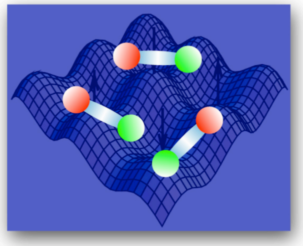
Renormalization group flow of spectral functions for ultracold quantum gases

Phys. Rev. A 83, 063620 (2011)

Richard Schmidt and Tilman Enss
(group of Wilhelm Zwerger)

Workshop on Renormalization Group Approach
from Ultra Cold Atoms to the Hot QGP
Kyoto, Japan
31.08.2011

Technische Universität München



DFG - FOR 801

Physik Department - T34

Content

the Fermi polaron

fRG for flowing spectral functions

analysis of the quasi-particle properties

rf-spectroscopy and experiment

outlook

Content

the Fermi polaron

fRG for flowing spectral functions

analysis of the quasi-particle properties

rf-spectroscopy and experiment

$\xrightarrow{\textbf{ERG 2010}}$ comments on derivative expansion and regulator dependence

outlook

the Fermi polaron

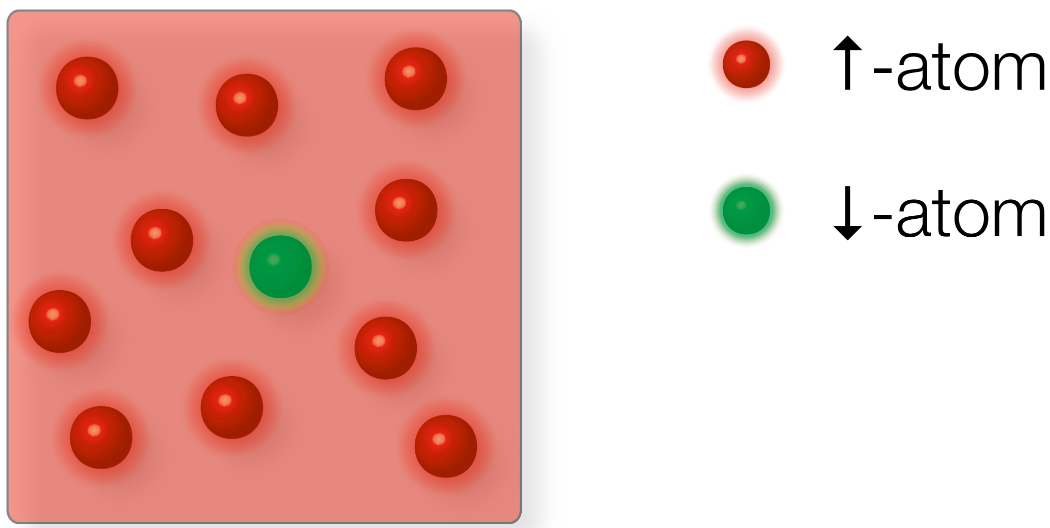
the Fermi polaron

Fermi polaron: single ↓-atom in ↑-Fermi gas

(ultracold Fermi gas $T \sim 100\mathrm{nK}$, non-relativistic)

limit of extreme population imbalance

⟶ SPIN BALANCED SYSTEM - BEC/BCS CROSSOVER: SEE TALK BY **MICHAEL SCHERER**



tunable (strong) interactions; characterized by scattering length a

→ simple system

→ nice benchmark system for non-perturbative methods

the Fermi polaron

two-body problem

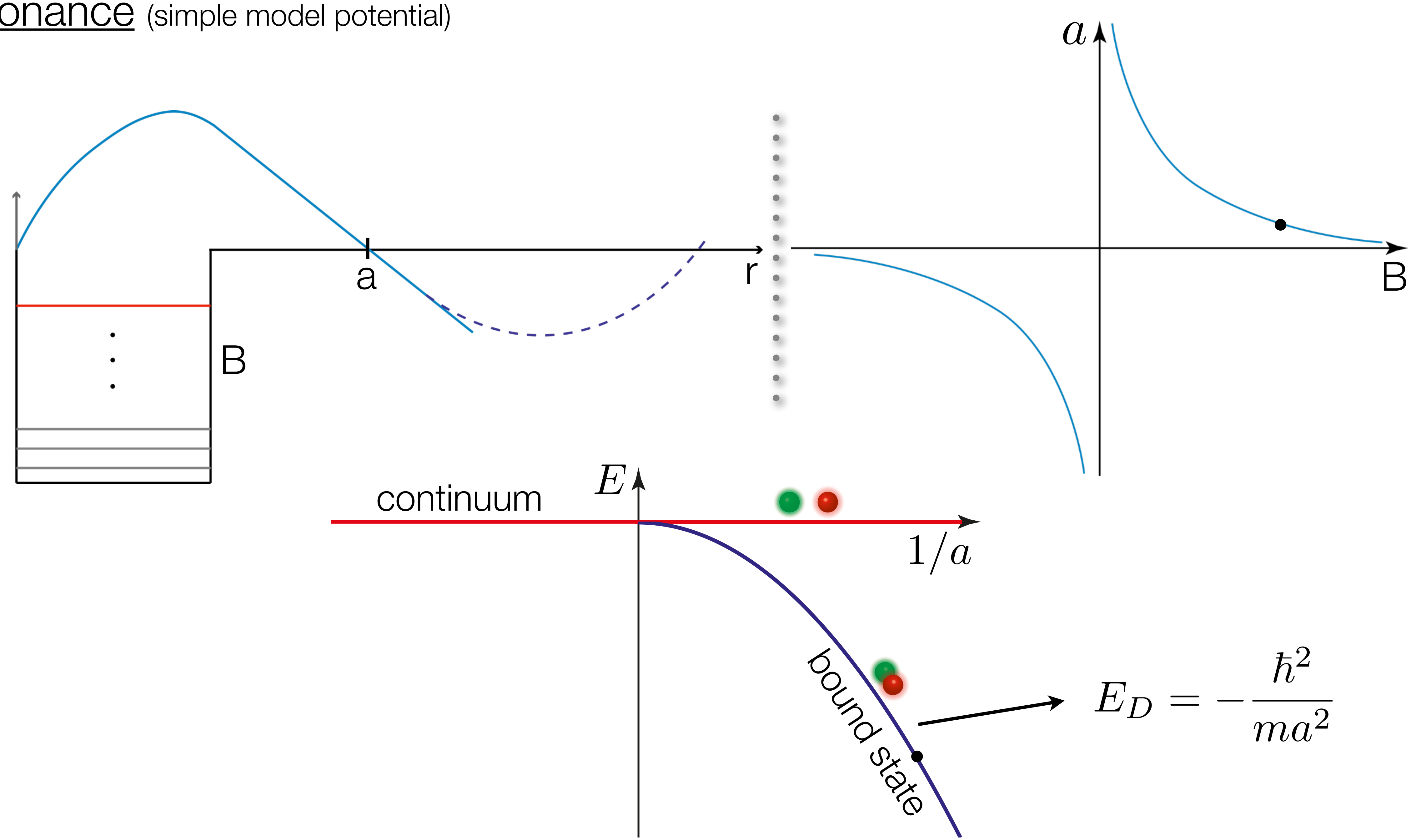
- s-wave interactions universally described by scattering length a
- scattering length a tunable via Feshbach resonances

the Fermi polaron

two-body problem

- s-wave interactions universally described by scattering length a
- scattering length a tunable via Feshbach resonances

<u>shape resonance</u> (simple model potential)



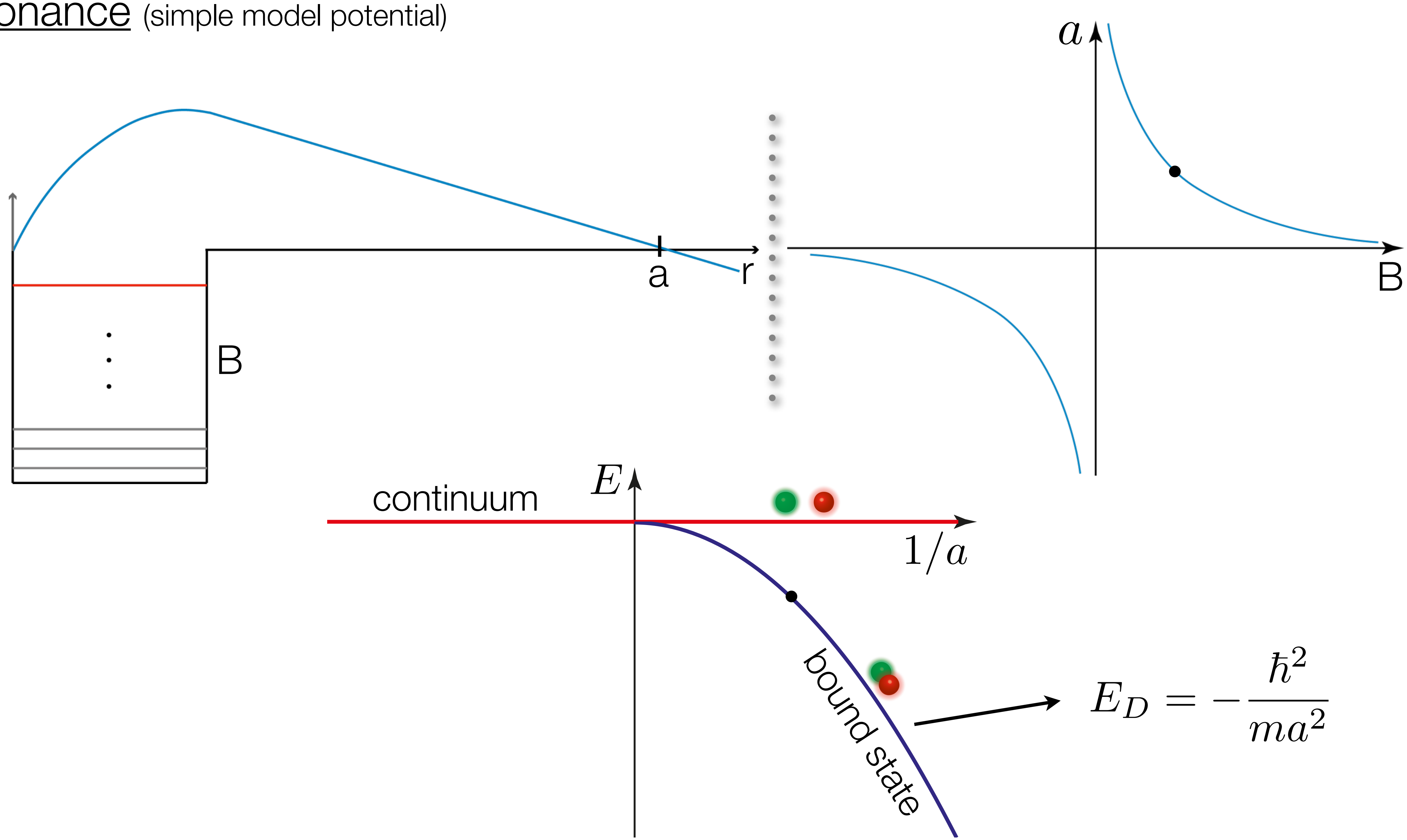
$$E_D = -\frac{\hbar^2}{ma^2}$$

the Fermi polaron

two-body problem

- s-wave interactions universally described by scattering length a
- scattering length a tunable via Feshbach resonances

<u>shape resonance</u> (simple model potential)

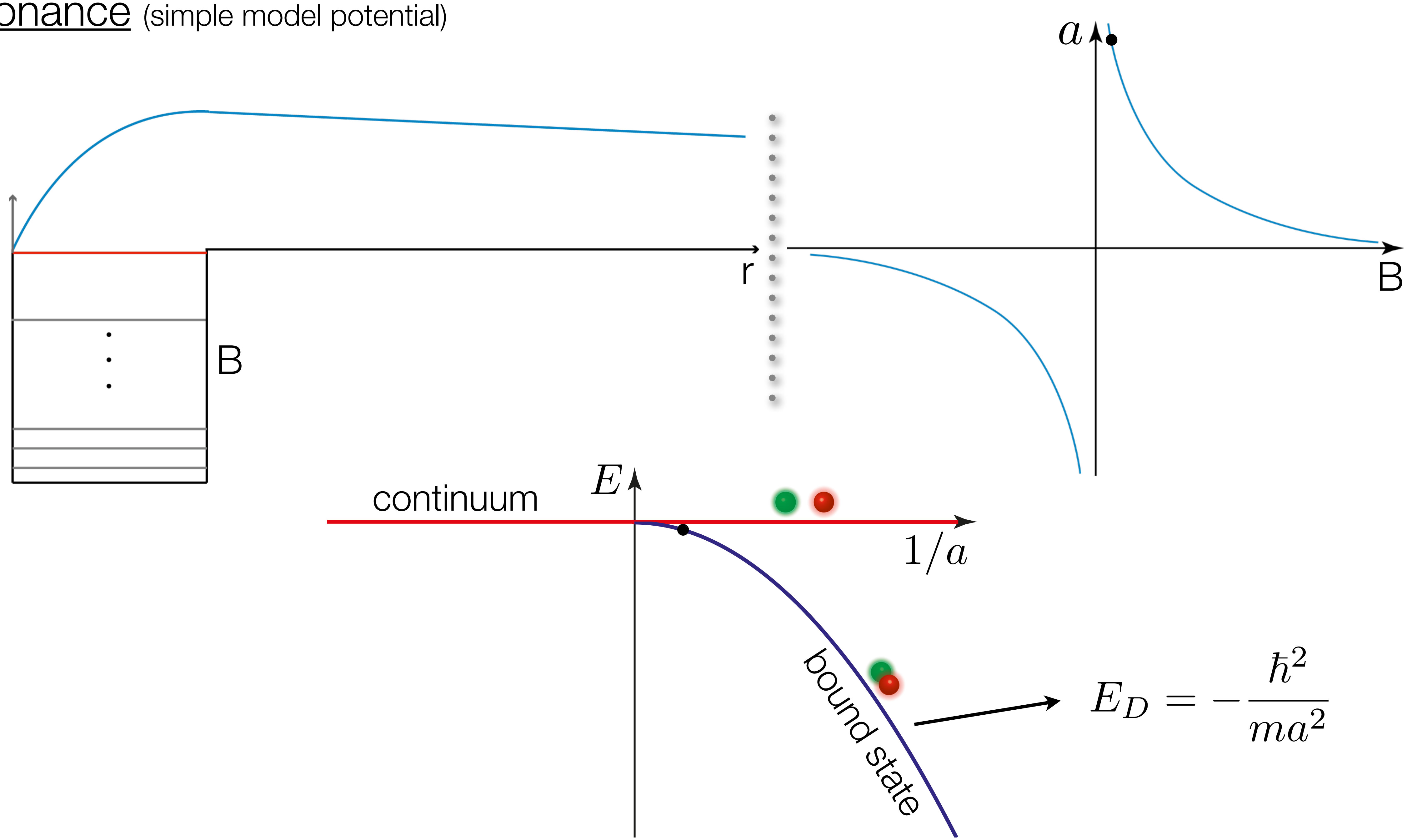


the Fermi polaron

two-body problem

- s-wave interactions universally described by scattering length a
- scattering length a tunable via Feshbach resonances

<u>shape resonance</u> (simple model potential)



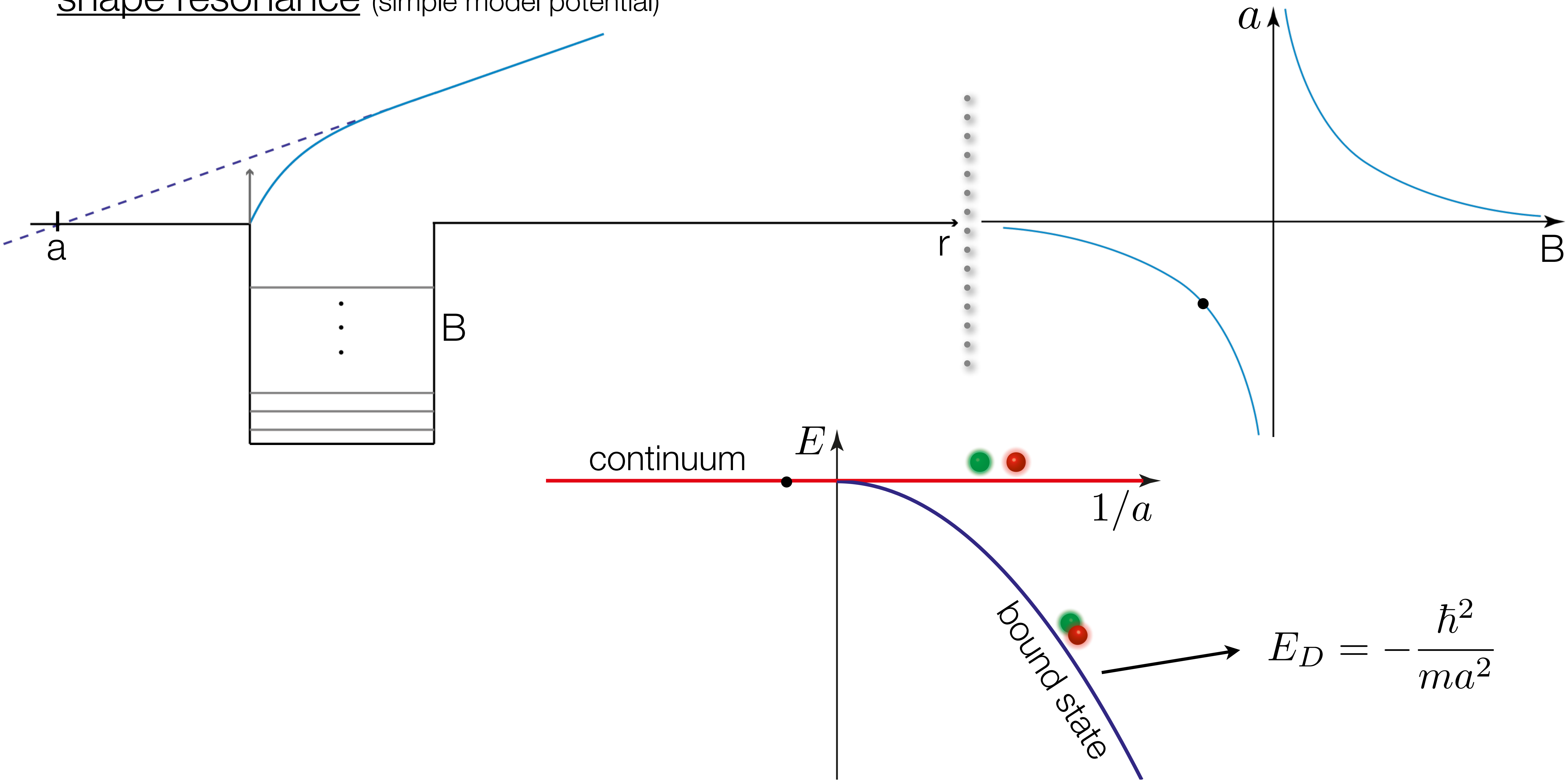
$$E_D = -\frac{\hbar^2}{ma^2}$$

the Fermi polaron

two-body problem

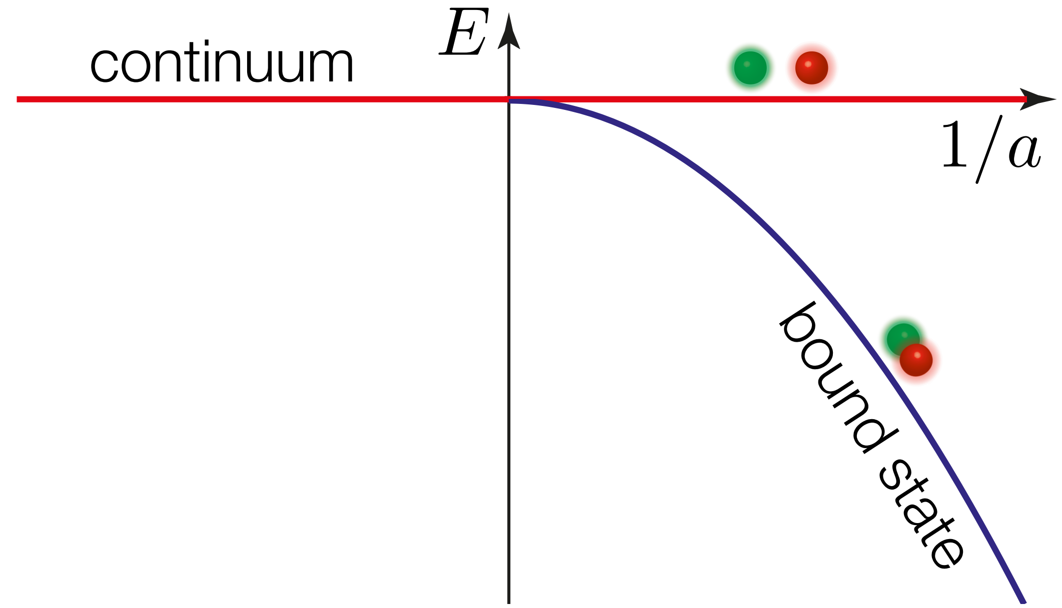
- s-wave interactions universally described by scattering length a
- scattering length a tunable via Feshbach resonances

<u>shape resonance</u> (simple model potential)



