With the FRG towards the QCD Phase diagram

Bernd-Jochen Schaefer

University of Graz, Austria

RG Approach from Ultra Cold Atoms to the Hot QGP

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Helmholtz Alliance Extremes of Density and Temperature: Cosmic Matter in the Laboratory (EMMI)

in collaboration with Yukawa Institute for Theoretical Physics (YITP)

Kyoto, Japan

QCD Phase Transitions

QCD $\longrightarrow$ two phase transitions:

1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \rightarrow SU_L(N_f) \times SU_R(N_f)$

   order parameter:

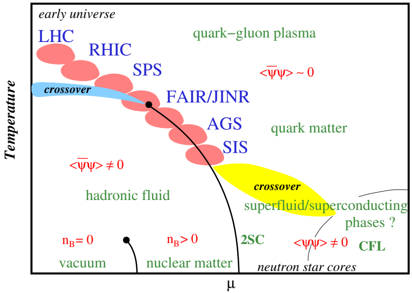
   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

2. de/confinement

   order parameter: Polyakov loop variable

   $$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase, } T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase, } T > T_c \end{cases}$$

   $$\Phi = \left\langle \mathrm{tr}_c \mathcal{P} \exp \left(i \int_0^\beta d\tau A_0(\tau, \vec{x}) \right) \right\rangle / N_c$$



At densities/temperatures of interest only model calculations available

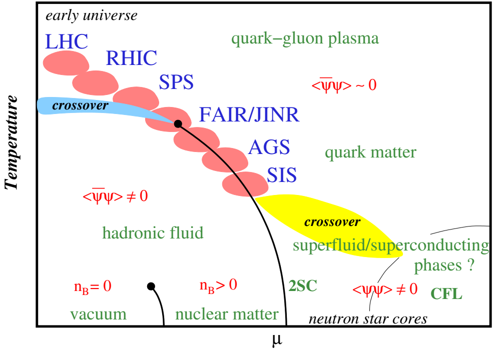
alternative: ➜ dressed Polyakov loop (dual condensate)

relates chiral and deconfinement transition $\longrightarrow$ spectral properties of Dirac operator

effective models:

1. Quark-meson model — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

The conjectured QCD Phase Diagram



At densities/temperatures of interest
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Open issues:

related to chiral & deconfinement transition

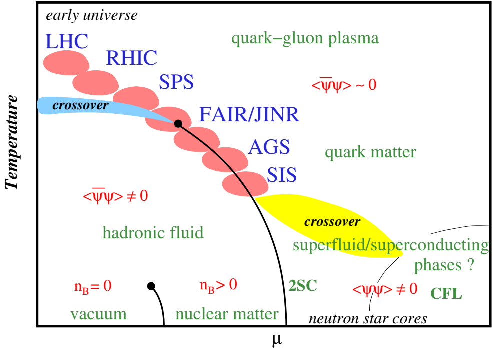
- ▷ existence of CEP? Its location?
- ▷ additional CEPs? How many?
- ▷ coincidence of both transitions at $\mu = 0$?
- ▷ quarkyonic phase at $\mu > 0$?
- ▷ chiral CEP/ deconfinement CEP?
- ▷ finite volume effects? $\rightarrow$ lattice comparison
- ▷ so far only MFA results **effects of fluctuations?** $\rightarrow$ size of crit. region

non-perturbative functional methods (FunMethods) $\rightarrow$ here: FRG

$\rightarrow$ complementary to lattice

- no sign problem $\mu > 0$
- chiral symmetry/fermions (small masses/chiral limit) ...

The conjectured QCD Phase Diagram



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non-perturbative functional methods (FunMethods) $\rightarrow$ here: FRG

$\rightarrow$ complementary to lattice

lattice versus FRG

$N_c = 2$ Polyakov-quark-meson-diquark (PQMD) model [Strodthoff, BJS, von Smekal; in prep. '11]

Outline

- **Three-Flavor Chiral Quark-Meson Model**
- **...with Polyakov loop dynamics**
- **Taylor expansions and generalized susceptibilities**
- **Functional Renormalization Group**
- **Finite volume effects**

$N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

Quark part with Yukawa coupling h:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - h\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

Meson part: scalar σ_a and pseudoscalar π_a nonet

meson fields: $M = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\mathcal{L}_{\text{meson}} = \text{tr}[\partial_\mu M^\dagger \partial^\mu M] - m^2 \text{tr}[M^\dagger M] - \lambda_1(\text{tr}[M^\dagger M])^2 - \lambda_2 \text{tr}[(M^\dagger M)^2] + c[\det(M) + \det(M^\dagger)]$$
$$+\text{tr}[H(M + M^\dagger)]$$

- explicit symmetry breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$
- $U(1)_A$ symmetry breaking implemented by 't Hooft interaction

➜ talk by Mario Mitter

Phase diagram for $N_f = 2 + 1 \quad (\mu \equiv \mu_q = \mu_s)$

- Model parameter fitted to (pseudo)scalar meson spectrum:
- one parameter precarious: $f_0(600)$ 'particle' (i.e. sigma)$\rightarrow$ broad resonance

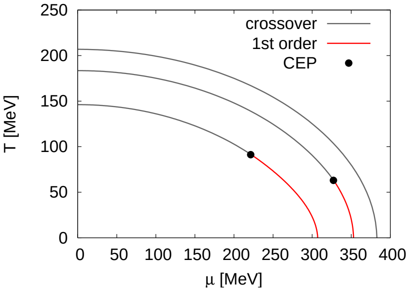
  PDG: mass=$(400 \ldots 1200)$ MeV

  we use fit values for m_σ between $(400 \ldots 1200)$ MeV

➜ existence of CEP depends on m_σ!

