New Applications of Renormalization Group Methods in Nuclear Physics

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In collaboration with:
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Kyoto, August 31, 2011

Renormalization Group Approach
from Ultra Cold Atoms to the Hot QGP

THE OHIO STATE UNIVERSITY

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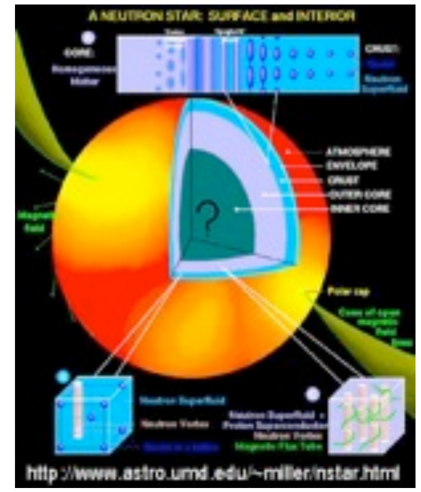
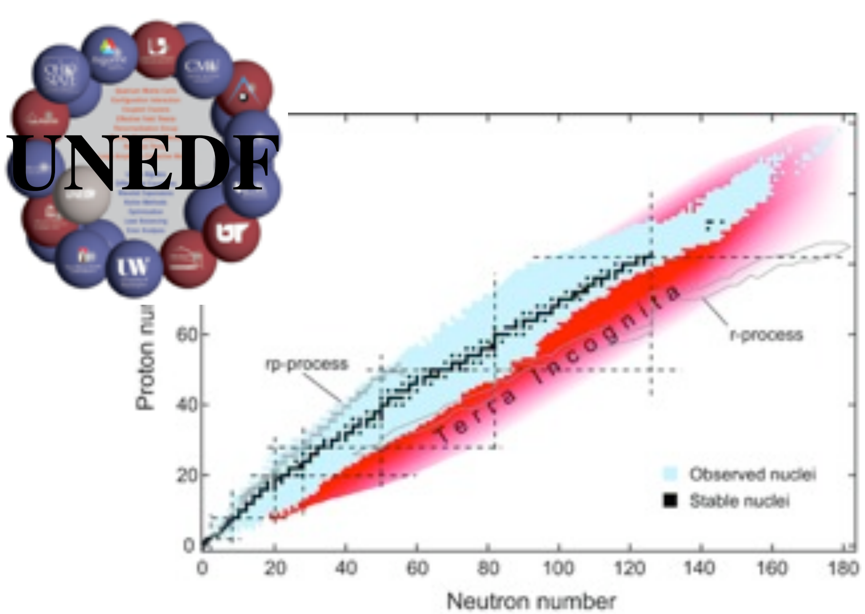
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

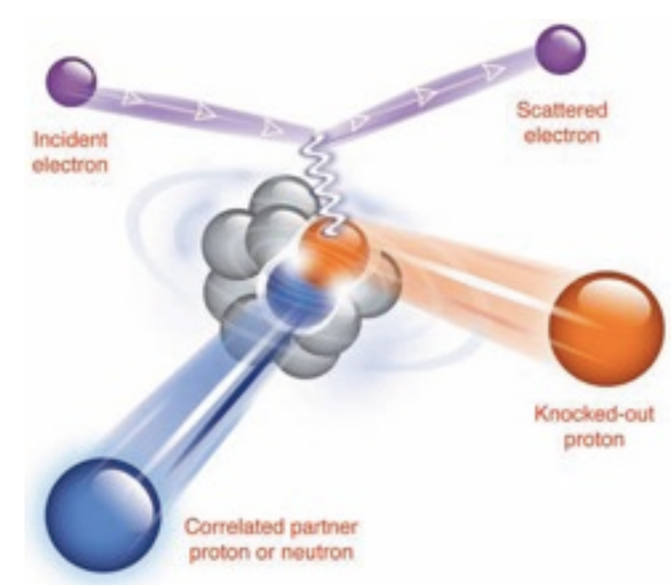


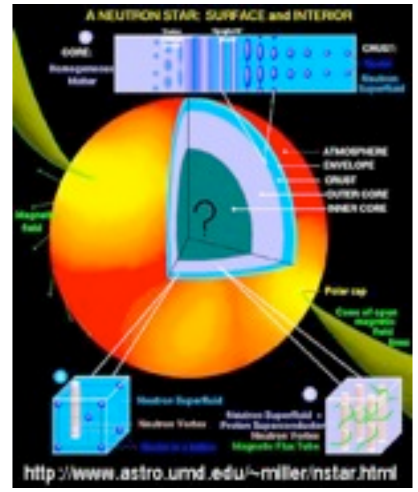
Nuclear equation of state
and astrophysical applications

Light and
neutron-rich nuclei



Correlations in
nuclear systems





Nuclear equation of state
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Unified microscopic
description!

Light and
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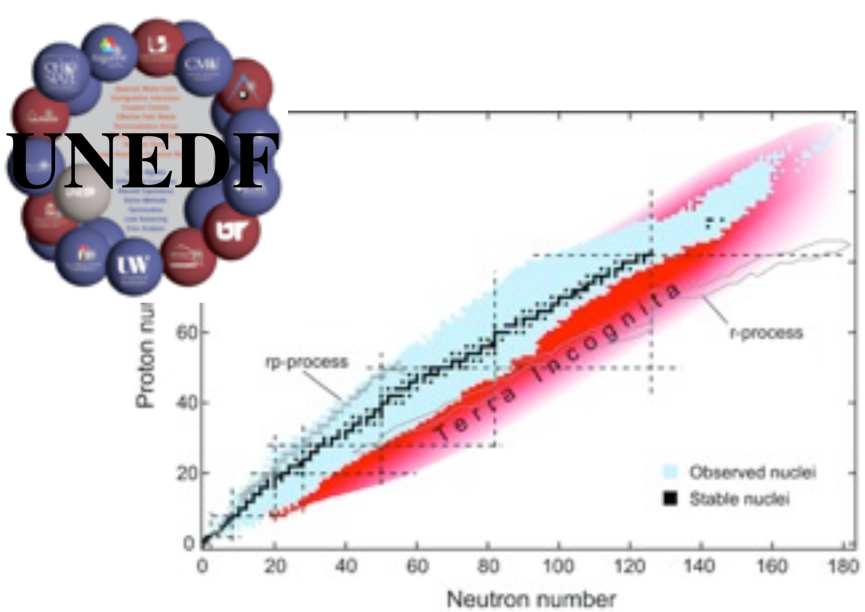

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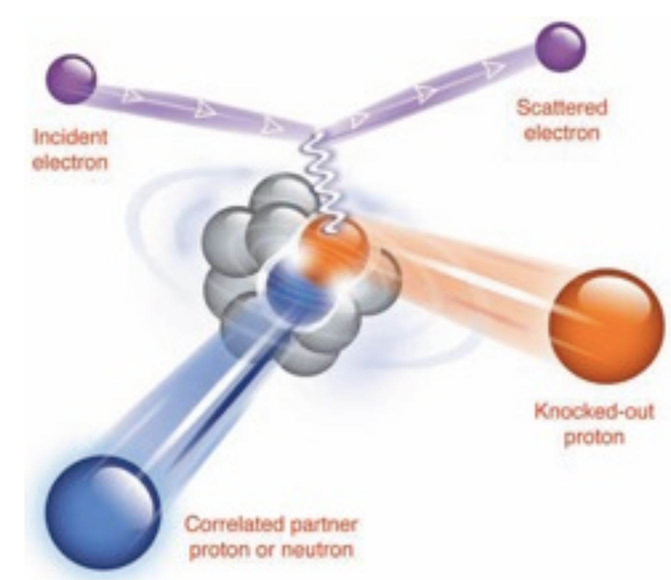
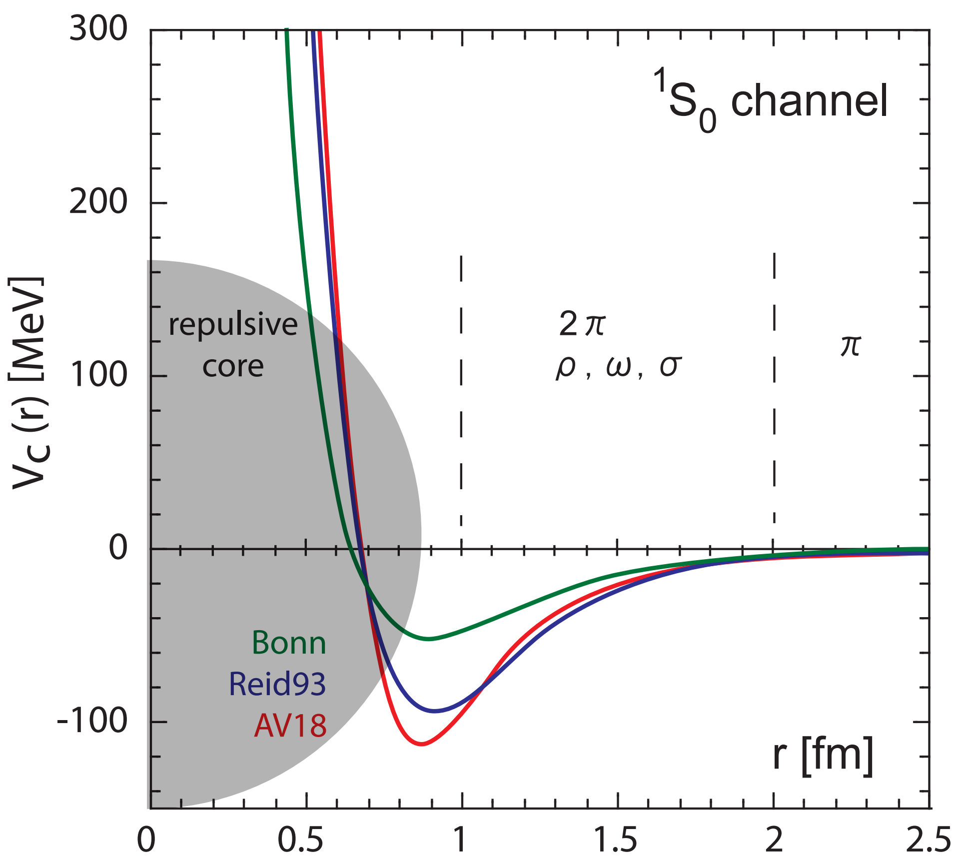
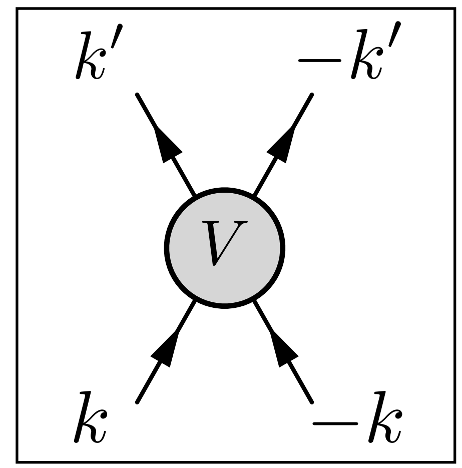
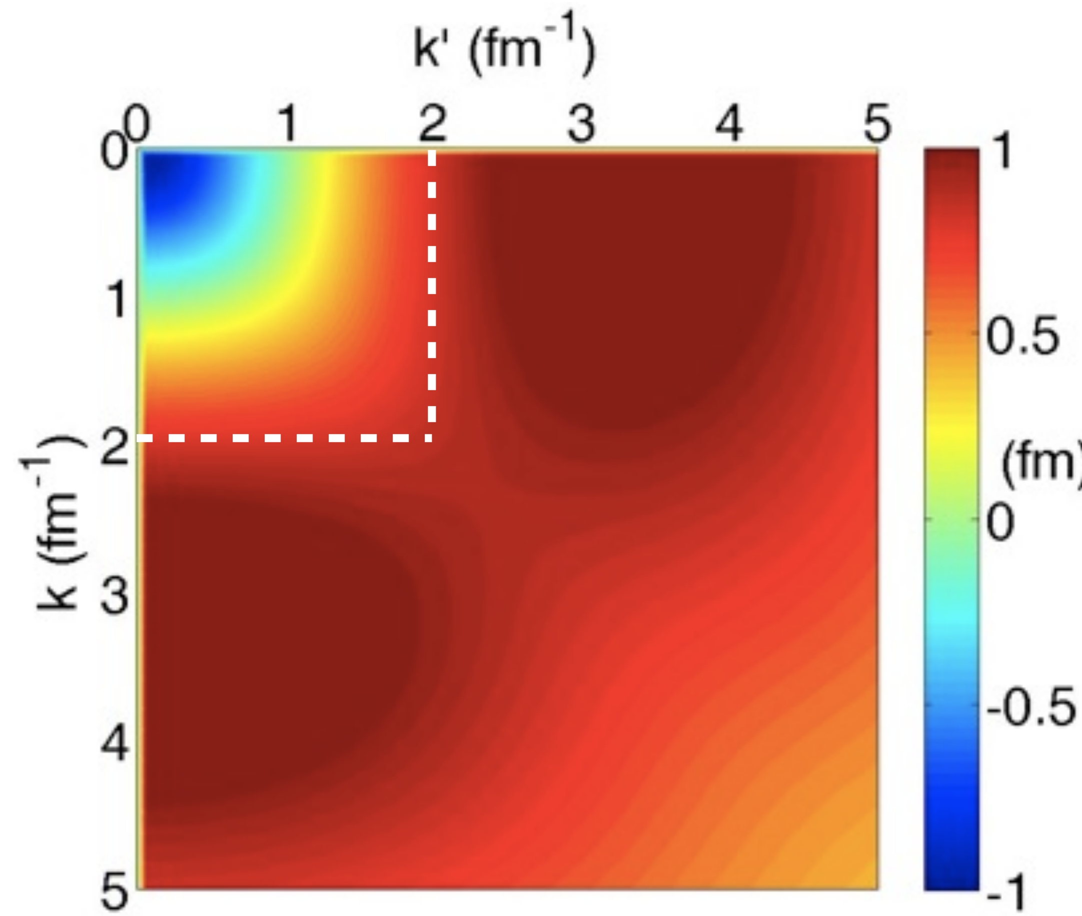


Problem: Traditional “hard” NN interactions





$$\langle k'|V|k\rangle \longrightarrow$$



- constructed to fit low-energy scattering data
- “hard” NN interactions contain repulsive core at small relative distance
- strong coupling between low and high-momentum components

$\Rightarrow$ nuclear many-body problem non-perturbative, hard to solve!

Claim:
Problems due to high resolution from interaction!

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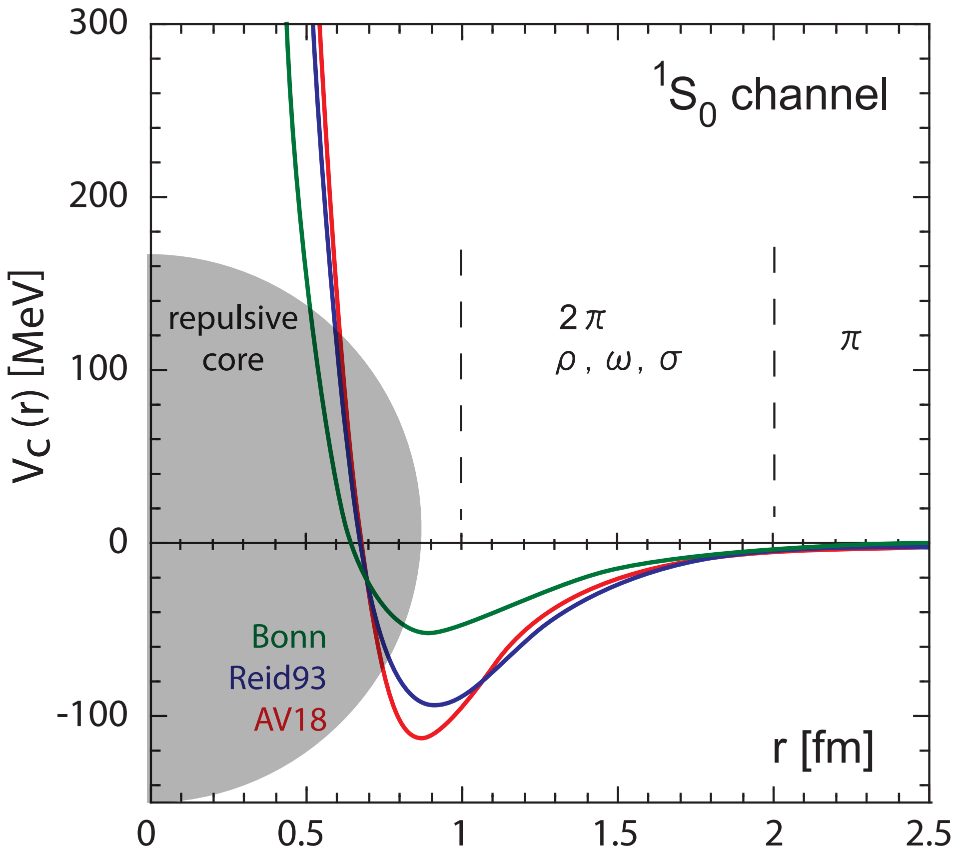

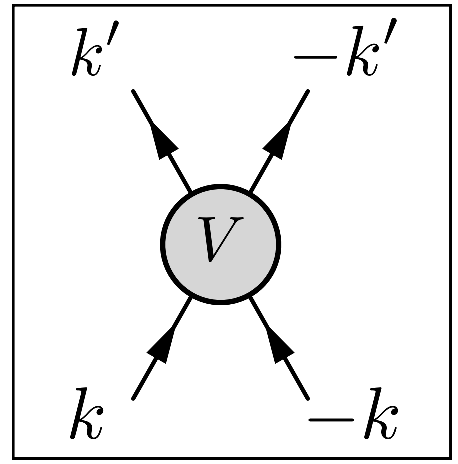

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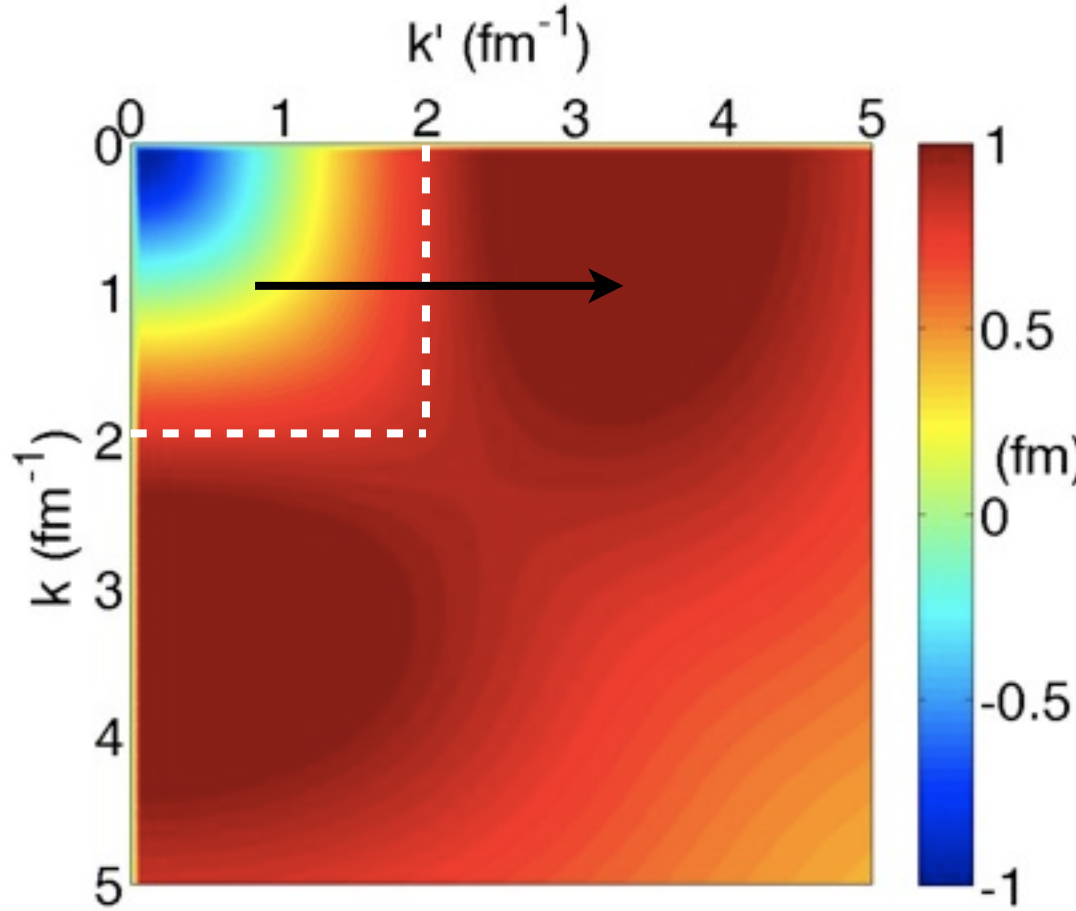

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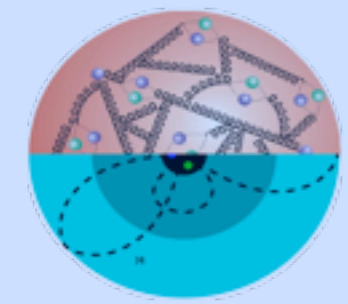
Resolution dependence of nuclear forces

QCD $\rightarrow$ Effective theory for NN, 3N, many-N interactions:

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{\mathrm{3N}}(\Lambda) + V_{\mathrm{4N}}(\Lambda) + \ldots$$

$\Lambda \gg \Lambda_{\mathrm{chiral}}$

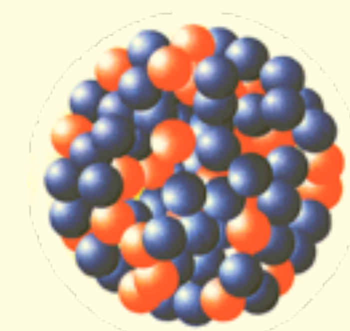
quarks+gluons/partons: $Q \gg m_\pi$



$\Lambda_{\mathrm{chiral}}$

typical momenta in nuclei: $Q \sim m_\pi$

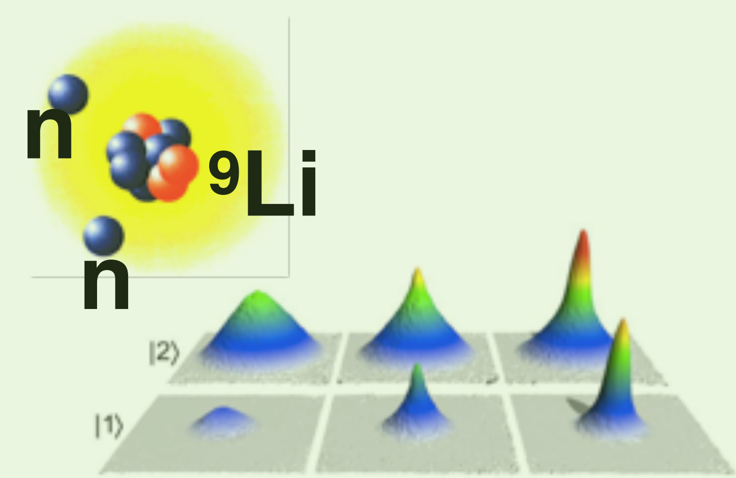
chiral EFT: nucleons interacting via pion exchanges and short-range contact interactions



$\Lambda_{\mathrm{pionless}}$

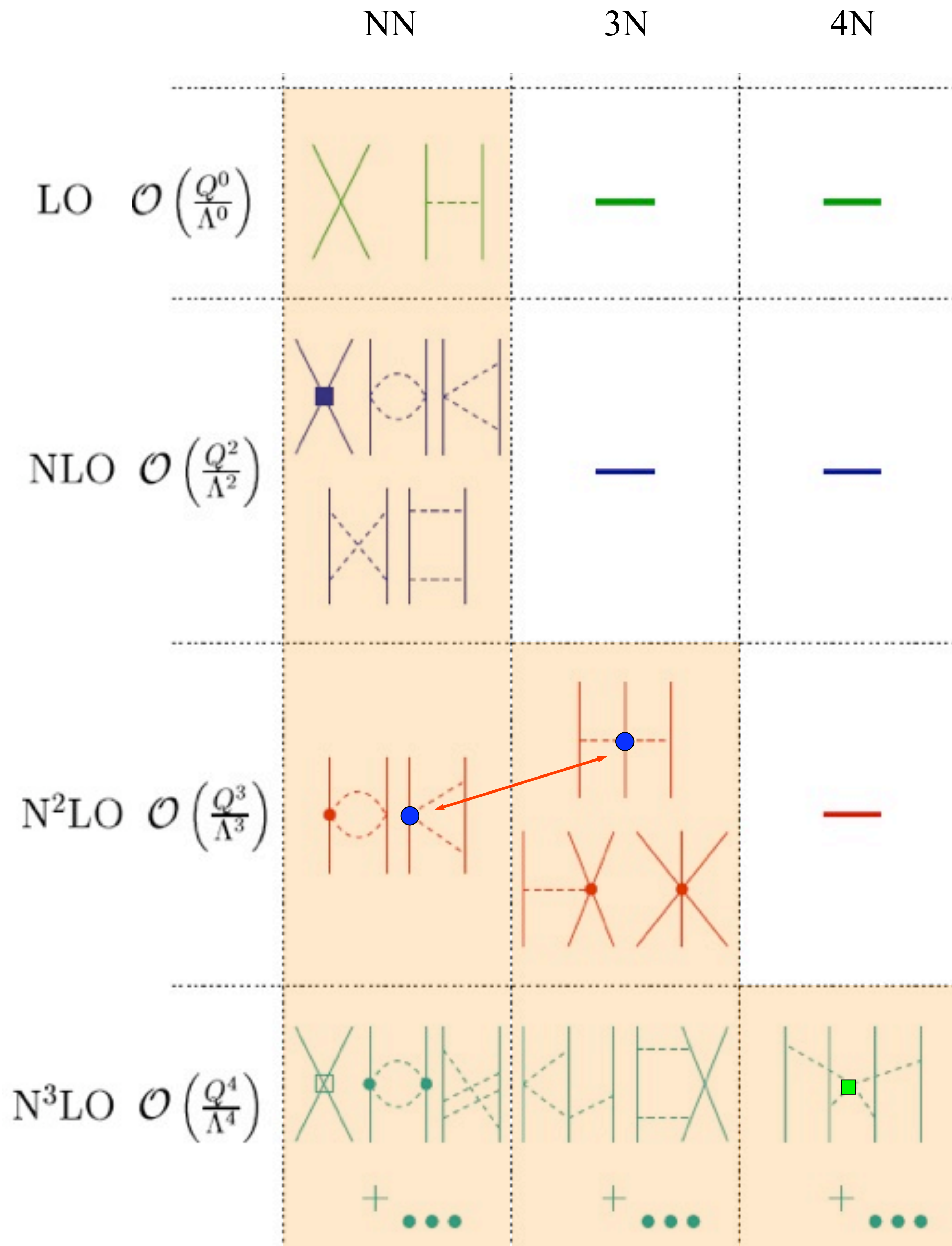
large scattering length physics: $Q \ll m_\pi$

pionless EFT: unitary regime, non-universal corrections

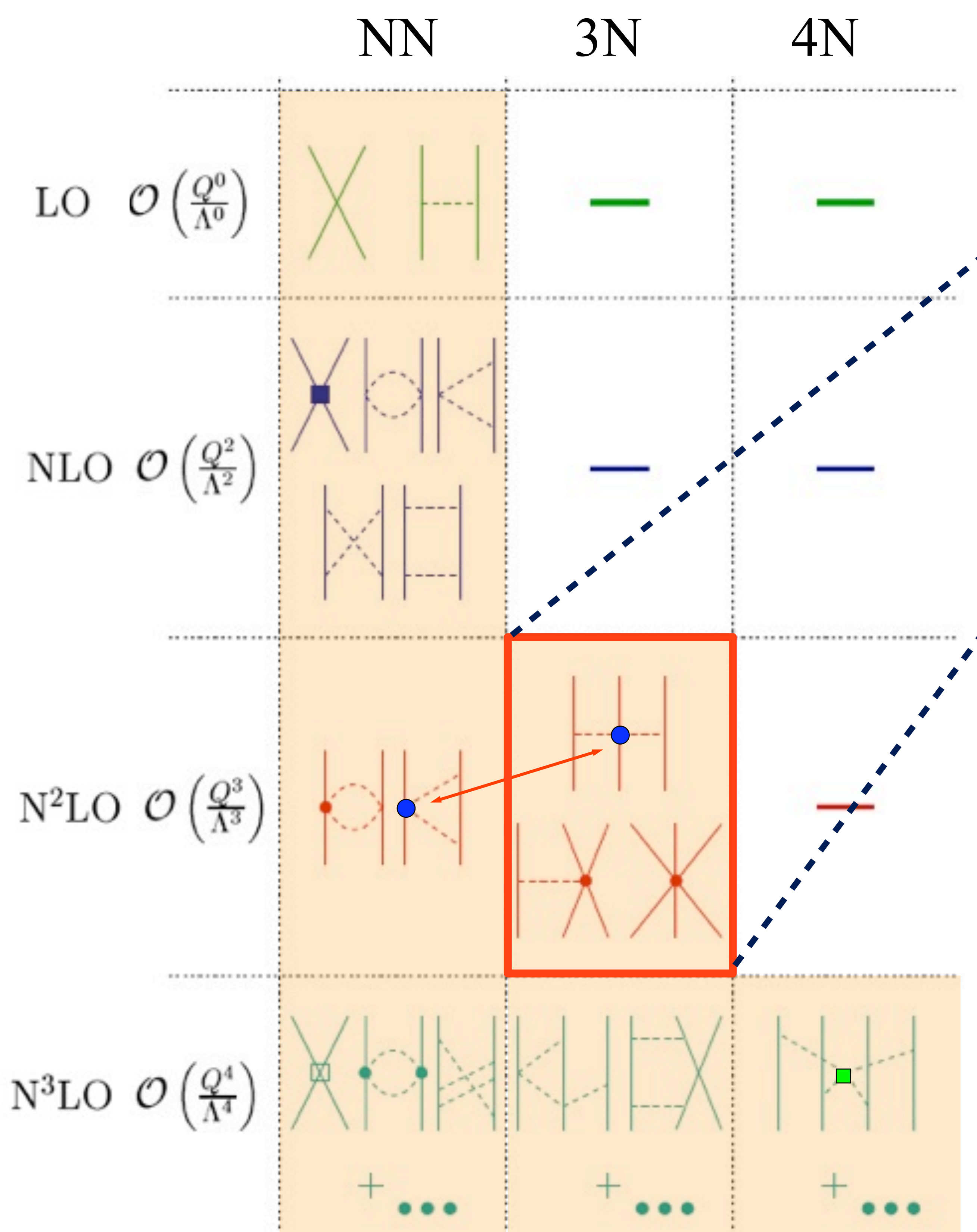


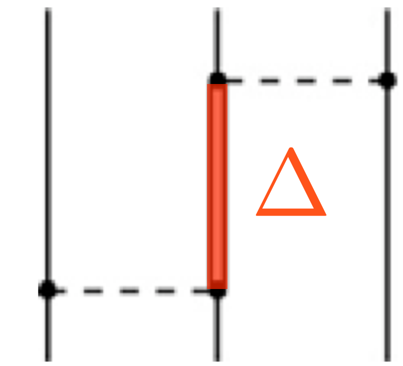
Basic concepts of chiral effective field theory

