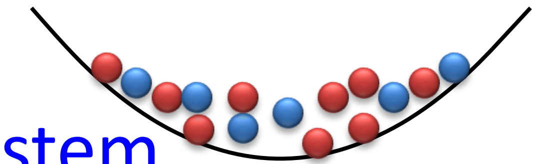
One-dimensional Fermi gases: Density-matrix renormalization group study of ground state properties and dynamics

30 August 2011
Masaki TEZUKA (Kyoto University)

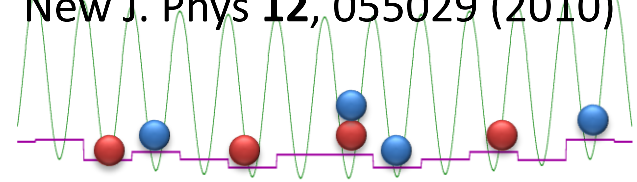
Plan of the talk

- Introduction
  - Low-dimensional cold atom systems
  - Density-matrix renormalization group
- Harmonically trapped imbalanced system
  - Larkin-Ovchinnikov state
- Optical lattice with disorder
  - Proving superfluidity by dynamics
- Collision dynamics
  - Non-classical reflectance
- Summary



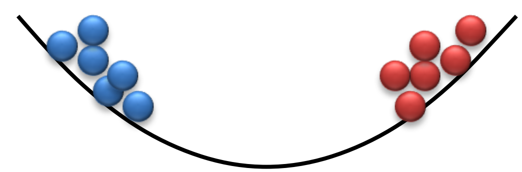
PRL **100**, 110403 (2008)

New J. Phys **12**, 055029 (2010)

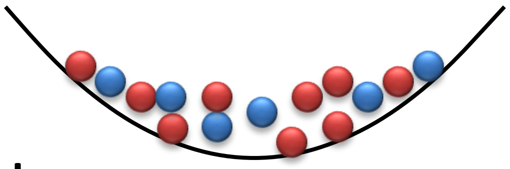


PRA **82**, 043613 (2010)

arXiv: 1107.0774



Collaborators

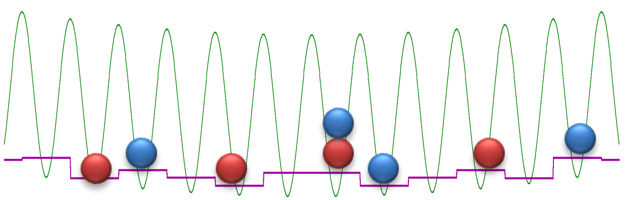


- Masahito Ueda (University of Tokyo)
  - Harmonically trapped imbalanced system

PRL **100**, 110403 (2008)
New J. Phys **12**, 055029 (2010)

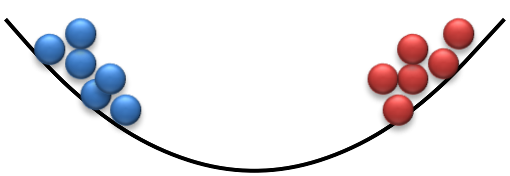
- Antonio M. García-García (Cambridge University)
  - Optical lattice with disorder

PRA **82**, 043613 (2010)

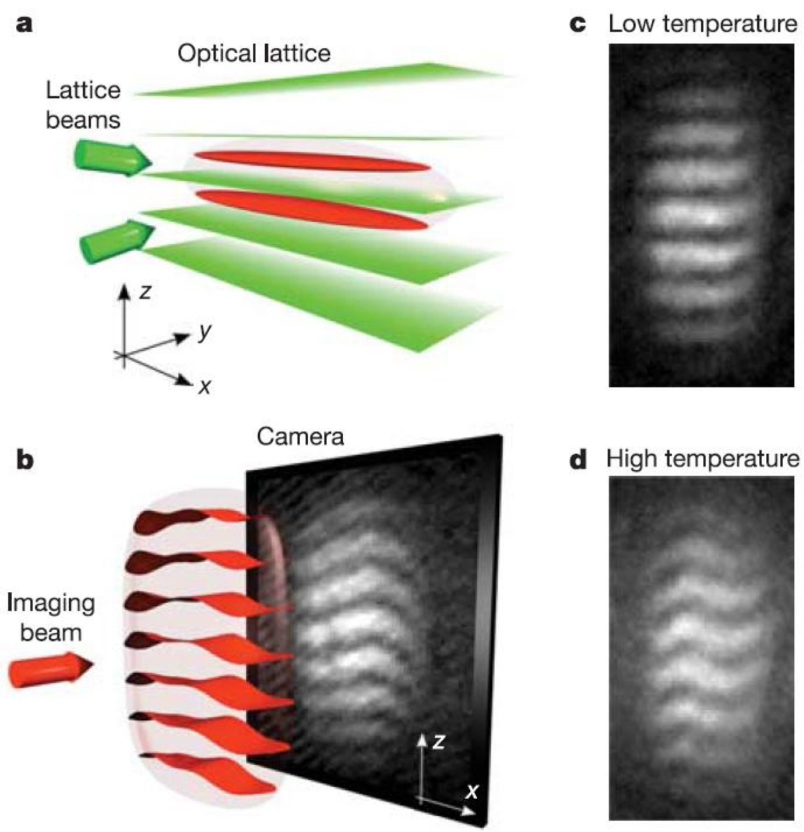


- Jun'ichi Ozaki (Kyoto University)
- Norio Kawakami (Kyoto University)
  - Collision dynamics

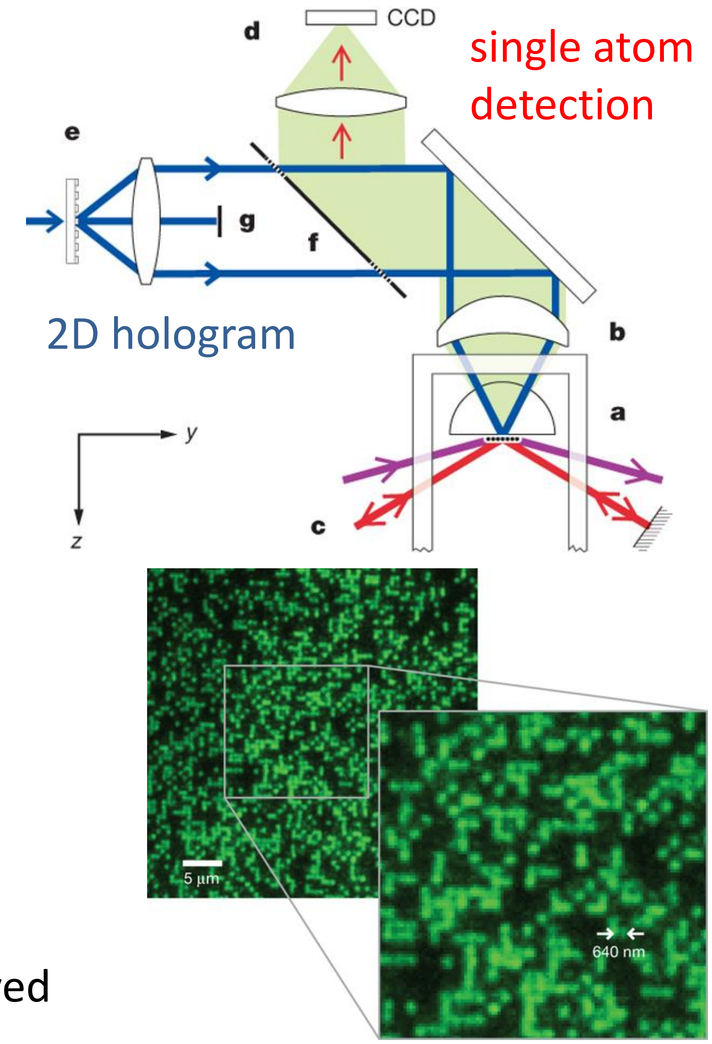
arXiv: 1107.0774



Low-dimensional cold atom systems



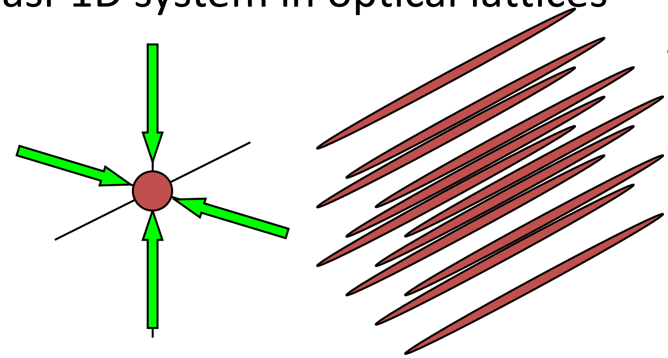
2D Bose-Einstein condensates (BEC) released and collide
➔Berezinskii-Kosterlitz-Thouless (BKT) transition is observed
Hadzibabic *et al.*: Nature **441**, 1118 (2006)



WS Bakr *et al.*: Nature **462**, 74 (2009)

Low-dimensional cold atom systems

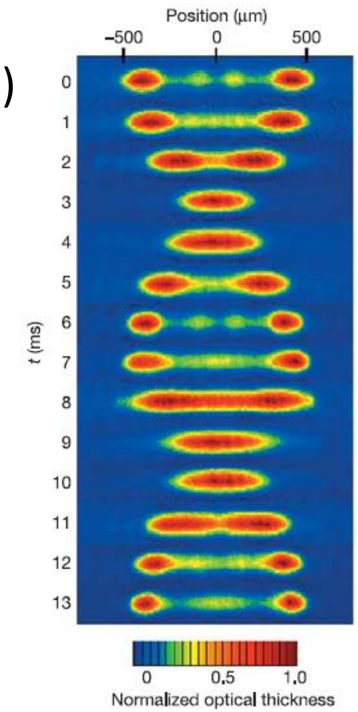
Quasi-1D system in optical lattices



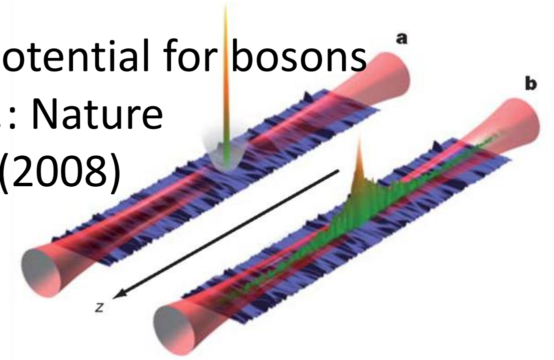
2D lattice of 1D systems

see *e.g.* I. Bloch, Nature Phys. (2005)

1D Bosons: absence of thermalization
Kinoshita et al.:
Nature **440**, 900 (2006)



Speckle potential for bosons
Billy *et al.*: Nature
453, 891 (2008)

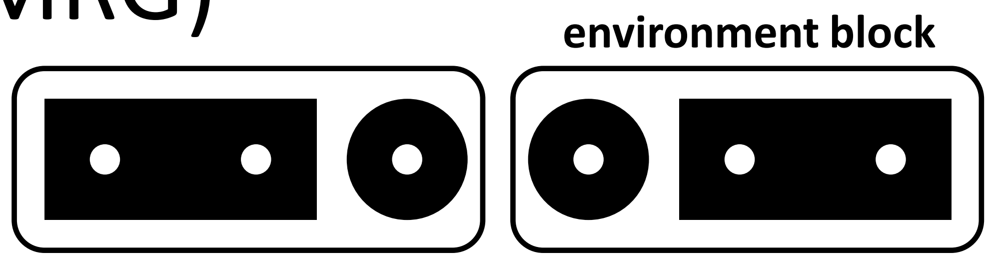


"1D" : Strong radial confinement
Radial level separation >> energy scale of axial motion
(Fermi energy in fermionic system)

- Low-dimensional but not necessarily solvable
- How to simulate numerically without uncontrollable approximations?

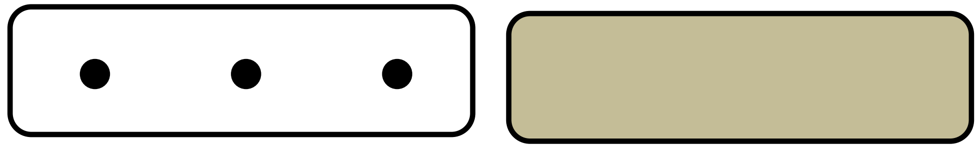
Density-Matrix Renormalization Group (DMRG)

System S = B + s + s′ + B′



$$\left|\Psi_{ijj'i'}\right\rangle = c_{ijj'i'} \sum_{ijj'i'} \left|\phi_i\right\rangle \left|\chi_j\right\rangle \left|\chi_{j'}\right\rangle \left|\phi_{i'}\right\rangle$$

Partial density matrix for the left block L ($k \equiv (ij)$)



$$\rho_{kk'} = \sum_{i'j'} \left|\Psi_{kj'i'}\right\rangle \left\langle \Psi_{kj'i'}\right| = \sum_{i'j'} c_{kj'i'} c^{*}_{k'j'i'}$$

Has all information on L = B + s when S is in the **target state** |Ψ>

Diagonalize ρ : states with larger eigenvalues are more important

cf. In NRG (numerical renormalization group) the lowest energy states are kept

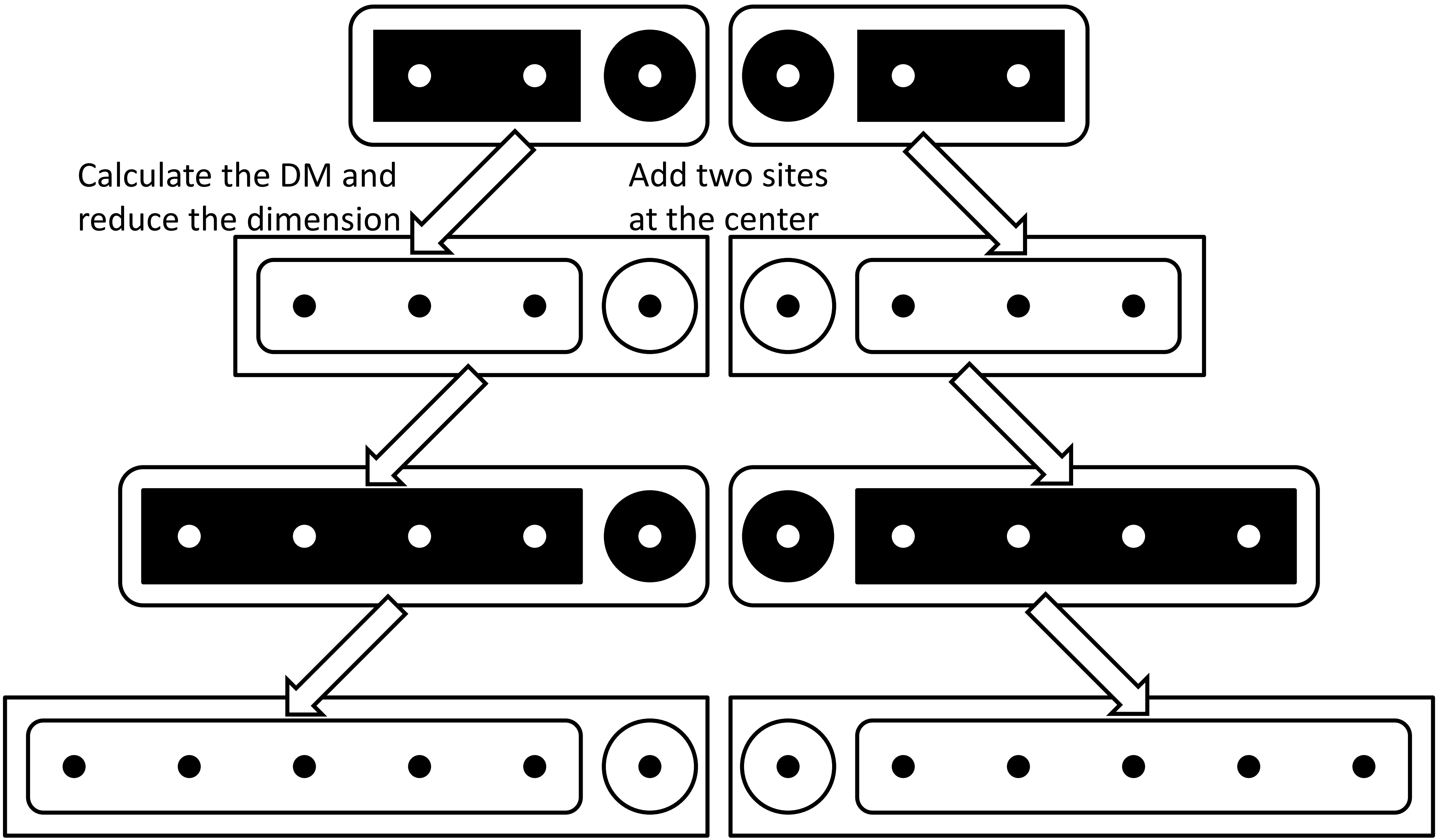
Reduce the dimension to *m*

of the Hilbert space for L

= “renormalization”

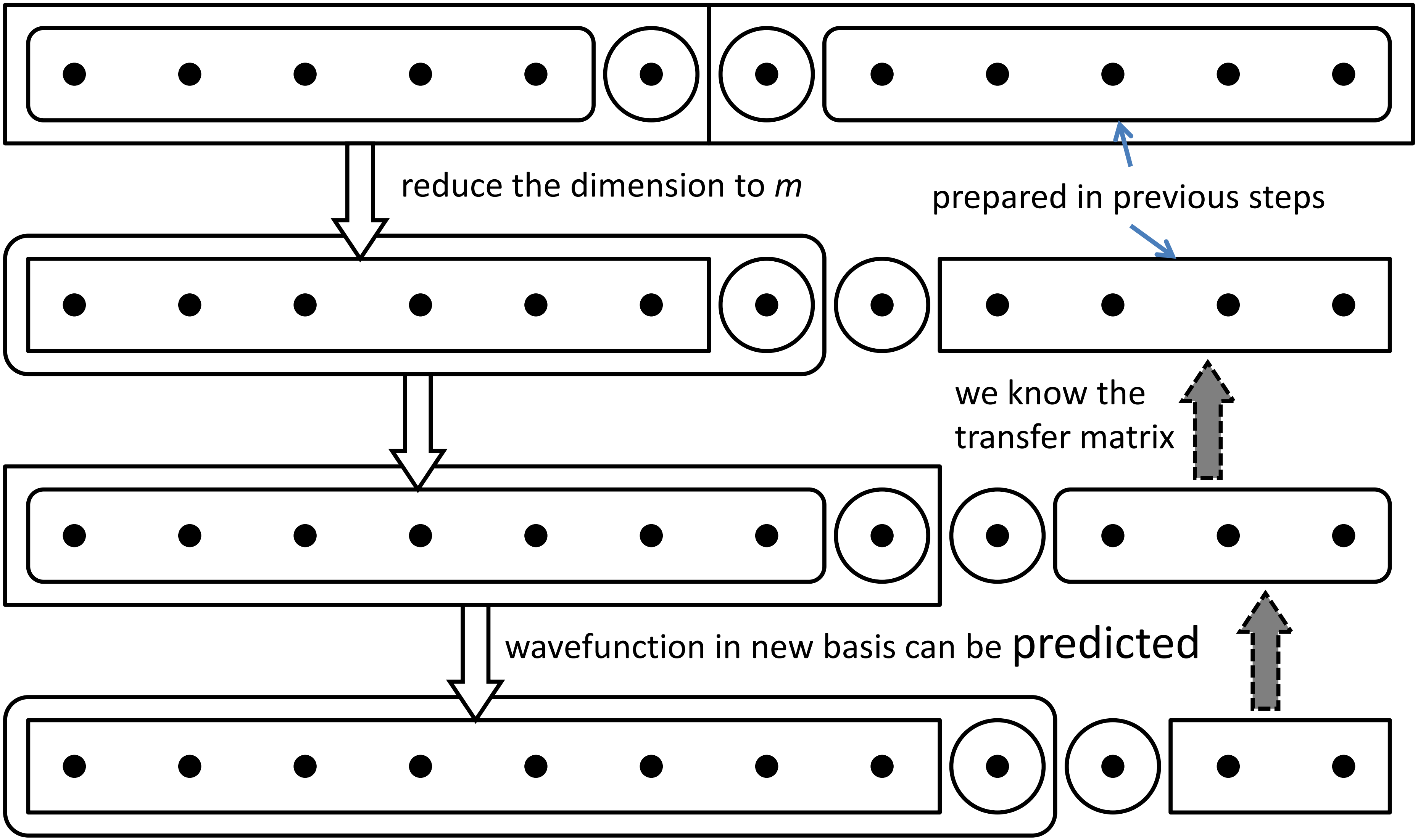
S. R. White: PRL (1992) PRB (1993)
Reviews:
Schollwöck: Ann. Phys. **326**, 96 (2011),
RMP **77**, 259 (2005)
Hallberg: Adv. Phys. **55**, 477 (2006)

Infinite system DMRG



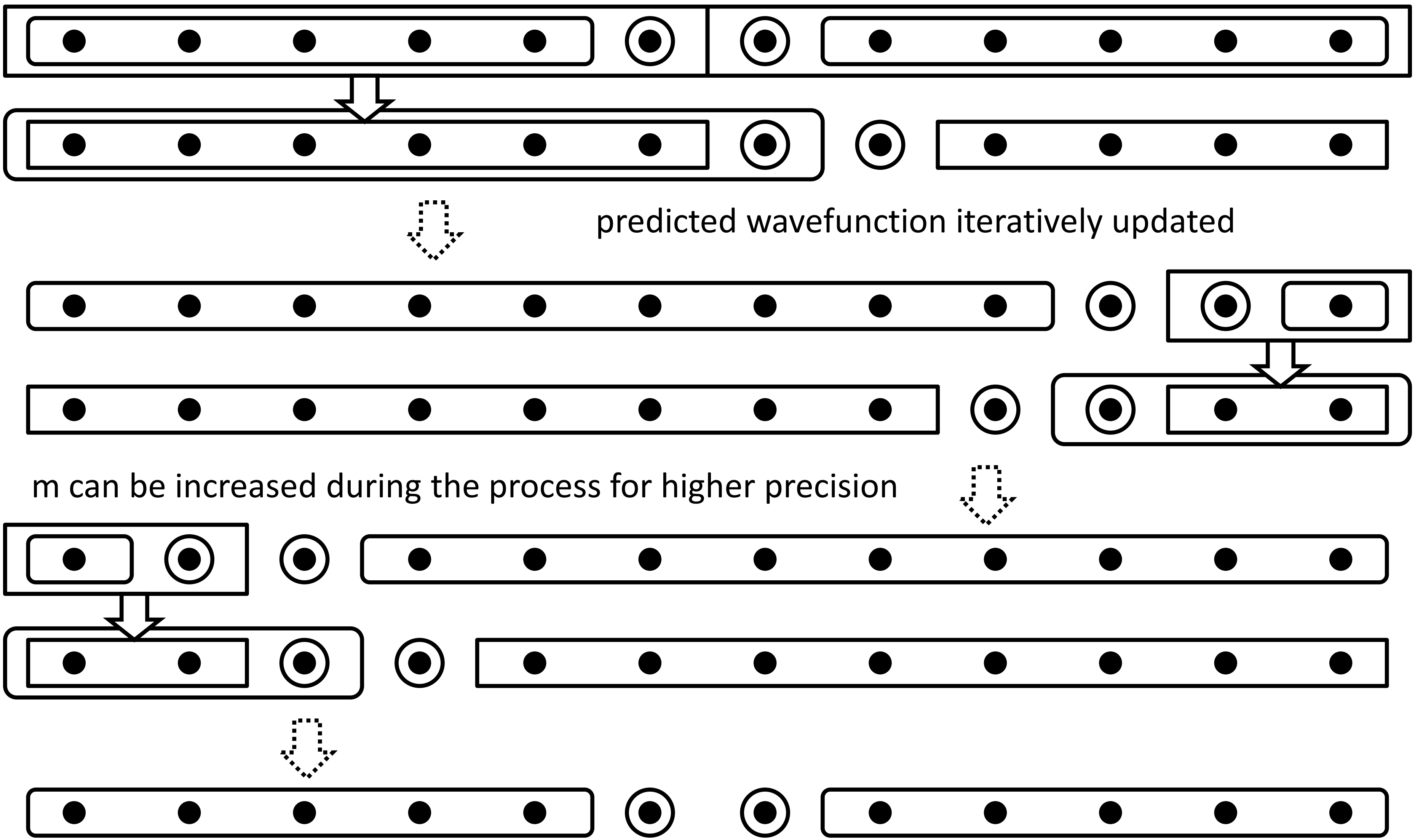
Iterate until the desired system size is reached

Finite system DMRG



One of the blocks can be made longer iteratively

Finite system DMRG



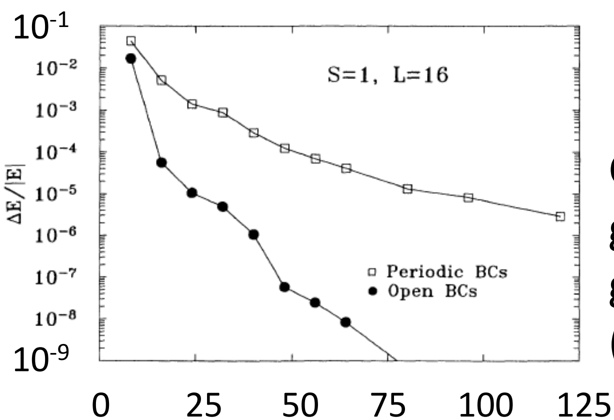
Iterate until physical quantities (*e.g.* energy) converge

Application of DMRG: Low-dimensional quantum systems

DMRG: variational method

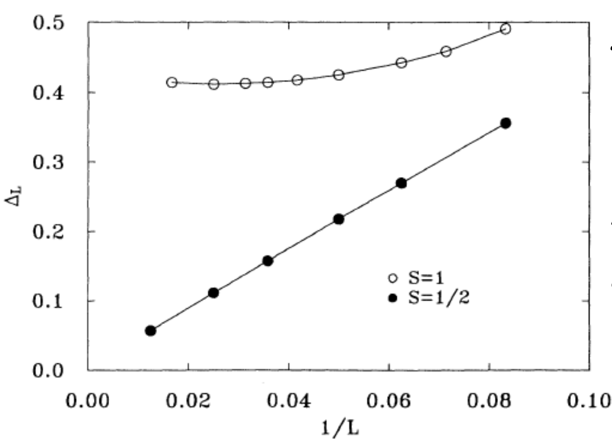
Error in ground state energy is positive, and decrease as # of states m is increased

1D Heisenberg model (White: PRB **48**, 10345 (1993))



$$H = \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

Conjecture:
gapped for spin $N \in \mathbf{N}$,
gapless for spin N+1/2
(Haldane PRL 1983)

