

Nonthermal fixed points in turbulent Bose gases

Boris Nowak



Jan Schole, Dénes Sexty, Thomas Gasenzer

Institut für Theoretische Physik
Ruprecht-Karls Universität Heidelberg

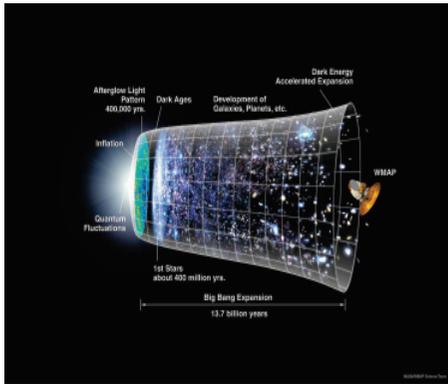
Philosophenweg 16 • 69120 Heidelberg • Germany

email: b.nowak@thphys.uni-heidelberg.de
www: www.thphys.uni-heidelberg.de/~gasenzer

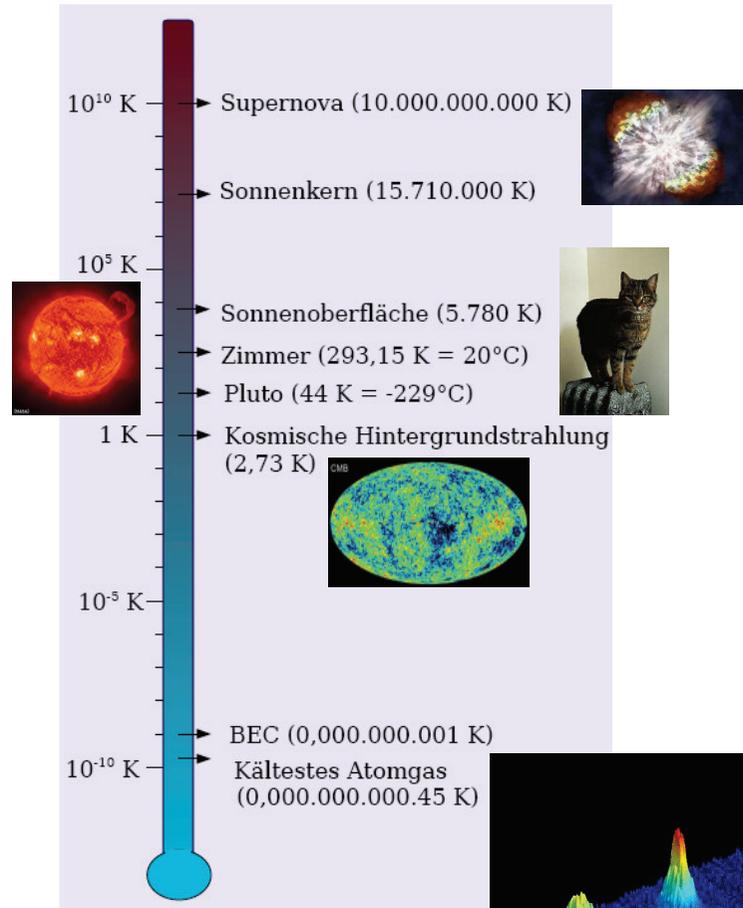


Center for
Quantum
Dynamics

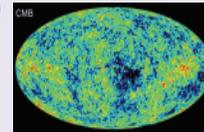
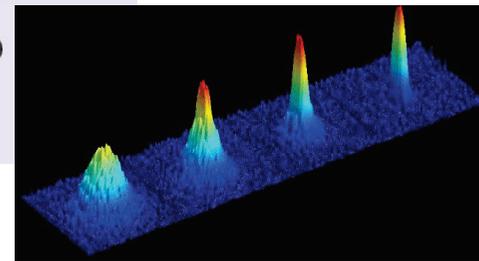
Nonequilibrium Quantum Gases



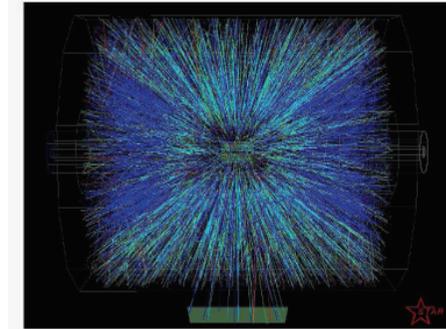
Early universe



Ultracold gases



Result of colliding two Gold nuclei (Relativistic Heavy Ion Collider, BNL):



Heavy-ion collisions



Nonequilibrium Dynamics



Initial state:
Far from equilibrium



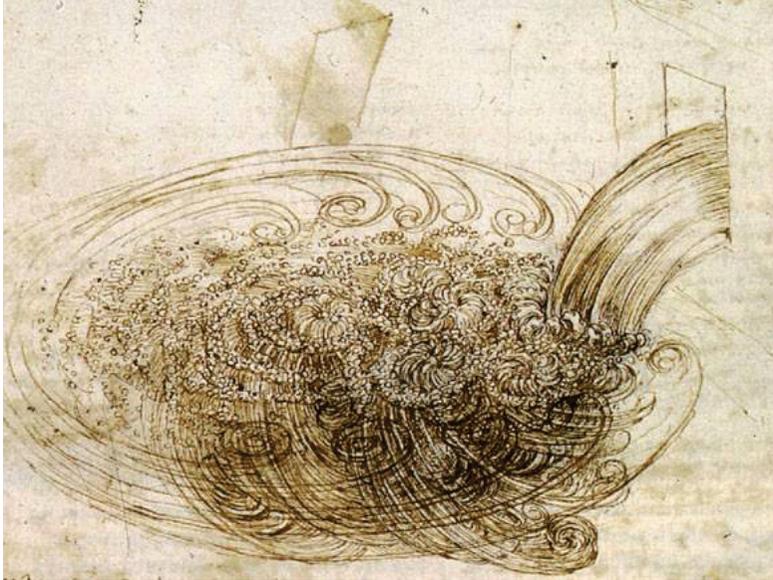
Transient state:
e.g. Turbulence
(Nonthermal fixed point)



