A Wilsonian view
on
the nuclear effective field theory

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based on the works in collaboration with
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Plan

- Introduction to NEFT
- Power counting and (Wilsonian) RG
- NEFT without pions
- NEFT with pions
- Summary

Introduction to NEFT

- NEFT: Nuclear Effective Field Theory
  - describes interactions among nucleons
  - is not a model
    - based on general principles of EFT, systematic, related to QCD through chiral symmetry (cf. phenomenological NN potentials)
    - An extension of chiral perturbation theory
  - (Nonrelativistic) nucleons (and pions) are explicit degrees of freedom

As an EFT

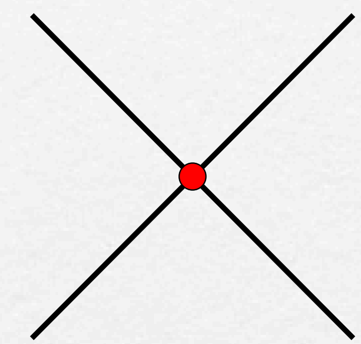
- has a finite range of validity, determined by a physical cutoff Λ_0 (≈ 400 MeV for NEFT with pions, ≈ m_π for NEFT without pions)
- has an infinite number of operators which should be organized by power counting rules
- (We limit ourselves to the NN systems.)

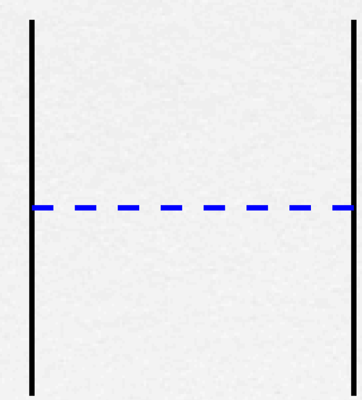
Peculiarity of S-waves

- Fine-tuned!
  - $^1S_0$ scattering length : -23.7 fm
  - $^3S_1$ scattering length : 5.4 fm
  - pion Compton wavelength: 1.4fm
- inherently nonperturbative
  - Existence of deuteron

Interactions

- local 4-nucleon operators with an arbitrary number of derivatives (contact operators)
  - All of them are irrelevant naively.
- pion exchange





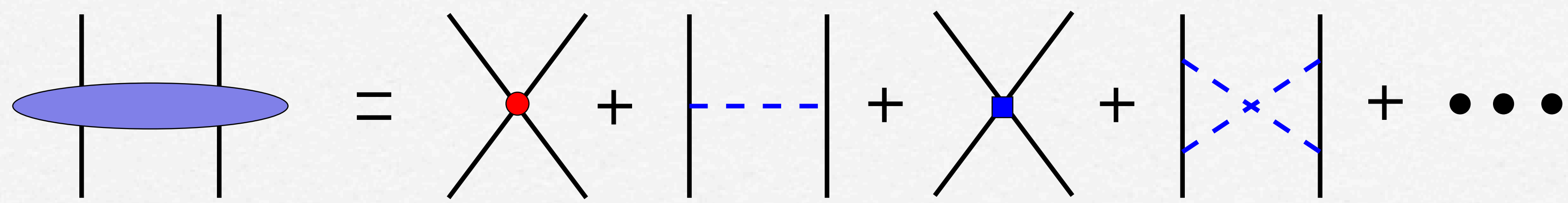
Nonperturbative features

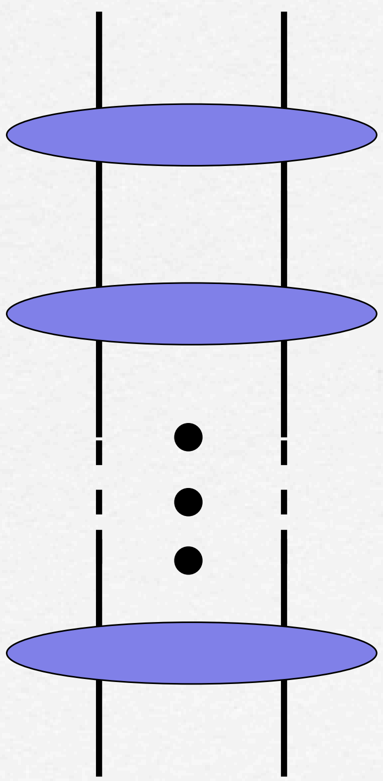
- The existence of the bound state (deuteron) suggests that a naive power counting (for the amplitudes) breaks down somewhere.
- The fine-tuning suggests the existence of a nontrivial fixed point.

Power counting

- Power counting is important, and is nontrivial in the NEFT.
- There are mainly two power counting schemes
  - Weinberg ('90)
  - Kaplan-Savage-Wise ('98), van Kolck ('99)

Weinberg’s power counting





- Construct an “effective potential” (sum of 2-nucleon irreducible parts) perturbatively on the basis of the naive dimensional analysis
- Substitute it into the Lippmann-Schwinger (or Schrödinger) equation (i.e., Solve nonperturbatively)

Pros and cons

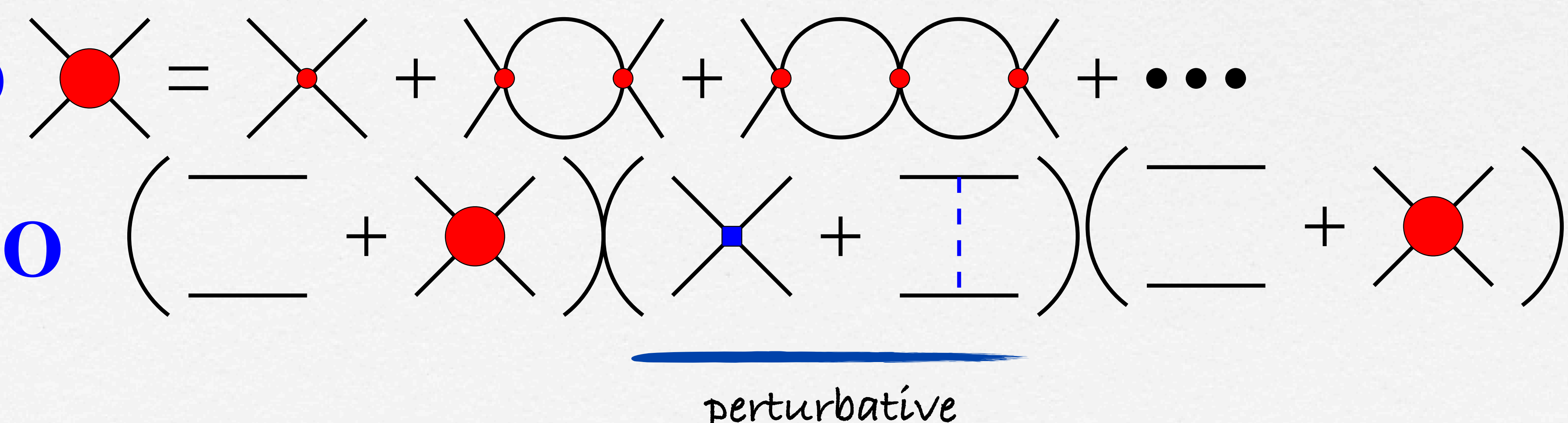
- Very successful numerically. There exist N^3LO calculations (Machleidt and Entem ('05), Epelbaum, Glockle, and Meißner('05)), which accurately reproduce NN scattering data.
- Power counting is inconsistent. Higher order counterterms are necessary to cancel the divergences in lower orders.
- The fine-tuning is not taken into account.
- Nonperturbative divergences (cutoff dependence) are not cancelled.