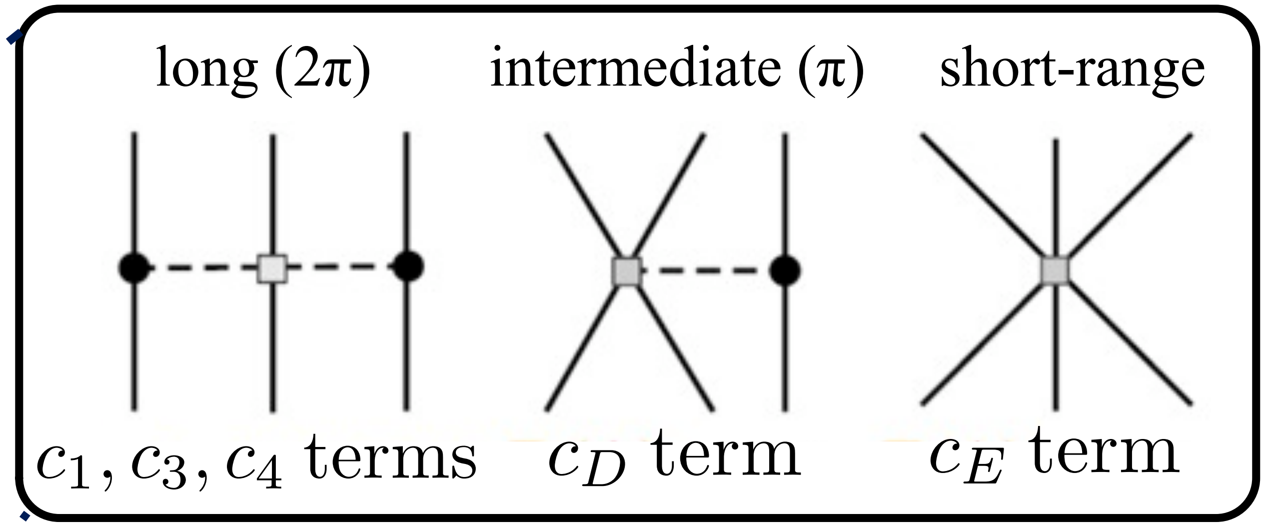
- choose effective degrees of freedom: here nucleons and pions
- short-range physics captured in few short-range couplings
- separation of scales: Q << Λ_b, breakdown scale Λ_b~500 MeV
- power-counting: expand in powers Q/Λ_b
- systematic: work to desired accuracy, obtain error estimates



Leading order chiral 3N forces







- large uncertainties in 2π coupling constants at present:

$$c_1 = -0.9^{+0.2}_{-0.5}\ ,\ c_3 = -4.7^{+1.5}_{-1.0}\ ,\ \ c_4 = 3.5^{+0.5}_{-0.2}$$

leads to theoretical uncertainties in many-body observables

- c_D and c_E have to be determined in $A \geq 3$ systems

Changing the resolution: The (Similarity) Renormalization Group

- goal: generate unitary transformation of "hard" Hamiltonian

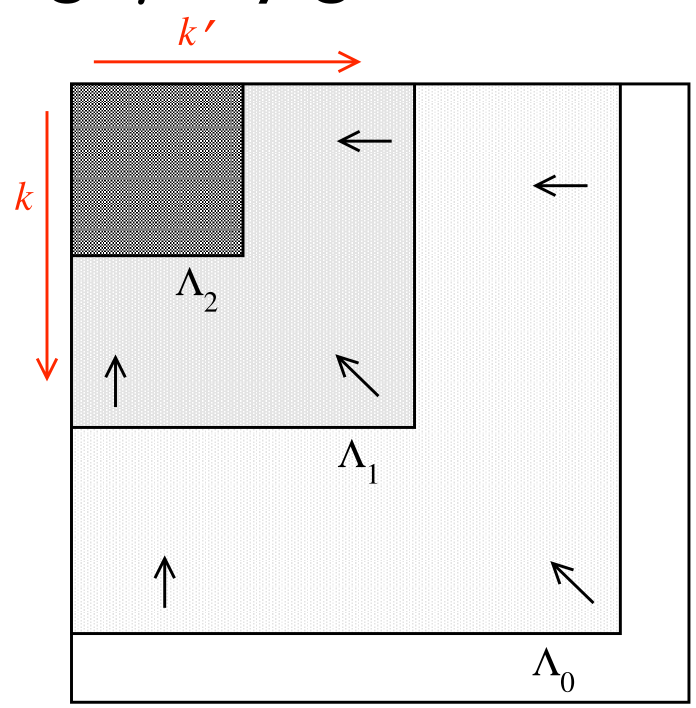
$$H_\lambda = U_\lambda H U_\lambda^\dagger \quad \text{with the resolution parameter } \lambda$$

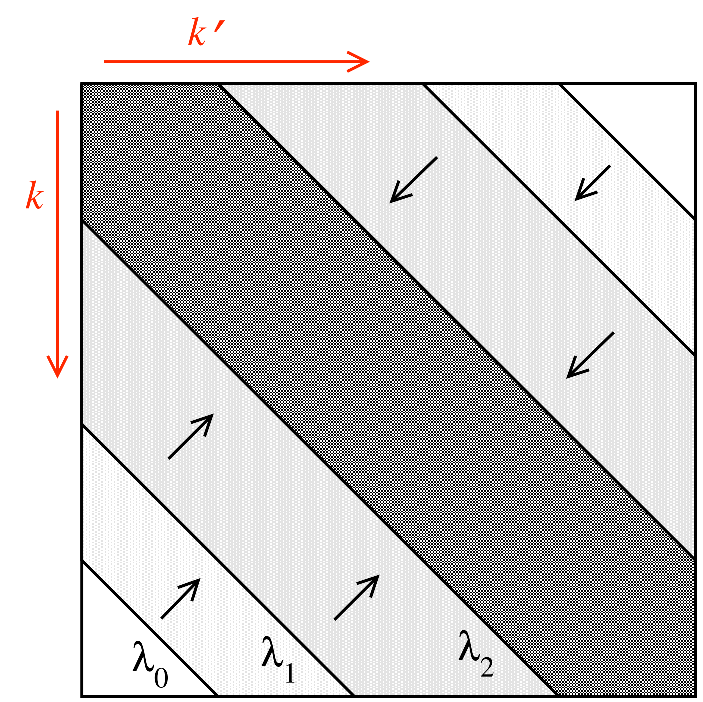
- change resolution in small steps: $$\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$$

- transformed wave functions and operators

$$|\psi_\lambda\rangle = U_\lambda |\psi\rangle \qquad O_\lambda = U_\lambda O U_\lambda^\dagger \quad \Rightarrow \quad \langle\psi| O |\psi\rangle = \langle\psi_\lambda| O_\lambda |\psi_\lambda\rangle$$

- specifying η_λ by generator G_λ: $\quad \eta_\lambda = [G_\lambda, H_\lambda]$

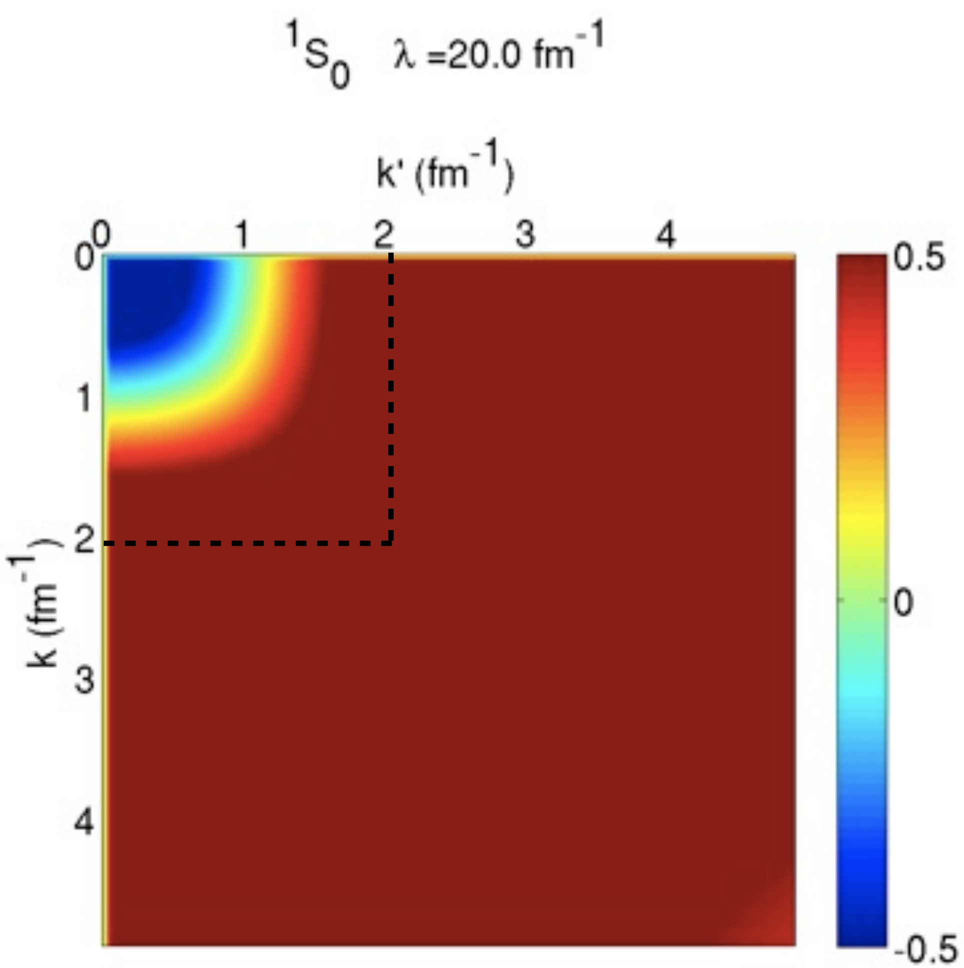


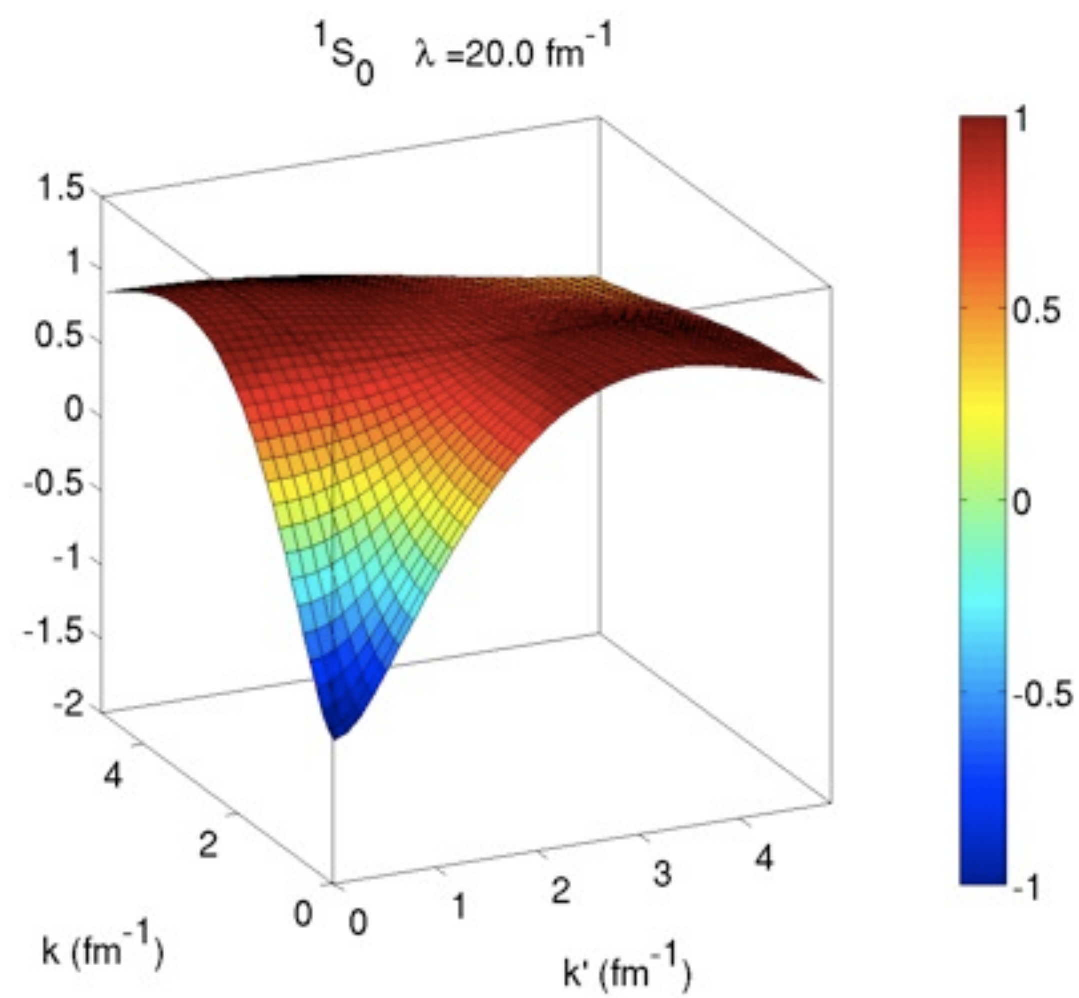


Changing the resolution: The (Similarity) Renormalization Group

- common choice for generator

  relative kinetic energy operator $G_\lambda = T$:





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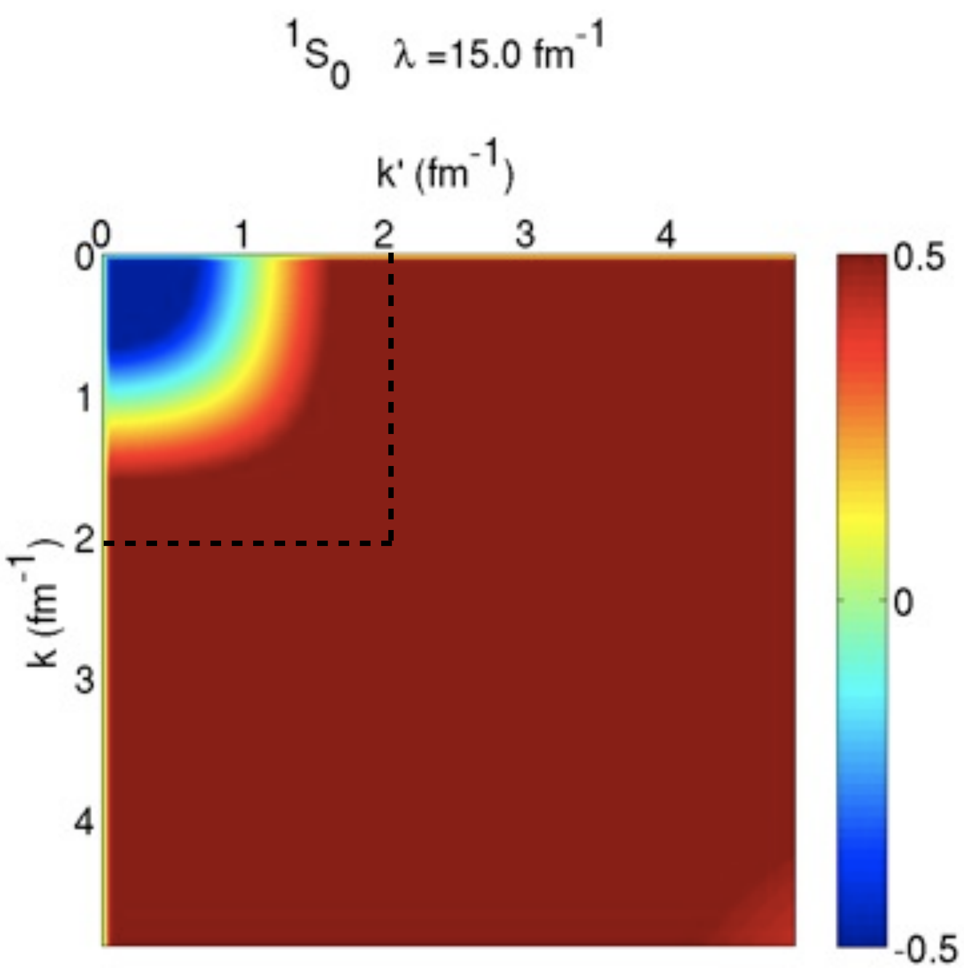

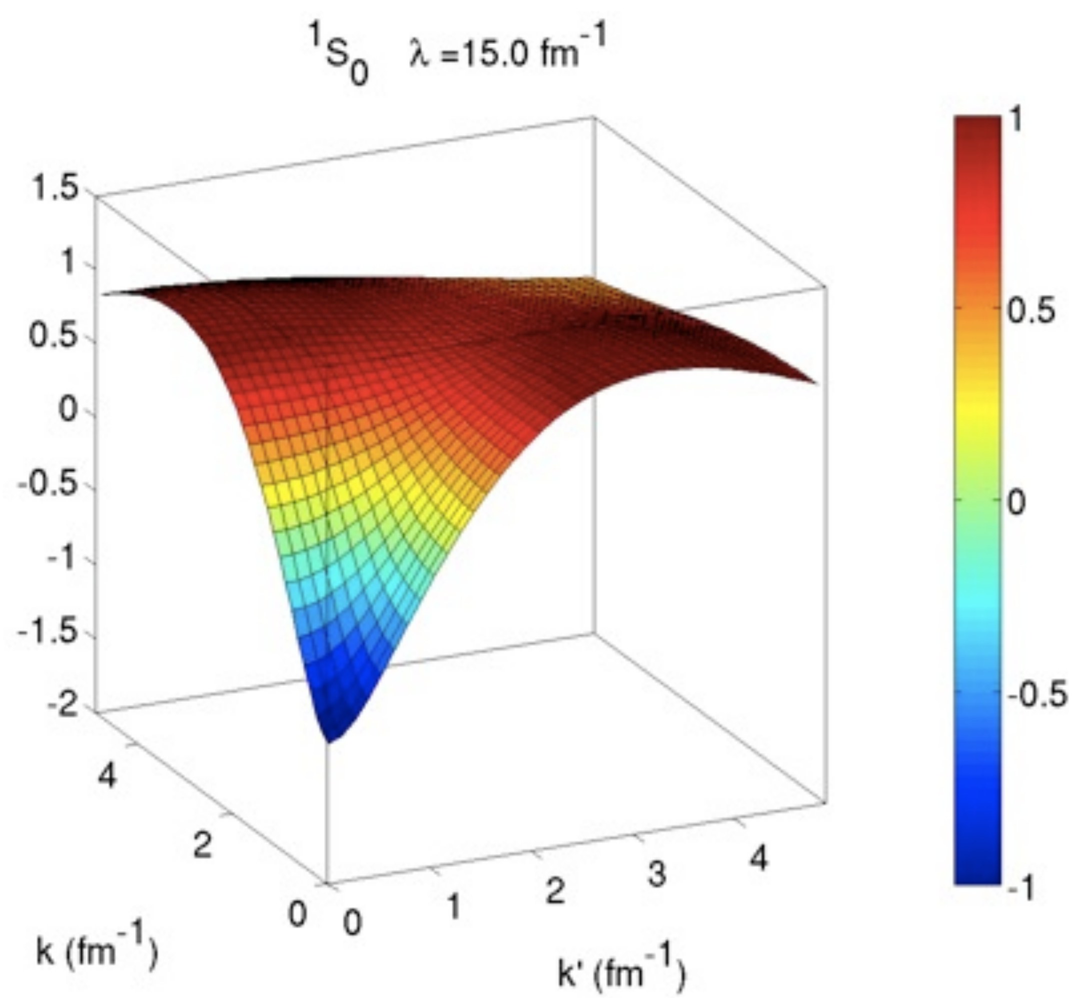

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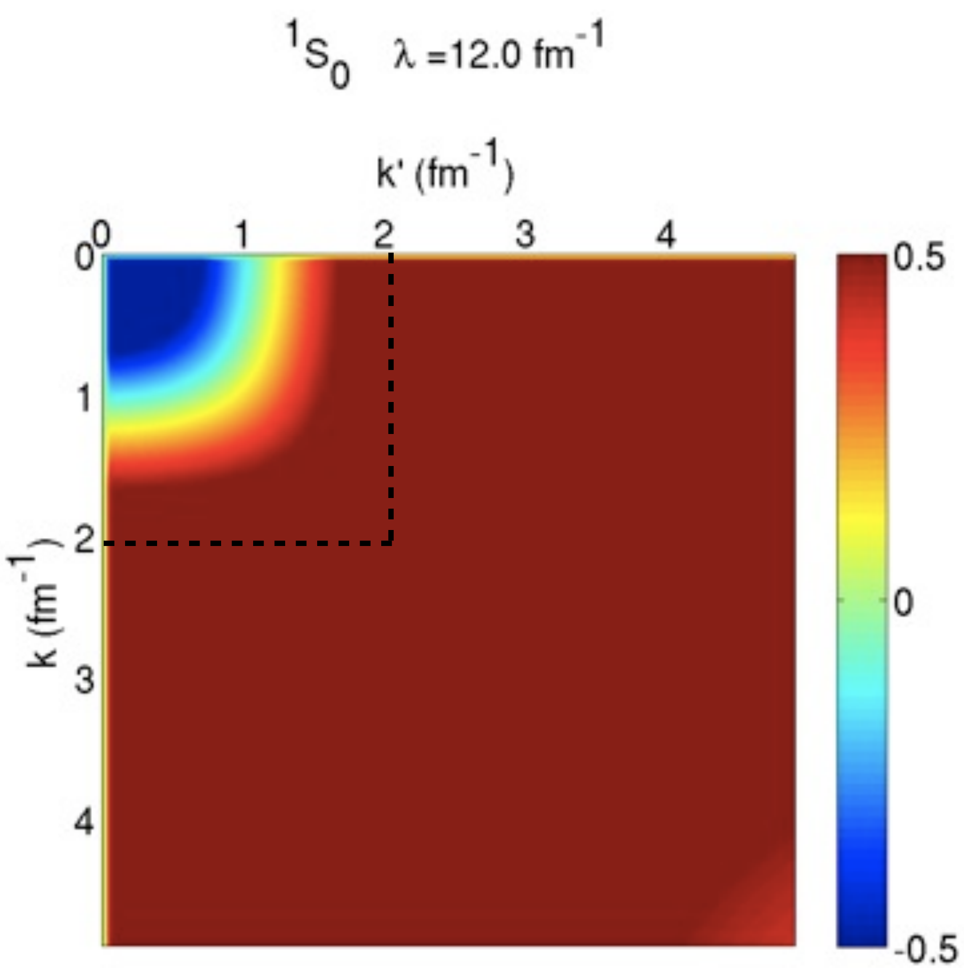

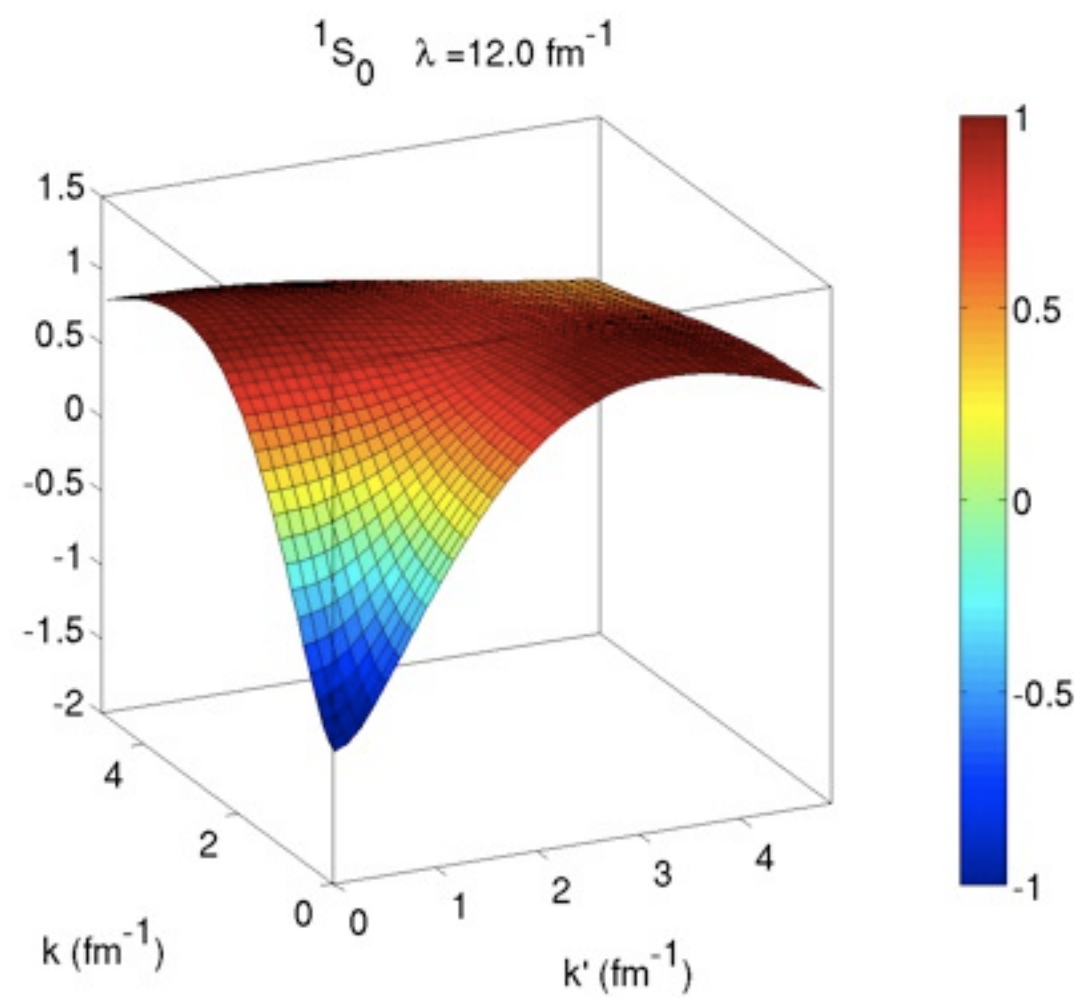

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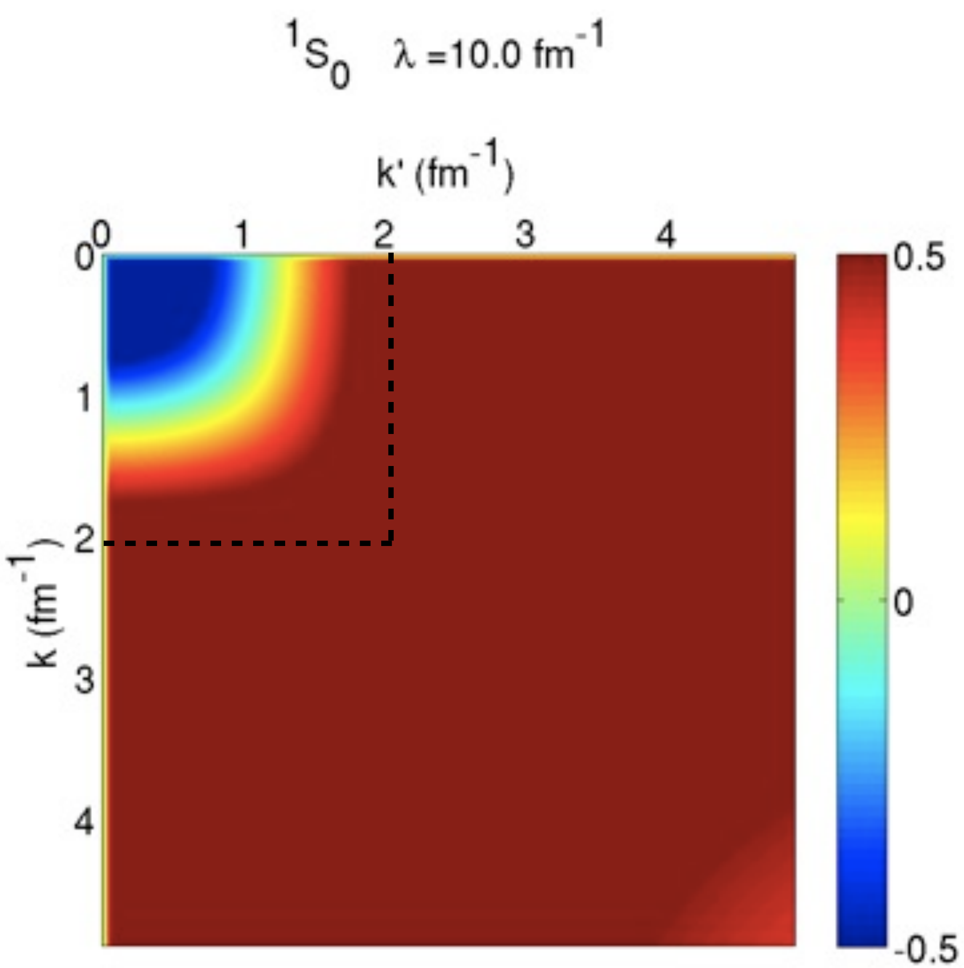