Final state:
Thermal equilibrium



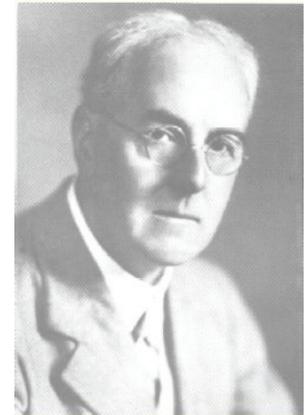
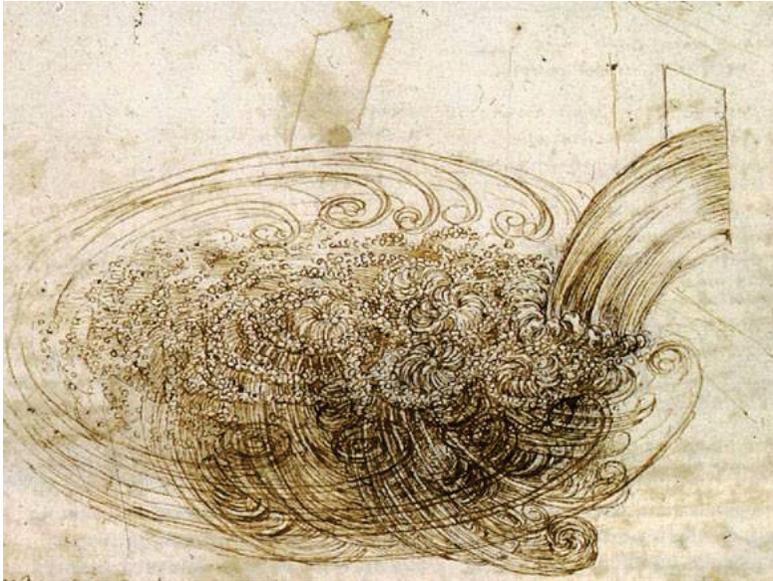
Classical Turbulence



Leonardo da Vinci
(1452 – 1519)



Richardson cascade



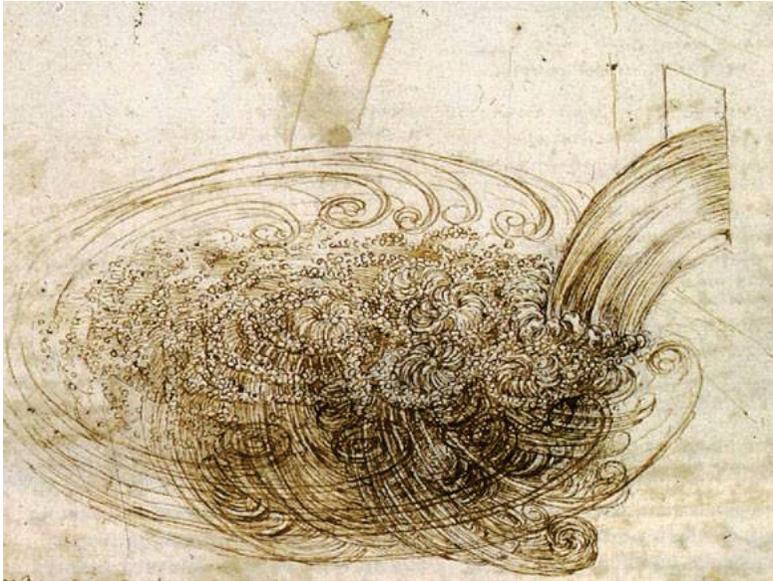
Lewis F. Richardson
(1881-1953)

Kinetic energy cascade:

large scales (source) → small scales (sink)



Kolmogorov theory



Andrey N. Kolmogorov
(1903-1987)

Kinetic energy cascade:

large scales (source) → small scales (sink)

Kolmogorov (1941): $E(k) \sim k^{-5/3}$

...Turbulence as dynamical critical phenomenon...



Turbulent ultracold Bose gas

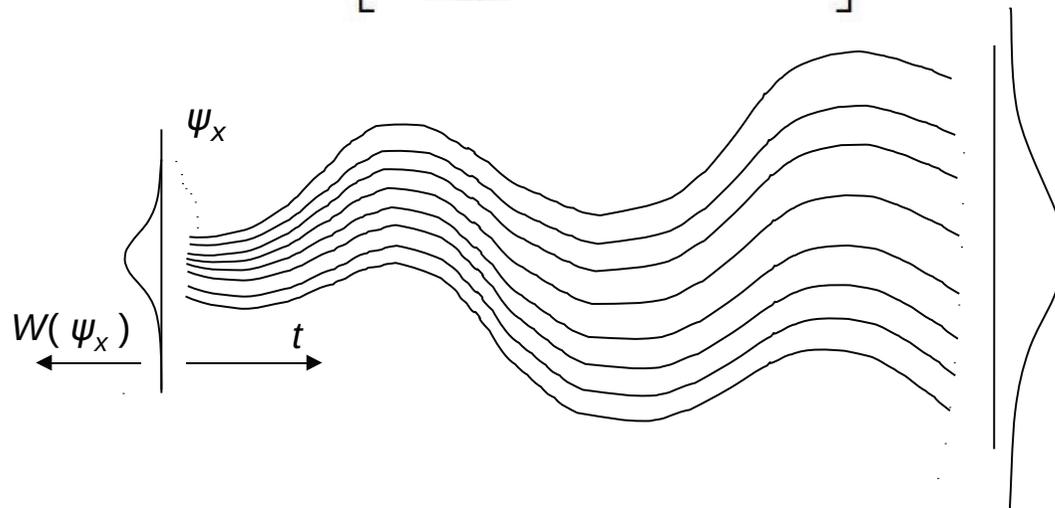
B. N., D. Sexty, T Gasenzer: Phys. Rev. B, Rapid Comm. (2011)

B. N., J. Schole, D. Sexty, T. Gasenzer: in preparation

Semiclassical simulations

Classical field equation for a nonrelativistic complex scalar field:

$$i\partial_t\psi(\mathbf{x}, t) = \left[-\frac{\partial_{\mathbf{x}}^2}{2m} + g|\psi(\mathbf{x}, t)|^2 \right] \psi(\mathbf{x}, t)$$



Radial occupation number:

$$n(\mathbf{k}) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k}) \rangle \quad \begin{array}{l} + \text{ path average} \\ + \text{ angle average} \end{array}$$

e.g. P. B. Blakie et al.: Adv Phys. (2008)



