Fluctuations around Bjorken flow and the onset of turbulent phenomena

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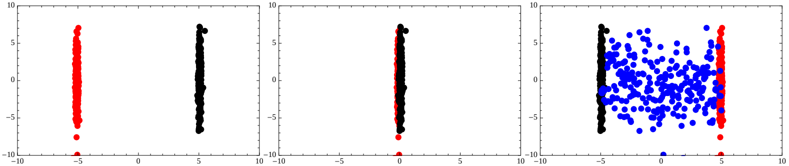
Heavy Ion Collisions



- ions are strongly Lorentz-contracted
- *some* medium is produced after collision
- medium expands in longitudinal direction and gets diluted

Evolution in time

- Non-equilibrium evolution at early times
  - initial state at from QCD? Color Glass Condensate? ...
  - thermalization via strong interactions, plasma instabilities, particle production, ...
- Local thermal and chemical equilibrium
  - strong interactions lead to short thermalization times
  - evolution from relativistic fluid dynamics
  - expansion, dilution, cool-down
- Chemical freeze-out
  - for small temperatures one has mesons and baryons
  - inelastic collision rates become small
  - particle species do not change any more
- Thermal freeze-out
  - elastic collision rates become small
  - particles stop interacting
  - particle momenta do not change any more

Fluid dynamic regime

- assumes strong interaction effects leading to local equilibrium
- fluid dynamic variables
  - thermodynamic variables: e.g. $T(x)$, $\mu(x)$
  - fluid velocity $u^\mu(x)$
- can be formulated as derivative expansion for $T^{\mu\nu}$
- hydrodynamics is universal!
- ideal hydro: needs equation of state $p = p(T, \mu)$ from thermodynamics
- first order hydro: needs also transport coefficients like viscosity $\eta = \eta(T, \mu)$ from linear response theory
- second order hydro: needs also relaxation times,...

Bjorken boost invariance

- How does the fluid velocity look like?
- Bjorkens guess: $v_3(x^0, x^1, x^2, x^3) = \frac{x^3}{x^0}$
- leads to an invariance under Lorentz-boosts in the x^3-direction
- use coordinates

$$\tau = \sqrt{(x^0)^2 - (x^3)^2}, \;\; x^1, \;\; x^2, \;\; y = \mathrm{arctanh}(x^3/x^0)$$

- fluid velocity

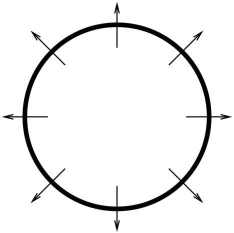
$$u^\mu = (u^\tau, u^1, u^2, u^y) = (\sqrt{1-(u^1)^2-(u^2)^2}, \; u^1, \; u^2, \; 0)$$

- thermodynamic scalars like energy density

$$\epsilon = \epsilon(\tau, x^1, x^2)$$

- remaining problem is 2+1 dimensional

Transverse Expansion



- for central collisions

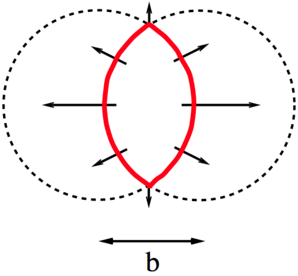
$$\epsilon = \epsilon(\tau, r)$$

- initial pressure gradient leads to radial flow

$$\begin{pmatrix} u^1 \\ u^2 \end{pmatrix} = \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} f(\tau, r)$$

- experimental signature not very distinct
  (particle momenta go in all directions)

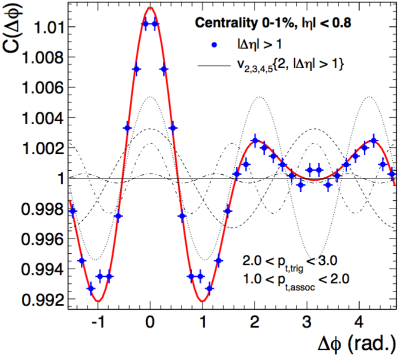
Elliptic flow



- non-central collisions lead to deviations from rotation symmetry
- pressure gradients larger in one direction
- larger fluid velocity in this direction
- more particles will fly in this direction
- can be quantified in terms of elliptic flow v_2

$$C(\Delta\phi) \sim 1 + 2\, v_2\ \cos(2\,\Delta\phi)$$

A puzzle: v_3 and v_5



(ALICE, arXiv:1105.3865, similar pictures also from CMS, ATLAS, Phenix)