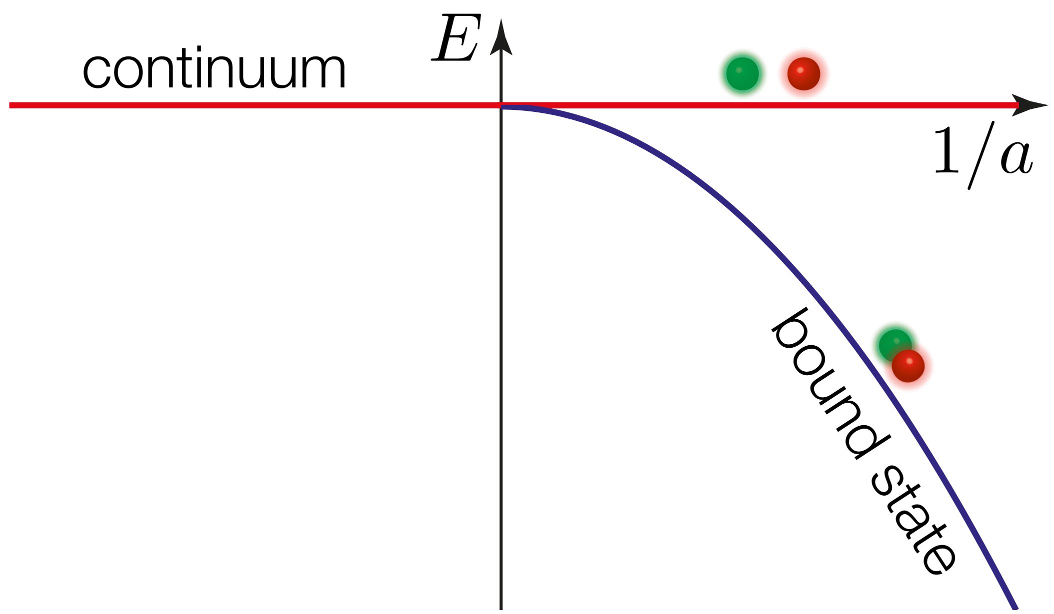
the Fermi polaron

two-body problem

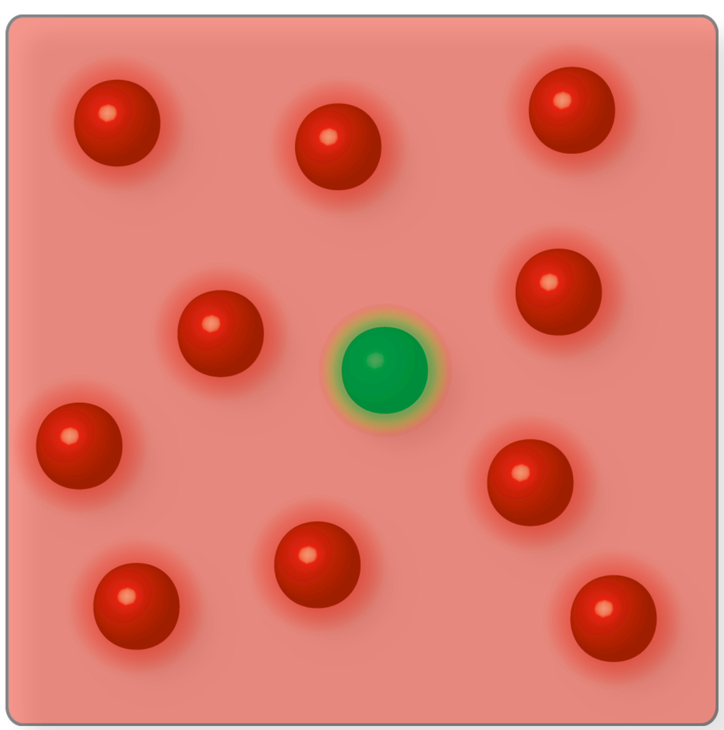


the Fermi polaron

two-body problem



finite density of ↑-atoms $\vartheta = (k_F a)^{-1}$

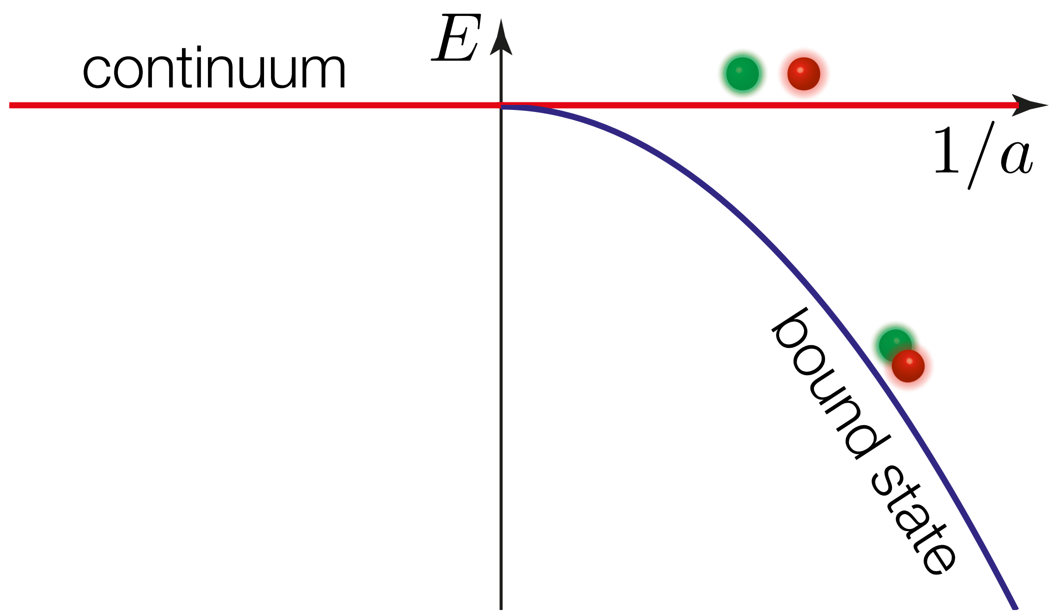


$\vartheta \ll -1$

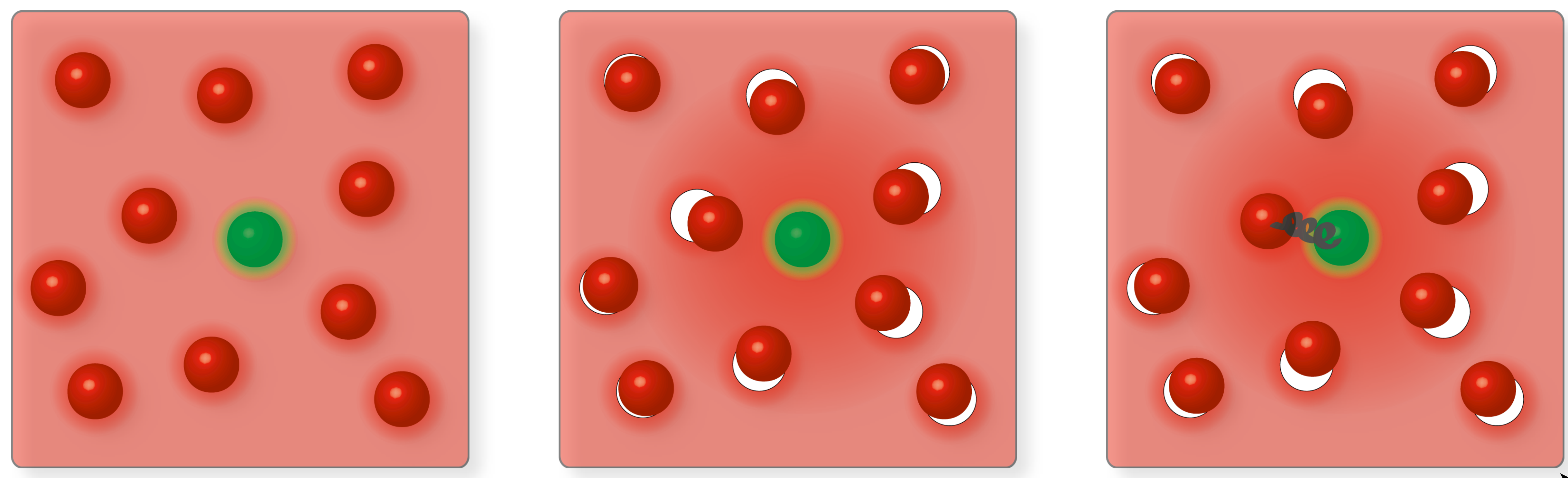
free propagation

the Fermi polaron

two-body problem



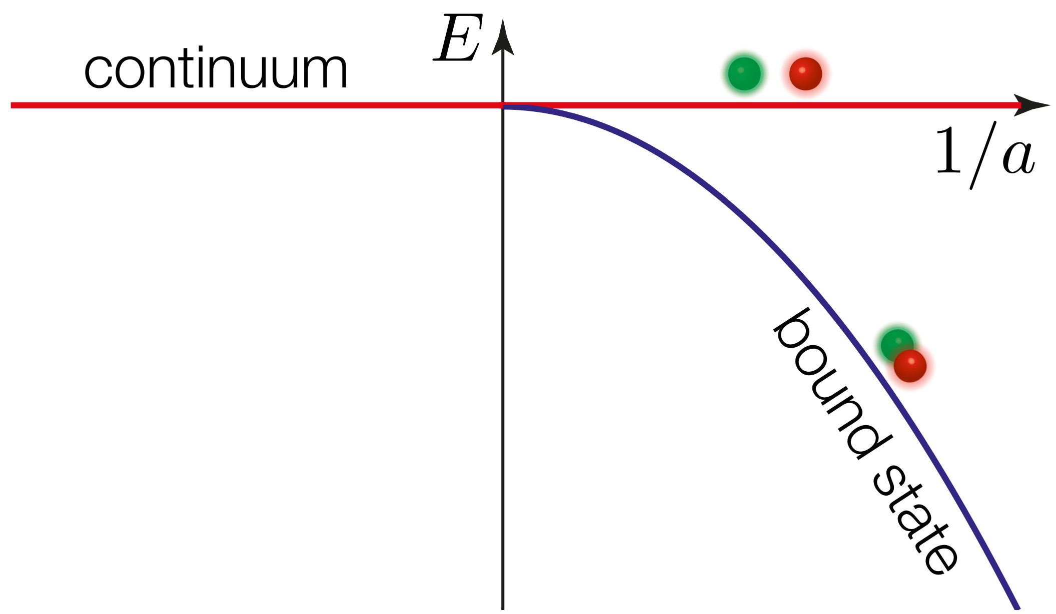
finite density of ↑-atoms $\vartheta = (k_F a)^{-1}$



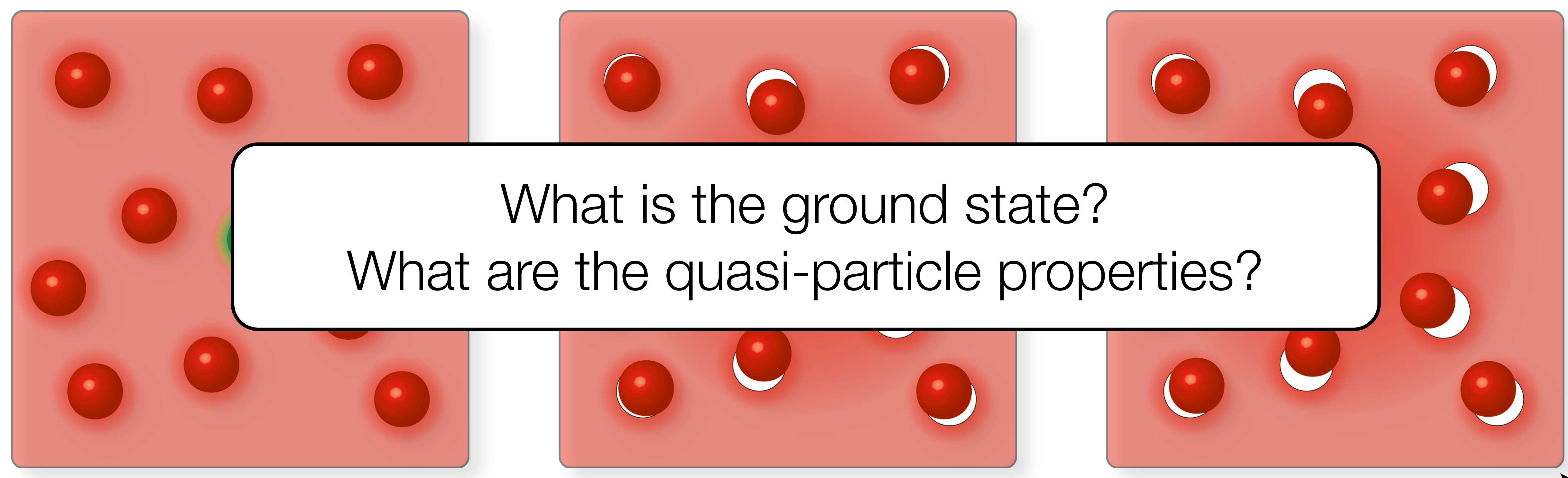
$\vartheta \ll -1$ free propagation | $\vartheta = 0$ renormalized quasiparticle | $\vartheta \gtrsim 1$ singlet bound state

the Fermi polaron

two-body problem



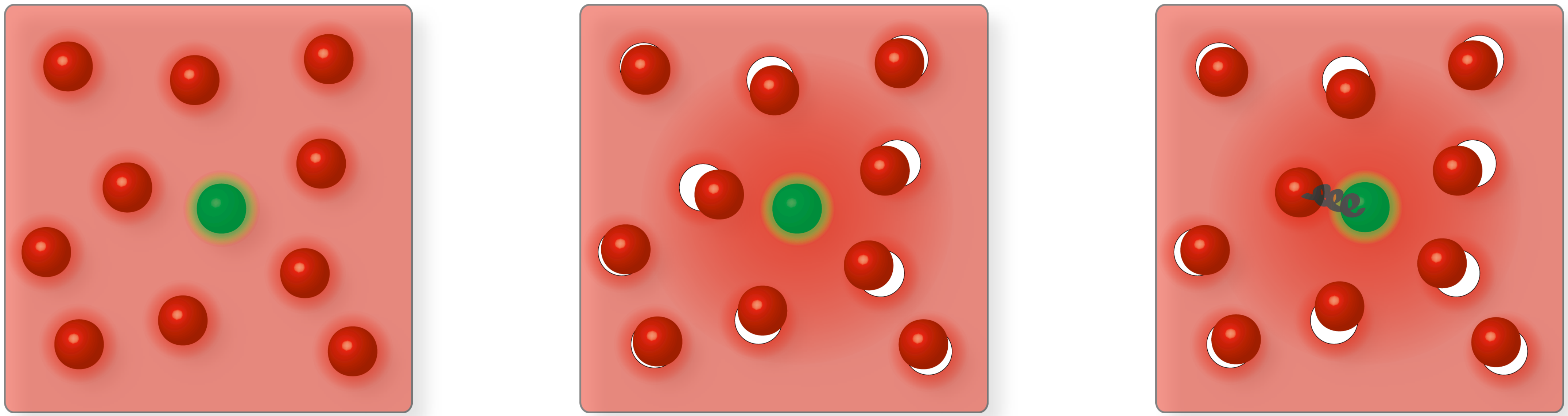
finite density of ↑-atoms $\vartheta = (k_F a)^{-1}$



$\vartheta \ll -1$	$\vartheta = 0$	$\vartheta \gtrsim 1$
free propagation	renormalized quasiparticle	singlet bound state

spectral function and quasi-particle properties

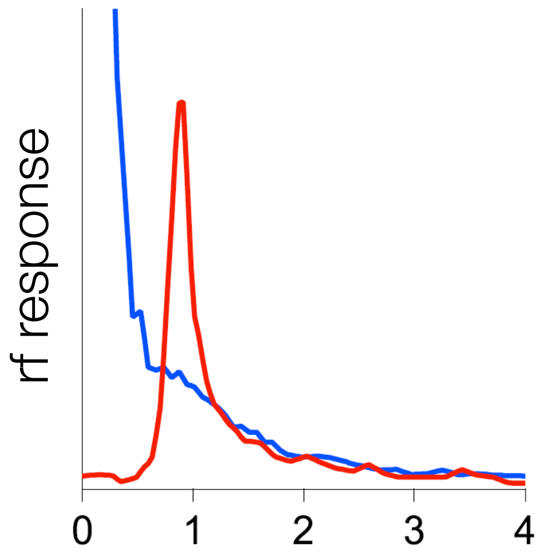
experiment: radio frequency spectroscopy

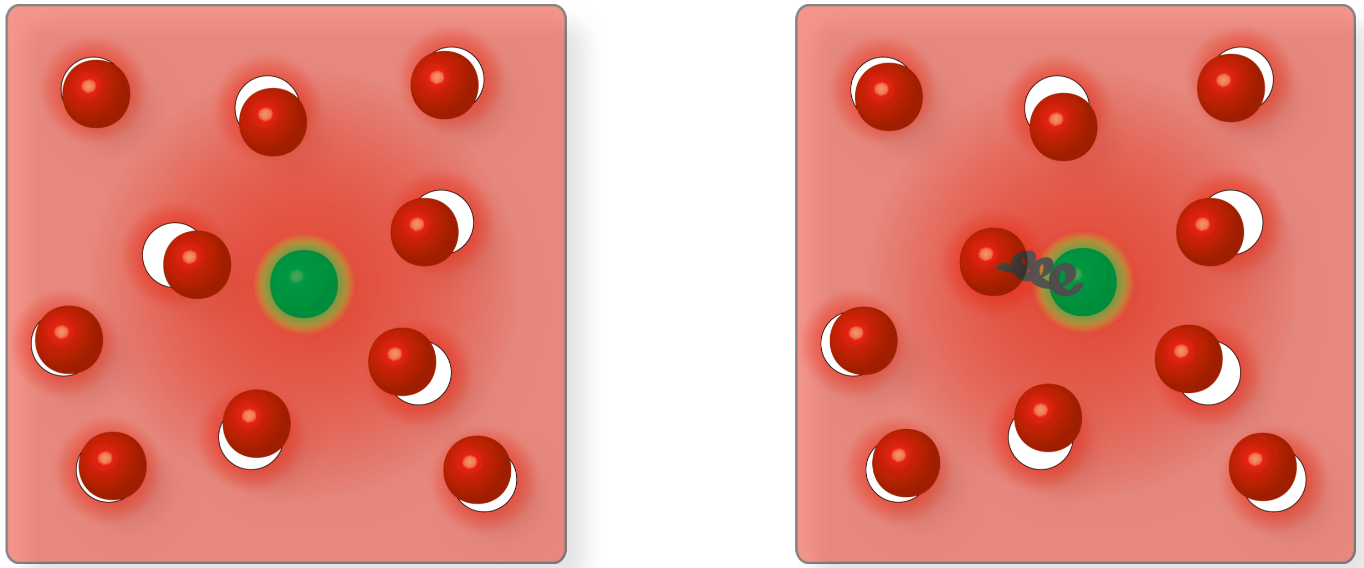


Zwierlein Group, MIT (2009)

spectral function and quasi-particle properties

experiment: radio frequency spectroscopy

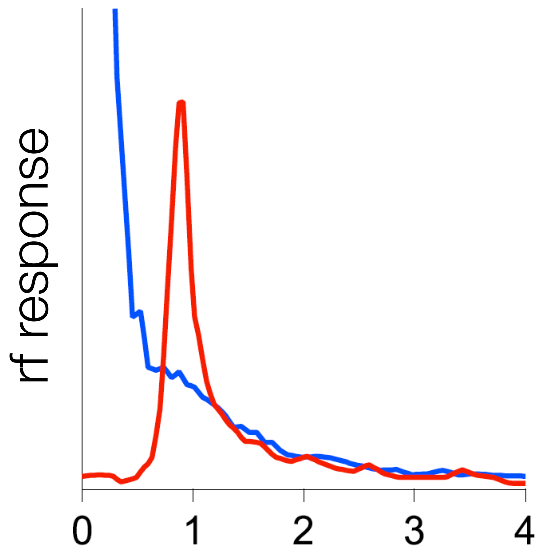


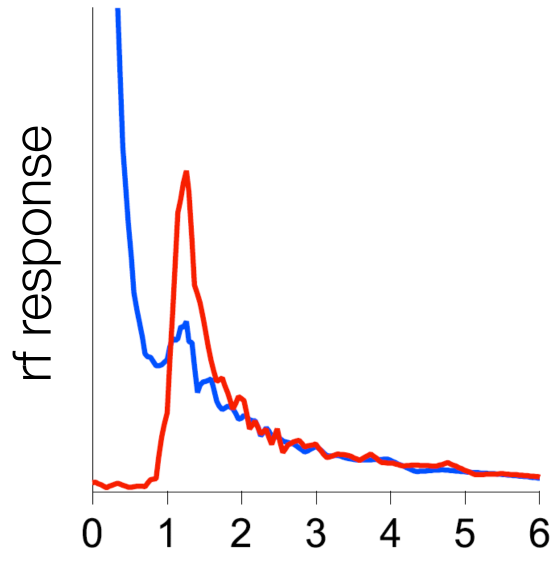


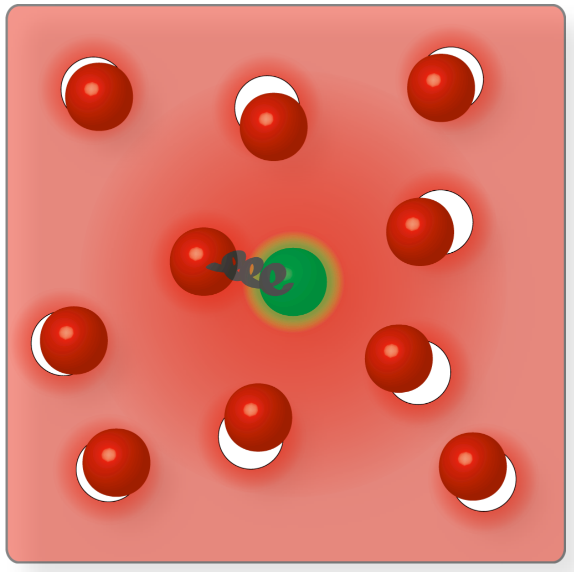
ZWIERLEIN GROUP, MIT (2009)

spectral function and quasi-particle properties

experiment: radio frequency spectroscopy



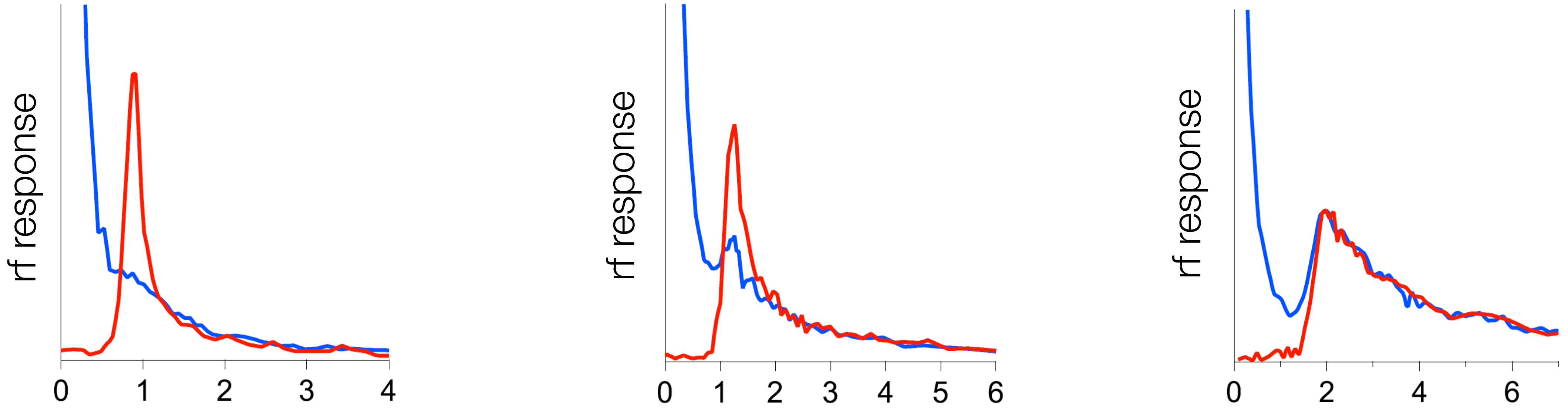




Zwierlein Group, MIT (2009)

spectral function and quasi-particle properties

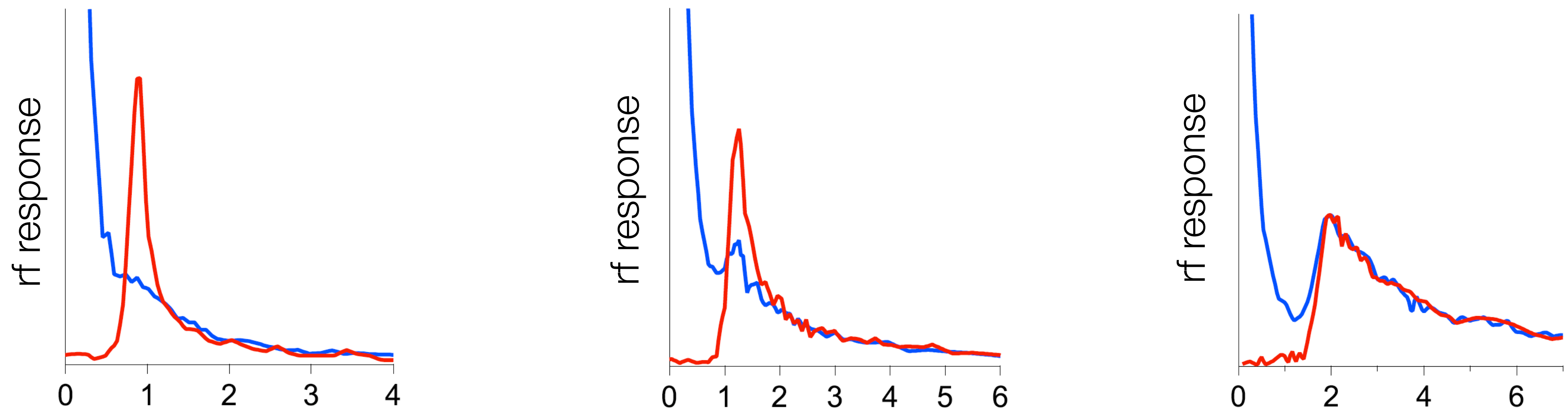
experiment: radio frequency spectroscopy