Example: $m_\sigma = 600$ MeV (lower lines), 800 and 900 MeV (here mean-field approximation)

with $U(1)_A$ **[BJS, M. Wagner '09]**

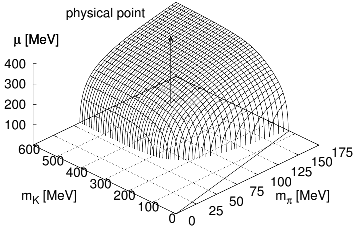


Chiral critical surface ($m_\sigma = 800$ MeV)

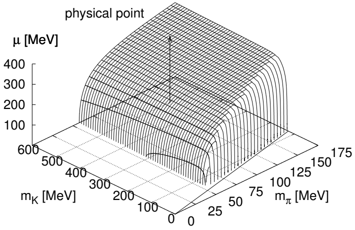
➜ standard scenario for $m_\sigma = 800$ MeV (as expected)

here: 't Hooft coupling μ-independent (might change if μ-dependence is considered)

with $U(1)_A$



without $U(1)_A$



[BJS, M. Wagner, '09]

non-standard scenario in PNJL with (unrealistic) large vector int. $\rightarrow$ bending of surface

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- ○ Three-Flavor Chiral Quark-Meson Model
- ○ **...with Polyakov loop dynamics**
- ○ Taylor expansions and generalized susceptibilities
- ○ Functional Renormalization Group
- ○ Finite volume effects

Polyakov-quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1. **polynomial Polyakov loop potential:** **Polyakov 1978, Meisinger 1996, Pisarski 2000**

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0\ )}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

2. **logarithmic potential:** **Rößner et al. 2007**

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2$ and $b(T) = b_3(T_0/T)^3$

3. **Fukushima** **Fukushima 2008**

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

a controls deconfinement b strength of mixing chiral & deconfinement

Polyakov-quark-meson (PQM) model

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$$b_2(T, T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

back reaction of the matter sector to the YM sector: N_f and μ-modifications

in presence of dynamical quarks: $T_0 = T_0(N_f, \mu, m_q)$ **BJS, Pawlowski, Wambach; 2007**

N_f	0	1	2	2 + 1	3
T_0 [MeV]	270	240	208	187	178

matter back reaction to the YM sector important at $\mu \neq 0$

for $\mu \neq 0$: $\bar{\phi} > \phi$ is expected

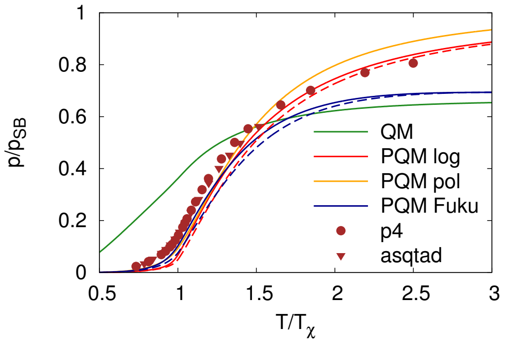
since $\bar{\phi}$ is related to free energy gain of antiquarks

in medium with more quarks $\rightarrow$ antiquarks are more easily screened.

QCD Thermodynamics $N_f = 2+1$

[BJS, M. Wagner, J. Wambach '10]

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations



- ▷ solid lines:
  PQM with lattice masses (HotQCD)
  $m_\pi \sim 220, m_K \sim 503$ MeV
- ▷ dashed lines:
  (P)QM with realistic masses

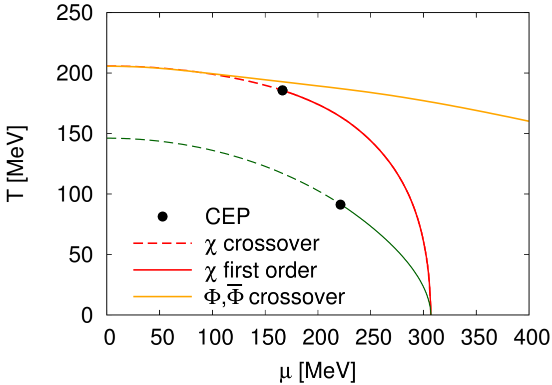
data taken from: [Bazavov et al. '09]

Stefan-Boltzmann limit: $\frac{p_{SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

$N_f = 2+1$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

chiral transition and 'deconfinement' coincide at small densities



- for PQM model

  (upper lines)
- for QM model

  (lower lines)

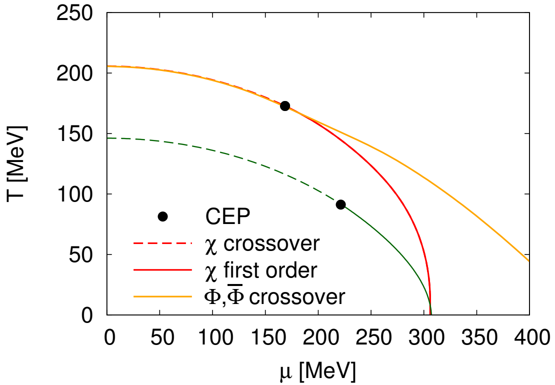
[BJS, M. Wagner; in prep. '11]

$N_f = 2$: [BJS, Pawlowski, Wambach; 2007]

$N_f = 2 + 1$ (P)QM phase diagrams

Summary of QM and PQM models in mean field approximation

chiral transition and 'deconfinement' coincide at small densities



- for PQM model

  (upper lines)

  with
  matter back reaction in Polyakov loop potential

  → shrinking of possible quarkyonic phase

- for QM model

  (lower lines)

[BJS, M. Wagner; in prep. '11]

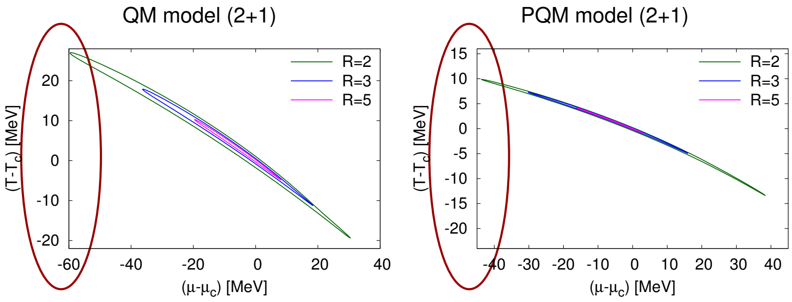
$N_f = 2$: [BJS, Pawlowski, Wambach; 2007]

Critical region

contour plot of **size of the critical region** around CEP

defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop



[BJS, M. Wagner; in preparation 11]

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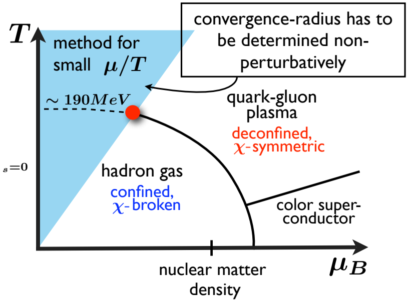
- ○ ...with Polyakov loop dynamics
- ○ **Taylor expansions and generalized susceptibilities**
- ○ Functional Renormalization Group
- ○ Finite volume effects