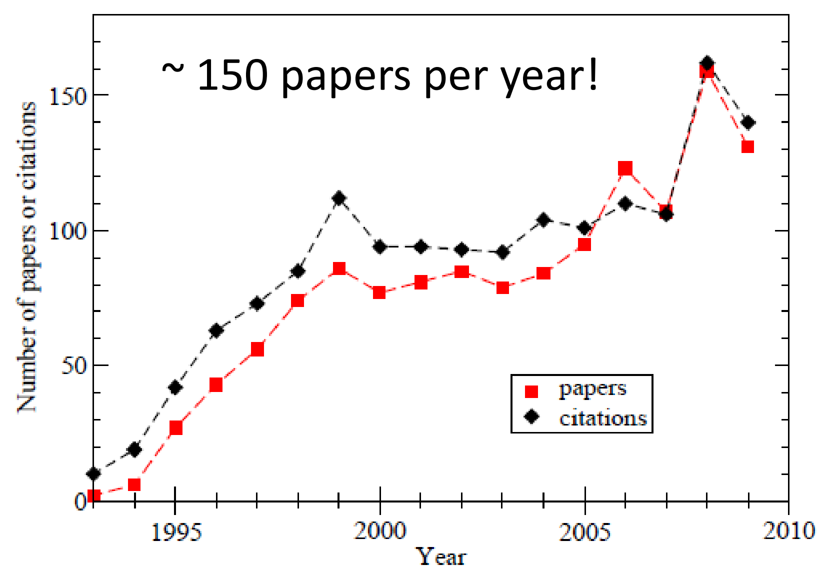


S=1 Haldane gap detected

An improved method gives 0.4104792485(4) (Ueda and Kusakabe: PRB **84**, 054446 (2011))

DMRG publications from 1994 to 2009

Annual number of
(a) published papers on the topic "density-matrix-renormalization" and
(b) citations to Steve White's original paper [PRL 69, 2863-2866 (1992)].
Data from the ISI *Web of Science* database at http://apps.isiknowledge.com

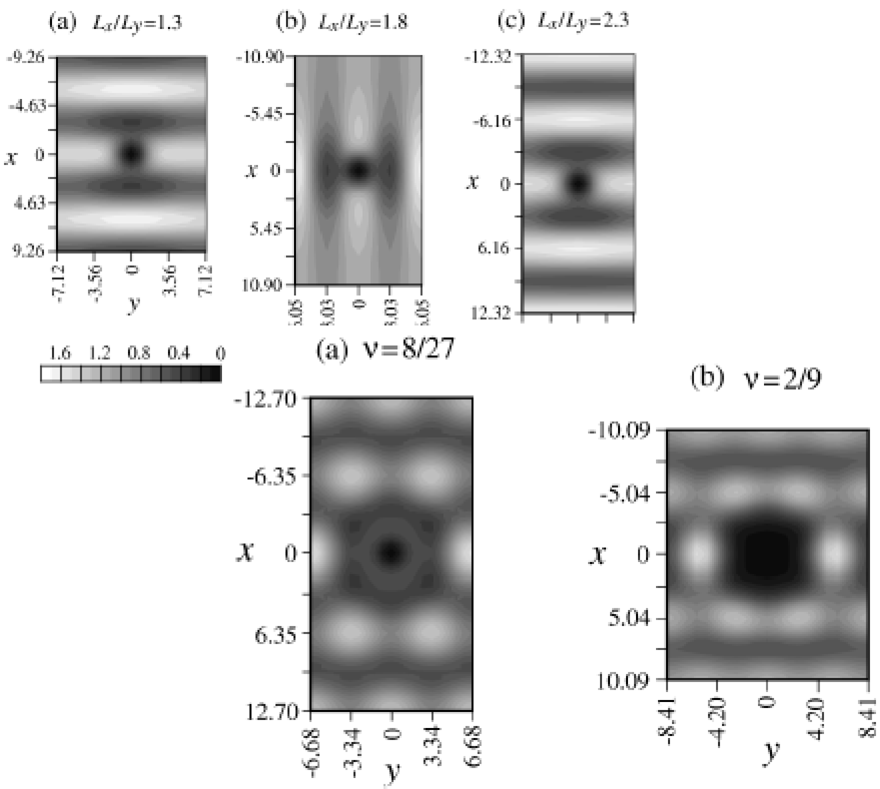


Eric Jeckelmann, April 16, 2010

http://www.itp.uni-hannover.de/~jeckelm/dmrg/paper_stat5.pdf

Application of DMRG: Low-dimensional quantum systems

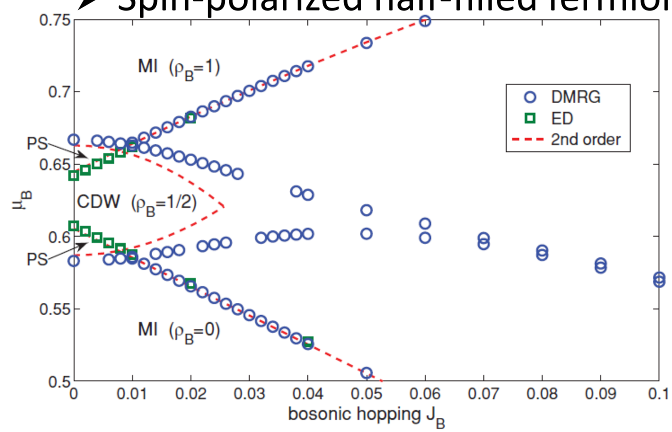
Fractional Quantum Hall systems
(Landau levels: effectively 1D)



Shibata and Yoshioka: PRL **86**, 5755 (2001)

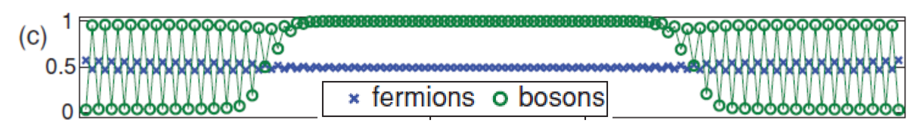
Mixtures of bosons and fermions

- Spin-polarized half-filled fermions + bosons

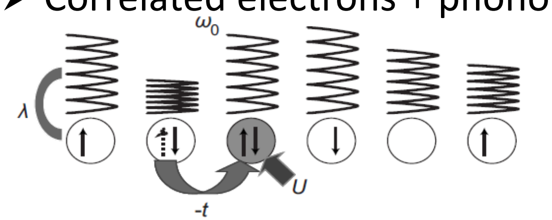


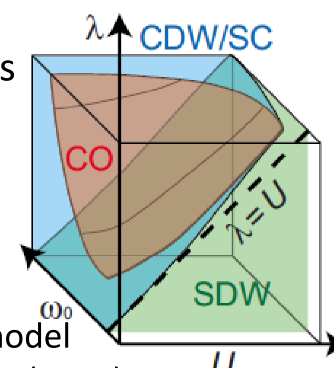
Mering and Fleischhauser: PRA **81**, 011603 (R) (2010)

Phase separation between CDW and Mott insulator



- Correlated electrons + phonons





Phase diagram of Hubbard-Holstein model
Tezuka, Arita and Aoki: PRB **76**, 155114 (2007)

Also, quantum chemistry, 2D & 3D classical systems, …

Two-component Fermi gas

Neutral atoms: bosons or fermions

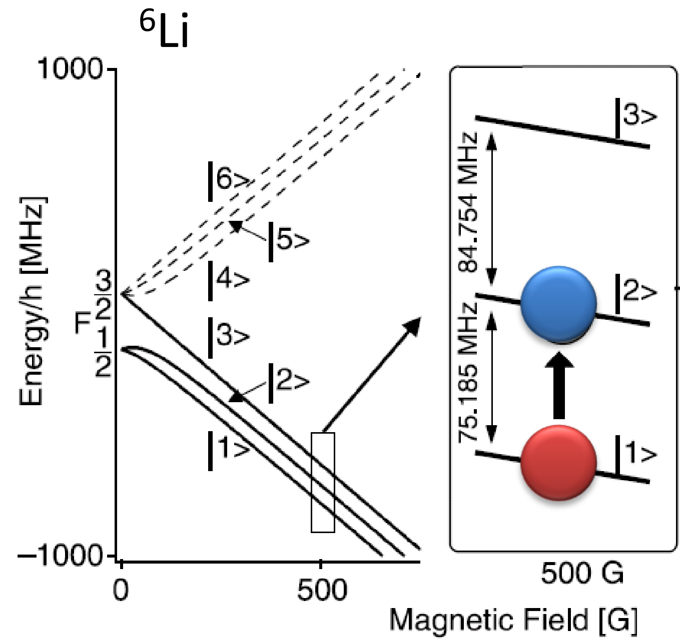
depending on parity of A + Z
(nucleon number + electron number)

Atom: fine structure, hyper fine structure

electron spin *S*,

orbital degrees of freedom,

nuclear spin *I*



Gupta *et al.*: Science **300**, 1723 (2003)

Fermions in two hyperfine states:

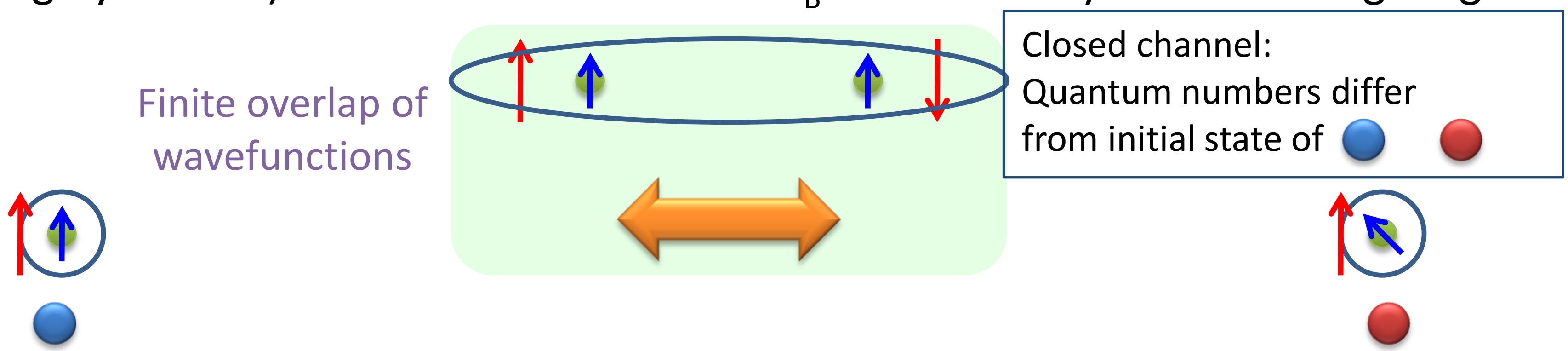
Loss due to three-body collisions is rare 🡐 Pauli principle

(pseudo-) spin population preserved

➔ Fermi gas with fixed spin imbalance can be realized

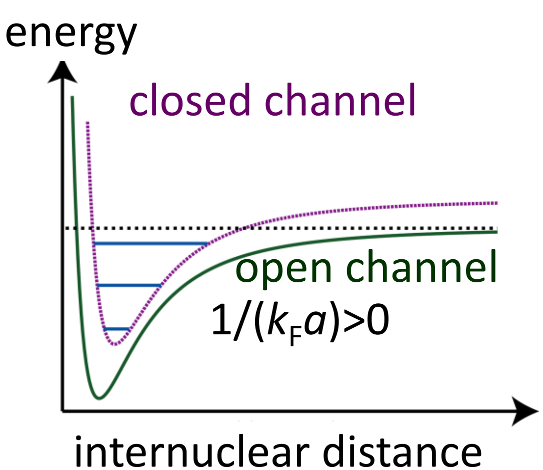
Feshbach resonance

A (highly excited) molecular state close to E_B=0 can modify the scattering length



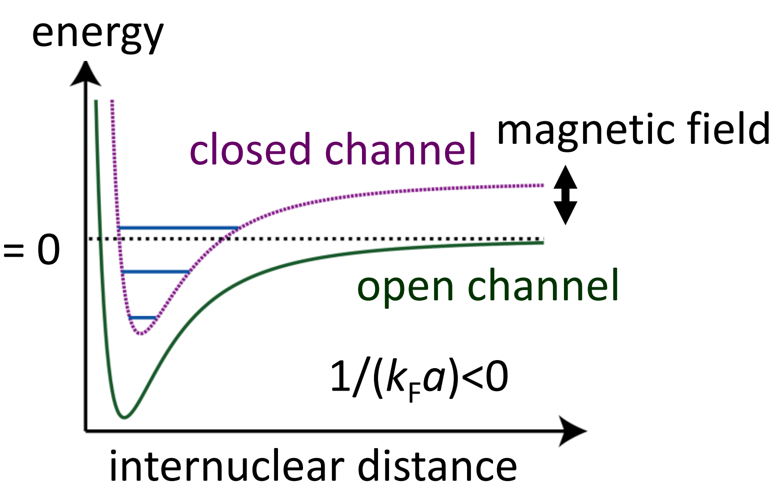
When the open and close channels differ in magnetic moment, we can utilize

<u>Rapid change of scattering length as function of magnetic field *B*</u>



Feshbach resonance:

One bound (=molecular) state has binding energy = 0



$1/(k_F a)=0$

B

➔ interaction (including sign) can be controlled

For equal number of atoms,

BEC-BCS crossover

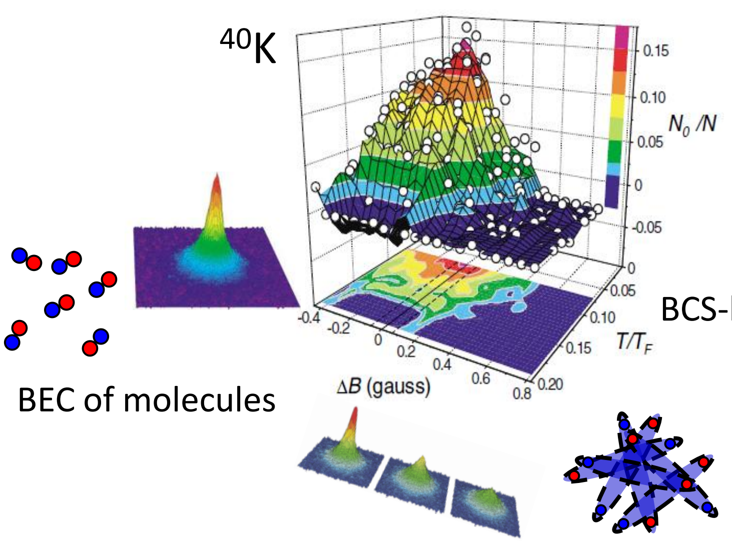
BEC : Bosons might break into fermions at energy Δ, but Δ is not correlated with T_c

BCS-type condensate : Pairing gap Δ is in proportion to T_c (density determines T_c)

BEC of diatomic molecules : smoothly connected with BCS condensate?

Theory: Eagles (1969), Leggett (1980), Nozières and Schmidt-Rink (1985), …

Experiment for the same number of Fermi atoms in two of the hyperfine states:



Greiner *et al.*: PRL (2003, 2004) Nature (2003)

➔ What happens when the numbers of two spins are not equal?

1) Harmonically trapped imbalanced system

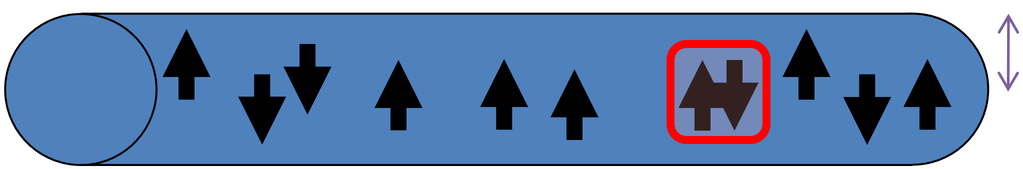
Motivation: condensation of population imbalanced fermions in elongated traps

Zwierlein *et al.*(MIT): Science **311**, 492 (2006)
Partridge *et al.*(Rice): Science **311**, 503 (2006)
➔ Discrepancy in cloud shape and maximum imbalance *P* for condensate

More recent experiment:
Nascimbène *et al.* (ENS): Nature **463**, 1057 (2010)

$P=(N_{\uparrow}-N_{\downarrow})/(N_{\uparrow}+N_{\downarrow})$

Q. What happens when the atom trap is essentially 1D?

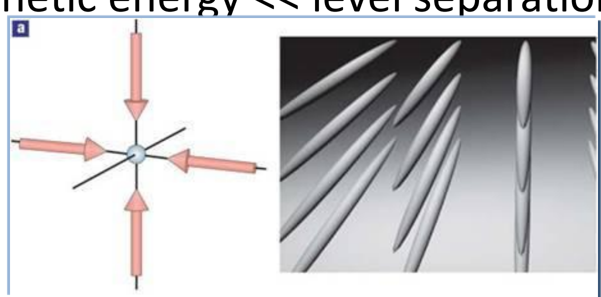


s-wave scatt. length ($\propto g^{-1}$)

$$a_{1D} = -\frac{a_{\perp}^2}{2a}\left(1 - C\,\frac{a}{a_{\perp}}\right)$$

Olshanii: PRL 1998

Kinetic energy << level separation of the radial trap

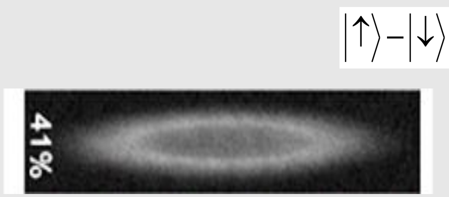
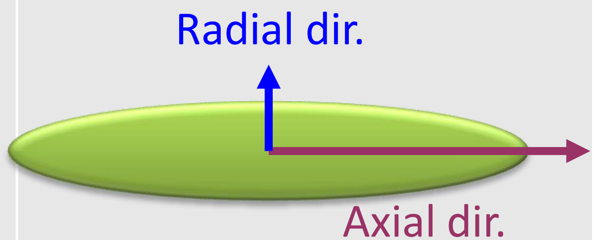
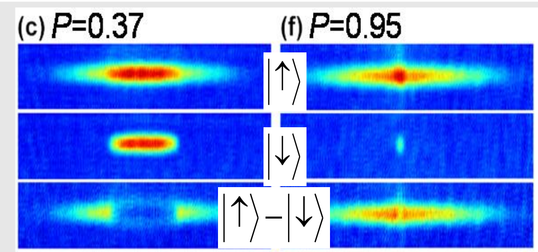


2D optical lattice → array of 1D traps realized
(Esslinger group, ETH Zürich; Hulet group, Rice)

I. Bloch: Nature Phys. (2005)

(Situation in 2006-2008)

3D: Controversy over experiments

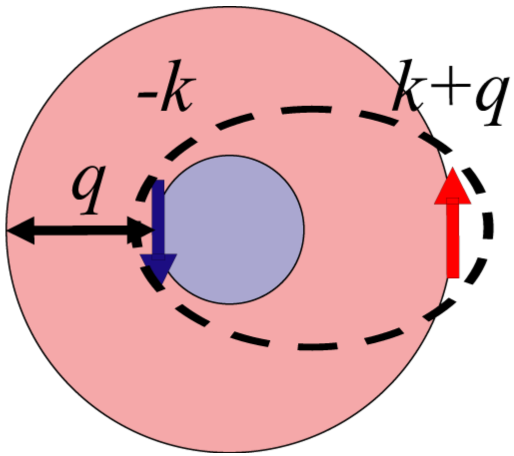
MIT		Rice
5 	Trap aspect ratio λ	50 
$\sim 10^7$	Number of ^{6}Li atoms N	$\sim 10^5$
~equipotential surface	Density distibution	significant deformation
 Shin *et al.*:PRL **97**, 030401(2006)		 Partridge *et al.*: PRL **97**, 190407 (2006)
P_{CC}=75-80%	Upper limit for imbalance P for condensation	P_{CC}>95%

$P=(N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$

$|\uparrow\rangle$: majority
$|\downarrow\rangle$: minority
(two hyperfine states of ^{6}Li)

→ What happens in 1D?

Pair with finite momentum: FF and LO states

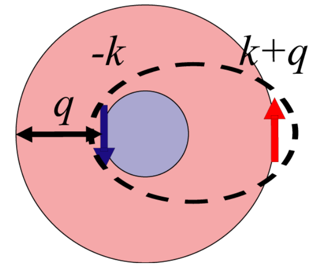


Different Fermi momentum for up and down spins

Pairs with non-zero momentum condense

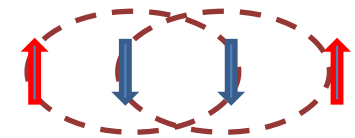
LO : density difference oscillates with wave number 2q

Fulde-Ferrell state

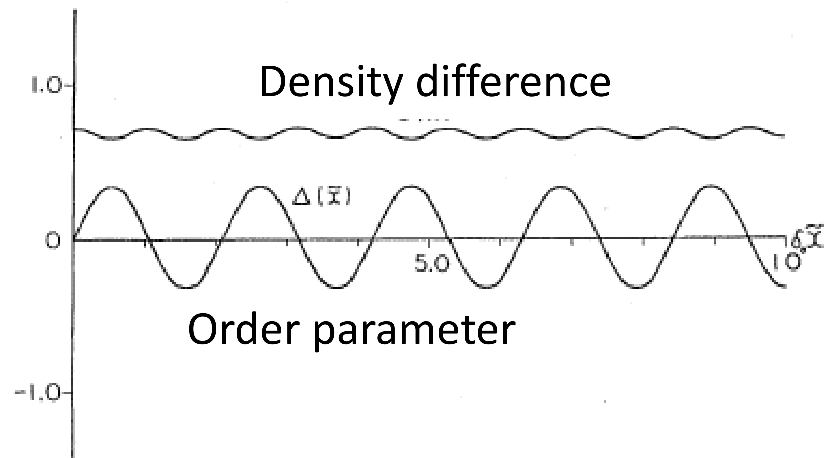


$\Delta \sim \exp(iqx)$

Larkin-Ovchinnikov state



$\Delta \sim \cos(qx)$



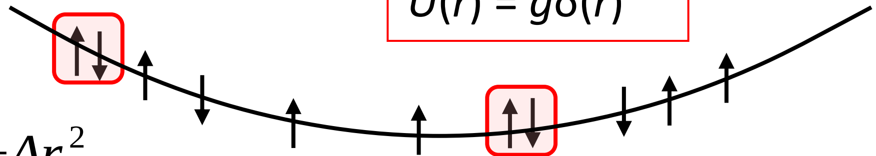
Machida and Nakanishi: PRB 30, 122 (1984)

Discretization of the space
➔ Apply DMRG

Confining potential

$\phi(r) = Ar^2$

$U(r) = g\delta(r)$



Kinetic energy

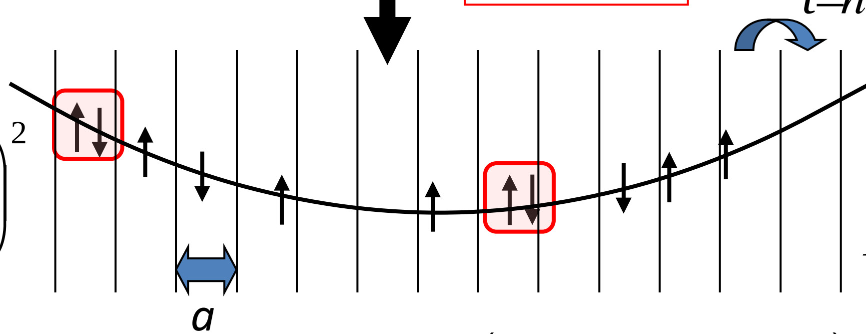
$K(k) = \hbar^2 k^2 / 2m$

$U = g / a$

$t = \hbar^2 / 2ma^2$

Site potential

$$\phi_i = Aa^2\left(i - \frac{L-1}{2}\right)^2$$



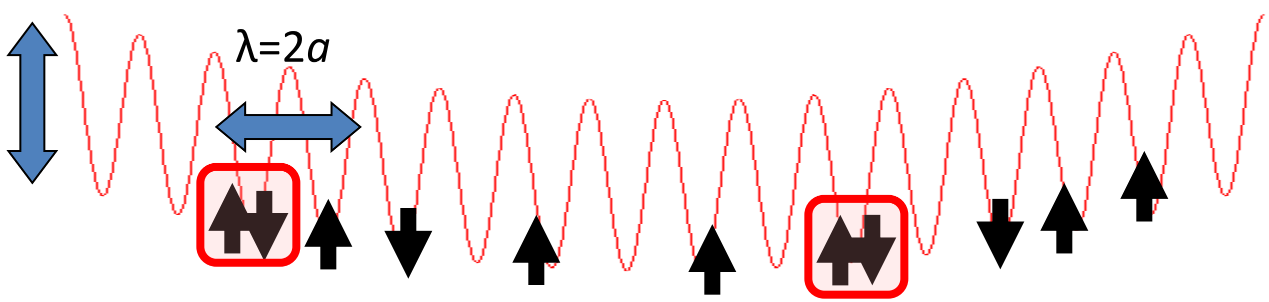
K. E. dispersion

$$K(k) = 2t(1-\cos ka) \approx ta^2k^2$$

L-site Hubbard model

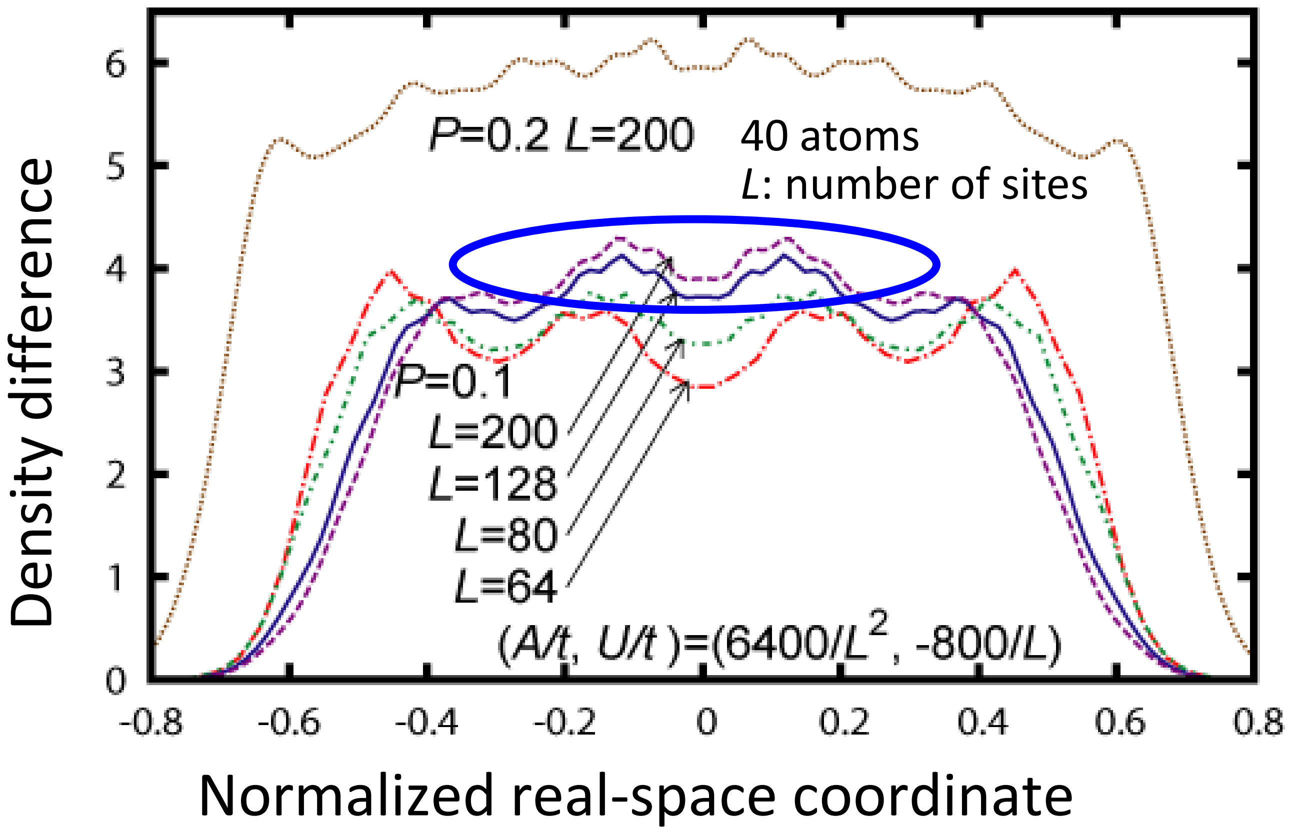
$$H = -t\sum_{\langle i,j\rangle,\sigma=\uparrow,\downarrow}\left(\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + \hat{c}^\dagger_{j\sigma}\hat{c}_{i\sigma}\right) + U\sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow} + \sum_{i,\sigma}\phi_i\hat{n}_{i\sigma}$$

cf. optical lattice systems



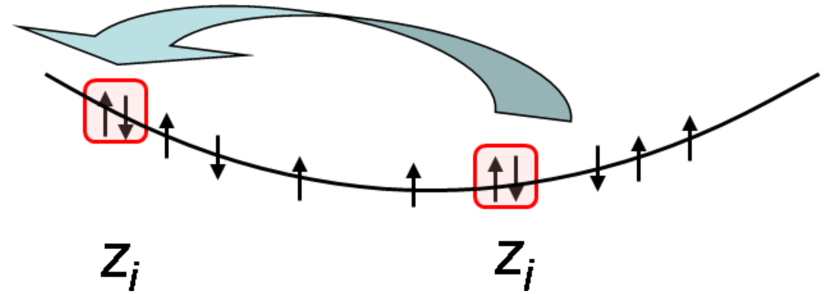
DMRG simulation of continuous system with the lattice introduced