ZWIERLEIN GROUP, MIT (2009)

spectral function and quasi-particle properties

experiment: radio frequency spectroscopy



ZWIERLEIN GROUP, MIT (2009)

theory:

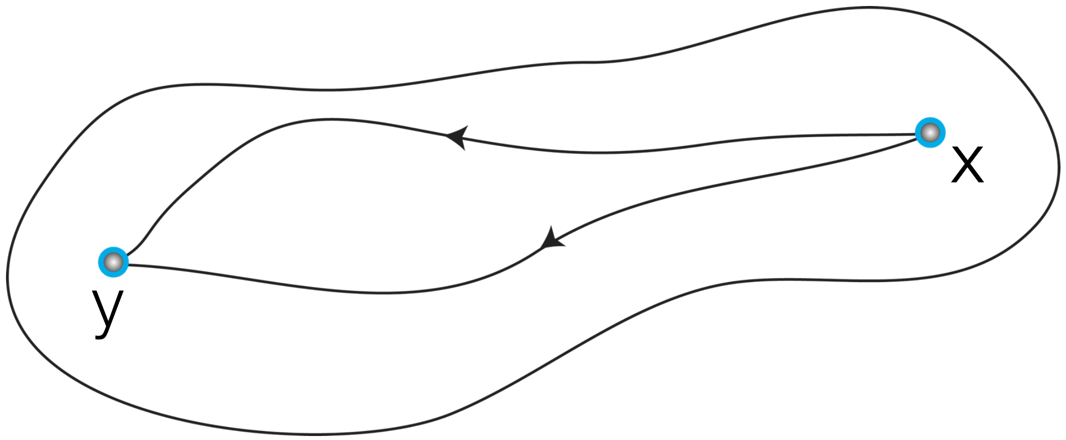
quasi-particle properties from Green's function: $G(x,y) = \left.\frac{\delta^2\Gamma[\phi]}{\delta\phi(x)\delta\phi(y)}\right|_{\phi=0}$

$\rightarrow$ spectral function: $\mathcal{A}(\omega,\mathbf{p}) = 2\mathrm{Im}G_R(\omega,\mathbf{p})$

Spectral function

Green's function - determines propagation of particle

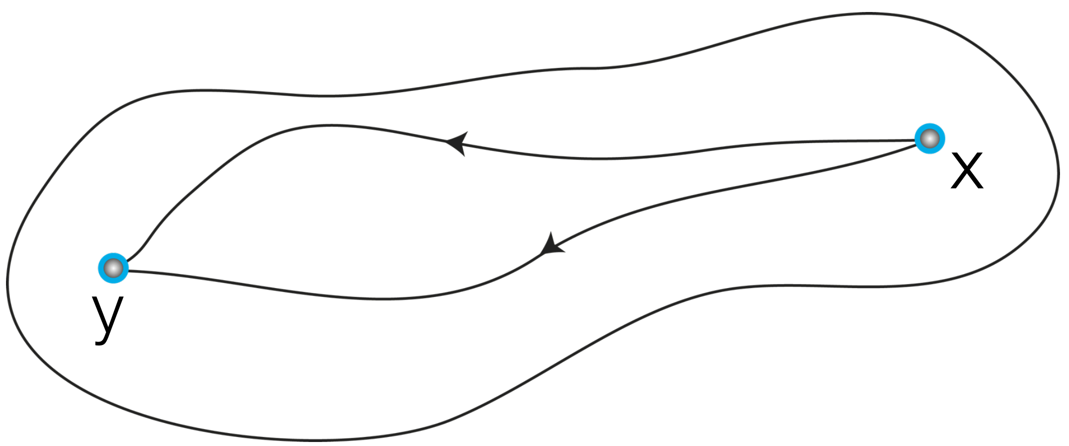
$$G(x,y) =< 0|\phi^*(y)\phi(x)|0 >$$



Spectral function

Green's function - determines propagation of particle

$$G(x,y) =< 0|\phi^*(y)\phi(x)|0 >$$



momentum space:

$$G(\omega,\mathbf{p}) = \int \frac{dE}{2\pi}\frac{\mathcal{A}(E,\mathbf{p})}{E-\omega}$$

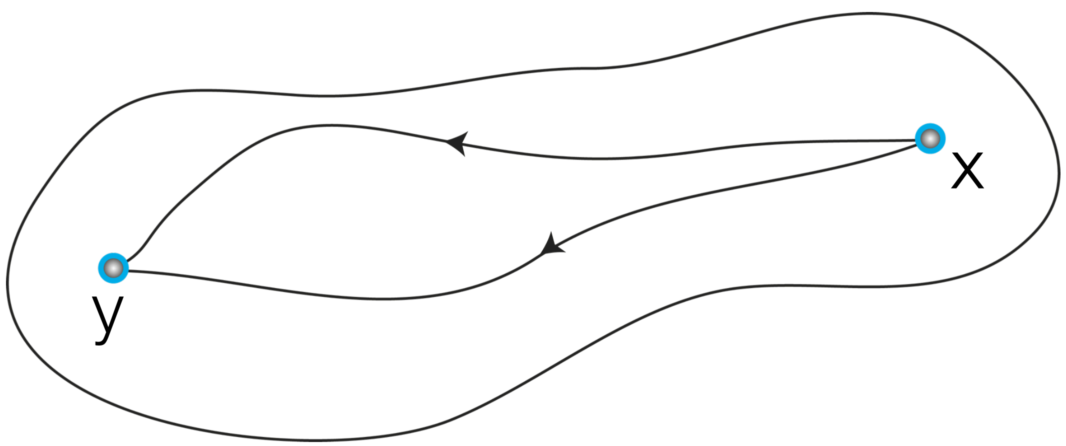
spectral function $\mathcal{A}(E,\mathbf{p})$:

- determines where particle can ‚live' in $E/\mathbf{p}$ plane
- determines dispersion

Spectral function

Green‘s function - determines propagation of particle

$$G(x,y) =< 0|\phi^*(y)\phi(x)|0 >$$

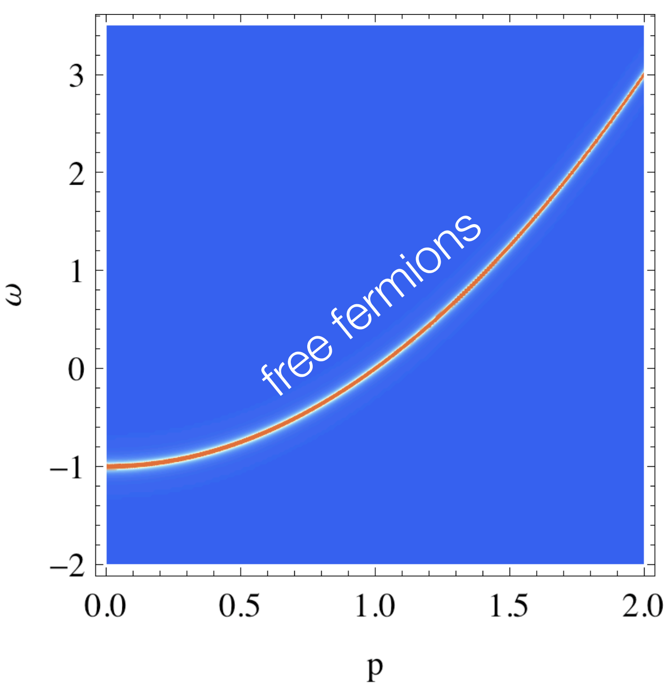


momentum space:

$$G(\omega,\mathbf{p}) = \int \frac{dE}{2\pi}\frac{\mathcal{A}(E,\mathbf{p})}{E-\omega} = \int \frac{dE}{2\pi}\frac{2\pi\delta(E-(\mathbf{p}^2-\mu))}{E-\omega} = \frac{1}{-\omega+\mathbf{p^2}-\mu}$$

spectral function $\mathcal{A}(E,\mathbf{p})$:

- determines where particle can ‚live‘ in $E/\mathbf{p}$ plane
- determines dispersion



fRG for flowing spectral functions

Fermi polaron: truncation

- two-component Fermi gas with contact interaction, classical (UV) action

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma]\psi_\sigma + g \int_{\mathrm{X}} \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow$$

$\boxed{2m = 1}$

non-relativistic

Fermi polaron: truncation

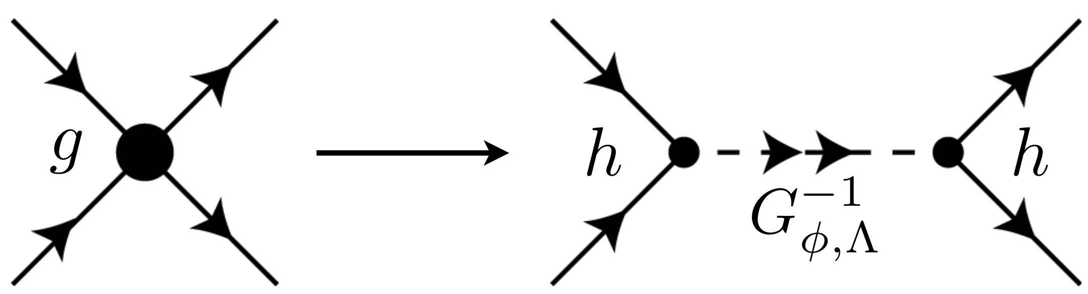
- two-component Fermi gas with contact interaction, classical (UV) action

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma]\psi_\sigma + g \int_X \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow \qquad \boxed{2m = 1}$$

non-relativistic

- Hubbard-Stratonovich transformation:
  interaction replaced by exchange of (virtual) molecule

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma]\psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1} \phi + h \int_X (\psi_\uparrow^* \psi_\downarrow^* \phi + h.c.)$$



Fermi polaron: truncation

- two-component Fermi gas with contact interaction, classical (UV) action

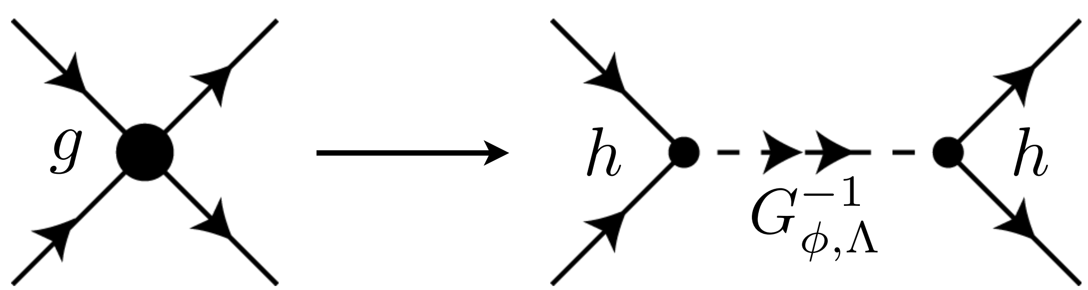
$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma]\psi_\sigma + g \int_X \psi_\uparrow^* \psi_\downarrow^* \psi_\downarrow \psi_\uparrow$$

$\boxed{2m = 1}$

non-relativistic

- Hubbard-Stratonovich transformation:
  interaction replaced by exchange of (virtual) molecule

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma]\psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1} \phi + h \int_X (\psi_\uparrow^* \psi_\downarrow^* \phi + h.c.)$$



- equivalent to original action (integrate out ϕ) for ($h \to \infty$)

$$g = -\frac{h^2}{G_{\phi,\Lambda}^{-1}}$$

Fermi polaron: truncation

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma] \psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1} \phi + h \int_X (\psi_\uparrow^* \psi_\downarrow^* \phi + h.c.)$$

- typical RG approach:
  - expansion of propagators in momentum and frequencies (derivative expansion)
  - good for critical phenomena, only a few couplings

$$\Gamma_{k,\phi,\mathrm{kin}} = \int_{\mathbf{p},\omega} \phi^* [-iA_k\omega + B_k \mathbf{p}^2 + C_k] \phi$$

Fermi polaron: truncation

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma]\psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1}\phi + h\int_X (\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)$$

- typical RG approach:
  - expansion of propagators in momentum and frequencies (derivative expansion)
  - good for critical phenomena, only a few couplings

$$\Gamma_{k,\phi,\mathrm{kin}} = \int_{\mathbf{p},\omega} \phi^*[-iA_k\omega + B_k\mathbf{p}^2 + C_k]\phi$$

- poor spectral resolution: only single (coherent) quasi-particle excitation

Fermi polaron: truncation

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^*[i\omega + p^2 - \mu_\sigma]\psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1}\phi + h\int_X (\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)$$

- typical RG approach:
  - expansion of propagators in momentum and frequencies (derivative expansion)
  - good for critical phenomena, only a few couplings

$$\Gamma_{k,\phi,\mathrm{kin}} = \int_{\mathbf{p},\omega} \phi^*[-iA_k\omega + B_k\mathbf{p}^2 + C_k]\phi$$

- poor spectral resolution: only single (coherent) quasi-particle excitation

Our approach:

$$\begin{aligned}\Gamma_k \quad &= \quad \int_{\mathbf{p},\omega} \left\{\psi_\uparrow^*[-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p})\phi\right\} \\ &+ \quad \int_{\vec{x},\tau} h(\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)\end{aligned}$$

we keep arbitrary momentum/frequency dependence:

- infinitely many couplings
- full spectral information

Fermi polaron: truncation

$$S = \int_P \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^* [i\omega + p^2 - \mu_\sigma]\psi_\sigma + \phi^* G_{\phi,\Lambda}^{-1}\phi + h \int_X (\psi_\uparrow^* \psi_\downarrow^* \phi + h.c.)$$

- typical RG approach:
  - expansion of propagators in momentum and frequencies (derivative expansion)
  - good for critical phenomena, only a few couplings

$$\Gamma_{k,\phi,\mathrm{kin}} = \int_{\mathbf{p},\omega} \phi^* [-iA_k\omega + B_k\mathbf{p}^2 + C_k]\phi$$

- poor spectral resolution: only single (coherent) quasi-particle excitation

Our approach:

$$\Gamma_k = \int_{\mathbf{p},\omega} \left\{ \psi_\uparrow^* [-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p})\phi \right\}$$
$$+ \int_{\vec{x},\tau} h(\psi_\uparrow^* \psi_\downarrow^* \phi + h.c.)$$

we keep arbitrary momentum/frequency dependence:

- infinitely many couplings
- full spectral information

similar work:	ELLWANGER ET AL.(1994/98), PAWLOWSKI ET AL. (2002), KATO (2004), FISCHER ET AL. (2004), BLAIZOT ET AL. (2006), BENITEZ, BLAIZOT ET AL. (2009/10)
exact calculations:	DIEHL, KRAHL, SCHERER (2008), MOROZ ET AL. (2009/10)

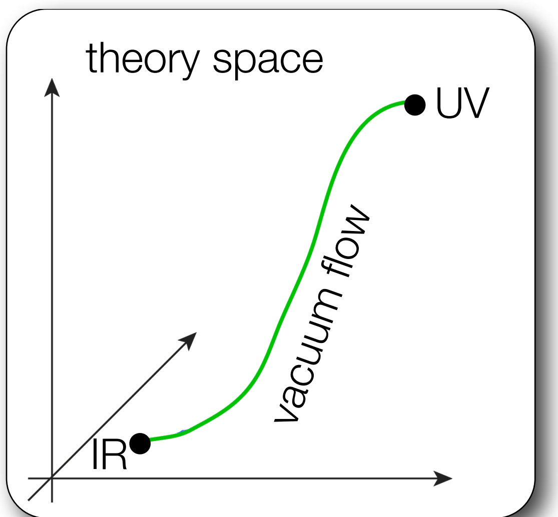
initial conditions

- ↑-atoms not renormalized by single ↓-atom: $P_\uparrow(\omega,\mathbf{p}) = -i\omega + \mathbf{p}^2 - \mu_\uparrow$

$$\begin{aligned}\Gamma_k &= \int_{\mathbf{p},\omega} \Big\{\psi_\uparrow^*[-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p})\phi\Big\} \\ &+ \int_{\vec{x},\tau} h(\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)\end{aligned}$$

initial conditions

- ↑-atoms not renormalized by single ↓-atom: $P_\uparrow(\omega, \mathbf{p}) = -i\omega + \mathbf{p}^2 - \mu_\uparrow$
- $G_{\phi,\Lambda}$ from two-body (vacuum) physics - ***exact*** calculation: reproduces Landau scattering amplitude in the IR

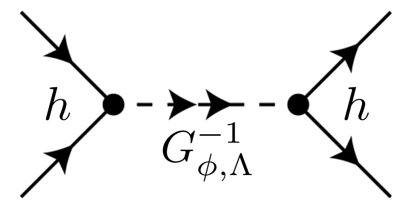


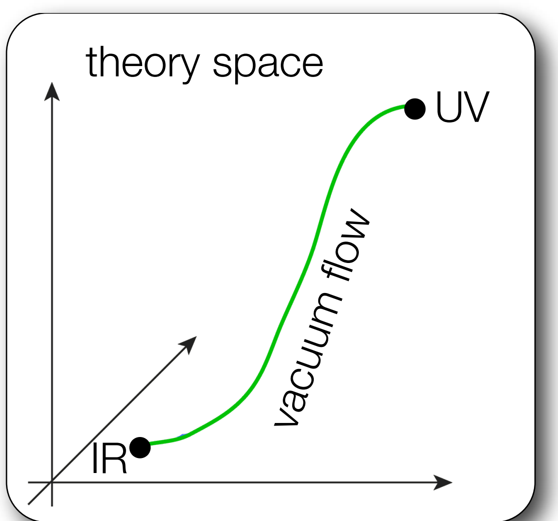
....

$$\Gamma_k = \int_{\mathbf{p},\omega} \left\{ \psi_\uparrow^* [-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p})\phi \right\} + \int_{\vec{x},\tau} h(\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)$$

initial conditions

- ↑-atoms not renormalized by single ↓-atom: $P_\uparrow(\omega, \mathbf{p}) = -i\omega + \mathbf{p}^2 - \mu_\uparrow$
- $G_{\phi,\Lambda}$ from two-body (vacuum) physics - ***exact*** calculation: reproduces Landau scattering amplitude in the IR





$$\boxed{f(q) = \frac{1}{-1/a - iq} = \frac{h^2}{8\pi} G^{\text{vac}}_{\phi,\text{R}}(\omega = 2q^2, \mathbf{p} = 0)}$$

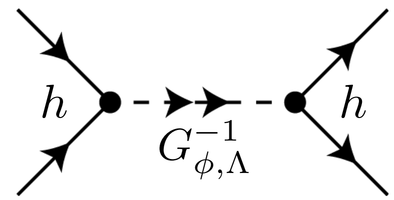
$$[G^{\text{vac}}_{\phi,\text{IR}}(\omega, \mathbf{p})]^{-1} = \frac{h^2}{8\pi}\left(-a^{-1} + \sqrt{-\frac{i\omega}{2} + \frac{\mathbf{p}^2}{4}}\right)$$

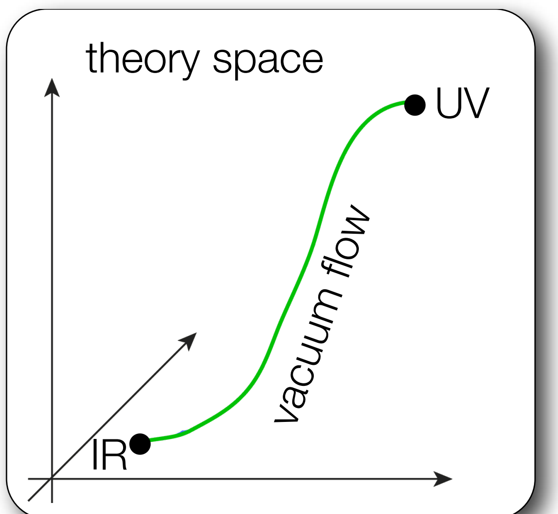
(not possible within a derivative expansion)

$$\begin{aligned}\Gamma_k \quad &= \quad \int_{\mathbf{p},\omega} \left\{\psi^*_\uparrow[-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi^*_\downarrow G^{-1}_{\downarrow,k}(\omega, \mathbf{p})\psi_\downarrow + \phi^* G^{-1}_{\phi,k}(\omega, \mathbf{p})\phi\right\} \\ &+ \quad \int_{\vec{x},\tau} h(\psi^*_\uparrow \psi^*_\downarrow \phi + h.c.)\end{aligned}$$

initial conditions

- ↑-atoms not renormalized by single ↓-atom: $P_\uparrow(\omega, \mathbf{p}) = -i\omega + \mathbf{p}^2 - \mu_\uparrow$
- $G_{\phi,\Lambda}$ from two-body (vacuum) physics - ***exact*** calculation: reproduces Landau scattering amplitude in the IR





$$\boxed{f(q) = \frac{1}{-1/a - iq} = \frac{h^2}{8\pi} G^{\mathrm{vac}}_{\phi,\mathrm{R}}(\omega = 2q^2, \mathbf{p} = 0)}$$

$$[G^{\mathrm{vac}}_{\phi,\mathrm{IR}}(\omega, \mathbf{p})]^{-1} = \frac{h^2}{8\pi}\left(-a^{-1} + \sqrt{-\frac{i\omega}{2} + \frac{\mathbf{p}^2}{4}}\right)$$

(not possible within a derivative expansion)

• derivative expansion in cold atom context:

BEC/BCS crossover MANCHESTER GROUP (BIRSE ET AL.), HEIDELBERG GROUP (WETTERICH ET AL.) SIMILAR: FRANKFURT GROUP (KOPITZ)