Finite density extrapolations $\mathbf{N_f = 2 + 1}$

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial (\mu/T)^n}\Bigg|_{\mu=0}$$



[C. Schmidt '09]

convergence radii:

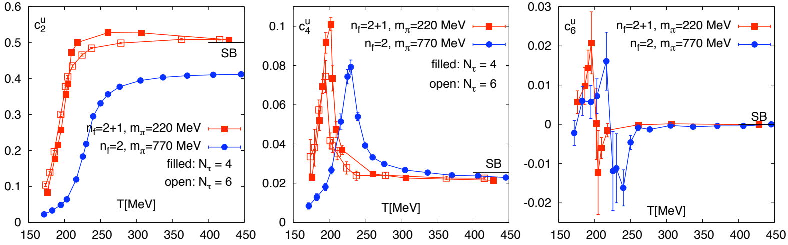
limited by first-order line?

$$\rho_{2n} = \left|\frac{c_2}{c_{2n}}\right|^{1/(2n-2)}$$

Finite density extrapolations $N_f = 2 + 1$

Taylor expansion:

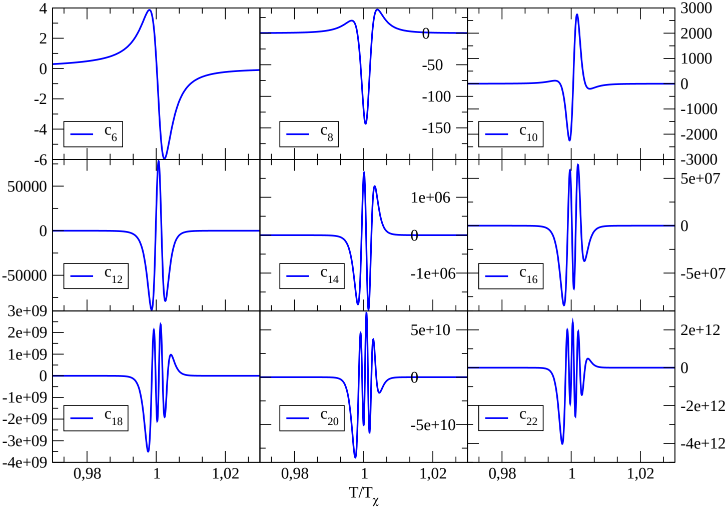
$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial (\mu/T)^n}\Bigg|_{\mu=0}$$



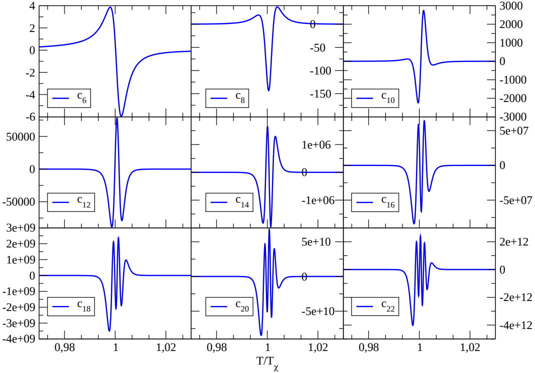
[Miao et al. '08]

New method: based on algorithmic differentiation **[M. Wagner, A. Walther, BJS, CPC '10]**

Taylor coefficients c_n numerically known to high order, e.g. $n = 22$



Taylor coefficients for $\mathrm{N_f = 2 + 1}$ PQM model



- ▷ this technique applied to PQM model
- ▷ investigation of convergence properties of Taylor series
- ▷ properties of c_n
  - oscillating
  - increasing amplitude
  - no numerical noise
  - small outside transition region
  - number of roots increasing
  - 26th order

[F. Karsch, BJS, M. Wagner, J. Wambach; arXiv:1009.5211]

Can we locate the QCD critical endpoint with the Taylor expansion ?

Taylor expansions

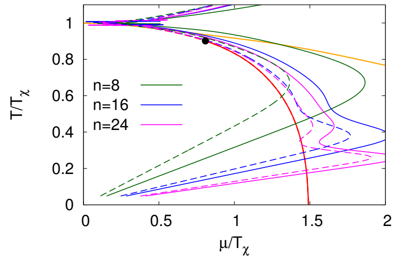
Findings:

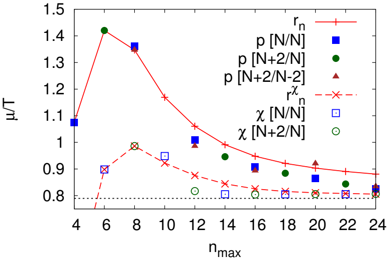
- simply Taylor expansion: slow convergence
  high orders needed
  disadvantage for lattice simulations

- Taylor applicable within convergence radius
  also for $\mu/T > 1$

$$r_{2n} = \left| \frac{c_{2n}}{c_{2n+2}} \right|^{1/2} , \; r^{\chi}_{2n} = \left| \frac{(2n+2)(2n+1)}{(2n+3)(2n+4)} \right|^{1/2} r_{2n+2}$$

Padé [N/N]



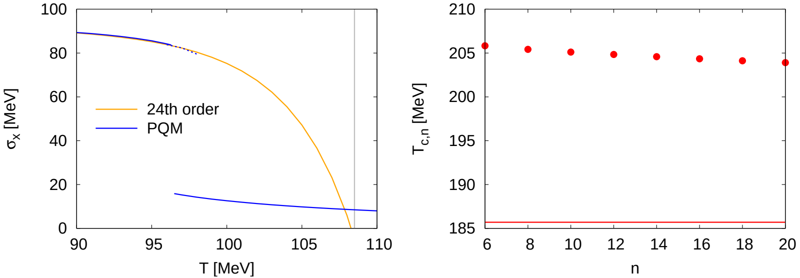


[F.Karsch, BJS, M.Wagner, J.Wambach; arXiv:1009.5211]

Taylor expansions

Findings:

- simply Taylor expansion: slow convergence
  high orders needed
  disadvantage for lattice simulations
- Taylor applicable within convergence radius
  also for $\mu/T > 1$
- but 1st order transition not resolvable
  expansion around $\mu = 0$



[F.Karsch, BJS, M.Wagner, J.Wambach; arXiv:1009.5211]

Generalized Susceptibilities

- Can we use these coefficients to locate CEP experimentally?