Smaller lattice spacing ➔ continuum limit approached



Density difference: shows oscillation incommensurate with lattice

Pair correlation and density distribution



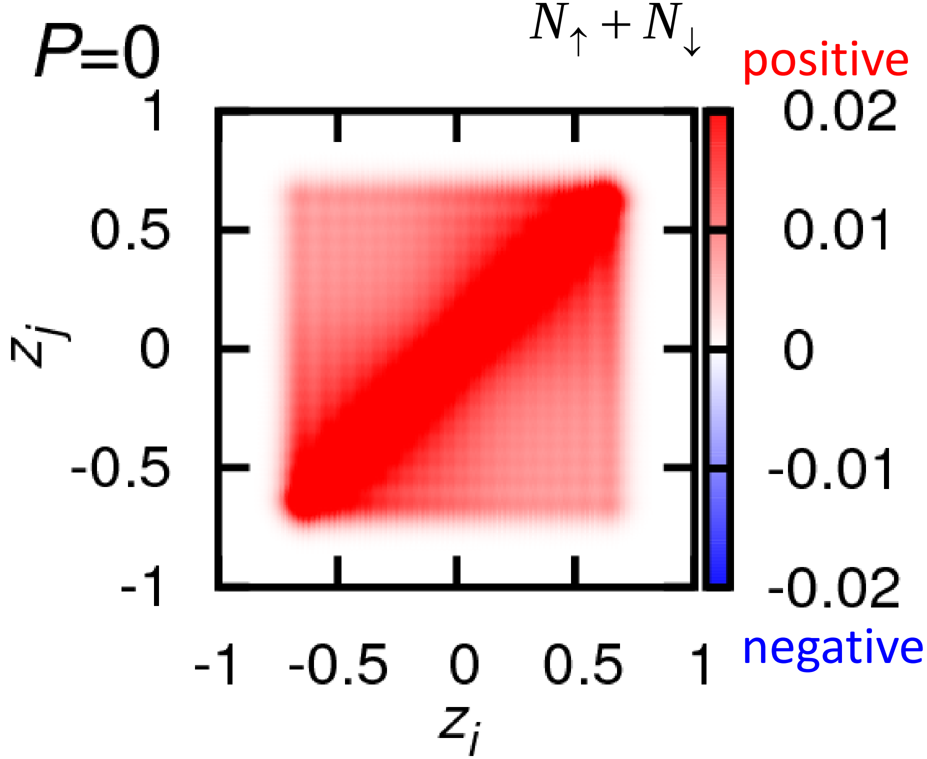
Pair correlation

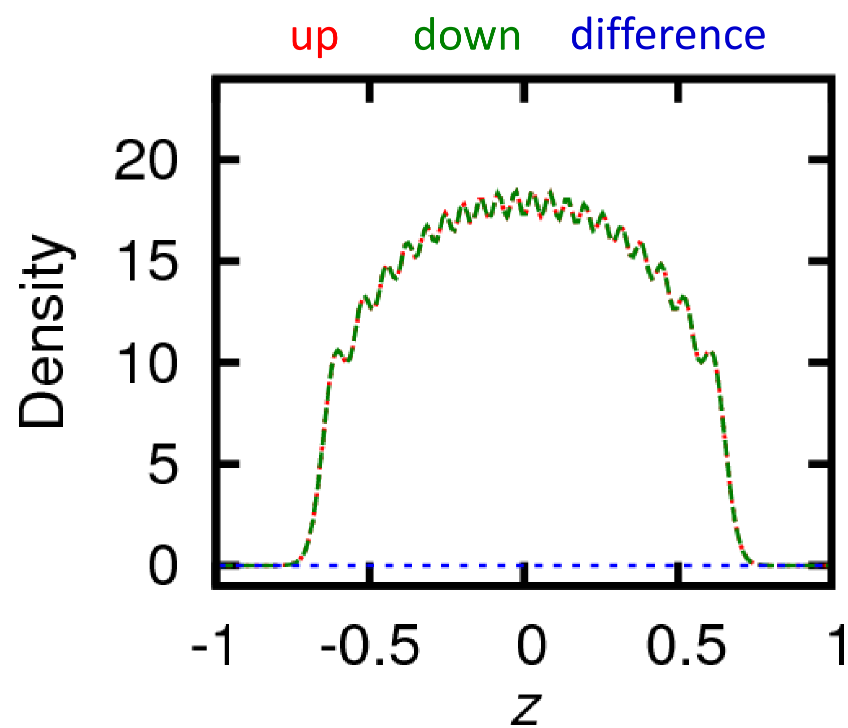
$$\left\langle \psi_0^{(N)} \left| \hat{c}_{i,\downarrow}^{\dagger} \hat{c}_{i,\uparrow}^{\dagger} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \right| \psi_0^{(N)} \right\rangle \approx \Delta(z_i)^* \Delta(z_j)$$

M. Tezuka and M. Ueda,
PRL 100, 110403 (2008)

imbalance parameter

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

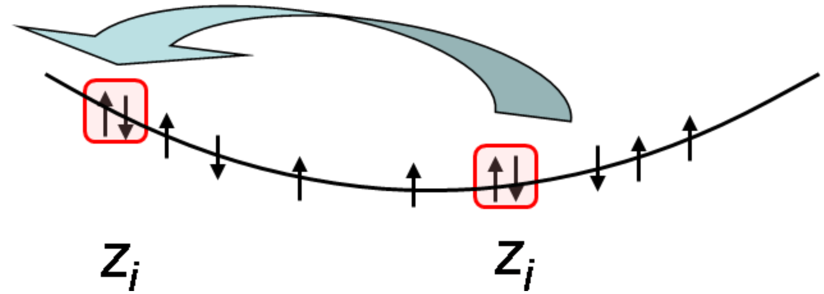




Pair correlation and density distribution

Pair correlation

$$\left\langle \psi_0^{(N)} \left| \hat{c}_{i,\downarrow}^{\dagger} \hat{c}_{i,\uparrow}^{\dagger} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \right| \psi_0^{(N)} \right\rangle \approx \Delta(z_i)^* \Delta(z_j)$$



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M. Tezuka and M. Ueda,
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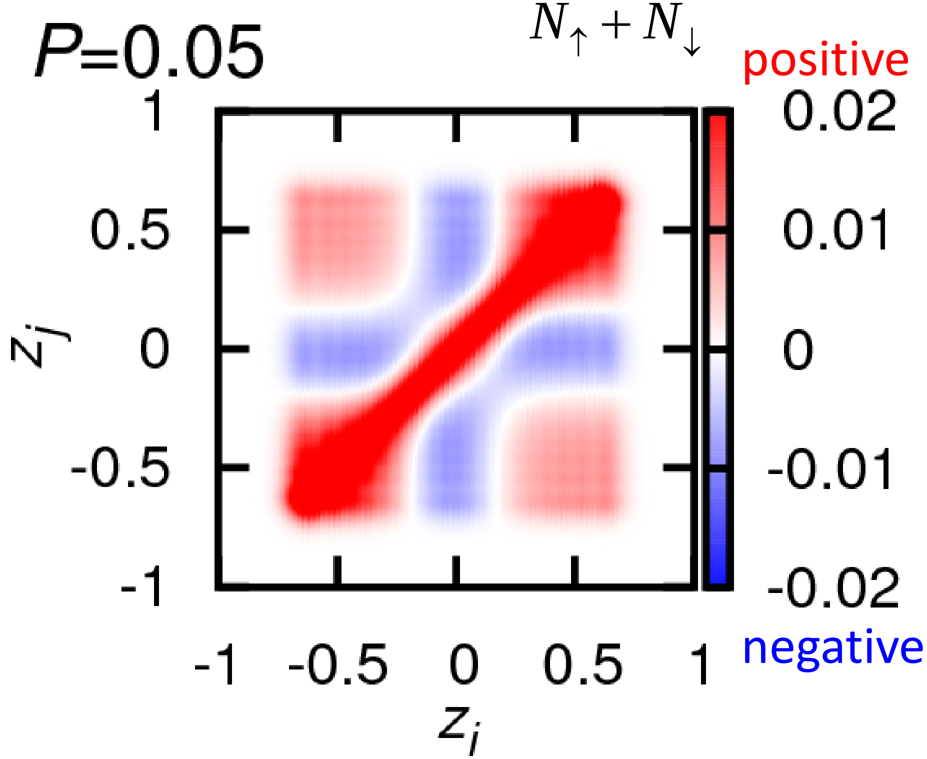
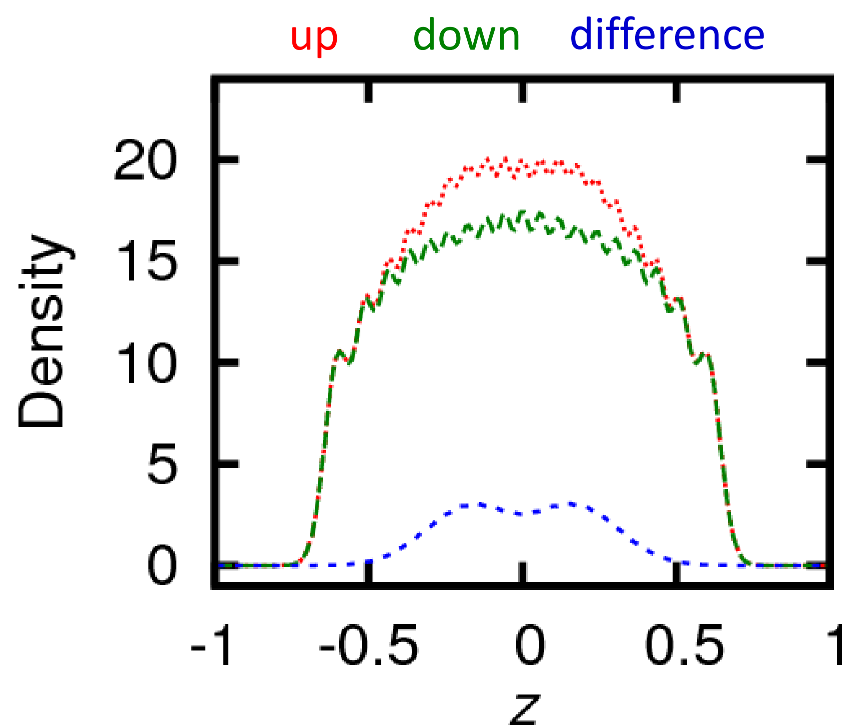
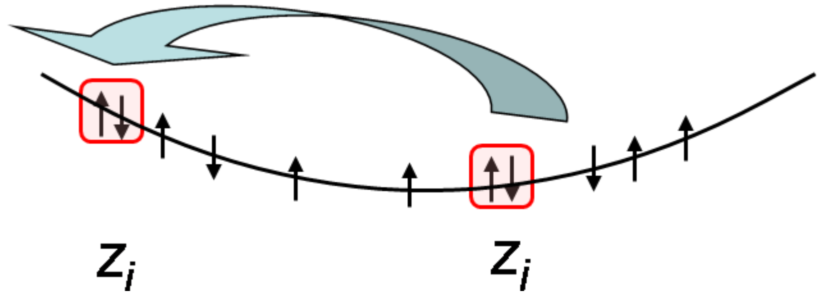




Pair correlation and density distribution

Pair correlation

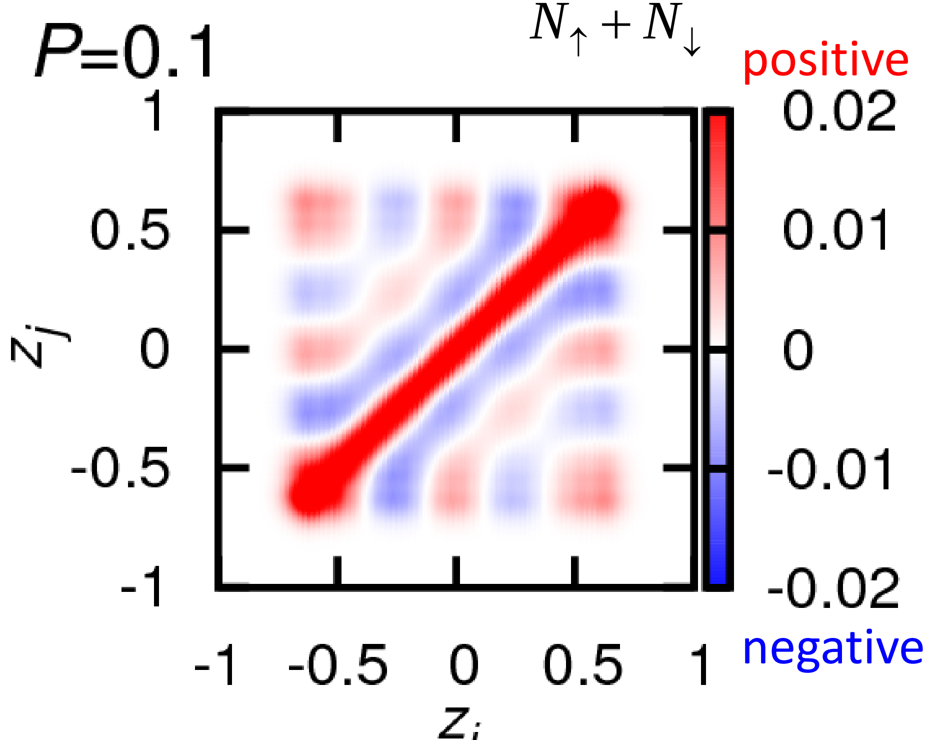
$$\left\langle \psi_0^{(N)} \right| \hat{c}^\dagger_{i,\downarrow}\hat{c}^\dagger_{i,\uparrow}\hat{c}_{j,\uparrow}\hat{c}_{j,\downarrow} \left| \psi_0^{(N)} \right\rangle \approx \Delta(z_i)^* \Delta(z_j)$$

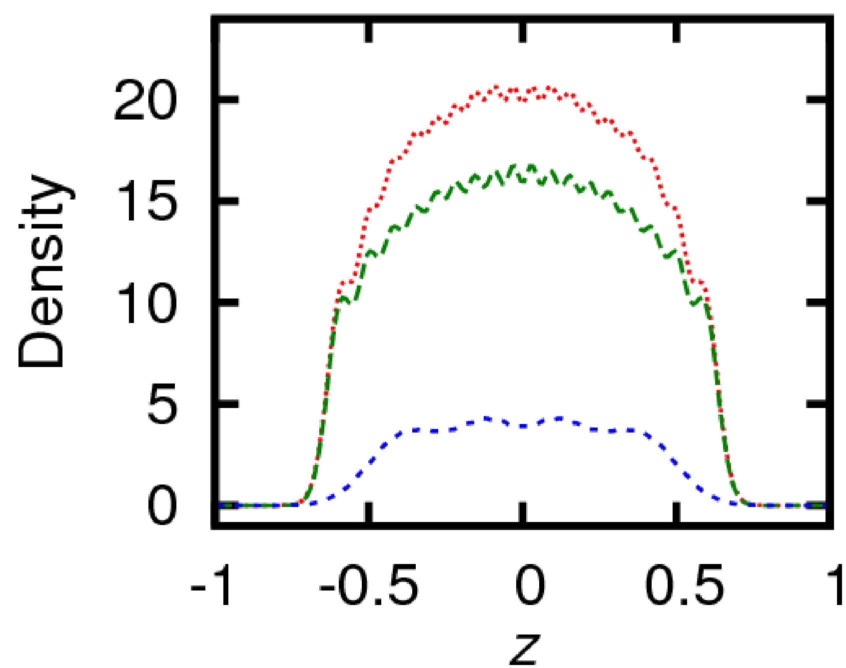


M. Tezuka and M. Ueda, PRL 100, 110403 (2008)

imbalance parameter

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

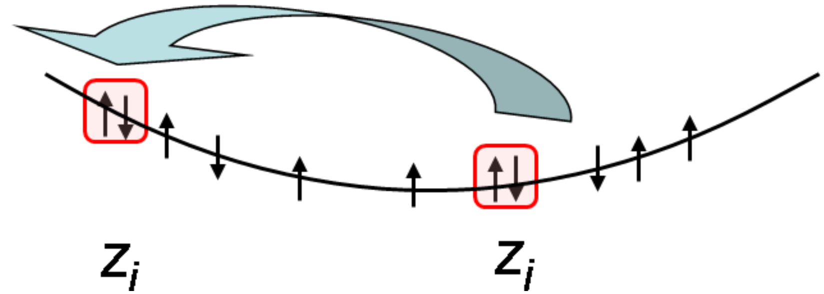




Pair correlation and density distribution

Pair correlation

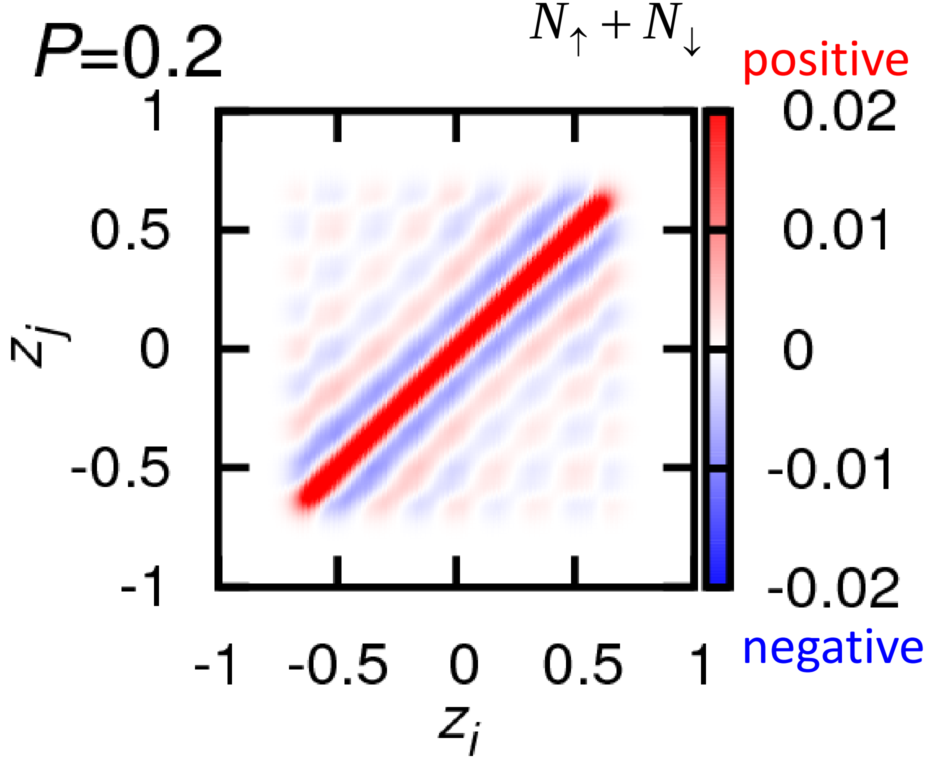
$$\left\langle \psi_0^{(N)} \middle| \hat{c}_{i,\downarrow}^{\dagger} \hat{c}_{i,\uparrow}^{\dagger} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \middle| \psi_0^{(N)} \right\rangle \approx \Delta(z_i)^* \Delta(z_j)$$

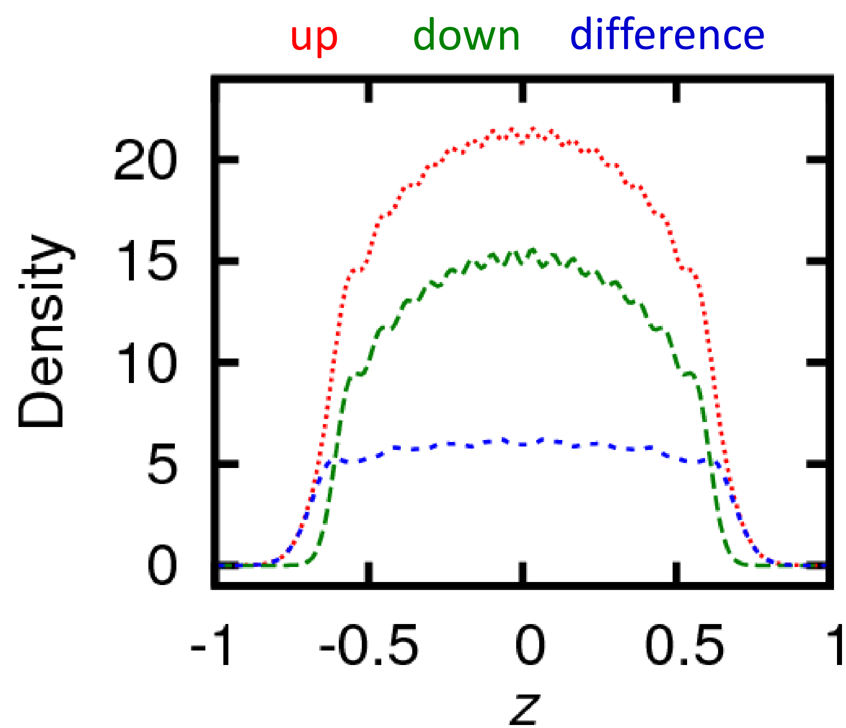


M. Tezuka and M. Ueda, PRL 100, 110403 (2008)

imbalance parameter

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

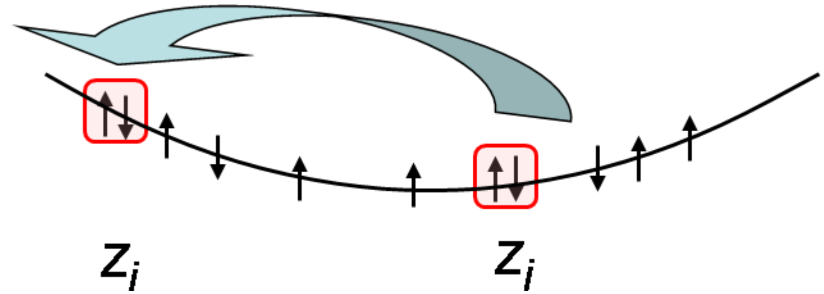




Pair correlation and density distribution

Pair correlation

$$\left\langle \psi_0^{(N)} \middle| \hat{c}^\dagger_{i,\downarrow} \hat{c}^\dagger_{i,\uparrow} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \middle| \psi_0^{(N)} \right\rangle \approx \Delta(z_i)^* \Delta(z_j)$$

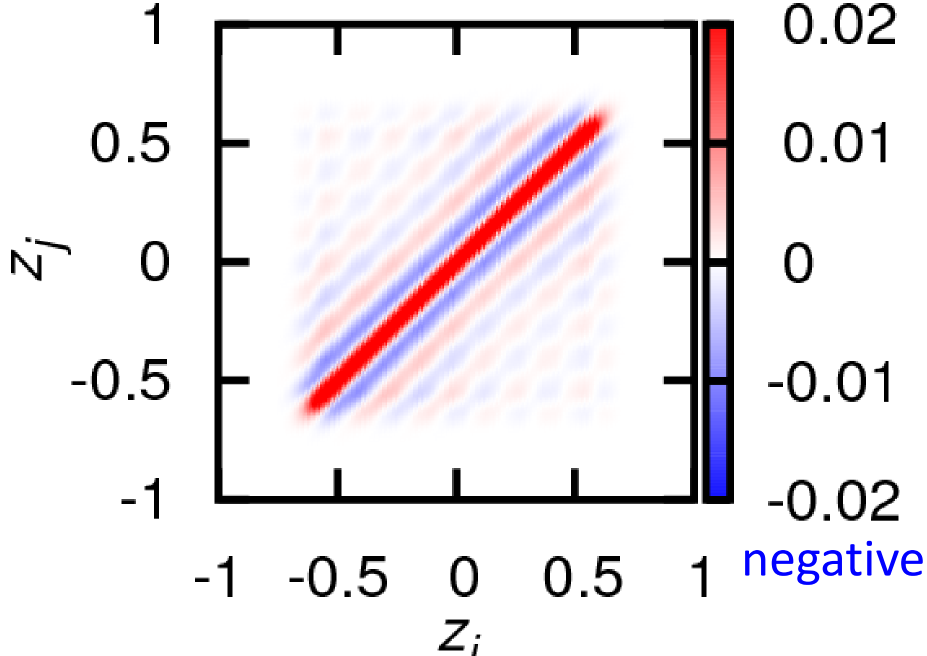


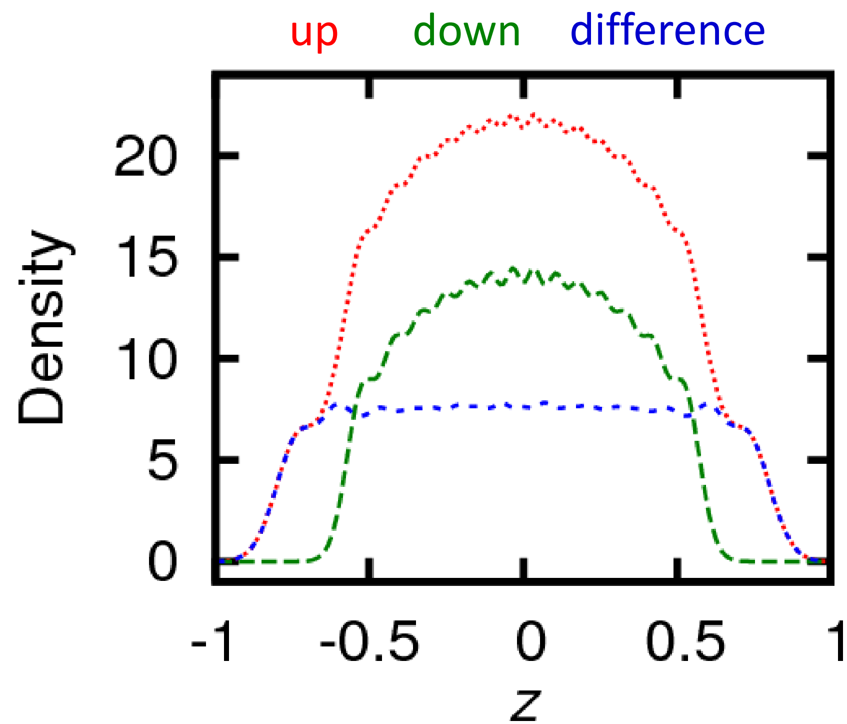
M. Tezuka and M. Ueda,
PRL 100, 110403 (2008)

imbalance parameter

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

P=0.3

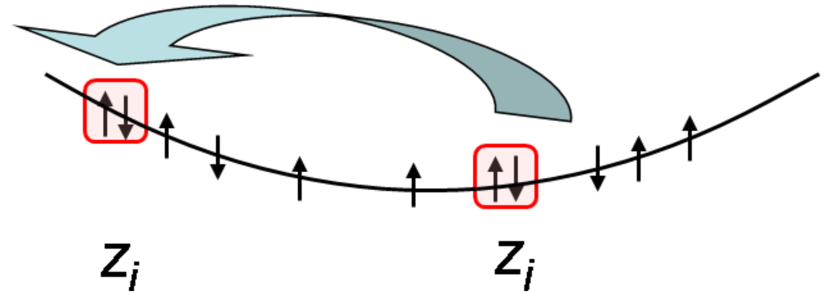




Pair correlation and density distribution

Pair correlation

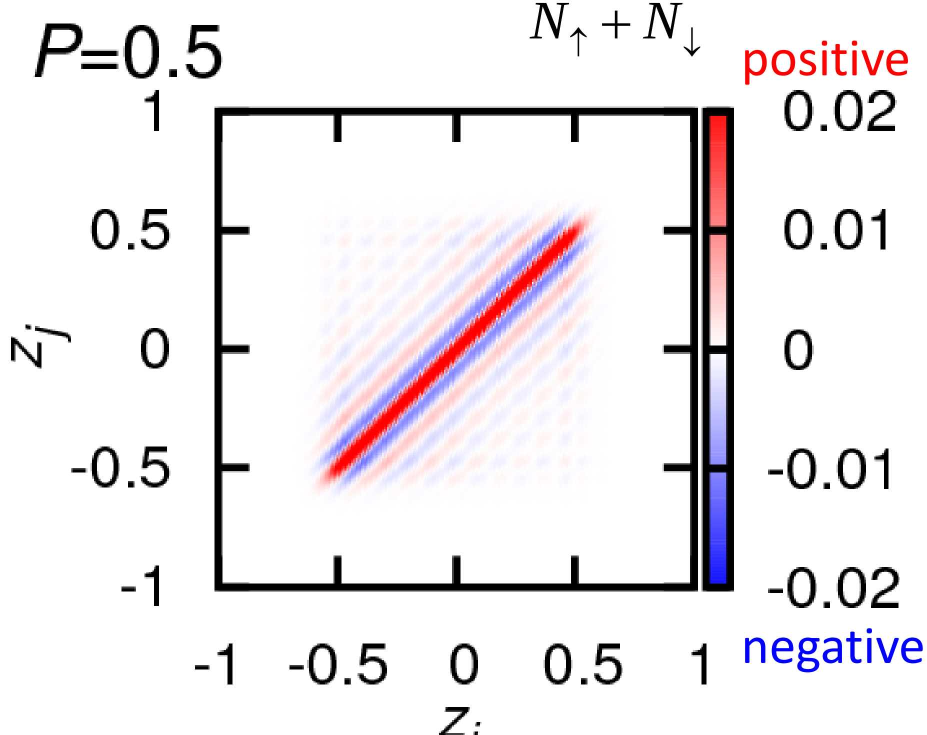
$$\left\langle \psi_0^{(N)} \left| \hat{c}^\dagger_{i,\downarrow} \hat{c}^\dagger_{i,\uparrow} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \right| \psi_0^{(N)} \right\rangle \approx \Delta(z_i)^* \Delta(z_j)$$