Movie 1

Phase Evolution in 2D
(single run)

$$\psi(\mathbf{x}, t) = \sqrt{n(\mathbf{x}, t)} \exp\{i\phi(\mathbf{x}, t)\}$$



Movie 2

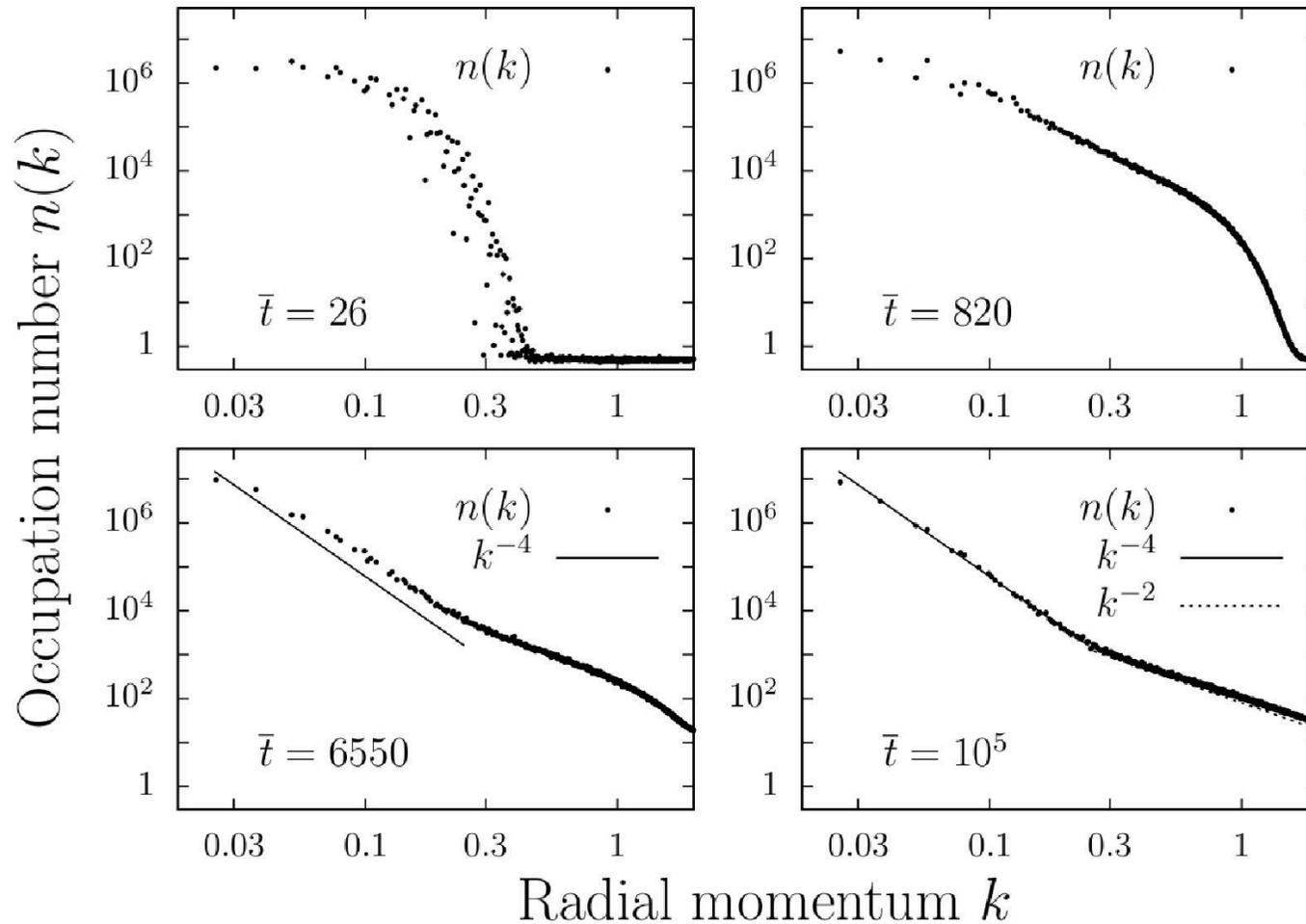
Vortex dynamics and spectrum in 2D (single run)

Radial occupation number:

$$n(k) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k}) \rangle \quad \begin{array}{l} (+ \text{ path average}) \\ + \text{ angle average} \end{array}$$



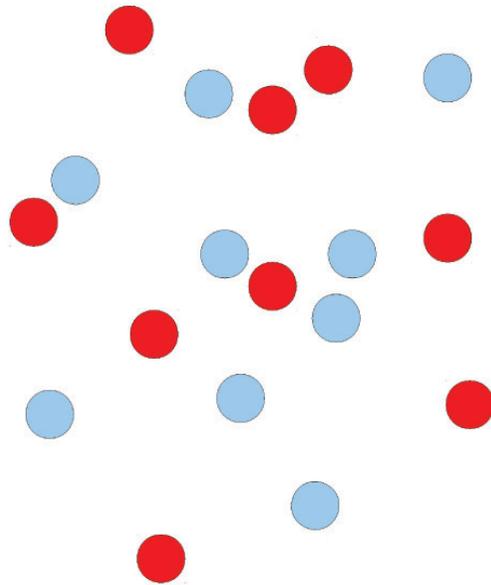
2D simulations



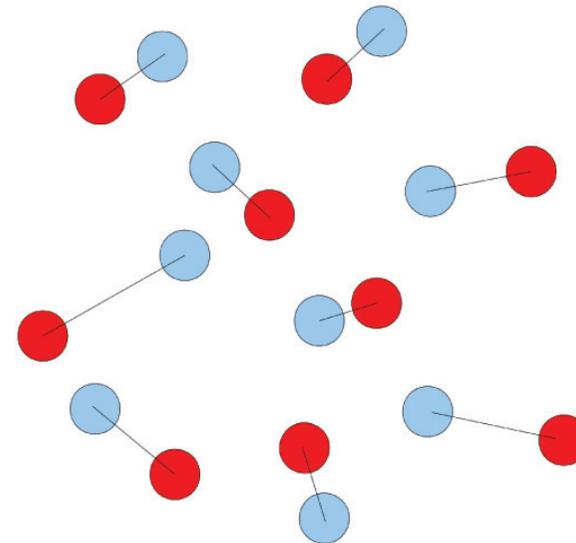
B. N., D. Sexty, T Gasenzer: Phys. Rev. B, Rapid Comm. (2011)