KSW power counting

- Only the contact operator without derivatives is iterated.
- Other interactions, even the pion exchanges are treated as perturbation.

LO

NLO

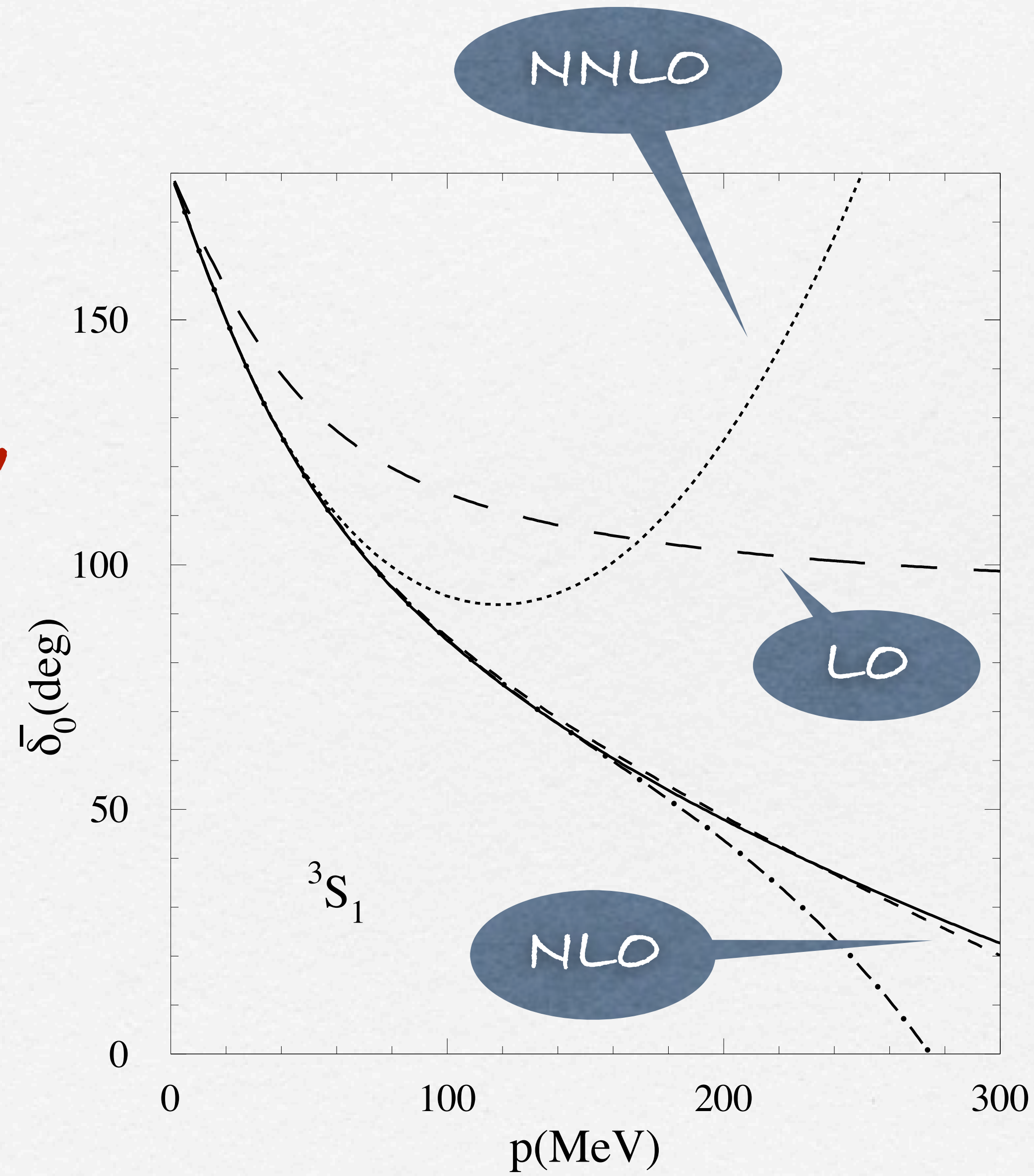


Pros and cons

- Analytic expressions for the amplitudes (and the phase shifts) can be obtained.
- Dimensional regularization with PDS (power divergence subtraction) may be used.
- The fine-tuning is taken into account.
- There is no inconsistency in the power counting.
- The EFT expansion however does not converge!

To be perturbative, or nonperturbative? That is a question!?

- Fleming et al. ('00) explicitly show that the EFT expansion with KSW power counting does not converge in several channels, including $^3S_1$ at the NNLO, due to the tensor force.



Why it fails?

- The tensor part of pion-exchange is singular ($\sim 1/r^3$) at short distances. Perhaps it must be treated nonperturbatively…?
  - Beane, Bedaque, Savage, and van Kolck ('02) propose that only the $1/r^3$ part of OPE should be treated nonperturbatively.
- What determines the correct power counting?

From "trials and errors" to a systematic way

- Usually, a power counting scheme is proposed and the justification of it comes from a good fit to the data.
  - It is sometimes hard to find the correct one when it fails.
- We would like to have a theoretical framework which determines possible power counting schemes.