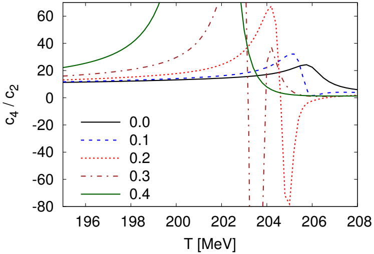
  Is there a memory effect that the system (HIC) has passed through the QCD phase transition?

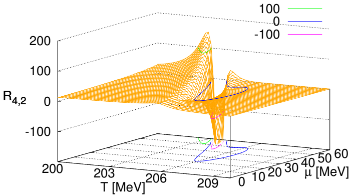
- Probing phase diagram with fluctuations of e.g. net baryon number

- Example: Kurtosis $R^B_{4,2} = \frac{1}{9}\frac{c_4}{c_2}$ $\rightarrow$ probe of deconfinement?

  It measures quark content of particles carrying baryon number B

- in HRG model $R_{4,2} = 1$ (always positive)





[BJS, M.Wagner; in prep. 11]

Generalized Susceptibilities

Fluctuations of higher moments (more sensitive to criticality)

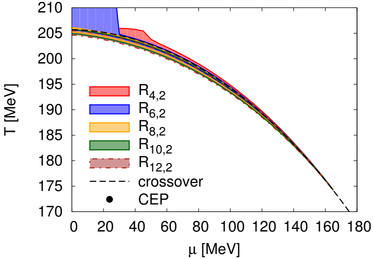
exhibit strong variation from HRG model

- $\rightarrow$ turn negative [Karsch, Redlich; 11] see also [Friman, Karsch, Redlich, Skokov; 11]

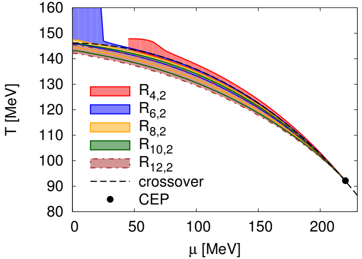
see talk by V. Skokov

- higher moments: $R^q_{n,m} = c_n/c_m$
- regions where $R_{n,m}$ are negative along crossover line in the phase diagram

PQM with $T_0(\mu)$



QM model



[BJS, M.Wagner; in prep. 11]

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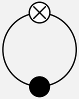
- Finite volume effects

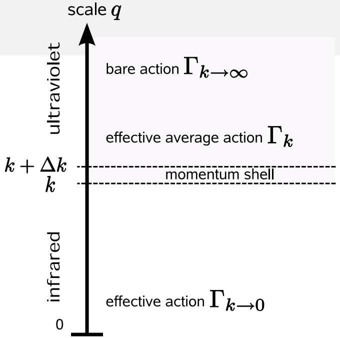
Functional RG Approach

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

FRG (average effective action) **[Wetterich '93]**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

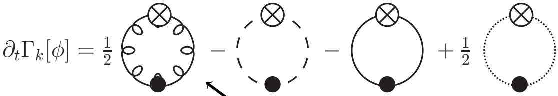
$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$ 



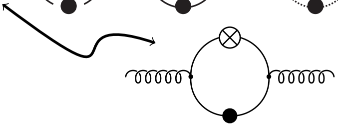
$\mathbf{T}_0(N_f,\mu)$ modification

full dynamical QCD FRG flow: fluctuations of
gluon, ghost, quark and meson (via hadronization) fluctuations

[Braun, Haas, Marhauser, Pawlowski; '09]



in presence of dynamical quarks
gluon propagator modified:



$\rightarrow$ pure Yang Mills flow + these modifications

pure Yang Mills flow
replaced by effective Polyakov loop potential:
(fit to YM thermodynamics)

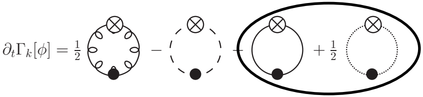
$$T_0 \leftrightarrow \Lambda_{QCD} \quad : \quad T_0 \to T_0(N_f, \mu, m_q)$$

[BJS, Pawlowski, Wambach; 2007]

Functional Renormalization Group

[Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$



$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k regulators

PQM truncation $N_f = 2$

[Herbst, Pawlowski, BJS; 2011]

see also [Skokov, Friman, Redlich; 2010]

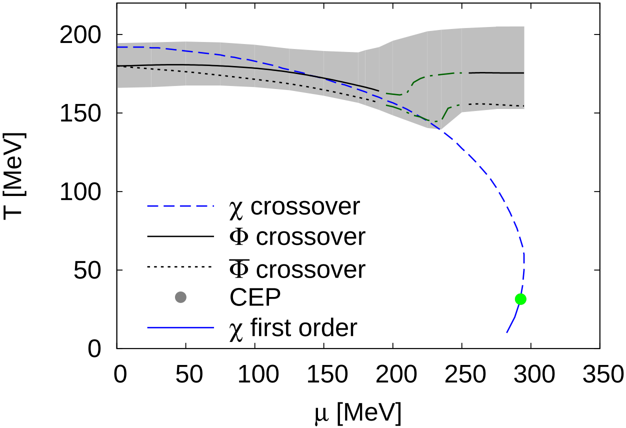
$$\Gamma_k = \int d^4x \left\{ \bar{\psi}\left(\not{D} + \mu\gamma_0 + ih(\sigma + i\gamma_5 \vec{\tau}\vec{\pi})\right)\psi + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \Omega_k[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] \right\}$$

Initial action at UV scale Λ:

$$\Omega_\Lambda[\sigma, \vec{\pi}, \Phi, \bar{\Phi}] = U(\sigma, \vec{\pi}) + \mathcal{U}(\Phi, \bar{\Phi}) + \Omega_\Lambda^\infty[\sigma, \vec{\pi}, \Phi, \bar{\Phi}]$$

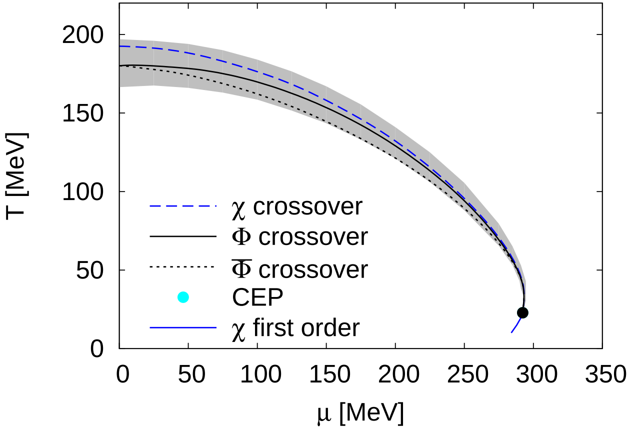
$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