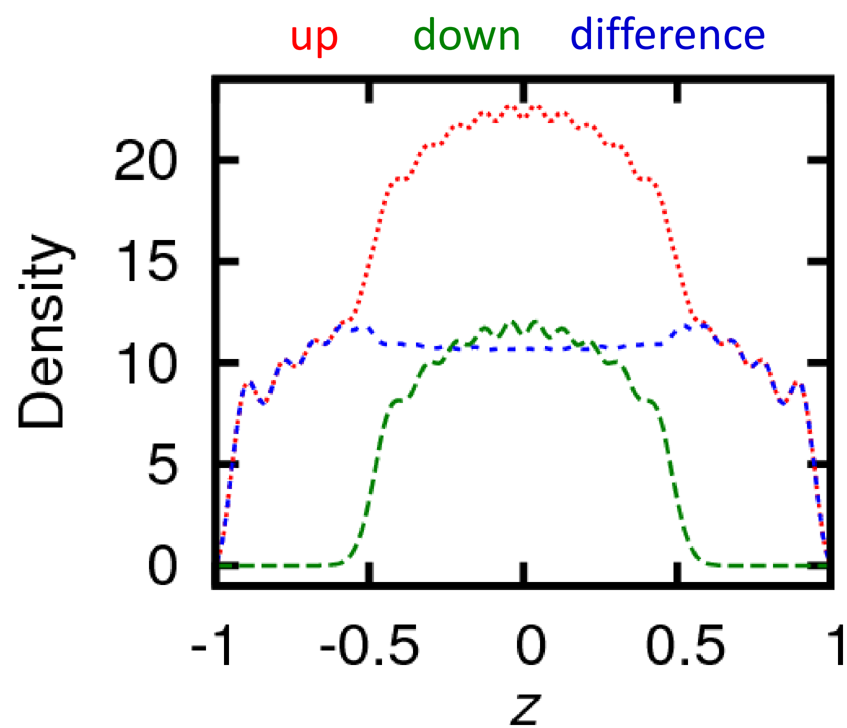


imbalance parameter

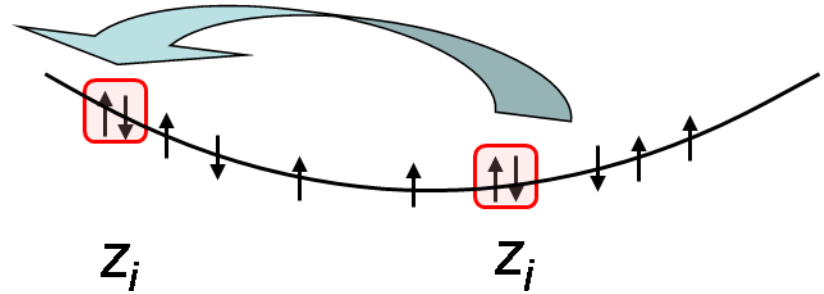
$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

M. Tezuka and M. Ueda,
PRL 100, 110403 (2008)





Pair correlation and density distribution



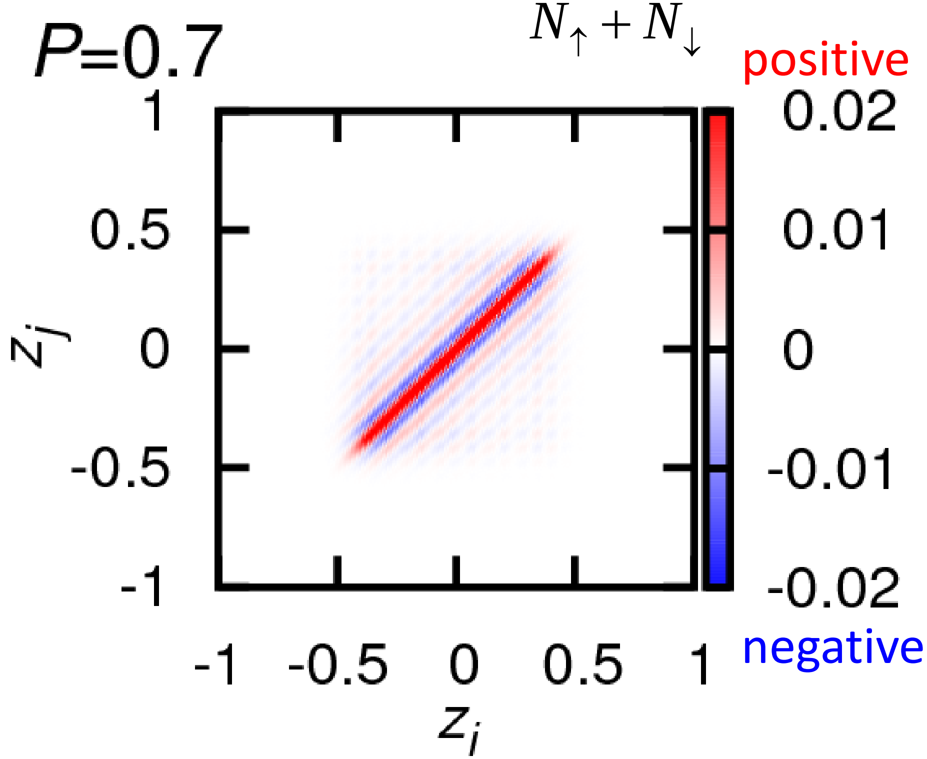
Pair correlation

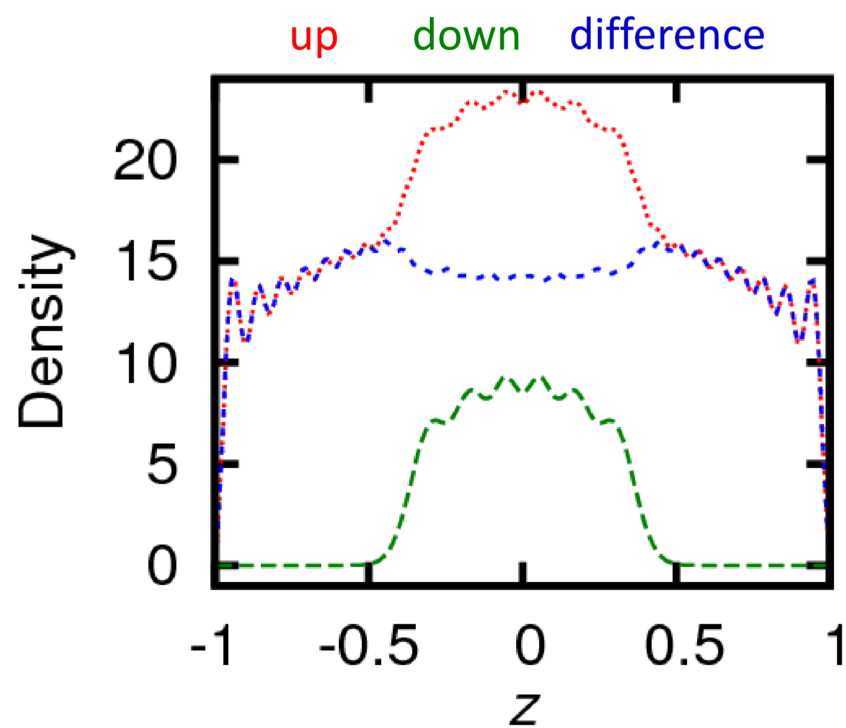
$$\left\langle \psi_0^{(N)} \middle| \hat{c}_{i,\downarrow}^{\dagger} \hat{c}_{i,\uparrow}^{\dagger} \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow} \middle| \psi_0^{(N)} \right\rangle \approx \Delta(z_i)^* \Delta(z_j)$$

M. Tezuka and M. Ueda,
PRL 100, 110403 (2008)

imbalance parameter

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$





Condensate? – two-body density matrix

N Fermions in *M* states:
Maximum possible eigenvalue = $N(M\text{-}N+2)/M \sim N$
(C.N. Yang, RMP 1962)
➔ Measure of pair condensation

$$\rho^{(2)}_{ii',jj'} \equiv \langle \psi_0 | \hat{c}^\dagger_{i',\downarrow} \hat{c}^\dagger_{i,\uparrow} \hat{c}_{j,\uparrow} \hat{c}_{j',\downarrow} | \psi_0 \rangle$$

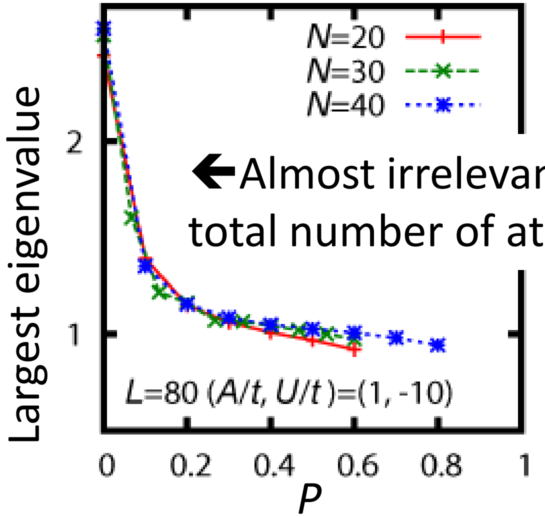
L^4 matrix elements for *L*-site chain

Diagonalize to obtain eigenvalue distribution

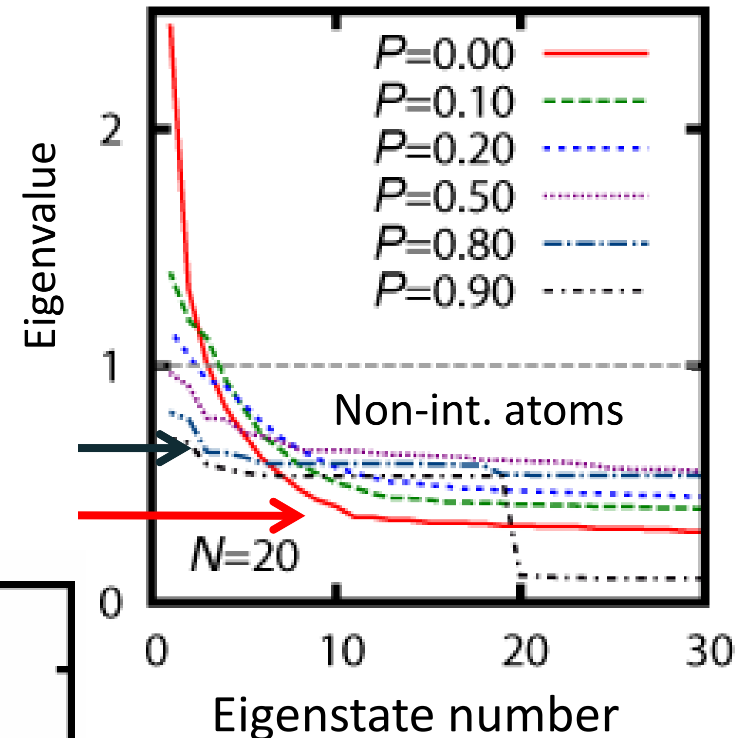
Eigenfunction: state occupied by (↑, ↓) pairs

cf. Condensate fraction for Bose gas 🡐 one-body DM

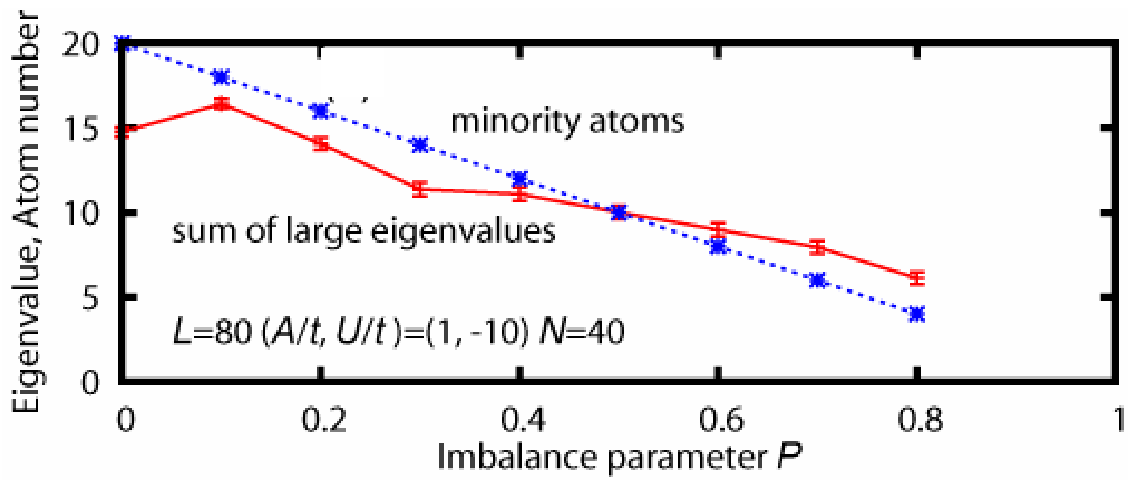
Eigenvalue distribution



← Almost irrelevant of total number of atoms



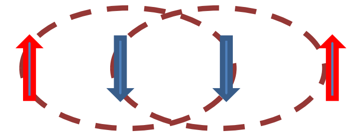
Kinks observed



M. Tezuka and M. Ueda:
PRL 100, 110403 (2008)

➔ Most of minority atoms contribute to quasi-condensate

LO condensate at trap center



Pair correlation:
periodic sign change

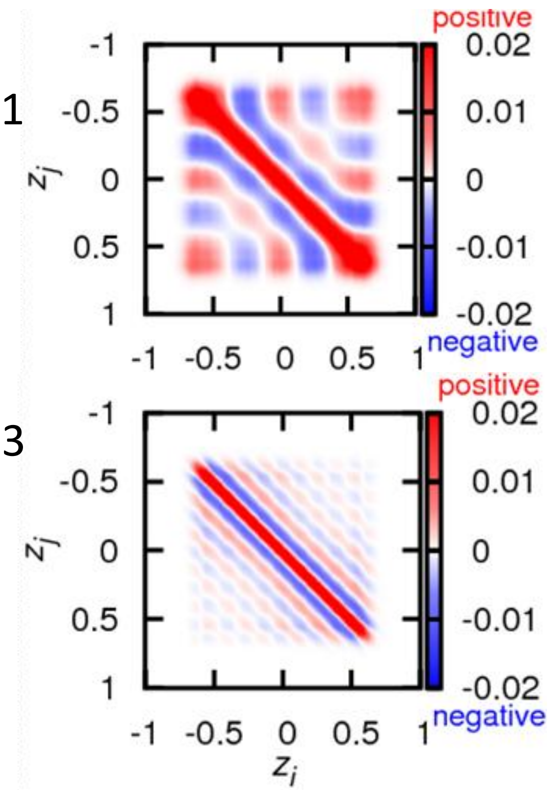
Population difference:
constant + oscillation

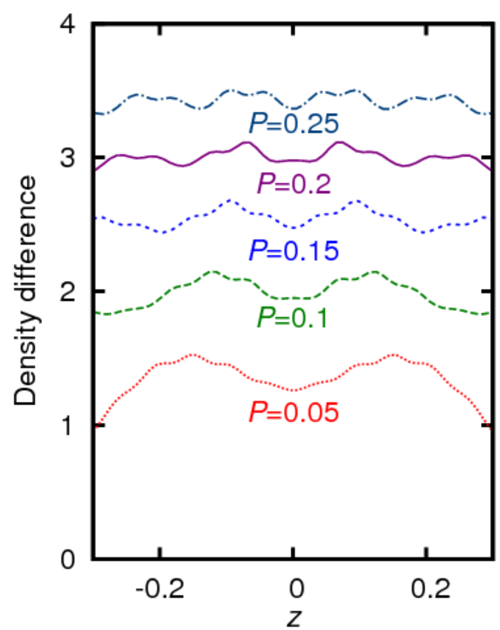
$\Delta k_{\mathrm{F}} = \pi \Delta n$ in 1D

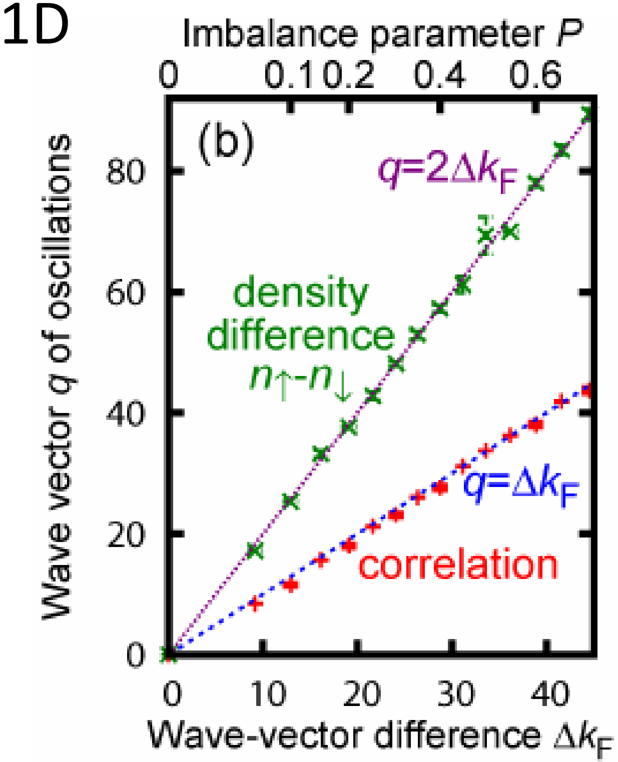
M. Tezuka and M. Ueda:
PRL 100, 110403 (2008)

P=0.1

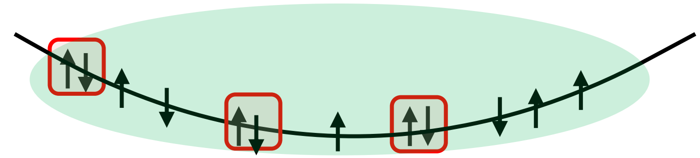
P=0.3

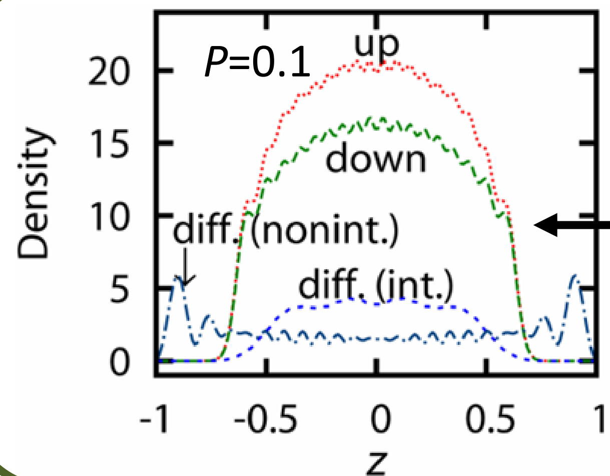




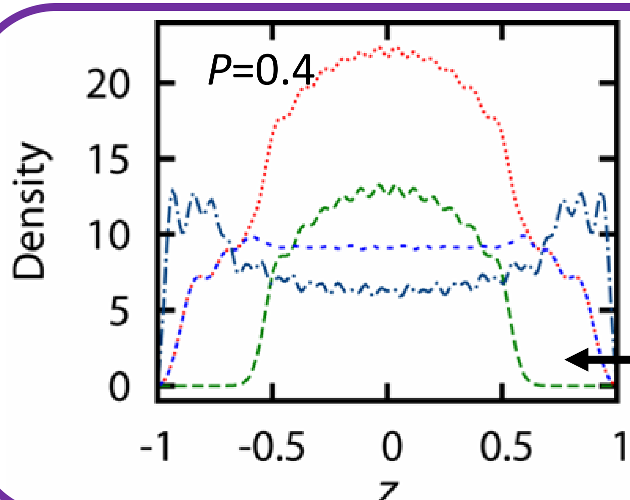


Phase diagram

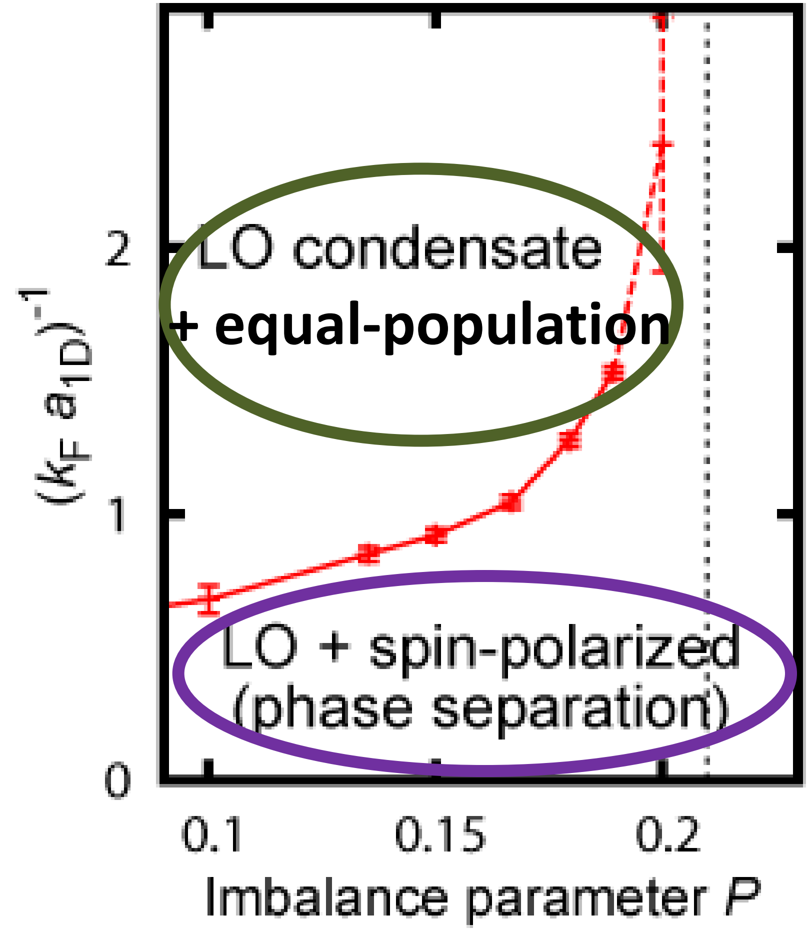


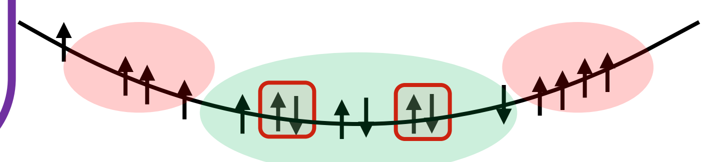


No up-only region



Up-only;
Phase sep.