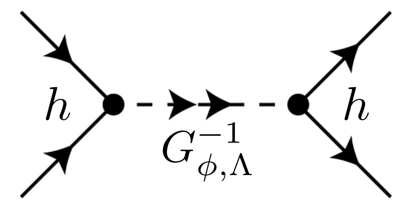
SU(3) Fermi gas, few-body physics FLOERCHINGER, MOROZ, RS, WETTERICH, PRA, ANN. PHYS, ... (2008-10)

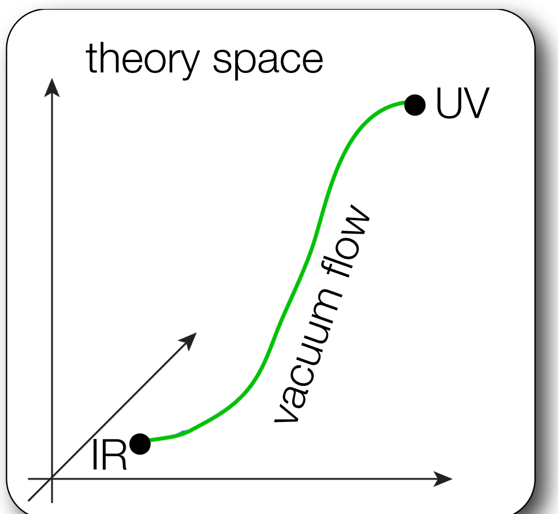
THREE-BODY LOSS IN ^{6}LI: FLOERCHINGER, SCHMIDT, WETTERICH PRA (2009)

REVIEW IN SPECIAL ISSUE OF FEW BODY SYSTEMS, SPRINGER (2011)

initial conditions

- $\uparrow$-atoms not renormalized by single $\downarrow$-atom: $\quad P_\uparrow(\omega,\mathbf{p}) = -i\omega + \mathbf{p}^2 - \mu_\uparrow$
- $G_{\phi,\Lambda}$ from two-body (vacuum) physics - ***exact*** calculation: reproduces Landau scattering amplitude in the IR





$$\boxed{f(q) = \frac{1}{-1/a - iq} = \frac{h^2}{8\pi} G^{\mathrm{vac}}_{\phi,\mathrm{R}}(\omega = 2q^2, \mathbf{p} = 0)}$$

$$[G^{\mathrm{vac}}_{\phi,\mathrm{IR}}(\omega,\mathbf{p})]^{-1} = \frac{h^2}{8\pi}\left(-a^{-1} + \sqrt{-\frac{i\omega}{2} + \frac{\mathbf{p}^2}{4}}\right)$$

(not possible within a derivative expansion)

$$\begin{aligned}\Gamma_k \quad &= \quad \int_{\mathbf{p},\omega} \left\{ \psi^*_\uparrow[-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi^*_\downarrow G^{-1}_{\downarrow,k}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G^{-1}_{\phi,k}(\omega,\mathbf{p})\phi \right\} \\ &+ \quad \int_{\vec{x},\tau} h(\psi^*_\uparrow \psi^*_\downarrow \phi + h.c.)\end{aligned}$$

determination of $\mu_\downarrow$

- bare down propagator: $P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -i\omega + \mathbf{p}^2 - \mu_\downarrow$
  - $\mu_\downarrow$ determines $\downarrow$-occupation
  - polaron: $\mu_\downarrow$ marks phase transition from zero to non-vanishing $\downarrow$-occupation

....

$$\begin{aligned}\Gamma_k &= \int_{\mathbf{p},\omega} \Big\{\psi_\uparrow^*[-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p})\phi\Big\} \\ &+ \int_{\vec{x},\tau} h(\psi_\uparrow^*\psi_\downarrow^*\phi + h.c.)\end{aligned}$$

determination of $\mu_\downarrow$

- bare down propagator: $P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -i\omega + \mathbf{p}^2 - \mu_\downarrow$
  - $\mu_\downarrow$ determines ↓-occupation
  - polaron: $\mu_\downarrow$ marks phase transition from zero to non-vanishing ↓-occupation
  - $\mu_\downarrow$: energy to add single ↓-atom to the system, here $N_\downarrow = 1$

$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

....

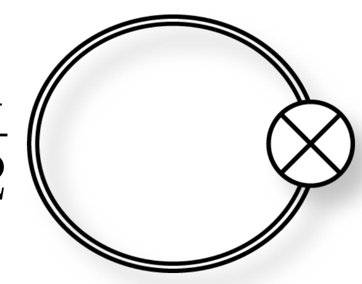
$$\begin{aligned}\Gamma_k \quad &= \quad \int_{\mathbf{p},\omega} \Big\{ \psi_\uparrow^* [-i\omega + \mathbf{p}^2 - \mu_\uparrow] \psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p}) \psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p}) \phi \Big\} \\ &+ \quad \int_{\vec{x},\tau} h(\psi_\uparrow^* \psi_\downarrow^* \phi + h.c.)\end{aligned}$$

flowing spectral functions

$$\Gamma_k = \int_{\mathbf{p},\omega} \left\{ \psi_\uparrow^* [-i\omega + \mathbf{p}^2 - \mu_\uparrow]\psi_\uparrow + \psi_\downarrow^* G_{\downarrow,k}^{-1}(\omega,\mathbf{p})\psi_\downarrow + \phi^* G_{\phi,k}^{-1}(\omega,\mathbf{p})\phi \right\}$$

$$+ \int_{\vec{x},\tau} h(\psi_\uparrow^* \psi_\downarrow^* \phi + h.c.)$$

$$\Downarrow$$

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_k R_k = \frac{1}{2}$$ 

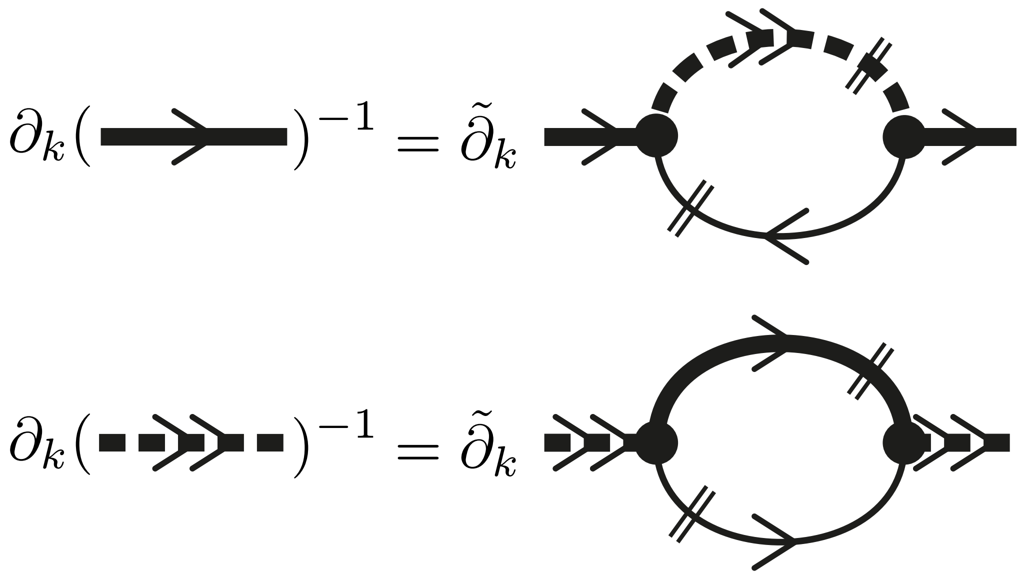
sharp regulators

$$G_{\downarrow,k}^c(\omega,\mathbf{p}) = \frac{\theta(|\mathbf{p}| - k)}{P_{\downarrow,k}(\omega,\mathbf{p})},$$

$$G_{\phi,k}^c(\omega,\mathbf{p}) = \frac{\theta(|\mathbf{p}| - k)}{P_{\phi,k}(\omega,\mathbf{p})},$$

$$G_{\uparrow,k}^c(\omega,\mathbf{p}) = \frac{\theta(|\mathbf{p}^2 - \mu_\uparrow| - k^2)}{P_{\uparrow,k}(\omega,\mathbf{p})}.$$

flowing spectral functions

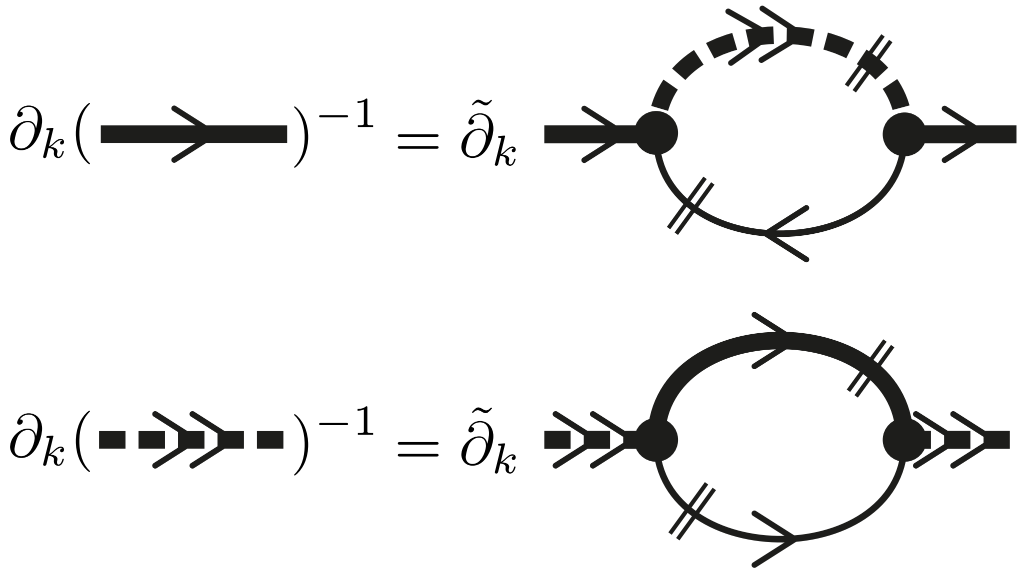


$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P)\, G^c_{\uparrow,k}(Q)$$

$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q)\, G^c_{\uparrow,k}(P-Q)$$

$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

flowing spectral functions



$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P) G^c_{\uparrow,k}(Q)$$

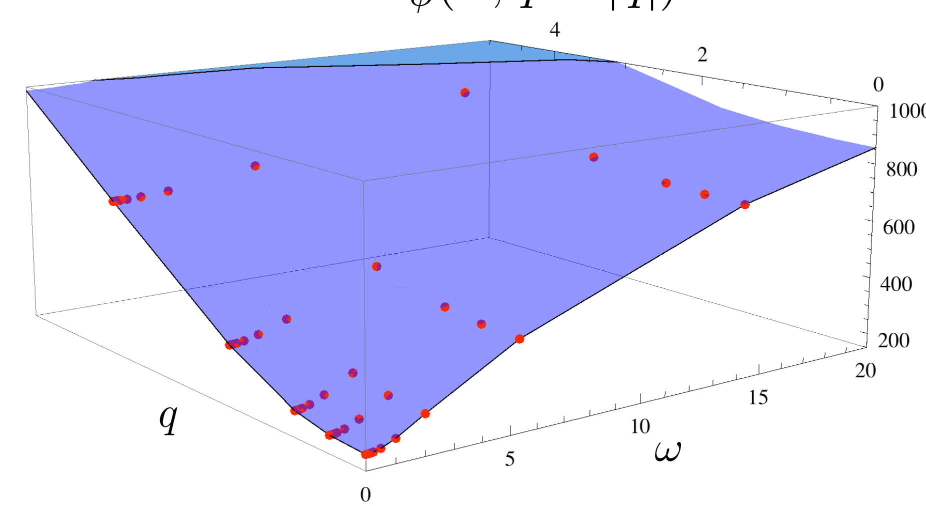
$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q) G^c_{\uparrow,k}(P-Q)$$

$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

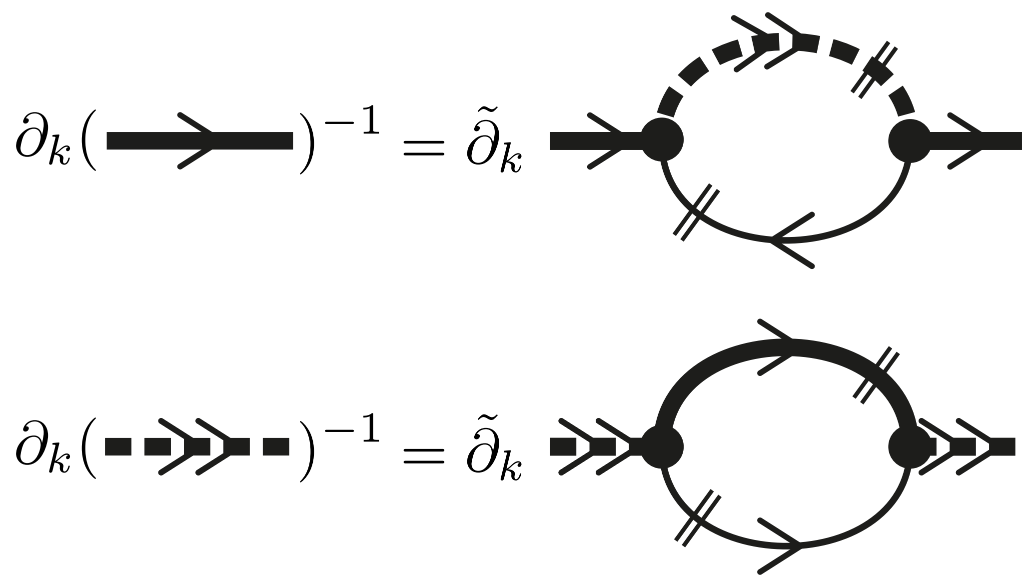
<u>Algorithm</u>

<u>Input</u>: At UV, $k = \Lambda$: Put bare propagators $P_\Lambda(\omega, \mathbf{p})$ on logarithmic w/q grid: $P_{ij} = P(\omega_i, p_j)$

$\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$



flowing spectral functions



$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P)\, G^c_{\uparrow,k}(Q)$$

$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q)\, G^c_{\uparrow,k}(P-Q)$$

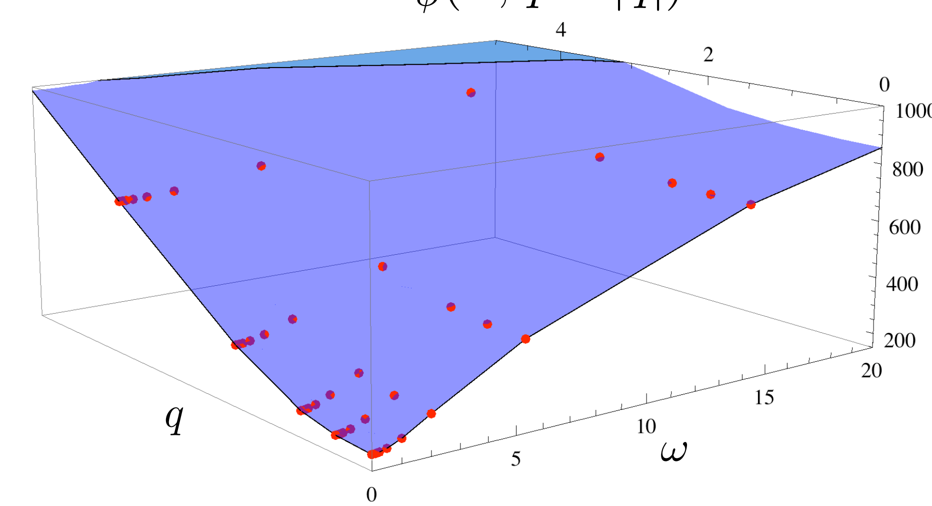
$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

Algorithm

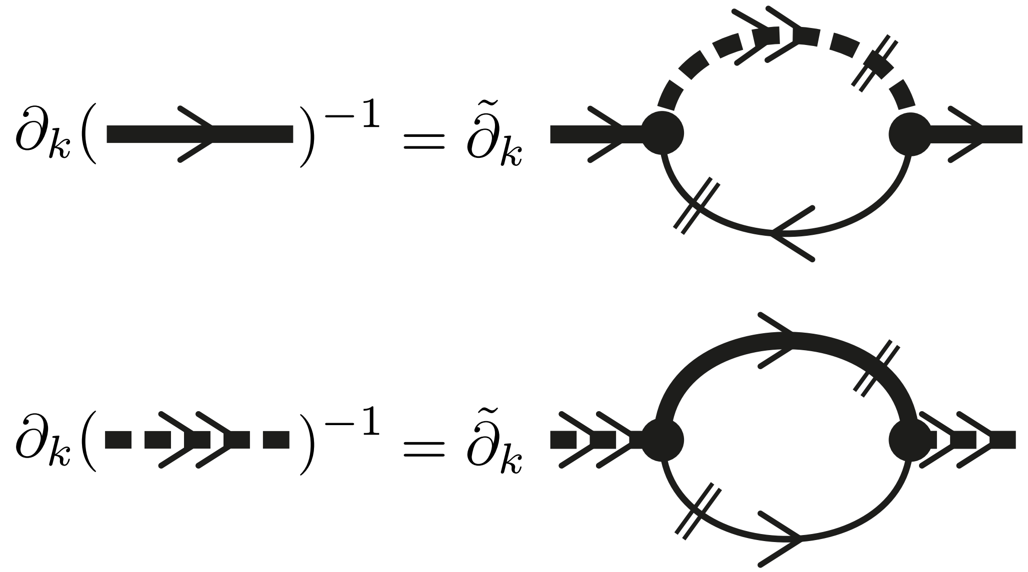
Input: At UV, $k = \Lambda$: Put bare propagators $P_\Lambda(\omega, \mathbf{p})$ on logarithmic w/q grid: $P_{ij} = P(\omega_i, p_j)$

1. RG scale k : Reconstruct $P(\omega, p)$ from bicubic spline interpolation of P_{ij}

$\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$



flowing spectral functions



$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P)\, G^c_{\uparrow,k}(Q)$$

$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q)\, G^c_{\uparrow,k}(P-Q)$$

$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

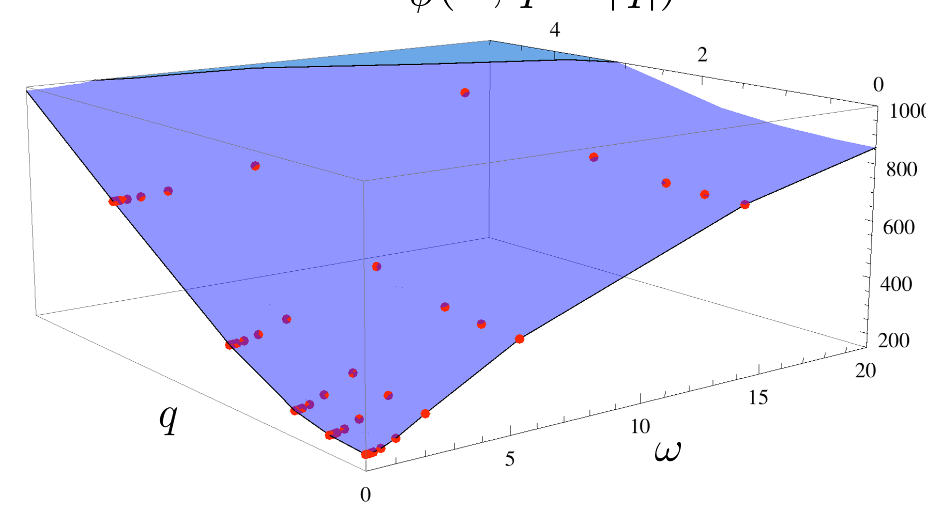
Algorithm

Input: At UV, $k = \Lambda$: Put bare propagators $P_\Lambda(\omega, \mathbf{p})$ on logarithmic w/q grid: $P_{ij} = P(\omega_i, p_j)$

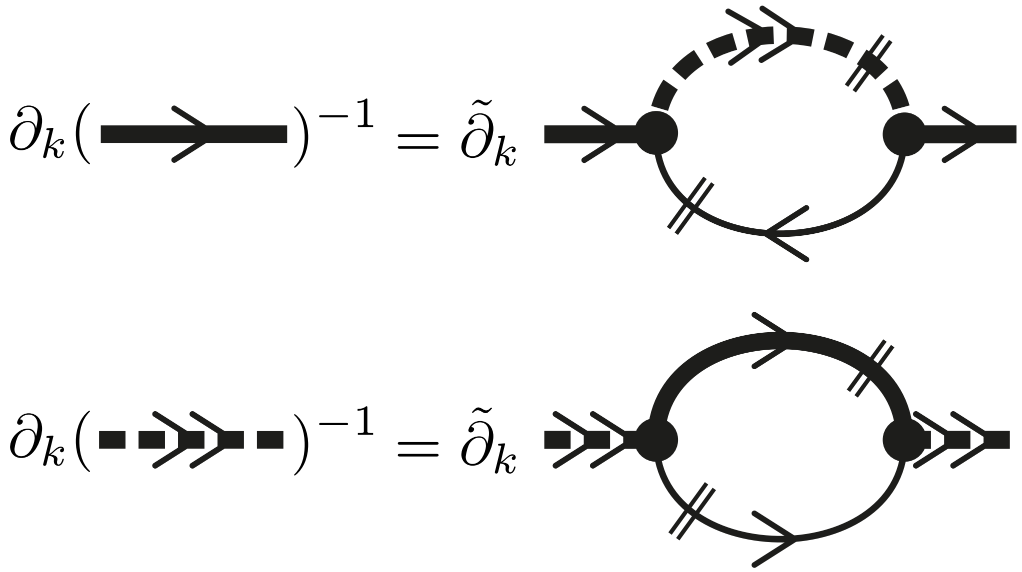
1. RG scale k : Reconstruct $P(\omega, p)$ from bicubic spline interpolation of P_{ij}

2. Plug $P(\omega, p)$ into flow equations

$\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$



flowing spectral functions



$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P) G^c_{\uparrow,k}(Q)$$

$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q) G^c_{\uparrow,k}(P-Q)$$

$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

Algorithm

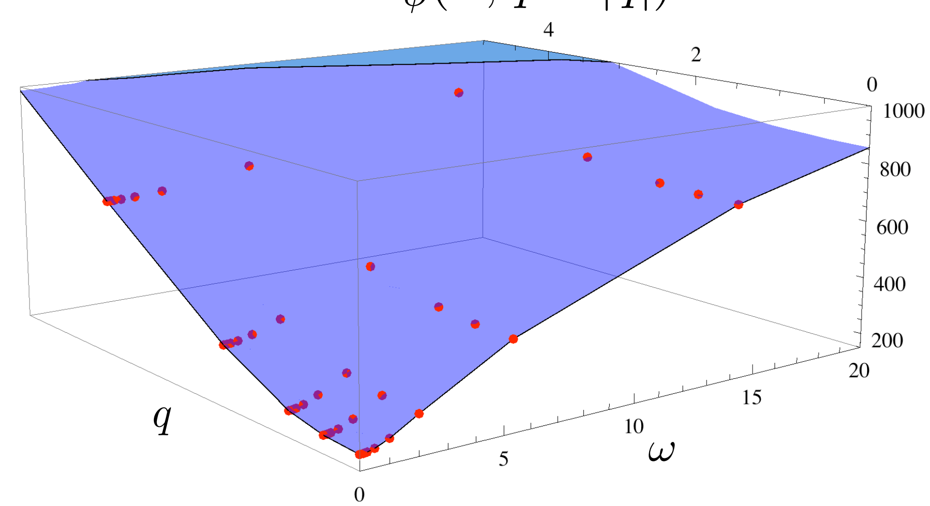
Input: At UV, $k = \Lambda$: Put bare propagators $P_\Lambda(\omega, \mathbf{p})$ on logarithmic w/q grid: $P_{ij} = P(\omega_i, p_j)$