Power counting and (Wilsonian) RG

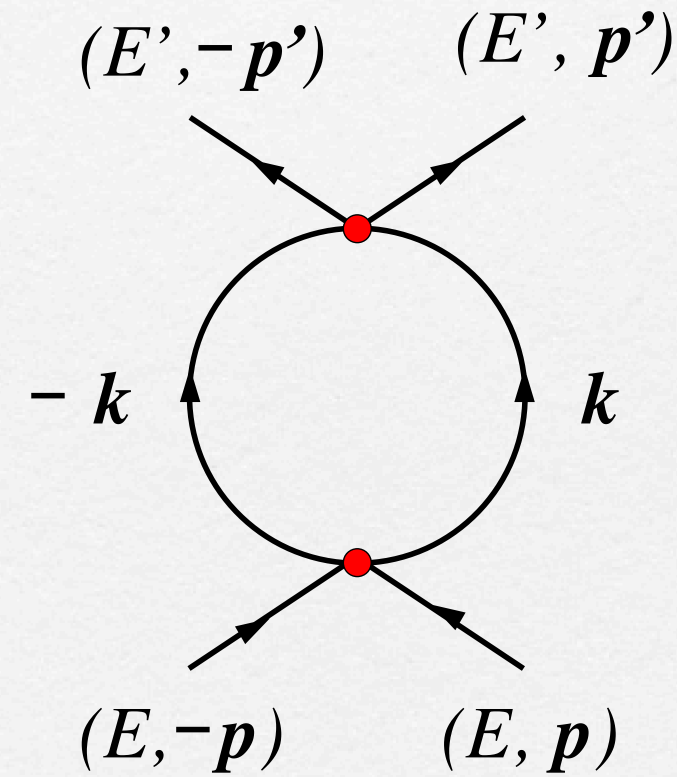
- What is power counting?
  - Order of magnitude estimate on the basis of dimensional analysis.
  - The importance of an operator is determined by its dimension.
  - Quantum theoretically, the scaling dimension should be considered.
  - It is obtained by a (Wilsonian) RG analysis.

Wilsonian RG for the NN systems

- Simplification for a nonrelativistic fermion system
  - Anti-fermions are not excited: States are divided into sectors parametrized by the nucleon number. (We are interested in the two-nucleon sector.)
  - Rotational symmetry: Each sectors are further divided into spin, (isospin,) and partial wave sectors. (We concentrate on the S-waves.)
  - Pauli principle: The number of operators with a definite number of derivatives are very limited.

Wilsonian RG for the NN systems

- Galilean invariance: The usual implementation of a cutoff function in the Eucledean propagators does not work. Rather, we cutoff the relative 3-momenta in the loops.
  - A sharp cutoff thus does not produce terms which are nonanalytic in external momenta.



$$\Lambda - d\Lambda < |\mathbf{k}| < \Lambda$$

Redundant operators

- A RG transformation generates all kinds of local operators which are consistent with symmetry.
- Operators such as $(N^T P^a N)^\dagger \left\{ N^T P^a \left(i\partial_t - \frac{\nabla^2}{2M} \right) \right\} N + h.c.$ are also considered, though they may be eliminated by the use of field redefinition.
- They are necessary for the cutoff independence of off-shell amplitudes.
- They may be viewed as "energy-dependent potential."

NEFT without pions

- Valid for $p \leq m_\pi$.
- Nucleons are the only explicit degrees of freedom. Pions (and other mesons, Deltas, etc.) are "heavy," so that their effects are represented as contact operators.
- A simplest field theory with a nontrivial fixed point...

Averaged action (up to O(p²))

$$\Gamma_\Lambda^{(\not\pi)} = \int d^4x \Bigg[N^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) N$$

$$\begin{cases} -C_0^{(S)}\mathcal{O}_0^{(S)} + C_2^{(S)}\mathcal{O}_2^{(S)} + 2B^{(S)}\mathcal{O}_2^{(SB)} \Bigg], & ({}^1S_0 \text{ channel}) \\ -C_0^{(T)}\mathcal{O}_0^{(T)} + C_2^{(T)}\mathcal{O}_2^{(T)} + 2B^{(T)}\mathcal{O}_2^{(TB)} + C_2^{(SD)}\mathcal{O}_2^{(SD)} \Bigg], & ({}^3S_1\text{–}{}^3D_1 \text{ channel}) \end{cases}$$

$$\mathcal{O}_0^{(S)} = \left(N^T P_a^{(S)} N\right)^\dagger \left(N^T P_a^{(S)} N\right),$$

$$\mathcal{O}_2^{(S)} = \left[\left(N^T P_a^{(S)} N\right)^\dagger \left(N^T P_a^{(S)} \overleftrightarrow{\nabla}^2 N\right) + h.c.\right],$$

$$\mathcal{O}_2^{(SB)} = \left[\left\{N^T P_a^{(S)} \left(i\partial_t + \frac{\nabla^2}{2M}\right) N\right\}^\dagger \left(N^T P_a^{(S)} N\right) + h.c.\right],$$

$$\mathcal{O}_0^{(T)} = \left(N^T P_i^{(T)} N\right)^\dagger \left(N^T P_i^{(T)} N\right),$$

$$\mathcal{O}_2^{(T)} = \left[\left(N^T P_i^{(T)} N\right)^\dagger \left(N^T P_i^{(T)} \overleftrightarrow{\nabla}^2 N\right) + h.c.\right],$$

$$\mathcal{O}_2^{(SD)} = \left[\left(N^T P_i^{(T)} N\right)^\dagger \left\{N^T \left(\overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j - \frac{1}{3}\delta_{ij} \overleftrightarrow{\nabla}^2\right) P_j^{(T)} N\right\} + h.c.\right],$$

$$\mathcal{O}_2^{(TB)} = \left[\left\{N^T P_i^{(T)} \left(i\partial_t + \frac{\nabla^2}{2M}\right) N\right\}^\dagger \left(N^T P_i^{(T)} N\right) + h.c.\right],$$

RGEs for $^1S_0$ (up to $O(p^2)$)