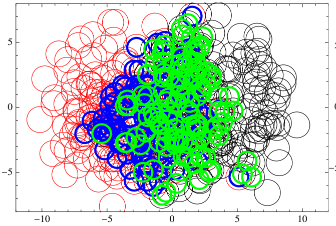
- quite generally, one can expand

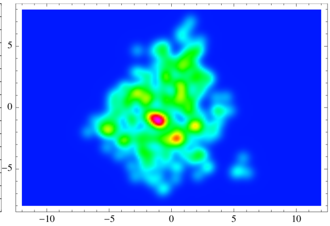
$$C(\Delta\phi) \sim 1 + \sum_{n=2}^{\infty} 2\, v_n\, \cos(n\, \Delta\phi)$$

- from symmetry reasons one expects naively $v_3 = v_5 = \ldots = 0$

Event-by-event fluctuations

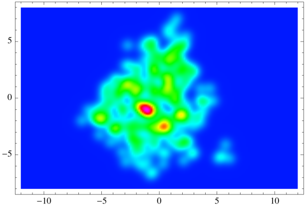
- argument for $v_3 = v_5 = 0$ is based on smooth energy density distribution
- there can be deviations from this due to event-by-event fluctuations
- for example using a Glauber model

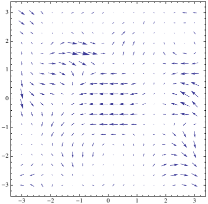




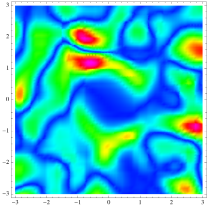
- this leads to sizeable v_3 and v_5

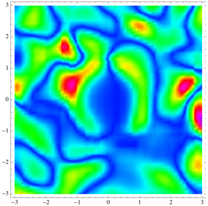
Velocity fluctuations





- also the velocity field will fluctuate at the initialization time τ_0
- take here transverse velocity for every participant to be Gaussian distributed with width $0.1c$
- vorticity $|\partial_1 u^2 - \partial_2 u^1|$ and divergence $|\partial_1 u^1 + \partial_2 u^2|$

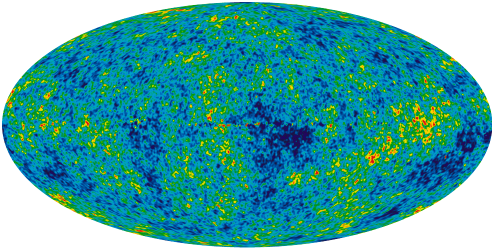


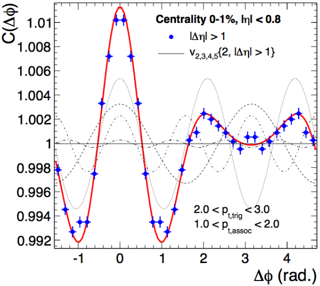


Why are fluctuations interesting?

- **Hydrodynamic fluctuations**: Local and event-by-event perturbations around the average of hydrodynamical fields:
    - energy density ϵ
    - fluid velocity u^μ
    - more general also: baryon number density n_B, ...
- measure for deviations from equilibrium
- contain interesting information from early times
- an be used to constrain thermodynamic and transport properties
- might affect other phenomena, e.g. jet quenching

Similarities to cosmic microwave background





- fluctuation spectrum contains info from early times
- many numbers can be measured and compared to theory
- detailed understanding of evolution needed
- could trigger precision era in heavy ion physics

Setup for treating fluctuations

- ensemble average over many events with fixed impact parameter b is described by smooth hydrodynamical fields

$$\bar{\epsilon} = \langle \epsilon \rangle$$
$$\bar{u}^\mu = \langle u^\mu \rangle$$

- fluctuations are added on top

$$\epsilon = \bar{\epsilon} + \delta\epsilon$$
$$u^\mu = \bar{u}^\mu + \delta u^\mu$$