1. RG scale k : Reconstruct $P(\omega, p)$ from bicubic spline interpolation of P_{ij}

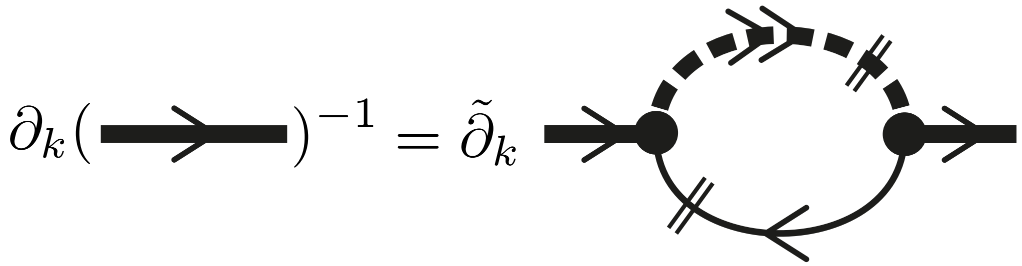
2. Plug $P(\omega, p)$ into flow equations

3. Calculate flow equations for the P_{ij}

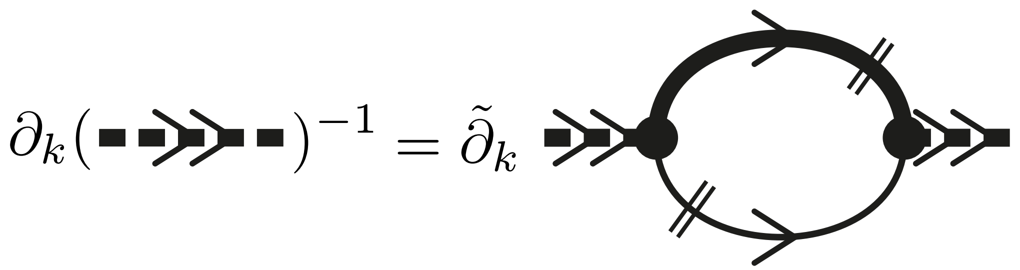
$\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$



flowing spectral functions



$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P)\, G^c_{\uparrow,k}(Q)$$



$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q)\, G^c_{\uparrow,k}(P-Q)$$

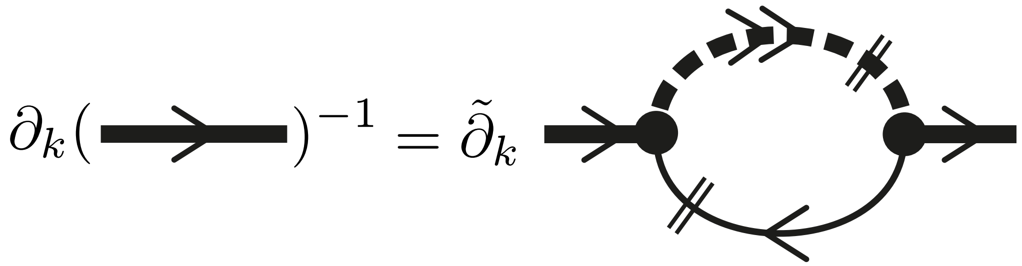
$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

Algorithm

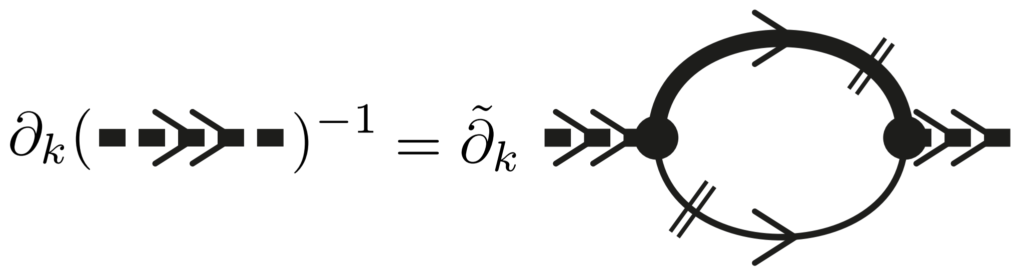
Input: At UV, $k = \Lambda$: Put bare propagators $P_\Lambda(\omega, \mathbf{p})$ on logarithmic w/q grid: $P_{ij} = P(\omega_i, p_j)$

1. RG scale k : Reconstruct $P(\omega, p)$ from bicubic spline interpolation of P_{ij}
2. Plug $P(\omega, p)$ into flow equations
3. Calculate flow equations for the P_{ij}
4. RG step: Adaptive Runga-Kutta evolution of flow equations $P_{ij}(k) \overset{\partial_k P_{ij}}{\longrightarrow} P_{ij}(k - \delta k)$

flowing spectral functions



$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P) G^c_{\uparrow,k}(Q)$$



$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q) G^c_{\uparrow,k}(P-Q)$$

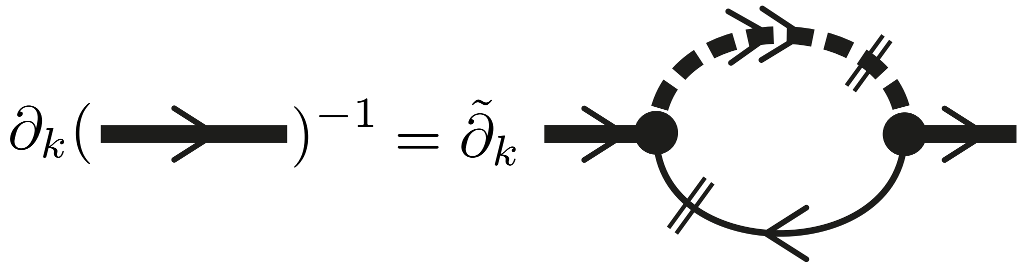
$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

Algorithm

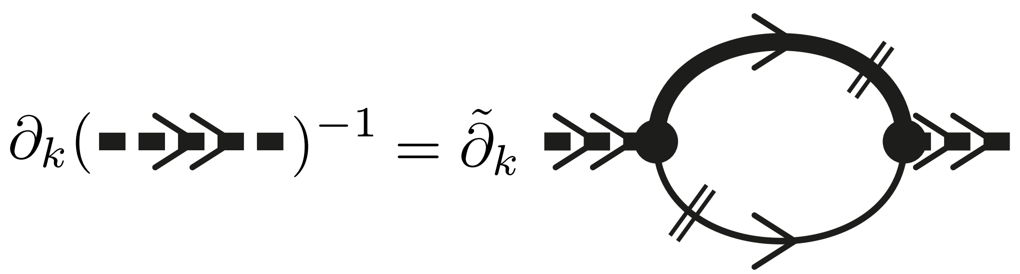
Input: At UV, $k = \Lambda$: Put bare propagators $P_\Lambda(\omega, \mathbf{p})$ on logarithmic w/q grid: $P_{ij} = P(\omega_i, p_j)$

1. RG scale k : Reconstruct $P(\omega, p)$ from bicubic spline interpolation of P_{ij}
2. Plug $P(\omega, p)$ into flow equations
3. Calculate flow equations for the P_{ij}
4. RG step: Adaptive Runga-Kutta evolution of flow equations $P_{ij}(k) \overset{\partial_k P_{ij}}{\longrightarrow} P_{ij}(k - \delta k)$
5. Stop flow when IR, $k = 0$, is reached

flowing spectral functions



$$\partial_k P_{\downarrow,k}(P) = h^2 \tilde{\partial}_k \int_Q G^c_{\phi,k}(Q+P) G^c_{\uparrow,k}(Q)$$



$$\partial_k P_{\phi,k}(P) = -h^2 \tilde{\partial}_k \int_Q G^c_{\downarrow,k}(Q) G^c_{\uparrow,k}(P-Q)$$

$$G^c_k \equiv \frac{1}{(P_k + R_k)}$$

Algorithm

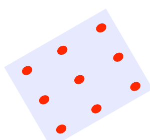
Input: At UV, $k = \Lambda$: Put bare propagators $P_\Lambda(\omega, \mathbf{p})$ on logarithmic w/q grid: $P_{ij} = P(\omega_i, p_j)$

1. RG scale k : Reconstruct $P(\omega, p)$ from bicubic spline interpolation of P_{ij}
2. Plug $P(\omega, p)$ into flow equations
3. Calculate flow equations for the P_{ij}
4. RG step: Adaptive Runga-Kutta evolution of flow equations $P_{ij}(k) \overset{\partial_k P_{ij}}{\longrightarrow} P_{ij}(k - \delta k)$
5. Stop flow when IR, $k = 0$, is reached

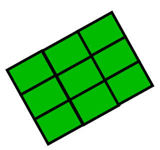
Output: Full quantum two-point Matsubara Green‘s functions

example: flow of the dimer propagator

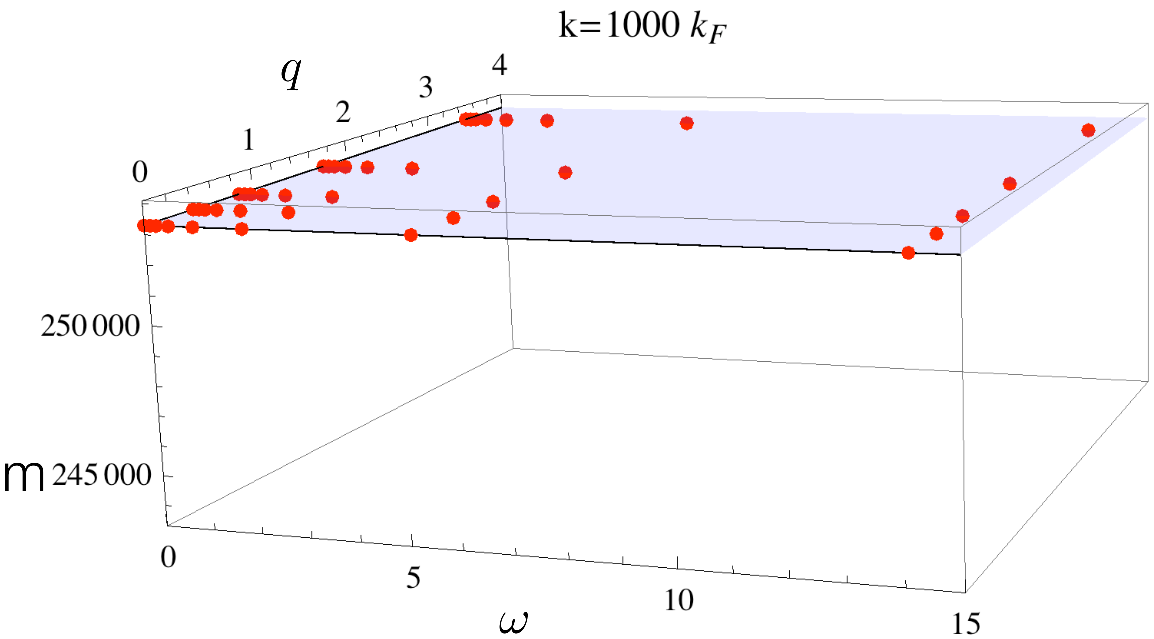
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$



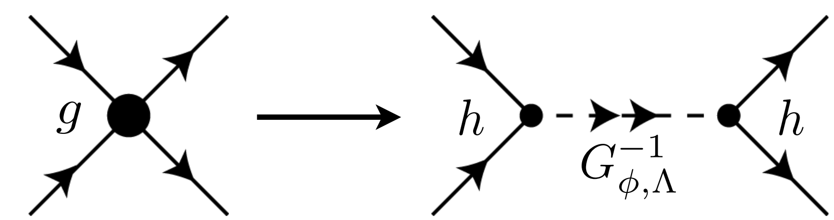
finite density



exact IR vacuum

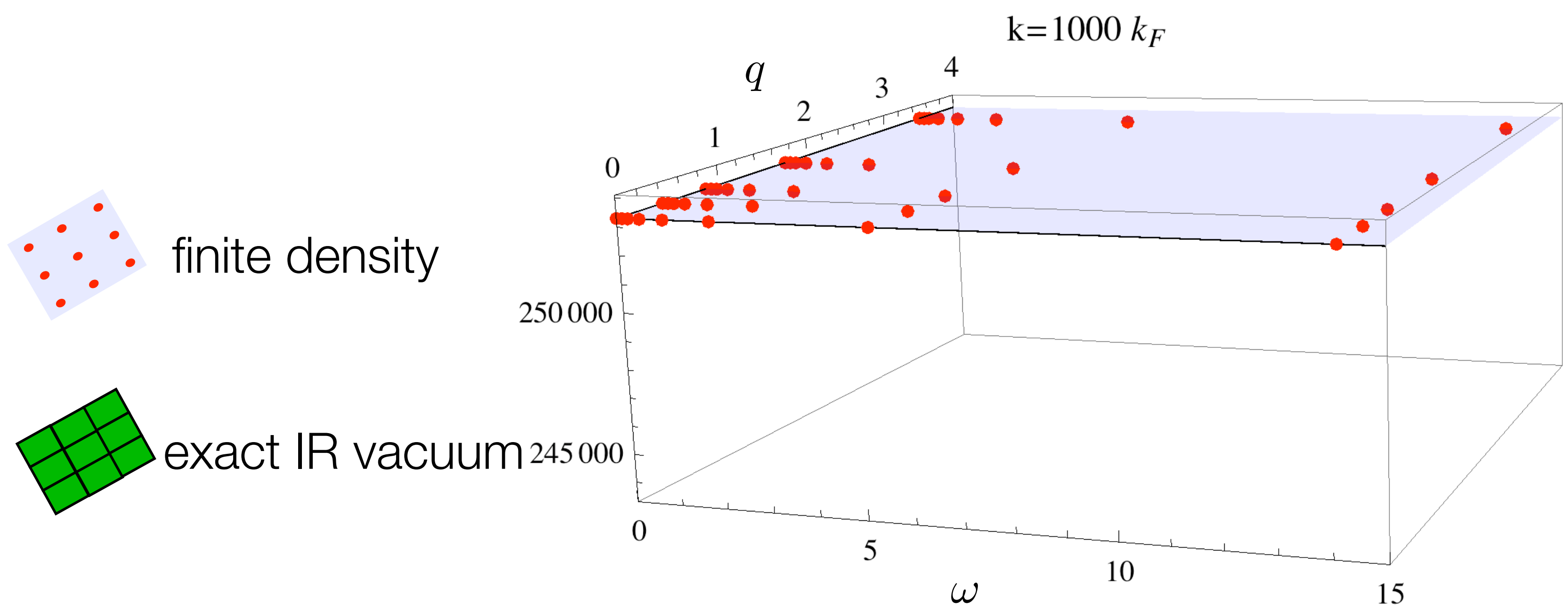


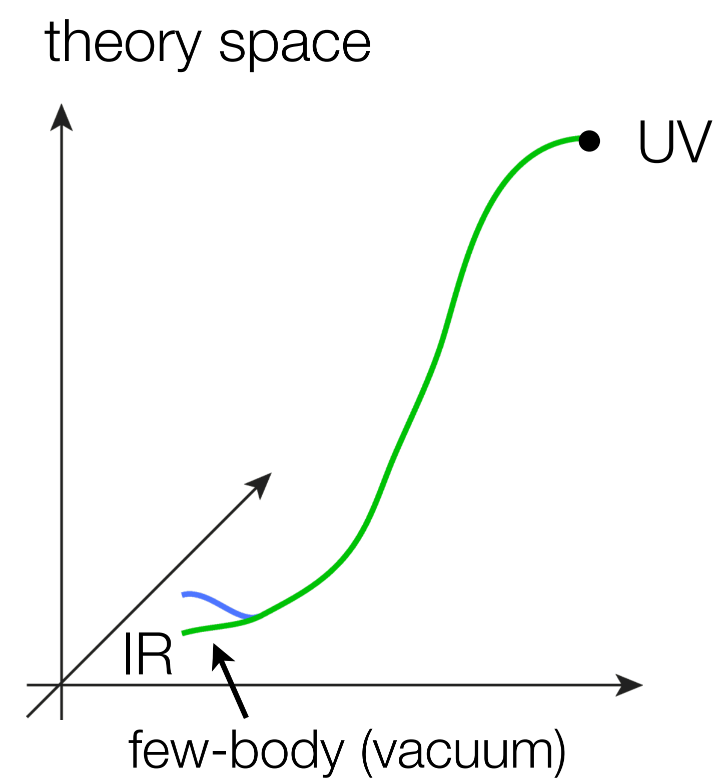
UV: no w/q dependence
→ contact interaction



example: flow of the dimer propagator

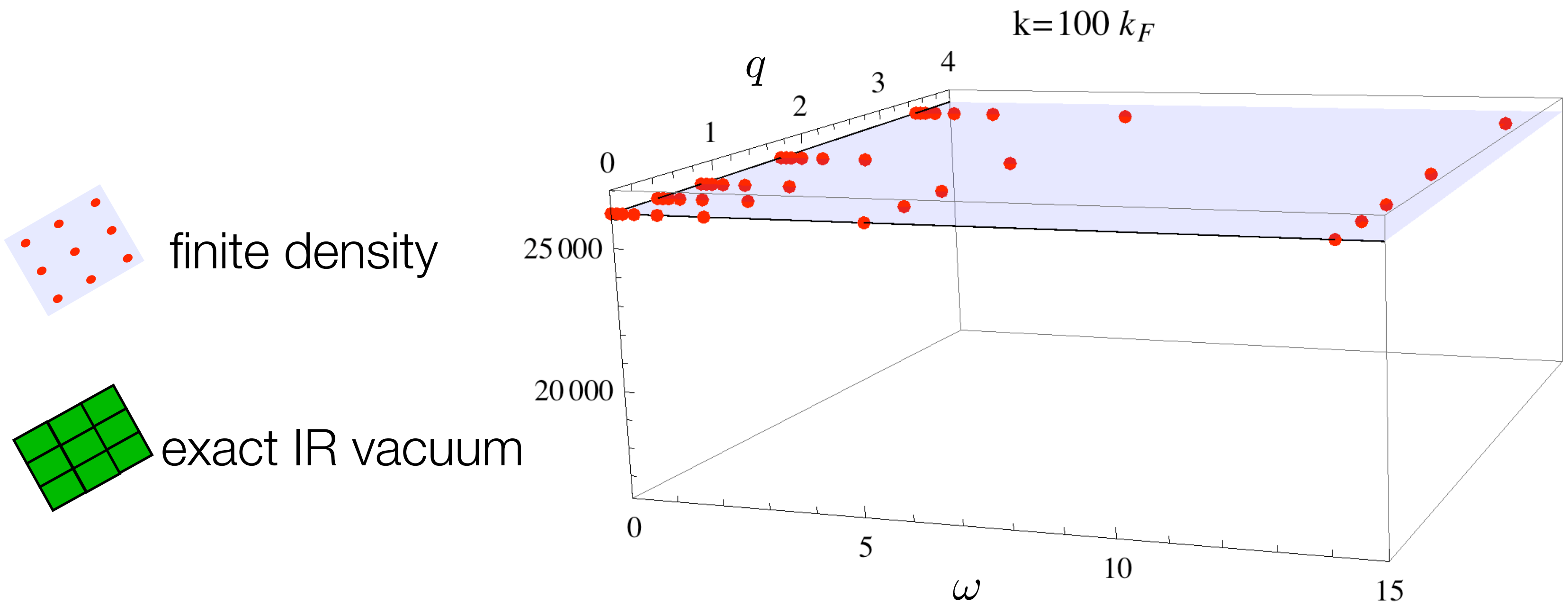
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$

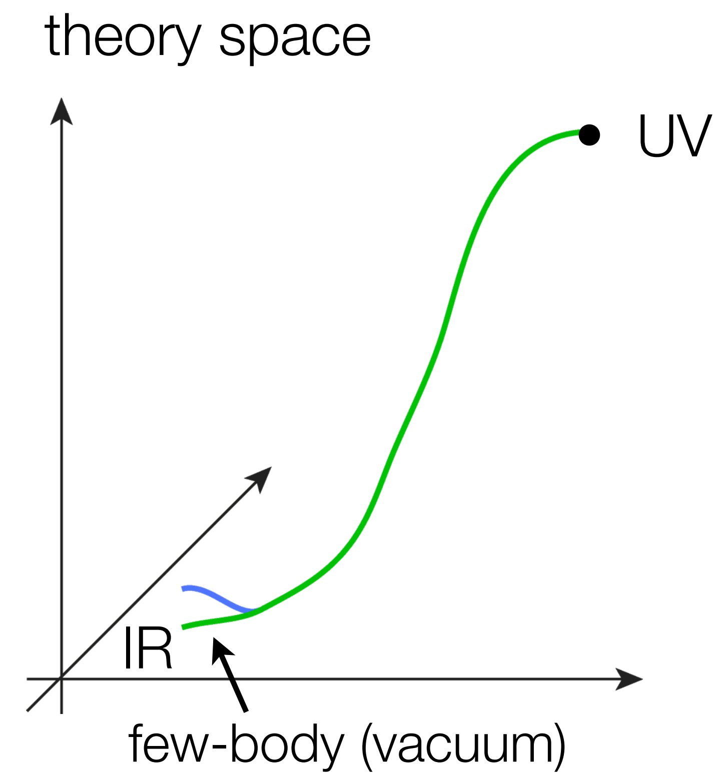




example: flow of the dimer propagator

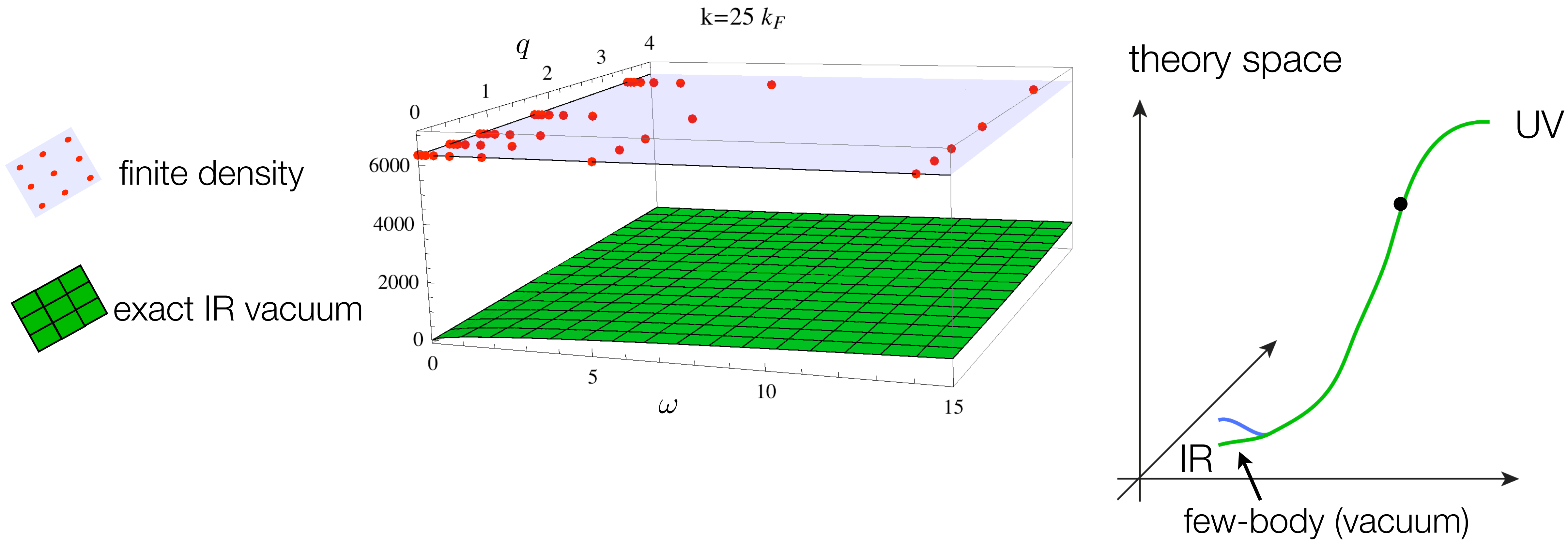
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$