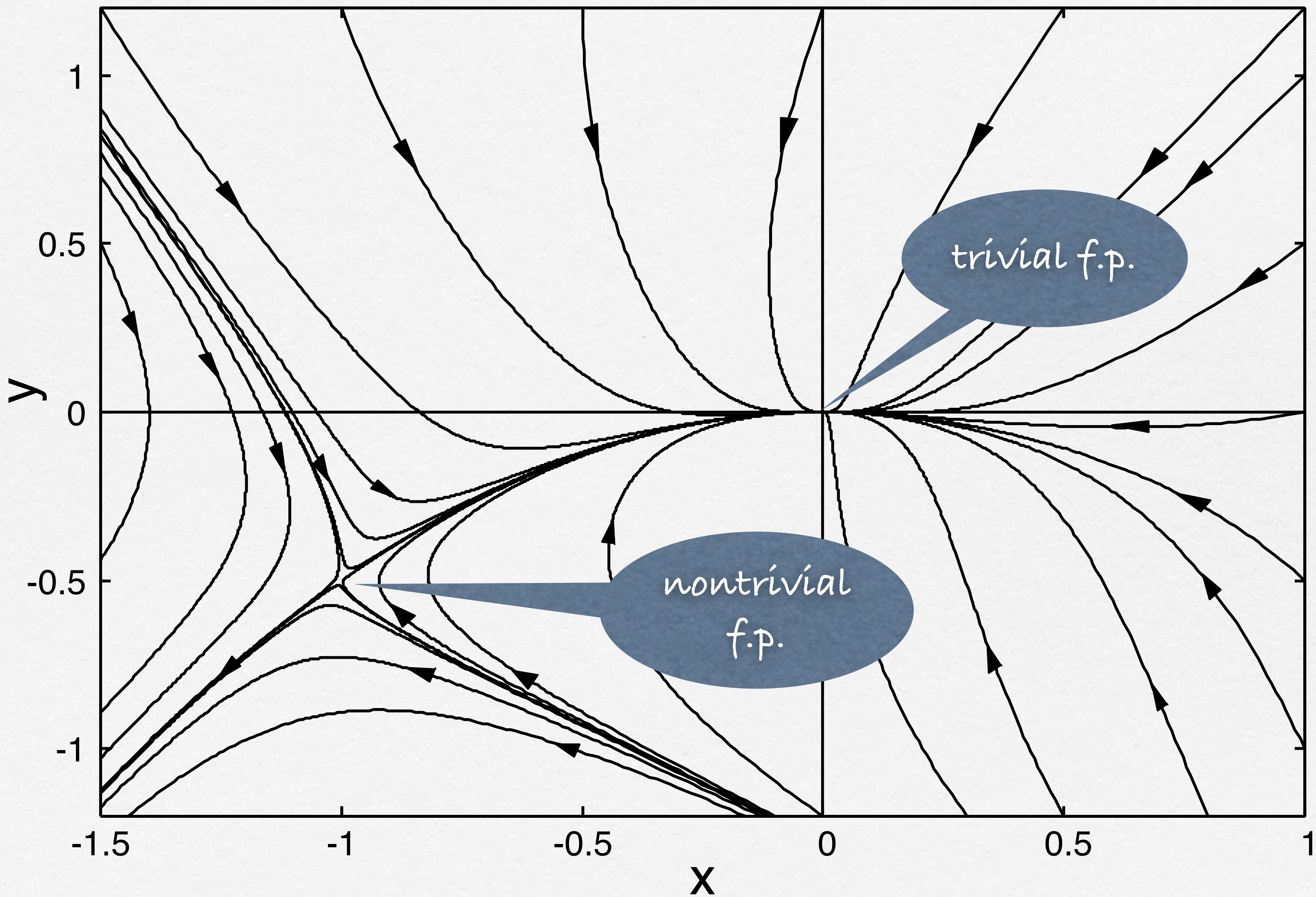
$$\frac{dx}{dt} = -x - (x+y+z)^2 \qquad t = \ln(\Lambda_0/\Lambda)$$

$$\frac{dy}{dt} = -3y - \left(\frac{1}{2}x^2 + 2xy + \frac{3}{2}y^2 + yz - \frac{1}{2}z^2\right)$$

$$\frac{dz}{dt} = -3z + \left(\frac{1}{2}x^2 + xy + \frac{1}{2}y^2 - xz - yz - \frac{3}{2}z^2\right)$$

- Operators mix.
- We have examined the case with operators up to $O(p^4)$, but the results do not change very much.

RG flow in a subspace



RGEs for $^3S_1$-$^3D_1$

$t = \ln(\Lambda_0/\Lambda)$

$$\frac{dx'}{dt} = -x' - (x' + y' + z')^2 - 2w'^2$$

$$\frac{dy'}{dt} = -3y' - \left(\frac{1}{2}x'^2 + 2x'y' + \frac{3}{2}y'^2 + y'z' - \frac{1}{2}z'^2\right) - w'^2$$

$$\frac{dz'}{dt} = -3z' + \left(\frac{1}{2}x'^2 + x'y' + \frac{1}{2}y'^2 - x'z' - y'z' - \frac{3}{2}z'^2\right) + w'^2$$