- here we use Bjorkens simplified model (infinite extend in transverse plane)

$$\bar{\epsilon} = \bar{\epsilon}(\tau)$$
$$\bar{u}^\mu = (1, 0, 0, 0)$$
$$u^\mu = \bar{u}^\mu + \left(\delta u^\tau, u^1, u^2, u^y\right)$$

Linearized equations for fluctuations

- consider only terms linear in $\delta\epsilon,\ (u^1, u^2, u^y)$
- decompose velocity field into
  - gradient term, described by divergence

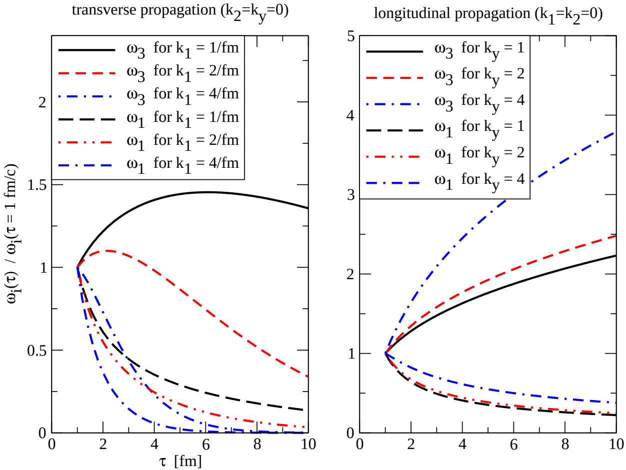
$$\theta = \partial_1 u^1 + \partial_2 u^2 + \partial_y u^y$$

  - rotation term, described by vorticity

$$\begin{aligned}\omega_1 &= \tau\,\partial_2 u^y - \frac{1}{\tau}\partial_y u^2 \\ \omega_2 &= \frac{1}{\tau}\partial_y u^1 - \tau\,\partial_1 u^y \\ \omega_3 &= \partial_1 u^2 - \partial_2 u^1\end{aligned}$$

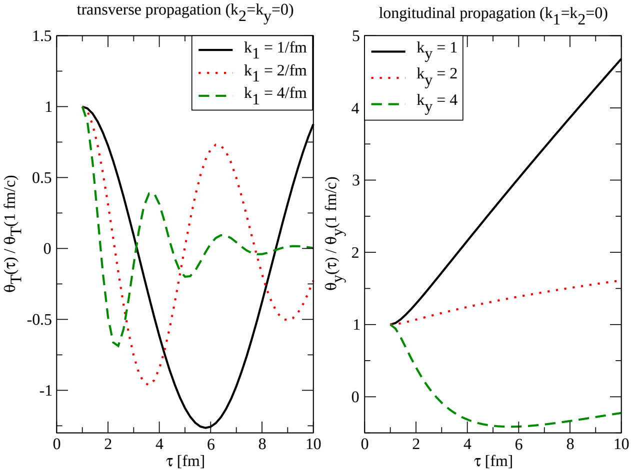
- θ and $\delta\epsilon$ are coupled: density or sound waves
- vorticity modes decouple from θ and $\delta\epsilon$

Vorticity modes



- solve equations in Fourier space $\omega_j = \omega_j(\tau, k_1, k_2, k_y)$
- short wavelength modes get damped by viscosity
- some modes can grow, however!

Sound modes



- oscillation for long times
- short wavelength modes get damped by viscosity
- also here some modes can grow!

Linear vs. non-linear evolution

- for linearized theory one can easily determine two-point correlation function at late times from the one at early times
- more difficult for non-linear evolution of fluctuations
- evolution equation for n-point functions couple: *"Closure problem"* in fluid dynamics literature
- needs more elaborate tools: functional techniques, numerical simulation, ...
- ... but here we do something else: we map the problem to another one!