Phase diagram $T_0 = 208$ MeV



[Herbst, Pawlowski, BJS; 2011]

Phase diagram $T_0(\mu), T_0(0) = 208$ MeV



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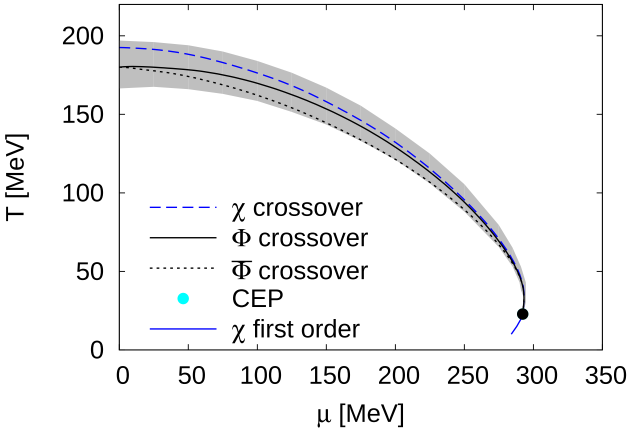
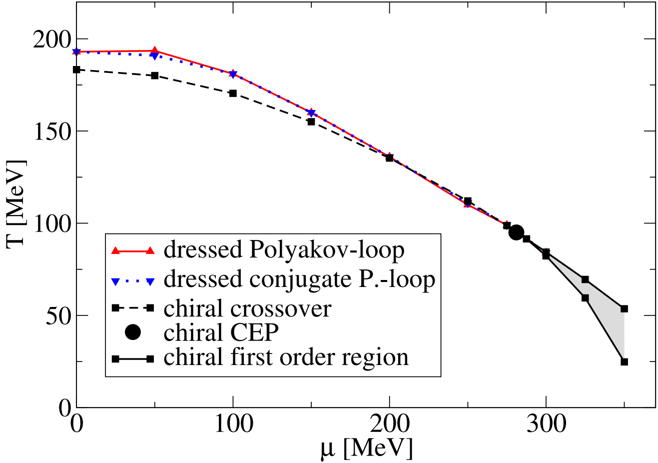
Phase diagram $T_0(\mu), T_0(0) = 208$ MeV



[Herbst, Pawlowski, BJS; 2011]

➜ CEP unlikely for small μ_B/T ➜ baryons

Phase diagram (DSE - HTL)



[Ch. S. Fischer, J. Luecker, J. A. Mueller; 2011]

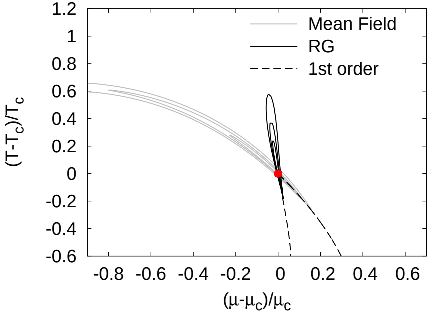
Critical region

similar conclusion if **fluctuations** are included

fluctuations via Functional Renormalization Group

comparison: $N_f = 2$ QM model

Mean Field $\leftrightarrow$ RG analysis



[BJS, Wambach '06]

Outline

- Three-Flavor Chiral Quark-Meson Model
- ...with Polyakov loop dynamics
- Taylor expansions and generalized susceptibilities
- Functional Renormalization Group
- **Finite volume effects**

Finite volume effects

▷ Finite Euclidean Volume: $L^3 \times 1/T$ — cf. talk by Bertram Klein

$$\int d^3p \to \left(\frac{2\pi}{L}\right)^3 \sum_{n_1 \in Z} \sum_{n_2 \in Z} \sum_{n_3 \in Z} \quad \text{with} \quad p^2 \to \frac{4\pi^2}{L^2}\left(n_1^2 + n_2^2 + n_3^2\right)$$

grand potential ($N_f = 2$ quark-meson truncation) **[A Tripolt, J Braun, B Klein, BJS; 2011]**

$$\partial_t \Omega_k = \mathcal{B}_p(k,L) \frac{k^5}{12\pi^2} \left[-\frac{2N_f N_c}{E_q} \left\{ \tanh\left(\frac{E_q - \mu}{2T}\right) + \tanh\left(\frac{E_q + \mu}{2T}\right) \right\} + \frac{1}{E_\sigma} \coth\left(\frac{E_\sigma}{2T}\right) + \frac{3}{E_\pi} \coth\left(\frac{E_\pi}{2T}\right) \right]$$

with $E_{\sigma,\pi,q} = \sqrt{k^2 + m^2_{\sigma,\pi,q}}$, $m_\sigma^2 = 2\Omega_k' + 4\sigma^2 \Omega_k''$, $m_\pi^2 = 2\Omega_k'$, $m_q^2 = h^2\sigma^2$

mode counting functions

$\mathcal{B}_p(k,L)$: periodic

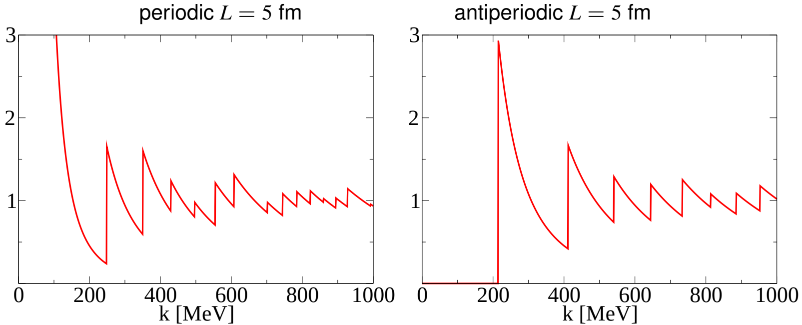
$\mathcal{B}_{ap}(k,L)$: antiperiodic

Finite volume effects

mode counting functions (periodic and antiperiodic (lower) boundary)

$$\mathcal{B}_p(k,L) = \frac{6\pi^2}{(kL)^3} \sum_{n_1,n_2,n_3 \in Z} \Theta\left(k^2 - \frac{4\pi^2}{L^2}\left(n_1^2+n_2^2+n_3^2\right)\right)$$

$$\mathcal{B}_{ap}(k,L) = \frac{6\pi^2}{(kL)^3} \sum_{n_1,n_2,n_3 \in Z} \Theta\left(k^2 - \frac{4\pi^2}{L^2}\left(\left(n_1+\frac{1}{2}\right)^2+\left(n_2+\frac{1}{2}\right)^2+\left(n_3+\frac{1}{2}\right)^2\right)\right)$$