$$\frac{dw'}{dt} = -3w' - (x' + y' + z')w'$$

- Very similar to the $^1S_0$ case

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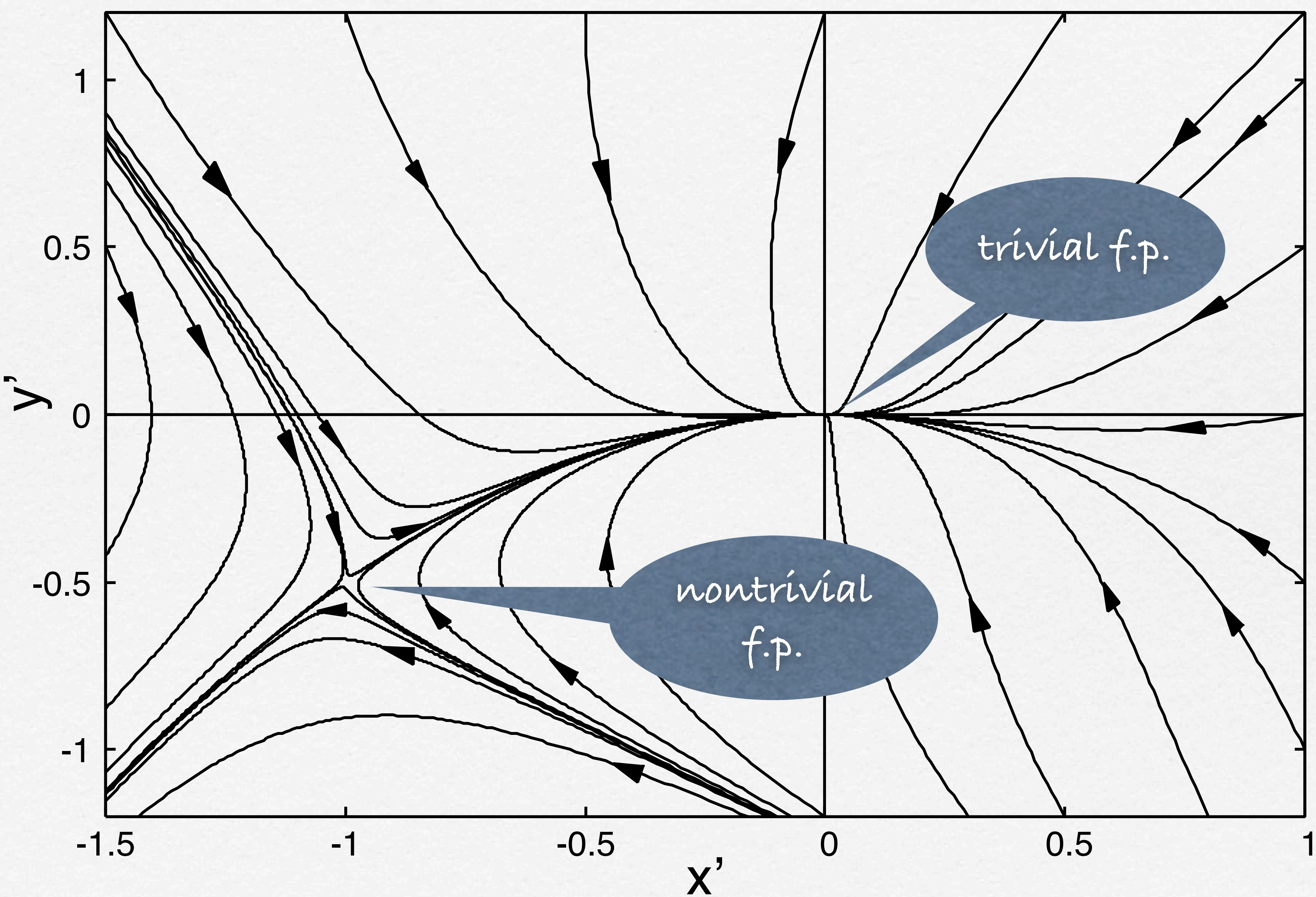
RG flow in a subspace



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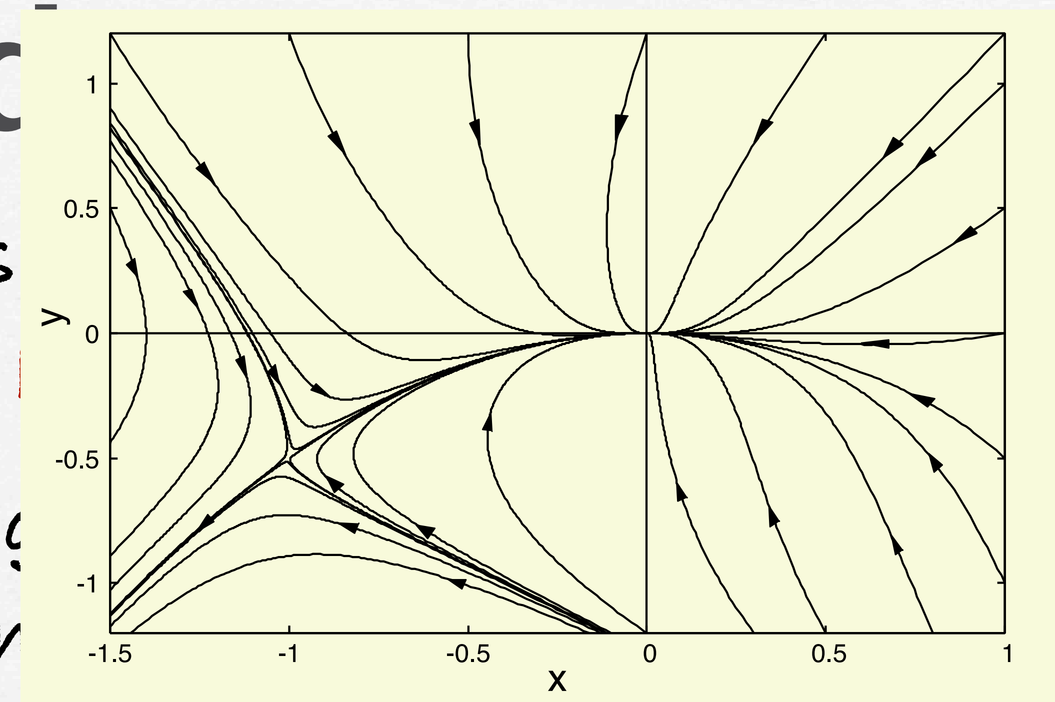
Fixed points and scaling dimensions

- Nontrivial fixed point:
  - $^1S_0$: $(x^*,y^*,z^*)=(-1,-1/2,1/2)$
  - $^3S_1$-$^3D_1$: $(x'^*,y'^*,z'^*,w'^*)=(-1,-1/2,1/2,0)$
- Scaling dimensions:
  - $^1S_0$: $(+1,-1,-2)$
  - $^3S_1$-$^3D_1$: $(+1,-1,-2,-2)$
- Only one combination of operators is relevant.
- Anomalous dimensions are +2. (cf. Birse et al. ('99))

What we found

- Our Wilsonian RG analysis
  counting very similar to the



- It is useful to use the languag
  phenomena for the NN system
  - There are strong-coupling and weak-coupling phases. $^1S_0$ channel is in the weak-coupling phase, whereas $^3S_1$-$^3D_1$ in the strong-coupling phase.
  - The inverse of the scattering length is the order parameter.

NEFT with pions

- valid up to $\Lambda_0 \approx 400$ MeV
- Nucleons and pions are explicit degrees of freedom. Their interactions are constrained by chiral symmetry.
- Pions propagate as relativistic particles.