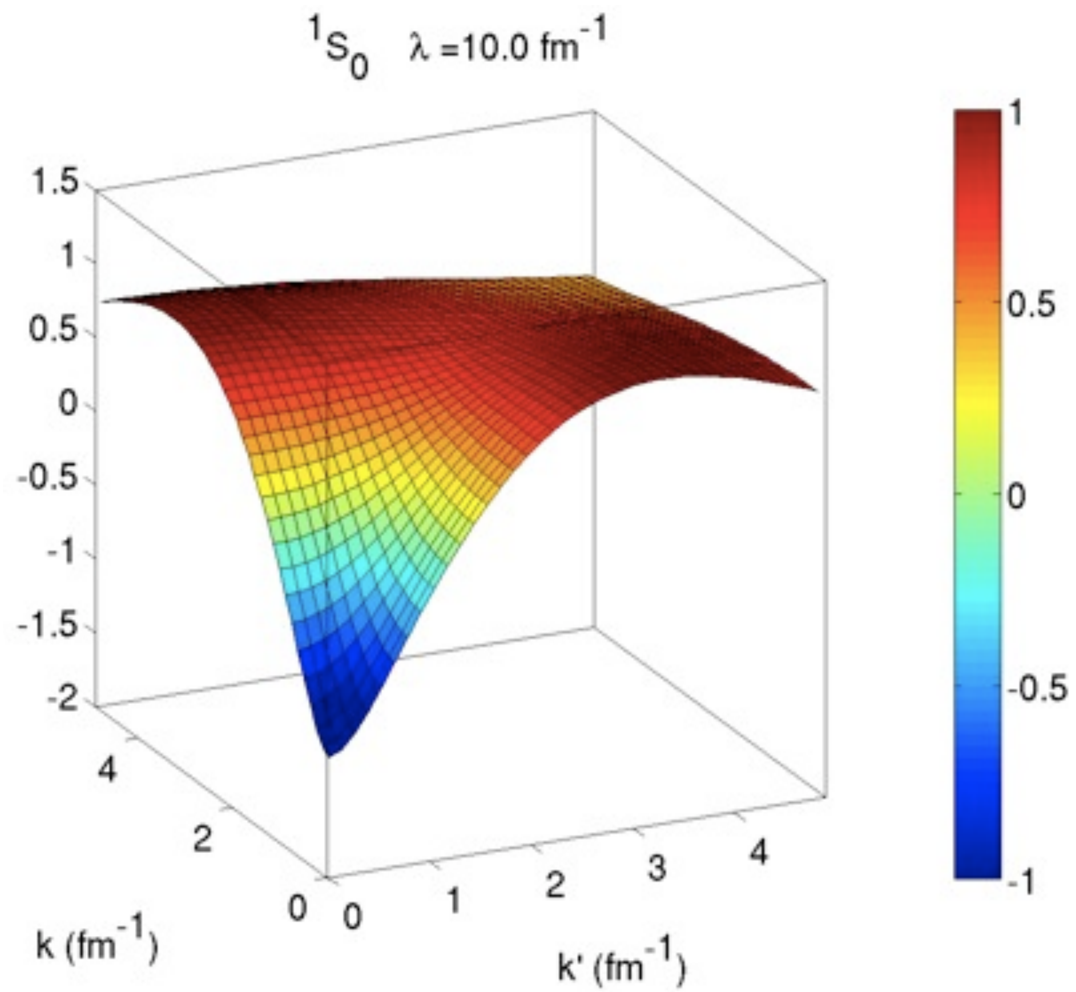

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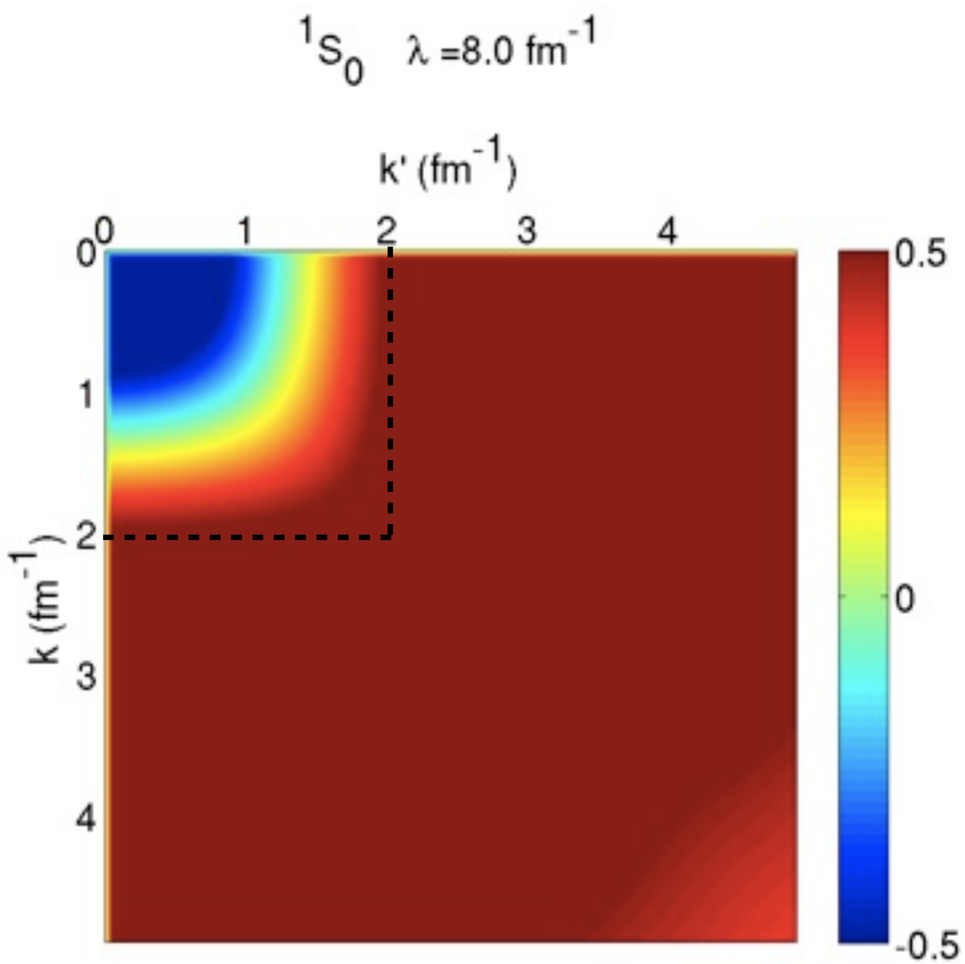

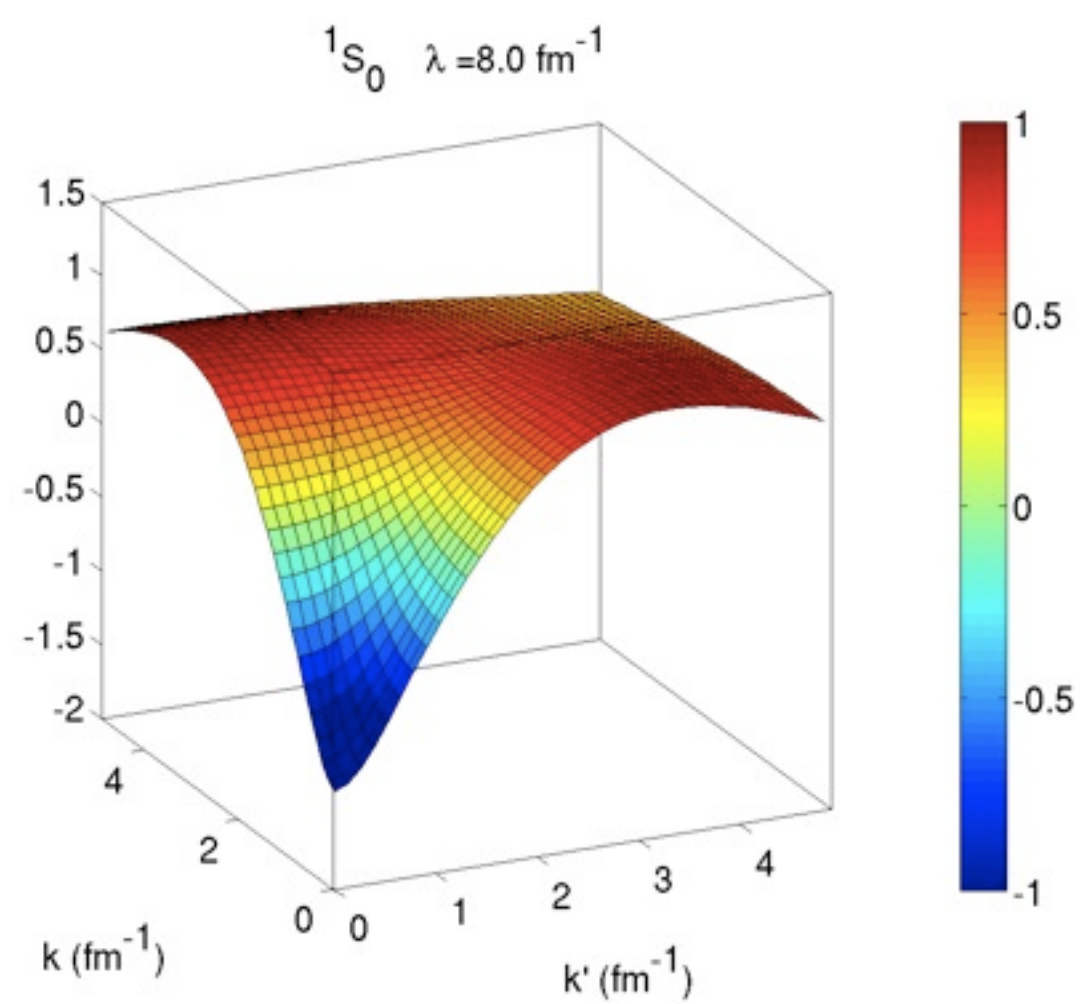

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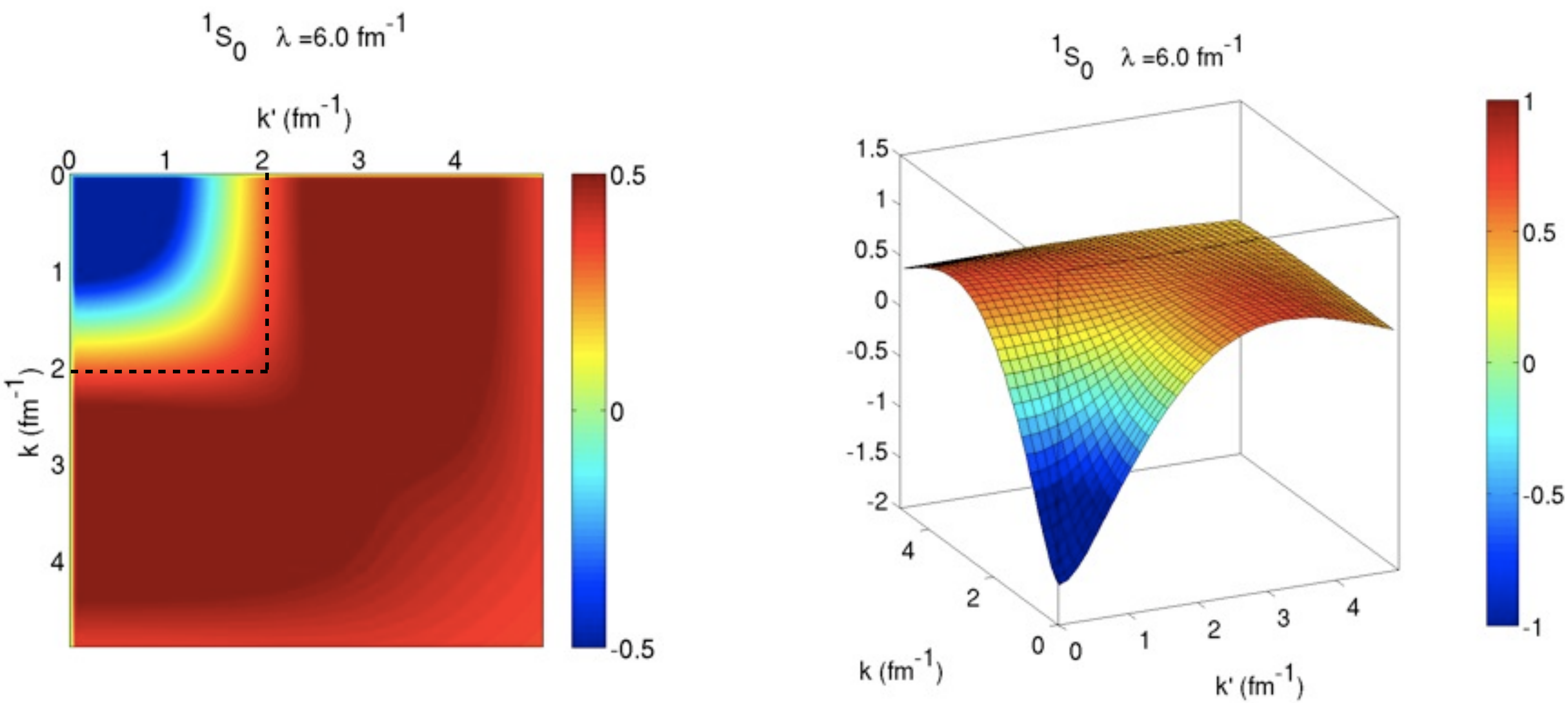

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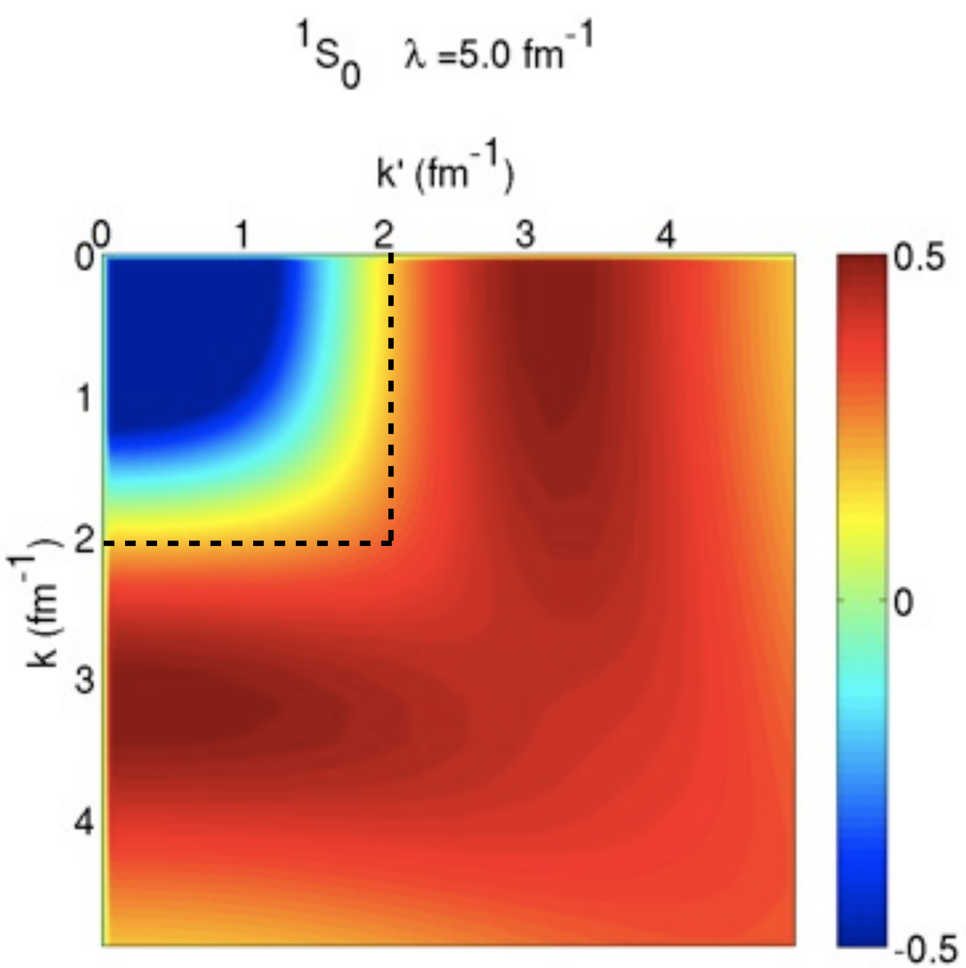

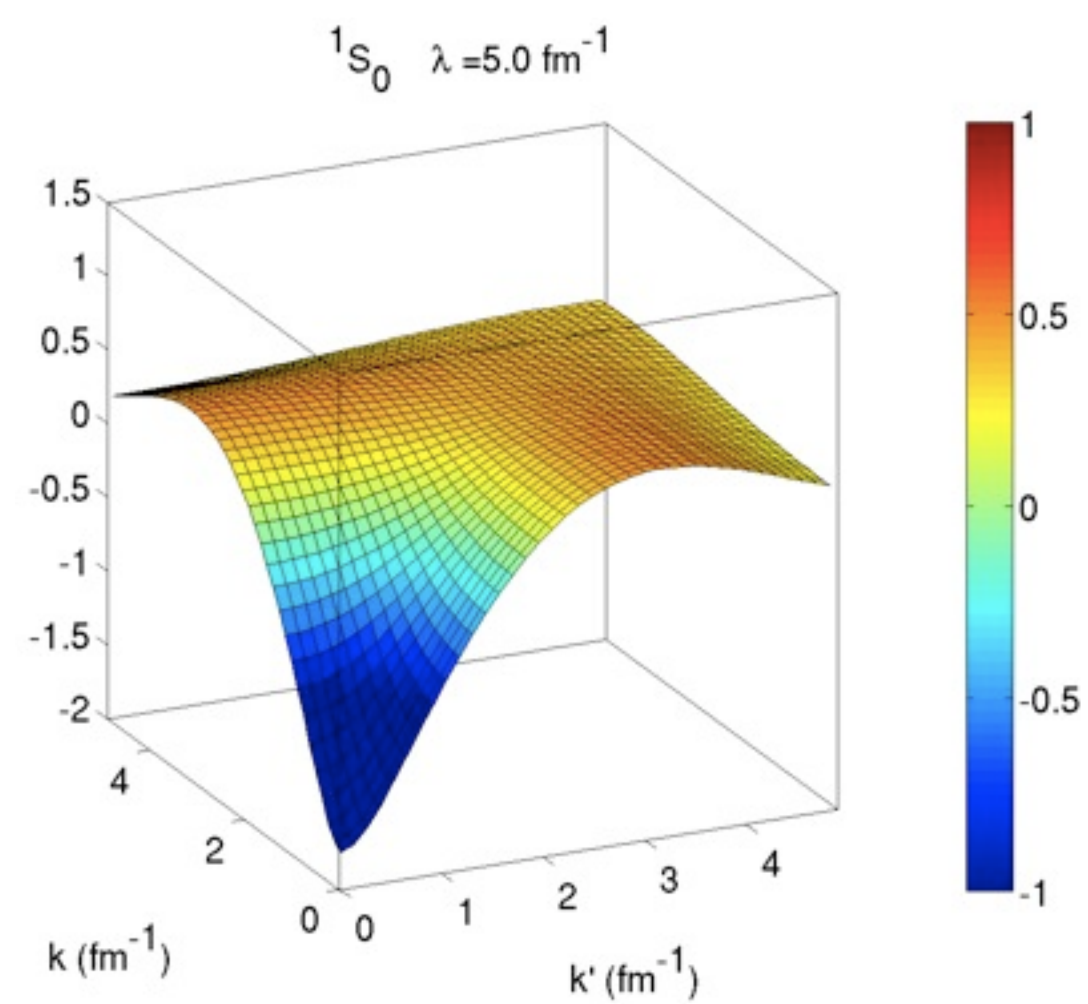

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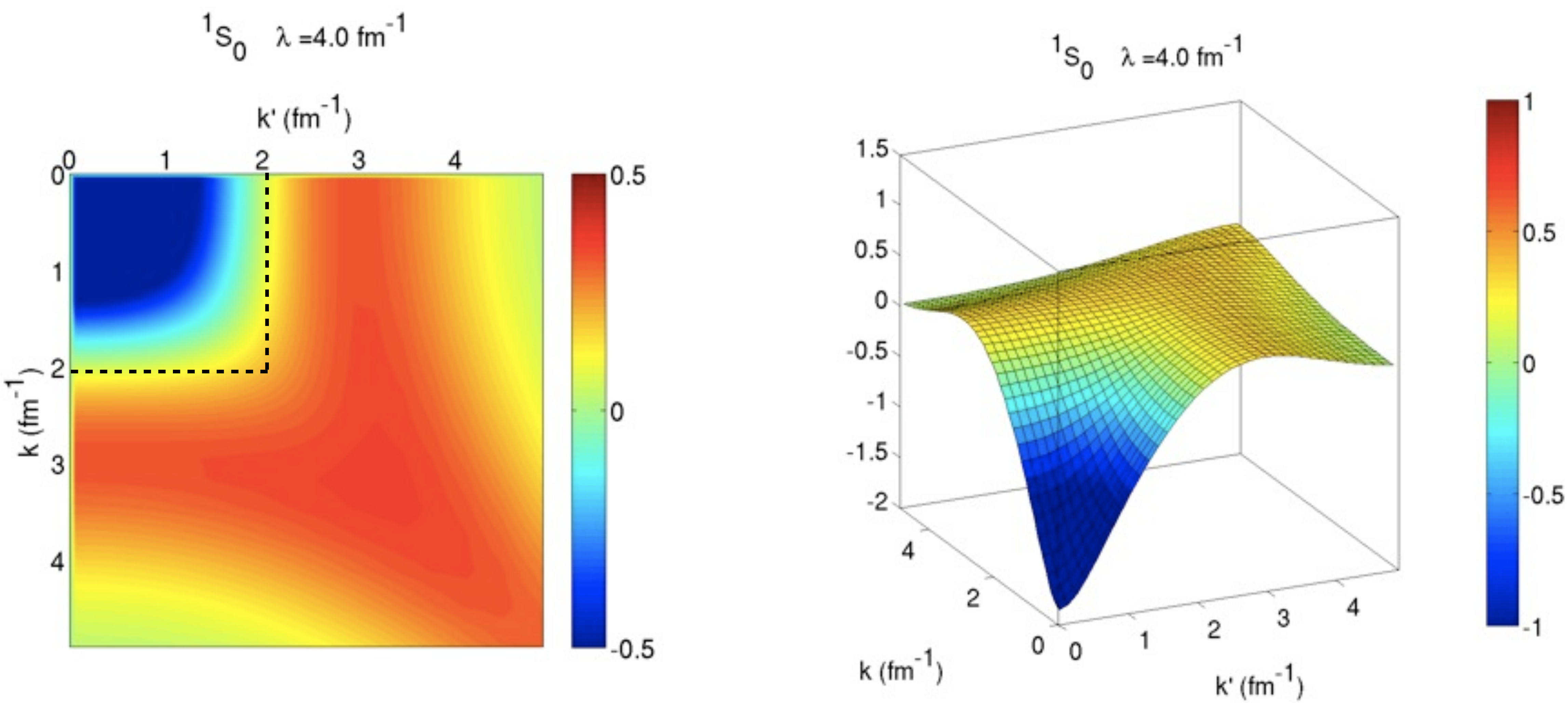

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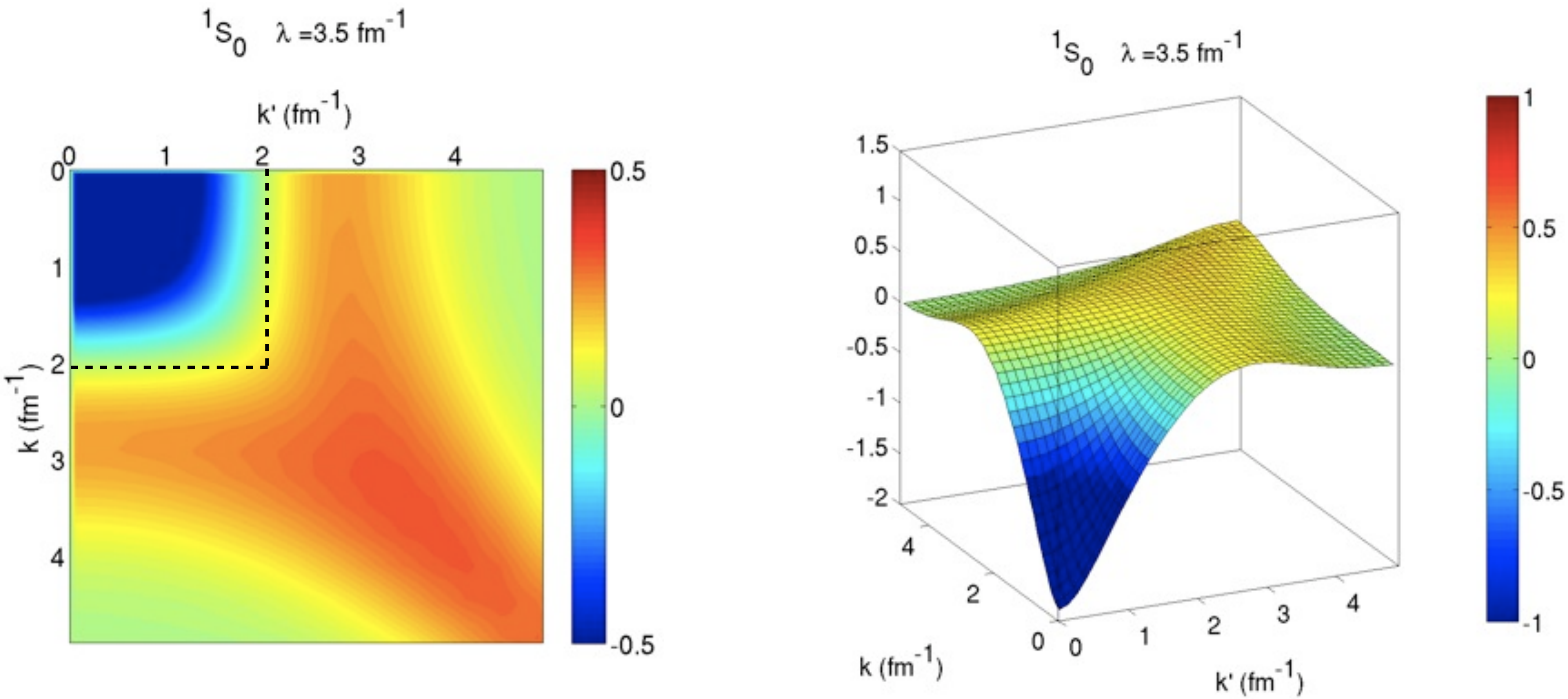

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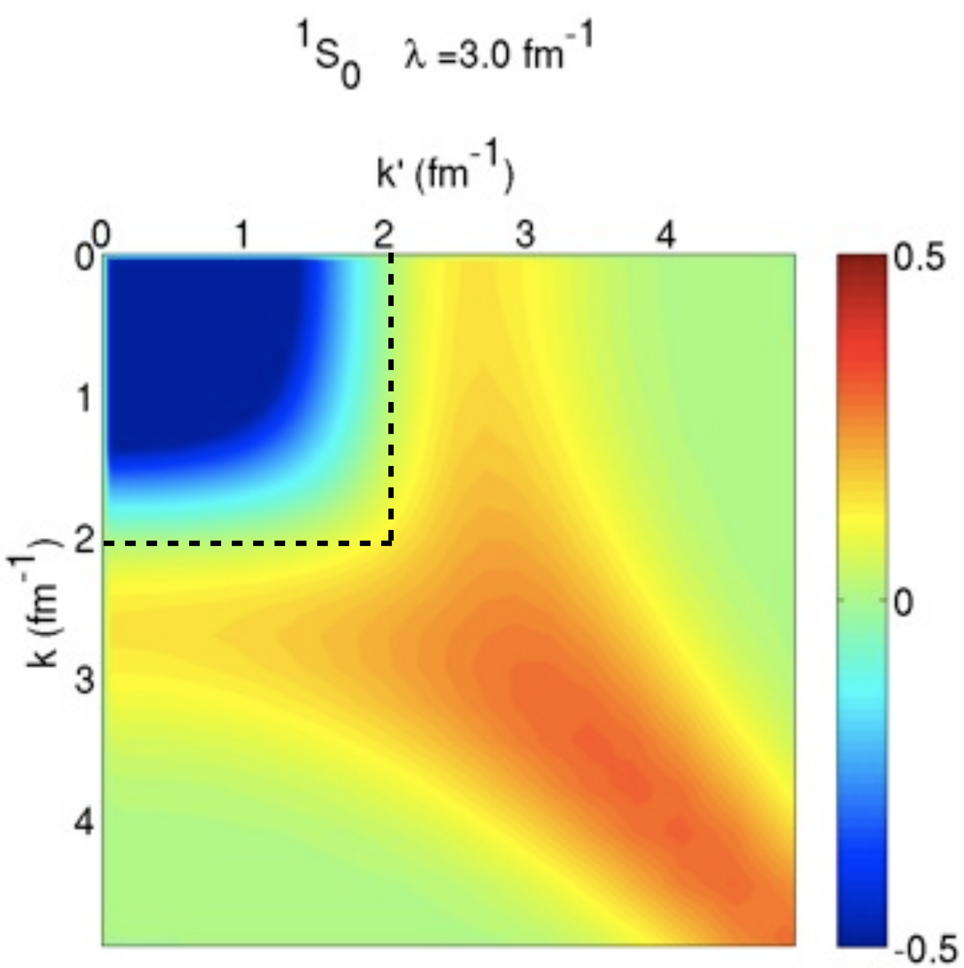

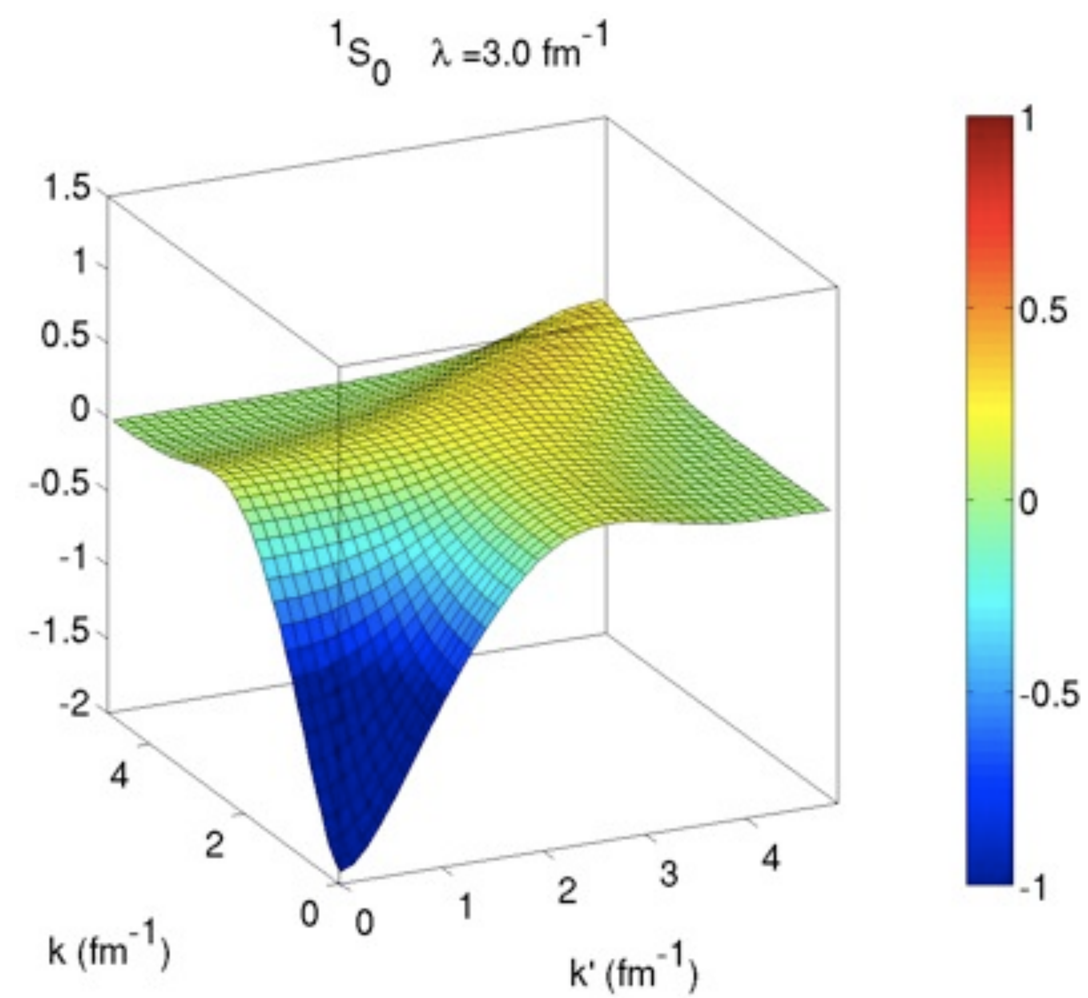

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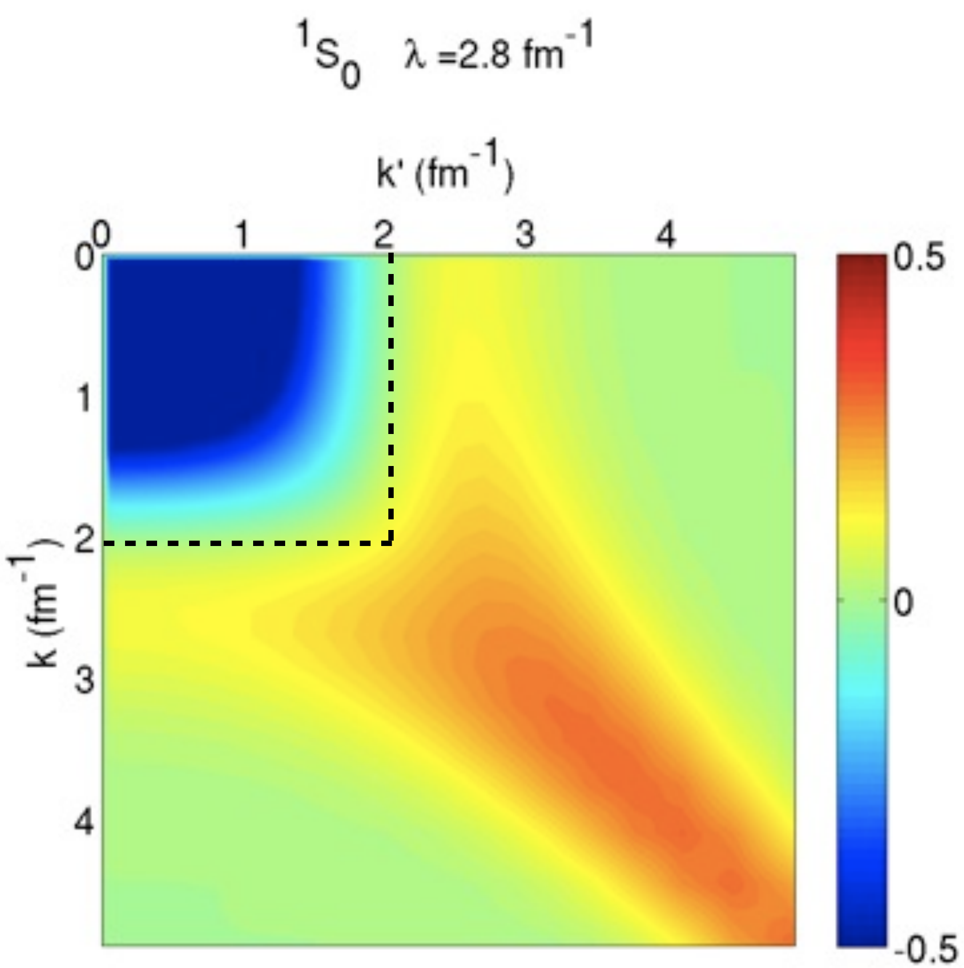