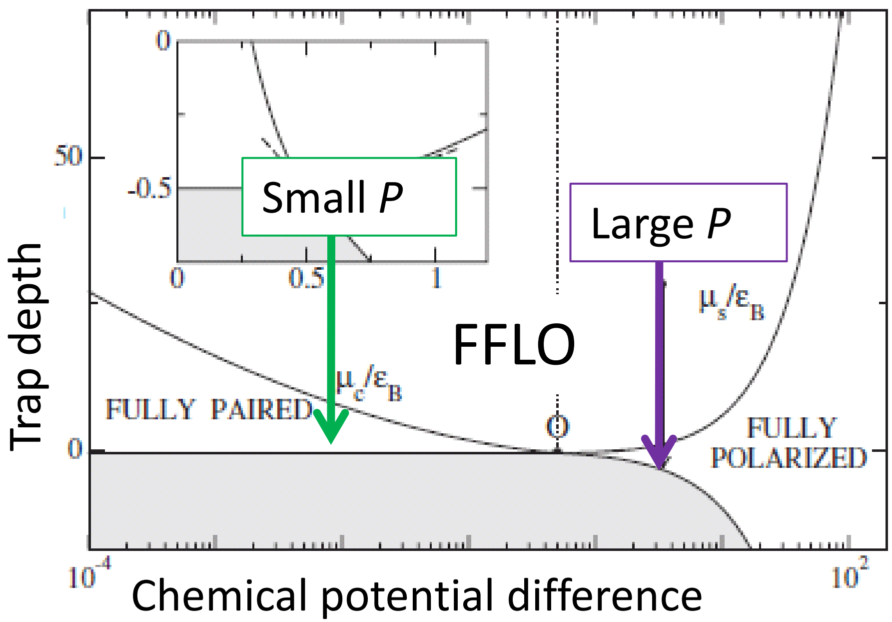


1D: LDA (local density approximation) results

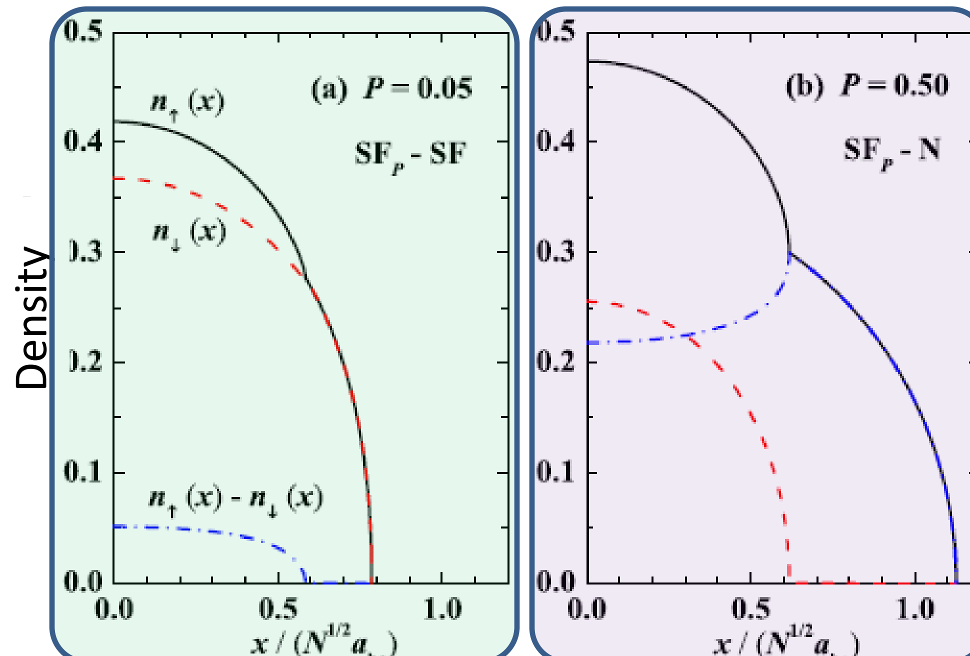
Exact solution for system without trap

(Yang's generalied Bethe ansatz ➔ Gaudin's integral equation)

Orso, PRL 98, 070402 (2007)



Hu, Liu and Drummond, PRL 98, 070403 (2007)

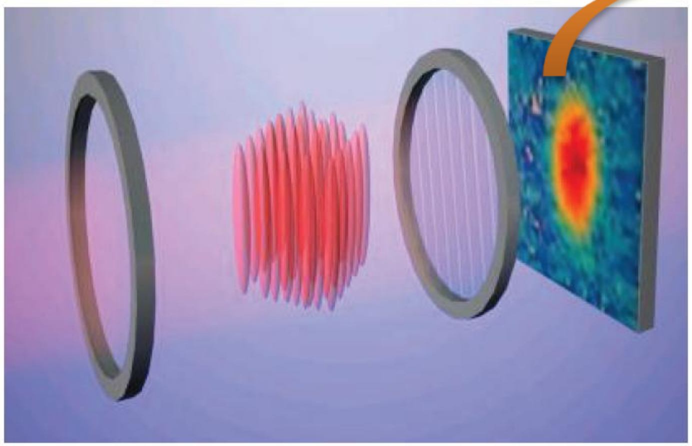


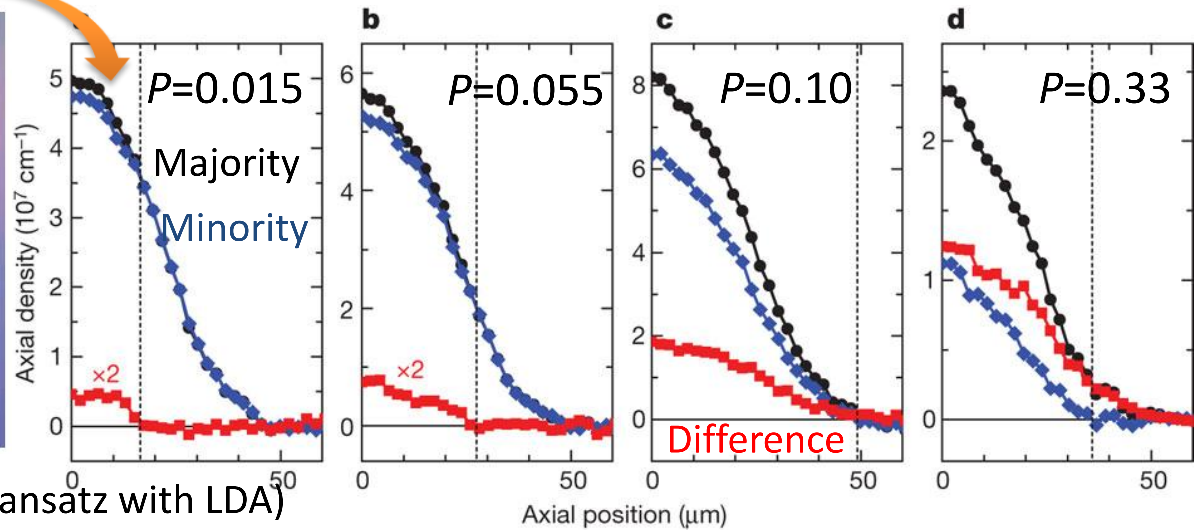
➔ Consistent with our DMRG results

1D Experiment (Rice group)

Density at central tube reconstructed

Liao *et al.*: Nature 467, 567 (Sep 2010)

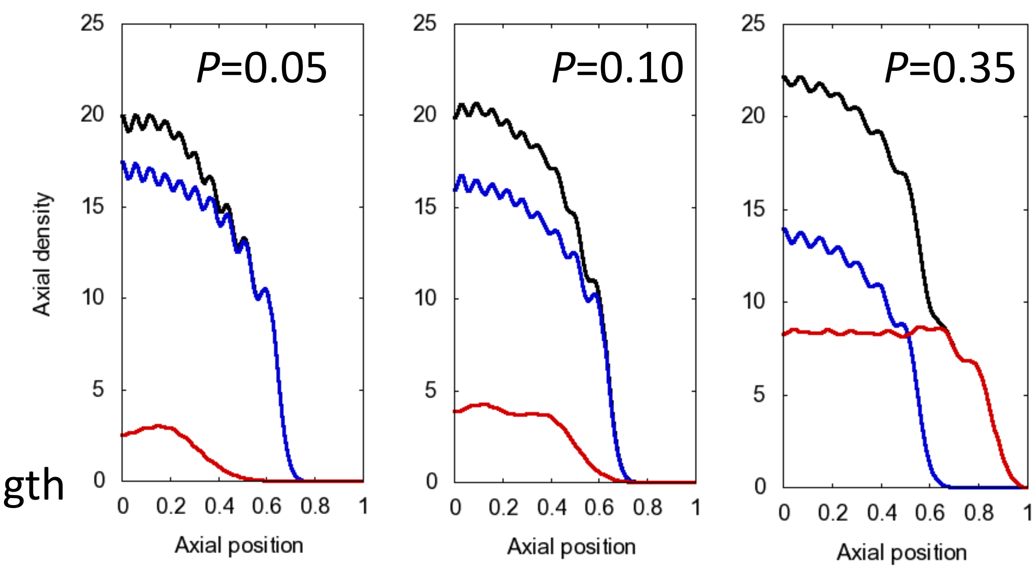


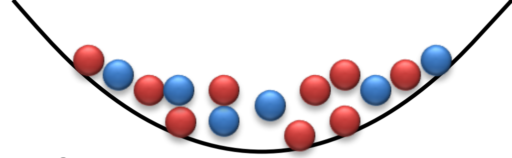


T/T_F ~ 0.15 (fit to thermodynamic Bethe ansatz with LDA)

Low enough for condensation?

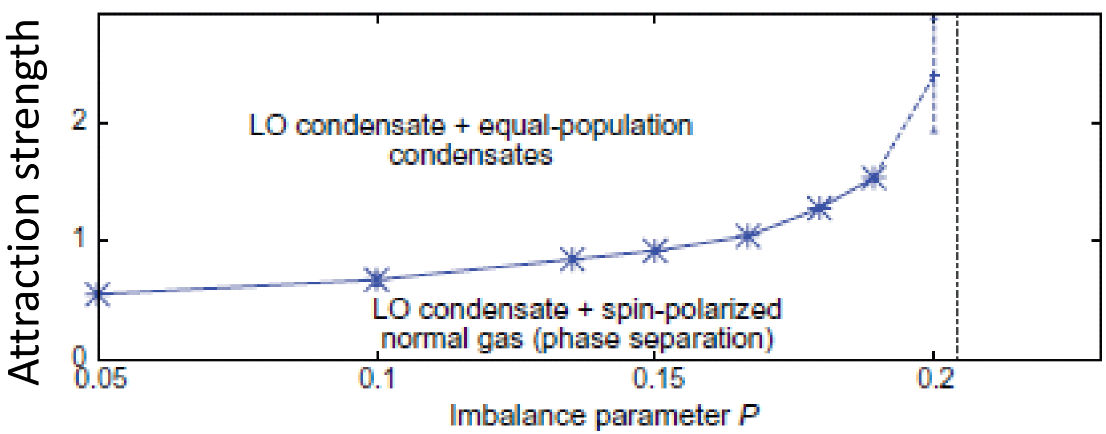
Our data at T=0 (interaction strength NOT tuned)

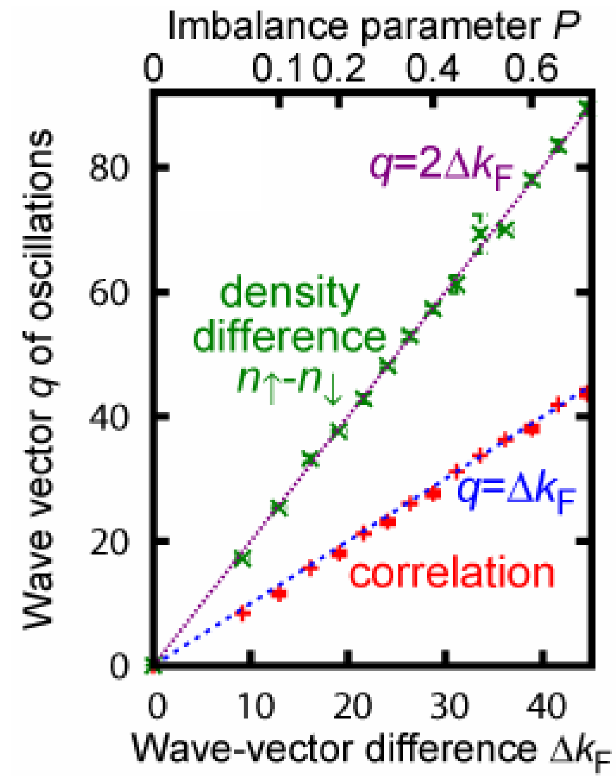




Conclusion: (1)
Population imbalance + harmonic trap

- Pairing? ➔ LO (quasi-) condensate
- Phase separation? ➔ Yes (LO at center)
- Upper limit in imbalance *P* for condensation? ➔ not observed





M. Tezuka and M. Ueda:
PRL 100, 110403 (2008); NJP 12, 055029 (2010)

see also:
Feiguin and Heidrich-Meisner: PRB 76, 220508R (2007); PRL 102, 076403 (2009) (Ladder);
Lüscher, Noack, and Läuchli: PRA 78, 013637 (2008); Batrouni *et al.*: PRL 100, 116405 (2008) (QMC)
Machida, Yamada, Okumura, Ohashi, and Matsumoto: PRA 77, 053614 (2008);
Rizzi *et al.*: PRB 77, 245105 (2008); Machida *et al.*: PRB 78, 235117 (2008)

2) Optical lattice with disorder

Disorder:
enhance?
suppress?

Optical trap shape

Superconductivity

Superfluidity

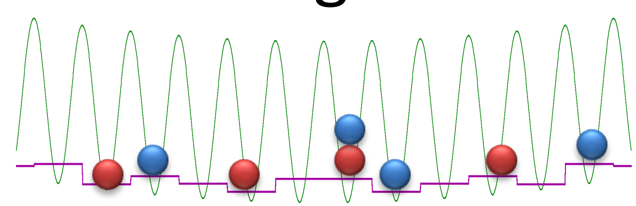
Interaction:

pairing force?
other phase?

Feshbach resonance

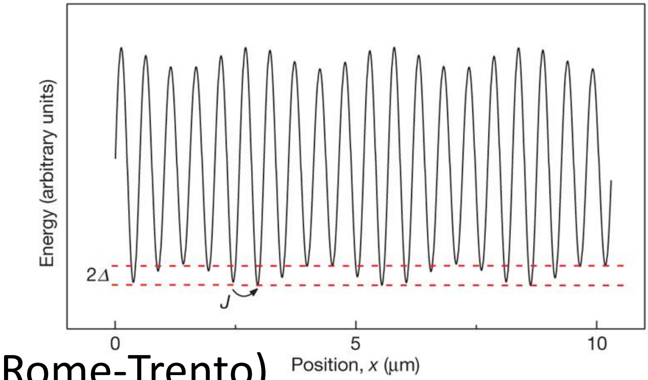
Population imbalance
= "magnetic field" : FFLO

Now widely controllable in cold Fermi atomic gases



- Optical lattice ← laser standing wave
  - Bichromatic lattice [Roati *et al.*: Nature **453**, 895 (2008)]
- Holographic potential imprinting in 2D
  - M. Greiner's group [Gillen *et al.*: PRA 2009; Bakr *et al.*: Nature 2009]

Experimental realization



- Optical lattice 🡐 laser standing wave
  - Bichromatic lattice
    - G. Roati *et al.*, Nature 453, 895 (2008) (Firenze-Rome-Trento)
- Another possibility: holographic potential imprinting in 2D
    - M. Greiner's group
      - J. I. Gillen *et al.*, PRA 80, 021602(R) (2009)
      - W.S. Bakr *et al.*, Nature 462, 74 (2009)

Our motivation: what happens in 1D?

- Quantum fluctuation suppresses true long range order (even for T=0)
- Finite system : can have condensate (superfluid)
- Is coherence length O(system size)?

⇨ Can be studied with numerically exact low-energy methods (Here we use DMRG)

Existing results

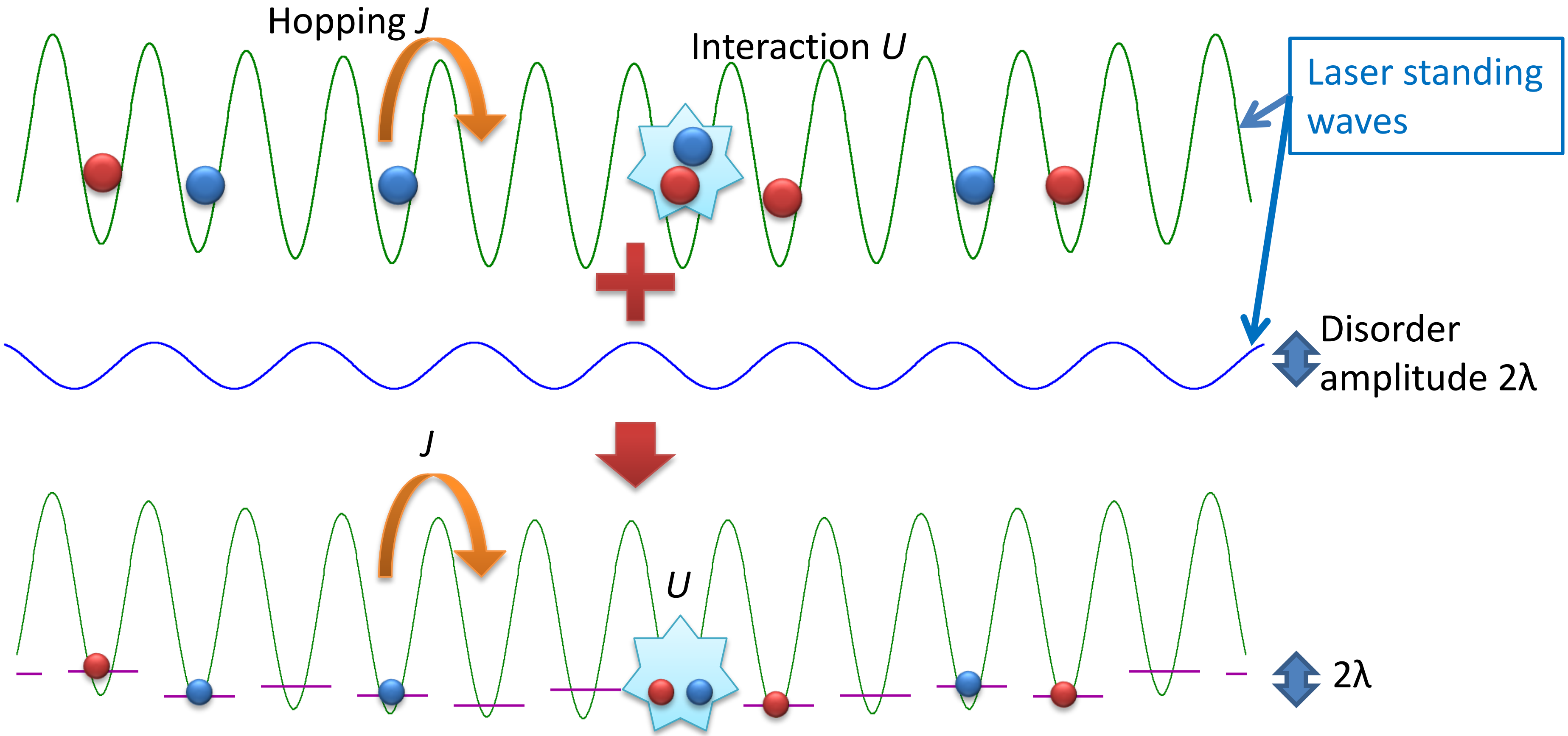
- Speckle potential (Gaussian random)
  - All eigenstates exponentially localized
    - L. Sanchez-Palencia *et al.*: PRL 98, 210401(2007)
    - A.M. García-García and E. Cuevas: PRB 79, 073104 (2009)
- Fibonacci potential ABAABABA ...
  - J. Vidal *et al.*: PRL 83, 3908 (1999), PRB 65, 014201 (2001); K. Hida: PRL 86, 1331 (2001)
  - Critical irrespective of the strength of λ
- Bichromatic potential

  "Aubry-Andre model"

  $$V(n) \equiv \lambda \cos(2\pi\omega n + \theta)$$

  - non-interacting : metal-insulator transition at $\lambda=2J$ (J: hopping)
  - Numerical studies of interacting systems
    - Bose Hubbard (DMRG): Deng *et al.*: PRA 78, 013625 (2008); Roux *et al.*: PRA 78, 023628 (2008)
    - Spinless Fermions : Chaves and Satija (ED, PRB 55, 14076 (1997)); Schuster *et al.* (PBC DMRG, PRB 65, 115114 (2002))

Quasiperiodic potential

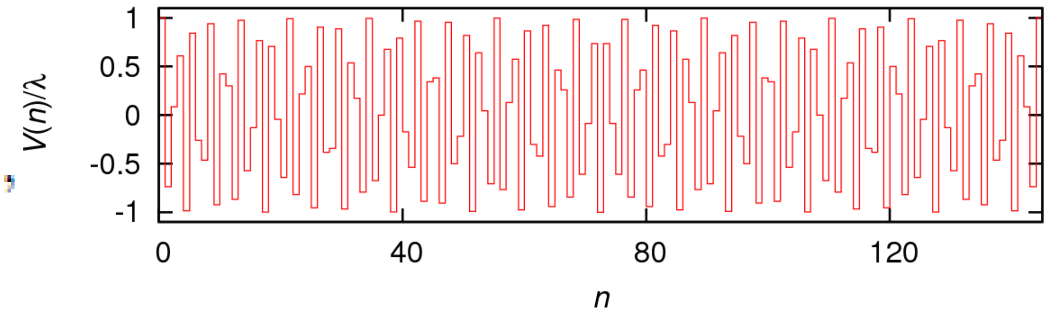


Modeled by a single-band Hubbard model with site level modification

Formulation: Hubbard model + quasiperiodic potential

J: unit of energy(=1)
U: negative for attractive interaction

$$\hat{H} = -J\sum_{i=1,\sigma}^{L-1}(\hat{c}^{\dagger}_{i+1,\sigma}\hat{c}_{i,\sigma} + \mathrm{h.c.}) + U\sum^{L-1}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} + \sum_{i=0}^{L-1}V(i)\hat{n}_i,$$

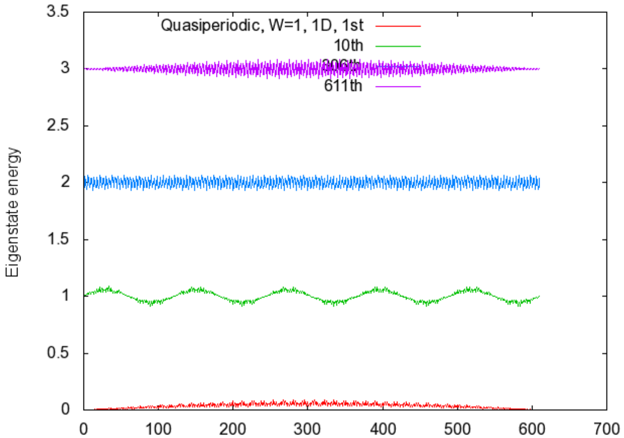


$$V(n) \equiv \lambda\cos(2\pi\omega n + \theta), \omega = F_k / F_{k+1} \cong (\sqrt{5}-1)/2, L = F_{k+1}+1$$
$$(F_0 = F_1 = 1, F_{k+2} = F_{k+1} + F_k)$$

- Ratio of consective Fibonacci numbers → golden ratio (=irrational number) as *k*→∞
- (N_σ, *L*) = (10,90), (26, 234), (42, 378) : ν=2/9
- Non-interacting case: all eigenstates become critial at λ=2*J*

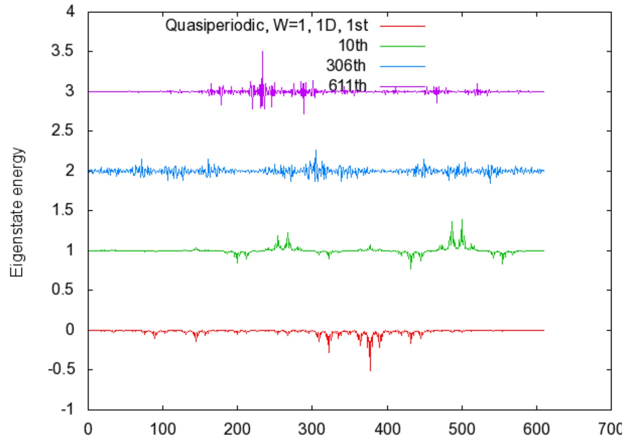
One-electron level scheme (non-interacting)

λ=1



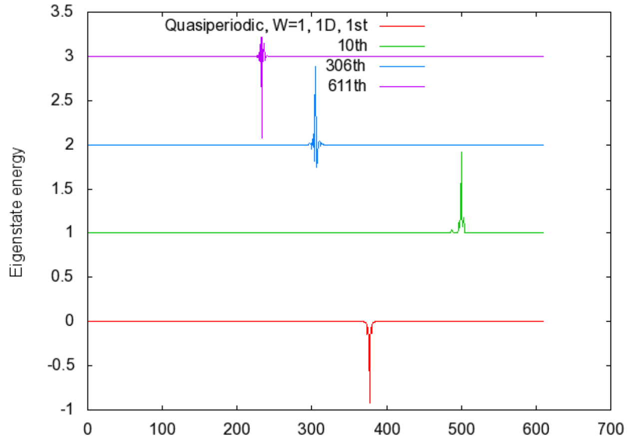
site number *i*

λ=2

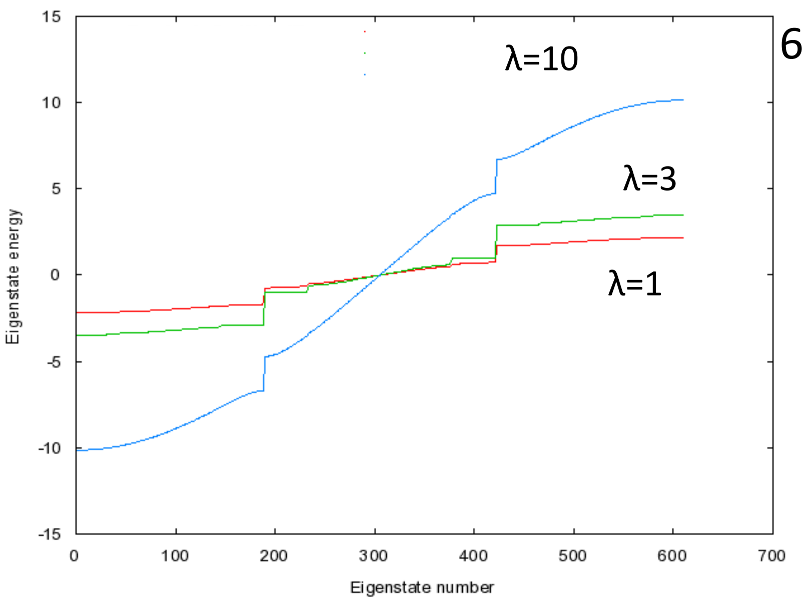


site number *i*

λ=3



site number *i*



611 sites; 1st, 10th, 306th, 611th eigenstate wavefunction

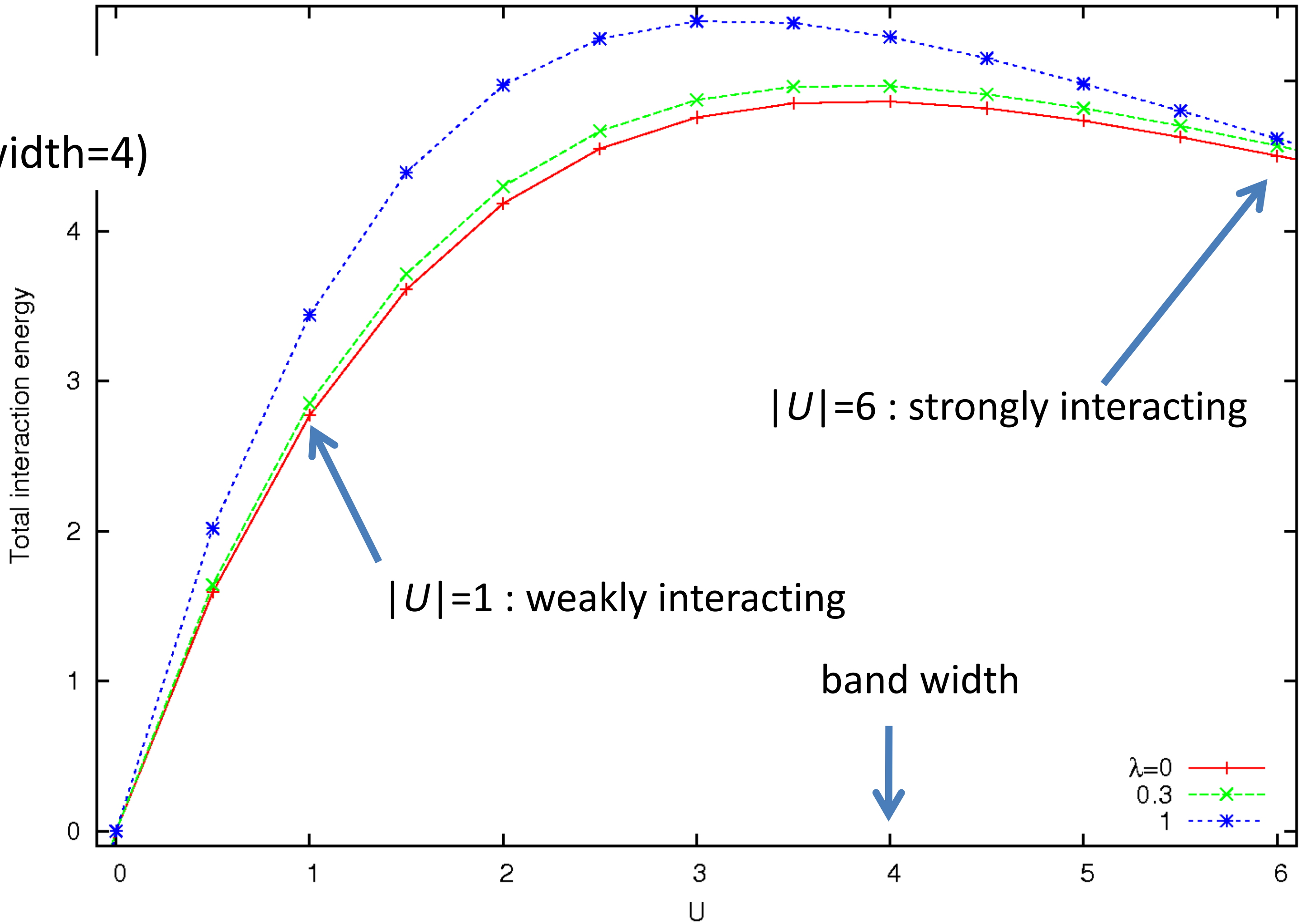
Fermions localize for λ>2 for $|U|=0$

(Energy spectrum is fractal and changes smoothly as λ is increased)

Interaction strength

Negative *U*: |interaction energy| linearly increase as |*U*| is increased
Positive *U*: it has a peak because double occupancy is suppressed as *U*→large

t=1
(band width=4)



How to detect pairing and delocalization?