Previous Wilsonian RG analysis by M. Birse ('06)

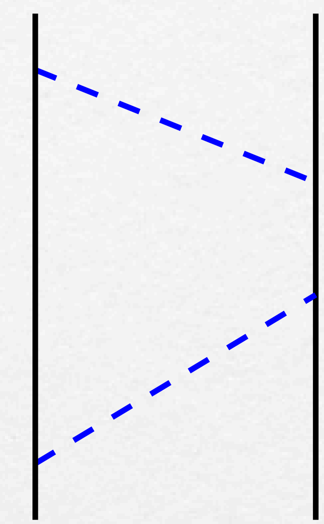
- using the distorted-wave RG formalism,
  - treats pion exchanges nonperturbatively.
  - does not introduce a cutoff (separation scale) for pion exchanges.
- finds that the scaling dimensions shifts by ±1/2 from those of the pionless case, even at the trivial fixed point.

Nonlinearly realized chiral symmetry

- We spent a lot of time for formulating a Wilsonian RG framework with exact chiral symmetry, but did not succeed...
  - The nonlinearly realized (global) symmetry is similar to local (gauge) symmetry.
  - We are interested in the flow itself.
- We eventually gave it up, and consider the leading order in the (p/M) expansion qualitatively.

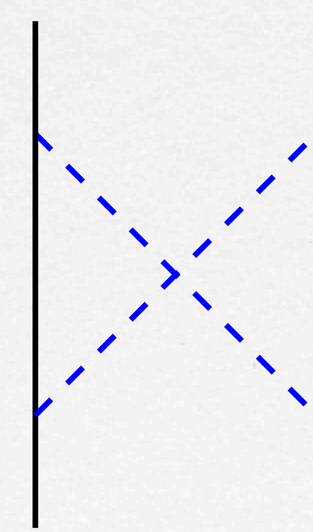
IR enhancement

- Consider one-loop diagrams with the 3-momentum integral being restricted to the shell-modes.



- The energy integration is dominated either by the nucleon poles or by the pion poles.

- The contribution of the nucleon poles in the 2-nucleon reducible diagrams get the enhancement factor (M/Λ). (cf. Weinberg ('90))



IR enhancement

- The IR enhancement is a consequence of the nonrelativistic kinematics. It is not directly related to the fine-tuning.
- The nucleon pole makes the pion propagator "instantaneous", i.e., the Yukawa potential.
- In the leading order in (M/Λ) expansion, the pions appear only as the instantaneous Yukawa potential.

$$\frac{i}{[-p^0+(\mathbf{p}+\mathbf{k})^2/2M]^2-\omega_{\mathbf{k}}^2} \xrightarrow{p^0-\mathbf{p}^2/2M \ll \Lambda^2/M \ll \Lambda} \frac{-i}{\mathbf{k}^2+m_\pi^2}$$

Reduced theory

- Because at leading order in (Λ/M), pions only contribute as the instantaneous Yukawa potential, and only the two-nucleon reducible one-loop diagrams should be considered, we may start with a reduced action.

Averaged action (up to O(p²))

$$\Gamma_\Lambda = \Gamma_\Lambda^{(\not\pi)} + \int d^4x \left\{ -D_2^{(c)} m_\pi^2 \mathcal{O}_0^{(c)} \right\}$$
$$+ \frac{g_A^2}{4f^2} \int dt \int d^3x\, d^3y \Big[\mathcal{O}^{(S)}(x,y) \nabla_x^2$$
$$+ \mathcal{O}_{ii}^{(T)}(x,y) \nabla_x^2$$
$$- 6\mathcal{O}_{ij}^{(T)}(x,y) \left(\partial_i^x \partial_j^x - \frac{1}{3}\delta_{ij} \nabla_x^2 \right) \Big] Y(|x-y|)$$

$$\mathcal{O}^{(S)}(x,y) = \left(N^T(x) P_a^{(S)} N(y) \right)^\dagger \left(N^T(y) P_a^{(S)} N(x) \right),$$

$$\mathcal{O}_{ij}^{(T)}(x,y) = \left(N^T(x) P_i^{(T)} N(y) \right)^\dagger \left(N^T(y) P_j^{(T)} N(x) \right),$$

L-OPE and S-OPE

- It is a general principle to represent physics beyond the cutoff as contact interactions.
- It usually refers to the effects of heavy particles.
- Pion is light, but its short-distance effects (OPE with momentum transfer larger than the cutoff) should also be represented as contact interactions.

L-OPE and S-OPE

- We will show :
  - A part of the short-distance part of OPE (S-OPE) is relevant, and should be treated nonperturbatively.
  - The long-distance part of OPE (L-OPE) should be treated as perturbation.
- The separation is important, since they behave differently.
- The introduction of the separation scale is discussed by Beane, Kaplan, and Vourinen ('09). But the philosophy is probably different.

RGEs for $^1S_0$ (assuming p, m_π < Λ)

$$\frac{dx}{dt} = -x - \left[x^2 + 2xy + y^2 + 2xz + 2yz + z^2\right]$$
$$- 2(x+y+z)\gamma - \gamma^2,$$
$$\frac{dy}{dt} = -3y - \left[\frac{1}{2}x^2 + 2xy + \frac{3}{2}y^2 + yz - \frac{1}{2}z^2\right]$$
$$- (x+2y)\gamma - \frac{1}{2}\gamma^2,$$
$$\frac{dz}{dt} = -3z + \left[\frac{1}{2}x^2 + xy + \frac{1}{2}y^2 - xz - yz - \frac{3}{2}z^2\right]$$
$$+ (x+y-z)\gamma + \frac{1}{2}\gamma^2,$$
$$\frac{du}{dt} = -3u - 2(x+y+z)(u-\gamma) - 2u\gamma + 2\gamma^2,$$

γ is a measure of the strength of OPE

RGEs for $^3S_1$-$^3D_1$ (assuming p, $m_\pi < \Lambda$)

$$\frac{dx'}{dt} = -x' - \left[x'^2 + 2x'y' + y'^2 + 2x'z' + 2y'z' + z'^2 + 2w'^2\right] - 2(x' + y' + z' - 4w')\gamma \underline{- 9\gamma^2},$$

$$\frac{dy'}{dt} = -3y' - \left[\frac{1}{2}x'^2 + 2x'y' + \frac{3}{2}y'^2 + y'z' - \frac{1}{2}z'^2 + w'^2\right] - (x' + 2y')\gamma \underline{+ \frac{7}{2}\gamma^2},$$

$$\frac{dz'}{dt} = -3z' + \left[\frac{1}{2}x'^2 + x'y' + \frac{1}{2}y'^2 - x'z' - y'z' - \frac{3}{2}z'^2 + w'^2\right] + (x' + y' - z' - 4w')\gamma \underline{+ \frac{9}{2}\gamma^2},$$

$$\frac{dw'}{dt} = -3w' - \left[x'w' + y'w' + z'w'\right] + \frac{1}{5}(2x' + 2y' + 2z' - 9w')\gamma + 2\gamma^2,$$

$$\frac{du'}{dt} = -3u' - 2(x' + y' + z')u' + 2(x' + y' + z' - 4w')\gamma - 2u'\gamma \underline{+ 18\gamma^2}.$$

Large coefficients!