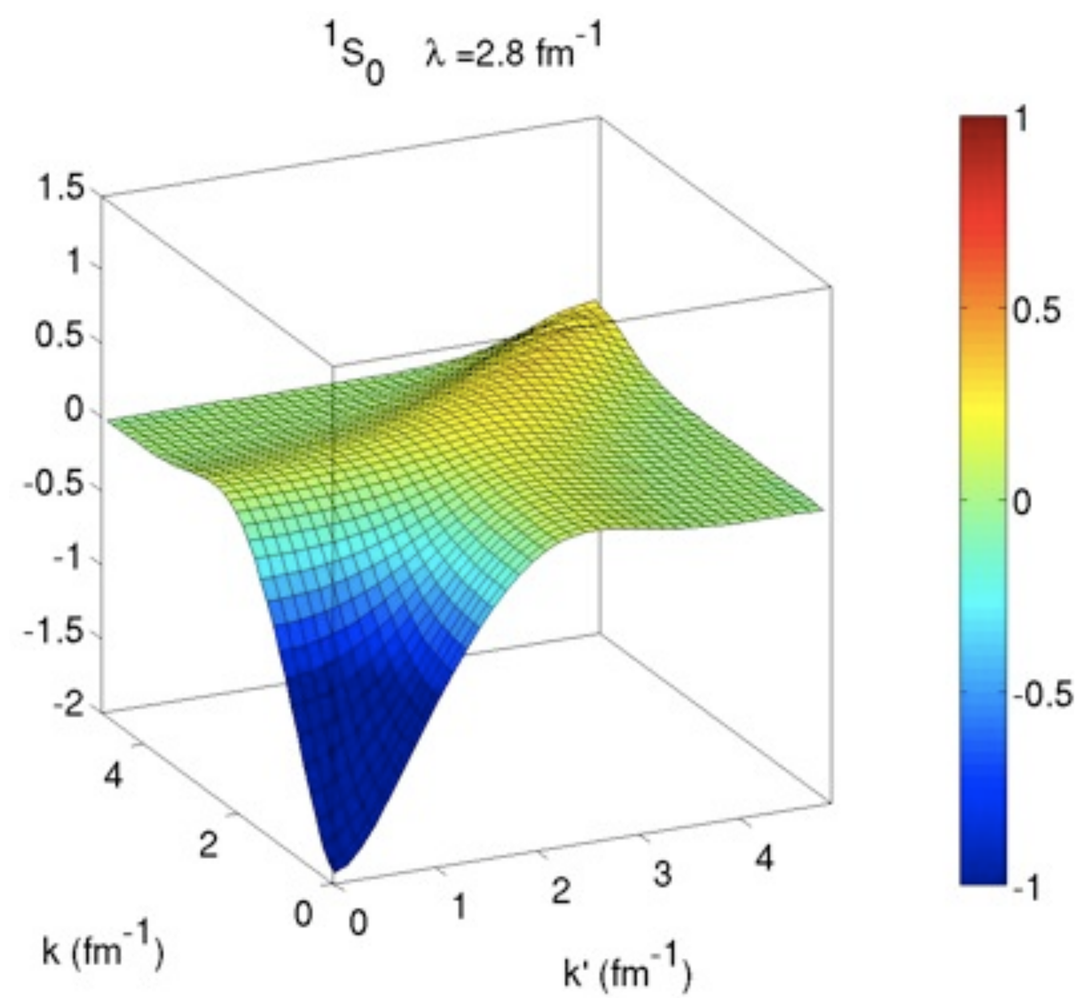

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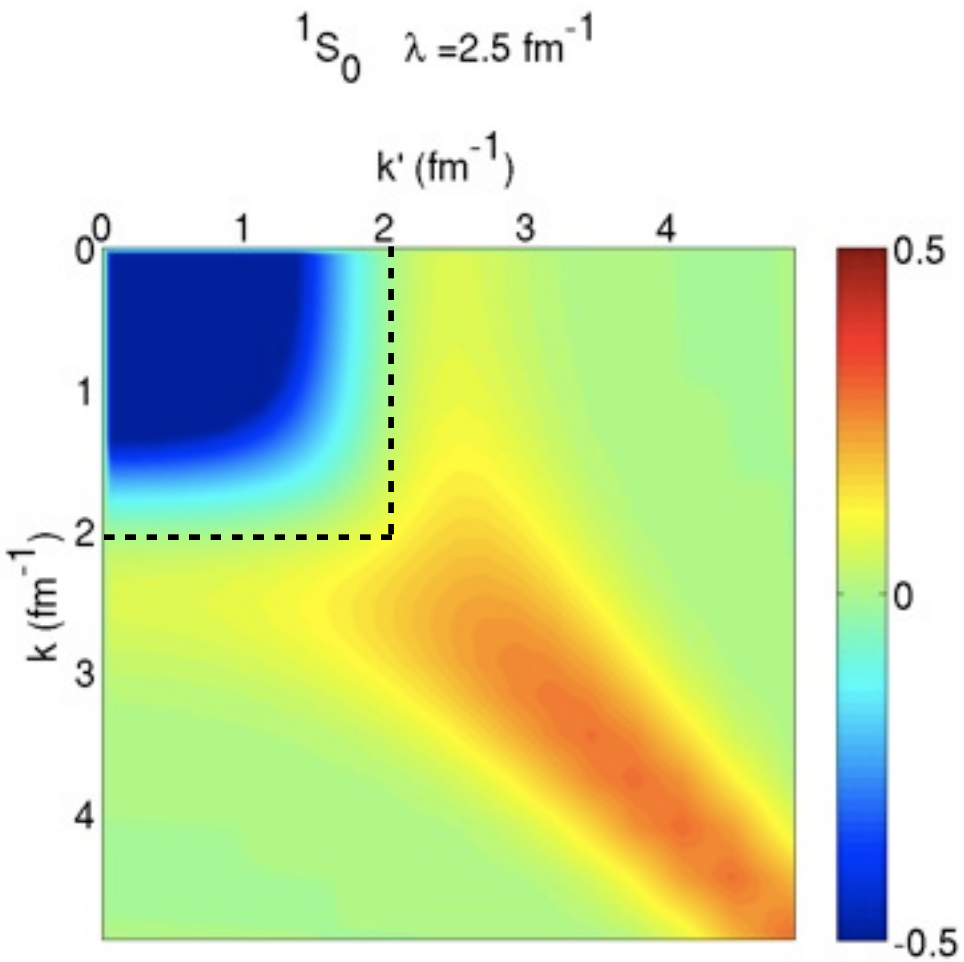

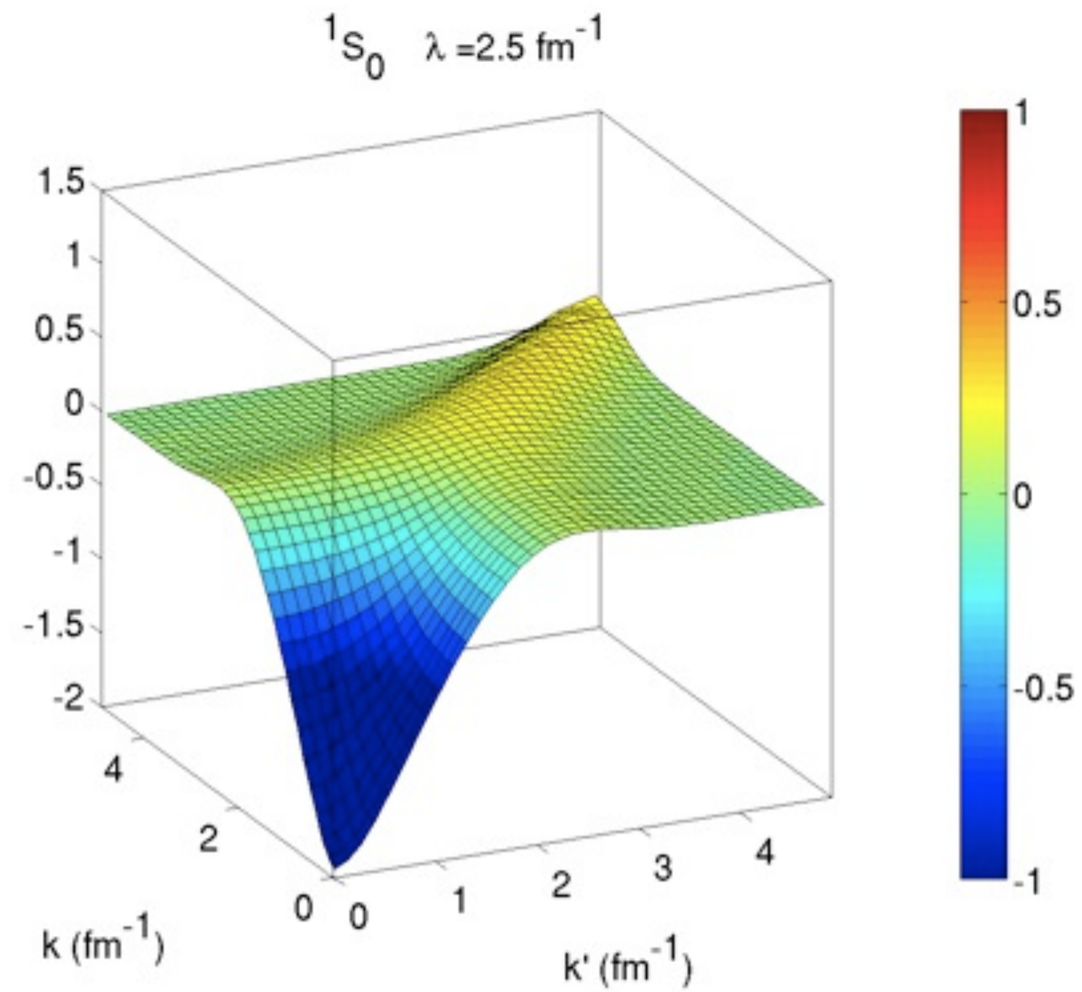

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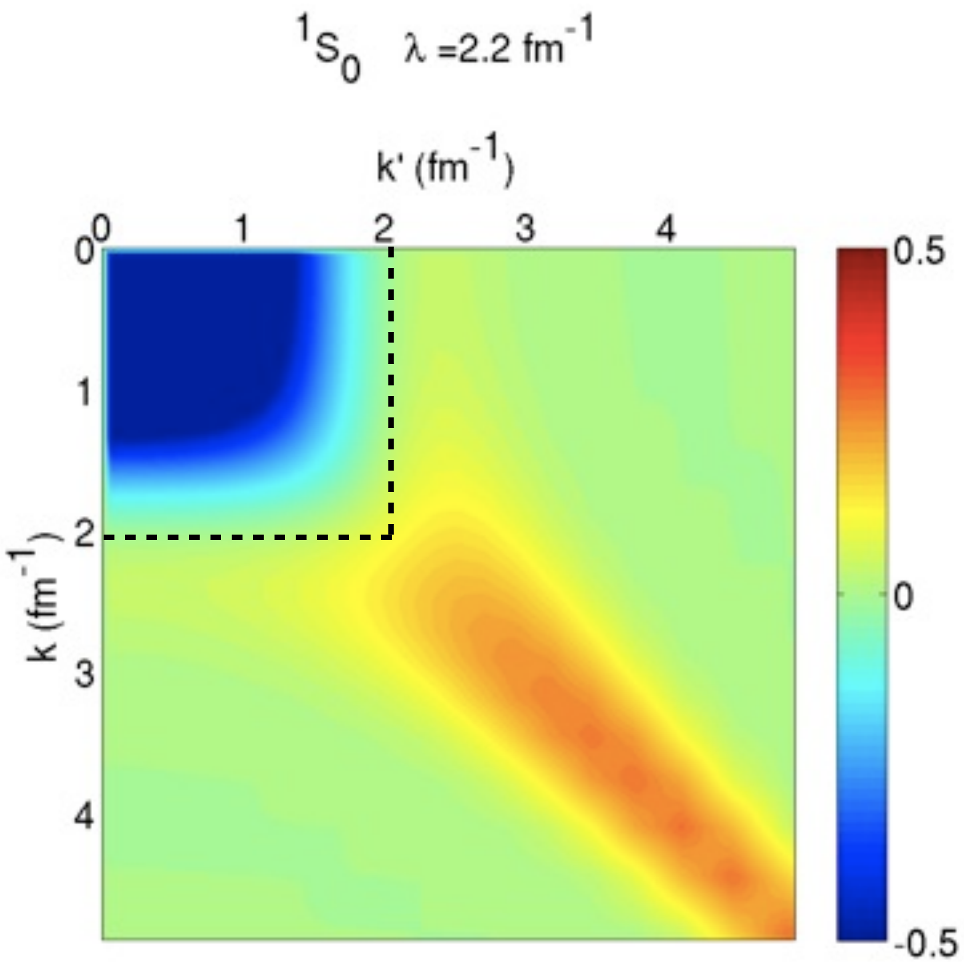

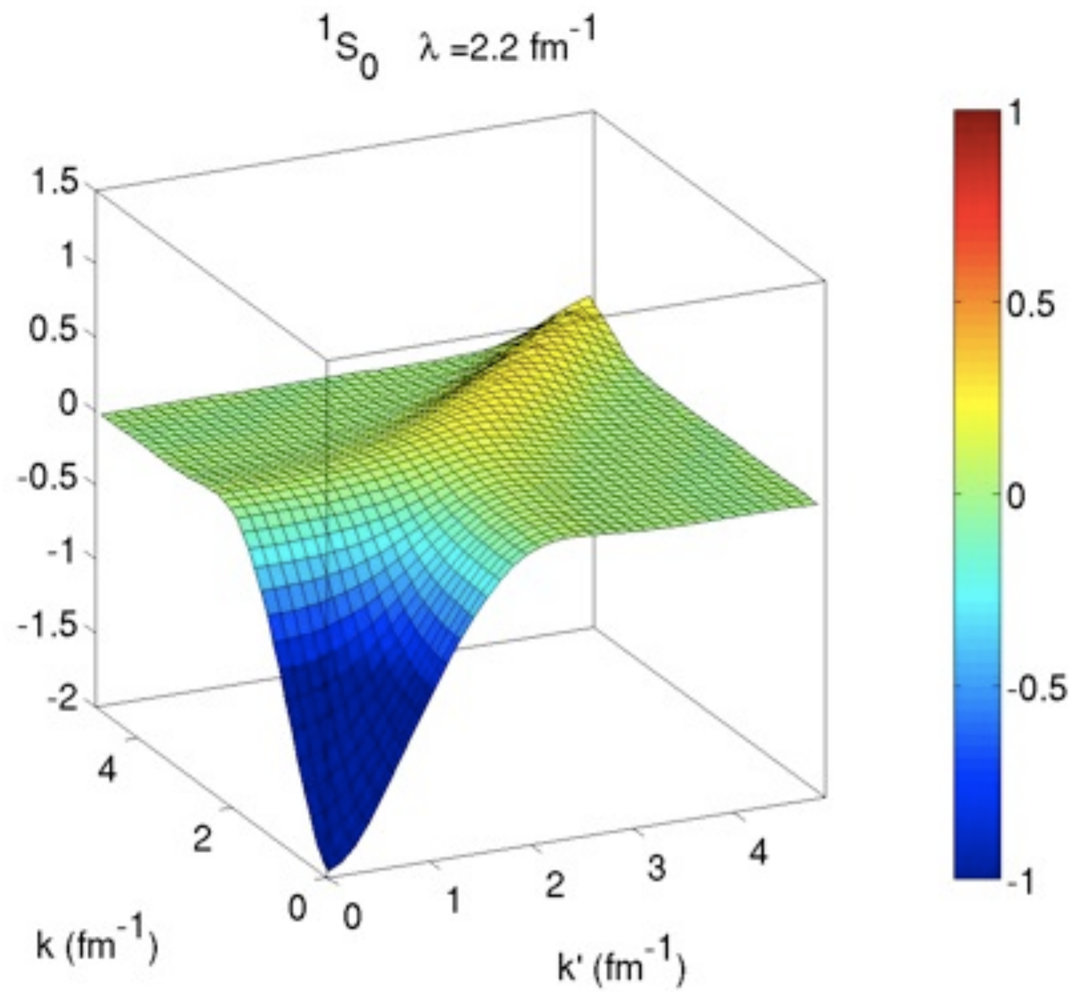

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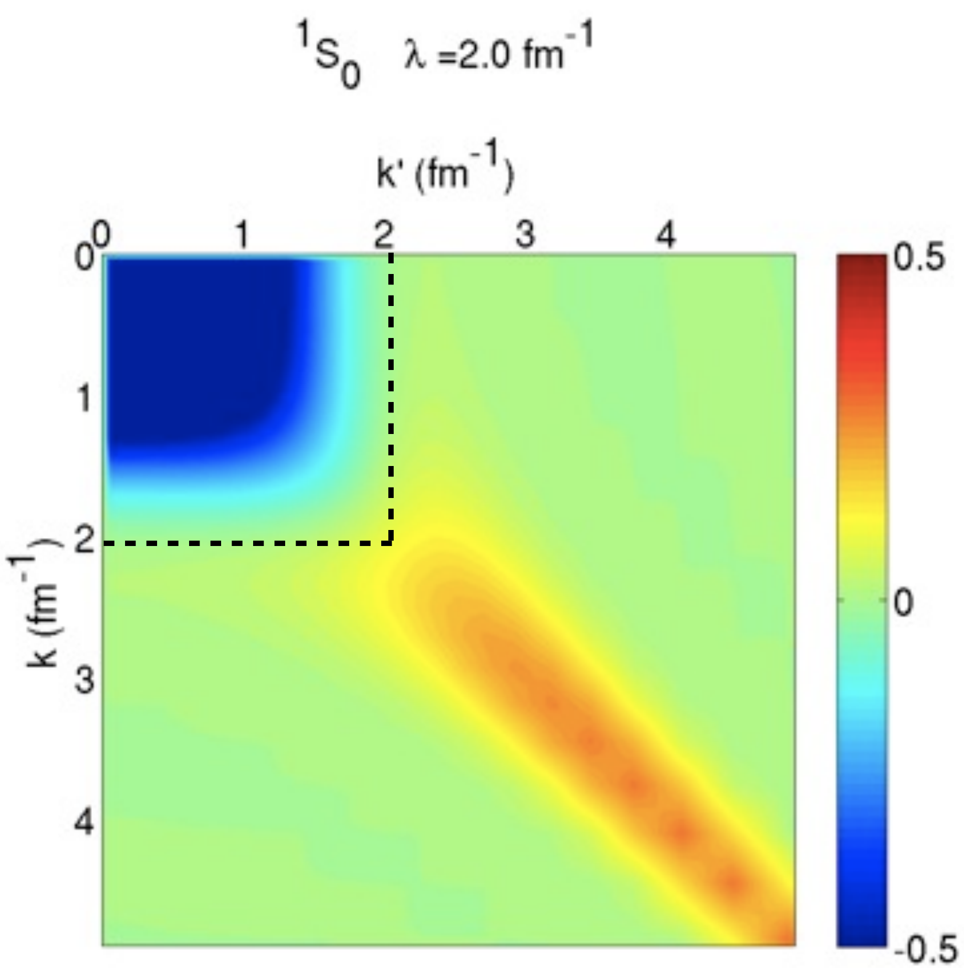

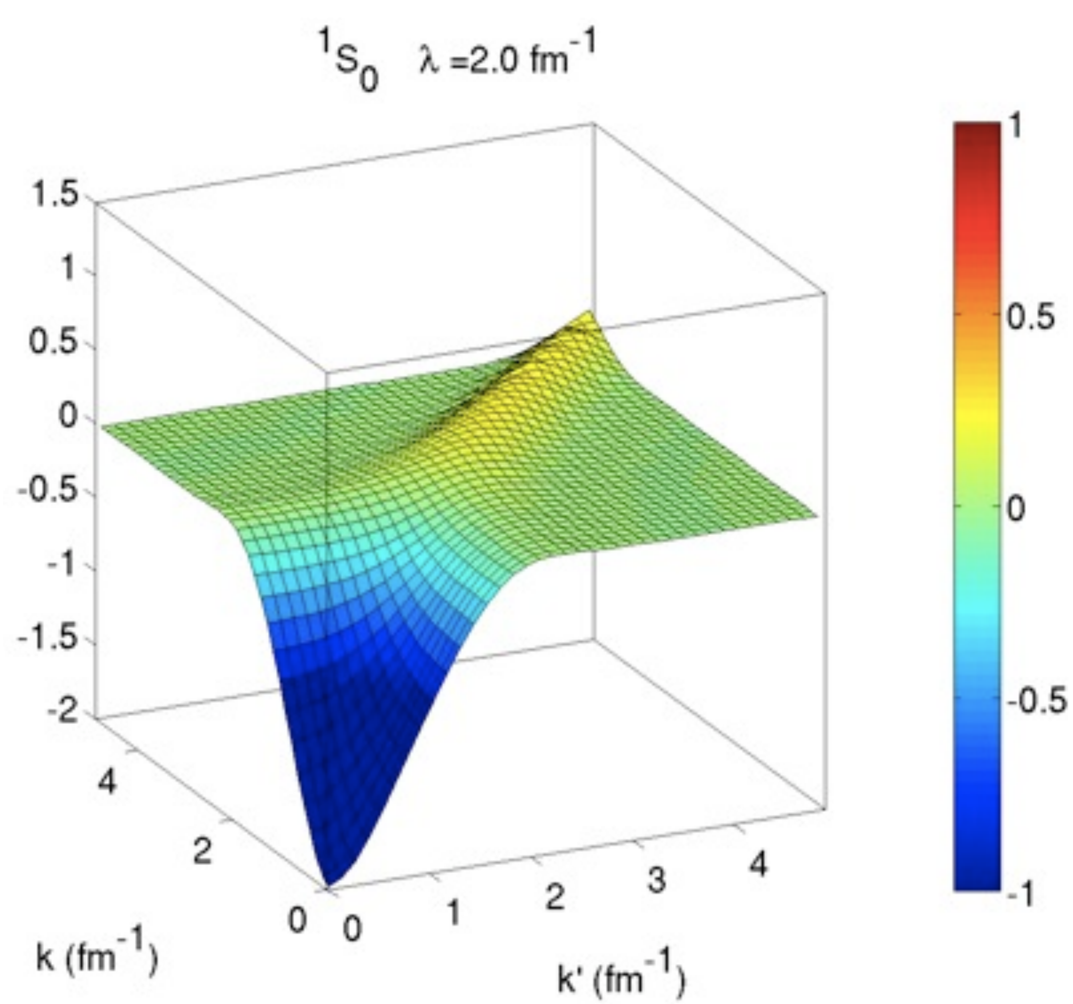

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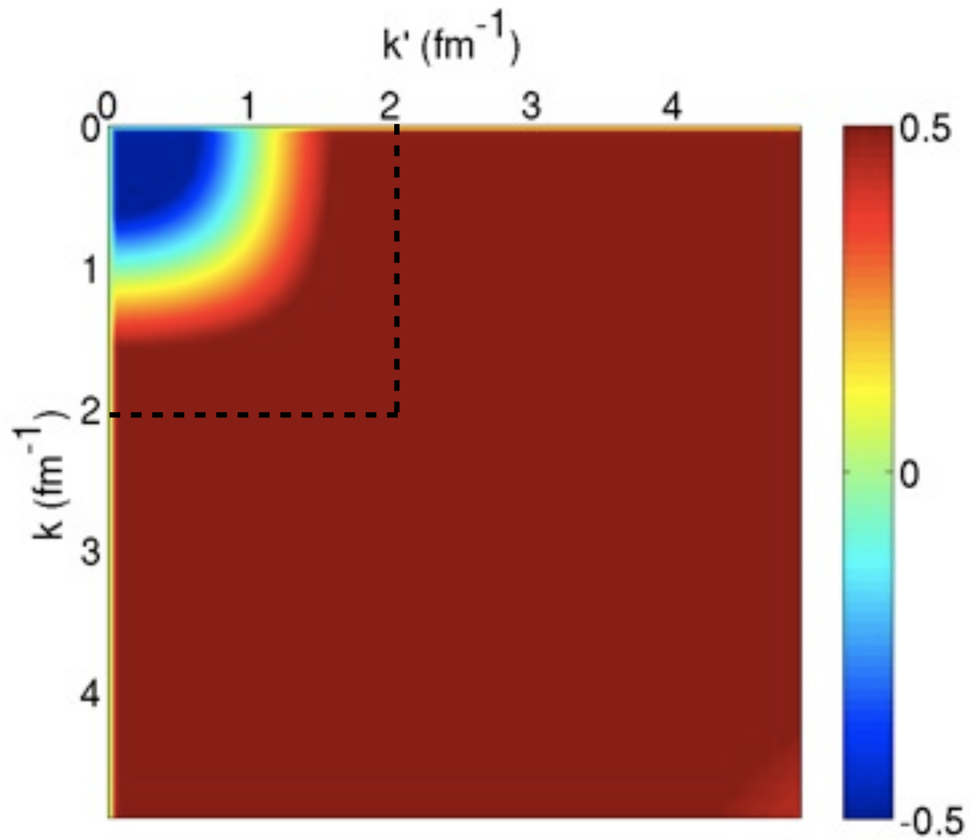
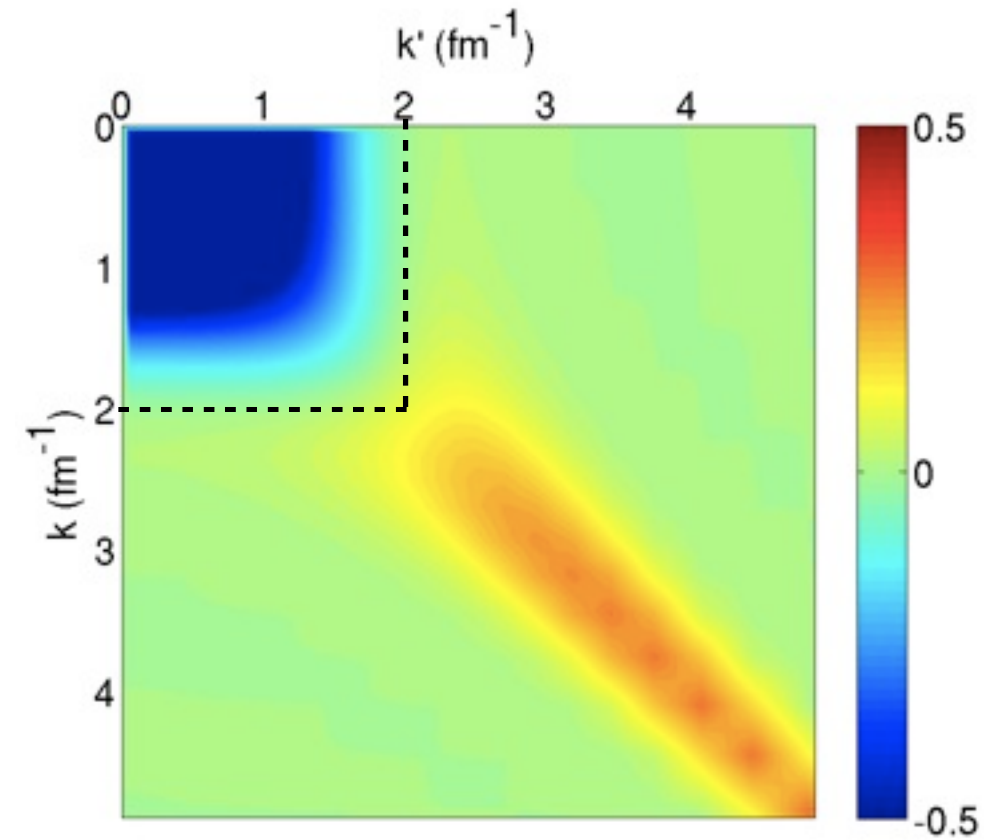




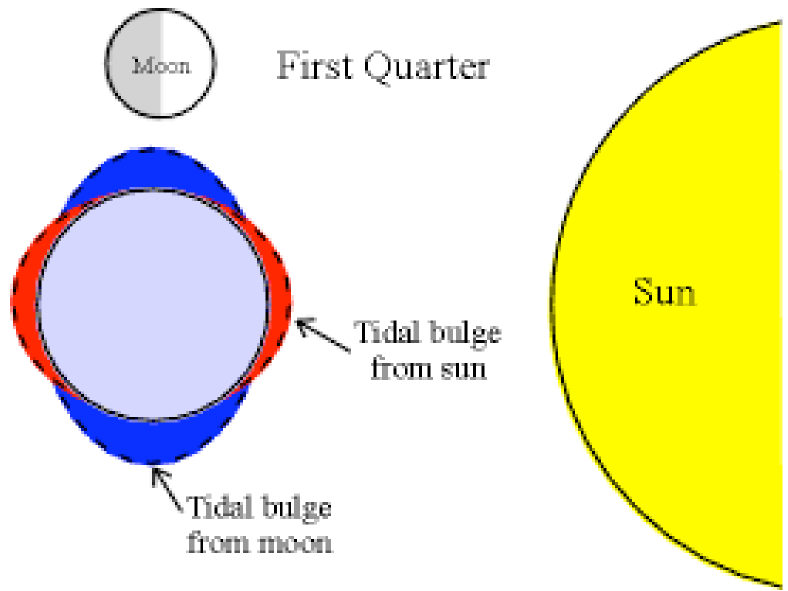
- elimination of coupling between low- and high momentum components, calculations much easier
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

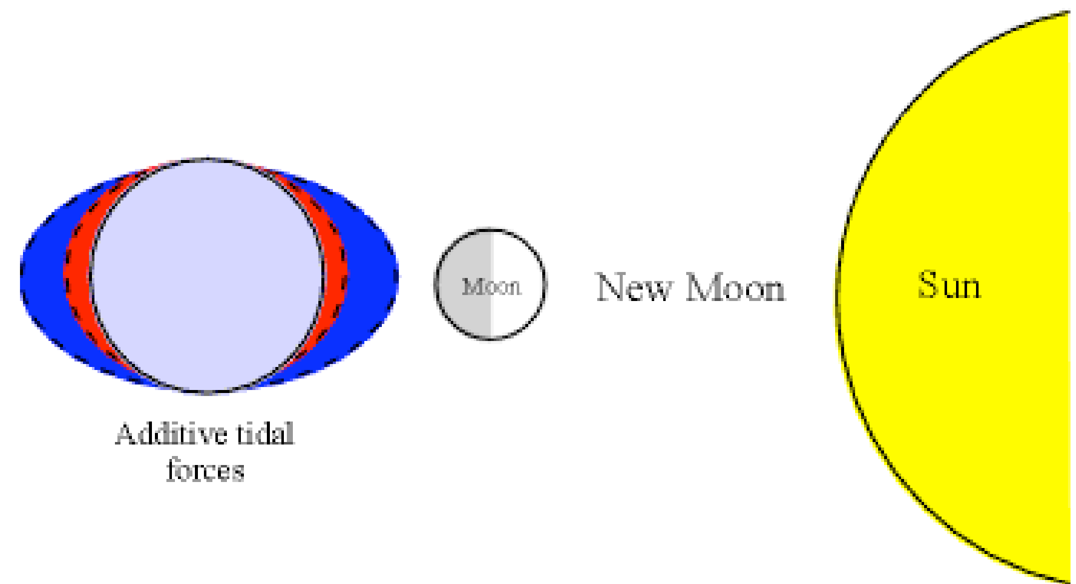
Not the full story:
RG transformation also changes three-body (and higher-body) interactions!

