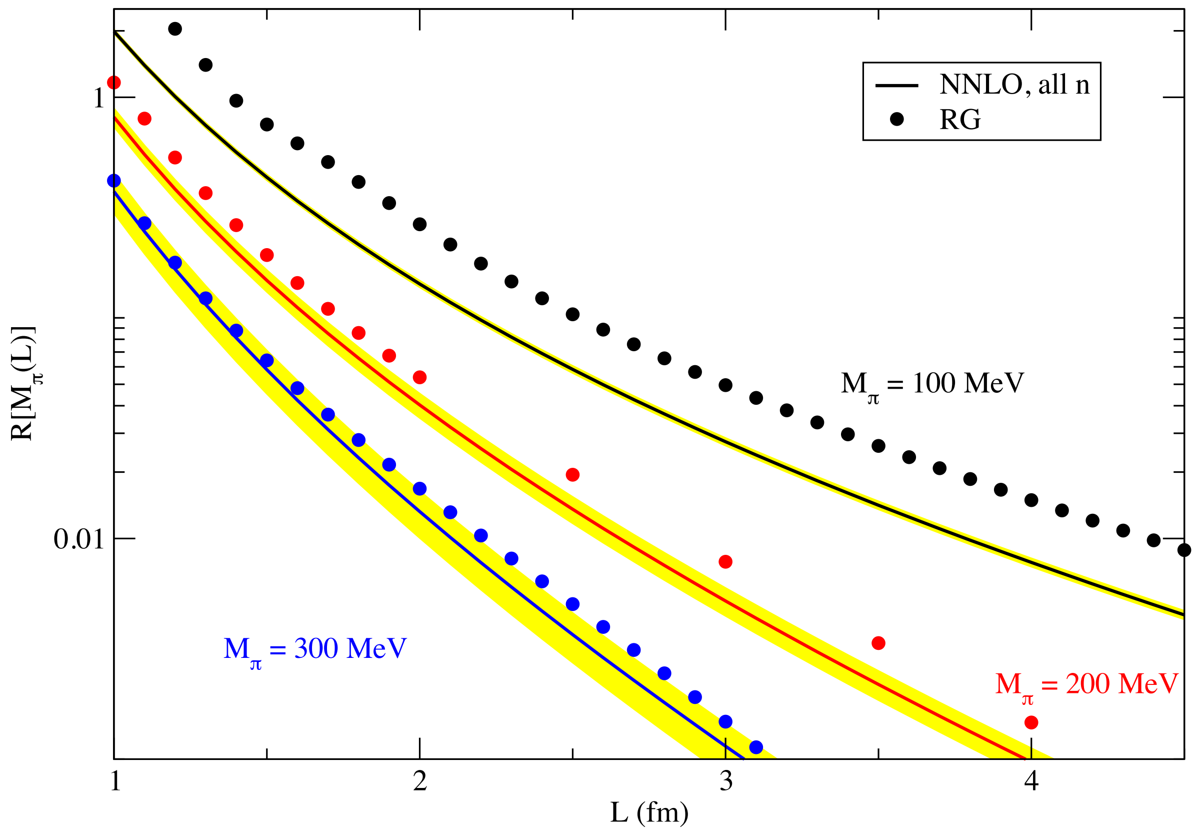
Quark contributions for a finite volume

- quark momentum modes contributing to the condensate (and the constituent quark mass) in a small finite volume
- enhancement of the zero-mode contribution ~1/V for **periodic** b. c.



Comparison to Chiral Perturbation Theory

- comparison of model results with **anti-periodic** boundary conditions from RG to ChPT in NNLO ChPT data thanks to G. Colangelo
  [G. Colangelo, S. Dürr, C. Haefeli, Nucl. Phys. B 271 (2005) 136.]



agreement only for **this** choice of boundary conditions!

keep boundary conditions in mind for the finite-volume analysis of lattice QCD results

Conclusions

- Scaling functions from the functional renormalization group for the analysis of the QCD chiral phase transition: Results from a model for the chiral phase transition

- Finite-size effects in lattice simulations can lead to significant deviations from expected scaling behavior

- Additional finite-volume effects for curvature of transition line is expected

- choice of spatial quark boundary condition is important!

Thanks to my collaborators

- Jens Braun, Universität Jena
- Piotr Piasecki, Technische Universität Darmstadt
- Bernd-Jochen Schaefer, Universität Graz
- Arno Tripolt, Universität Graz