Pairing

On-site pair correlation function:
easy to calculate with DMRG
Depends on the site potentials of the site pair

$$\Gamma(i,r) \equiv \left\langle \hat{c}^{\dagger}_{i+r,\downarrow} \hat{c}^{\dagger}_{i+r,\uparrow} \hat{c}_{i,\uparrow} \hat{c}_{i,\downarrow} \right\rangle$$

Averaged equal-time pair structure factor

Sum of pair correlation for all lengths

➔ average over sites

$$P_s \equiv \left\langle \sum_r \Gamma(i,r) \right\rangle_i$$

Increasing function of *L*
if decay of correlation is slow

cf. Hurt *et al.*: PRB 72, 144513 (2005);
Mondaini *et al.*: PRB 78, 174519 (2008)

Delocalization

Phase sensitivity: requires (anti-)periodic condition [see *e.g.* Schuster *et al.*: PRB 65, 115114 (2002)]
Hard to calculate within DMRG (not open BC) in large systems (OK for small systems)

$$I_E \equiv \left(\sum_i \left(\langle \hat{n}_i \rangle_{N+1,N+1} - \langle \hat{n}_i \rangle_{N,N} \right)^2 \right)^{-1}$$

Inverse participation ratio (IPR)

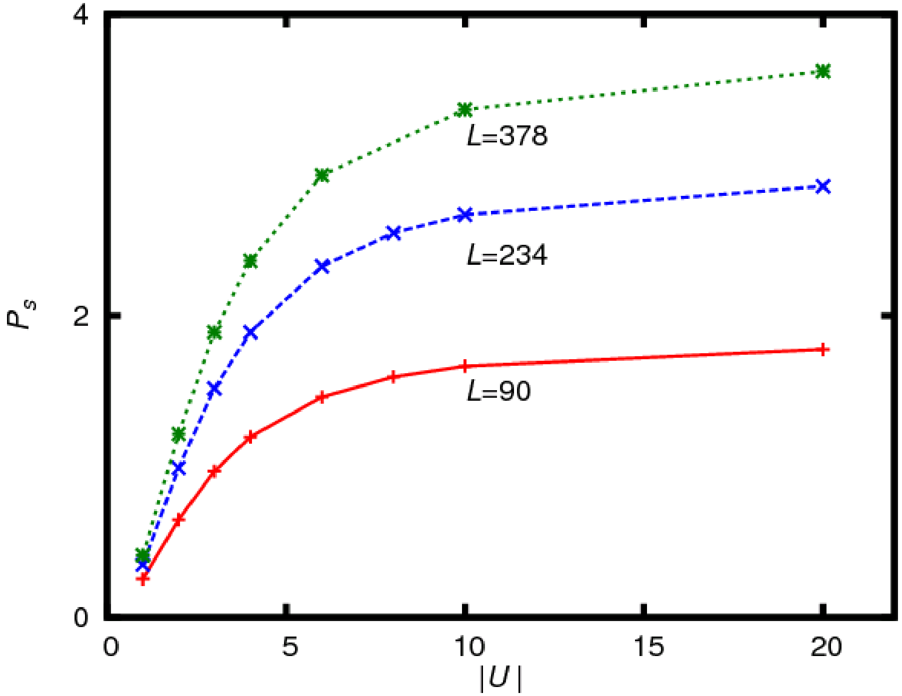
Add 2 atoms ➔ How uniformly is the population increase distributed?

Compare between different system sizes

The case without disorder (λ=0)

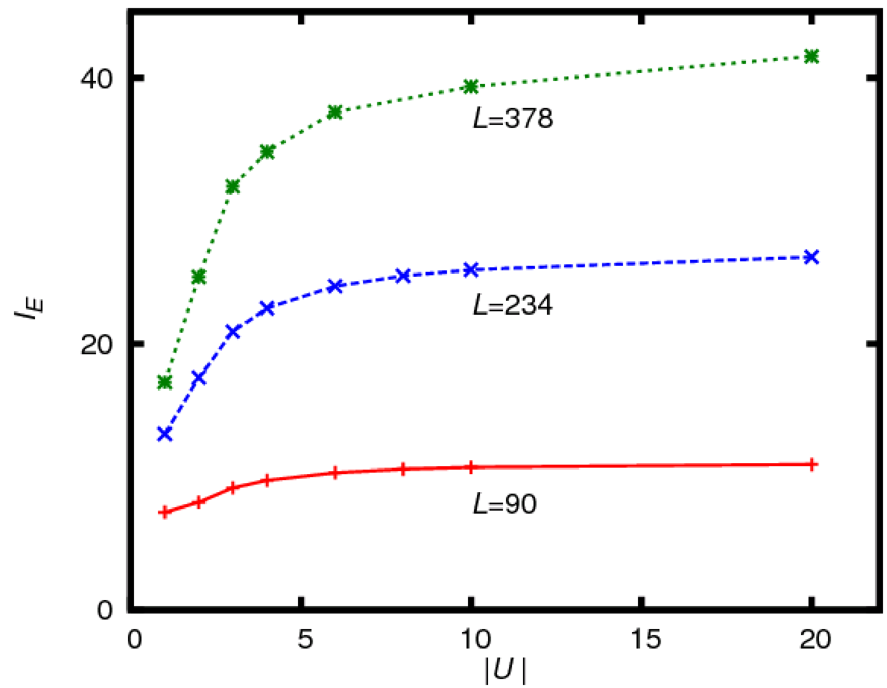
Pair structure factor

indicator of global (quasi long-range) superfluidity



Inverse participation ratio

indicator of atom delocalization

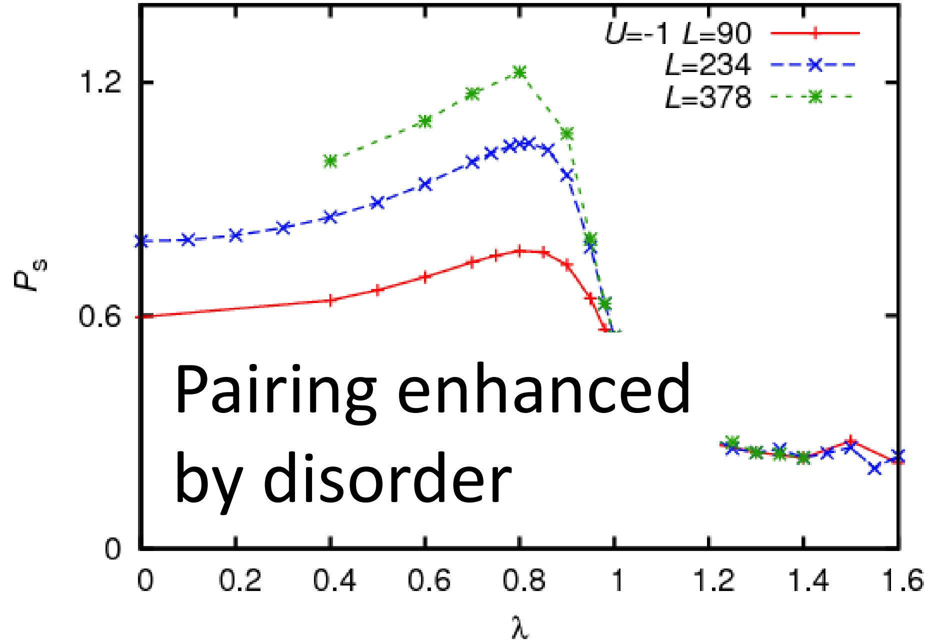


Both increase with $|U|$, and system size L

Weak $|U|$

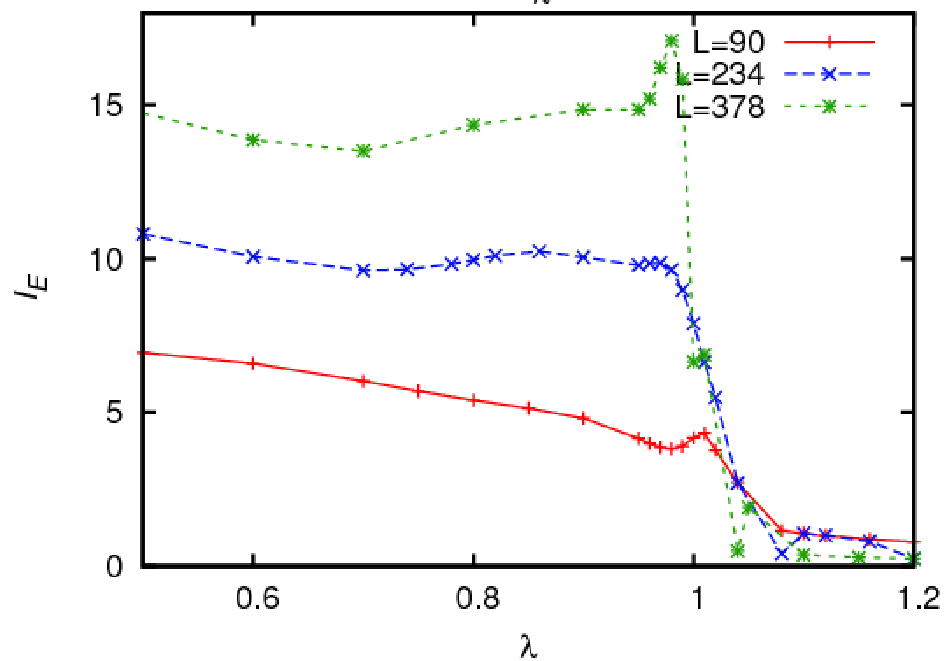
Tezuka and García-García:
PRA 82, 043613 (2010)

Pair structure factor



U=-1 :
Quasi long-range
pairing disappears
(λ_p~0.95) before
localization (λ_c~1.00)

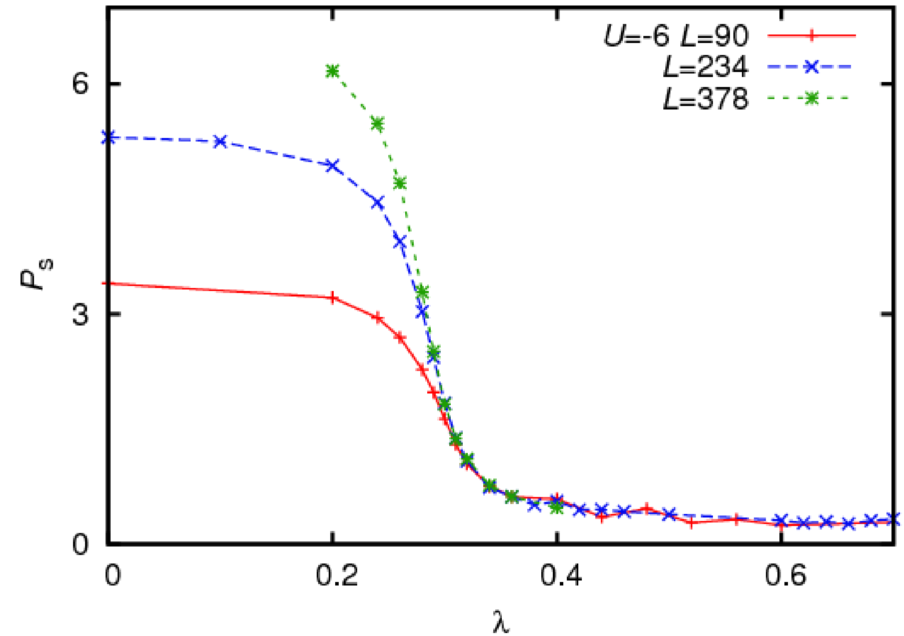
Inverse participation ratio



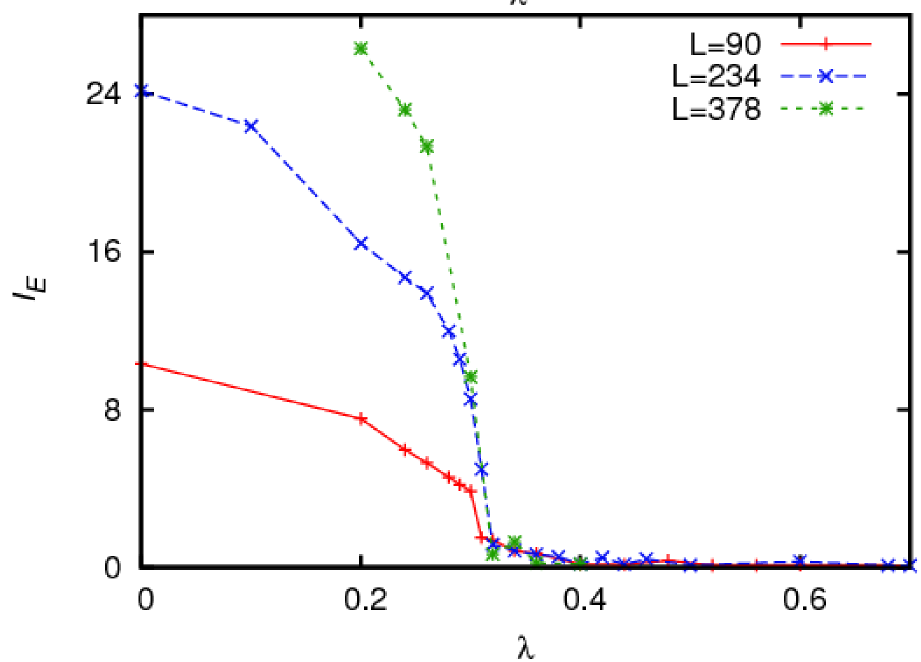
Larger $|U|$

Tezuka and García-García: PRA 82, 043613 (2010)

Pair structure factor



U=-6 :
Quasi long-range pairing disappears at localization (λ_c~0.30)



Inverse participation ratio

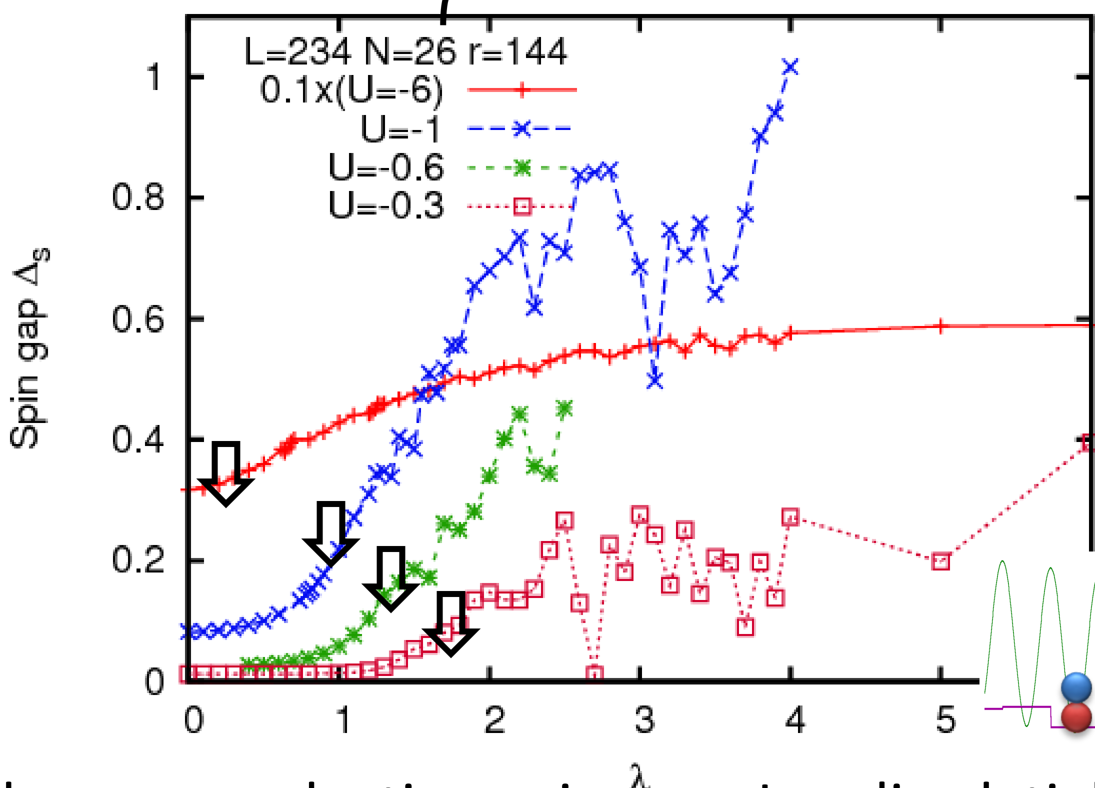
Tezuka and Garcia-Garcia: PRA 82, 043613 (2010)

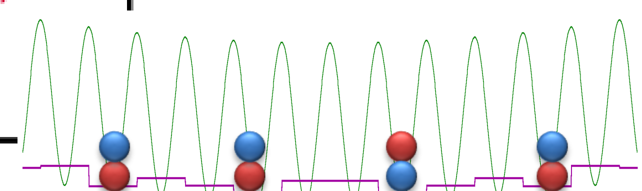
Spin gap

Minimum energy to break a pair by spin flipping

$$\Delta_s \equiv E_0(n+1, n-1) - E_0(n, n)$$

Continues to increase even after λ_c





Delocalized, superconducting pairs below sc-metal transition

Localized, tightly bound pairs at large λ

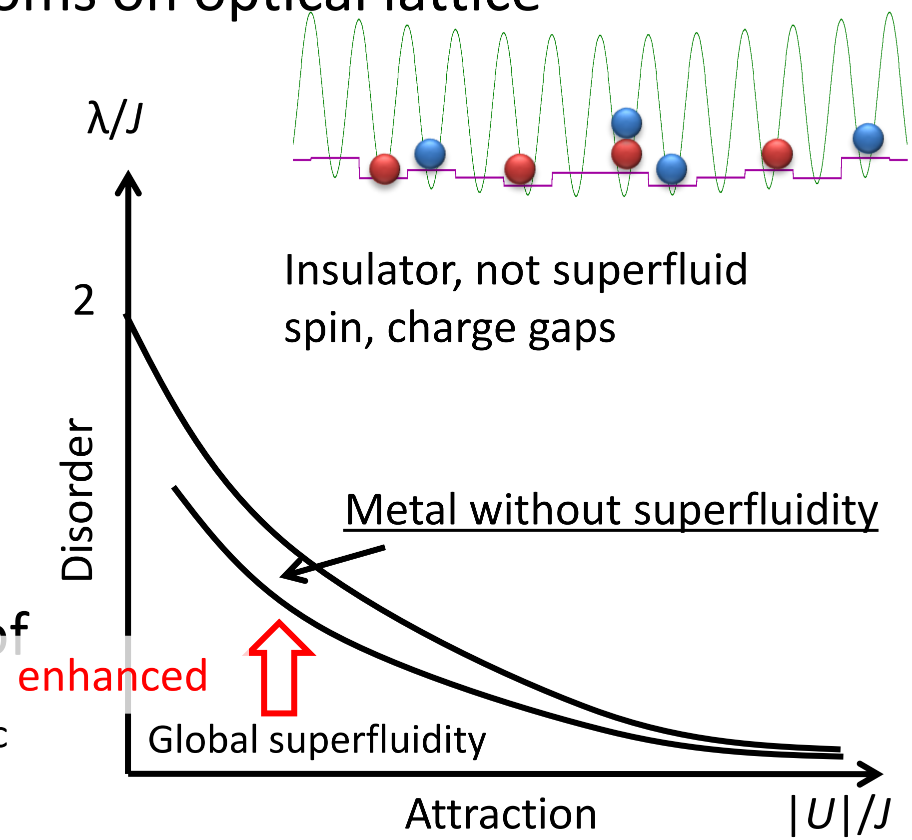
M. Tezuka and A. M. Garcia-Garcia: PRA 82, 043613 (2010)

Schematic phase diagram

- Effect of coexisting disorder (bichromatic potential) and short-range attractive interaction
  - Studied for 1D fermionic atoms on optical lattice

- For strong attraction ($|U| \gg J$), pairing decreases as disorder λ is increased, and localizes at ~ insulating transition λ_c

- For weaker attraction ($|U| \sim J$), pairing has a peak as a function of disorder λ, but localizes before λ_c



What about dynamics?

- Many experiments observe the dynamics of the atomic clouds after release from a trap

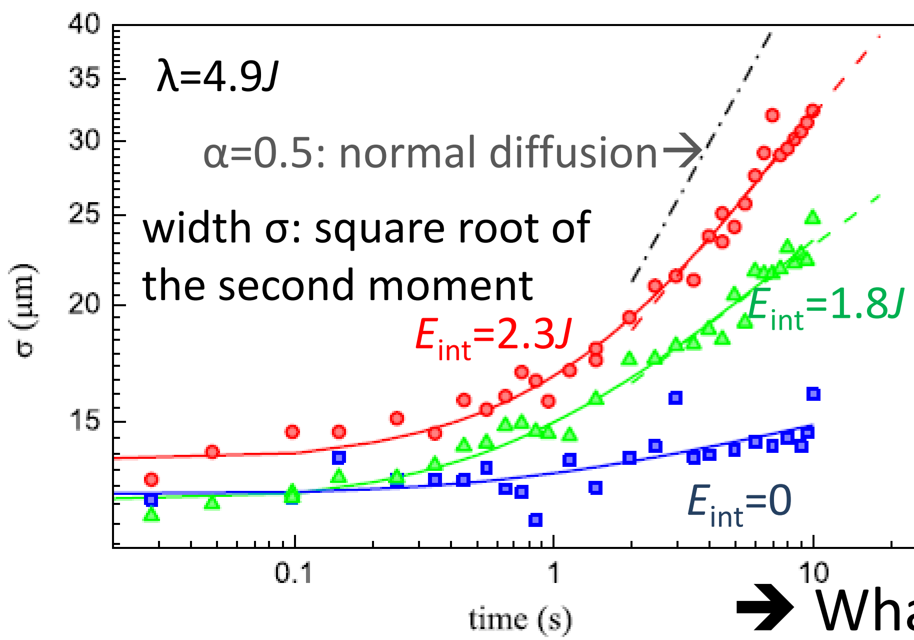
Bosons: E. Lucioni *et al.* (LENS, Florence): PRL 106, 230403 (2011)

Subdiffusion observed in bichromatic lattice (3D)

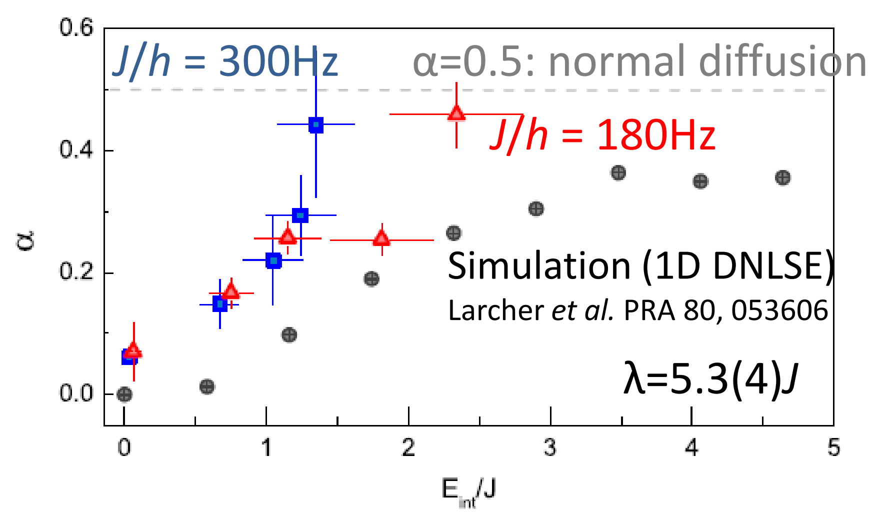
$V(x) = V_1\cos^2(k_1 x) + V_2\cos^2(k_2 x)$, $k_1=2\pi/(1064.4\text{nm})$, $k_2=2\pi/(859.6\text{nm})$

50 thousand ^{39}K atoms, almost spherical trap switched off at t=0

Initially a=280a_0 (repulsive), λ~3J (localized) ➔ tuned to final value within 10 ms



$\sigma(t) = \sigma_0(1 + t/t_0)^\alpha$



➔ What happens for interacting fermions?

Does the phase depend on filling? What do we see?