Why are there 3N forces?

Classical analog

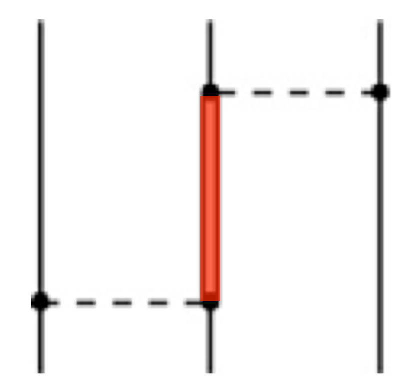
Tidal effects lead to 3N forces in earth-sun-moon system:





- force between earth and moon depends on the position of sun
- tidal deformations are internal excitations

- nucleons are composite particles, can also be excited
- change of resolution changes the excitations that can be described explicitly ⟶ change of 3N force
- three-nucleon forces are crucial at low resolution!

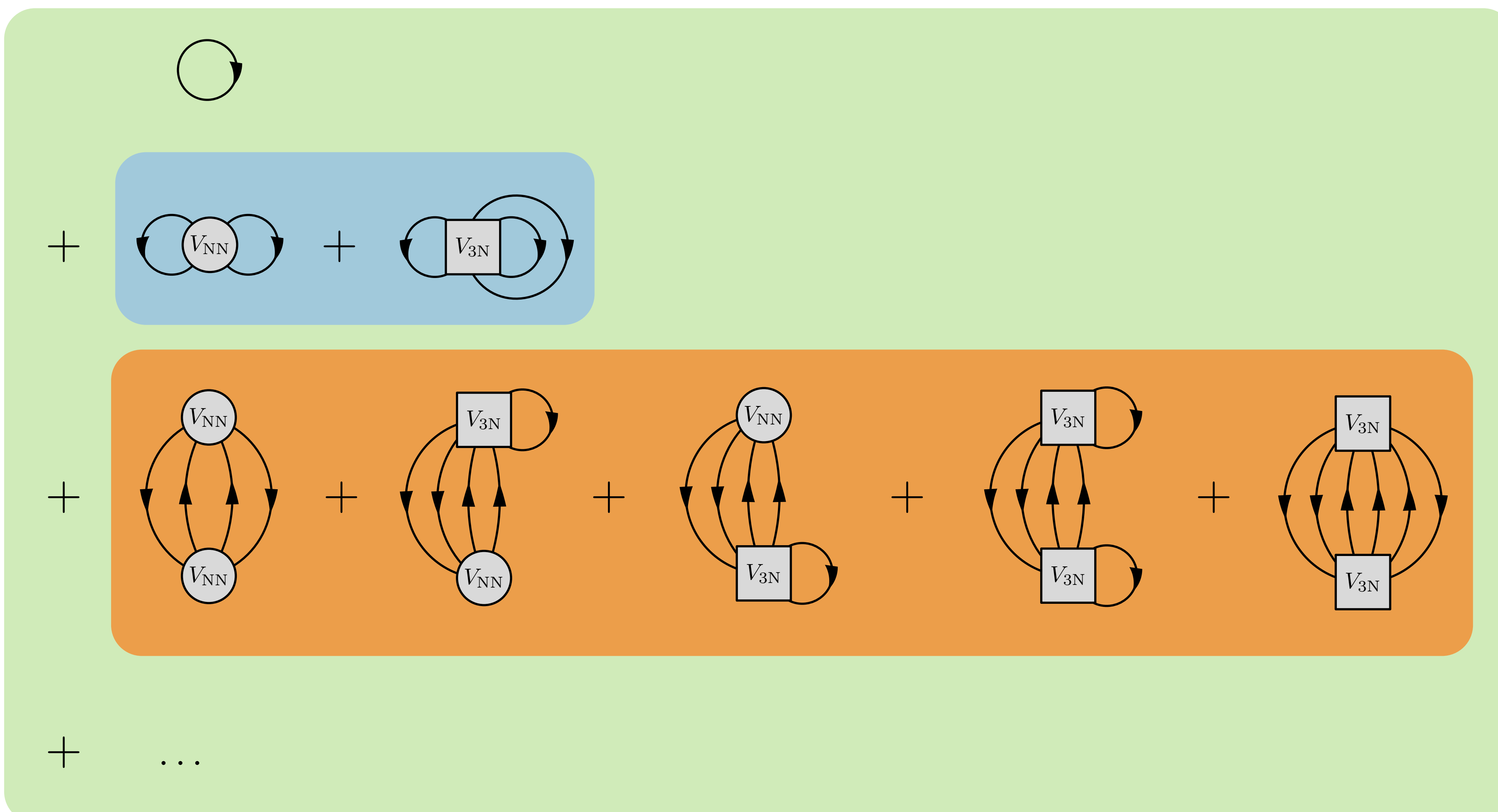


Equation of state: Many-body perturbation theory

central quantity of interest: energy per particle E/N

$$H(\lambda) = T + V_{\rm NN}(\lambda) + V_{\rm 3N}(\lambda) + \ldots$$

$E =$



kinetic energy

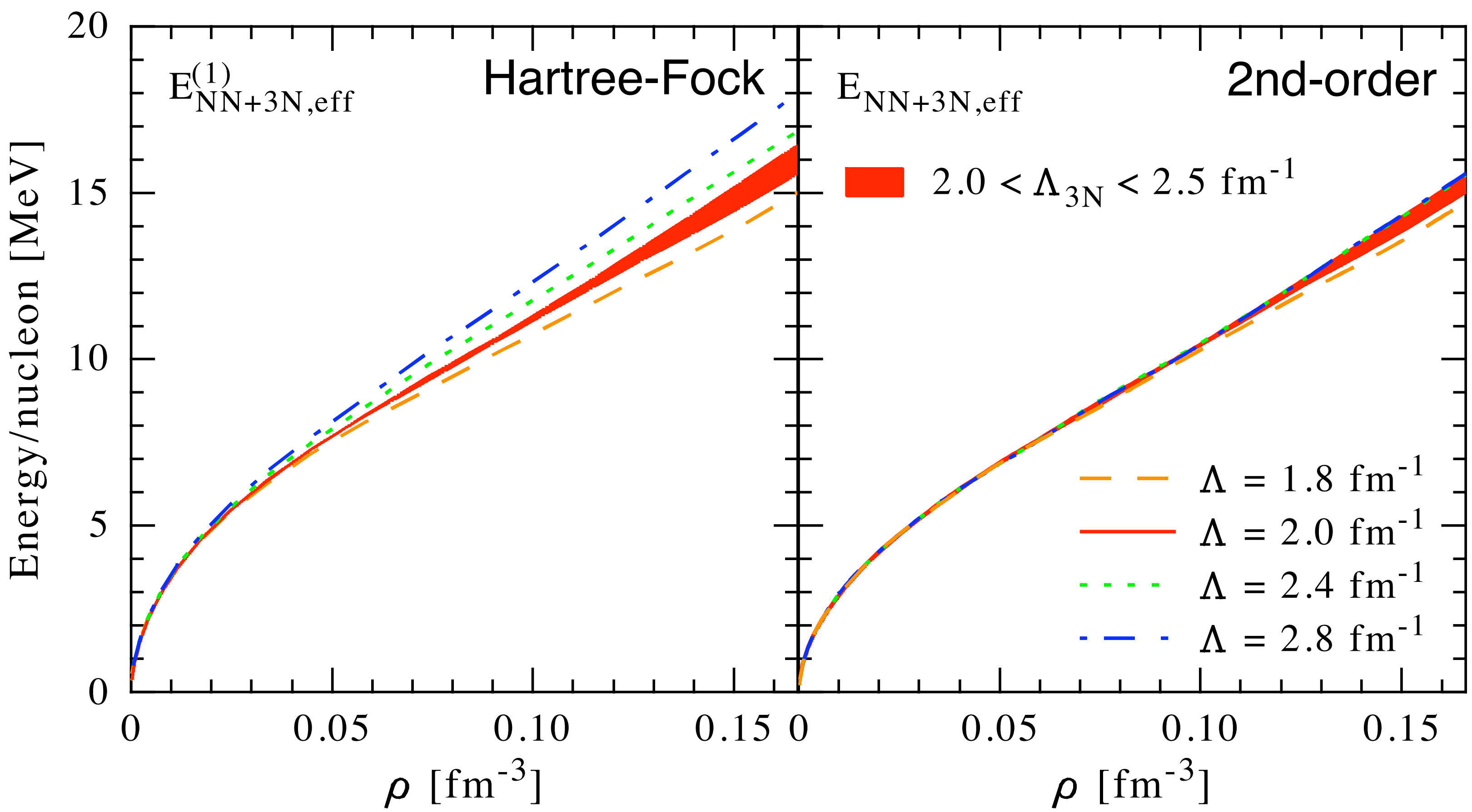
Hartree-Fock

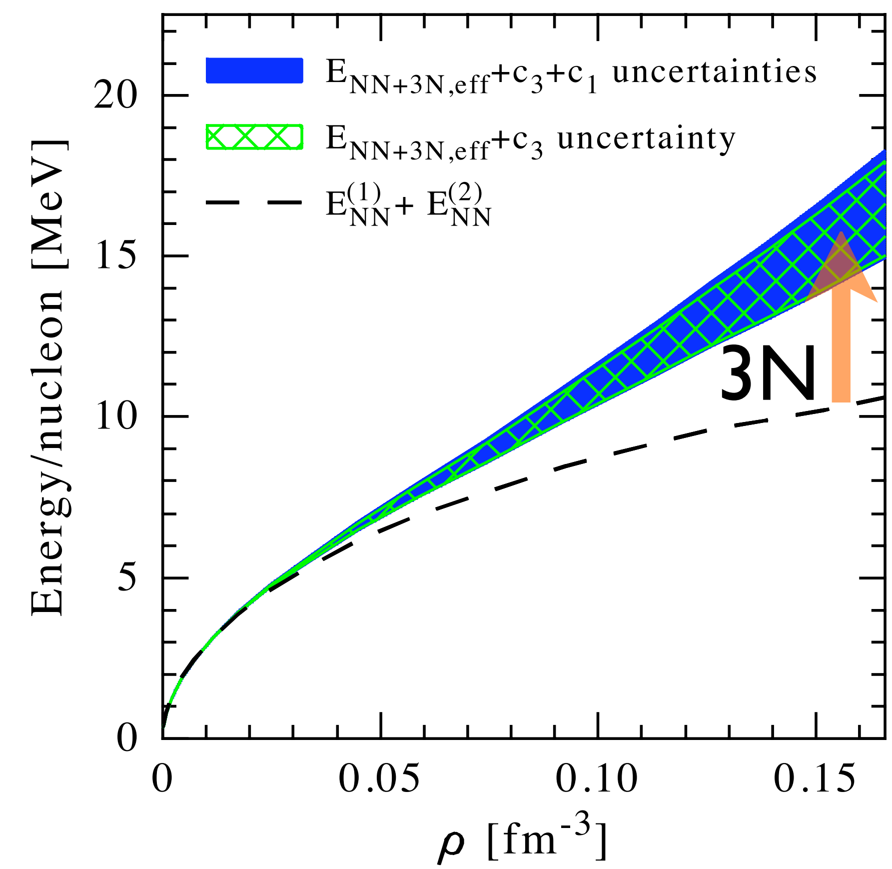
2nd-order

3rd-order
and beyond

- “hard” interactions require non-perturbative summation of diagrams
- with low-momentum interactions much more perturbative
- inclusion of 3N interaction contributions!
- use chiral interactions as initial input for RG evolution

Equation of state of pure neutron matter





KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence

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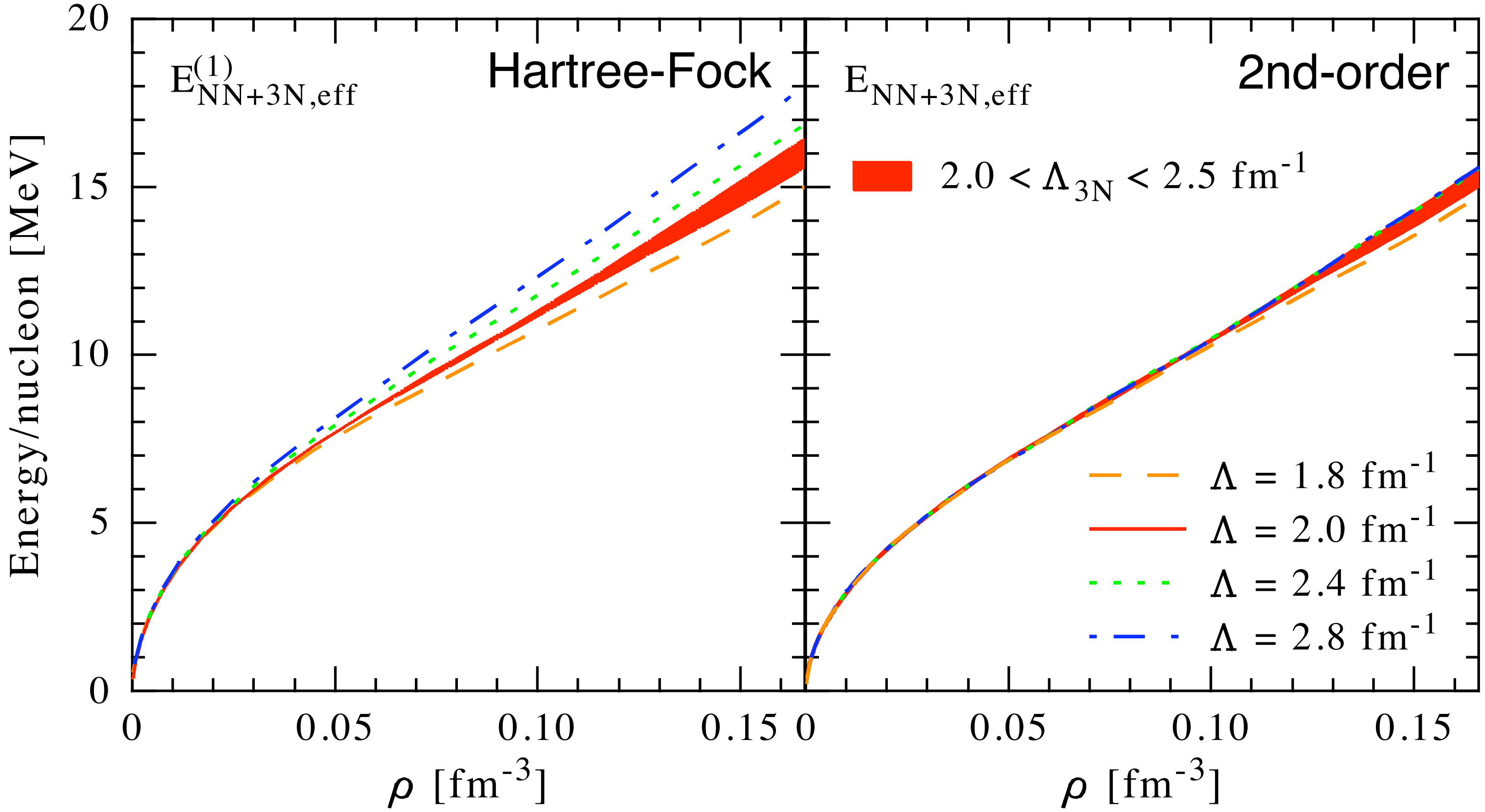

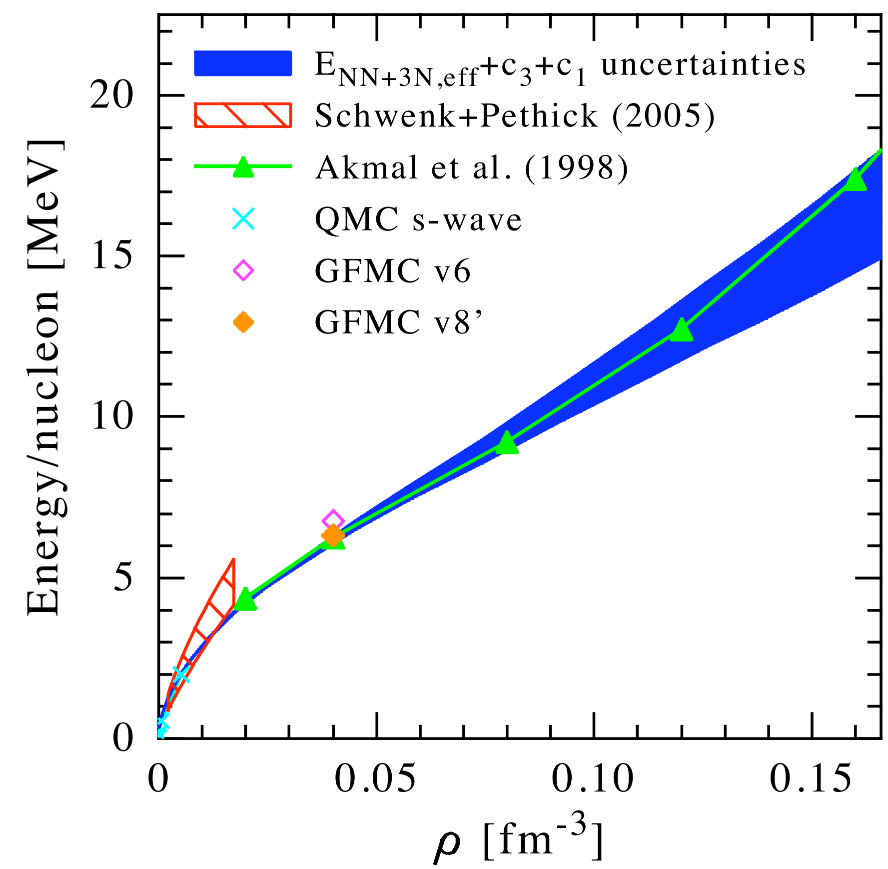

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- good agreement with other approaches (different NN interactions)

Neutron matter: Symmetry energy

$$E(\rho, \alpha = 1) = -a_V + \frac{K_0}{18\rho_0^2}(\rho - \rho_0)^2 + S_2(\rho)$$

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_0^2}(\rho - \rho_0)$$

c_1 [GeV]	c_3 [GeV]	a_4 [MeV]	p_0 [MeV fm^{-3}]
-0.81	-3.2	31.7	$2.4/2.5$
-0.81	-5.7	33.7	$2.9/3.0$
-0.7	-3.2	31.7	$2.4/2.5$
-1.4	-5.7	34.5	$3.3/3.4$

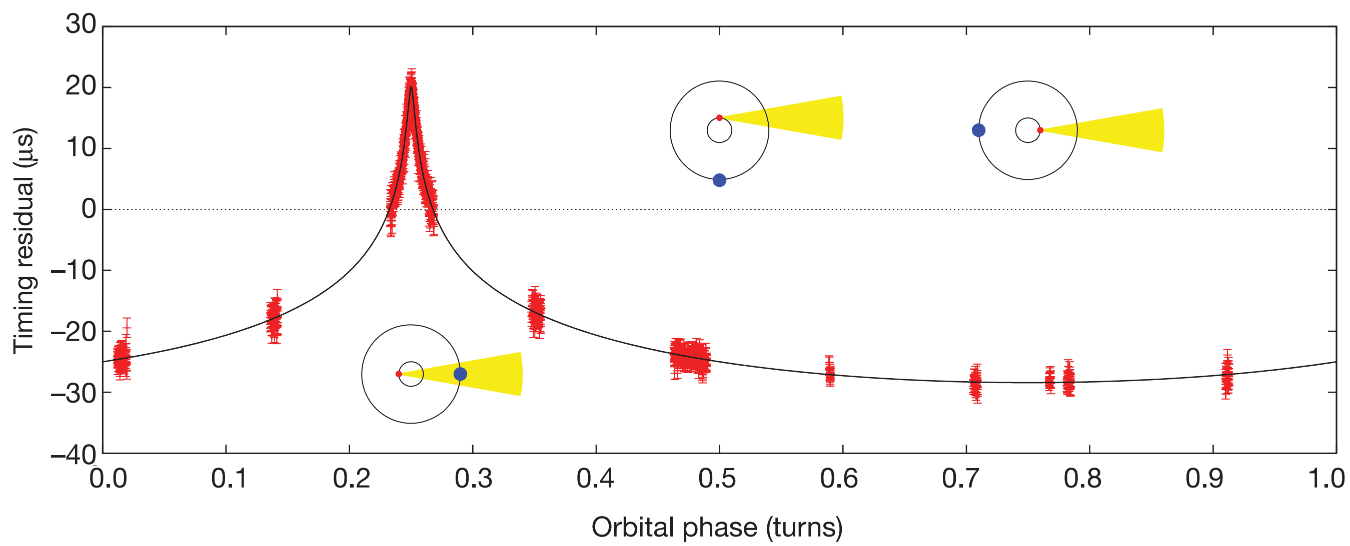
- uncertainties in c_i couplings lead to uncertainties in symmetry energy
- given the experimental constraint $a_4 = 30 \pm 4\,\mathrm{MeV}$ smaller absolute values of c_3 seem to be preferred from our results

Constraints on the nuclear equation of state (EOS)

nature

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]



Demorest et al., Nature 467, 1081 (2010)

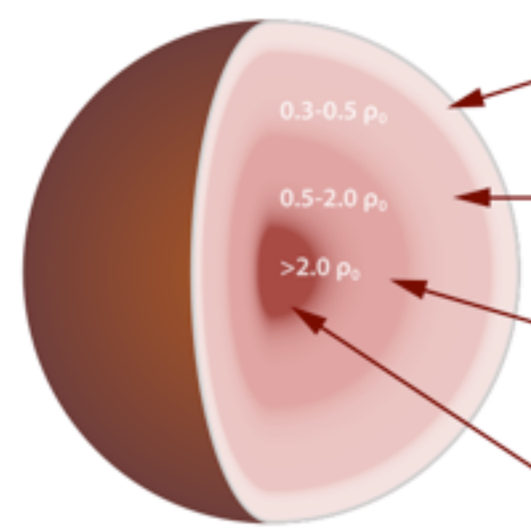
Credit: NASA/Dana Berry

$$M_{\rm max} = 1.65 M_\odot \rightarrow 1.97 \pm 0.04\, M_\odot$$

Structure of a neutron star is determined by Tolman-Oppenheimer-Volkov (TOV) equation:

$$\frac{dP}{dr} = -\frac{GM\epsilon}{r^2}\left[1+\frac{P}{\epsilon c^2}\right]\left[1+\frac{4\pi r^3 P}{Mc^2}\right]\left[1-\frac{2GM}{c^2 r}\right]^{-1}$$

crucial ingredient: energy density $\epsilon = \epsilon(P)$



Neutron star radius constraints

Problem: Solution of TOV equation requires EOS up to very high densities.
Radius of a typical NS ($M \sim 1.4\, M_{\odot}$) theoretically not well constrained.

But: Radius of NS is relatively insensitive to high density region.

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incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise
high-density extensions of EOS:

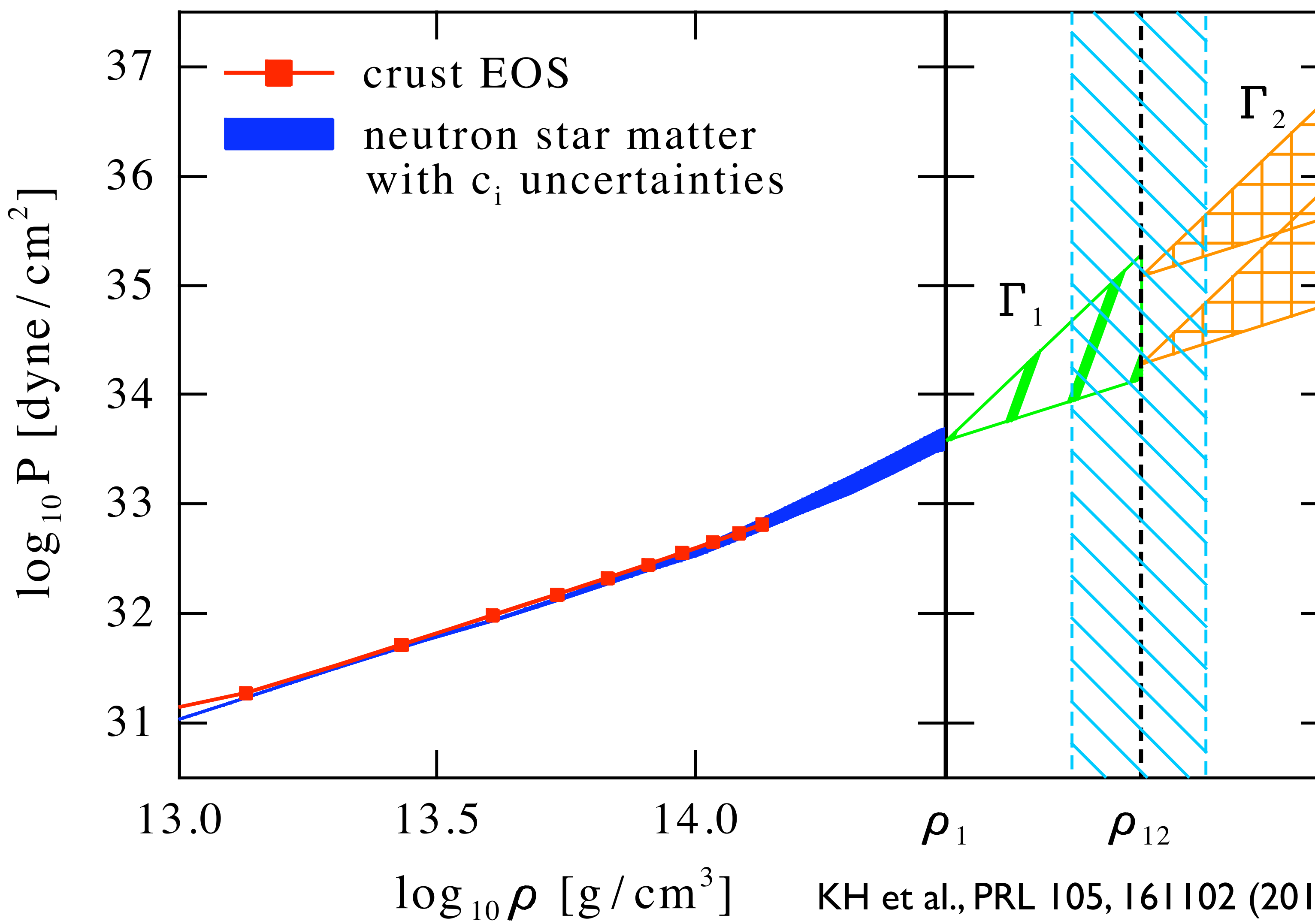
- use polytropic ansatz

$$p \sim \rho^{\Gamma}$$

- range of parameters

$$\Gamma_1, \rho_{12}, \Gamma_2$$

limited by physics!