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Dimensionless coupling constant for pion exchange

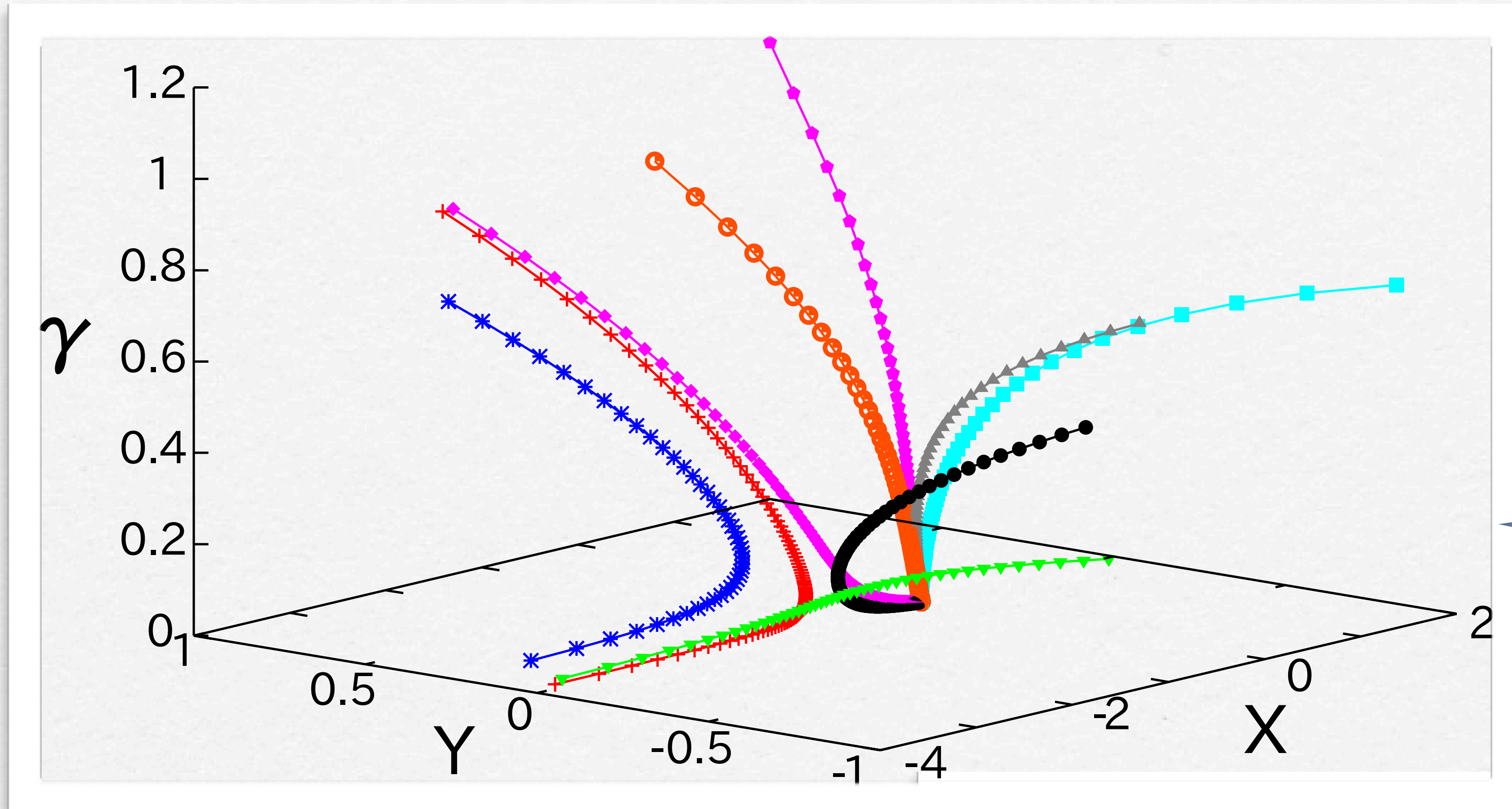
$$\gamma \equiv \frac{M\Lambda}{2\pi^2}\left(\frac{g_A}{2f}\right)^2$$

- There are other ways of defining the corresponding dimensionless coupling.
- The self-similarity property of RGEs (the cutoff should not appear in β functions) fixes this ambiguity.