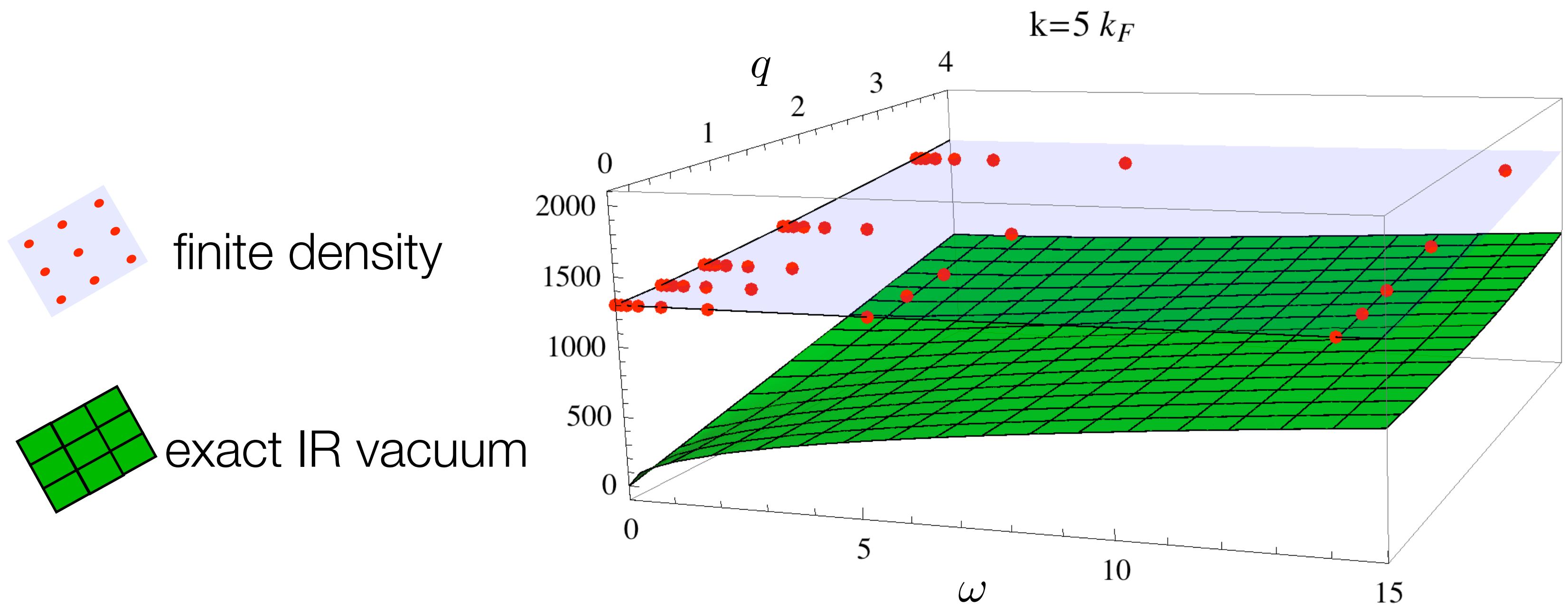
example: flow of the dimer propagator

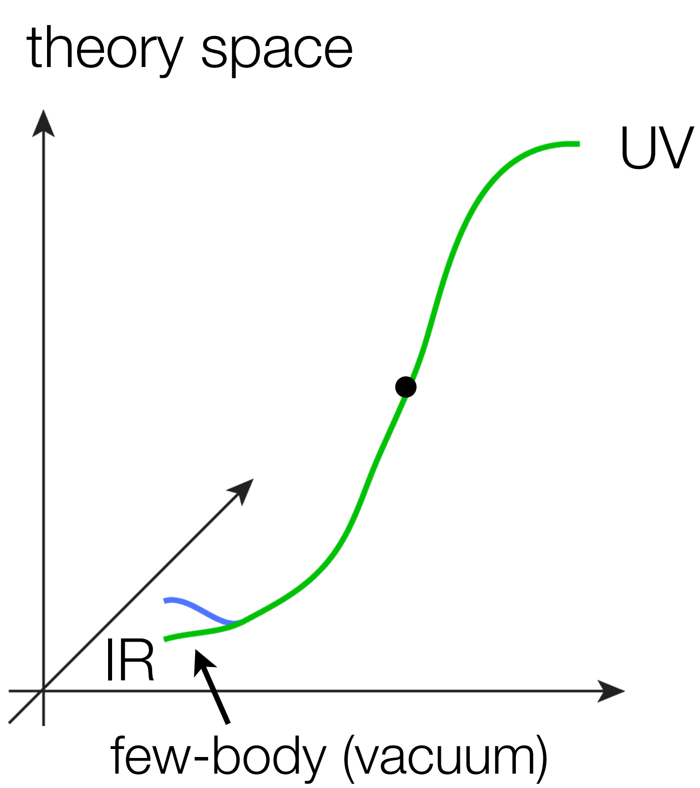
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$



example: flow of the dimer propagator

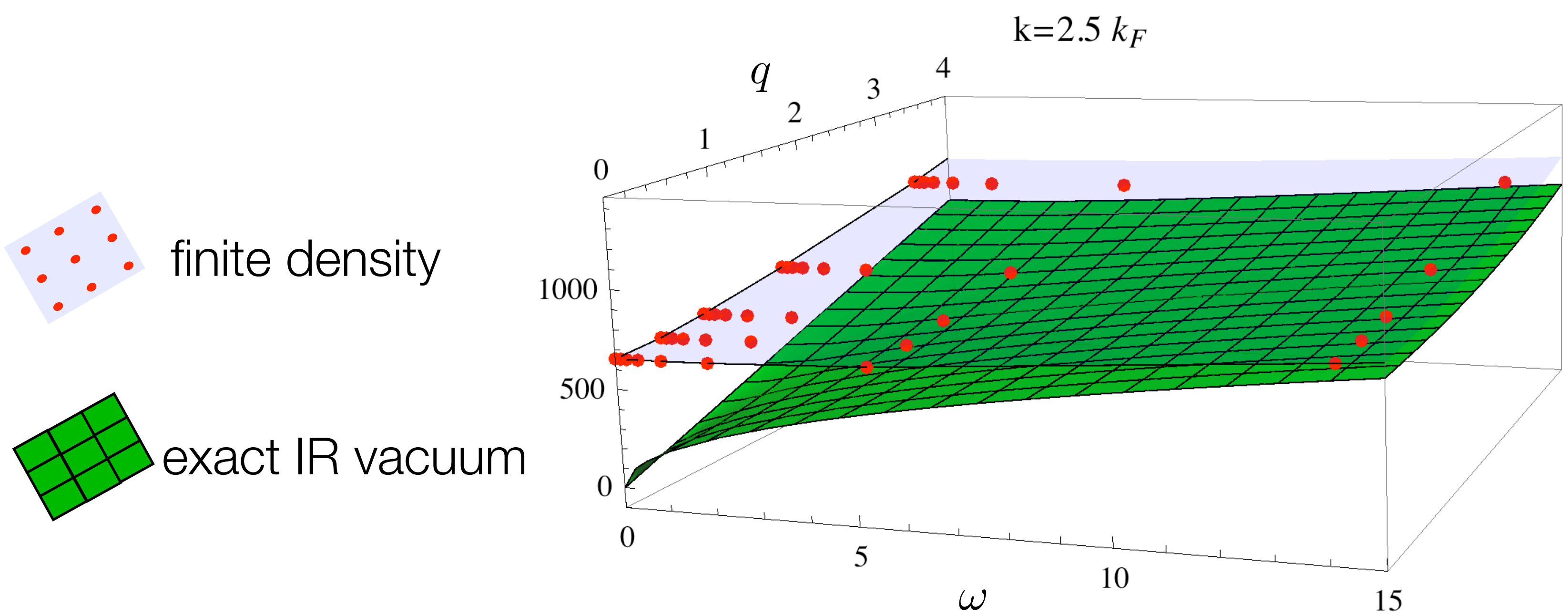
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$

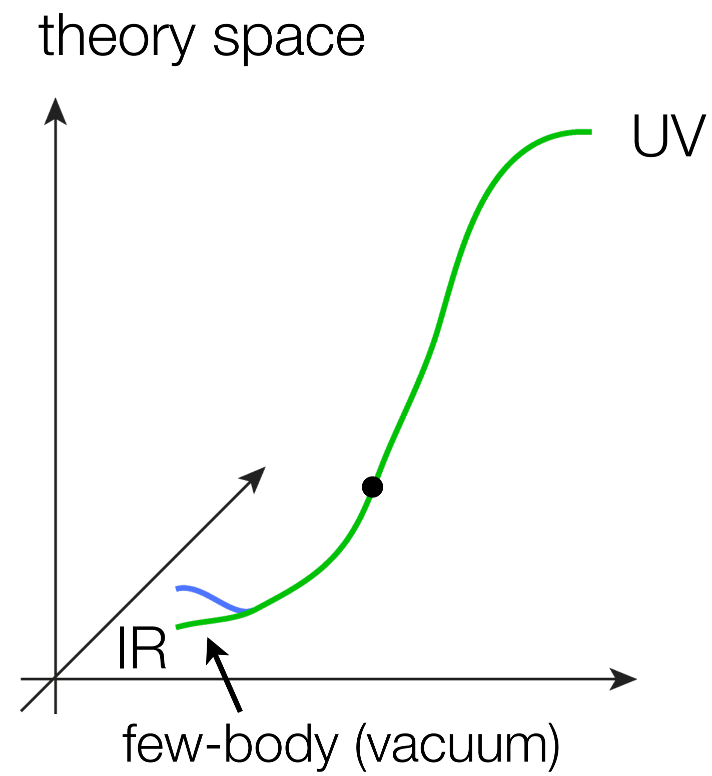




example: flow of the dimer propagator

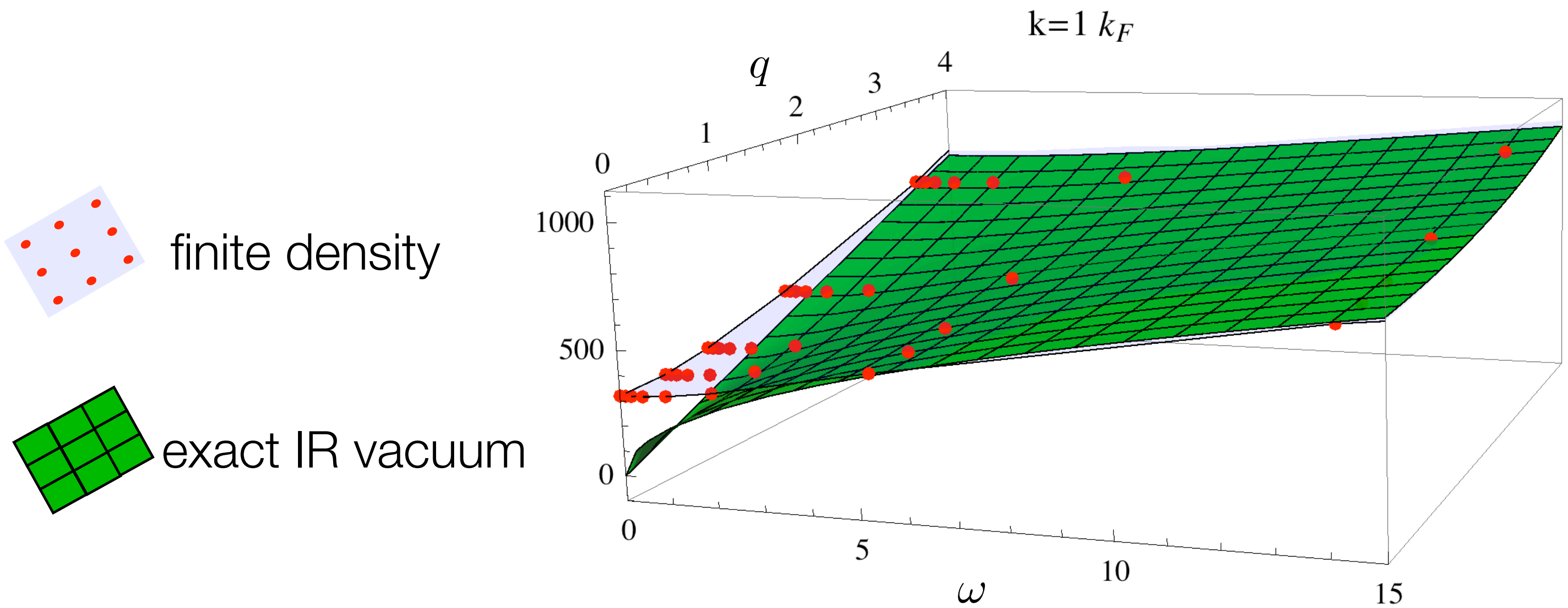
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$

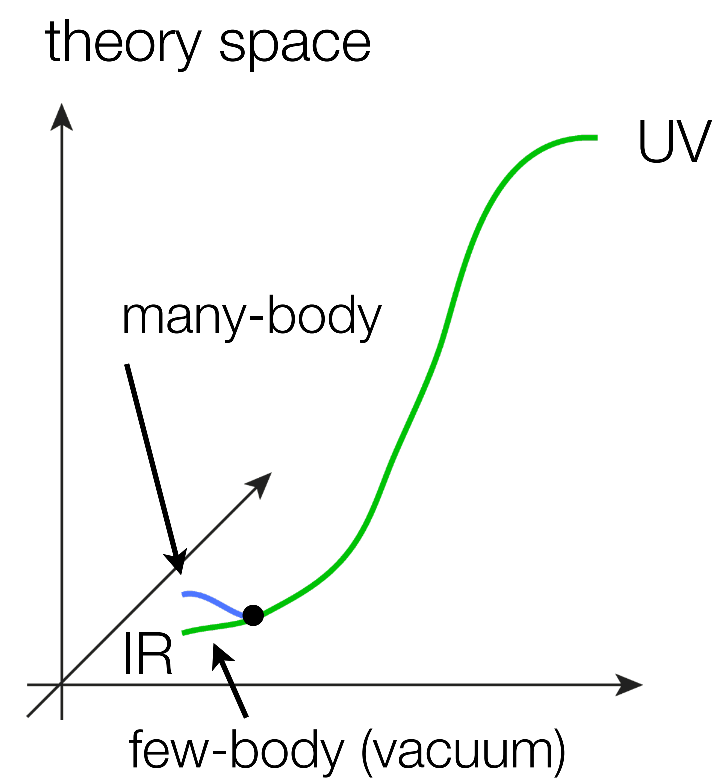




example: flow of the dimer propagator

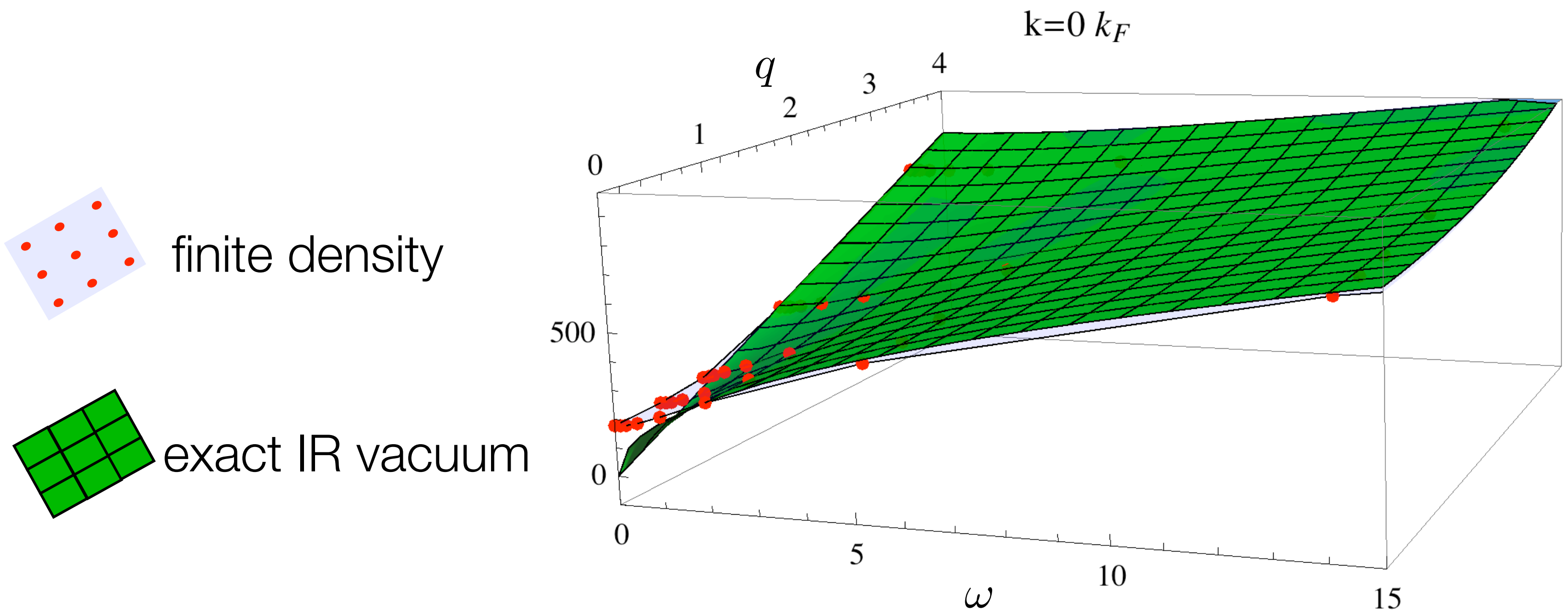
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$

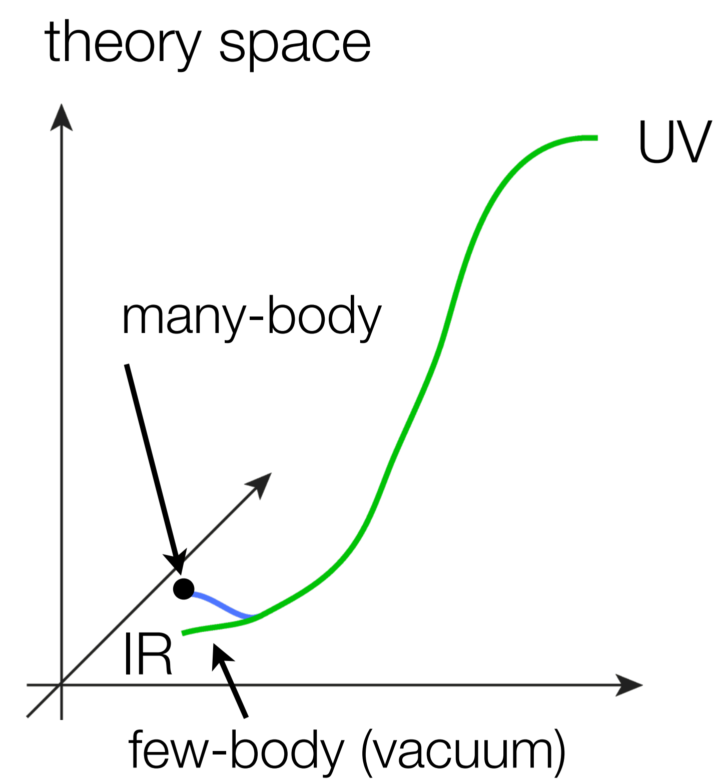




example: flow of the dimer propagator

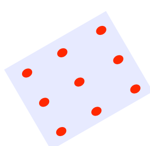
RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$