If the phase diagram is sensitive to the filling → Density decrease after release may induce (de)localization

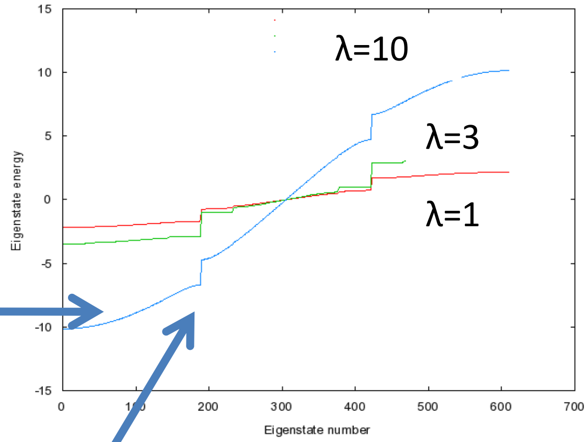
Intuitively,

The density is decreased as the atoms flow to the outer side of the system; density of states at the Fermi surface changes (not monotonously)

⇔

But we checked

The ground state phase diagram: does not depend strongly up to filling ~ 0.31 (per spin per site)



Small change of density of states does not affect λ_{loc} strongly

Gap at ~31% filling

One parameter scaling theory

Abrahams *et al.*: PRL 42, 673 (1979)
see also Garcia-Garcia and Wang: PRL 100, 070603 (2008)

L : system size, δ : (one-particle) mean level spacing

Dimensionless conductance
$g(L) = E_T / \delta$
E_T : Thouless energy
(related to typical time for particle to travel L)

(d>2)D Normal metal : $g(L) \propto L^{d-2} \to \infty$
$\because E_T \propto L^{-2}$, $\delta \propto L^{-d}$

--- Metal-insulator transition : $g(L) = g_c$ ---

Insulator : $g(L) \propto \exp(-L/\xi) \to 0$

$\langle x^2(t) \rangle \propto t^{\alpha}$

Motion slowed down,
~ $L^{2/\alpha}$ time to propagate L,
$E_T^{-1} \propto t^{-2/\alpha}$, $\alpha<1$

Multifractal spectrum with
Hausdorff dimension d_H
➔ $\delta \propto L^{-d/d_H}$

MIT should occur at $\alpha = 2d_H/d = 2d_H$
(has been checked for the non-interacting case;
Artuso et al.: PRL 68, 3826 (1992);
Piechon et al.: PRL 76, 4372 (1996))

Localization length ξ:
should diverge as $|\lambda-\lambda_c|^{-\nu}$
(ν=1 at U=0)

Near metal—insulator transition

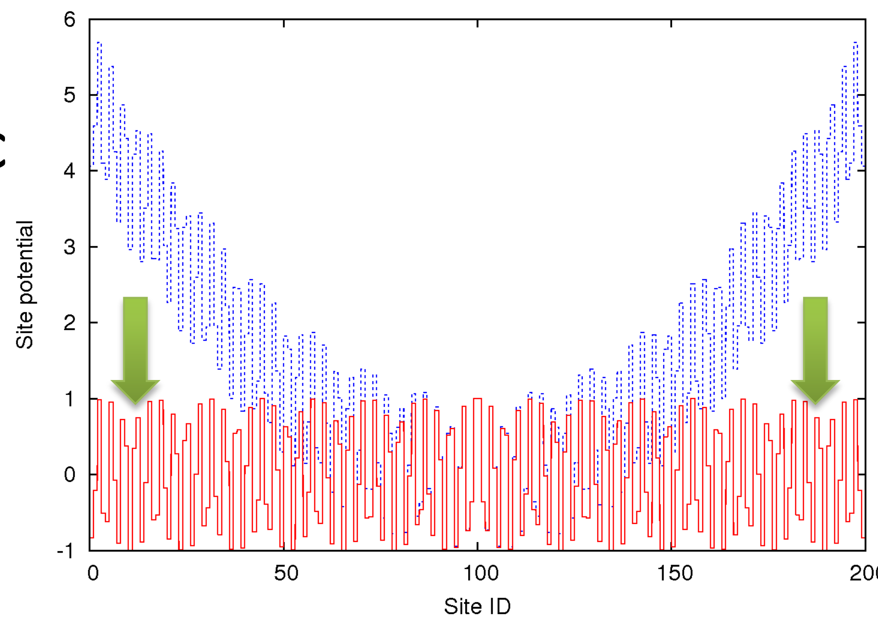
Disordered 1D system, $U < 0$

	$\|U\| \rightarrow 0$	intermediate $\|U\|$	$\|U\| \rightarrow \infty$
Diffusion $\langle x^2(t) \rangle \propto t^\alpha$	$\alpha \sim 1$ brownian motion	?	$\alpha \sim 2$ ballistic motion
Hausdorff dimension of the spectrum d_H	$d_H \sim 0.5$ see *e.g.* Artuso *et al.*: PRL 68, 3826 (1992)	One parameter scaling $\alpha = 2d_H$ at MIT	$d_H \sim 1$ Not fractal
Localization length close to transition $\xi \propto \|\lambda - \lambda_c\|^{-\nu}$	$\nu \sim 1$	Our conjecture $\nu = 1/(2d_H) = 1/\alpha$?	$\nu \sim 1/2$ mean-field like; similar to Cayley tree

➔ Let us numerically check by studying the dynamics

Setup (1)

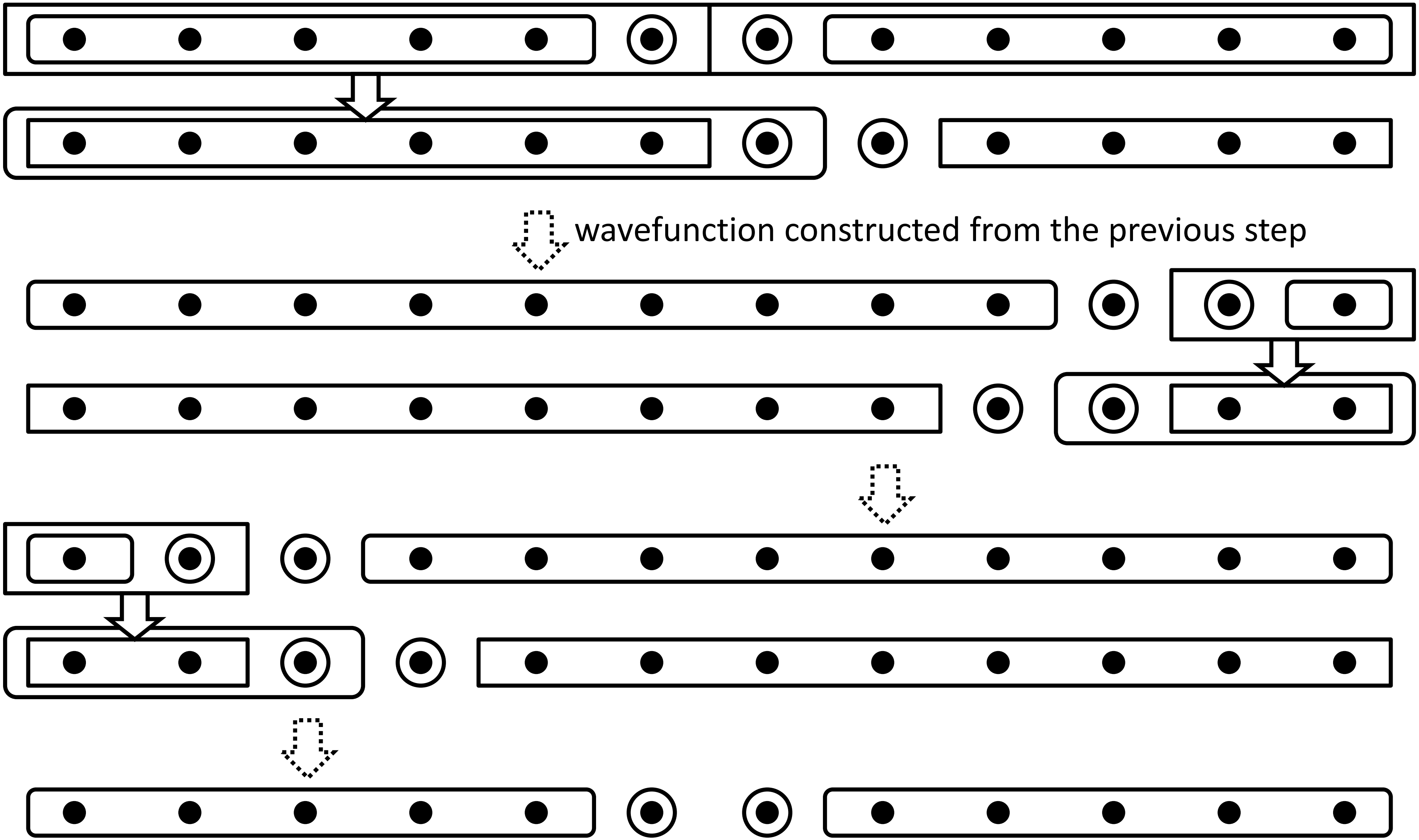
- Optical lattice + incommensurate potential (Aubry-André model)
- On-site attractive interaction
- Initially trapped in a harmonic potential



- Remove the harmonic potential but keep the incommensurate potential on: what happens?

➔ Study by time-dependent DMRG

Finite system DMRG



Iterate until physical quantities (*e.g.* energy) converge

Time-dependent DMRG

White and Feiguin: PRL 93, 076401 (2004)

Application of exp(-iτ$H_{i,j}$) is almost exact if $H_{i,j}$ only affects neighboring sites i, j

T/τ finite system iterations to reach time T

$$\hat{H} = \hat{H}_{1,2} + \hat{H}_{2,3} + \cdots + \hat{H}_{L-2,L-1} + \hat{H}_{L-1,L}$$

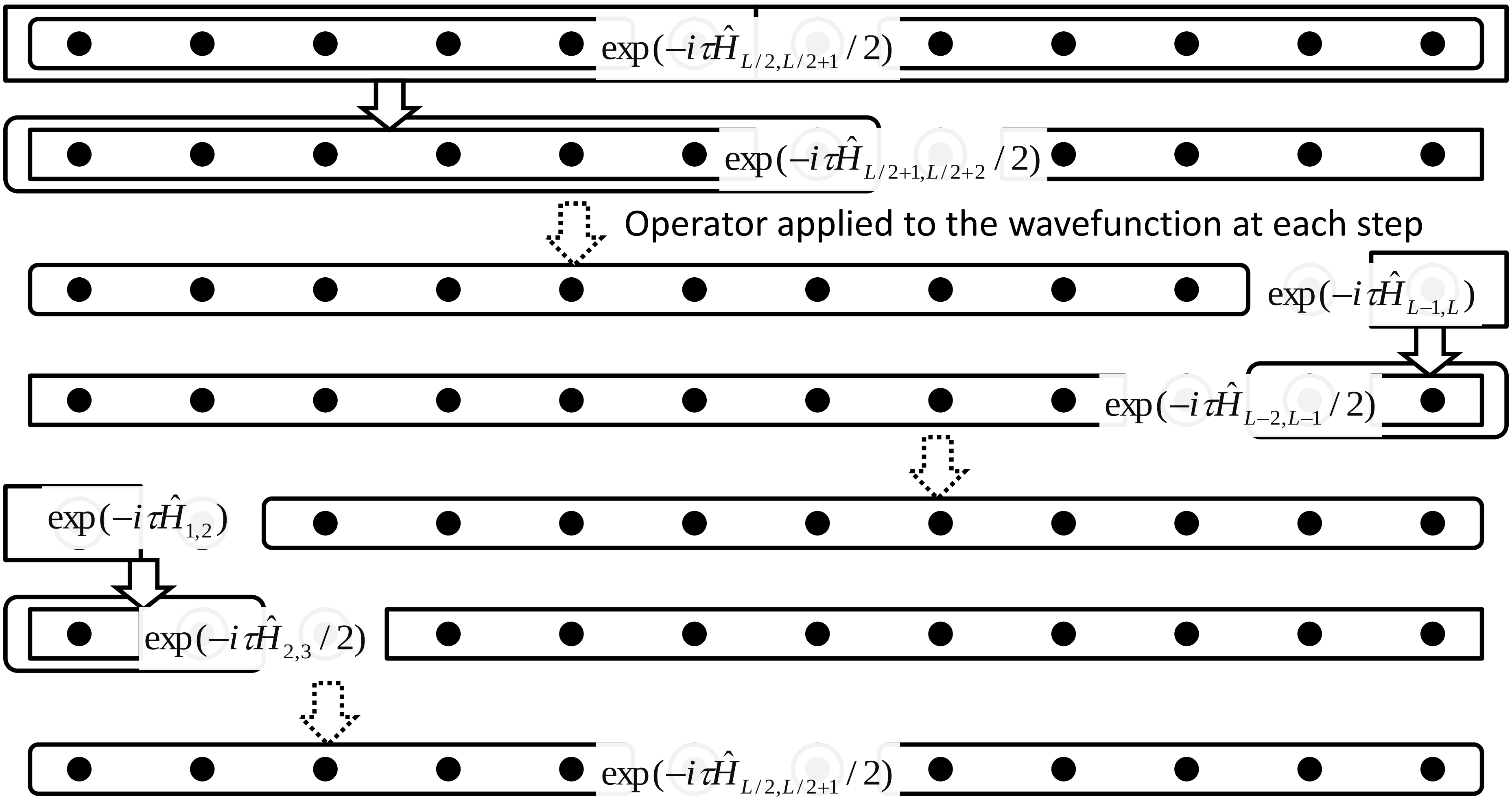
Suzuki-Trotter decompotision

$$\exp(-i\tau\hat{H})$$

$$= \exp(-i\tau\hat{H}_{L/2,L/2+1}/2)\cdots\exp(-i\tau\hat{H}_{2,3}/2)\exp(-i\tau\hat{H}_{1,2})$$

$$\exp(-i\tau\hat{H}_{2,3}/2)\cdots\exp(-i\tau\hat{H}_{L-2,L-1}/2)\exp(-i\tau\hat{H}_{L-1,L})$$

$$\exp(-i\tau\hat{H}_{L-2,L-1}/2)\exp(-i\tau\hat{H}_{L/2,L/2+1}/2) + O(\tau^3)$$



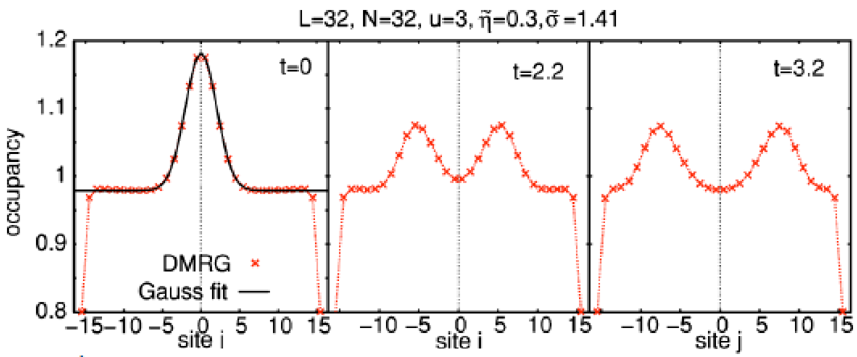
Time-dependent DMRG: other schemes

Cazalilla and Marston: PRL 88, 256403 (2002)
Luo et al.: PRL 91, 049701 (2003)
Vidal: PRL 93, 040502 (2004)
Feiguin and White: PRB 72, 020404 (2005)
Dutta and Ramasesha: PRB 82, 035115 (2010)

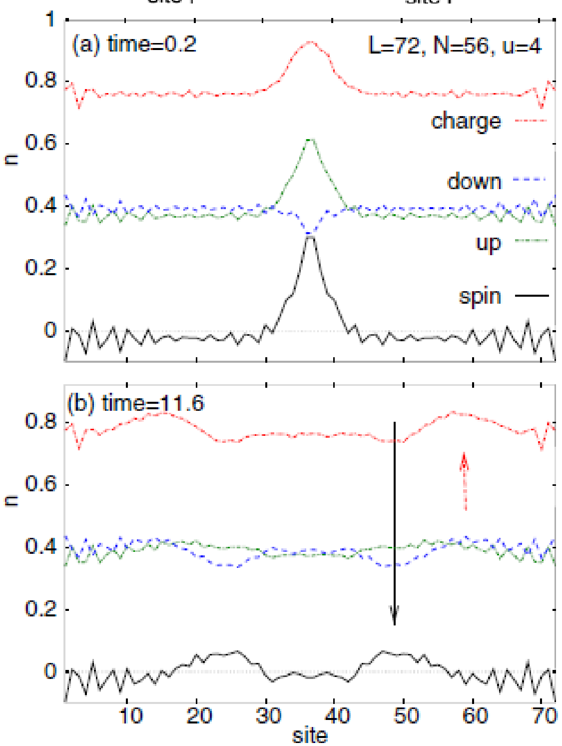
Time-evolving block decimation (TEBD) has also been applied to cold atom systems

e.g. Macroscopic quantum tunneling between different supercurrent states in a ring
Danshita and Polkovnikov: PRB 82, 094304 (2010)

Applications of time-dependent DMRG on cold atom systems

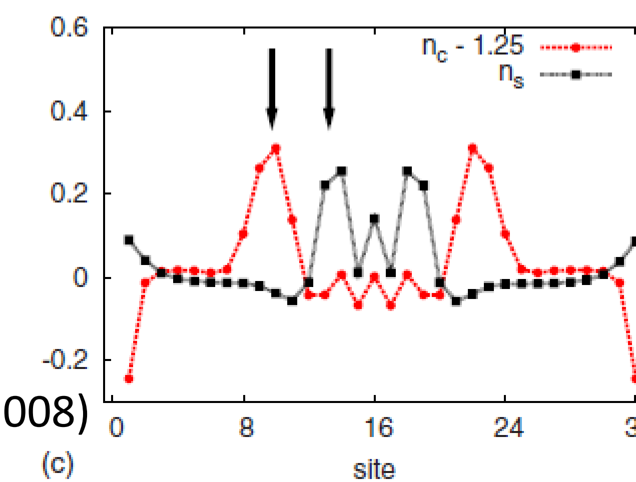


Propagating 1D density waves of bosons in optical lattice
Kollath *et al.*: PRA 71, 053606 (2005)

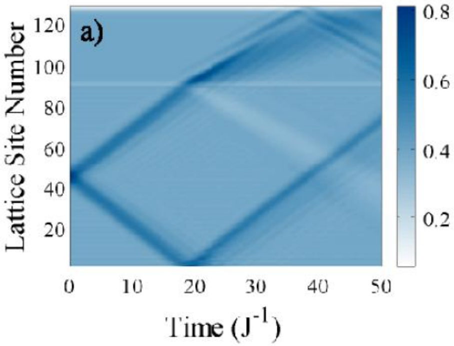


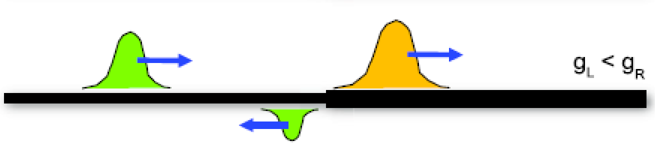
Spin-charge separation
in S=1/2 Hubbard model
Kollath et al.: PRL 95, 176401 (2005)

Spin-charge separation
in two-component Bose-Hubbard model
Kleine et al.: PRA 77, 013607 (2008)

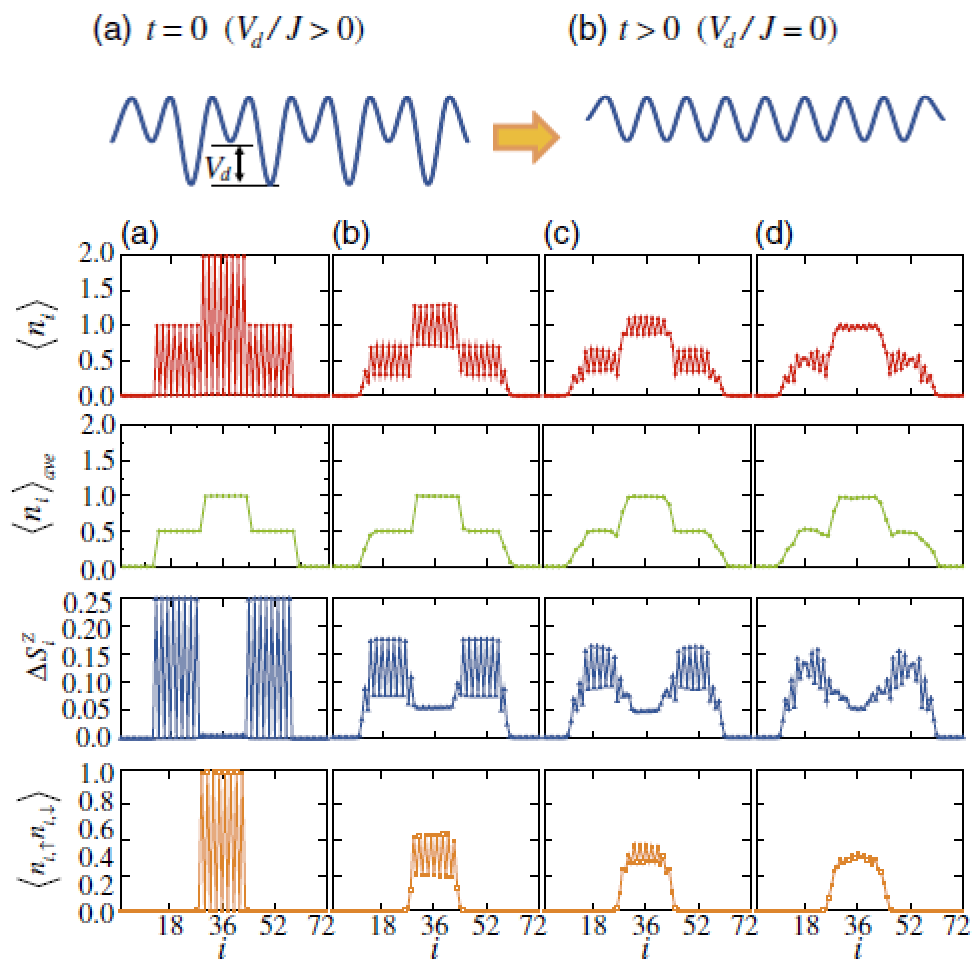


Applications of time-dependent DMRG on cold atom systems





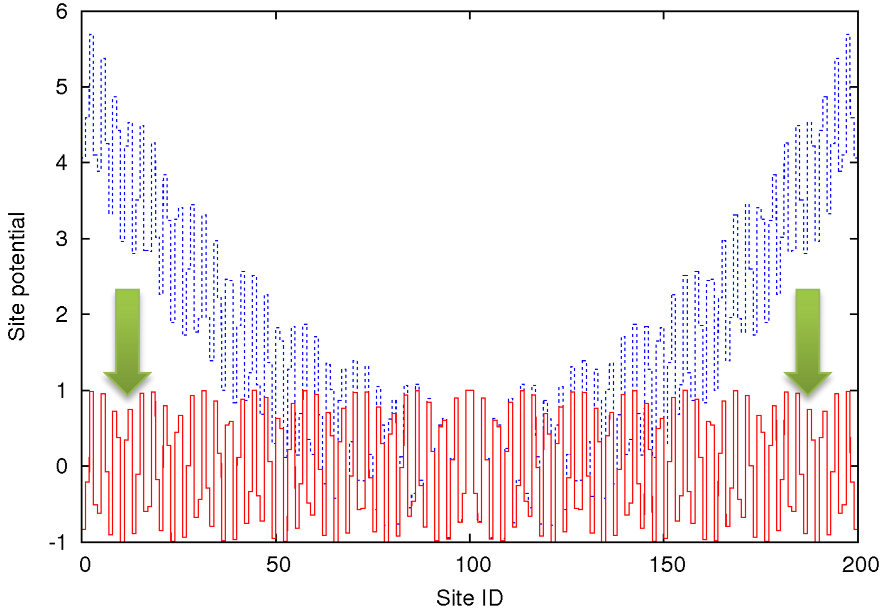
Andreev-like reflection in bosons
Daley *et al.*: PRL 100, 110404 (2008)



Optical superlattice quenched
Yamamoto et al.: JPSJ 78, 123002 (2009)

Setup (1)

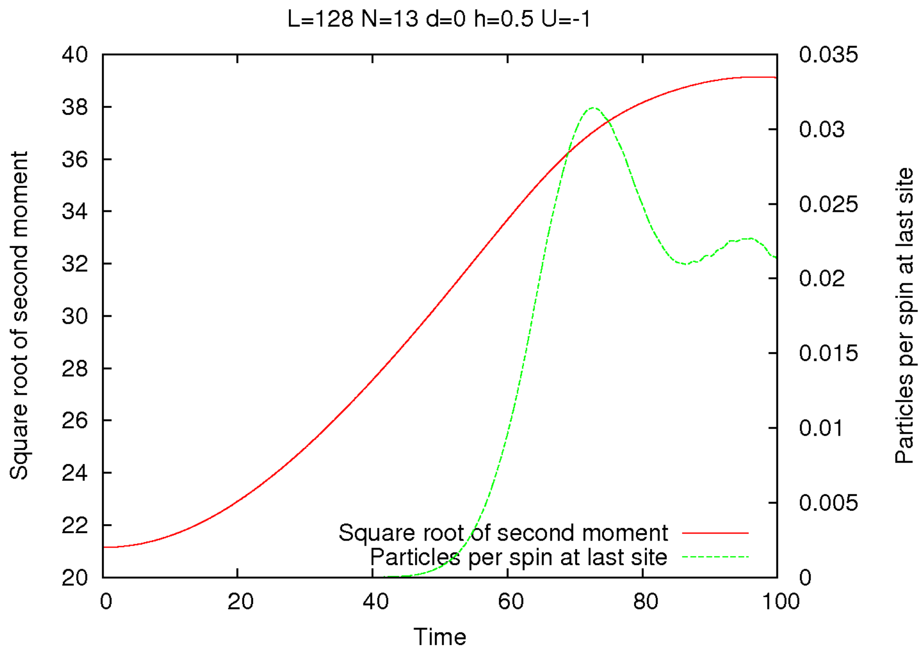
- Optical lattice + incommensurate potential (Aubry-André model)
- On-site attractive interaction
- Initially trapped in a harmonic potential



- Remove the harmonic potential but keep the incommensurate potential on: what happens?