2D statistics of vortices



$$n_k \sim k^{-4}$$



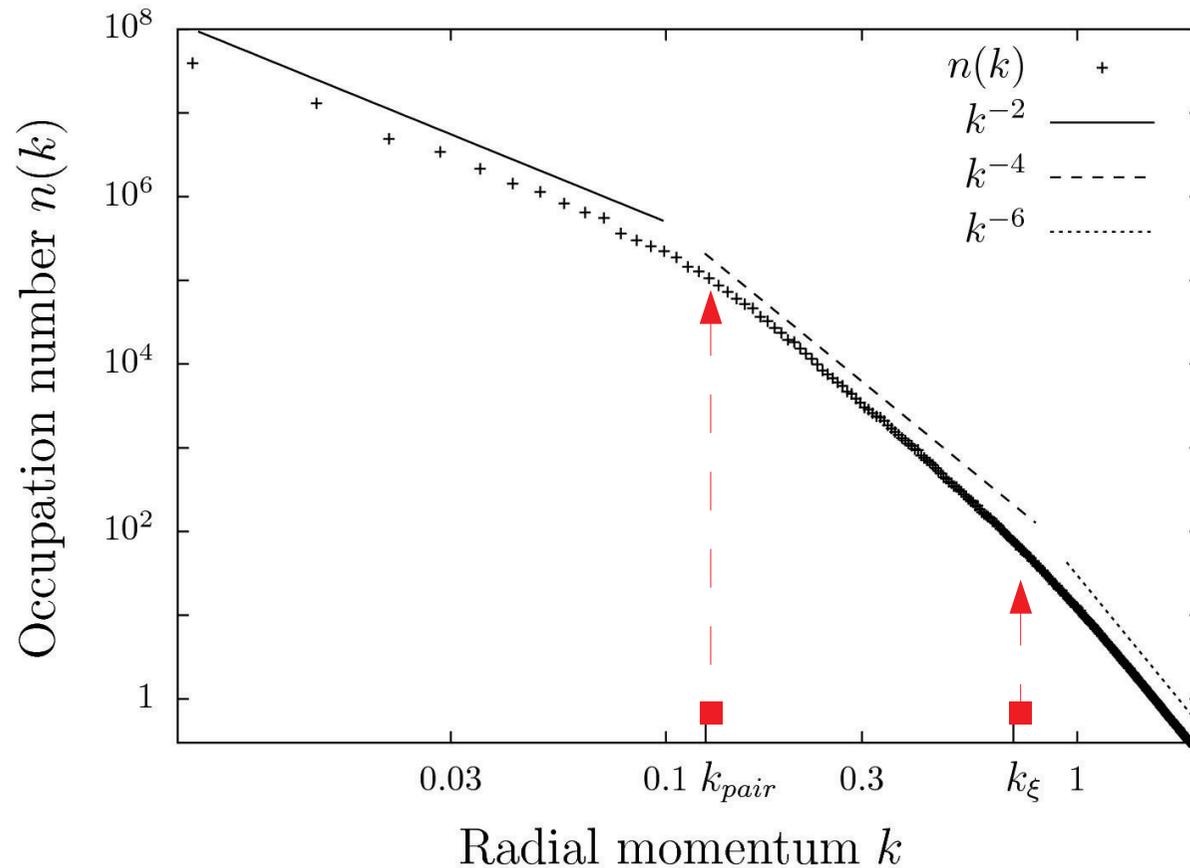
$$\begin{aligned} n_k &\sim k^{-2} & k < k_{\text{pair}} \\ n_k &\sim k^{-4} & k > k_{\text{pair}} \end{aligned}$$