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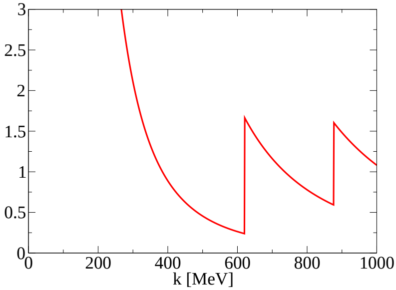
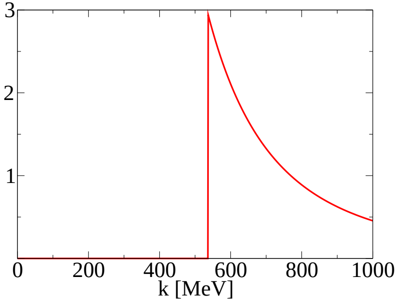
periodic $L = 2$ fm



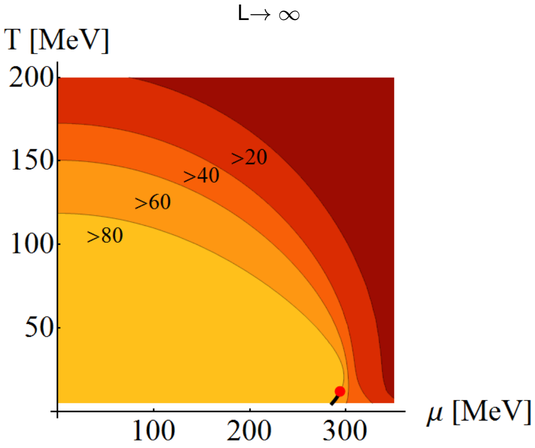
antiperiodic $L = 2$ fm



Finite volume effects

[A. Tripolt, J. Braun, B. Klein, BJS, in preparation ’11]

preliminary results for **periodic** BC



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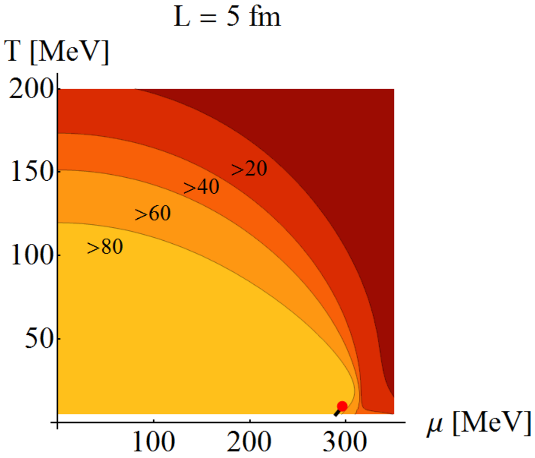

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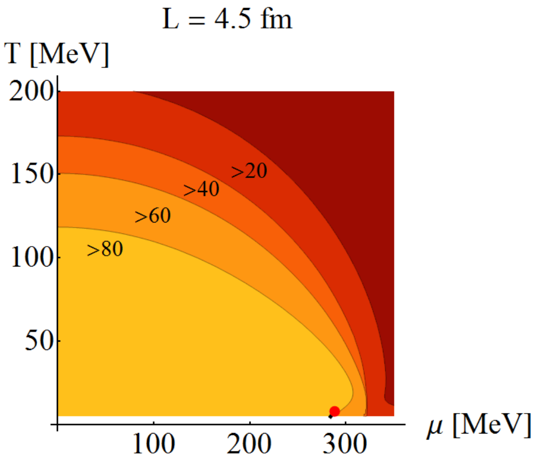

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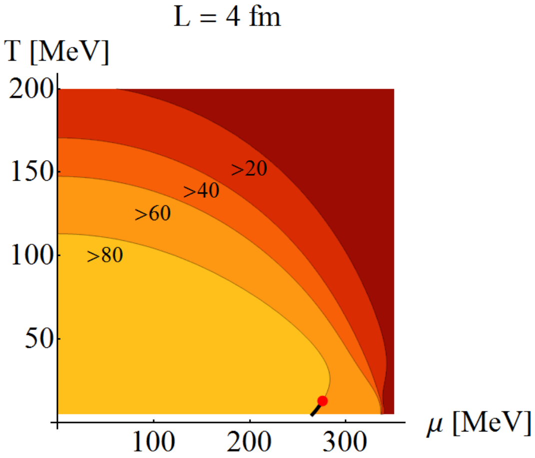

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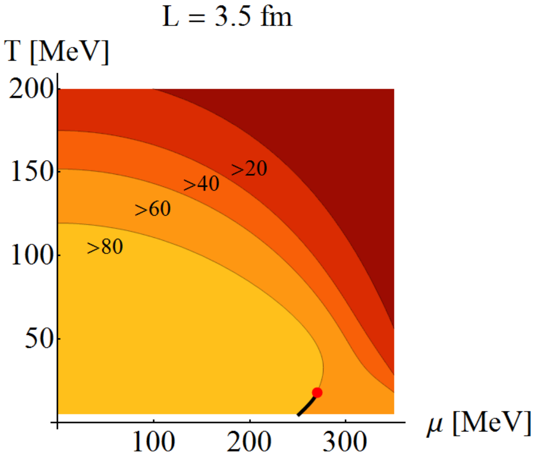