KH et al., PRL 105, 161102 (2010)

Neutron star radius constraints

use the constraints:

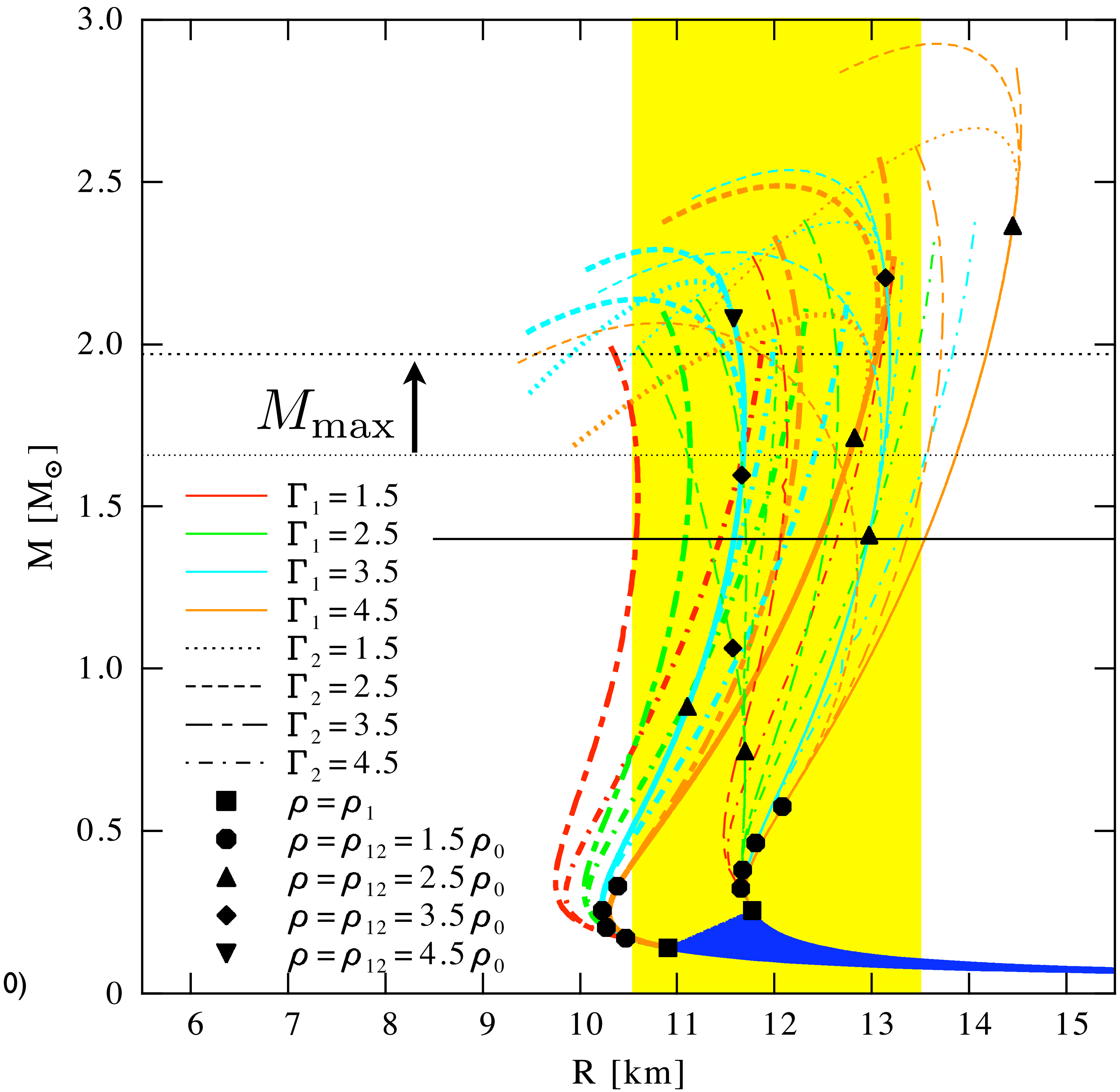
recent NS observation

$$M_{\max} > 1.97\, M_\odot$$

causality

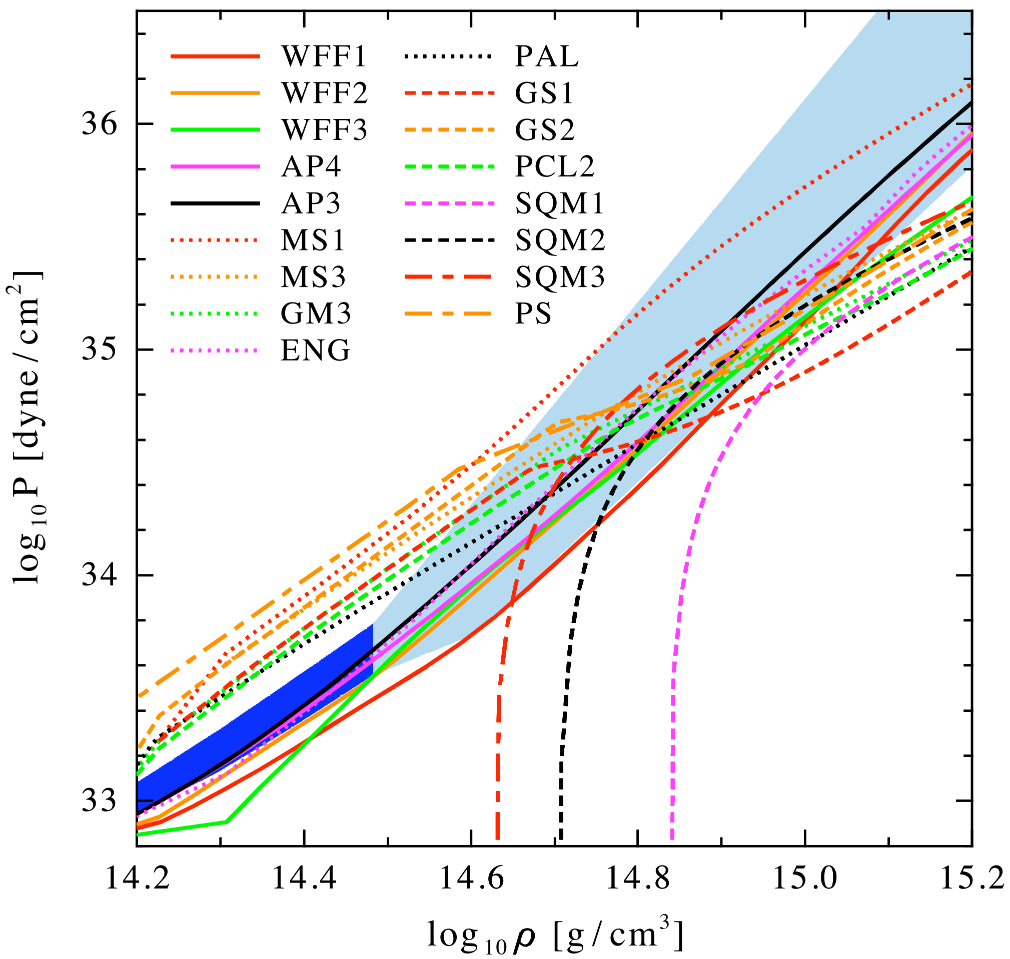
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH et al., PRL 105, 161102 (2010)

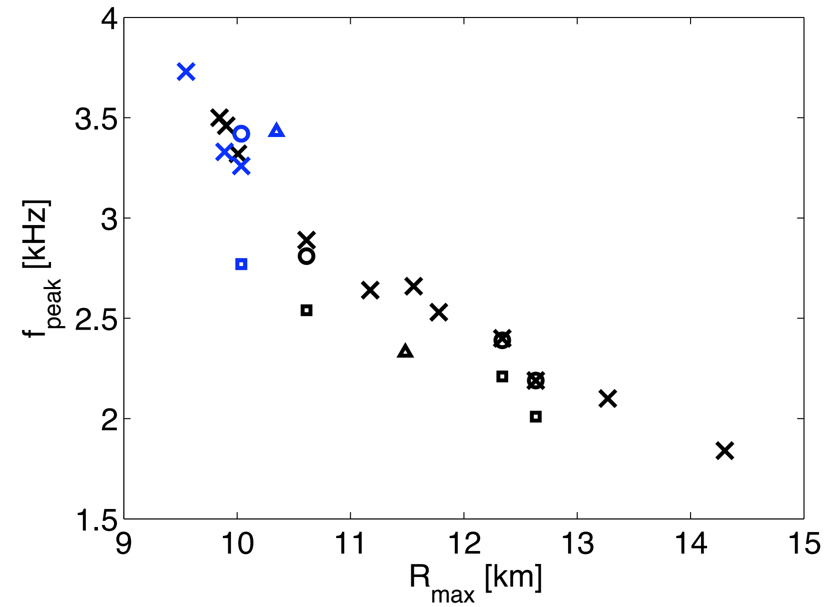


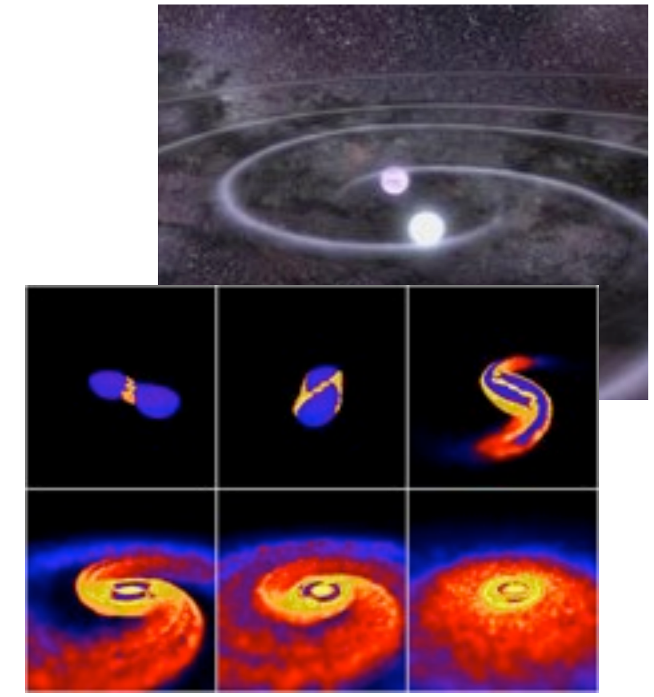
- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint after incorporating crust corrections: $10.5 - 13.5\,\mathrm{km}$

Gravitational wave signals from neutron star binary mergers

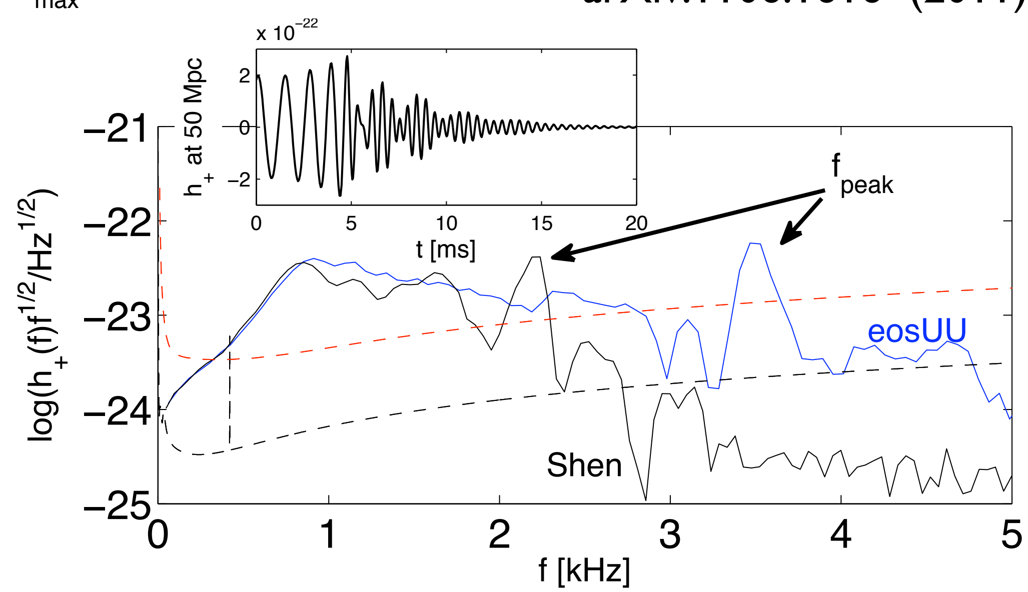


KH et al., PRL 105, 161102 (2010)



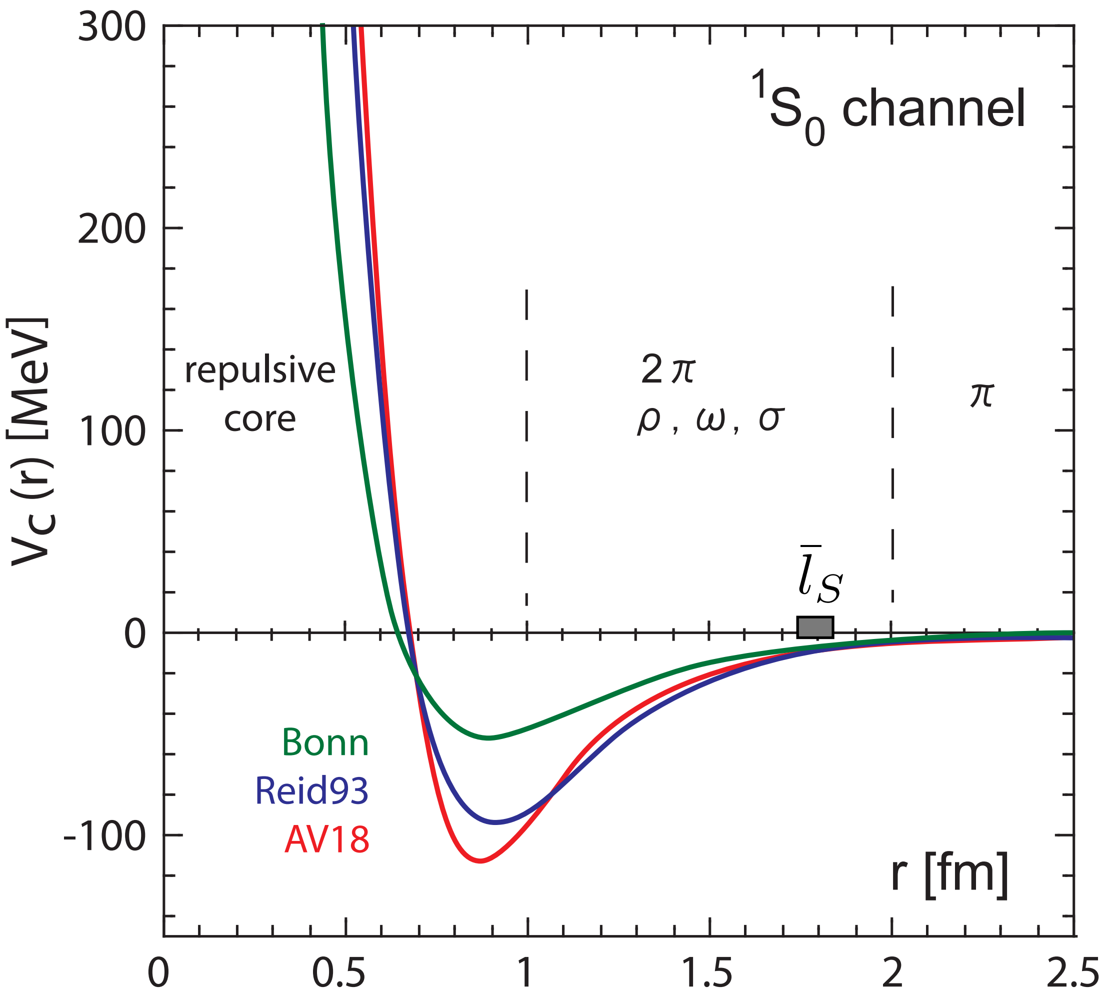


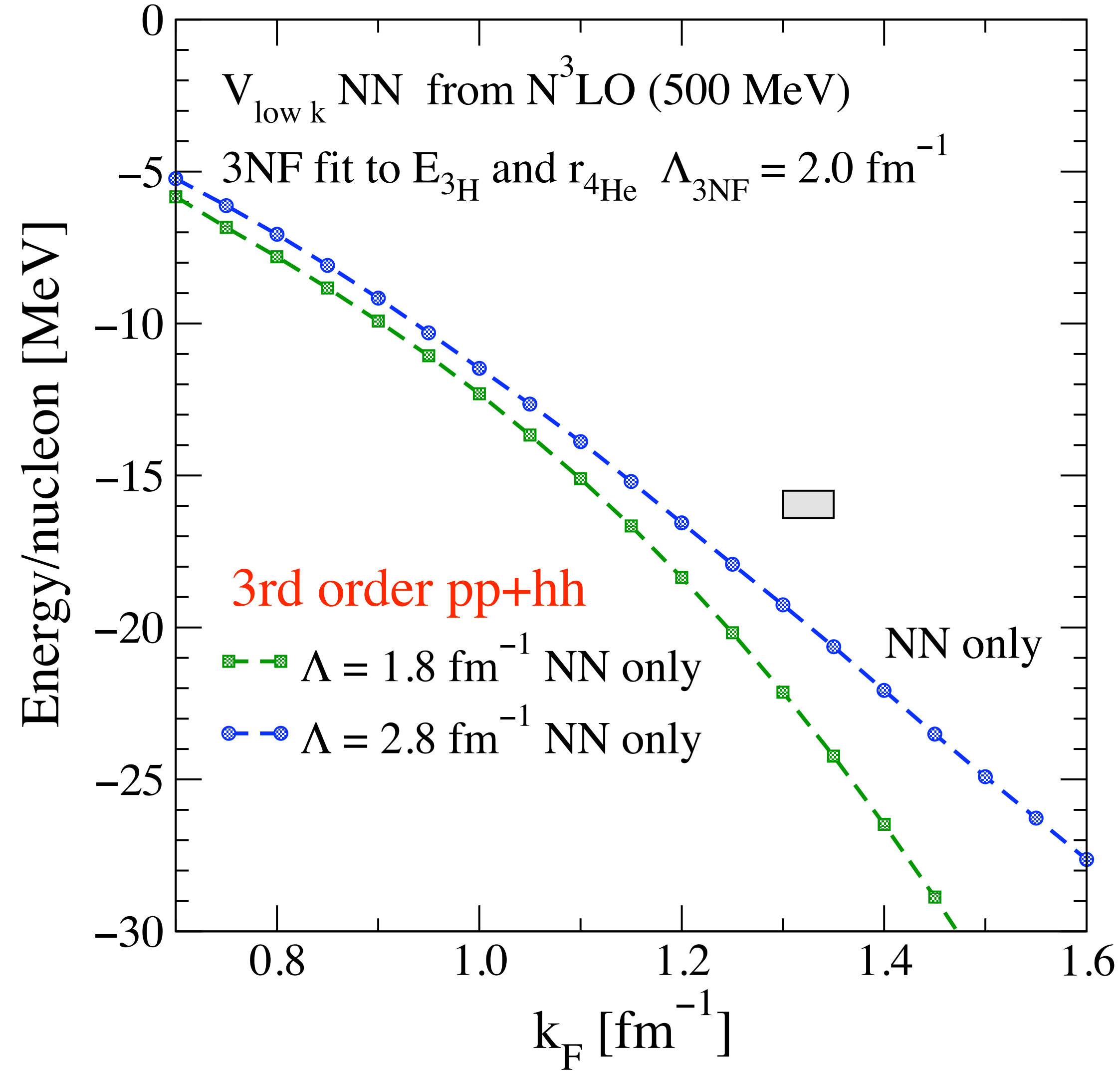
Bauswein and Janka, arXiv:1106.1616 (2011)



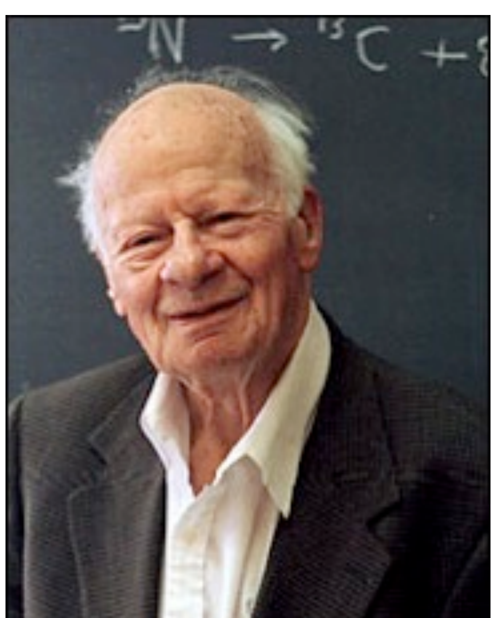
- high-density part of nuclear EOS only loosely constrained
- simulations of NS binary mergers show strong correlation between between $f_{\rm peak}$ of the GW spectrum and $R_{\max}$ of the corresponding EOS
- measuring $f_{\rm peak}$ is key step for constraining chiral EOS systematically at large ρ

Equation of state of symmetric nuclear matter, nuclear saturation





KH et al., PRC(R) 83, 031301 (2011)



"Very soft potentials must be excluded because they do not give saturation;
they give too much binding and too high density. In particular, a substantial tensor force is required."

Hans Bethe (1971)

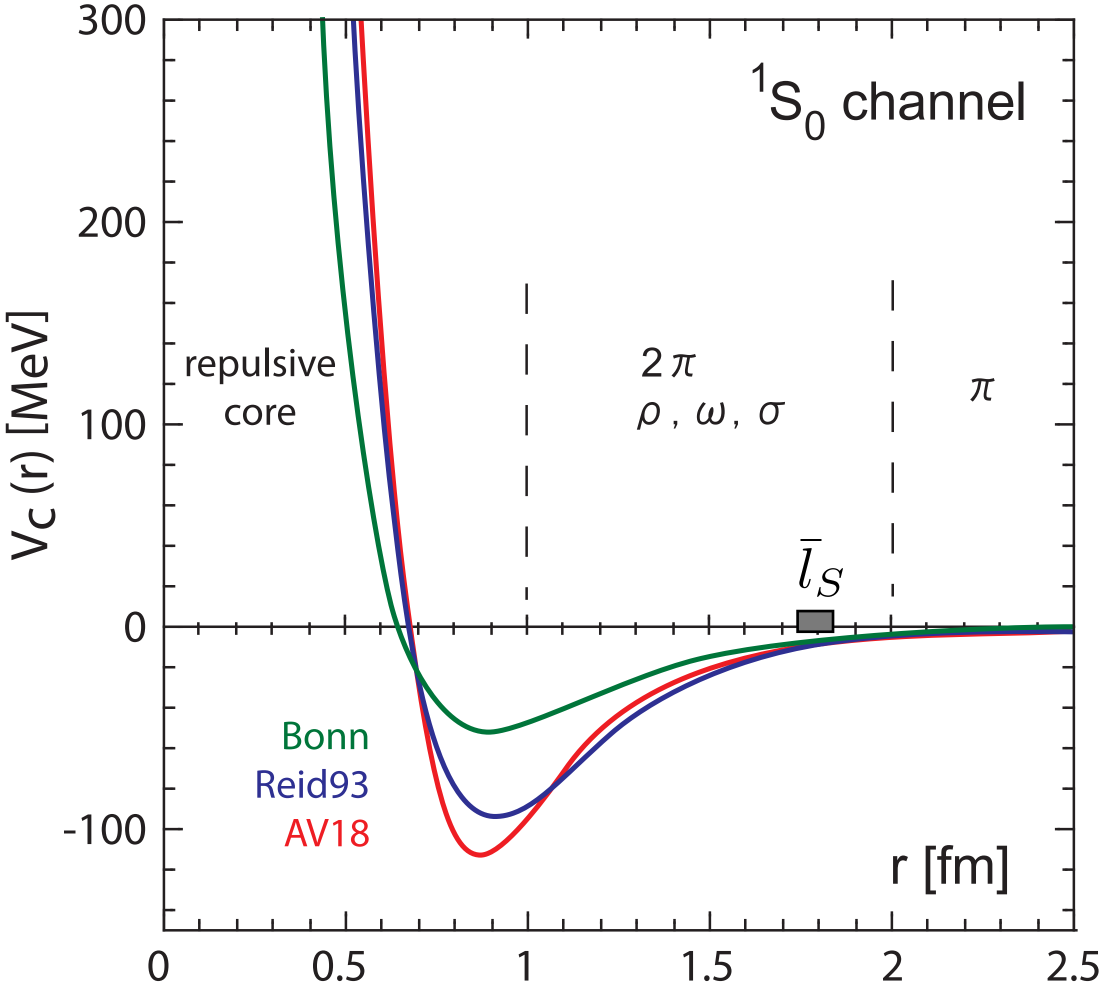
empirical nuclear saturation properties

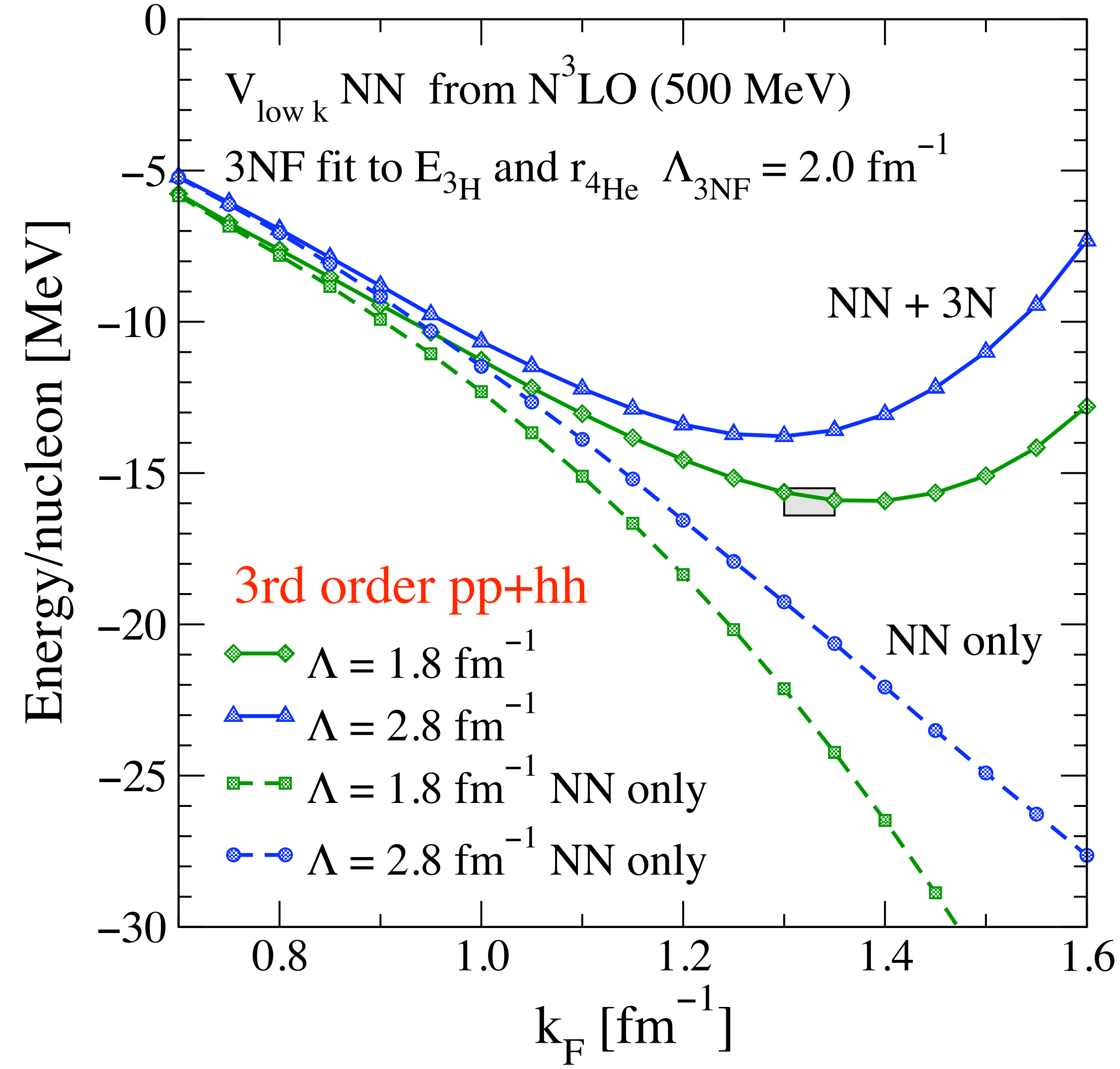
$$n_S \sim 0.16\,\text{fm}^{-3}$$

$$E_{\text{binding}}/N \sim -16\,\text{MeV}$$

$$\bar{l}_S \sim 1.8\,\text{fm}$$

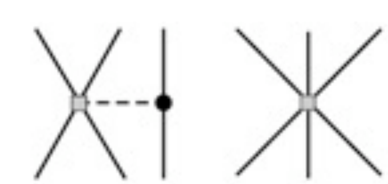
Equation of state of symmetric nuclear matter, Nuclear saturation



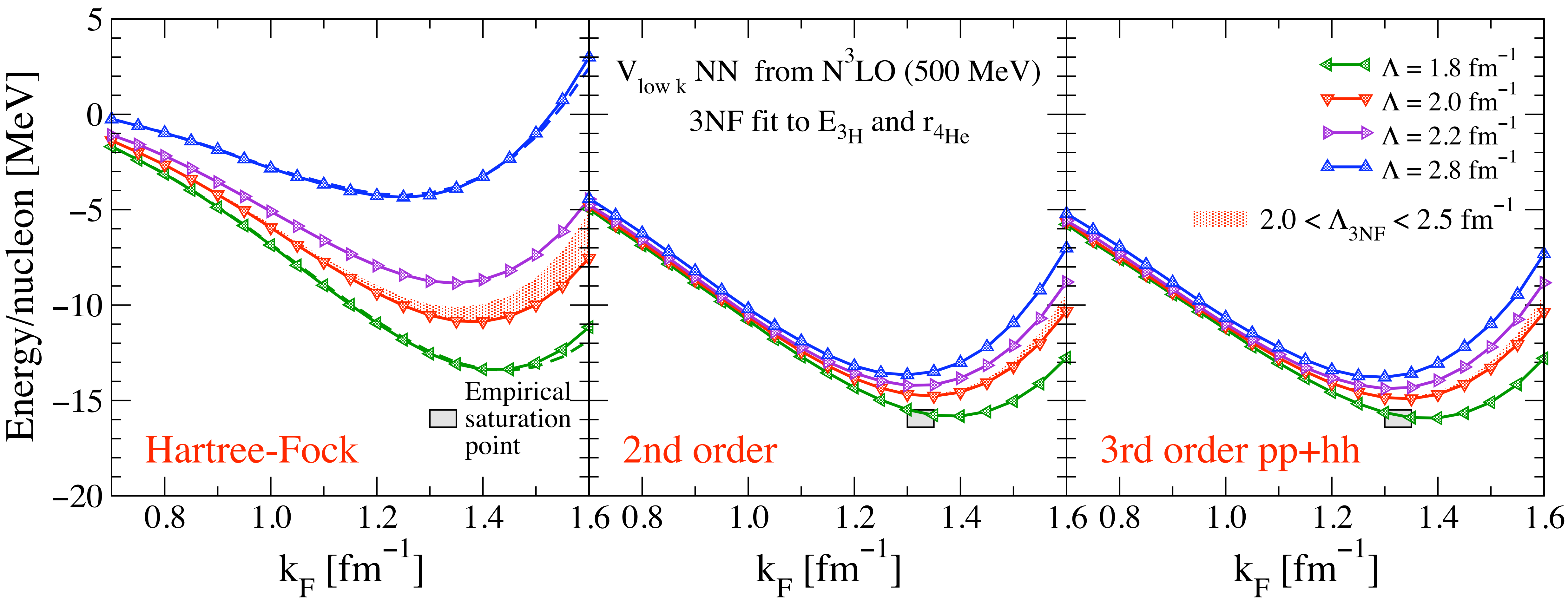


KH et al., PRC(R) 83, 031301 (2011)

- nuclear saturation delicate due to cancellations of large kinetic and potential energy contributions
- 3N forces are essential! 3N interactions fitted to $^3\mathrm{H}$ and $^4\mathrm{He}$ properties



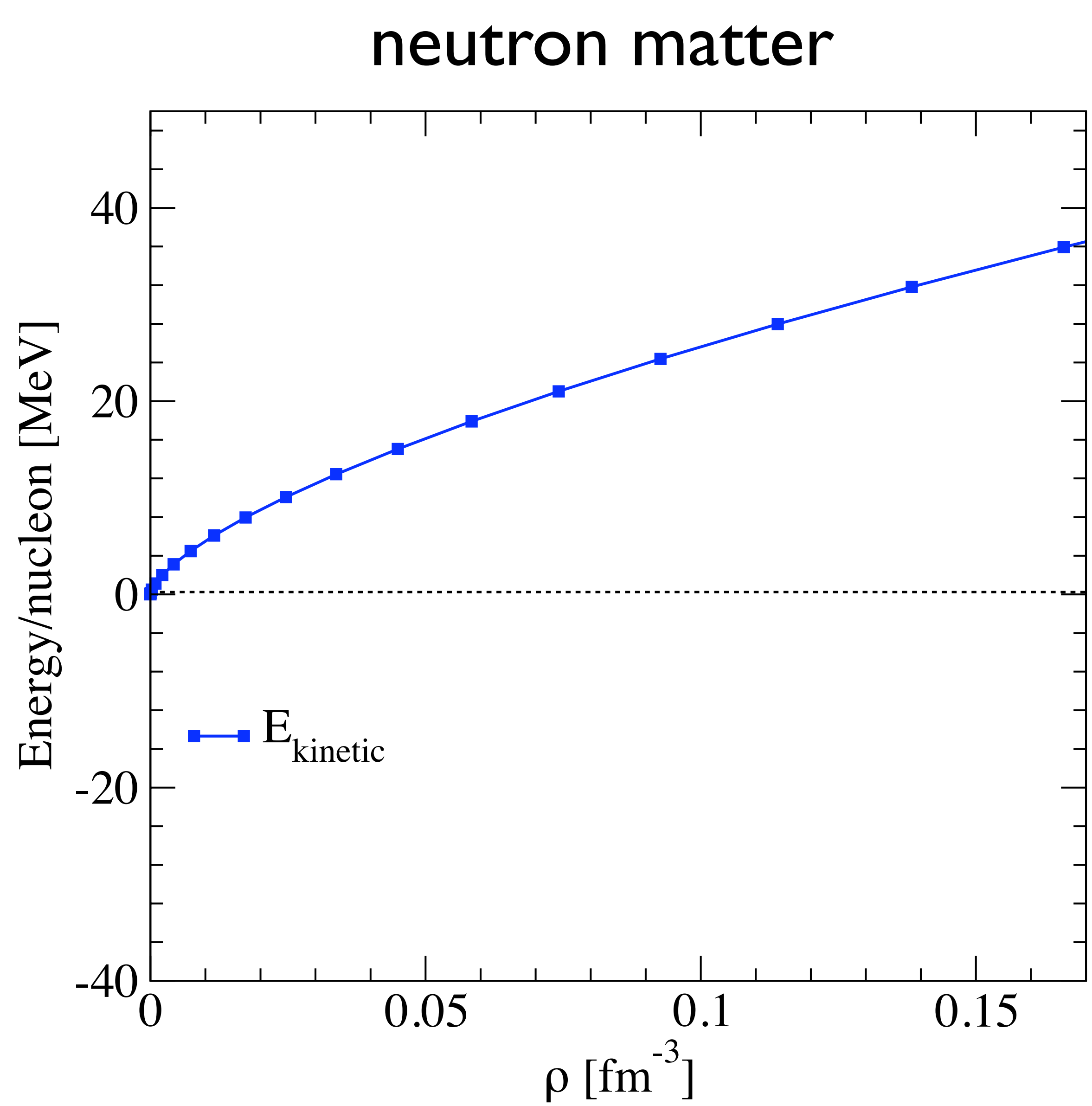
Equation of state of symmetric nuclear matter, Nuclear saturation