B. N., J. Schole, D. Sexty, T. Gasenzer: in preparation

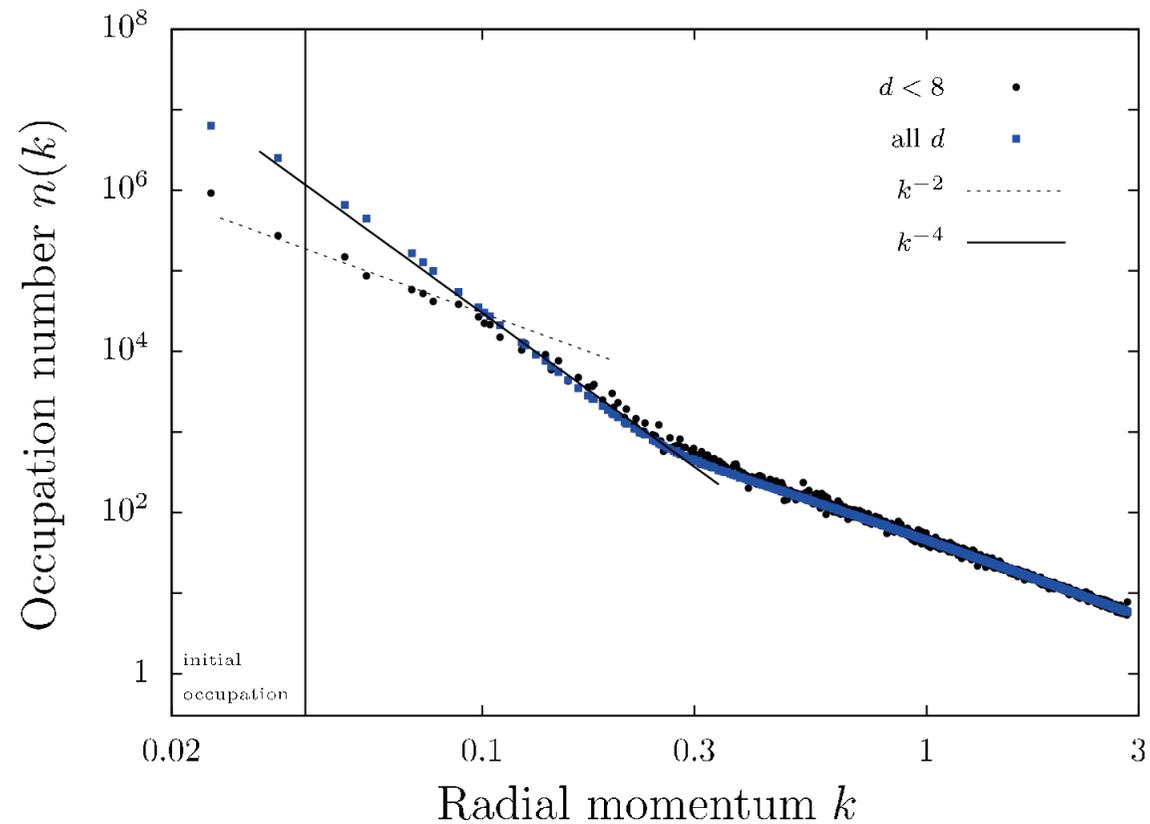


Scaling transition in 2D

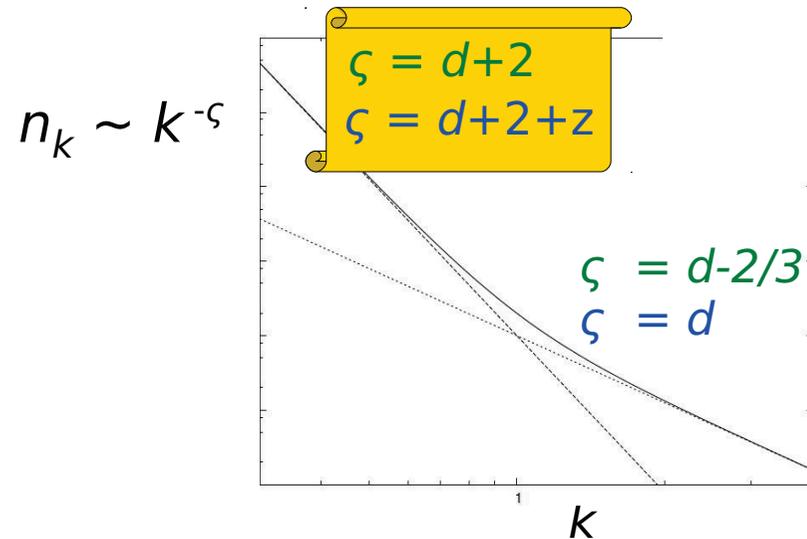
- pairing and vortex core size



Pair-scaling in simulations



