Universality far from equilibrium

(Nonthermal fixed points as quantum amplifier)

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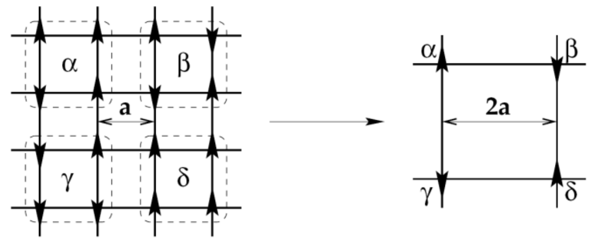
Renormalization group fixed points

RG: ‘microscope‘ with varying resolution of length scale



Denote coarse graining length as inverse characteristic momentum scale

$$\sim 1/k$$

Fixed point: physics looks the same for ‘all‘ resolutions, i.e. becomes independent of *k* (in rescaled units)

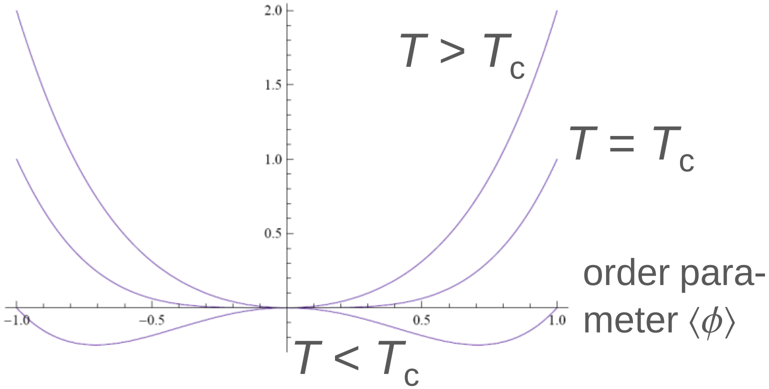
Typically not for *all* resolutions:

- IR fixed point for $k \to 0$
- UV fixed point for $k \to$ ‘∞‘

Examples in and out of equilibrium

a) Thermal equilibrium IR fixed points

effective potential



… associated to second-order phase transitions:

$$\langle\phi\phi\rangle(p) \sim 1/p^{2-\eta}$$

$$\omega(p) \sim p^z$$

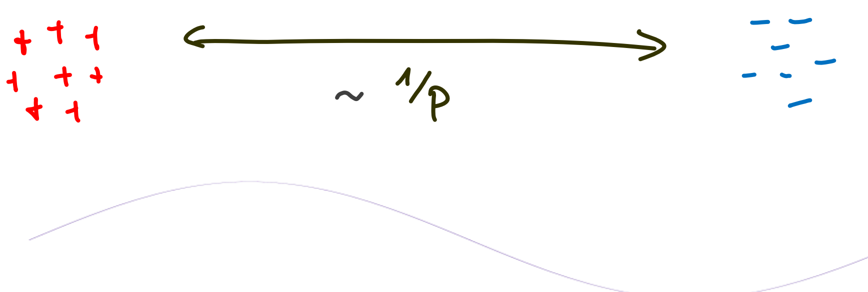
power-law exponents

- η: anomalous dimension
- z: dynamical exponent

b) Far from equilibrium IR fixed points

Important candidate: stationary transport of conserved charges

… IR fixed points associated to diverging time scales in thermalization process



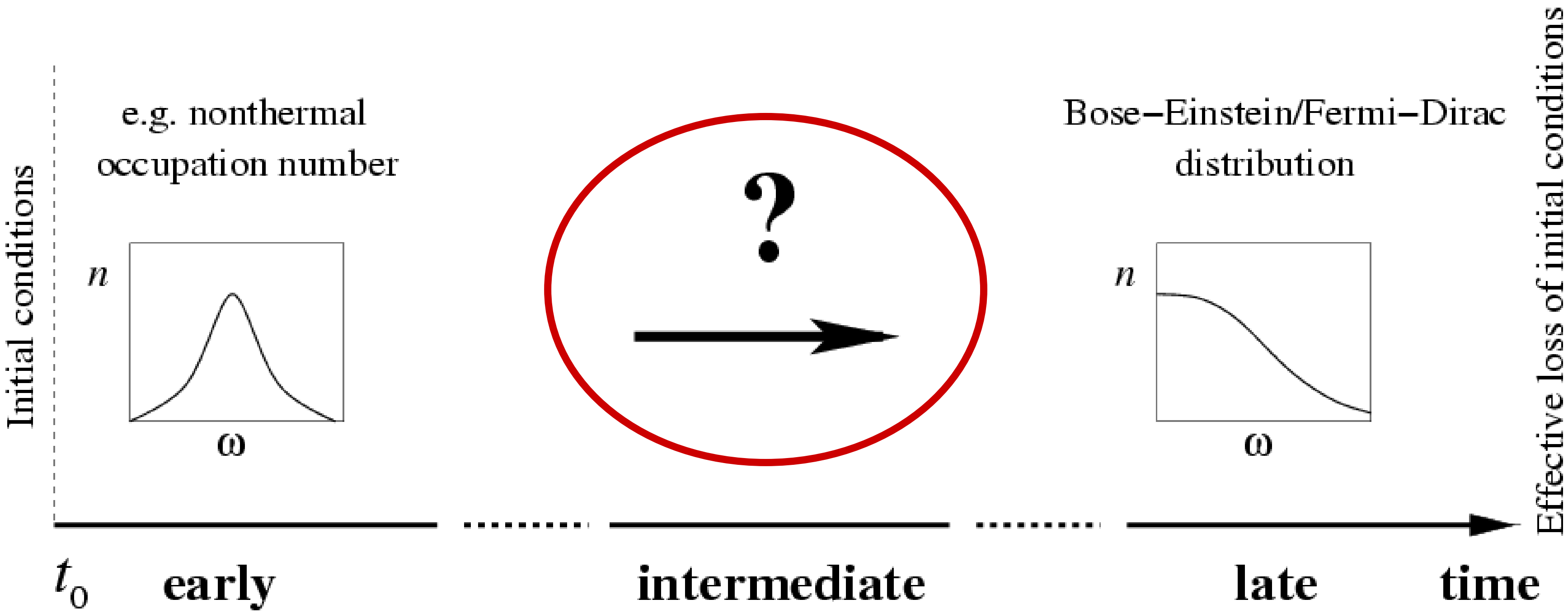
$$\lim_{p\to 0} \partial_t n(t,p) = 0$$

$$n(p) \sim 1/p^\kappa$$

- κ: occupation number exponent

Digression: diverging time scales

Thermalization process in quantum many-body systems, schematically:



- Characteristic nonequilibrium time scales? ***Relaxation? Instabilities?***
- Diverging time scales? E.g. associated to

- ***critical slowing down*** near second-order phase transitions, or
- far from equilibrium IR scaling solutions? → ***nonthermal fixed points***

Experimental example:

Modulation instability and capillary wave turbulence

Instability leads to breaking of waves & weak Kolmogorov wave turbulence

(perturbative!)