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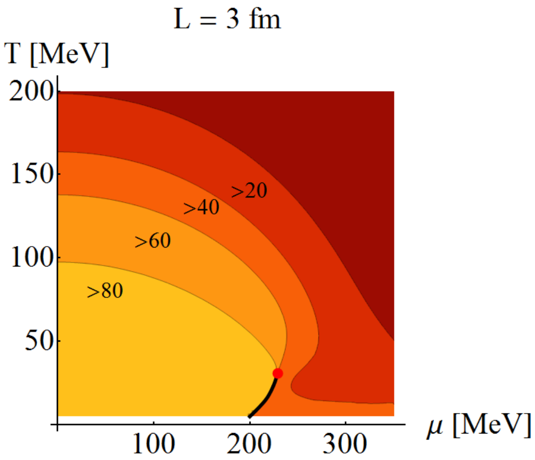

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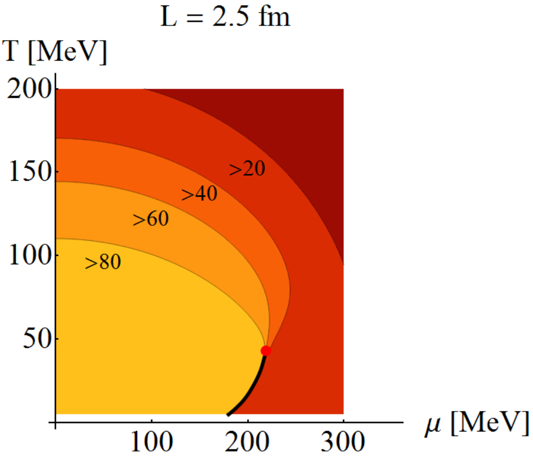

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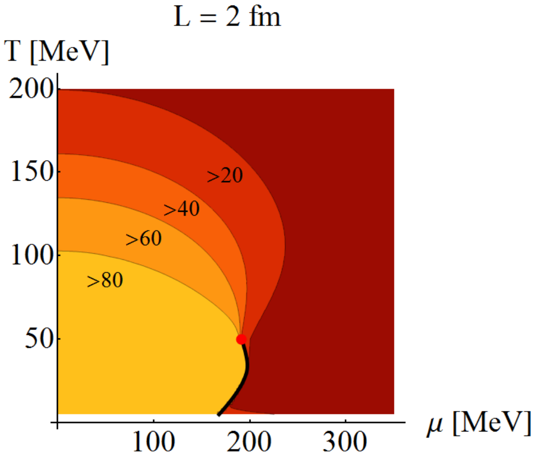

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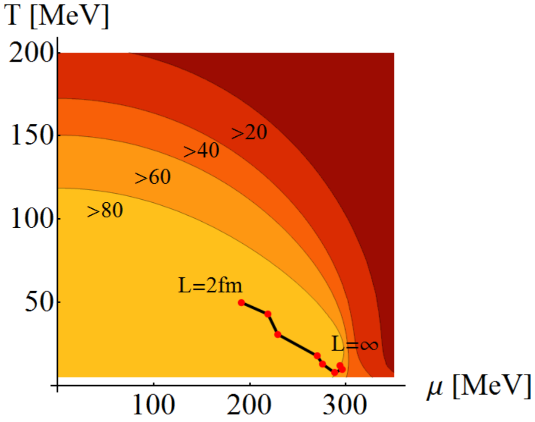
movement of the CEP ($L \to \infty \ldots 2$ fm)



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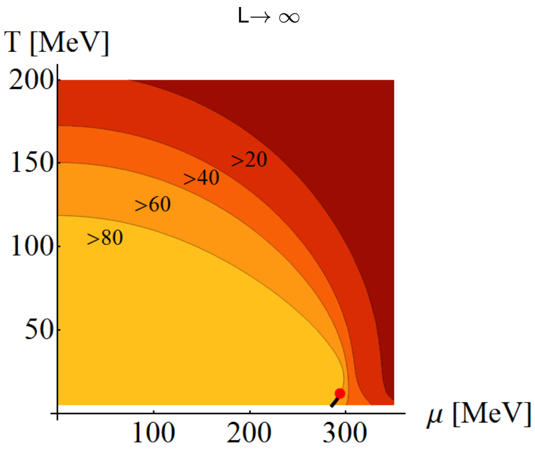

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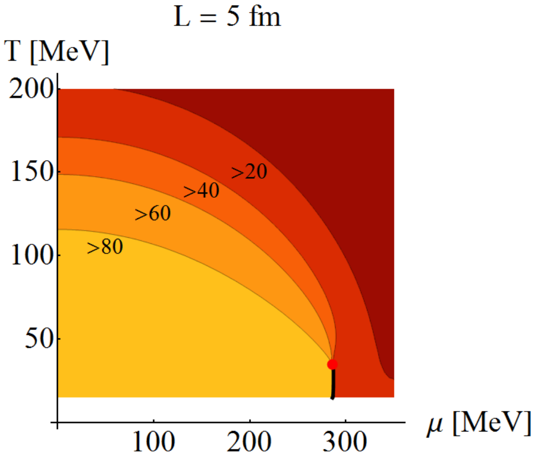

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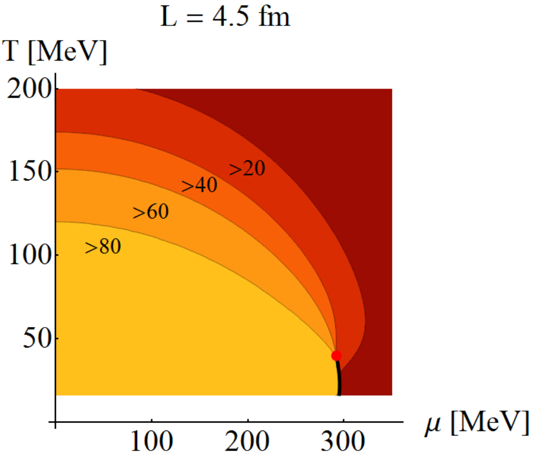

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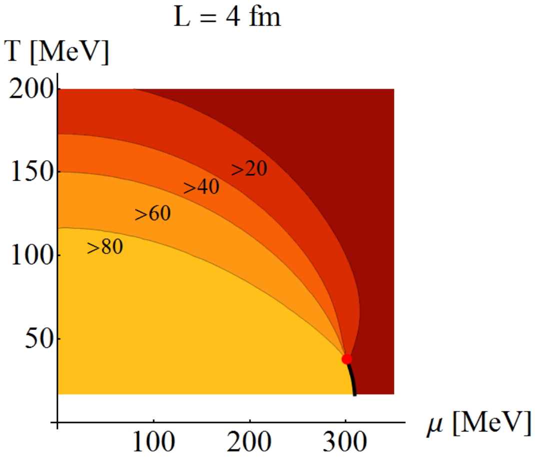