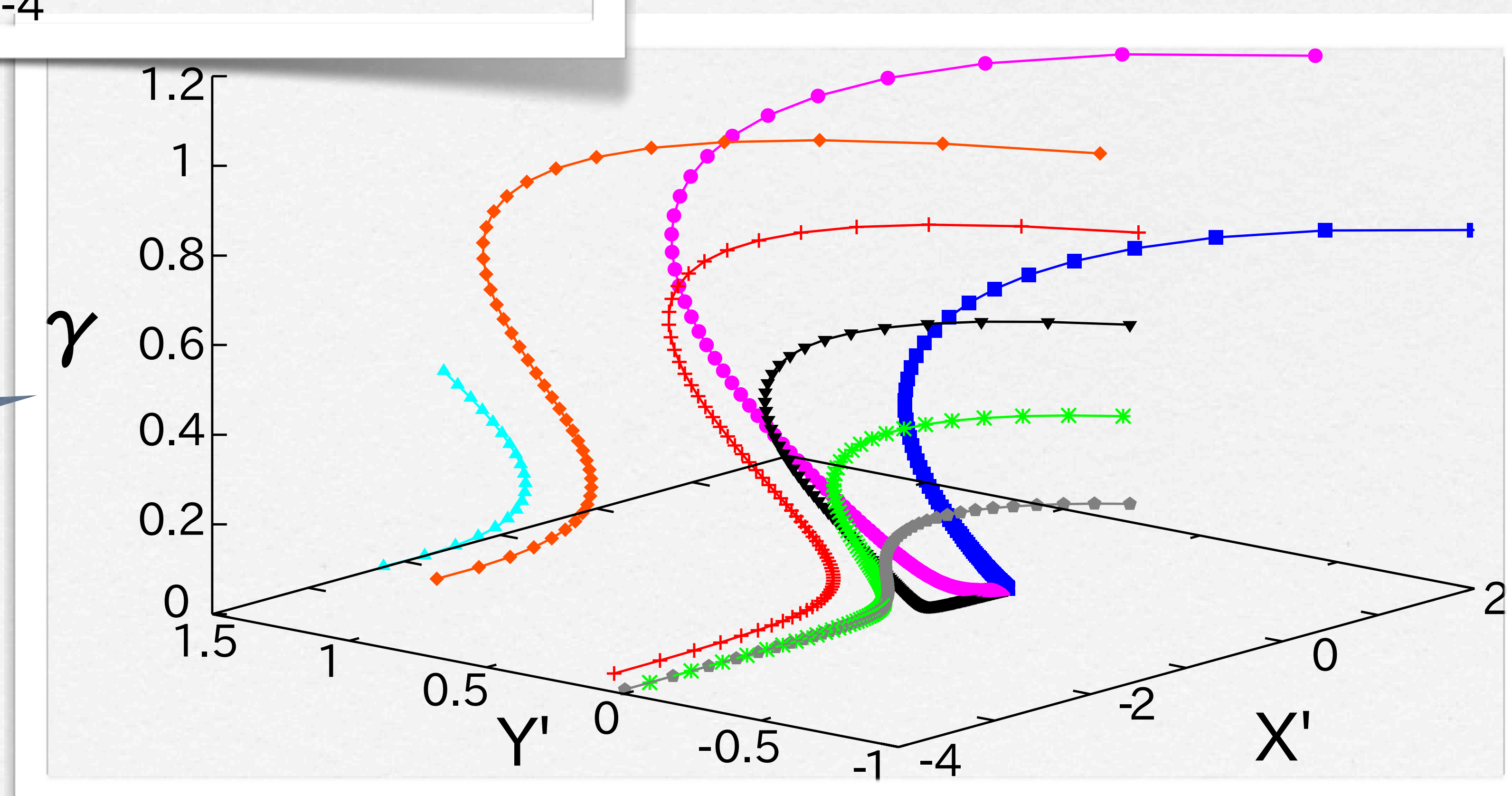
$$\frac{d}{dt}\gamma = -\gamma$$

RG flows

in subspaces



spin singlet



spin triplet

Fixed points and scaling dimensions

- Nontrivial fixed point:
  - $^1S_0$: $(x^*, y^*, z^*, u^*, \gamma^*) = (-1, -1/2, 1/2, 0, 0)$
  - $^3S_1$-$^3D_1$: $(x'^*, y'^*, z'^*, u'^*, w'^*, \gamma^*) = (-1, -1/2, 1/2, 0, 0, 0)$
- Scaling dimensions:
  - $^1S_0$: $(+1, -1, -1, -2)$
  - $^3S_1$-$^3D_1$: $(+1, -1, -1, -2, -2)$
- The nontrivial fixed points are identified with those in the NEFT without pions.
- The scaling dimensions are essentially the same as those in the NEFT without pions.

It means:

- The inclusion of pions (L-OPE) does not affect the existence (and the location) of the nontrivial fixed point, nor changes the scaling dimensions.
- The tensor force affects only the details of the flow (strong dragging).
- Pions (L-OPE) become less and less important at lower energies.

For Λ less than m_π

- We do not expand the contributions in powers of (m_π/Λ) anymore. Instead, we now expand them in powers of (Λ/m_π) and (p/m_π).
- The diagrams are essentially the same, but the RGEs changes.
- The dimensionless coupling for the pion exchange is also changed in accordance with the self-similarity property.

RGEs for $^1S_0$

(assuming $p < \Lambda < m_\pi$)

$$\tilde{\gamma} = \frac{\Lambda^2}{m_\pi^2}\gamma, \qquad \tilde{u} = \frac{m_\pi^2}{\Lambda^2}u.$$

$$\frac{d\chi}{dt} = (\text{first line of the pionless one with } x \to \chi) - 2(\chi + y + z)\tilde{\gamma} - \tilde{\gamma}^2,$$

$$\chi \equiv x + \tilde{u}$$

$$\frac{dy}{dt} = (\text{first line of the pionless one with } x \to \chi) - (2\chi + 3y + z)\tilde{\gamma} - \frac{3}{2}\tilde{\gamma}^2,$$

$$\frac{dz}{dt} = (\text{first line of the pionless one with } x \to \chi) + (\chi + y - z)\tilde{\gamma} + \frac{1}{2}\tilde{\gamma}^2,$$

$$\frac{d\tilde{\gamma}}{dt} = -3\tilde{\gamma}$$

Smoothly connected to the NEFT without pions

- The nontrivial fixed point can be identified with that in the NEFT without pions, with x being replace with X.
- The pion exchange (L-OPE) is much more irrelevant.

Summary

- We have performed Wilsonian RG analyses of the NEFT with and without pions in the NN systems in the S-waves.
- There is only one relevant operator, though the other operators also get large anomalous dimensions.
- Our analysis suggests a power counting very similar to KSW's.

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Summary

- The separation of pion exchanges into two parts, S-OPEs and L-OPEs is necessary for the consistent Wilsonian treatment. It is also important, because they behave differently.
- The inclusion of pions does not affect the existence (and the location) of the nontrivial fixed point, nor the scaling dimensions.
- Our results are very different from those obtained by M. Birse.

Complementary (or, hybrid) regularization

- Dimensional regularization is useful because operator mixing is minimal. But it does not have a separation scale.
- A regularization with an explicit separation scale is necessary to separate pion exchanges into two parts, but causes operator mixing, and makes the practical calculations intractable.

Complementary (or, hybrid) regularization

- A new regularization scheme
  - PDS for contact interactions μ
  - Gaussian damping for L-OPE λ

Schematically

$$\frac{k^2}{k^2+m_\pi^2} \longrightarrow \underbrace{1_{\rm PDS}}_{\text{S-OPE}} - \underbrace{\frac{m_\pi^2}{k^2+m_\pi^2}}_{\text{L-OPE}} e^{-k^2/\lambda^2}$$

We think

$\mu \approx \lambda$

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