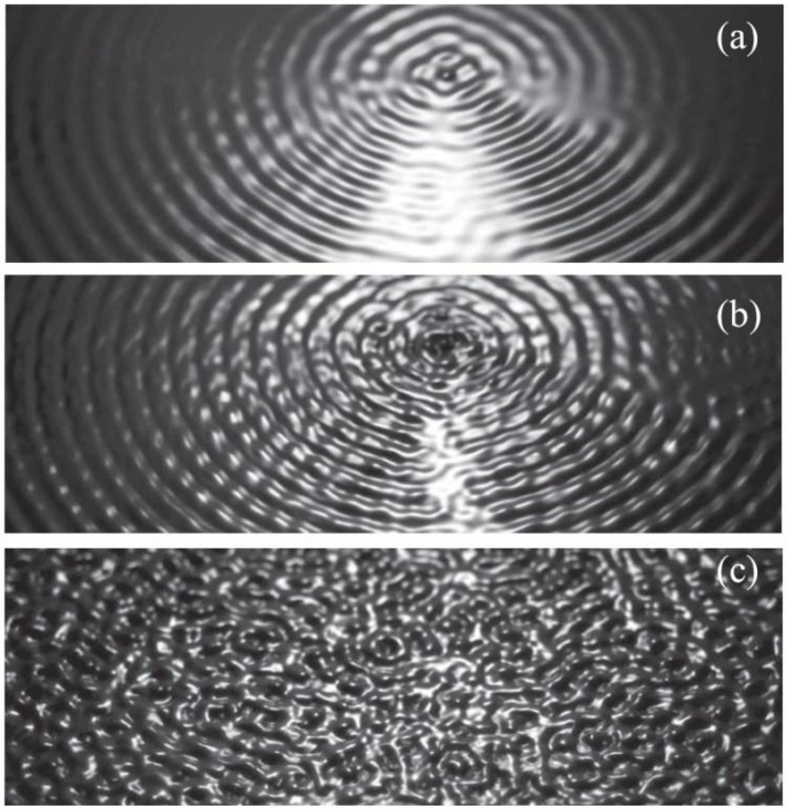
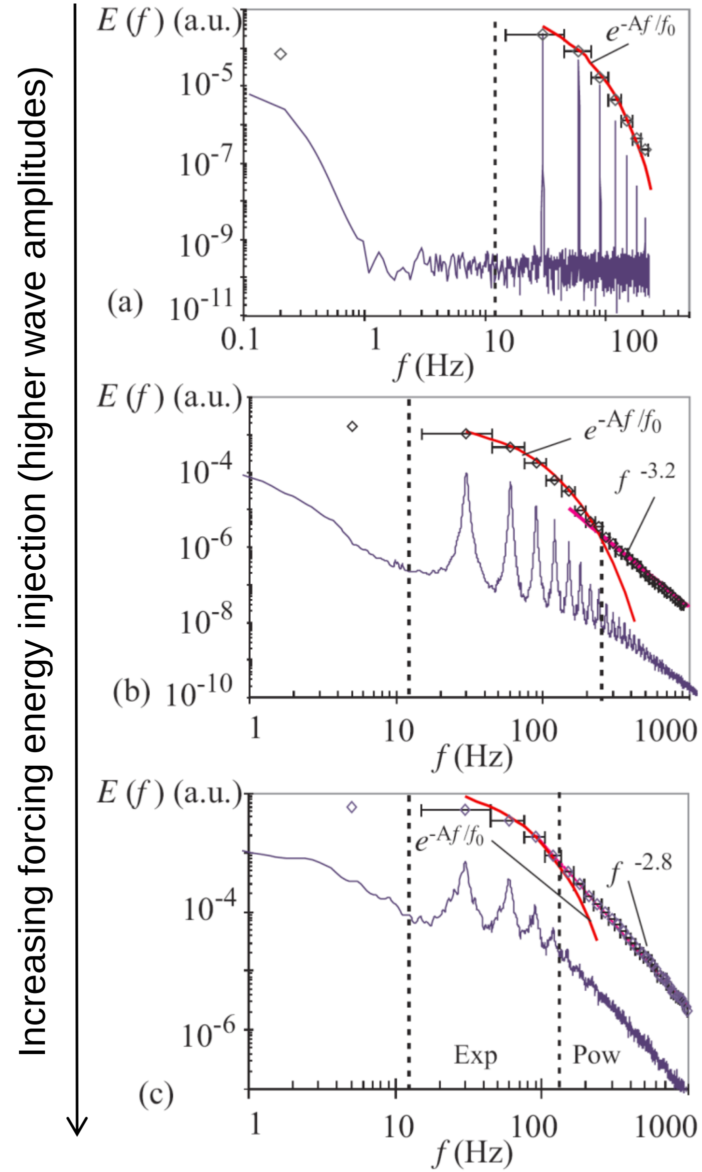


Fig. 1: (Colour on-line) Snapshots of the wave field evolution during the startup: (a) $t = 0.125\,\mathrm{s}$, (b) $t = 0.25\,\mathrm{s}$, (c) $t = 0.625\,\mathrm{s}$.

Xia, Shats, Punzmann, EPL91 (2010) 14002



Energy injection limited by droplet formation!

Fig. 2: (Colour on-line) Frequency spectra of capillary waves at different accelerations. (a) $\Delta a = 0.5g$, (b) $\Delta a = 1.4g$ and (c) $\Delta a = 2.1g$. Open diamonds show the spectral powers E_f of each harmonics.

Digression: weak wave turbulence

E.g. Boltzmann equation for *number conserving* 2↔2 scattering

$$\frac{\mathrm{d}n_1}{\mathrm{d}t} = \int \frac{\mathrm{d}^3 p_2}{(2\pi)^3 2E_2} \int \frac{\mathrm{d}^3 p_3}{(2\pi)^3 2E_3} \int \frac{\mathrm{d}^3 p_4}{(2\pi)^3 2E_4}$$

$$\times \quad \underbrace{\delta^3(p_1 + p_2 - p_3 - p_4)}_{\text{momentum conservation}} \underbrace{\delta(E_1 + E_2 - E_3 - E_4)}_{\text{energy conservation}} (2\pi)^4 \underbrace{|M|^2}_{\sim\lambda^2 \ll 1}$$

$$\times \left(\underbrace{n_3 n_4 (1 + n_1)(1 + n_2)}_{\text{“gain“ term}} - \underbrace{n_1 n_2 (1 + n_3)(1 + n_4)}_{\text{“loss“ term}} \right)$$

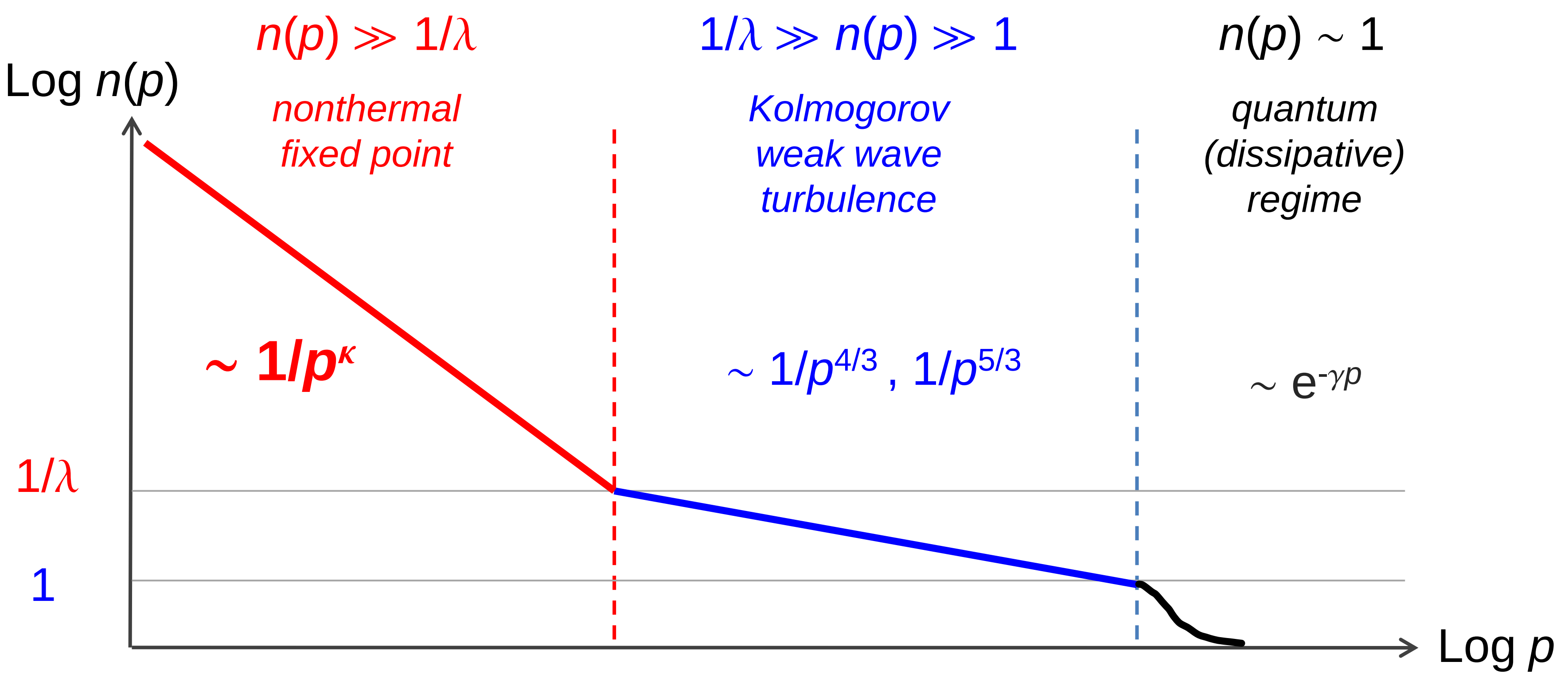
has different stationary solutions in the (classical) regime $n(p) \gg 1$:

1. $n(p) = 1/(e^{\beta\omega(p)} - 1)$ — thermal equilibrium
2. $n(p) \sim 1/p^{4/3}$ — turbulent *particle* cascade
3. $n(p) \sim 1/p^{5/3}$ — *energy* cascade

Kolmogorov, Proc. USSR Acad. Sci., 30 (1941) 299

Solutions 2 and 3 are limited to the “window“ $1/\lambda \gg n(p) \gg 1$, since for $n(p) \gg 1/\lambda$ the n↔m scatterings for n,m=1,..,∞ are as important as 2↔2