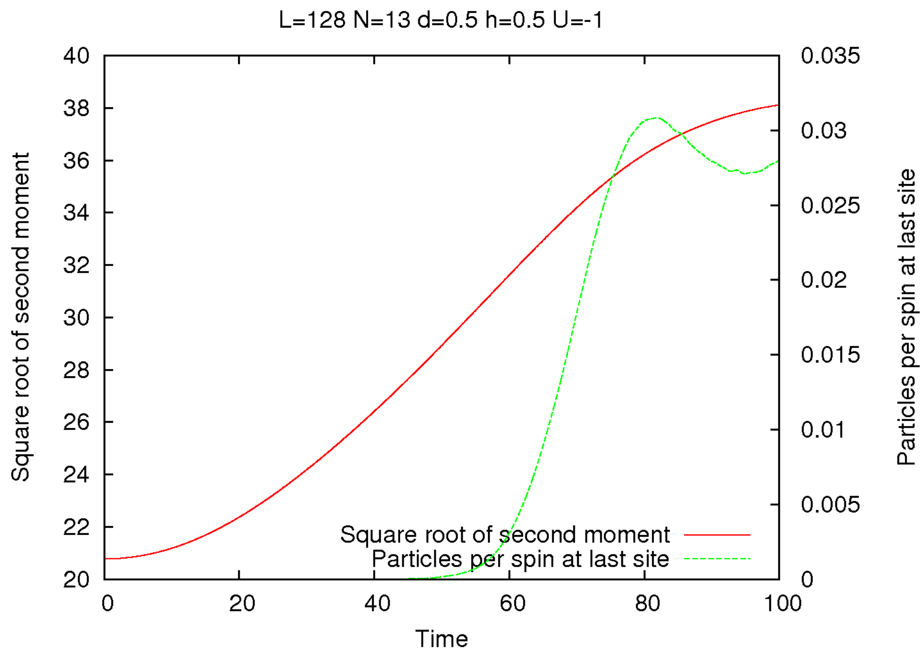
➔ Study by time-dep. DMRG for Hubbard model

Non-interacting case



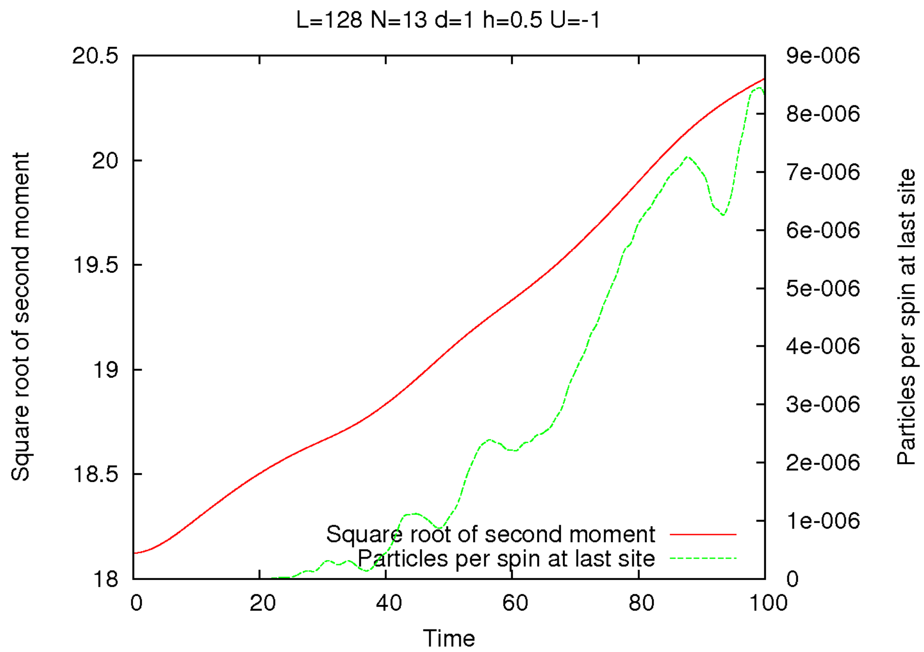
Smooth diffusion until bouncing back
at (artificial) system boundary

λ=0.5 < $\lambda_c(U=-1)$



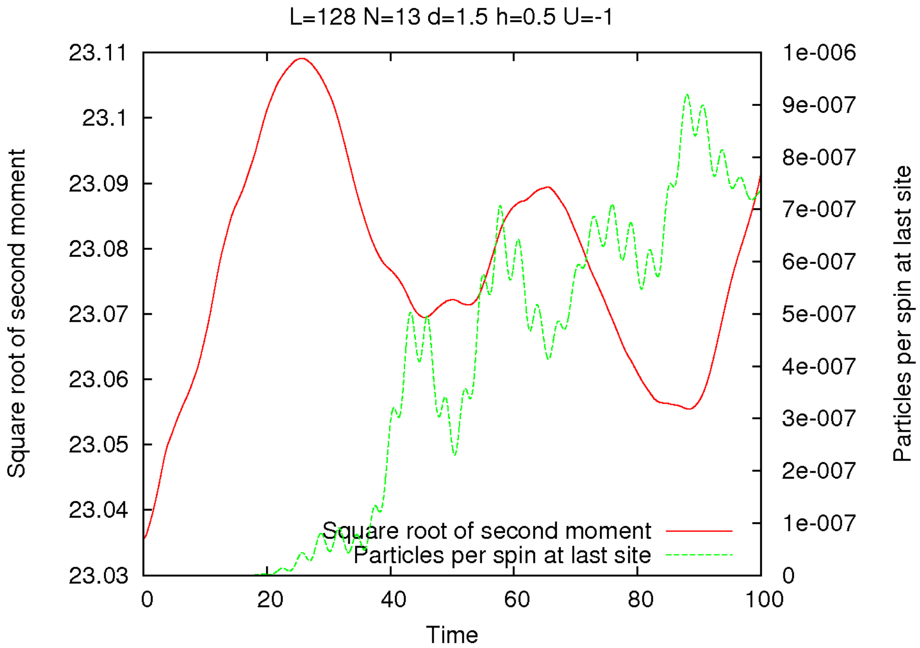
Atoms quickly flow to both sides

$\lambda=1.0 \sim \lambda_c(U=-1)$



Significantly slower diffusion

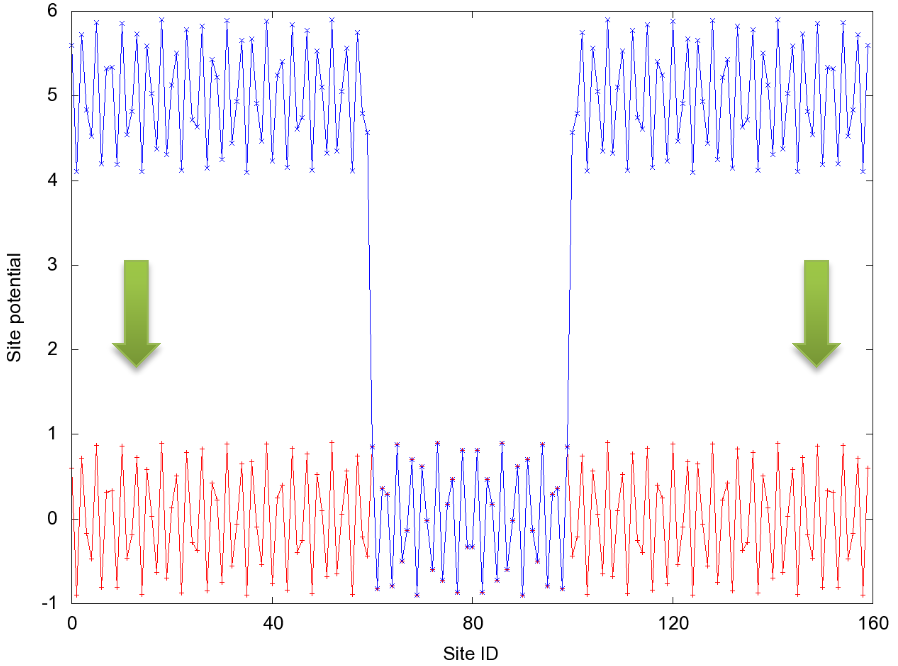
$\lambda=1.5 > \lambda_c(U=-1)$



Atoms move only locally

Setup (2)

- Optical lattice + incommensurate potential (Aubry-André model)
- On-site attractive interaction
- Initially trapped in a box potential



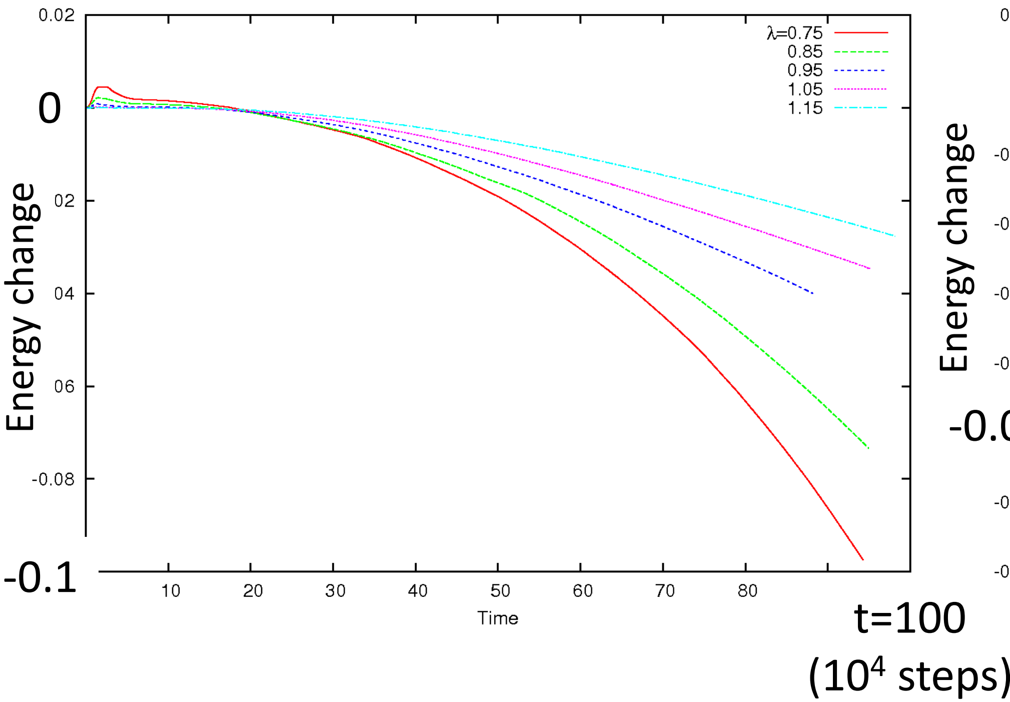
- Remove the box potential but keep the incommensurate potential on: what happens?

→ Study by time-dep. DMRG for Hubbard model

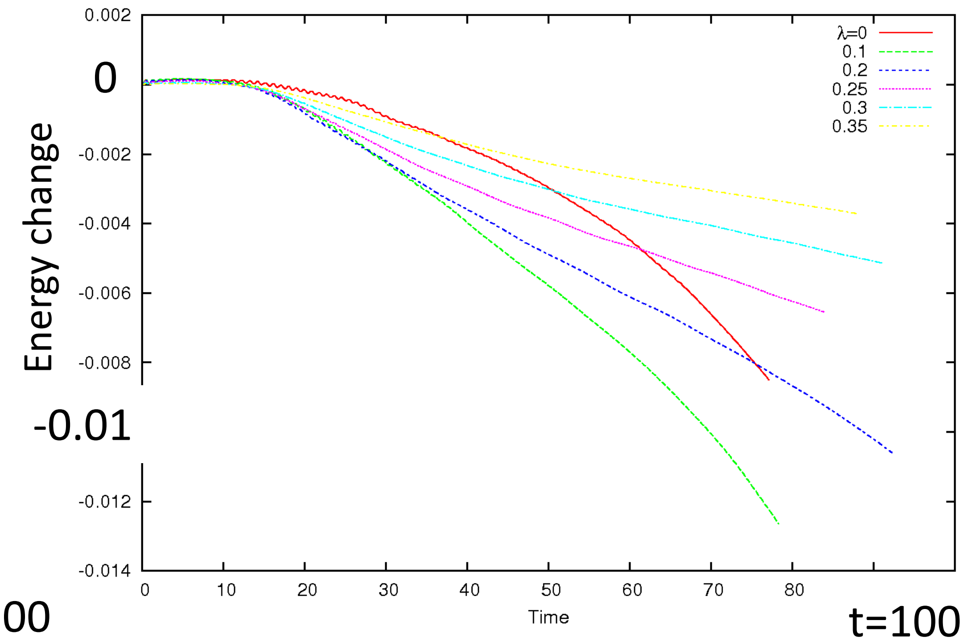
Check: energy is nearly preserved

160 sites, 12+12 fermions, Δt = 0.01

U=-1



U=-6



Suppressed motion for $\lambda > \lambda_c$

200 sites, 18 up and 18 down fermions, U=-1

$\lambda_c \sim 1.0$

λ=0.6

λ=0.9

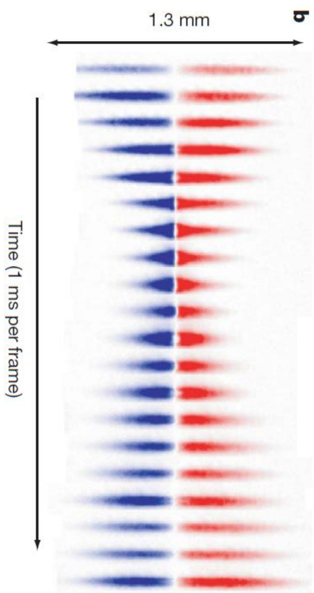
λ=1.1

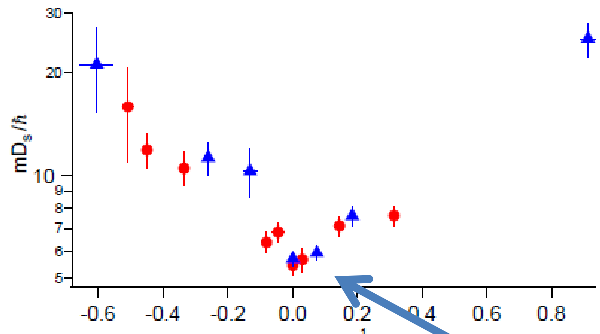
J. Ozaki, M. Tezuka, and N. Kawakami: arXiv: 1107.0774

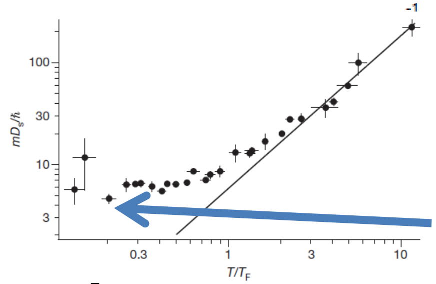
3) Collision dynamics

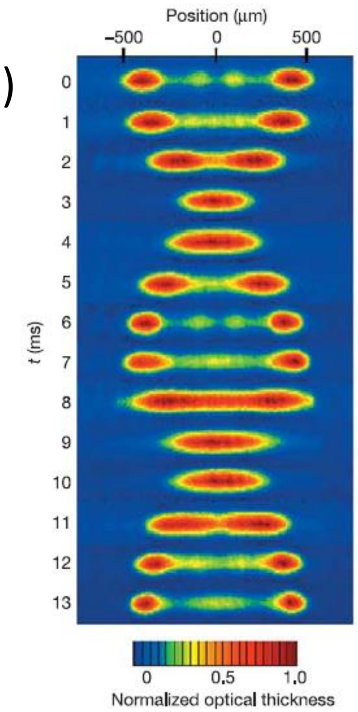
Elongated two-component Fermi gas:
up and down spins released from separate traps to collide ("Little Fermi Collider")

cf. 1D Bosons: absence of thermalization
Kinoshita et al.:
Nature 440, 900 (2006)

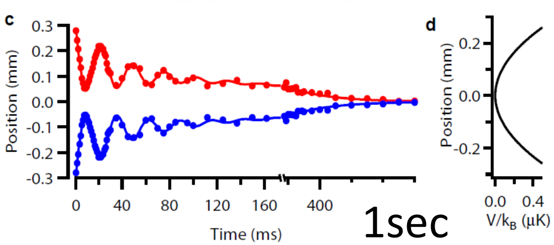








Spin diffusion constant minimum at Feshbach resonance; converges as $T \rightarrow 0$



Motion of center of mass

What happens at 1D?

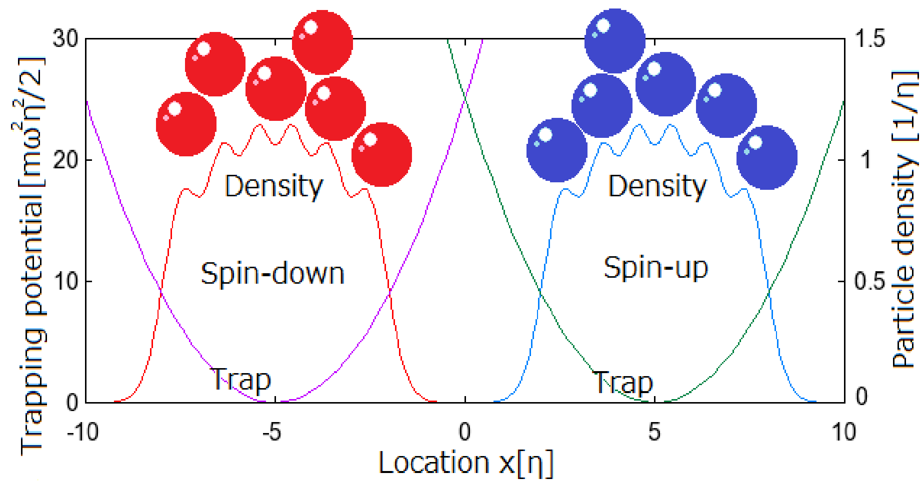
"Universal spin transport in a strongly interacting Fermi gas"
Sommer *et al.*: *Nature* **472** 201 (2011)

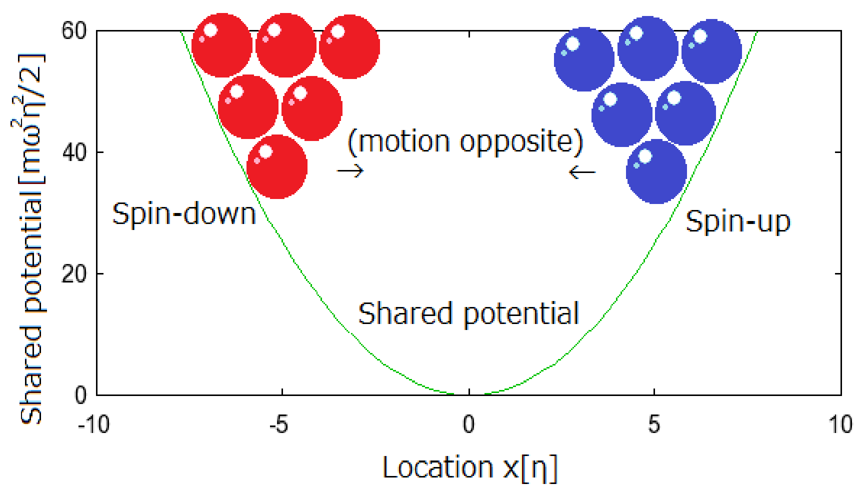
Motivation

What kind of many-body effects are observed during a single collision between two one-dimensional fermion clusters?

- A spin-dependent harmonic trap
- Quenched to a shared potential
- The fermions collide at the trap center

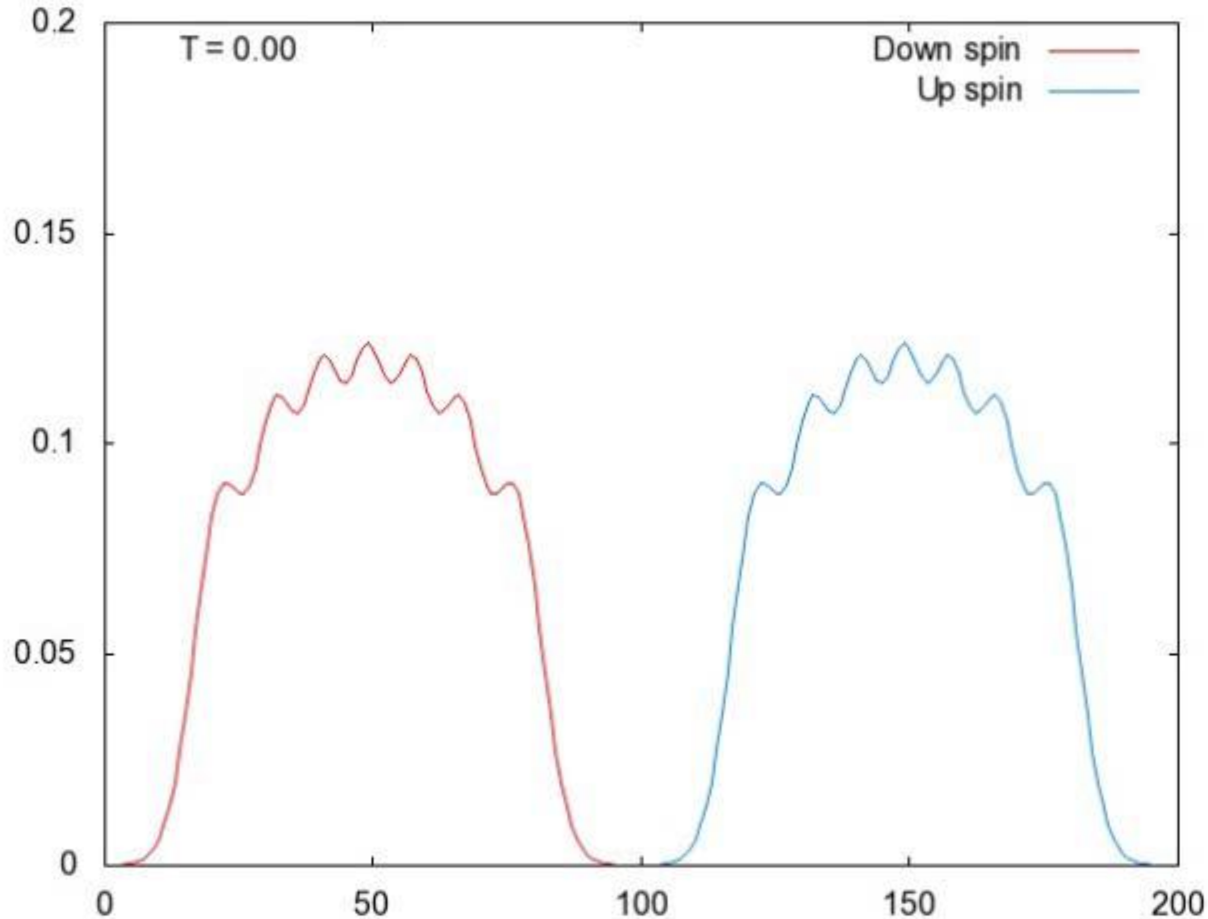
Model: Hubbard model
Method: time-dependent DMRG



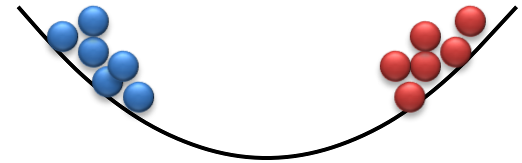


$U/J = 0.80$

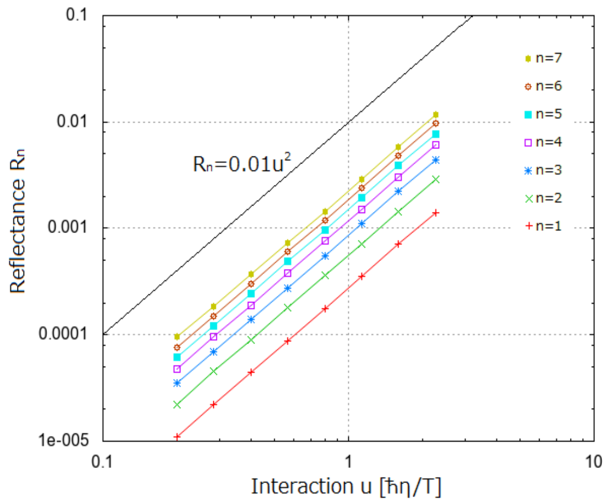
Example
(7+7 atoms, ~55% reflectance)

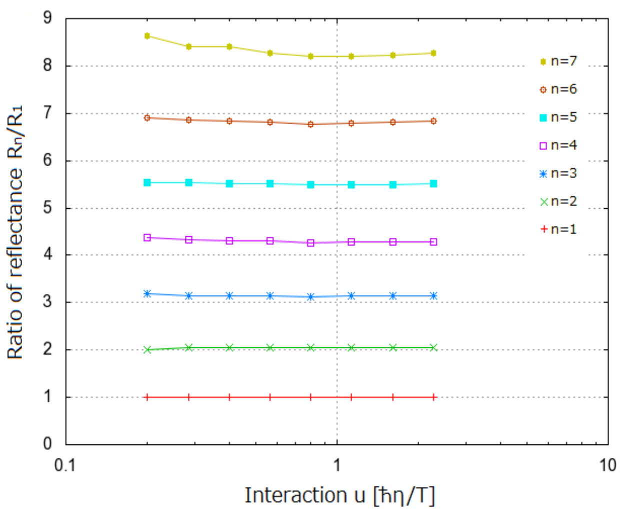


Weak interaction: most of atoms are not reflected



particle reflectance for $n + n$ atoms R_n: $R_1 \propto u^2$ $(u \to 0)$





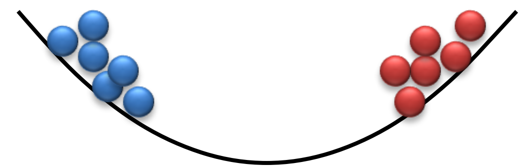
Quasi-classical model

Quasi-classical model:
a series of one-to-one collisions between two types of *independent* classical particles

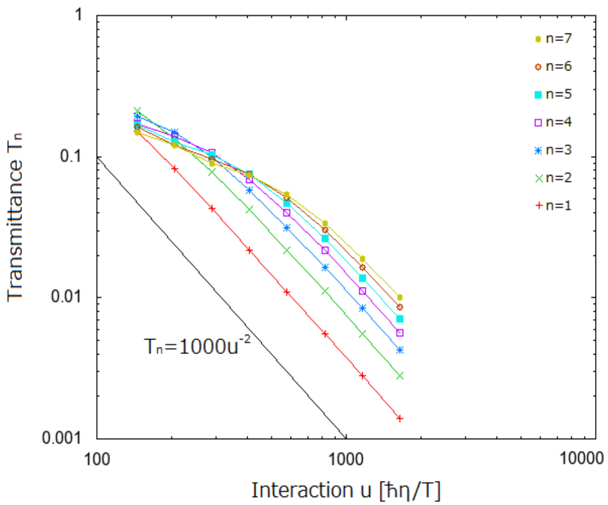
(i) $u \to 0$:
n^2 times of independent spin-up and spin-down collisions
$\Rightarrow R^{qc}_n = n^2 R_1/n = nR_1$

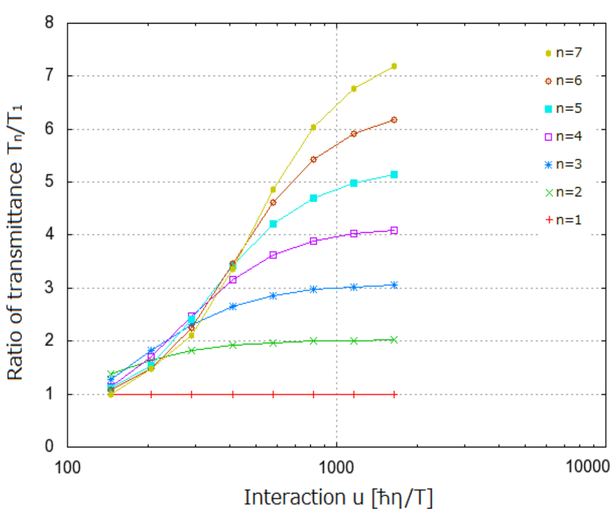
Consistent with the simulation

Strong interaction: most of atoms are reflected back



Transmittance for $n + n$ atoms $T_n : T_1 \propto u^{-2}$ $(u \to \infty)$, $R_n + T_n = 1$





Quasi-classical model

Quasi-classical model:
a series of one-to-one collisions between two types of *independent* classical particles

(ii) $u \to \infty$:
n atoms collide successively against n atoms $\Rightarrow T^{qc}_n = nT_1/n = T_1$

Inconsistent with the simulation!

What is going on?
The work is in progress.

Summary

- Application of DMRG for static and dynamic behavior of Fermi cold atom gases in 1D
  - Population-imbalanced gas in harmonic trap: FFLO-like condensate PRL **100**, 110403 (2008); New J. Phys **12**, 055029 (2010)
  - Quasiperiodic disorder
    - Can enhance condensation for weak attraction PRA **82**, 043613 (2010)
    - Trap-release dynamics close to metal-insulator transition: anomalous diffusion observed arXiv: 1107.0774
  - Collision of spin clusters: more atoms pass through than quasi-classically expected = emergent many-body behavior
- There is much more to explore with powerful numerical methods!