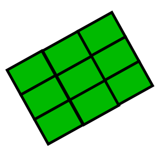


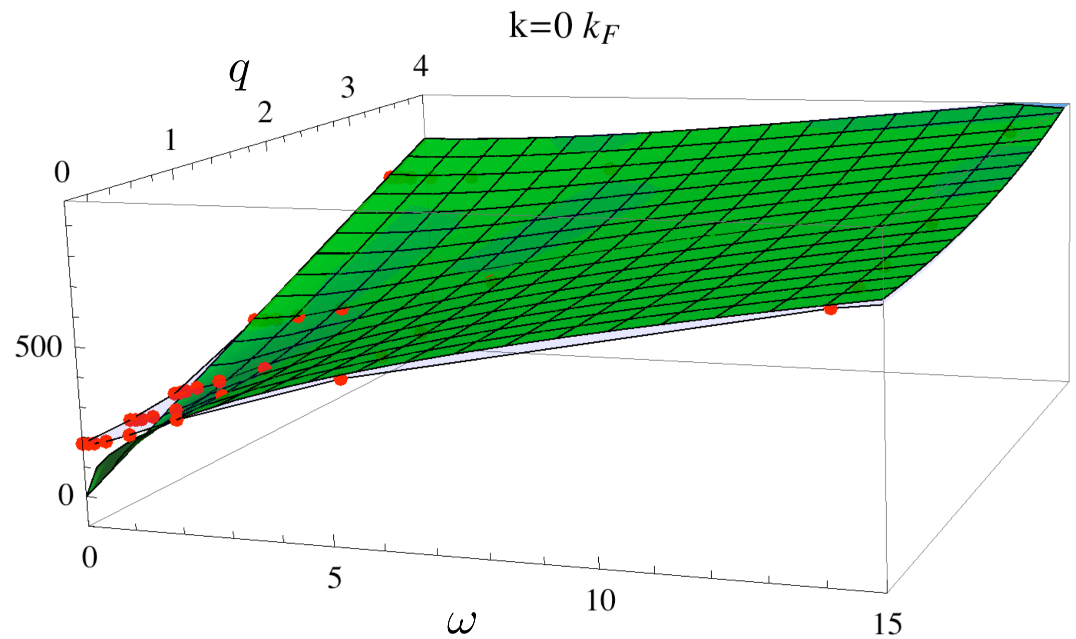


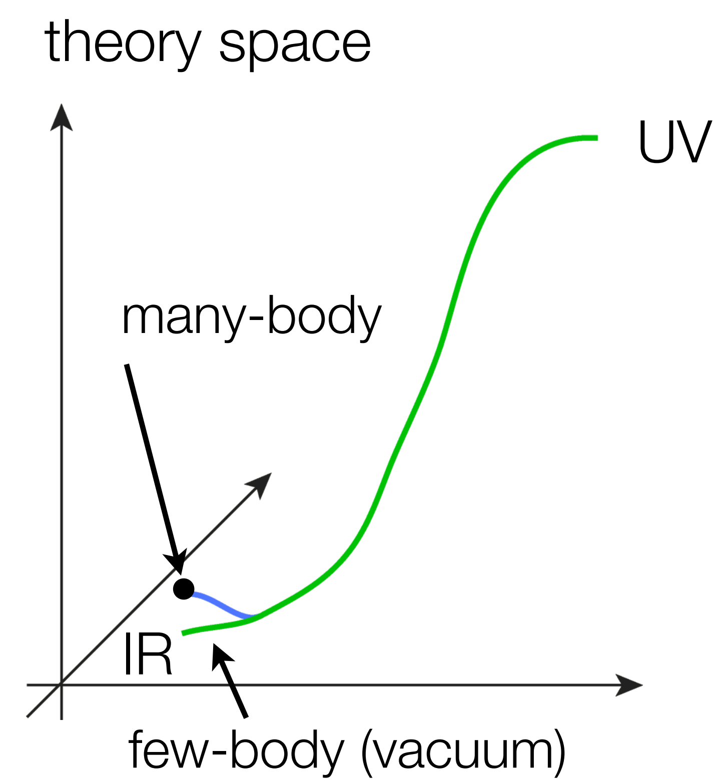
example: flow of the dimer propagator

RG Flow of $\mathrm{Re}\,P_\phi(\omega, q = |\vec{q}|)$


finite density


exact IR vacuum

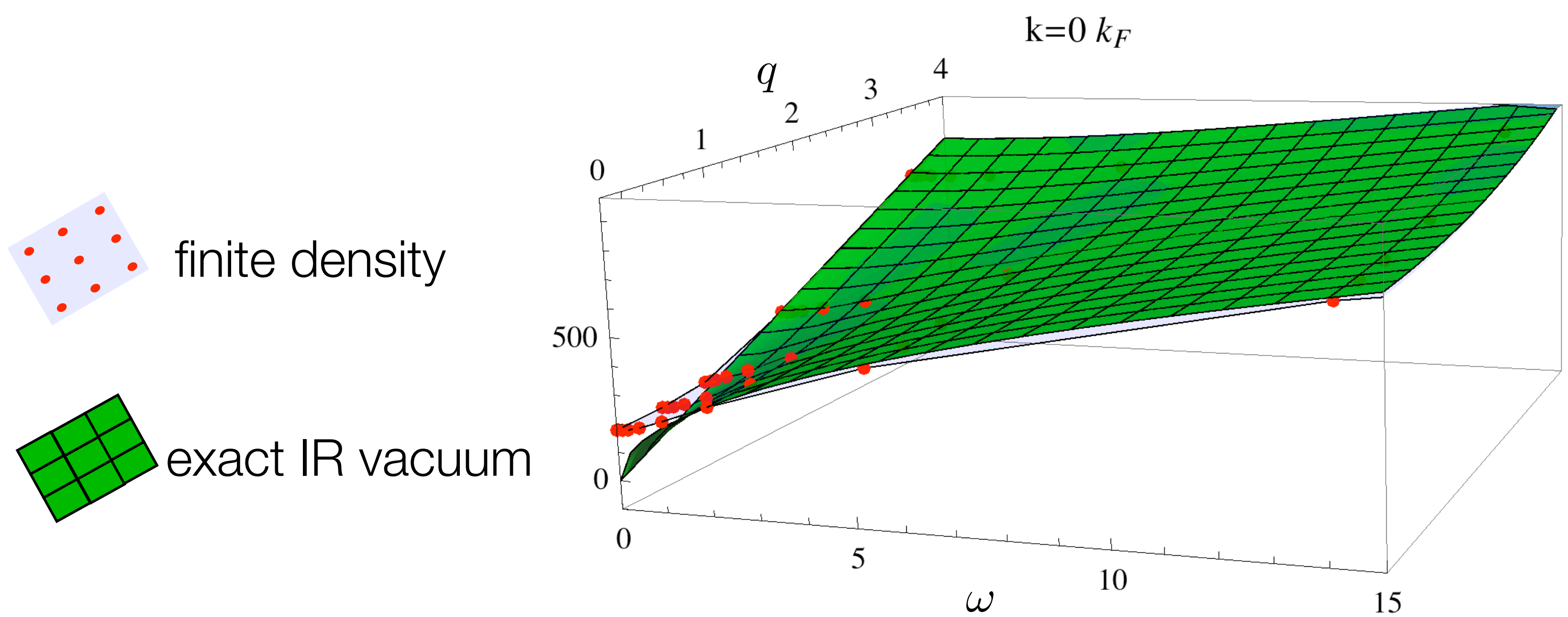


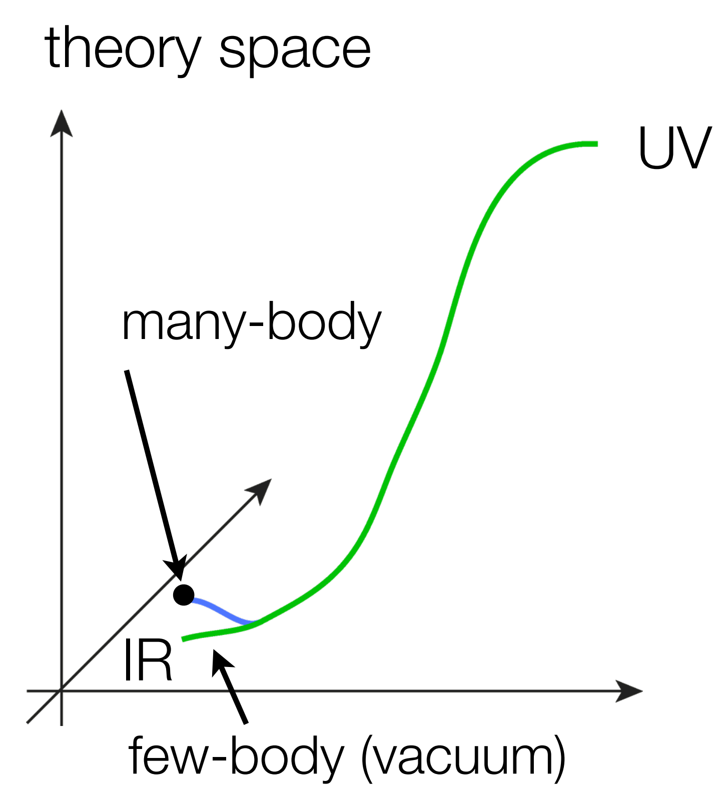


→ similarities to nice idea for RG flow of atoms in lattice
Racon, Depuis (2011)

example: flow of the dimer propagator

RG Flow of $\mathrm{Re}\, P_\phi(\omega, q = |\vec{q}|)$





→ similarities to nice idea for RG flow of atoms in lattice
Racon, Depuis (2011)

End of the flow (IR, $k = 0$)

imaginary time

$$P^{ij,k=0}_{\downarrow,\phi}$$

Analytical continuation
$\omega \to -i\omega$
Padé

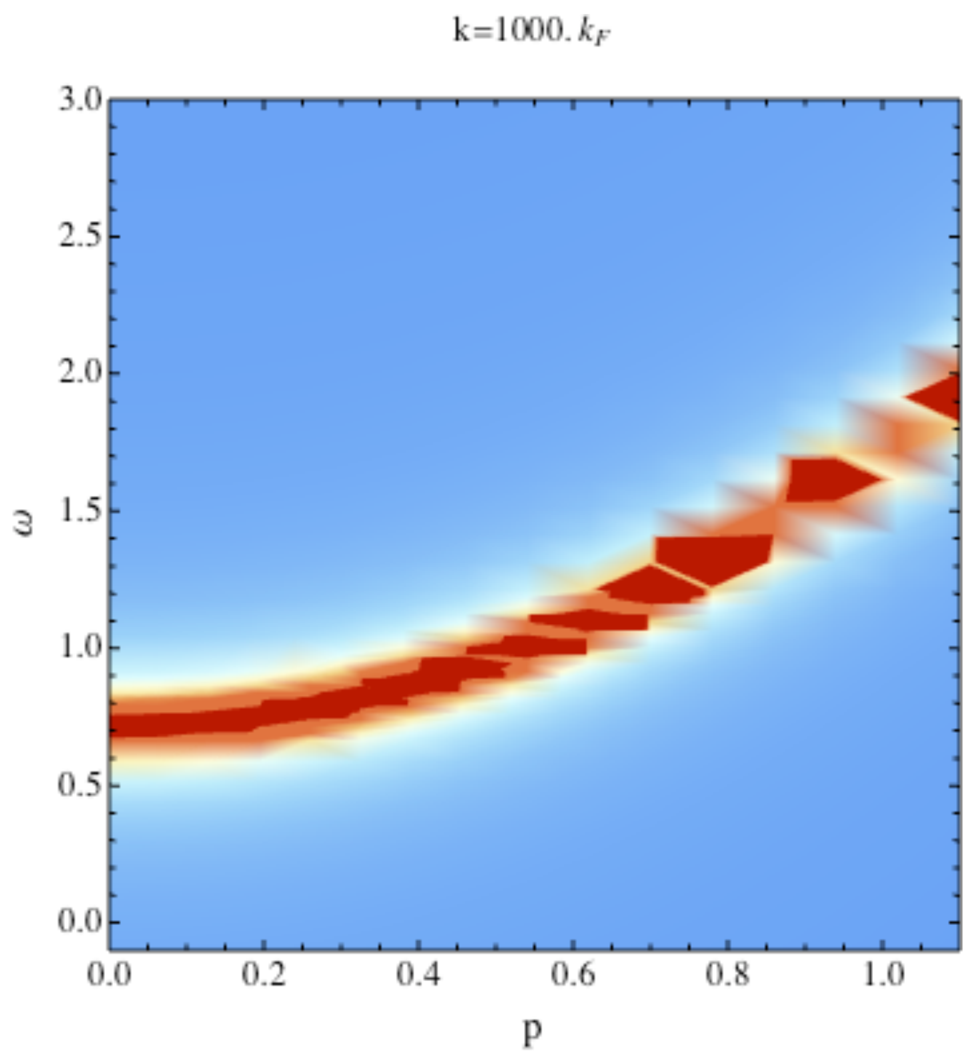
real time

$$G^R_{\downarrow,\phi}(\omega, \mathbf{p}) \longrightarrow \mathcal{A}_{\downarrow,\phi}(\omega, \mathbf{p}) = 2\,\mathrm{Im} G^R_{\downarrow,\phi}(\omega, \mathbf{p})$$

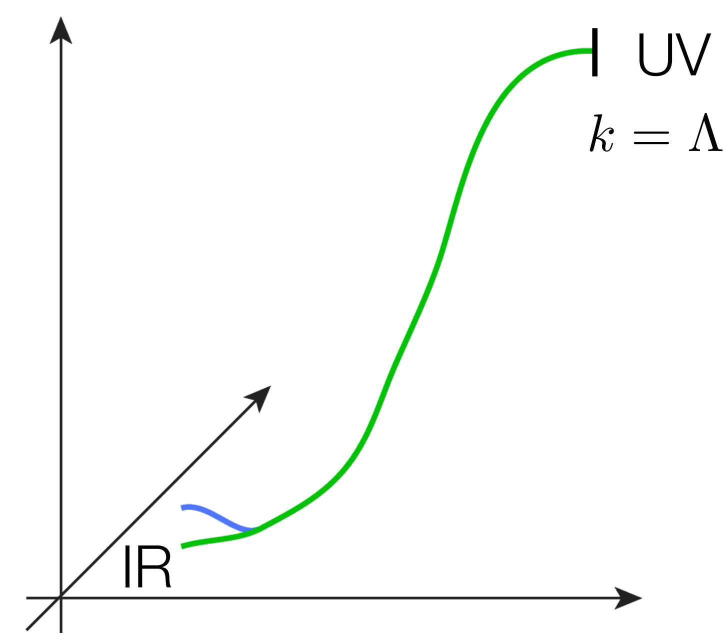
analysis of the quasi-particle properties

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$



theory space



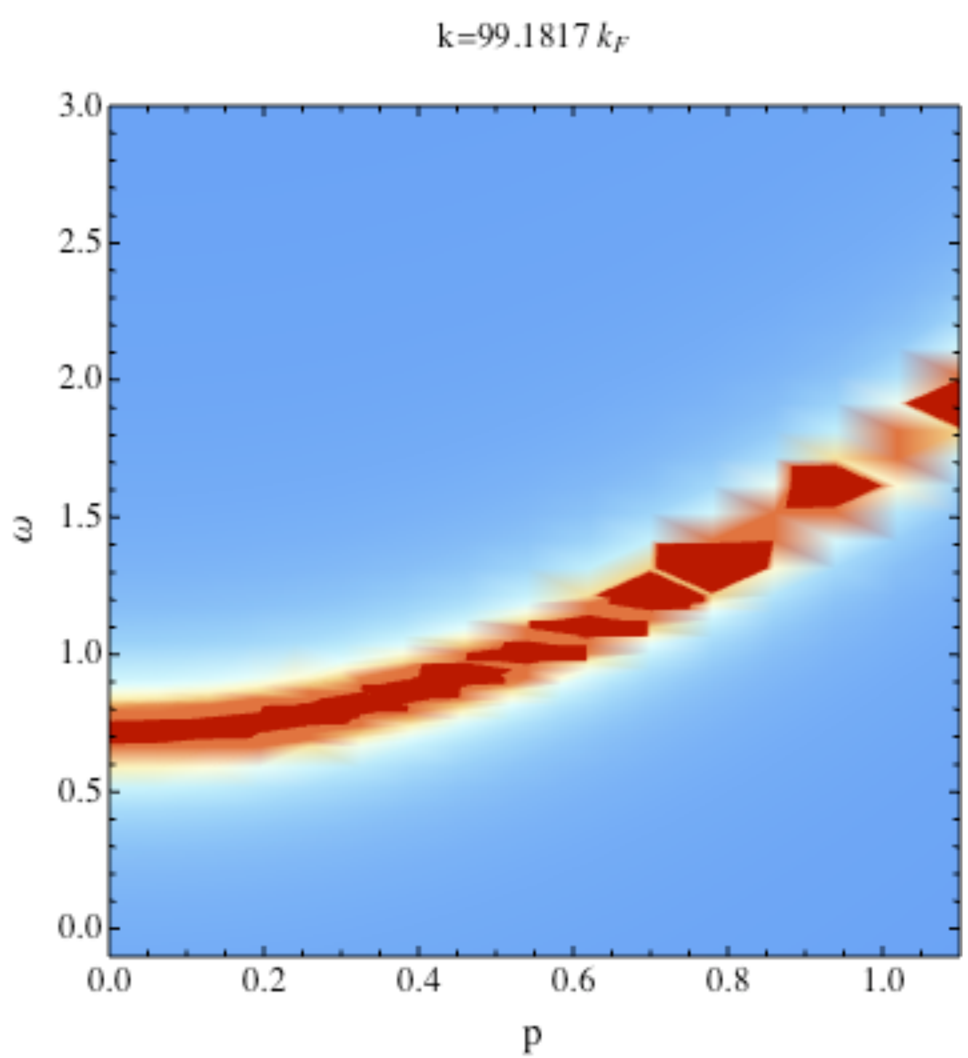
$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

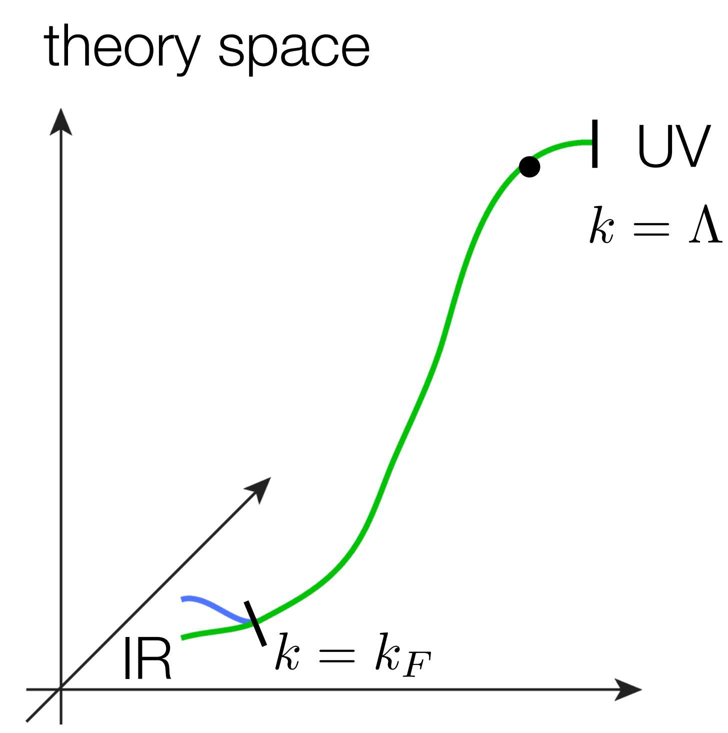
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

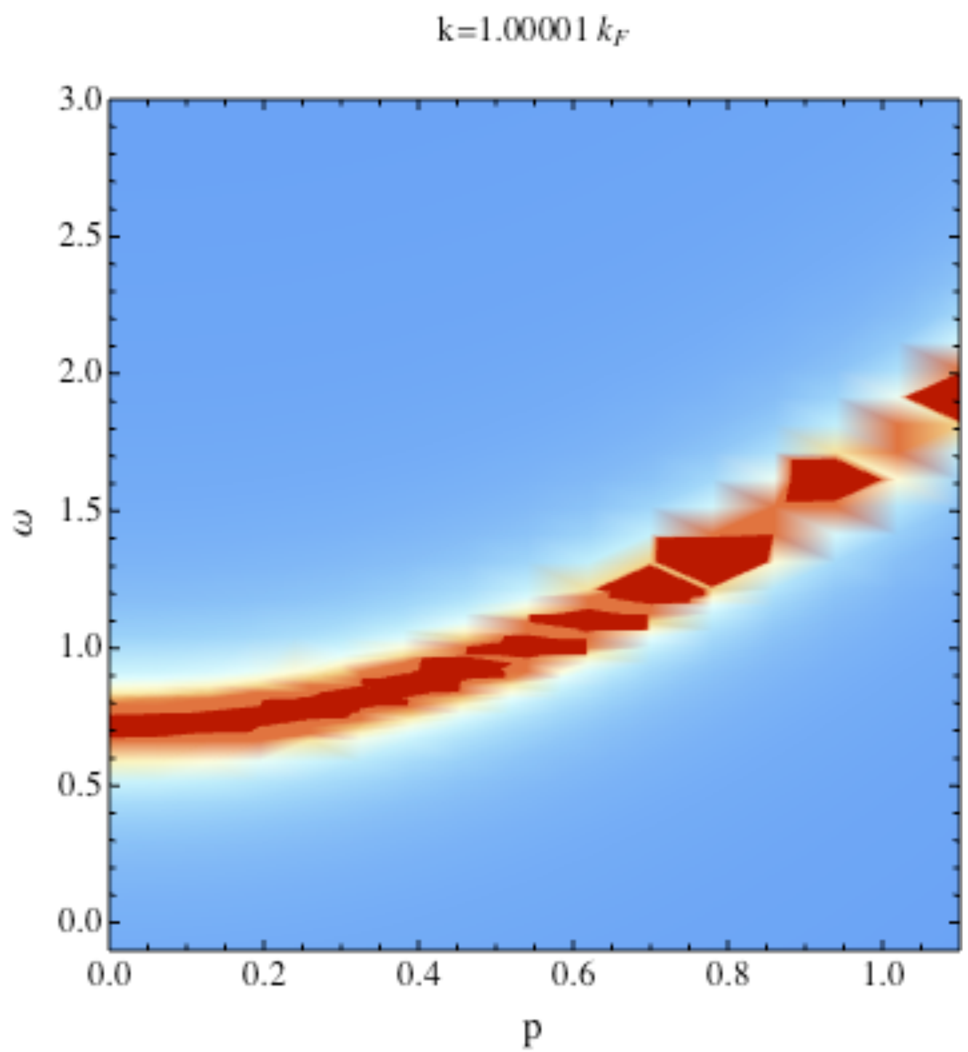
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

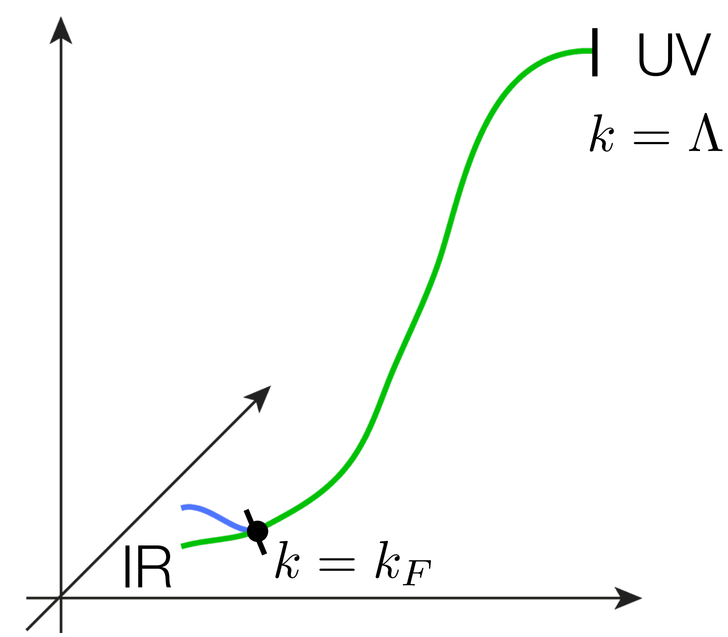
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