Weak vs. strong wave turbulence



Predicted scaling exponents for nonperturbative regime:

a) $n(p) \sim 1/p^4$ strong turbulence *particle* cascade

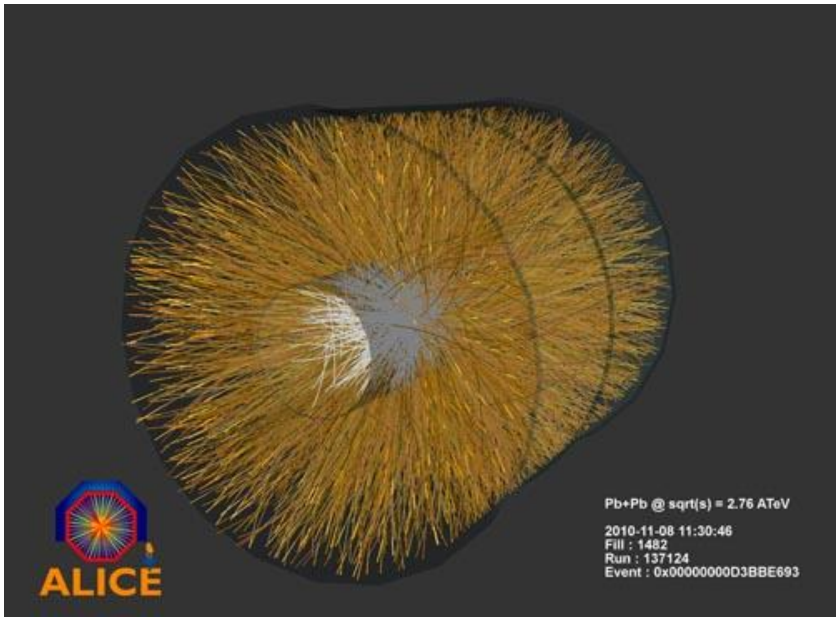
b) $n(p) \sim 1/p^5$ *energy* cascade

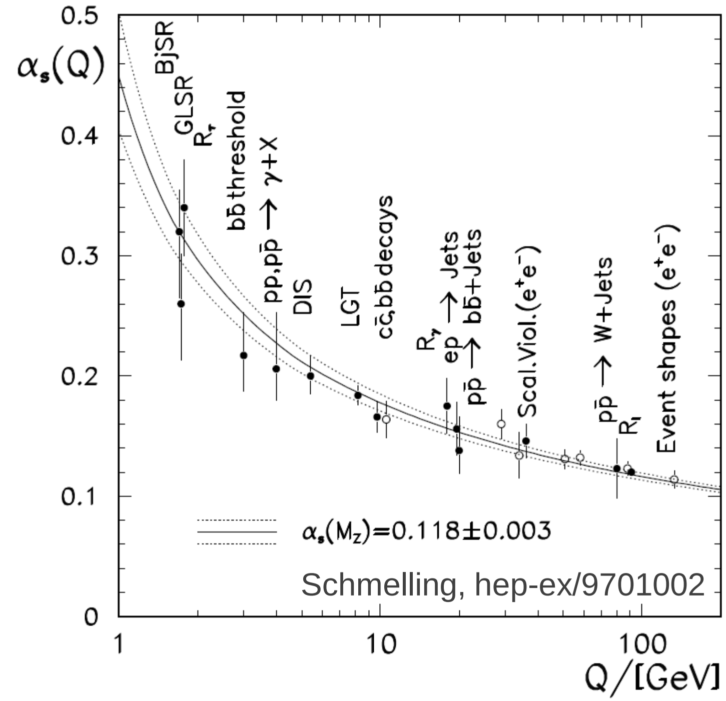
Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603

Experimental verification requires sufficient energy injection!

Experimental candidates for *nonperturbatively* large densities

1) Relativistic heavy-ion collisions explore strong interaction matter starting from a transient ***nonequilibrium*** state



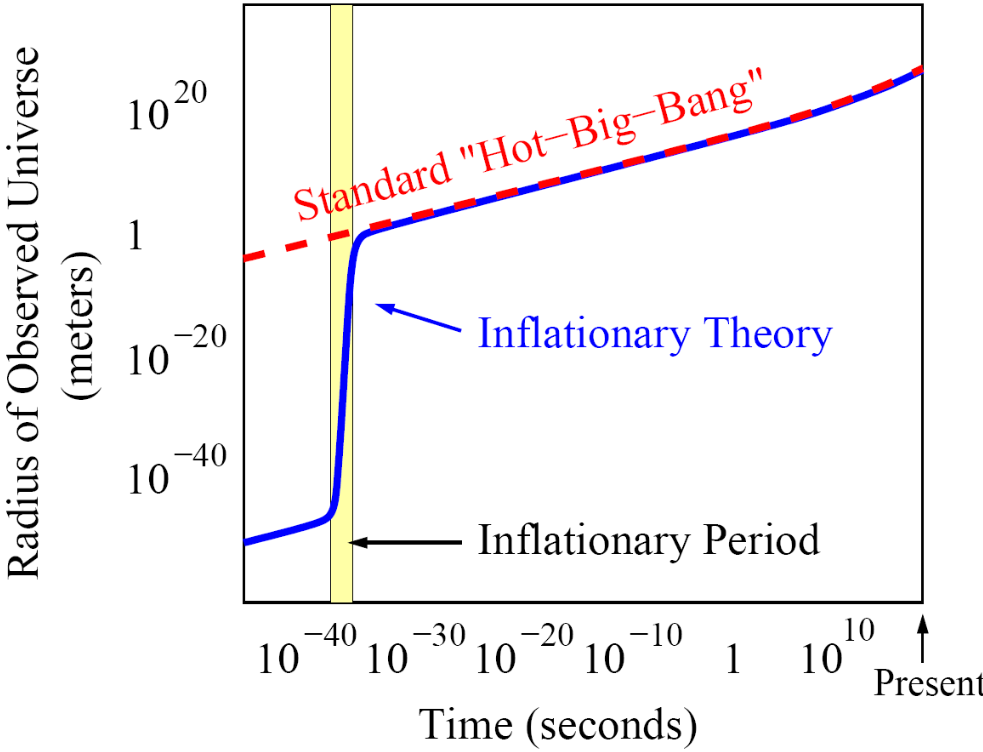


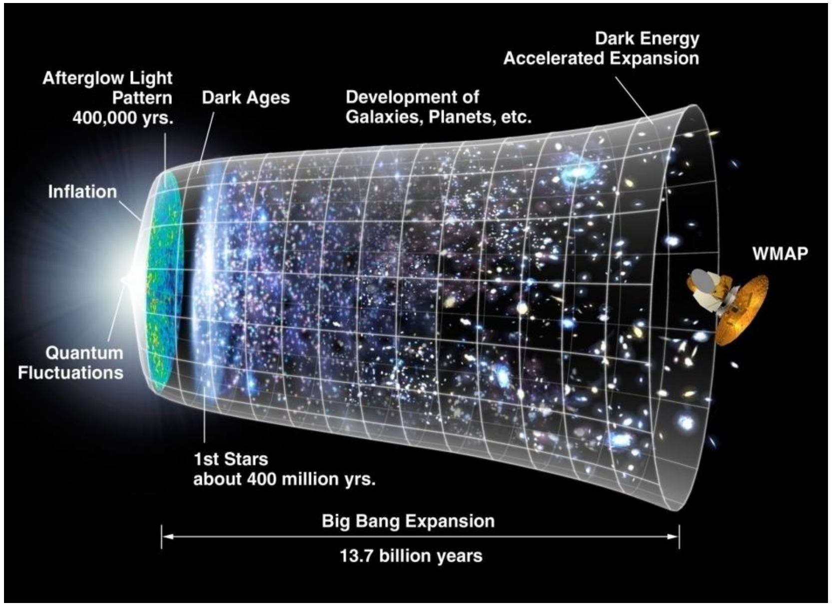
CGC: Energy density of gluons with typical momentum Q_s (at time $1/Q_s$)

$$\epsilon \sim \frac{Q_s^4}{\alpha_s}$$ i.e. occupation numbers $$n(p \lesssim Q_s) \sim \frac{1}{\alpha_s}$$ ***nonperturbative!***

→ see also last week's talks, this conference (jointly with TFT workshop)

2) Early universe at the end of inflation, schematic evolution:





- Energy density of matter (~ a^{-3}) and radiation (~ a^{-4}) dilutes by expansion

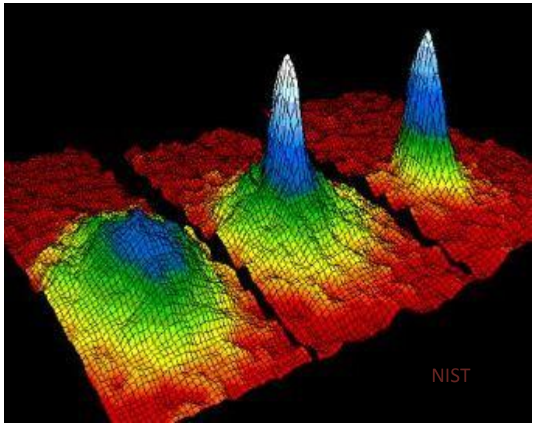
→ energy at the end of inflation stored in homogeneous field with amplitude

$$\phi_0 \sim \frac{M_0}{\sqrt{\lambda}} \quad \text{i.e. after preheating} \quad n(p \lesssim M_0) \sim \frac{1}{\lambda}$$

nonperturbatively large in quartic self-coupling $\lambda \simeq 10^{-12}$!