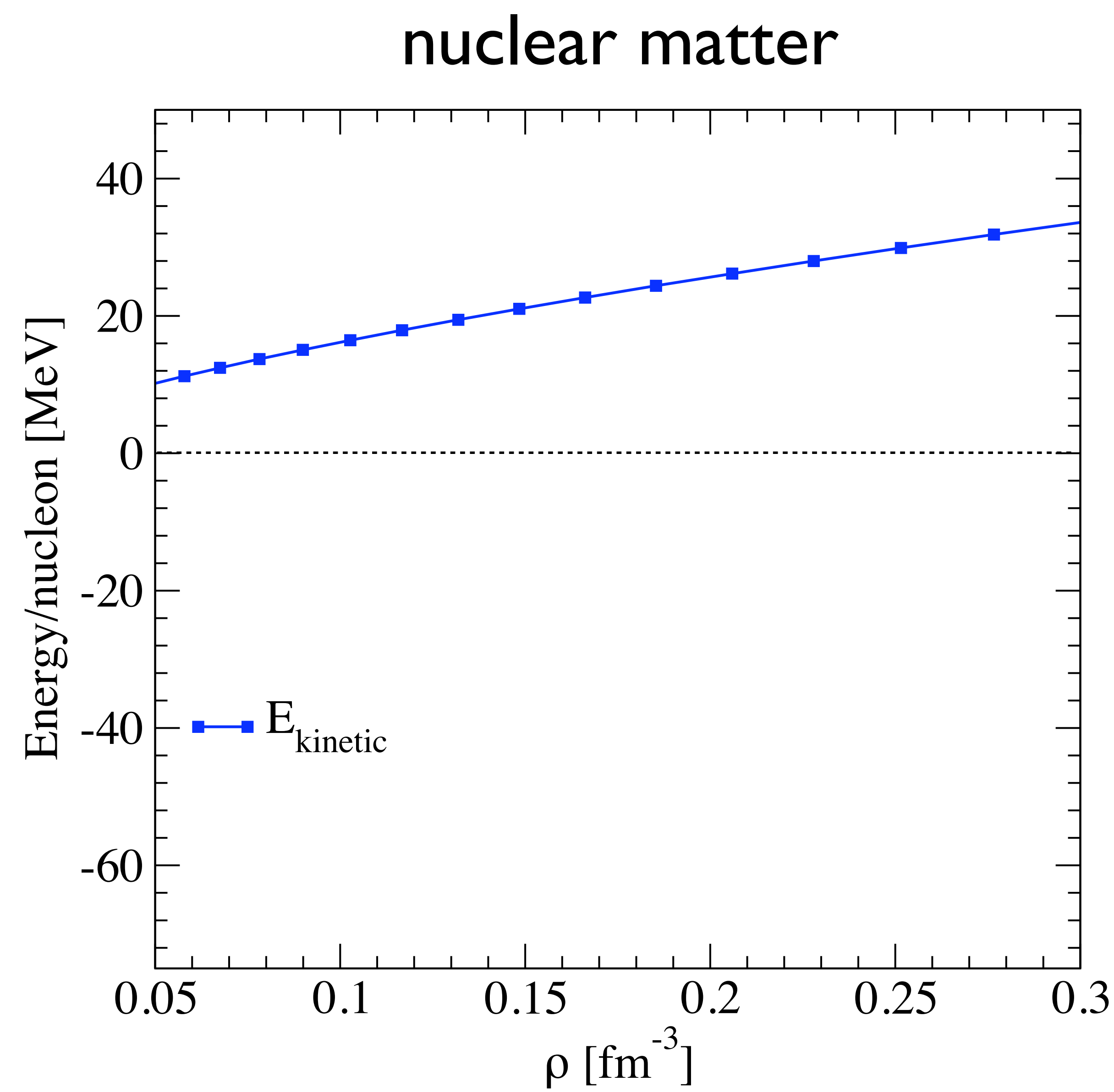


KH et al., PRC(R) 83, 031301 (2011)

- saturation point consistent with experiment, without new free parameters
- cutoff dependence at 2nd order significantly reduced
- 3rd order contributions small
- cutoff dependence consistent with expected size of 4N force contributions

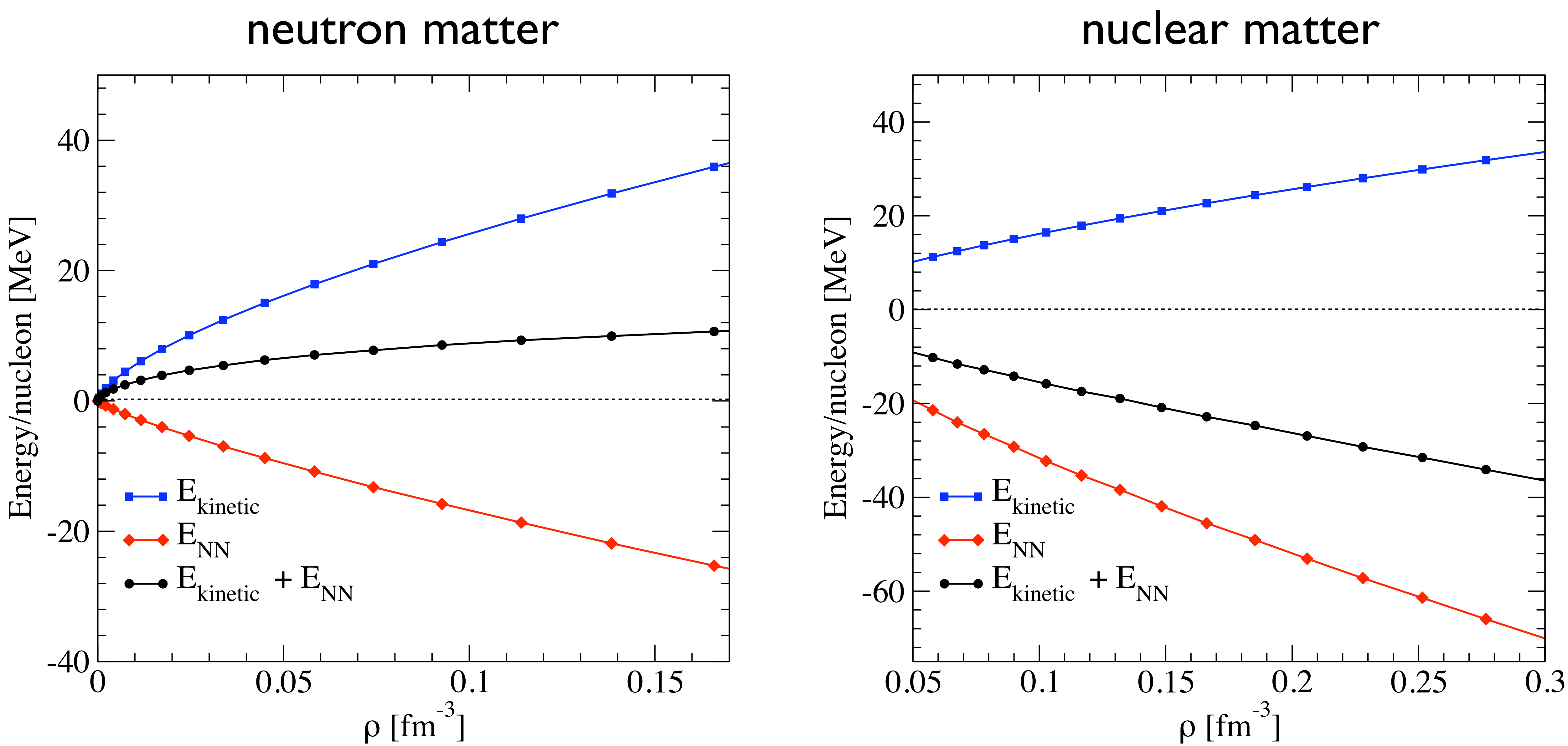
Hierarchy of many-body contributions





- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

Hierarchy of many-body contributions



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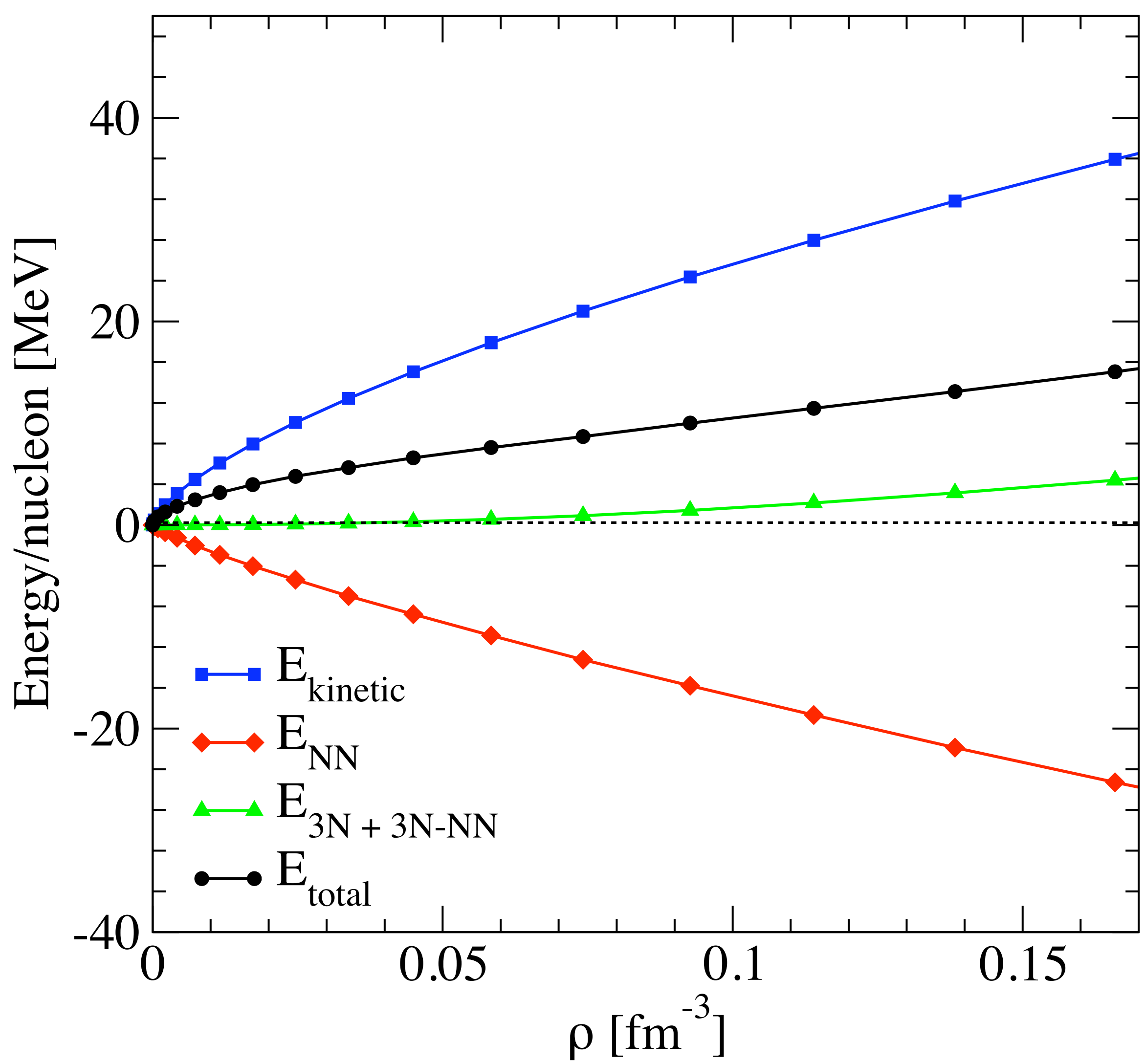
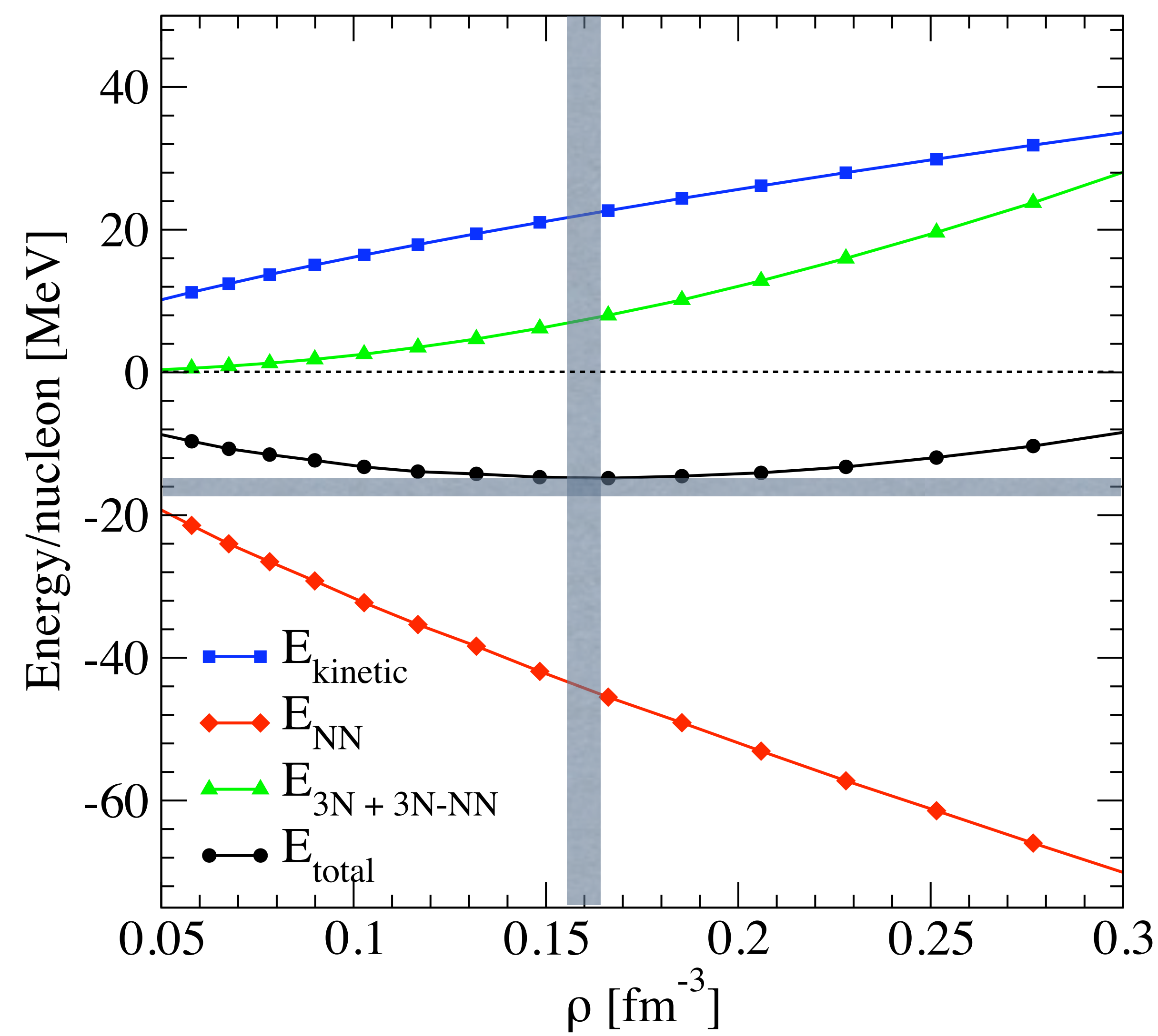
Hierarchy of many-body contributions

neutron matter



nuclear matter



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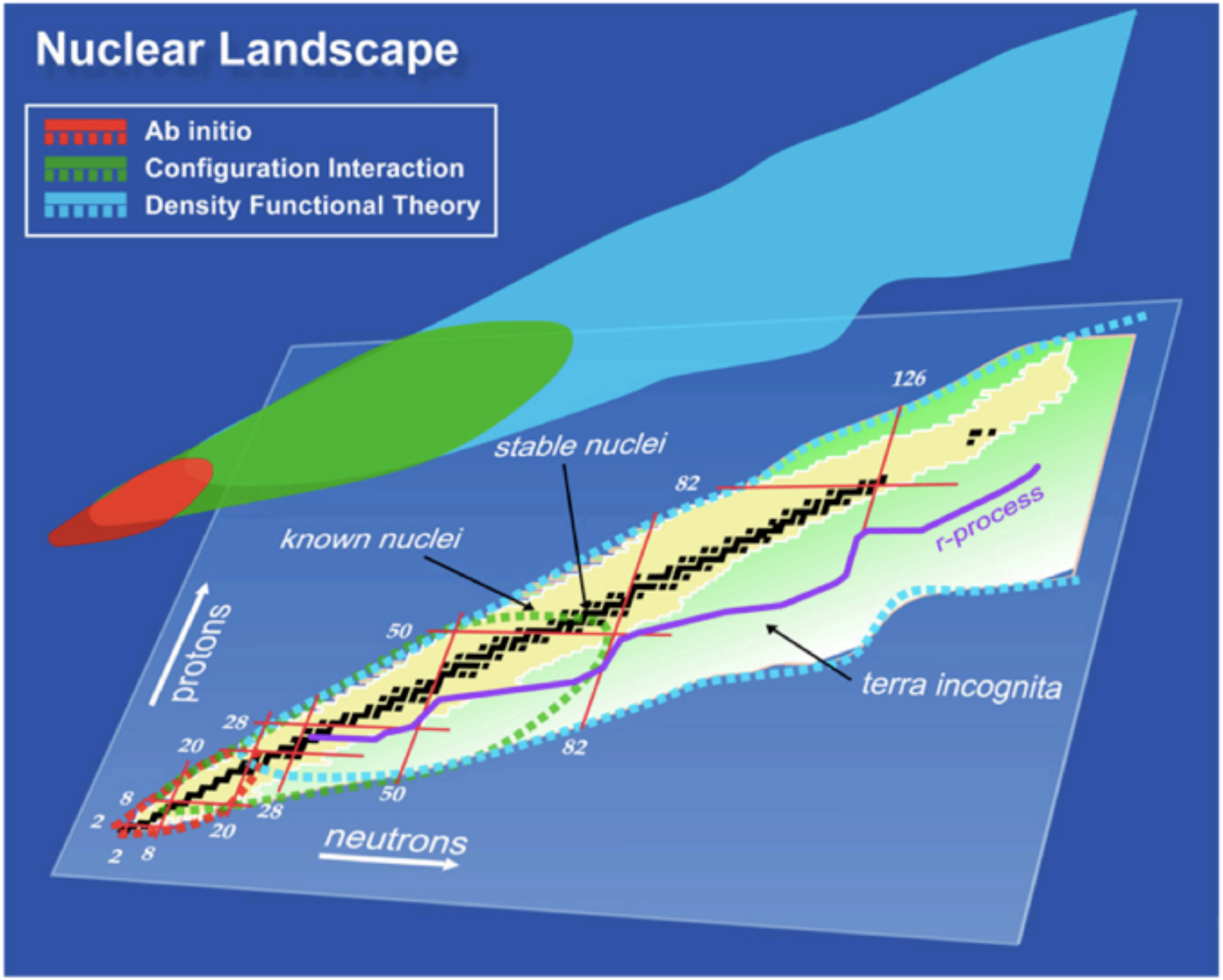
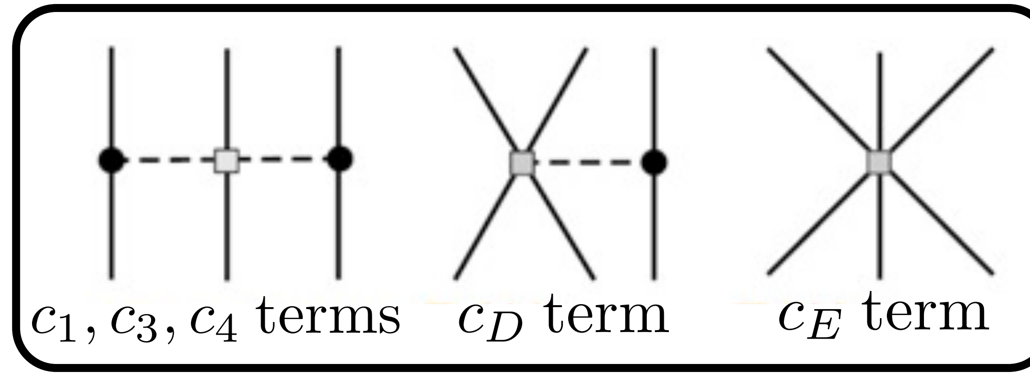
SRG-evolved NN + 3N interactions in light nuclei



consider ab-initio calculations of light nuclei

based on SRG-evolved interactions

SRG evolution of 3N interactions



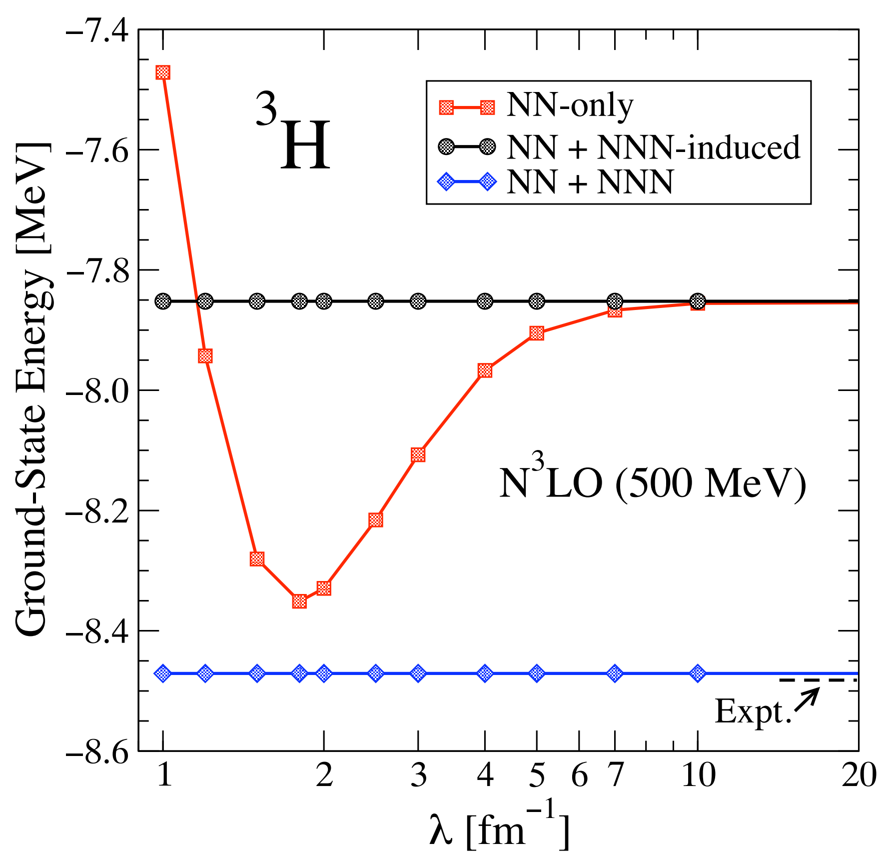
- So far:

fit intermediate (c_D) and short-range (c_E) 3NF couplings to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

- Ideal case: evolve 3NF consistently to lower resolution

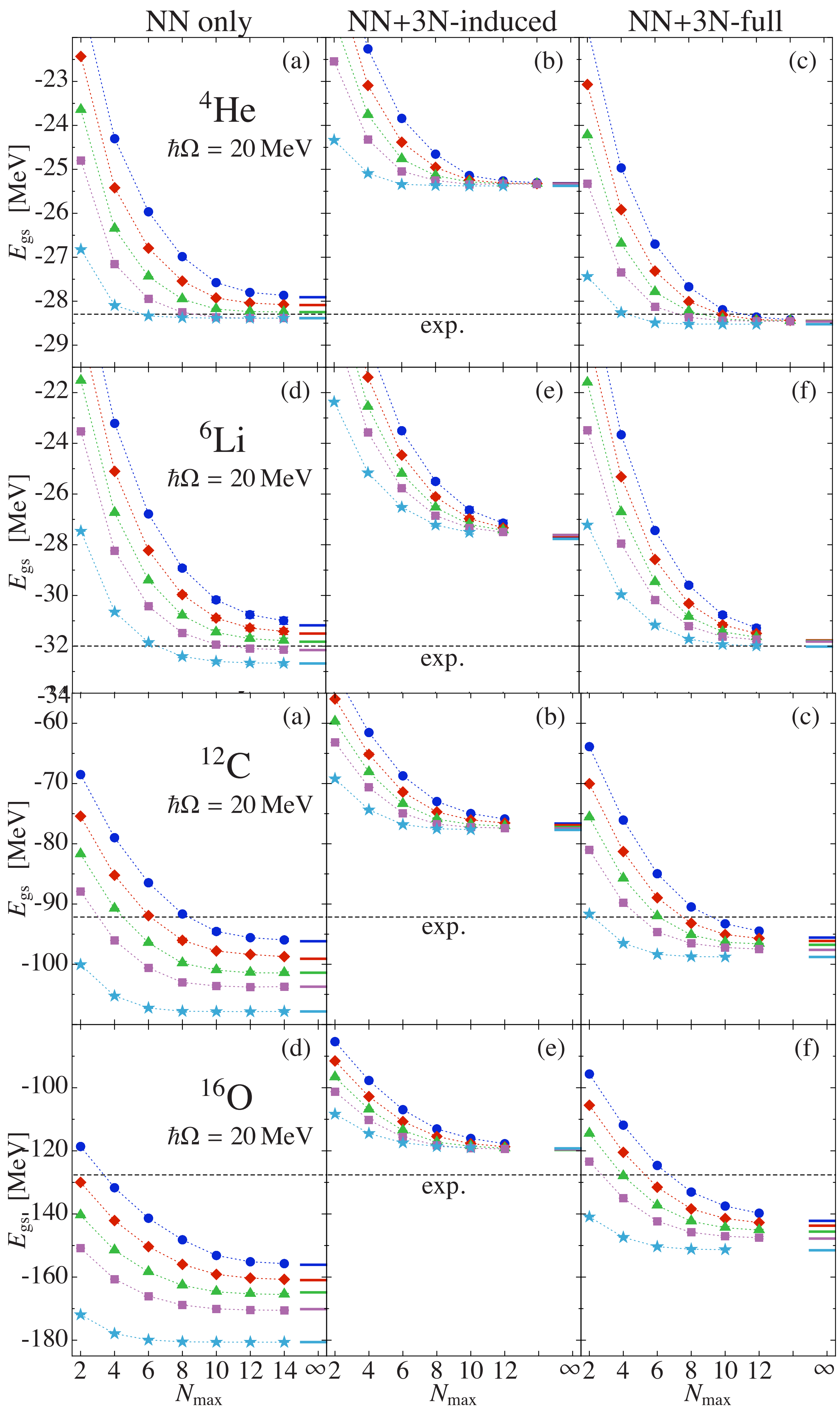
⋆ has been achieved using oscillator basis states, promising results in very light nuclei; problems in heavier nuclei, not suitable for use in infinite systems



Jurgenson, Navratil, Furnstahl, PRL 103, 082501 (2009)

Indications of significant 4N contributions?

ab initio calculations
(no-core shell model) of light nuclei:



small resolution dependence

significant resolution dependence!
(due to long-range part
of initial 3N interaction)

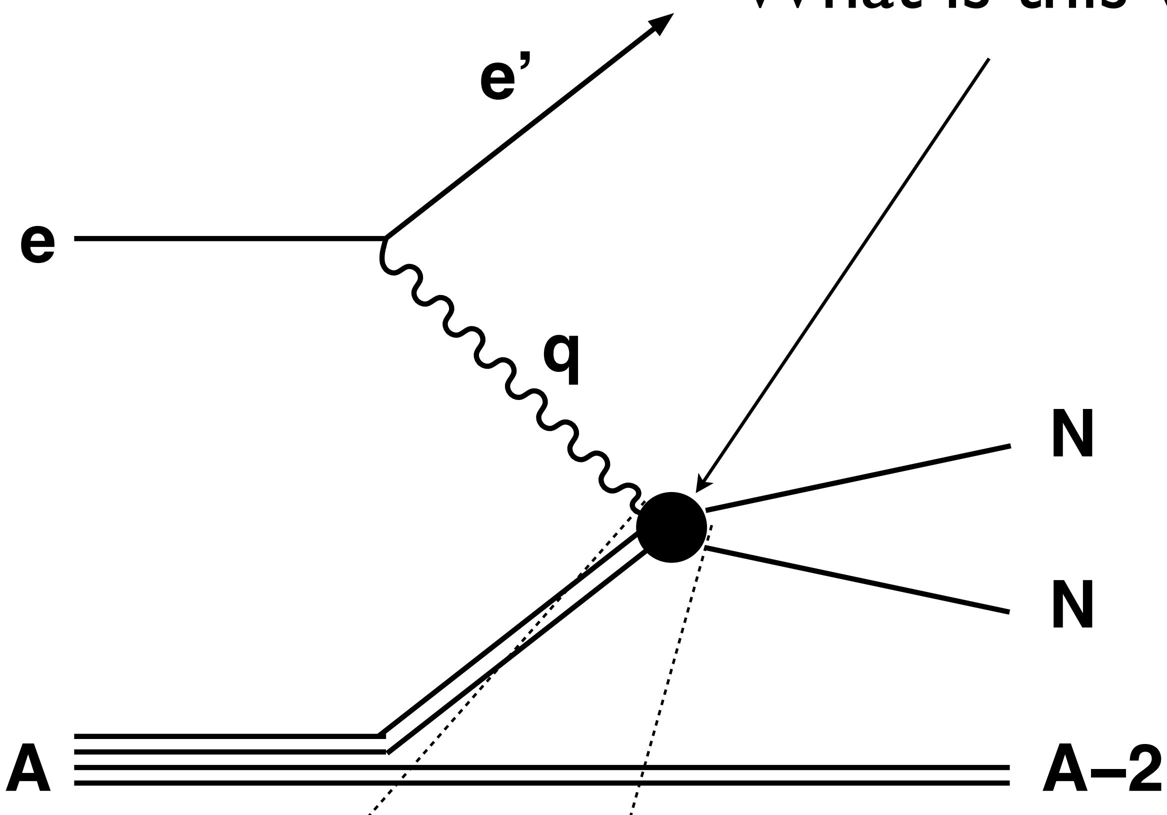
Roth et. al, PRL 107, 072501 (2011)

SRG evolution of 3N interactions

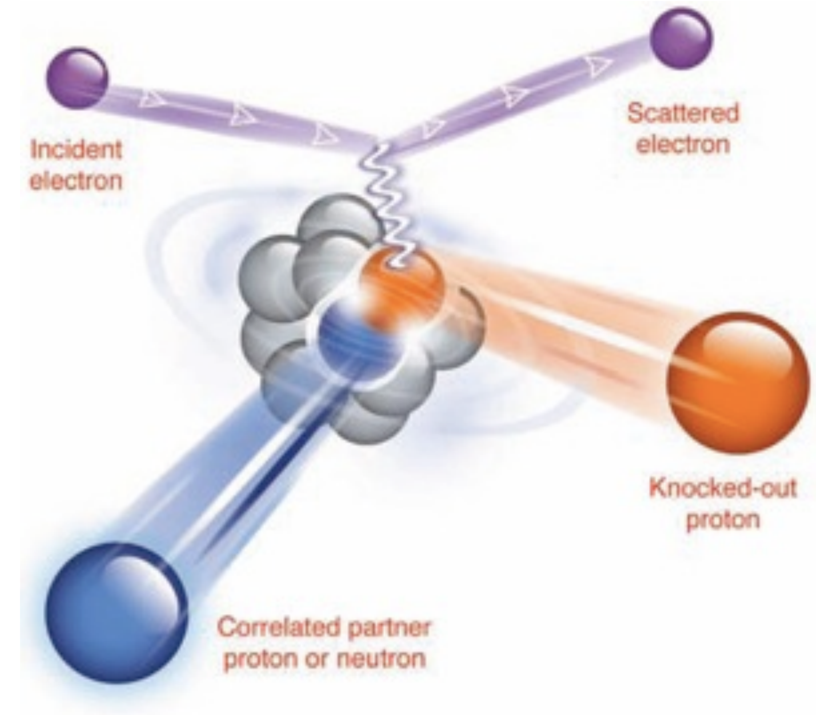
- contradicts our nucleonic matter results!
- convergence problems in RG evolution of 3N interactions in oscillator basis?
- current project: evolve 3N interaction in plane-wave basis
- similar technology to solving the A=3 Schroedinger (Faddeev) equations
- allows systematic investigation of flow of low-energy couplings and provides matrix elements suitable for finite nuclei and infinite-matter calculations
- makes it possible to study the evolution of operators like densities

Correlations in nuclear systems

What is this vertex?