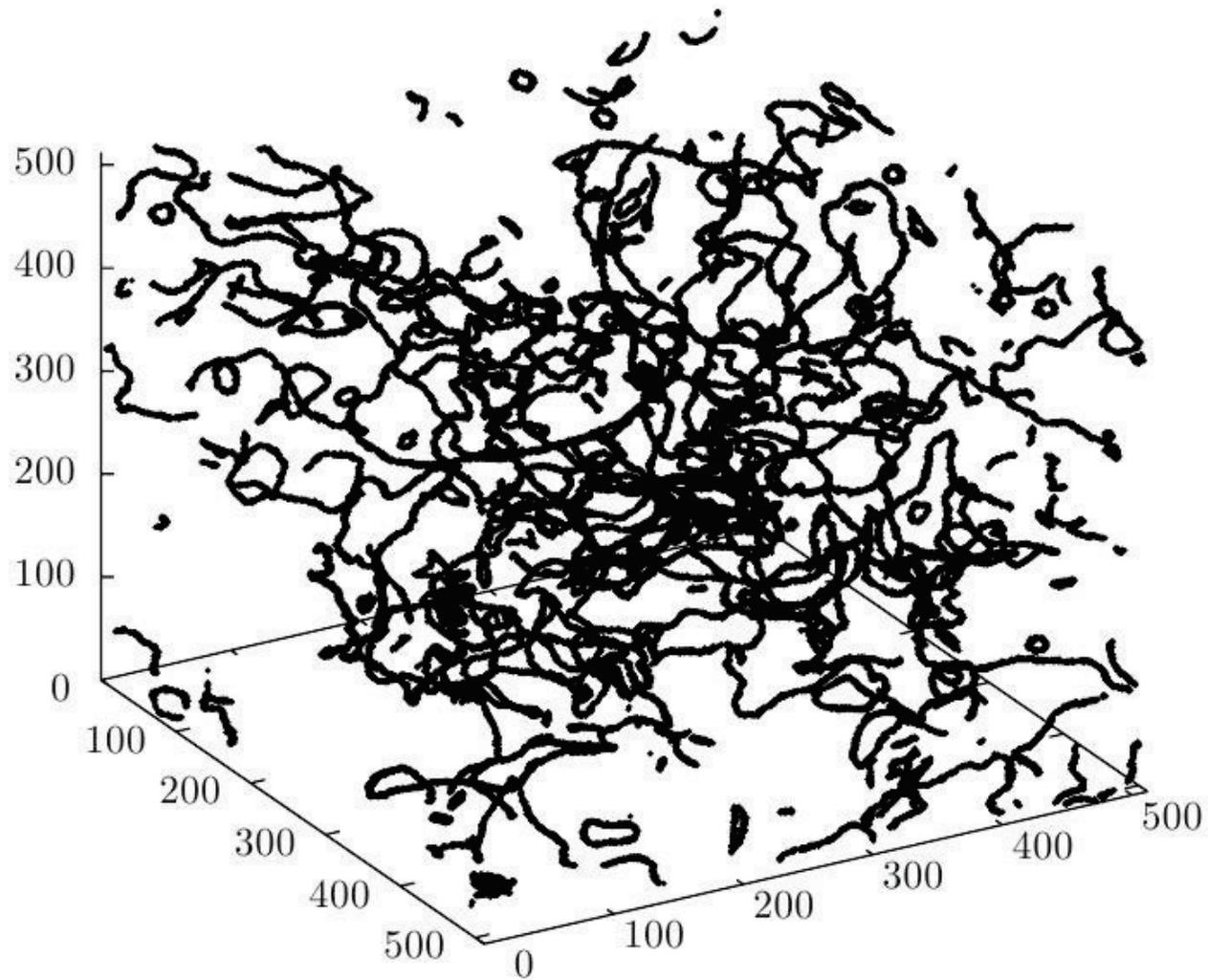
Nonthermal fixed points



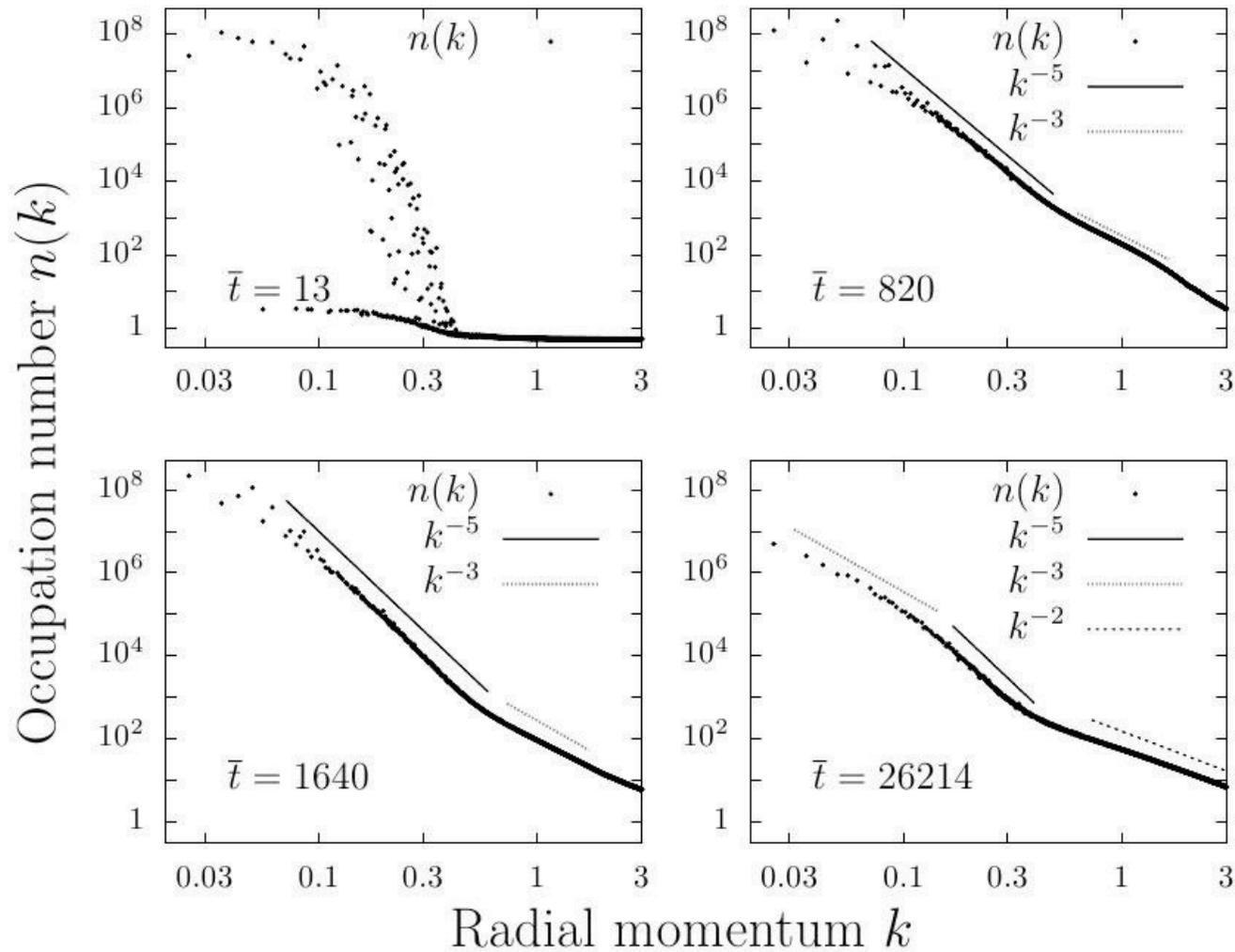
J. Berges, A. Rothkopf, J. Schmidt, PRL (2008)
C. Scheppach, J. Berges, T. Gasenzer, PRA (2010)