3) Ultracold quantum gases

...atoms at nanokelvins



$$H = \int d^3x \left\{ -\Psi_x^\dagger \frac{\nabla^2}{2m} \Psi_x + \frac{g}{2} \Psi_x^\dagger \Psi_x^\dagger \Psi_x \Psi_x \right\}$$

- $mg/4\pi$ determined by s-wave scattering length
- diluteness condition:

$$\tilde{g} \equiv (N/V)^{1/3} (mg/4\pi) \ll 1$$

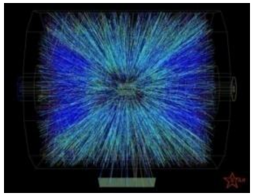
- nonperturbatively large occupation number of momentum mode *p* ***if***

$$n(p) = \langle \Psi^\dagger \Psi \rangle (p) \sim \frac{1}{\tilde{g}}$$

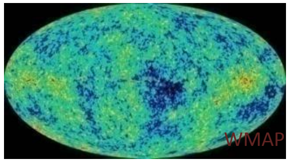
→ see also talk by Boris Nowak, this conference

Universality far from equilibrium

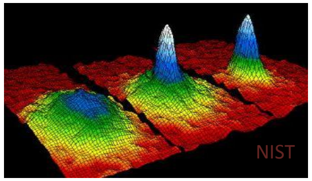
Heavy-ion collisions
(~100 MeV)



Early-universe (p)reheating
after inflation (~10^{16} GeV)



Cold quantum gas dynamics
(~10^{-13} eV)



Nonequilibrium instabilities

Nonthermal fixed points

Very different microscopic dynamics (instabilities) can lead to same macroscopic *scaling phenomena for bosons* (strong turbulence)

Berges, Rothkopf, Schmidt ‘08; Berges, Hoffmeister ‘09; Scheppach, Berges, Gasenzer ‘10; Carrington, Rebhan ‘10; Nowak, Sexty, Gasenzer ’11; Berges, Sexty ’11; …

Nonperturbative functional descriptions

Γ

1. Two-particle irreducible expansions

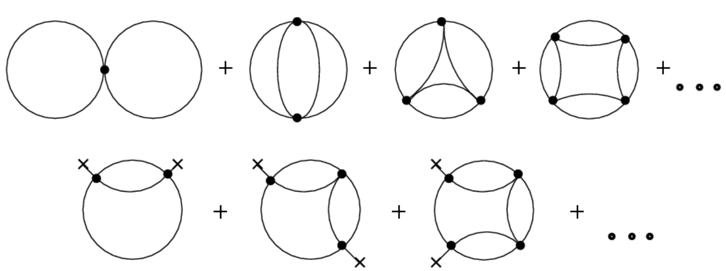
$$\Gamma[\phi, G] = S[\phi] + \frac{1}{2}\mathrm{Tr}\ln G^{-1} + \frac{1}{2}\mathrm{Tr}\, G_0^{-1}(\phi) G + \Gamma_2[\phi, G]$$

$$\delta\Gamma/\delta G = 0$$

Luttinger, Ward ´60; Baym ´62; Cornwall, Jackiw, Tomboulis ´74,…

2PI-1/N to NLO: ($\lambda(\phi_a\phi_a)^2$)

$$\Gamma_2[\phi, G] \;=$$



Berges ´02; Aarts, Ahrensmeier, Baier, Berges, Serreau ´02

2. Functional renormalization group

$$\Gamma_k[\phi] = \Gamma[\phi, G] + \frac{1}{2}\mathrm{Tr}\, G R$$

$$R = R(k)$$

Wetterich '93

$$\frac{\partial \Gamma_k[\phi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\, G \frac{\partial R}{\partial k}$$

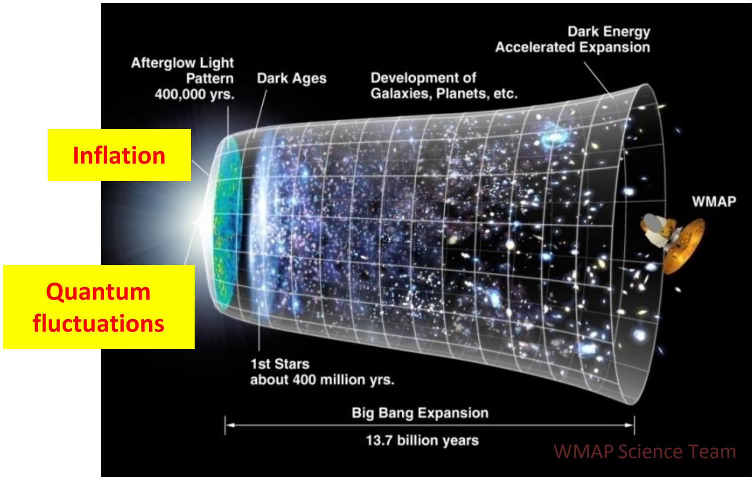
→ Real-time functional renormalization group

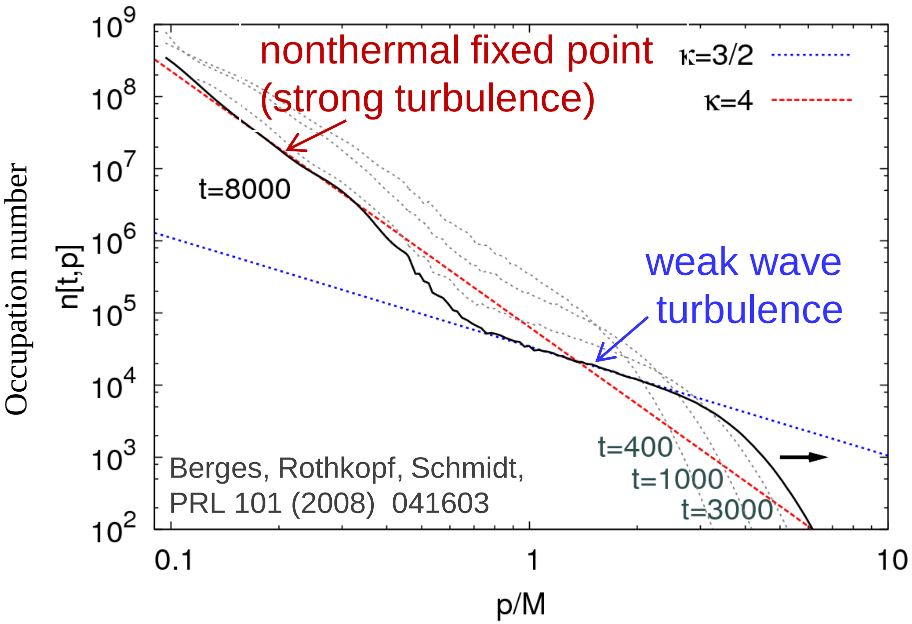
Canet, Delamotte, Deloubriere, Wschebor '04; Mitra, Takei, Kim, Millis '06; Gezzi, Pruschke, Meden '07; Jacobs, Meden, Schoeller '07; Gasenzer, Pawlowski '08; Berges, Hoffmeister '09; Kloss, Kopietz '10;…

+ 3. classical lattice simulations

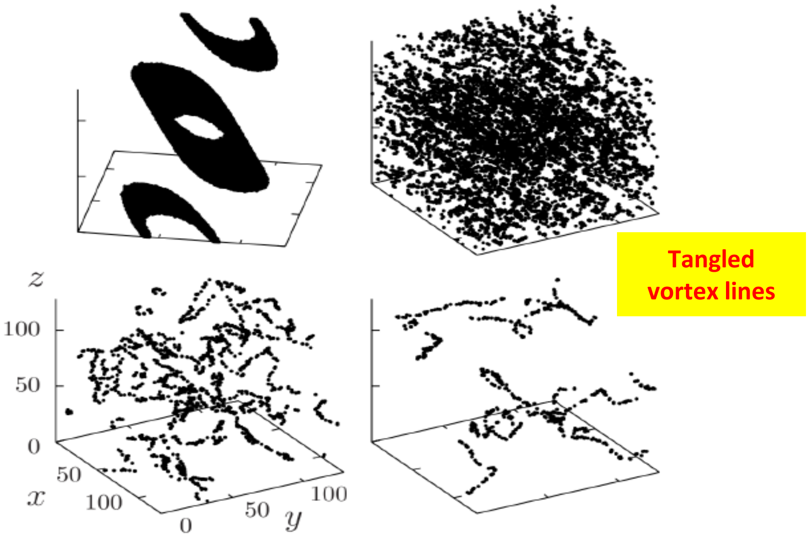
Strong turbulence: $\lim_{p\to 0} n(p) \sim \frac{1}{p^4}$ ← *universal scaling exponent*

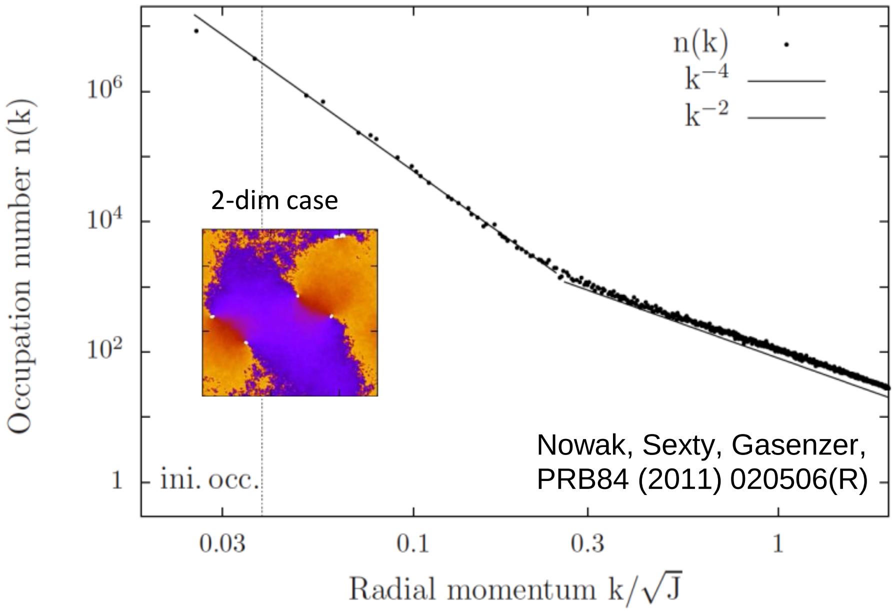
- Reheating dynamics after chaotic inflation





- Superfluid turbulence in a cold Bose gas





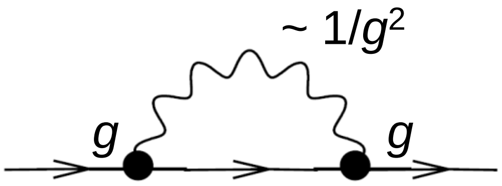
Nonthermal fixed points as quantum amplifier

Berges, Gelfand, Pruschke, PRL 107 (2011) 061301

- Fermions: $n_\psi(\text{p}) \leq 1$ (Pauli principle)

  → *no* classical-statistical approximation
  → *dramatic enhancement of genuine quantum effects*
  if coupled to nonperturbatively occupied bosons!