Limits of linearized theory

linear approximation works for:

- energy density

$$\frac{\delta\epsilon}{\bar{\epsilon}} \ll 1$$

- velocity field

$$\mathsf{Re} \ll 1$$

large Reynolds number $\mathsf{Re} \gg 1$ leads to turbulence!

typical numbers: $T = 0.3\,\mathsf{Gev}$, $l = 5\,\mathsf{fm}$, $u_T = 0.1c$

$$\Rightarrow \quad \mathsf{Re} \approx \frac{1}{\eta/s} \approx \mathcal{O}(10)$$

Small Mach number

$$\mathrm{Ma} = \frac{\sqrt{u_1 u^1 + u_2 u^2 + u_y u^y}}{c_S} \ll 1$$

- turbulent motion can be described as “compression-less”

$$\theta = \partial_1 u^1 + \partial_2 u^2 + \partial_y u^y = 0$$

- sound modes decouple from vorticity, they are much faster
- this does not mean that there are no sound waves present

Change of variables

- kinematic viscosity, essentially independent of time

$$\nu_0 = \frac{\eta}{s\, T_{\mathsf{Bj}}(\tau_0)}$$

- new time variable (*not* laboratory time)

$$t = \frac{3}{4\tau_0^{1/3}}\, \tau^{4/3} \qquad\qquad \partial_t = \left(\frac{\tau_0}{\tau}\right)^{1/3} \partial_\tau$$

- rescaled velocity field

$$v_j = \left(\frac{\tau_0}{\tau}\right)^{1/3} u_j$$

- temperature field

$$d = \left(\frac{\tau_0}{\tau}\right)^{2/3} \ln\left(\frac{T}{T_{\mathsf{Bj}}(\tau)}\right)$$

Compression-less flow

this leads to

$$\partial_t v_j + \sum_{m=1}^{2} v_m \partial_m v_j + \frac{1}{\tau^2} v_y \partial_y v_j + \partial_j d$$
$$-\nu_0 \left(\partial_1^2 + \partial_2^2 + \frac{1}{\tau^2} \partial_y^2 \right) v_j = 0.$$

- index $j = 1, 2, y$
- solenoidal constraint

$$\partial_1 v_1 + \partial_2 v_2 + \frac{1}{\tau^2} \partial_y v_y = 0$$

- for large times τ effectively *two-dimensional Navier-Stokes*!

Reynolds numbers

assume

- typical velocity in transverse direction v_T
- typical velocity in rapidity direction v_y
- typical transverse length scale l
- typical rapidity difference Δy
- kinematic viscosity $\nu_0 = \frac{\eta}{s\,T}$

define

$$\mathrm{Re}^{(T)} = \frac{v_T l}{\nu_0}, \qquad \mathrm{Re}^{(y)} = \frac{v_y\, l^2}{\nu_0\, \Delta y} \frac{1}{\tau^2} \qquad \text{for} \qquad \frac{l}{\tau \Delta y} \ll 1$$

and

$$\mathrm{Re}^{(T)} = \frac{v_T\, \tau^2\, \Delta y^2}{\nu_0\, l}, \qquad \mathrm{Re}^{(y)} = \frac{v_y\, \Delta y}{\nu_0} \qquad \text{for} \qquad \frac{l}{\tau \Delta y} \gg 1$$

Turbulence in $d = 2$

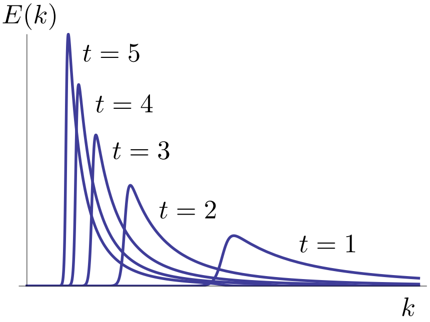
KRAICHNAN (1967):

- vorticity is conserved for $\nu_0 \to 0$
- scaling theory of forced turbulence in $d = 2$
- inverse cascade of energy to small wave numbers
- cascade of vorticity to large wave numbers

$$E(k) \sim k^{-3}$$

- qualitatively different to $d = 3$, emerges here dynamically

Decaying turbulence in $d = 2$



BATCHELOR (1969):

- scaling theory of decaying turbulence in $d = 2$

$$E(t,k) = \lambda^3 \, t \; f(k \, \lambda \, t) \qquad \text{with} \qquad \lambda^2 = \langle \vec{v}^2 \rangle = \text{const.}$$