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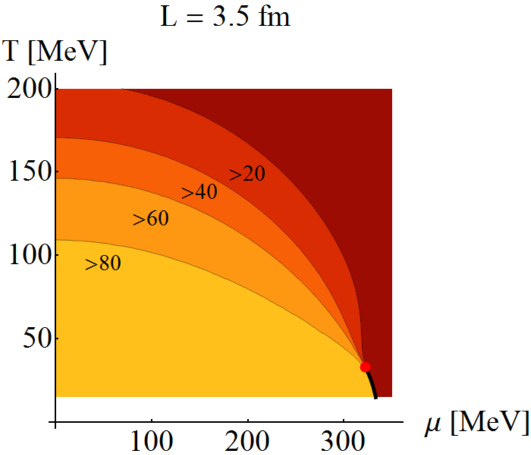

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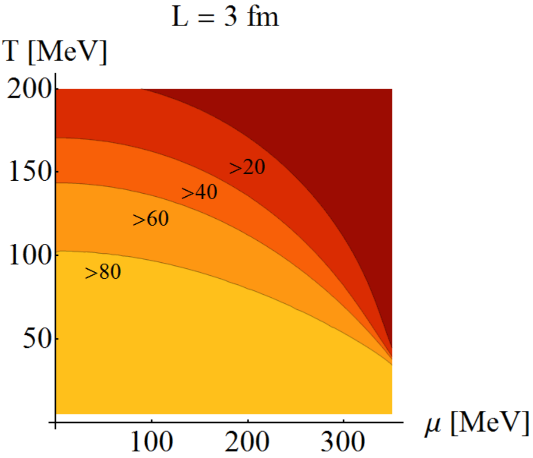

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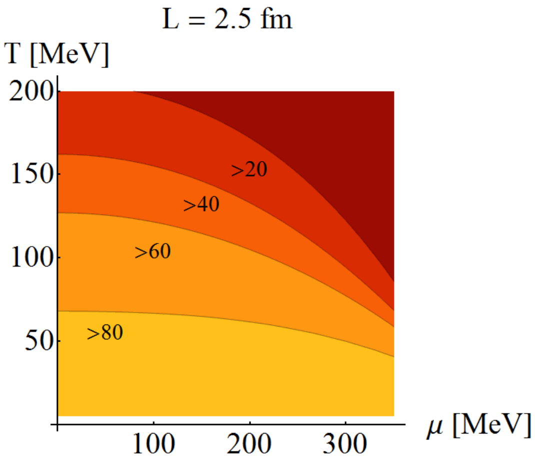

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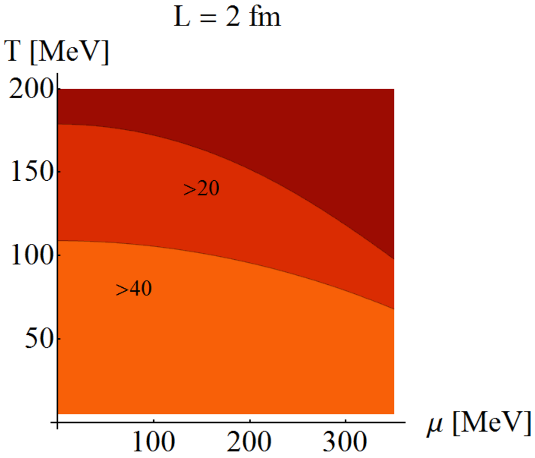

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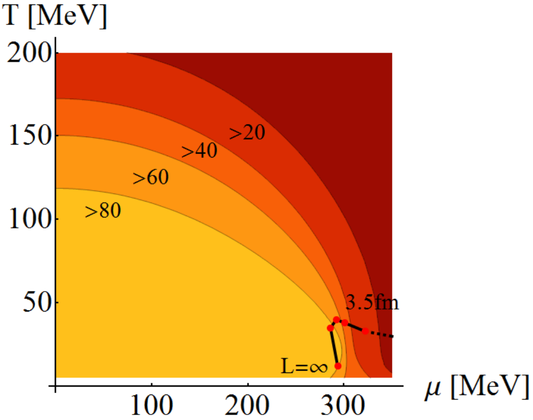


Finite volume effects

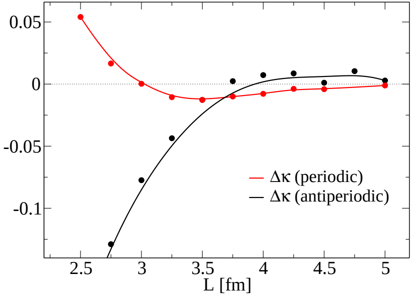
[A. Tripolt, J. Braun, B. Klein, BJS, in preparation '11]

- curvature κ

$$\frac{T_\chi(L,\mu)}{T_\chi(L,0)} = 1 - \kappa(L)\left(\frac{\mu}{\pi T_\chi(L,0)}\right)^2 + \ldots$$

- relative change

$$\Delta\kappa(L) = \frac{\kappa(L) - \kappa(\infty)}{\kappa(\infty)}$$



Summary

- $N_f = 2$ and $N_f = 2+1$ chiral (Polyakov)-quark-meson model study

➜ Mean-field approximation and FRG

➜ **fluctuations are important**

functional approaches (such as the presented FRG) are suitable and controllable tools to investigate the QCD phase diagram and its phase boundaries

Findings:

▷ **matter back-reaction to YM sector**: $T_0 \Rightarrow T_0(N_f, \mu)$

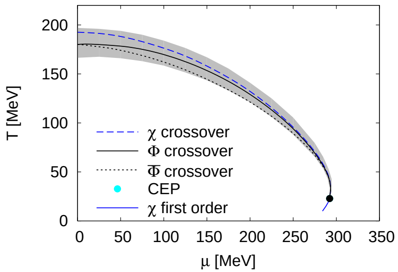
▷ **FRG with PQM truncation**: Chiral & deconfinement transition **coincide** for $N_f = 2$ with $T_0(\mu)$-corrections

▷ similar conclusion for $N_f = 2+1$

size of the critical region around CEP smaller

▷ Finite volume effects

▷ Higher moments



Outlook:

▷ FRG methods suitable → test lattice predictions (such as finite volume or $N_c = 2, \ldots$)

▷ include **glue dynamics** with FRG ➜ towards full QCD