E.g. dressed fermion self-energy correction in QCD:



$\sim \; O(1)$

All fermion quantum corrections 'saturate' parametrically to order one!

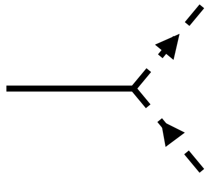
Requires real-time lattice simulations with dynamical fermions in QCD!

→ test claims with simpler model, where 2PI as well as lattice works

Model

- generic interaction of Yukawa type for N_f massless Dirac fermions:

$$-\frac{g}{N_f}\bar{\psi}_i\left(\frac{1-\gamma^5}{2}\Phi^\dagger_{ij}+\frac{1+\gamma^5}{2}\Phi_{ij}\right)\psi_j$$


g

→ couples left- and right-handed components

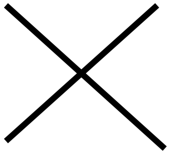
$$\psi_L=\frac{1-\gamma^5}{2}\psi \quad , \quad \psi_R=\frac{1+\gamma^5}{2}\psi$$

i.e. acting like a mass term for $\langle\, \Phi \,\rangle \neq 0$

- we consider N_f = 2 with symmetry group $SU_L(2)\times SU_R(2)\sim O(4)$

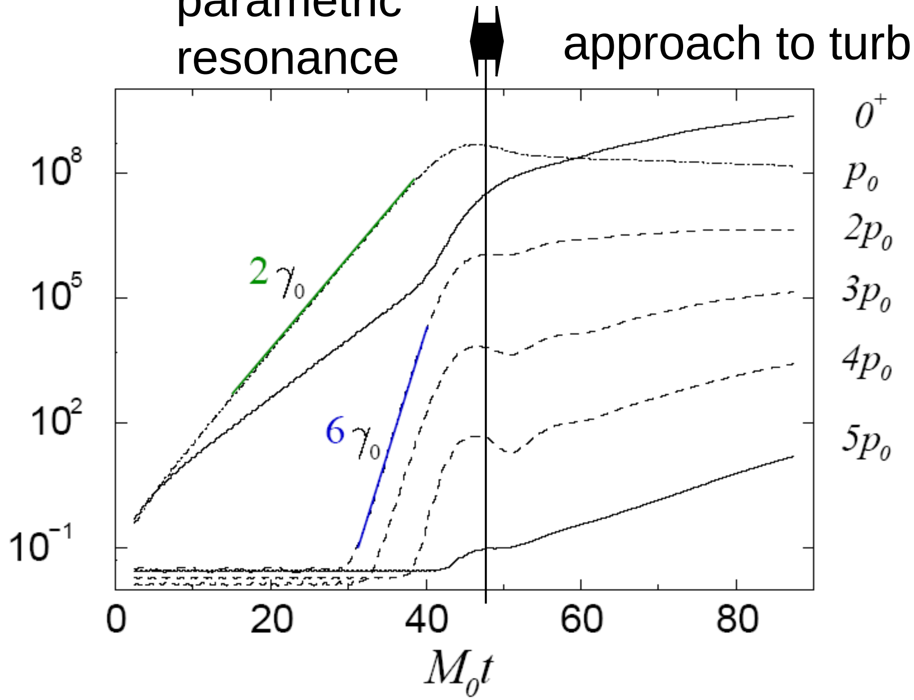
$$\Phi=\frac{1}{2}\left(\sigma+i\vec{\pi}\vec{\tau}\right)$$

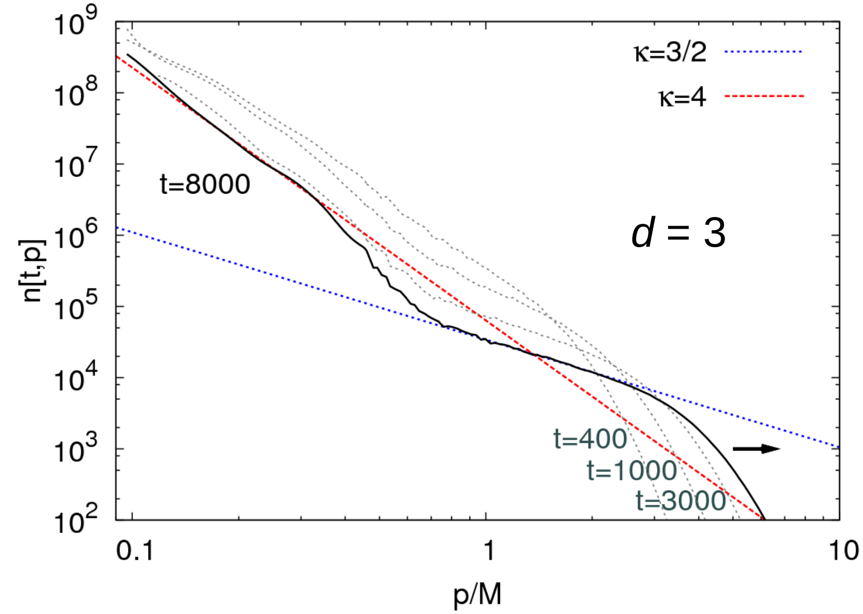
→ *N*=4 component linear σ-model with quartic self-interaction

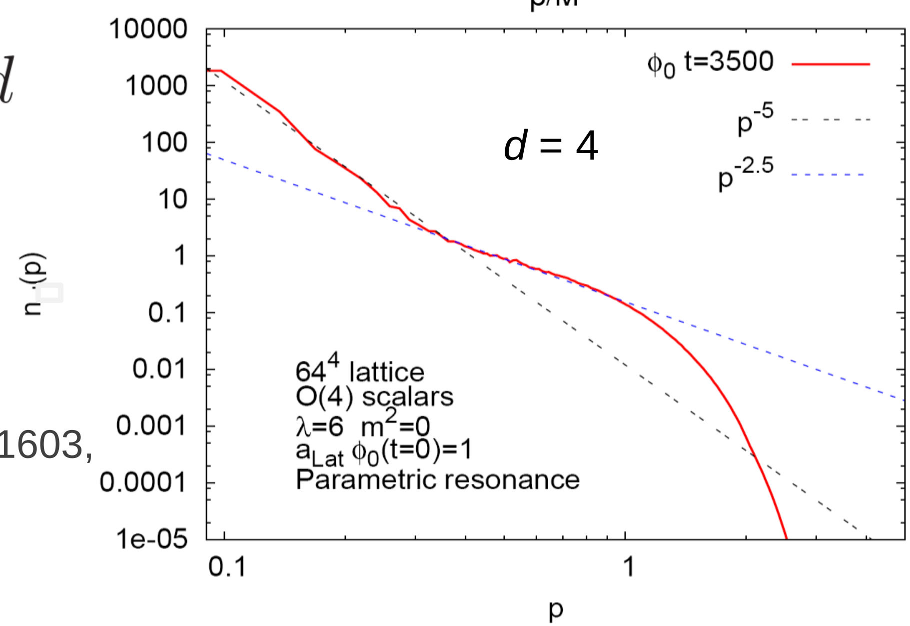

λ

Bosonic sector ($g = 0$)

parametric resonance ⟷ approach to turbulence:







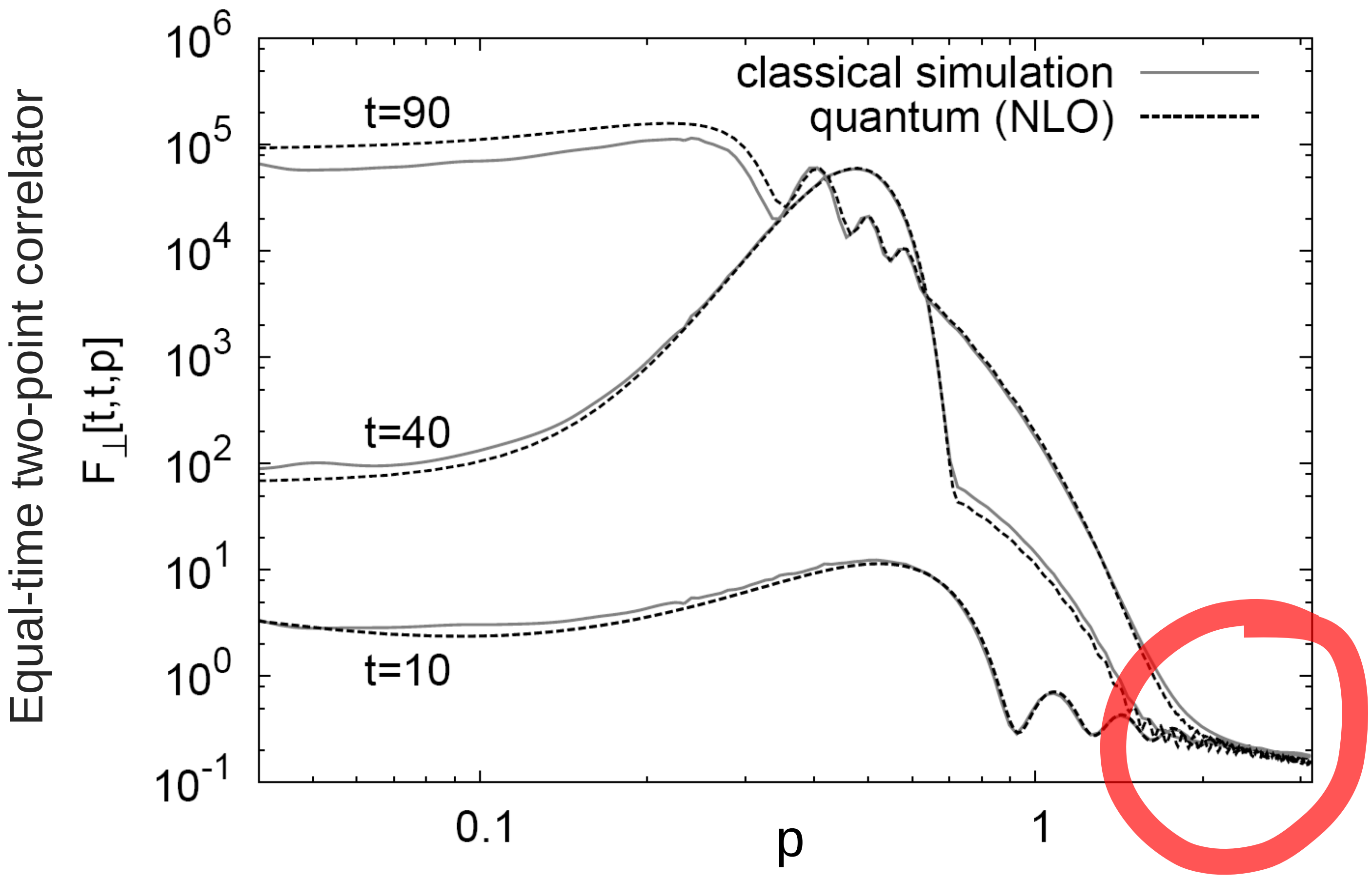
$n(t,p) \sim p^{-\kappa}$ with $\kappa = -\eta + z + d$

→ $\kappa = 4$ for $d = 3$,
$\kappa = 5$ for $d = 4$ ✓ IR

for $z = 1$ (relativistic), $\eta = 0$

Berges, Rothkopf, Schmidt, PRL 101 (2008) 041603,
Berges, Hoffmeister, NPB 813 (2009) 383,
Berges, Sexty, PRD 83 (2011) 085004

Comparing classical to quantum (g=0)



Practically no *bosonic* quantum corrections at the end of instability

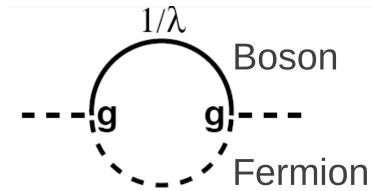
Accurate nonperturbative description by quantum 2PI-1/*N* to NLO

Fermions: failure of semi-classical approach

LO: $$iD^{-1}_{0,ij}(x,y) = \left[i\gamma^\mu \partial_\mu - m_\psi - \frac{g}{N_f}\phi(t) \right] \delta^{(4)}(x-y)\,\delta_{ij}$$

Baacke, Heitmann, Pätzold, *PRD* 58 (1998) 125013; Greene, Kofman, *PLB* 448 (1999) 6; Giudice, Peloso, Riotto, Tkachev, *JHEP* 9908 (1999) 014; Garcia-Bellido, Mollerach, Roulet, JHEP *0002* (2000) 034; …

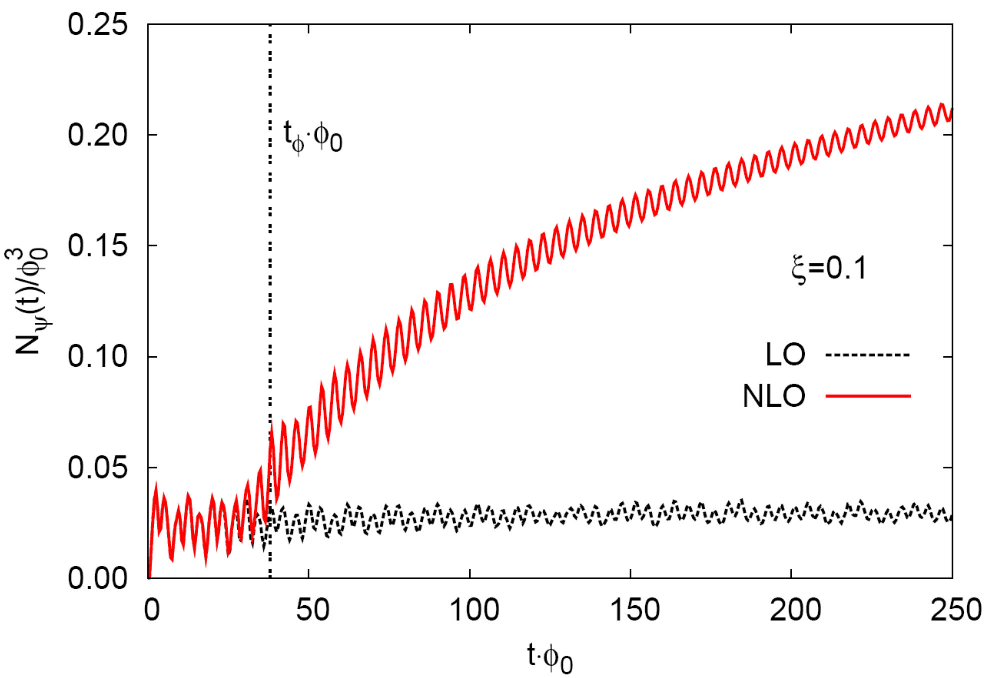
2PI-NLO: +



$\sim \frac{g^2}{\lambda}$

small self-coupling λ leads to *large* corrections!