3D simulations

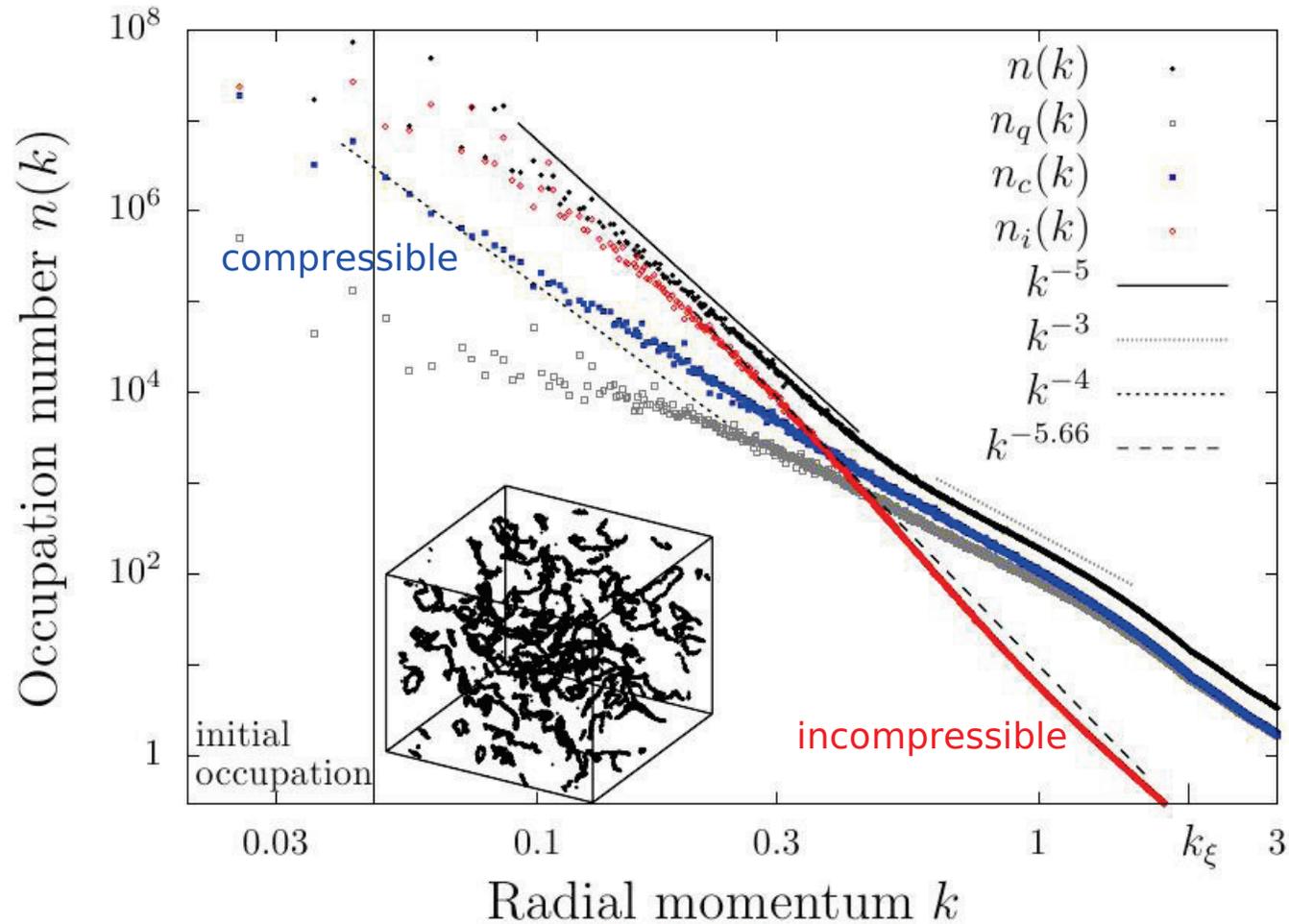


3D simulations



Scaling in 3D

$$E(k) = \omega(k)k^{d-1}n(k)$$

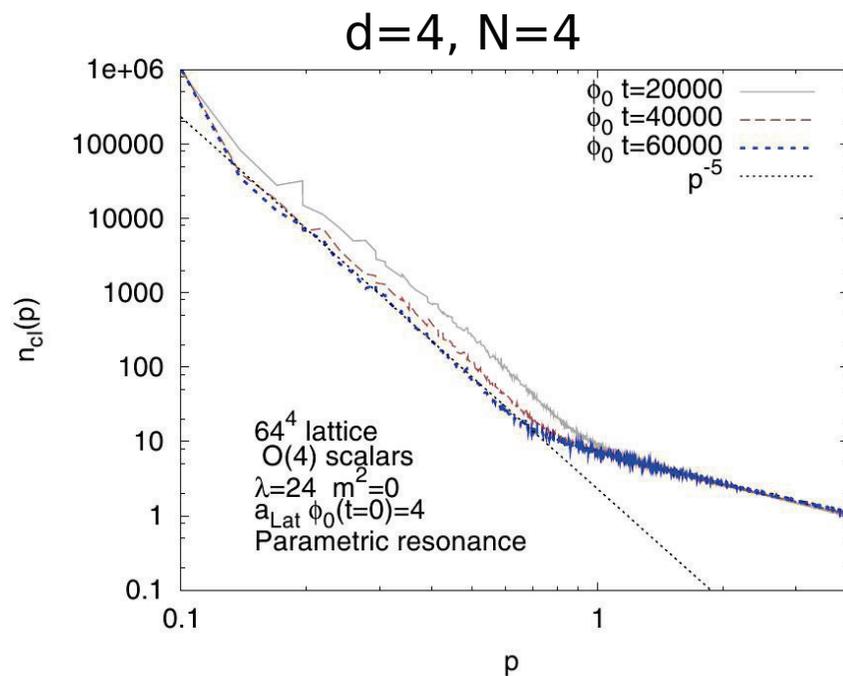


Turbulent relativistic Bose gas

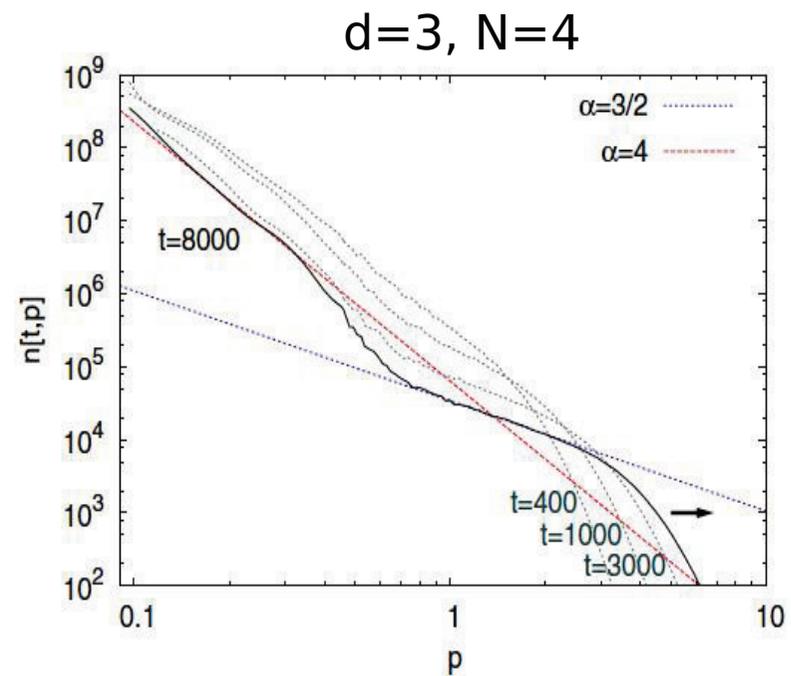
D. Sexty, B. N., T. Gasenzer: arXiv: 1108.0541 [hep-ph]

Relativistic simulations $O(N)$ in $d+1$ dim.

Classical field equation:
$$\left[\partial_t^2 - \Delta + \Phi^2 \right] \Phi_a = 0$$