Higinbotham, arXiv:1010.4433

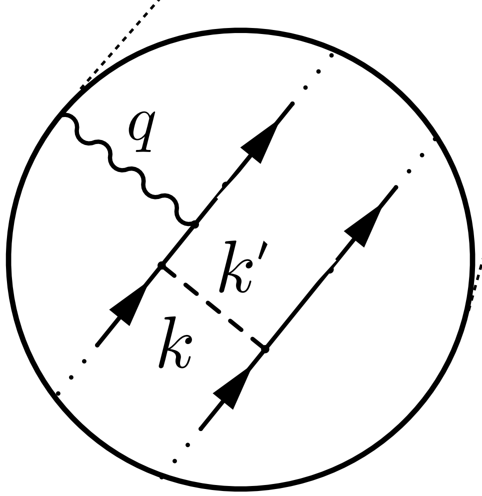


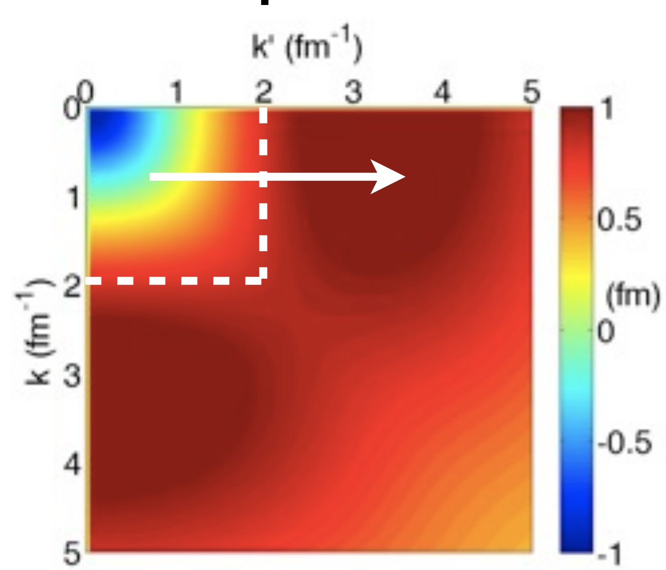
- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

Subedi et al., Science 320, 1476 (2008)

Explanation in terms of low-momentum interactions?

Short-range-correlation interpretation:



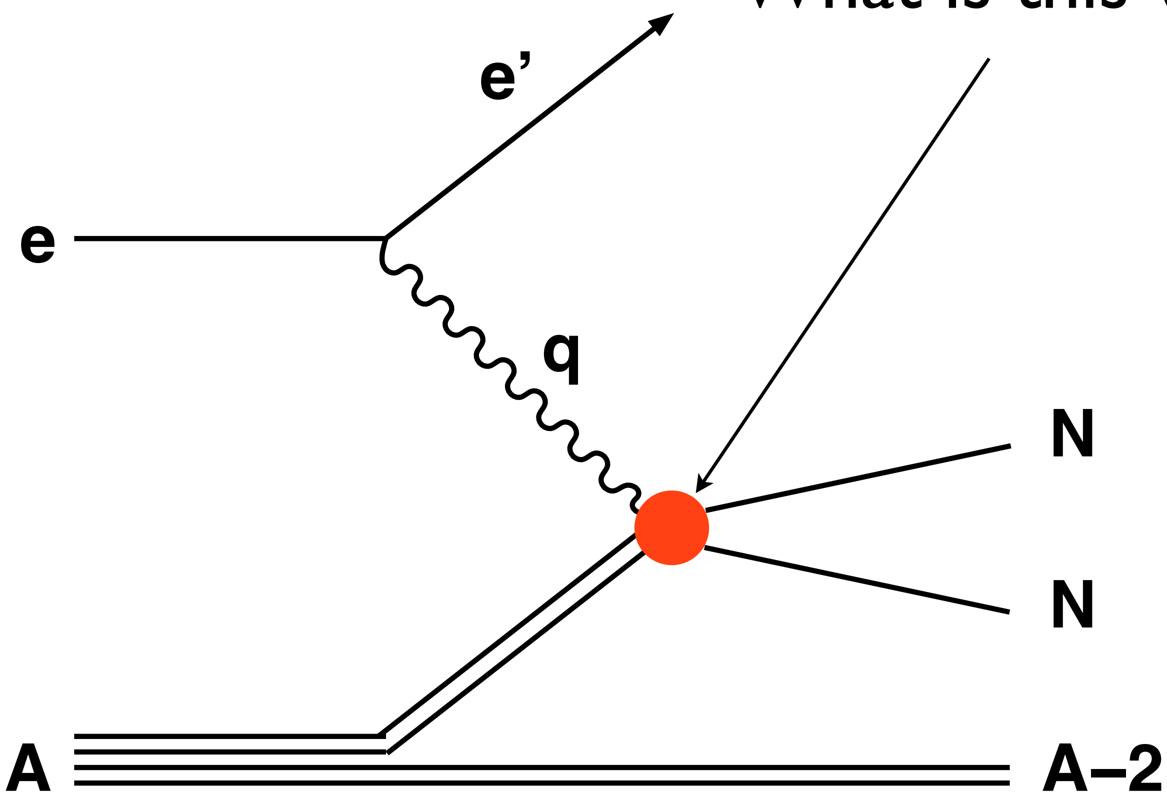


k : low rel. momentum

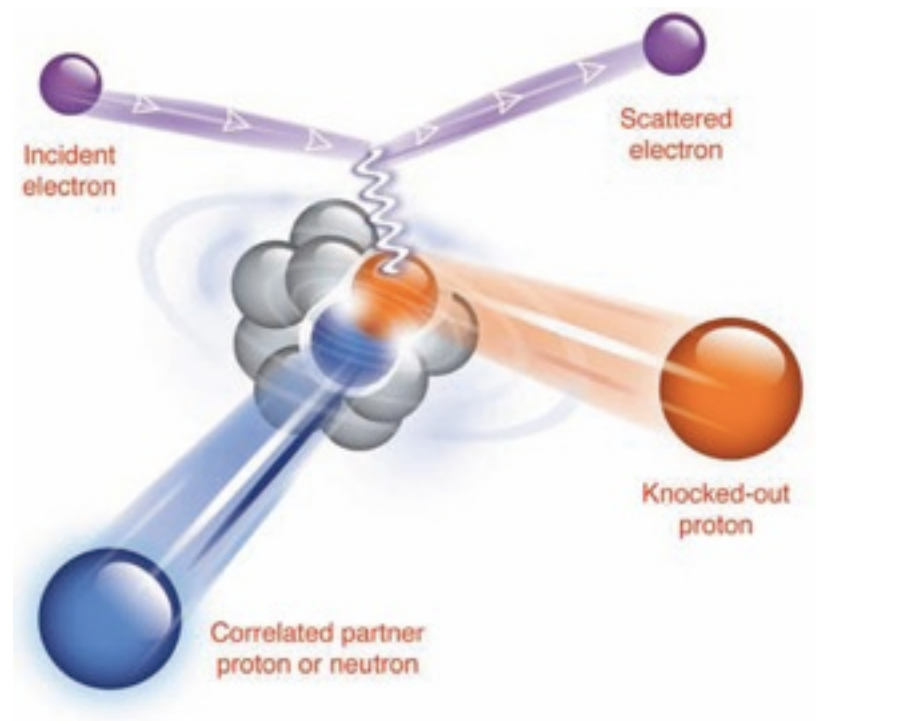
k' : high rel. momentum

Correlations in nuclear systems

What is this vertex?



Higinbotham, arXiv:1010.4433



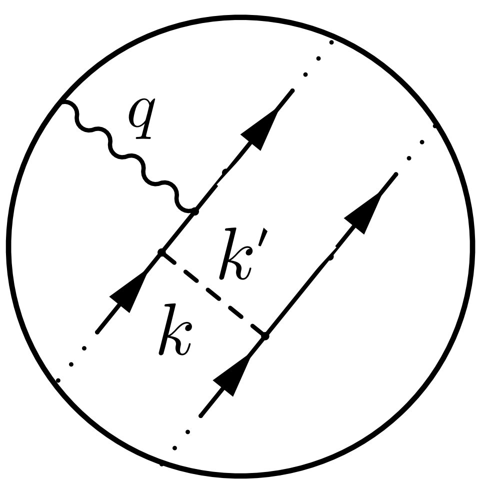
- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

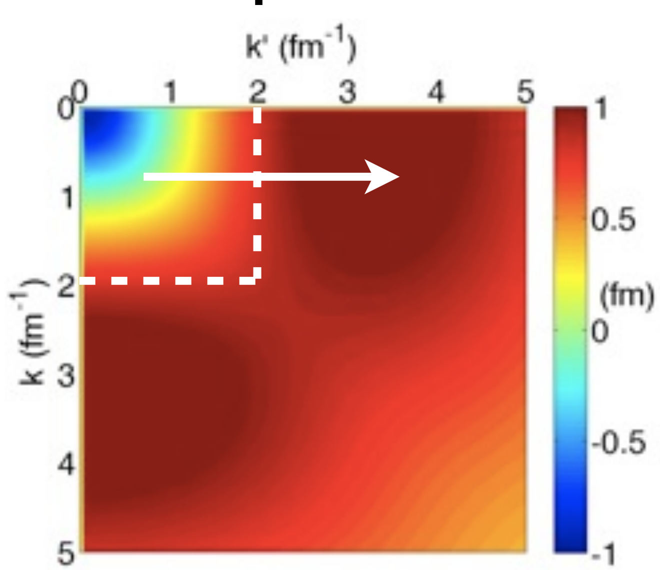
Subedi et al., Science 320, 1476 (2008)

Explanation in terms of low-momentum interactions?

Vertex depends on the resolution!
RG provides systematic way to calculate such processes at low resolution.

Short-range-correlation interpretation:

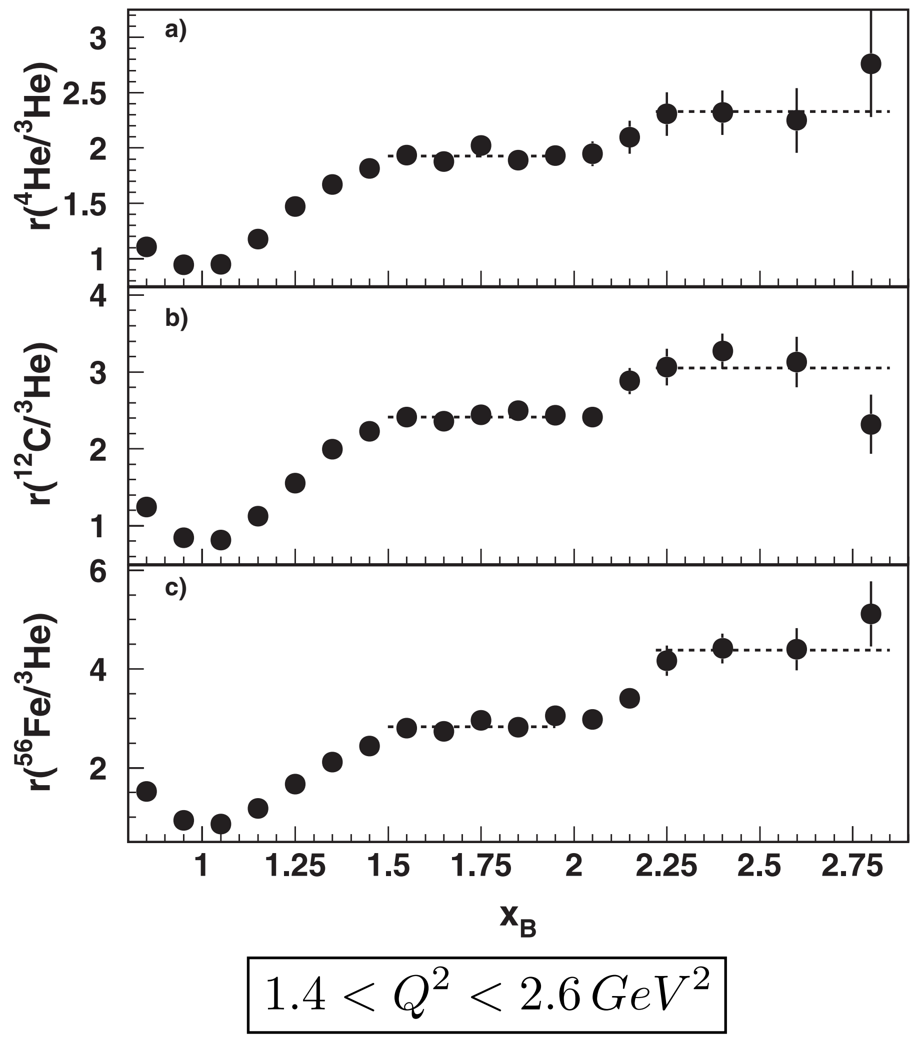




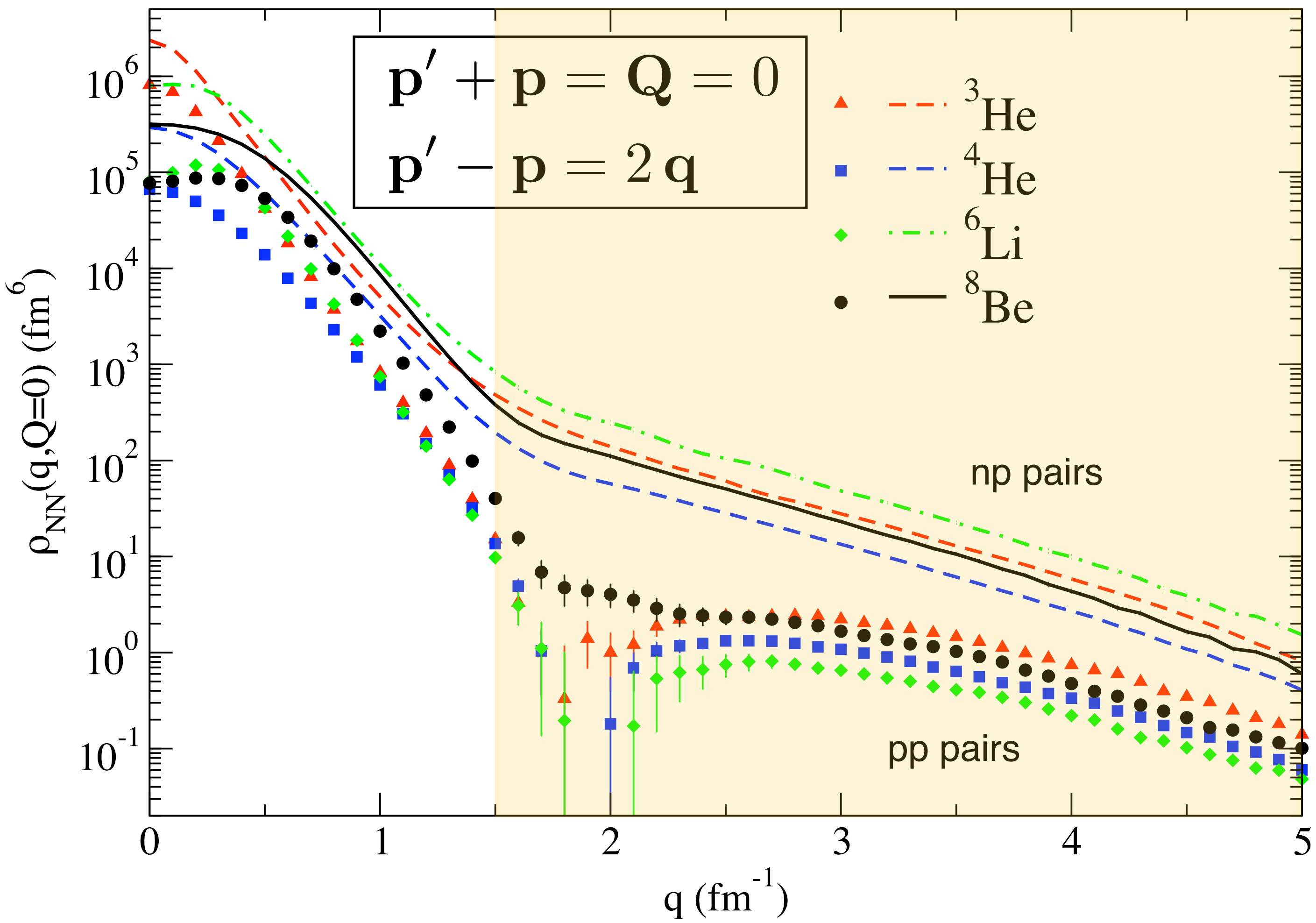
k : low rel. momentum

k' : high rel. momentum

Scaling in nuclear systems



Egiyan et al. PRL 96, 1082501 (2006)



Schiavilla et al., PRL 98, 132501 (2007)

- scaling behavior of momentum distribution function:

  $\rho_{\rm NN}(q, Q=0) \approx C_A \times \rho_{\rm NN, Deuteron}(q, Q=0)$ at large q

- dominance of np pairs over pp pairs
- “hard” (high resolution) interaction used, calculations hard!
- dominance explained by short-range tensor forces

Nuclear scaling at low resolution

$\langle \psi_\lambda | O_\lambda | \psi_\lambda \rangle$ **factorizes** into a low-momentum structure and a **universal** high momentum part if the initial operator only weakly couples low and high momenta $\longrightarrow$ explains scaling!

key: $U_\lambda(k, q) \approx K(k)Q(q)$ for $k < \lambda$ and $q \gg \lambda$

factorization!

That leads to:

$$
\begin{aligned}
\langle \psi_\lambda | \, O_\lambda \, | \psi_\lambda \rangle &= \int_0^\lambda dk \, dk' \int_0^\infty dq \, dq' \psi^\dagger(k) U_\lambda(k, q) O(q, q') U_\lambda(q', k') \psi_\lambda(k) \\
&\approx \int_0^\lambda dk \, dk' \psi^\dagger(k') \left[\int_0^\lambda dq \, dq' K(k) K(q) O(q, q') K(q') K(k') + I_{QOQ} K(k) K(k') \right] \psi^\dagger(k)
\end{aligned}
$$

with the **universal** quantity:

$$
I_{QOQ} = \int_\lambda^\infty dq \, dq' Q(q) O(q, q') Q(q')
$$

valid if initial operator weakly couples low and high momenta

Nuclear scaling at low resolution

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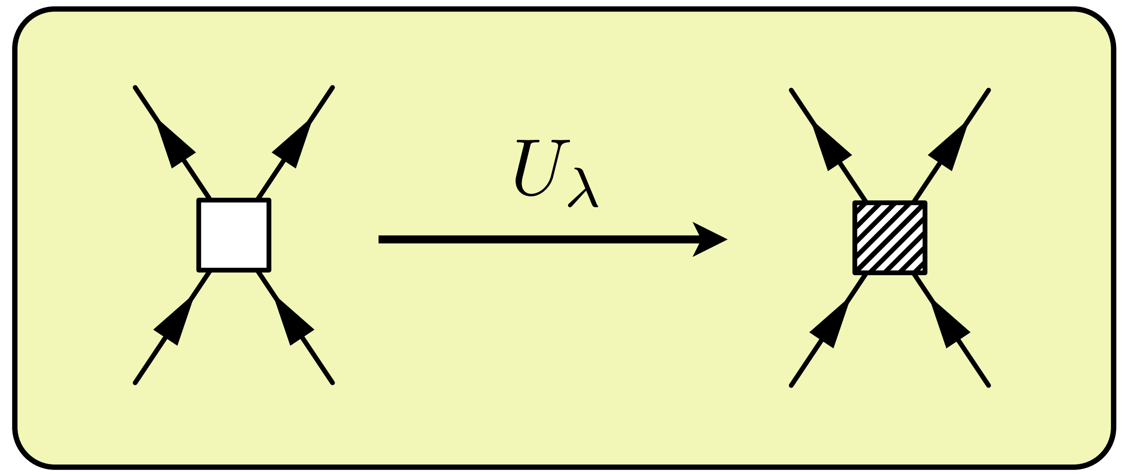
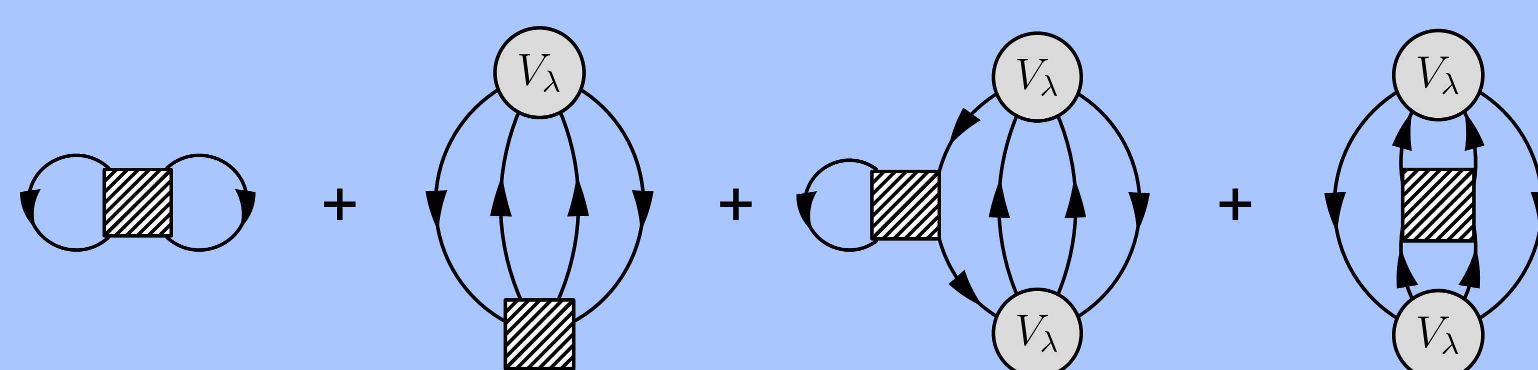
RG transformation of pair density operator (induced many-body terms neglected):

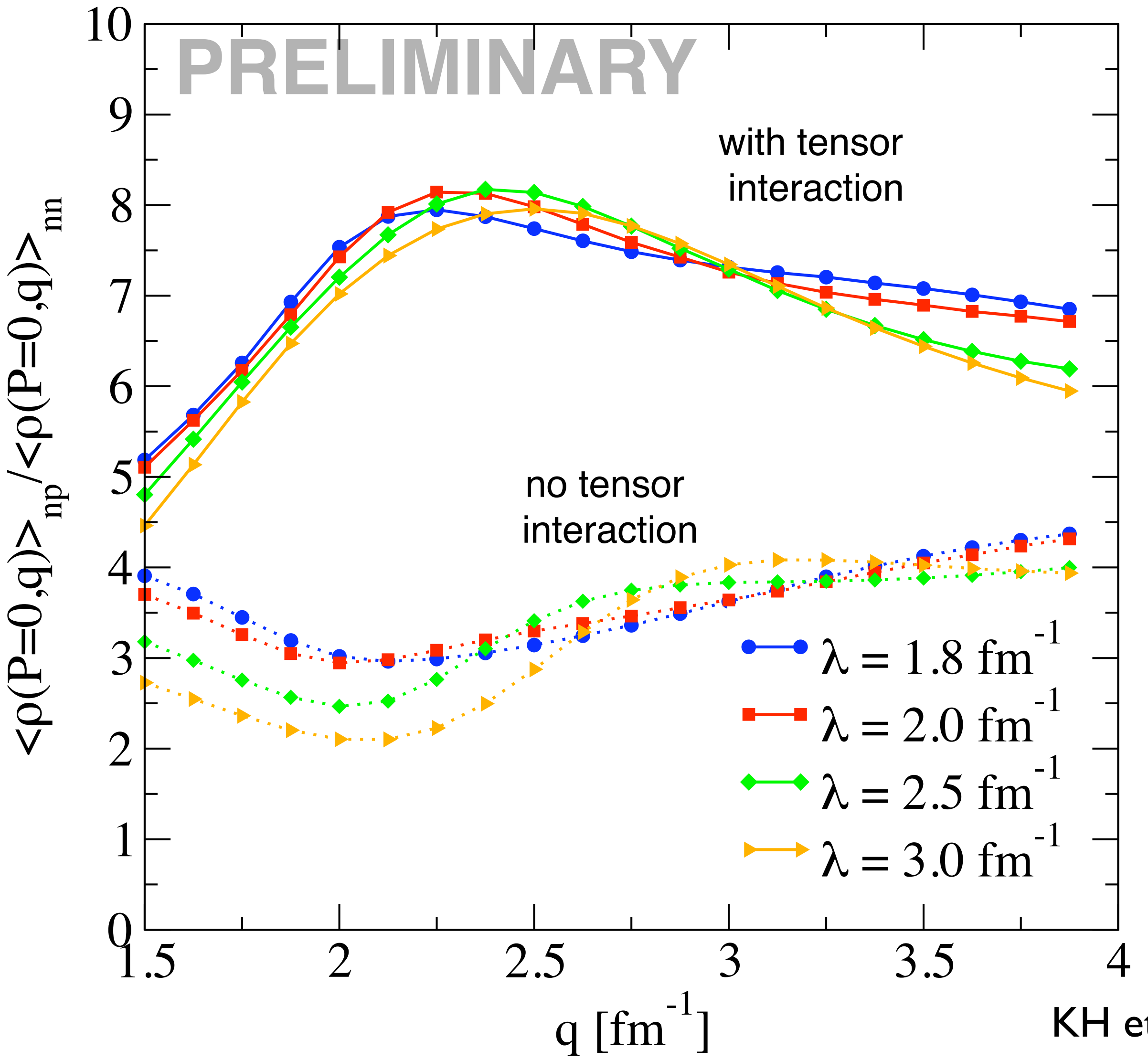


"simple" calculation of pair density at low resolution in nuclear matter:

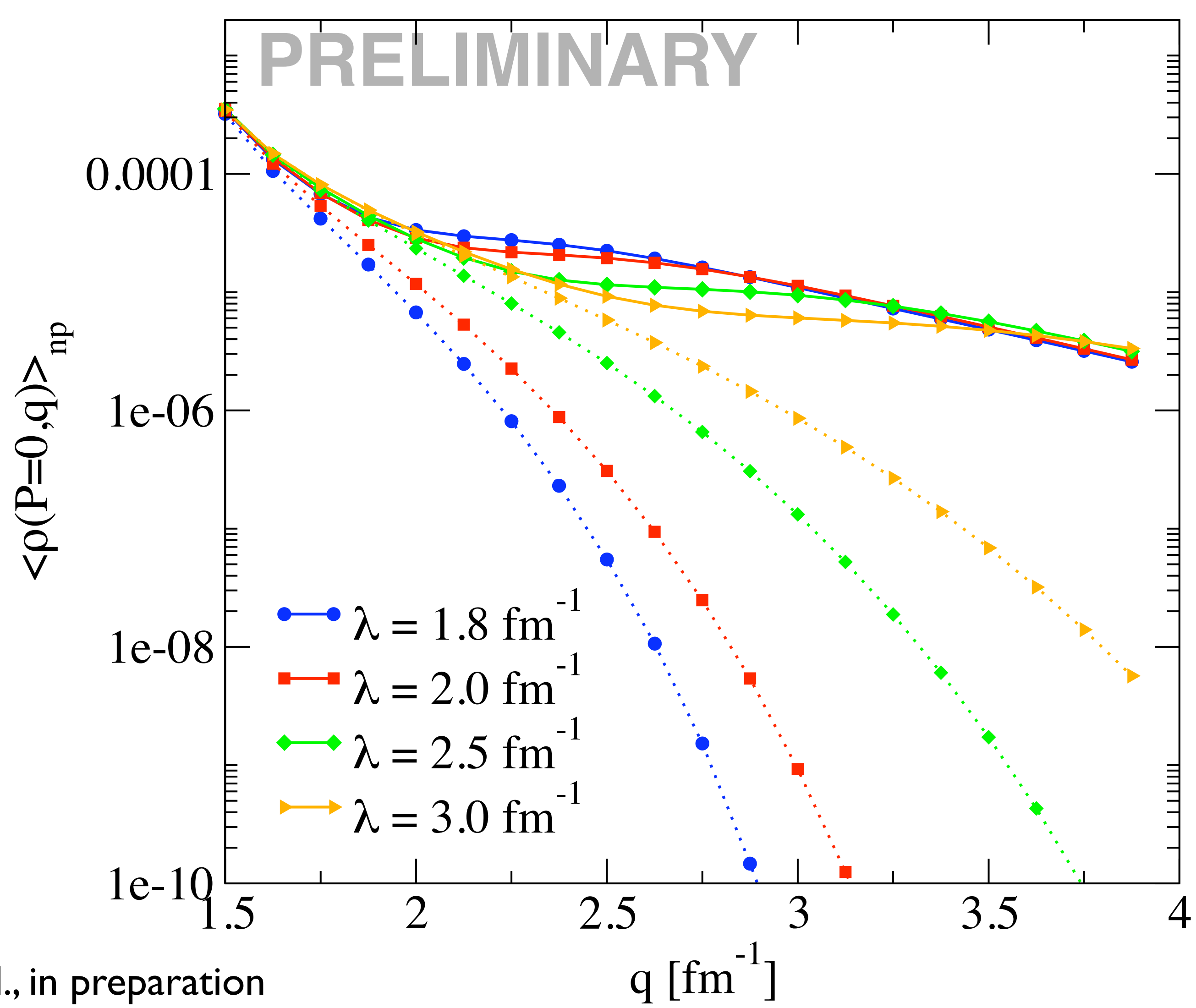
$\langle \rho(\mathbf{P}, \mathbf{q}) \rangle$ =



Nuclear scaling at low resolution



KH et al., in preparation



- pair-densities approximately resolution independent
- significant enhancement of np pairs over nn pairs due to tensor force
- reproduction of previous results using a “simple” calculation at low resolution!

High-resolution experiments can be explained by low-resolution methods!
Opens door to study other electro-weak processes and higher-body correlations.

Summary

- low-resolution interactions allow simpler calculations for nuclear systems
- observables invariant under changes in resolution scale, interpretation can change!
- chiral EFT provides systematic framework for constructing nuclear Hamiltonians
- 3N interactions are essential at low resolution
- nuclear matter equation of state consistent with empirical constraints
- constraints for the nuclear equation of state and radii of neutron stars