Berges, Gelfand, Pruschke, PRL 107 (2011) 061301



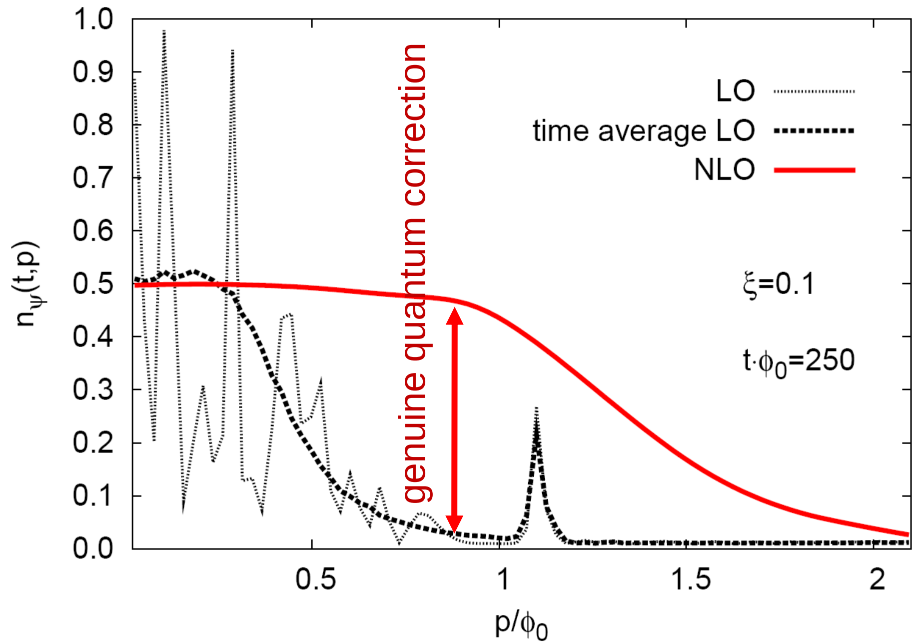
Parametric resonance preheating

$m_\psi = 0$

$$\xi \equiv g^2/\lambda$$

$$\phi = \phi_0 \sqrt{6N_s/\lambda}$$

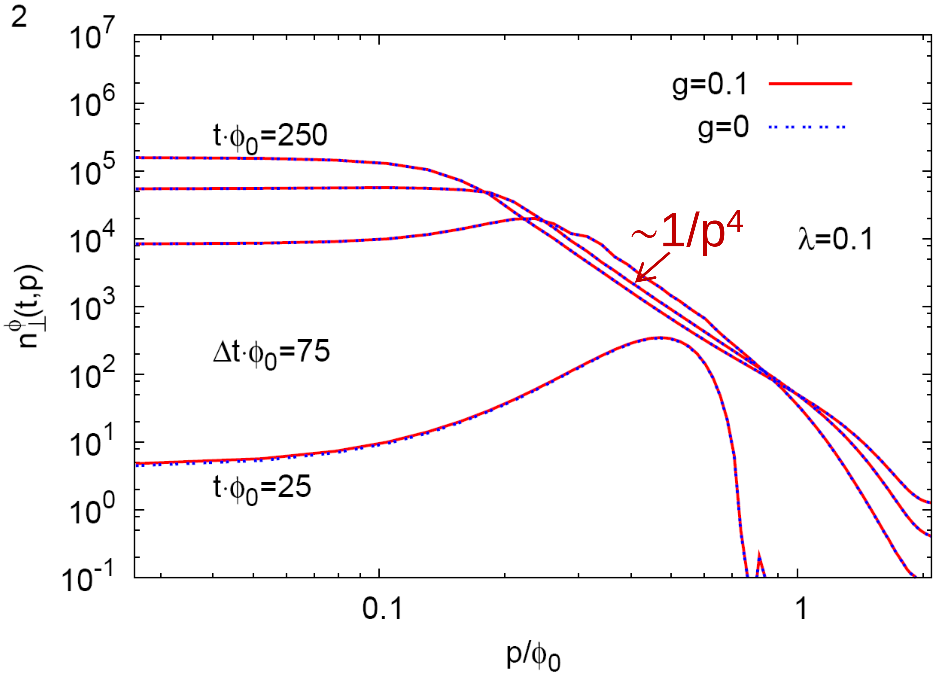
Occupation number distributions



Fermions

IR fermions thermally occupied

Bosons



Bosons still far from equilibrium

Nonequilibrium fermion spectral function

$$\rho(x,y) = i \left\langle \{\psi(x), \bar{\psi}(y)\} \right\rangle$$

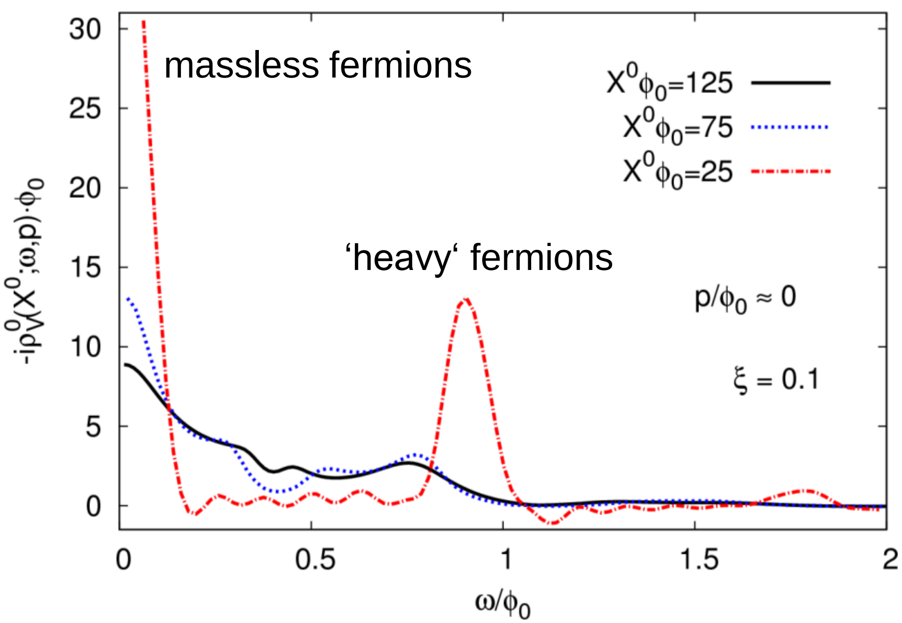
$$\rho_V^\mu = \frac{1}{4}\mathrm{tr}\,(\gamma^\mu \rho)$$ vector components

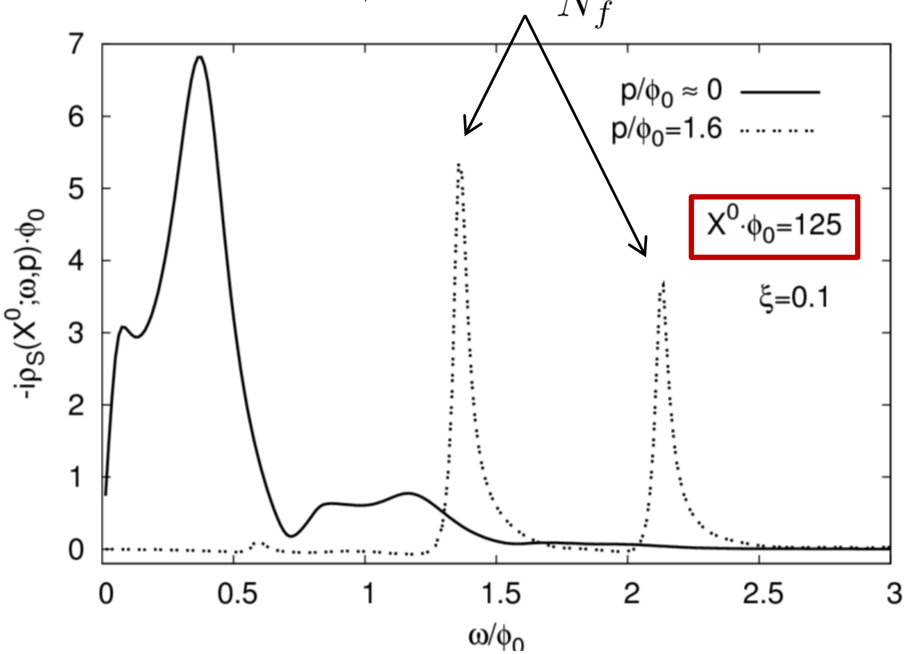
$$\rho_S = \frac{1}{4}\mathrm{tr}\,(\rho)$$ scalar component

quantum field anti-commutation relation: $-i\rho_V^0(t,t;\mathbf{p}) = 1$

Wigner transform: ($X^0 = (t + t')/2$)

$$M_\psi^{\mathrm{eff}}(t) \simeq \pm \frac{g}{N_f}|\phi(t)|$$





Lattice simulations with dynamical fermions

Consider general class of models including lattice gauge theories with covariant coupling to fermions:

$$\mathcal{L} = \frac{1}{2}\partial\Phi^*\partial\Phi - V(\Phi) + \sum_k^{N_f} \left[i\bar{\Psi}_k\gamma^\mu\partial_\mu\Psi_k - \bar{\Psi}_k(MP_L + M^*P_R)\Psi_k \right]$$

with $M = m - g\Phi(x)$, $P_L = \frac{1}{2}(1-\gamma^5)$, $P_R = \frac{1}{2}(1+\gamma^5)$

$$\int \prod_k D\Psi_k^+ D\Psi_k e^{i\int \mathcal{L}(\Phi,\Psi^+,\Psi)} \Longrightarrow \boxed{\partial_x^2\Phi(x) + V'(\Phi(x)) + N_f J(x) = 0}$$

$$J(x) = J^{\mathrm{S}}(x) + J^{\mathrm{PS}}(x)$$

$$J^{\mathrm{S}}(x) = -g\left\langle\bar{\Psi}(x)\Psi(x)\right\rangle = g\mathrm{Tr}\, D(x,x)\,,$$

$$J^{\mathrm{PS}}(x) = -g\left\langle\bar{\Psi}(x)\gamma^5\Psi(x)\right\rangle = g\mathrm{Tr}\, D(x,x)\gamma^5$$

For classical Φ(*x*) the exact equation for the fermion *D*(*x*,*y*) reads:

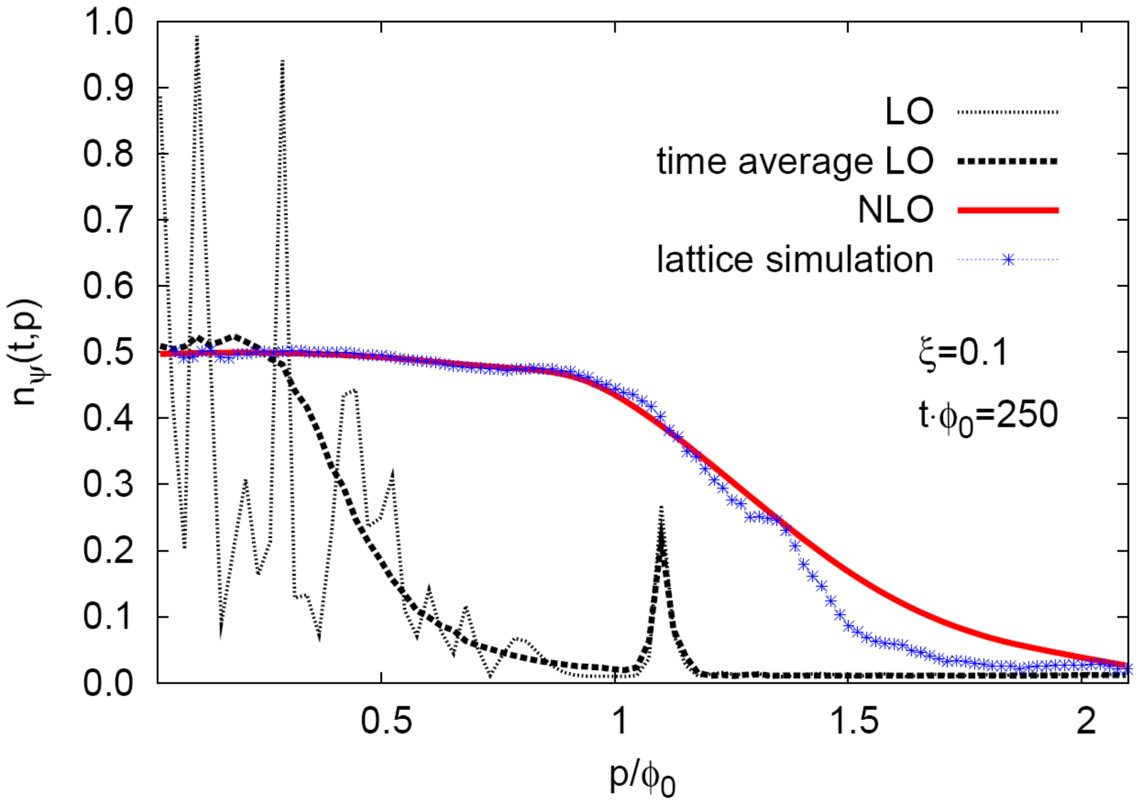
$$\boxed{(i\gamma^\mu\partial_{x,\mu} - m + g\mathrm{Re}\,\Phi(x) - ig\mathrm{Im}\,\Phi(x)\gamma^5)D(x,y) = 0}$$