J. Berges, D. Sexty: PRB (2011)

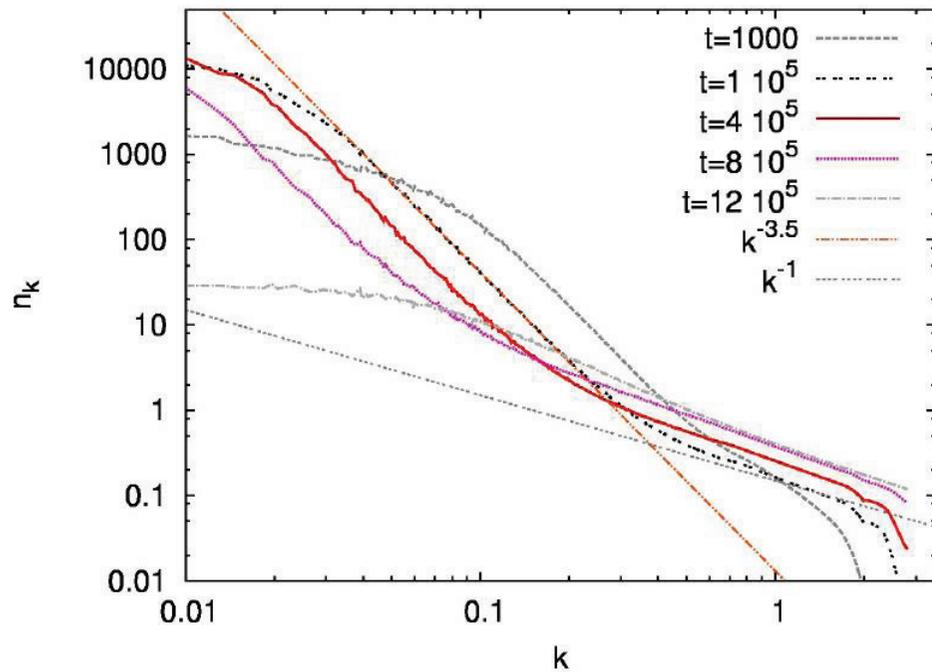


J. Berges, A. Rothkopf, J. Schmidt: PRL (2008)

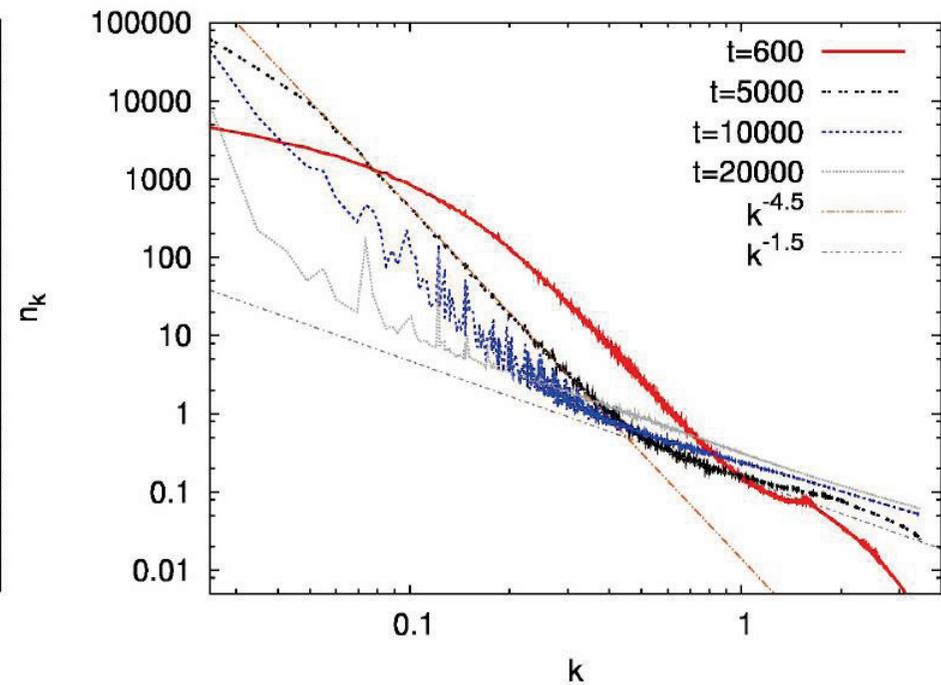


Relativistic simulations $O(N)$ in $d+1$ dim.

$d=2, N=2$



$d=3, N=2$



D. Sexty, B. N., T. Gasenzer: arXiv: 1108.0541 [hep-ph]



Movie 3

Early Universe:
Relativistic scalar field in 2D
(single run)

$$|\Psi(x,t)|$$

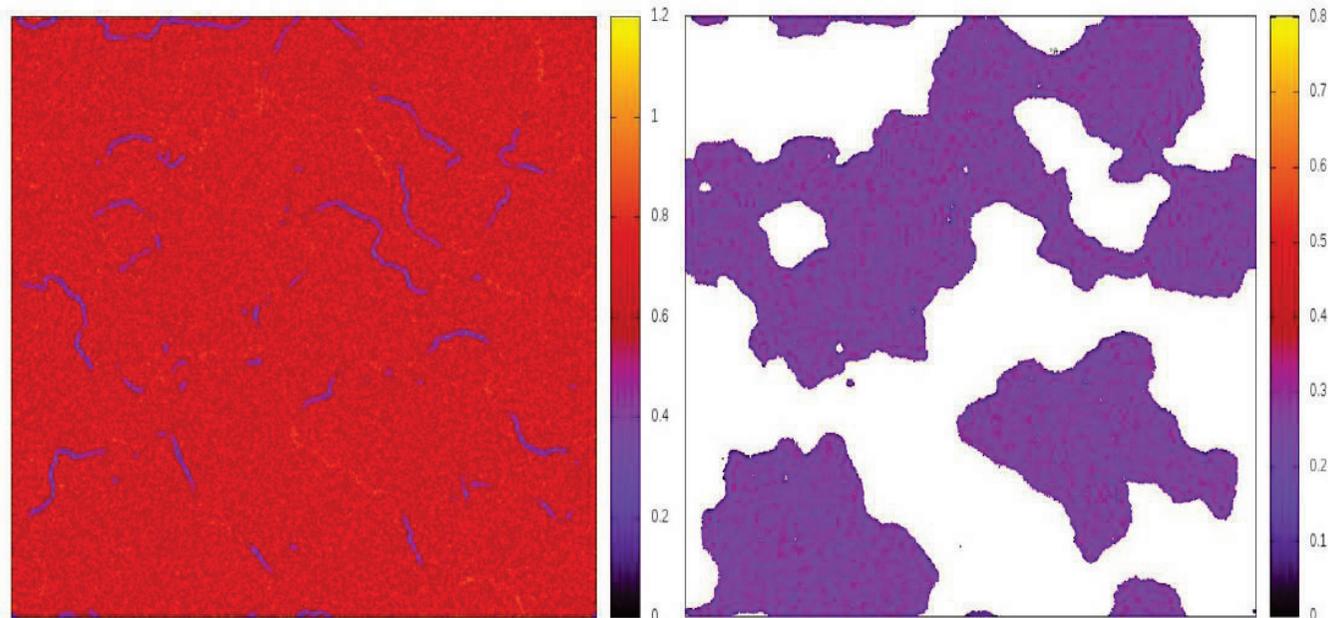


Charge Separation

Modulus of complex field $|\varphi|$

vs.

mean charge distribution



$$j_{\mu}(x) = \Phi_1(x)\partial_{\mu}\Phi_2(x) - \Phi_2(x)\partial_{\mu}\Phi_1(x).$$



Charge Separation