- turbulent motion goes to smaller and smaller wave numbers

One-particle spectra

- fluctuations in fluid fields modify the one-particle spectra

$$\frac{dN}{d^3p} = \frac{dN_0}{d^3p} + \frac{d\delta N_1}{d^3p}\langle (u^1)^2\rangle + \frac{d\delta N_2}{d^3p}\langle (u^y)^2\rangle + \frac{d\delta N_3}{d^3p}\langle (T - T_{\mathsf{fo}})^2\rangle$$

- explicit expressions can be derived
- effect qualitatively similar to viscosity
- depends only on a few numbers

Two-particle spectra

- correlation function of particles with momenta $\vec{p}_A$ and $\vec{p}_B$

$$C(\vec{p}_A, \vec{p}_B) = \frac{\frac{dN}{d^3p_A d^3p_B}}{\frac{dN}{d^3p_A}\ \frac{dN}{d^3p_B}}.$$

- effect of hydro fluctuations on this can be calculated
- particular interesting are *identical particles* (e.g. pions with equal charge and spin)
- $C(\vec{p}_A, \vec{p}_B)$ can probe *correlation functions* of hydrodynamic fields at different space-time points
- characteristic power-law decay with $|\vec{p}_A - \vec{p}_B|$ in turbulent situation
- allows in principle to test Kraichnans law $E(k) \sim k^{-3}$

Effects on macroscopic motion of fluid

- turbulent fluctuations might affect macroscopic motion
  - modified equation of state
  - modified transport properties
- anomalous, turbulent or eddy viscosity
  - proposed by Asakawa, Bass, Müller (2006) for plasma turbulence and Romatschke (2007) for fluid turbulence
  - could become negative in $d = 2$ (Kraichnan (1976))
  - depends on detailed state of turbulence – not universal
  - gradient expansion needs separation of scales
- more work needed

Summary

- fluid fluctuations are very interesting!
- some modes even grow, others are damped
- hydrodynamical fluctuations on expanding medium can become turbulent
- evolution laws can be mapped to two-dimensional Navier-Stokes equation for late times
- turbulence has interesting effects on the two-particle spectrum

Backup

Little Bang vs. Big Bang

Heavy Ions

Bjorken model

$$x^\mu = (\tau, x^1, x^2, y)$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & \tau^2 \end{pmatrix}$$

$$\epsilon_0(\tau), \quad u_0^\mu = (1,0,0,0)$$

$+$ hydrodyn. fluctuations

$$\epsilon = \epsilon_0(\tau) + \epsilon_1(\tau, x^1, x^2, y)$$

$$u^\mu = u_0^\mu + u_1^\mu(\tau, x^1, x^2, y)$$

Cosmology

Friedmann-Robertson-Walker

$$x^\mu = (t, x^1, x^2, x^3)$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & a(t) & & \\ & & a(t) & \\ & & & a(t) \end{pmatrix}$$

$$\epsilon_0(t), \quad u_0^\mu = (1,0,0,0)$$

$+$ hydrodyn. fluctuations

$$\epsilon = \epsilon_0(t) + \epsilon_1(t, x^1, x^2, x^3)$$

$$u^\mu = u_0^\mu + u_1^\mu(t, x^1, x^2, x^3)$$

$+$ gravity fluctuations

Turbulence in $d = 3$

fully developed turbulence

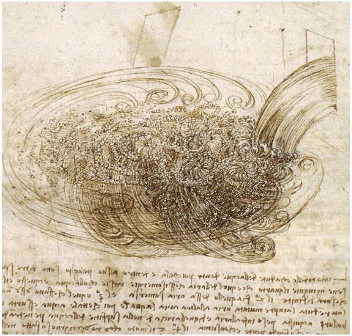
$$\text{Re} \to \infty$$

dissipated energy per unit time

$$\frac{d}{dt}\langle \vec{v}^2 \rangle = -\nu_0 \left\langle (\vec{\nabla} \times \vec{v})^2 \right\rangle = -\varepsilon$$

RICHARDSON (1922):

Big whorls have little whorls,
Which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscosity.



L. DA VINCI (CA. 1500)

KOLMOGOROV (1941):

$$E(k) \sim \varepsilon^{2/3} k^{-5/3}$$

with

$$\tfrac{1}{2}\langle \vec{v}^2 \rangle = \int_0^\infty dk\, E(k)$$