Very costly (4 4 N^3 N^3)! Use low-cost fermions of Borsanyi & Hindmarsh!

Aarts, Smit, NPB 555 (1999) 355

PRD 79 (2009) 065010

Real-time dynamical fermions in 3+1 dimensions!



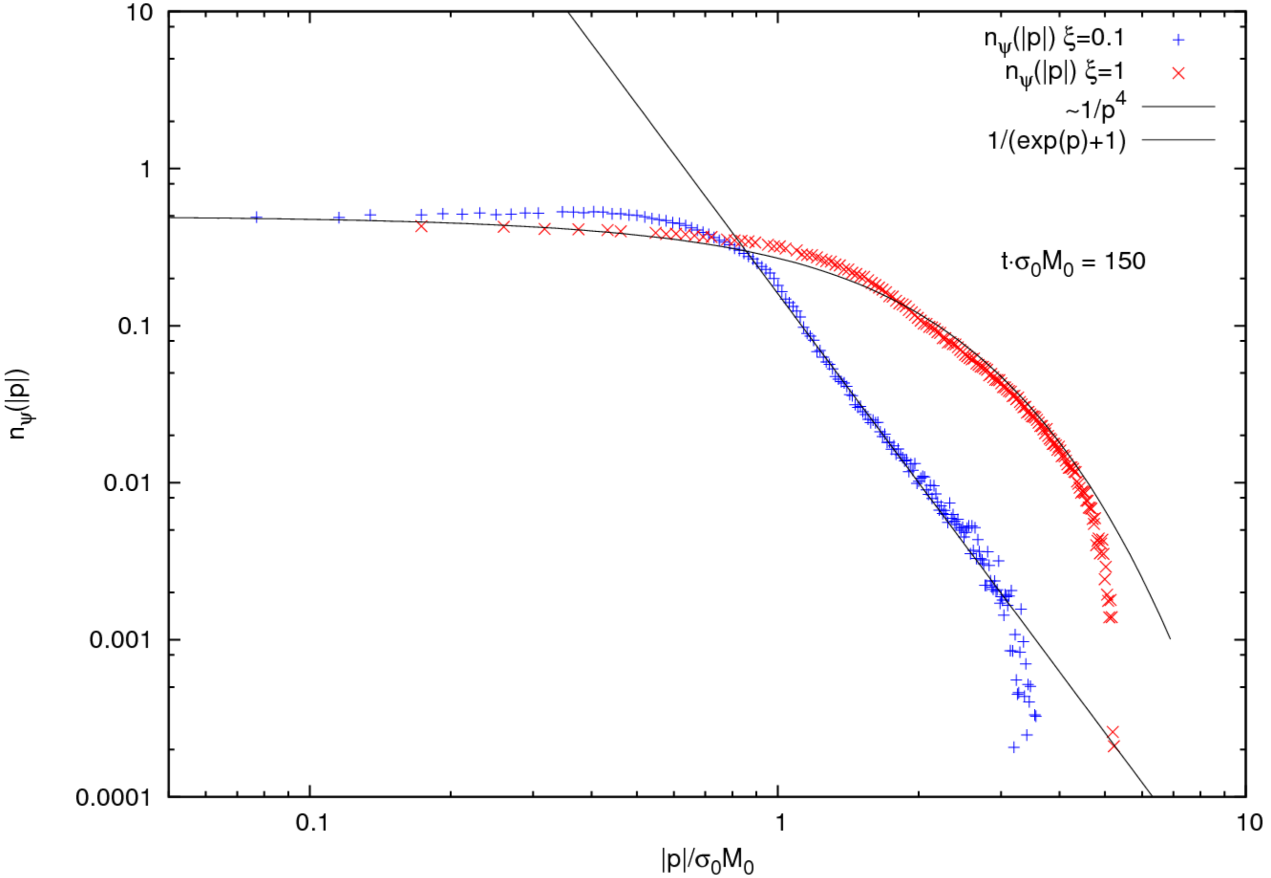
- Wilson fermions on a 64^3 lattice Berges, Gelfand, Pruschke, PRL 107 (2011) 061301
- Very good agreement with NLO quantum result (2PI) for $\xi \ll 1$
  (differences at larger *p* depend on Wilson term → larger lattices)
- Lattice simulation can be applied to $\xi \sim 1 \rightarrow SU(N)$ gauge theory

Preliminary results for $\xi = 1$

- comparison of $\xi = g^2/\lambda = 0.1$ and $\xi = 1$ (relevant for QCD)

log-log plot this time!

with Gelfand, Sexty, in preparation



→ substantially enhanced fermion production with early approach to Fermi-Dirac distribution in wide momentum range!

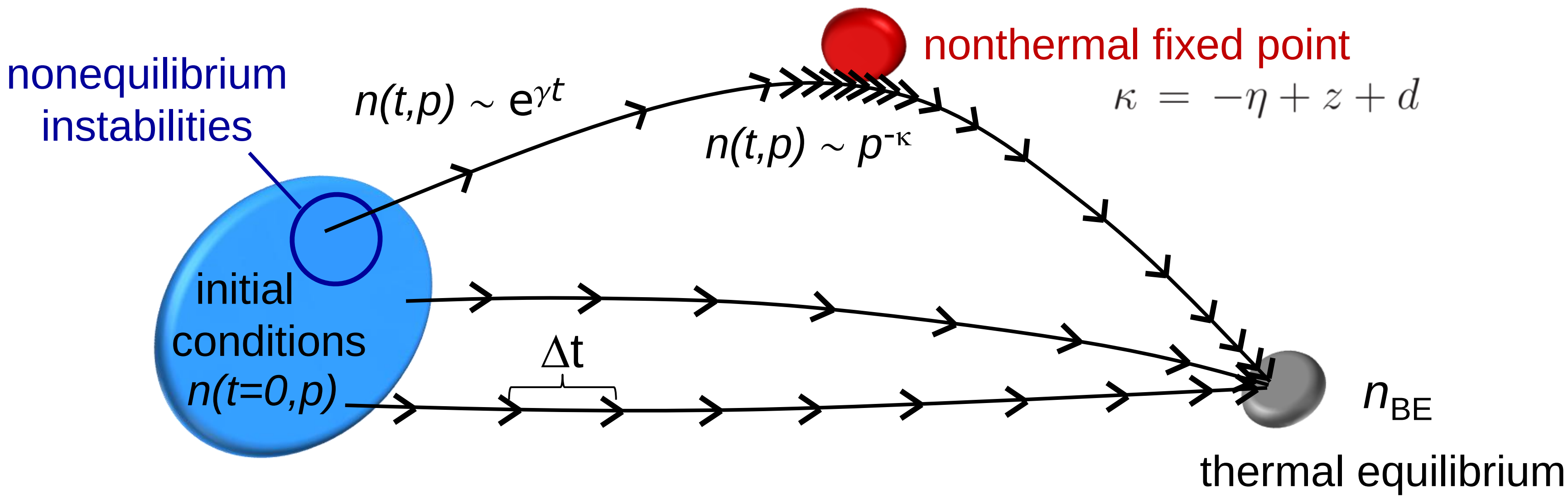
Conclusions

- nonthermal fixed points provide powerful classification for *universal far from equilibrium properties* of theories

- they *show strongly enhanced fluctuations* as compared to thermal equilibrium (e.g. $1/p^4$ as compared to thermal $1/p$)

- *genuine quantum effects for fermions are dramatically amplified* in the presence of nonthermal fixed points for bosons

- *heavy ion collisions, cold quantum gases* (and maybe early universe via gravity waves) are promising candidates to discover them



Nonthermal fixed points:

- approached from substantial class of initial conditions (no fine tuning!)
- properties independent of details of the underlying microscopic theory
- critical slowing down can substantially delay *thermalization*

Fermions:

- dramatically enhanced fermion production from quantum corrections (thermally occupied in the IR while bosons are still far from equilibrium)
- strongly coupled ($\xi \sim 1$) fermions required to speed-up thermalization of bosons