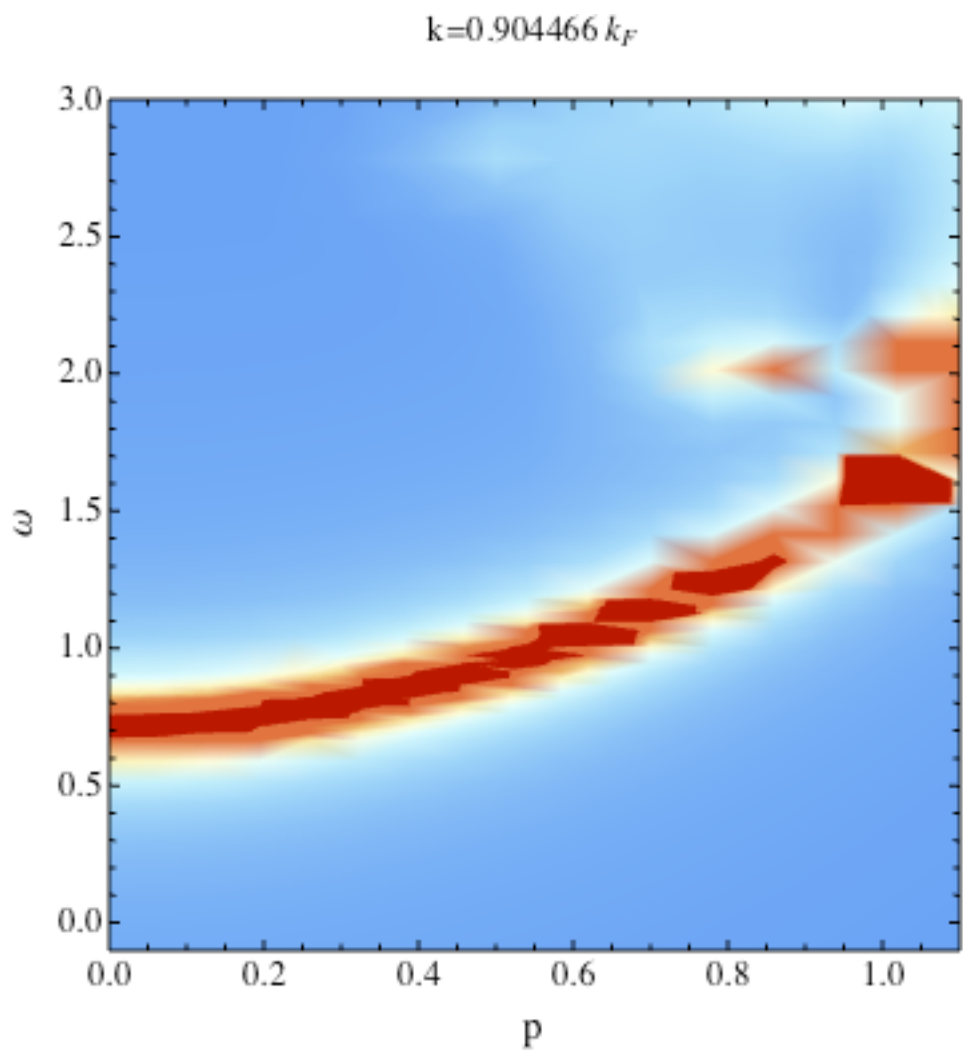
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

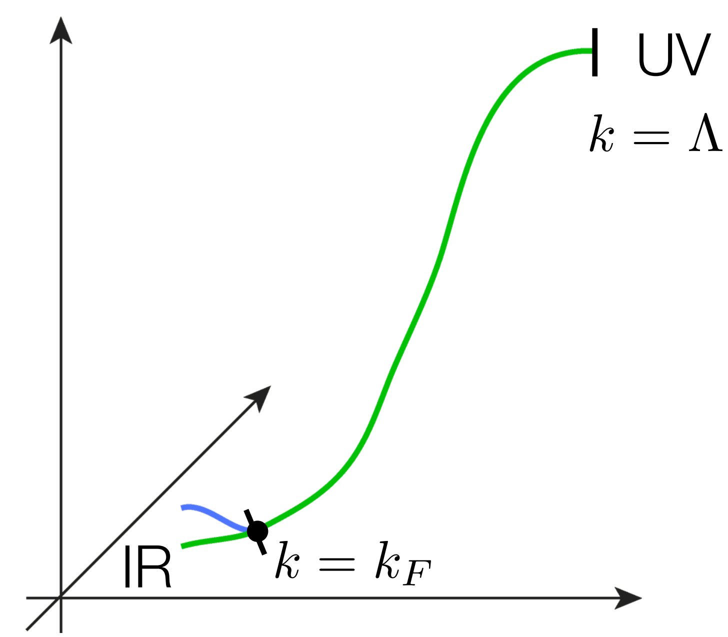
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

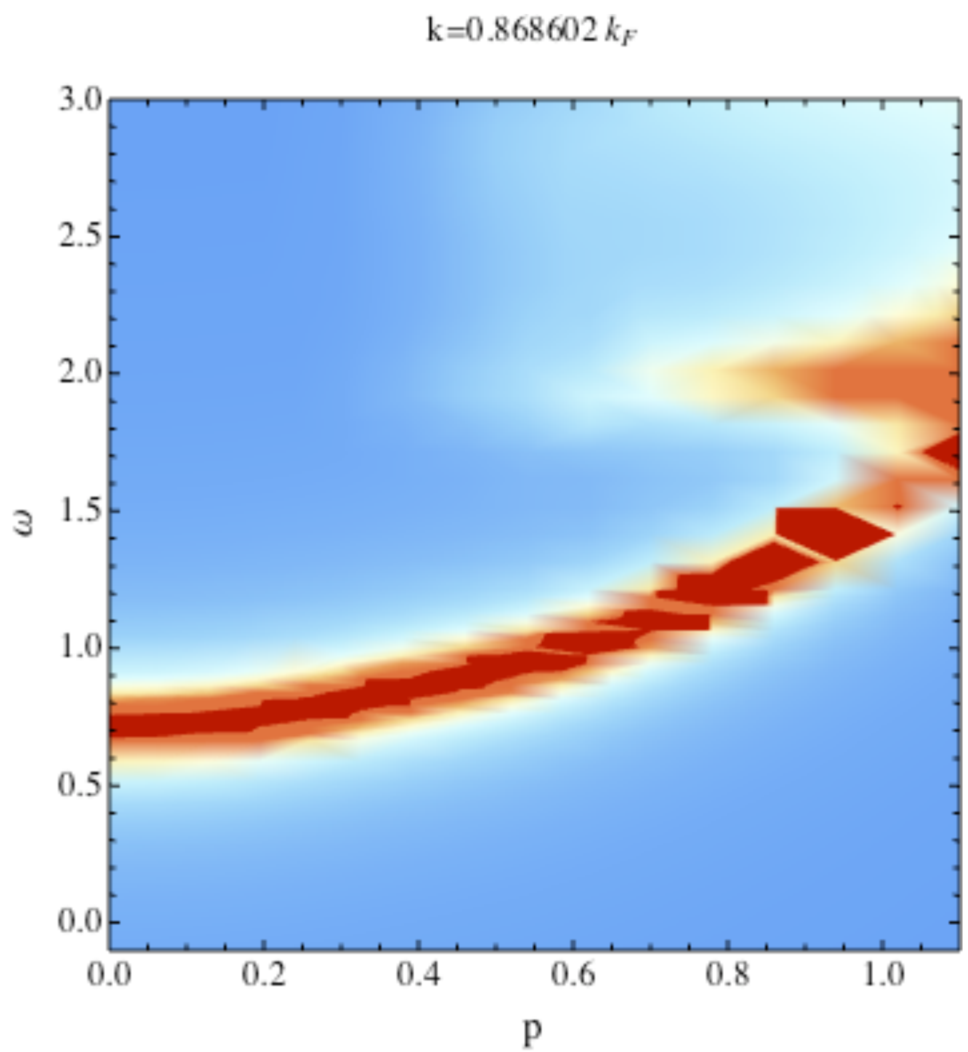
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

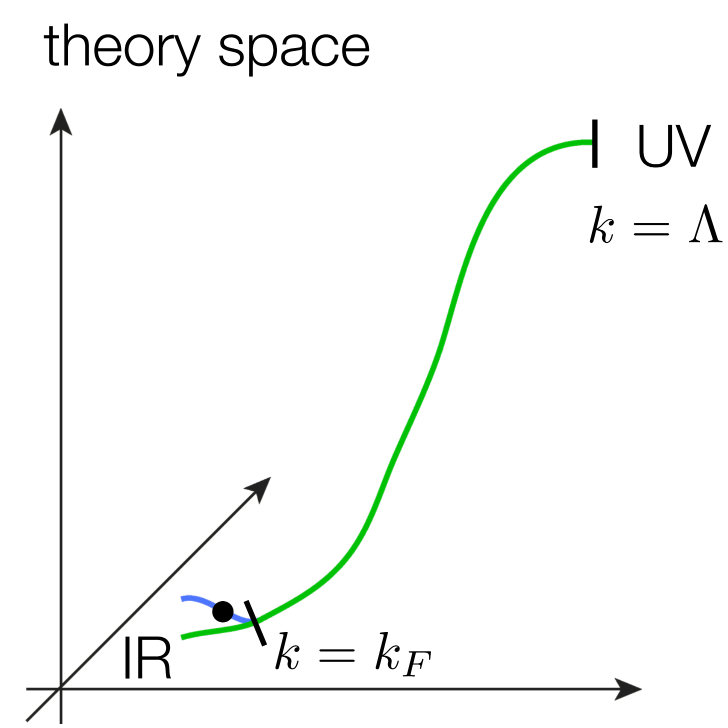
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

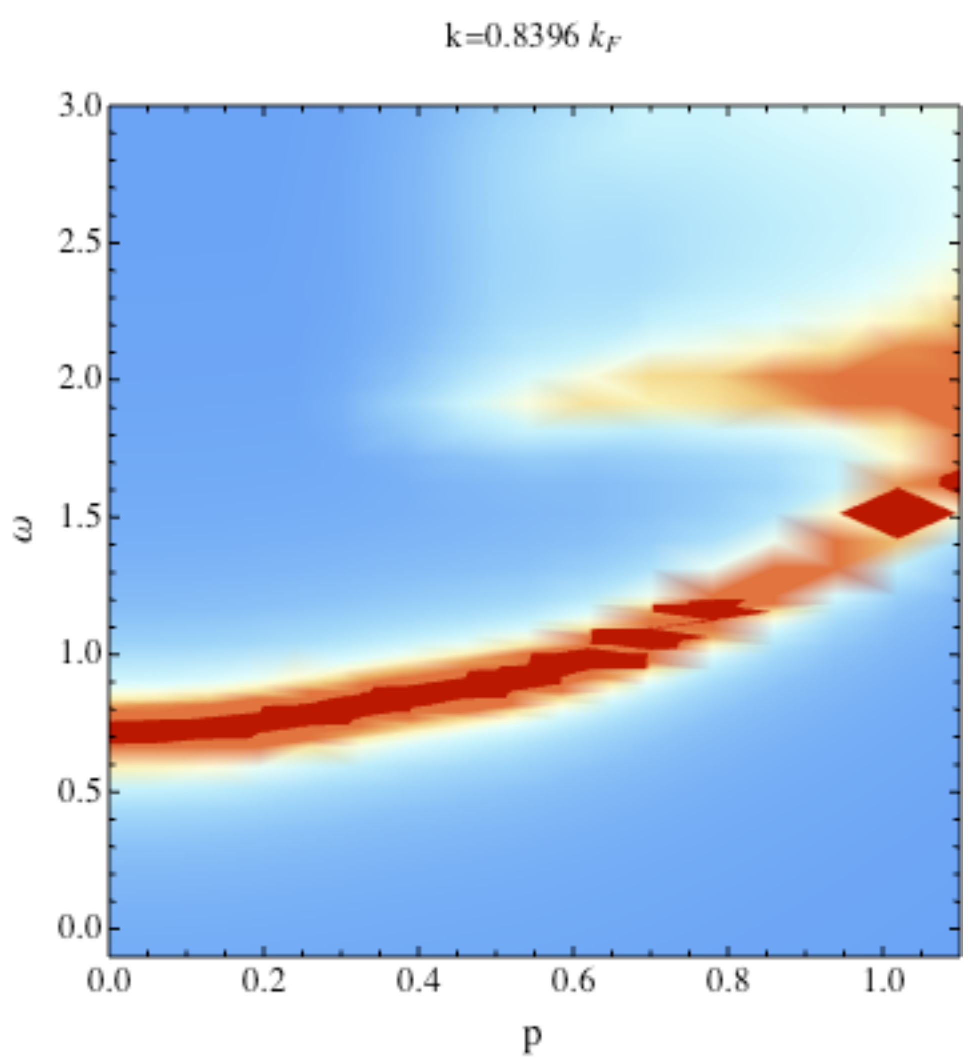
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

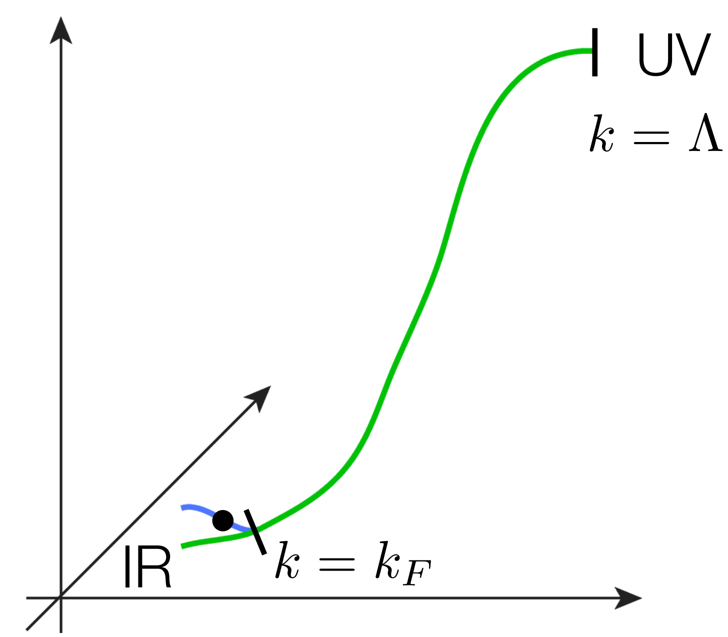
$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$



theory space



$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

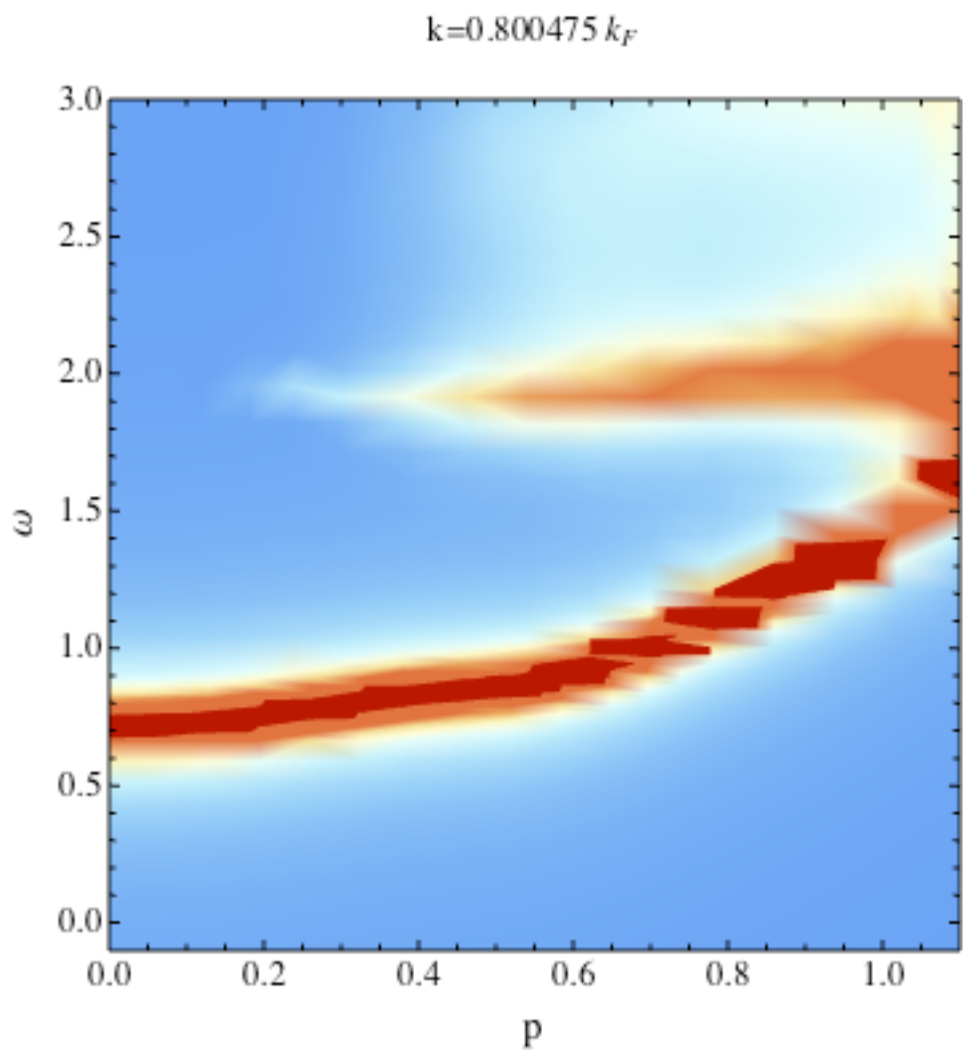
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

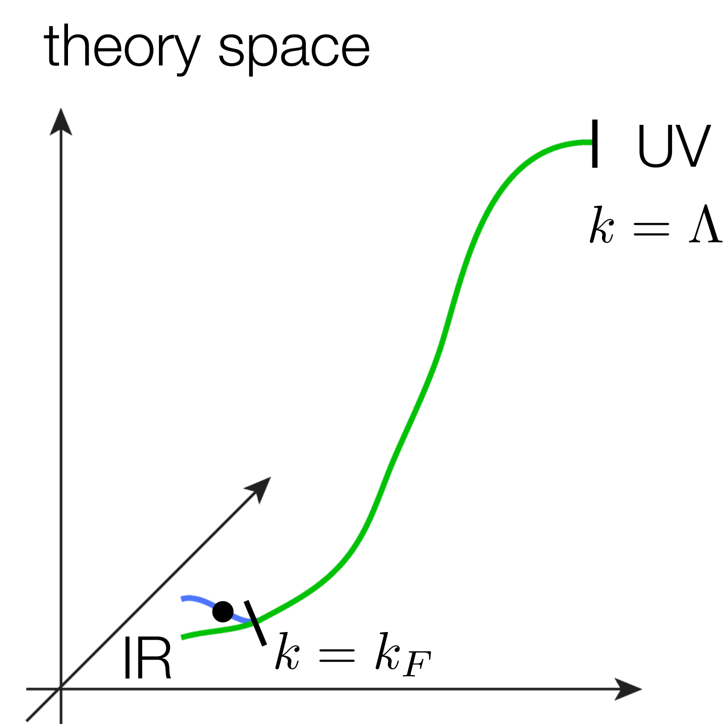
$\longrightarrow$ ground state energy

$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

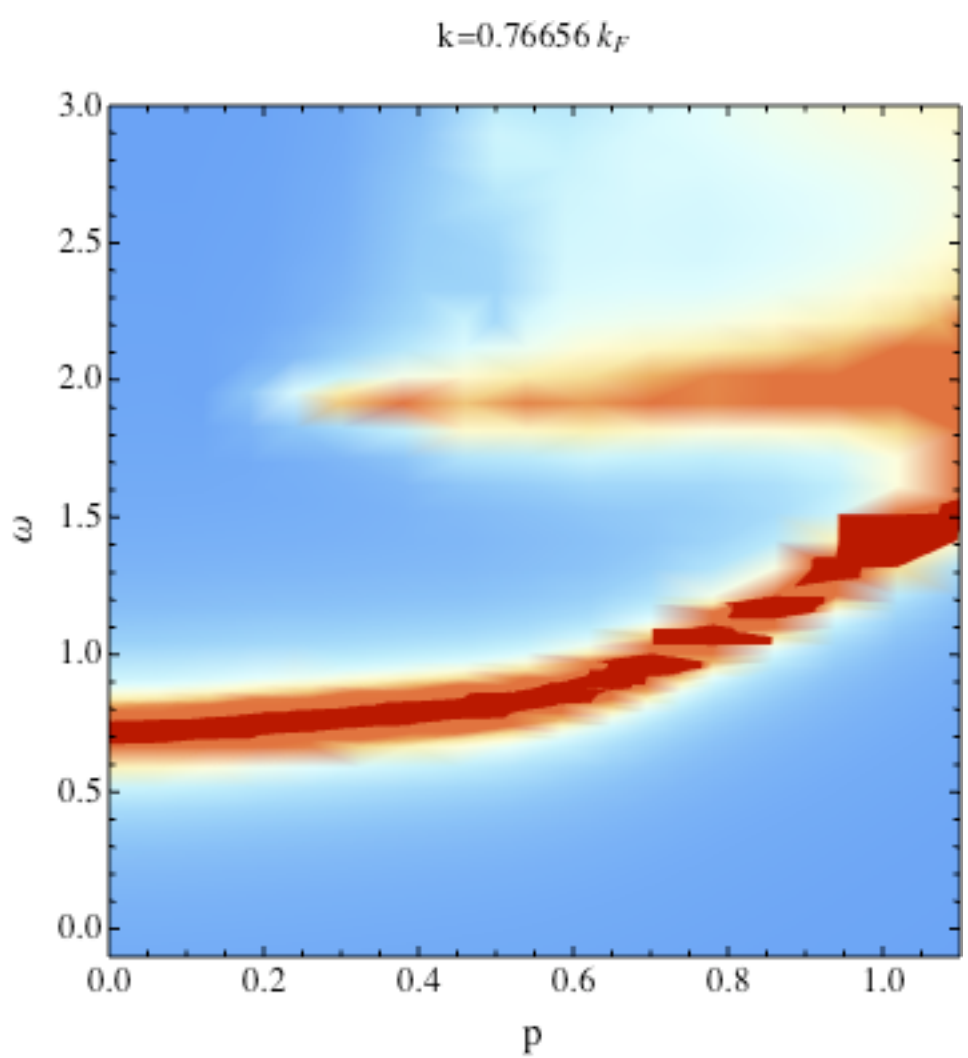
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

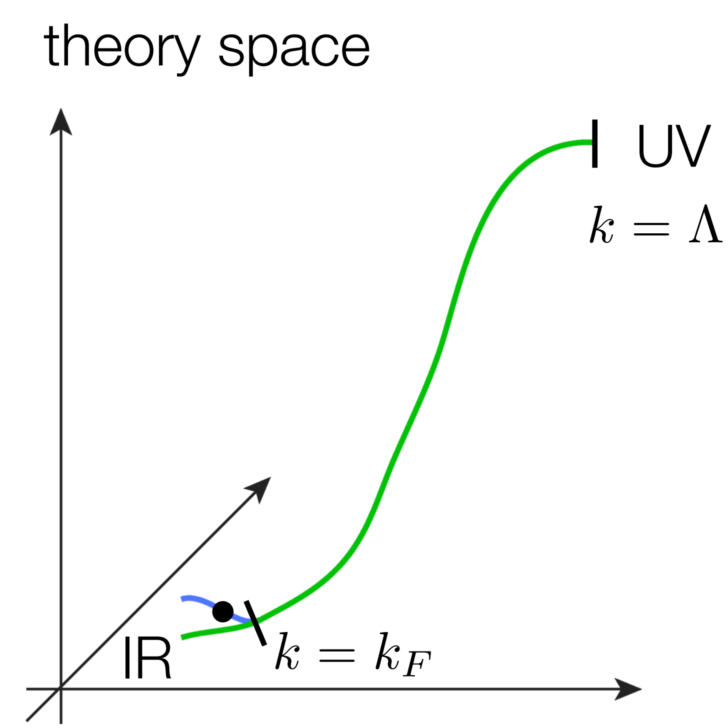
$\longrightarrow$ ground state energy

$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

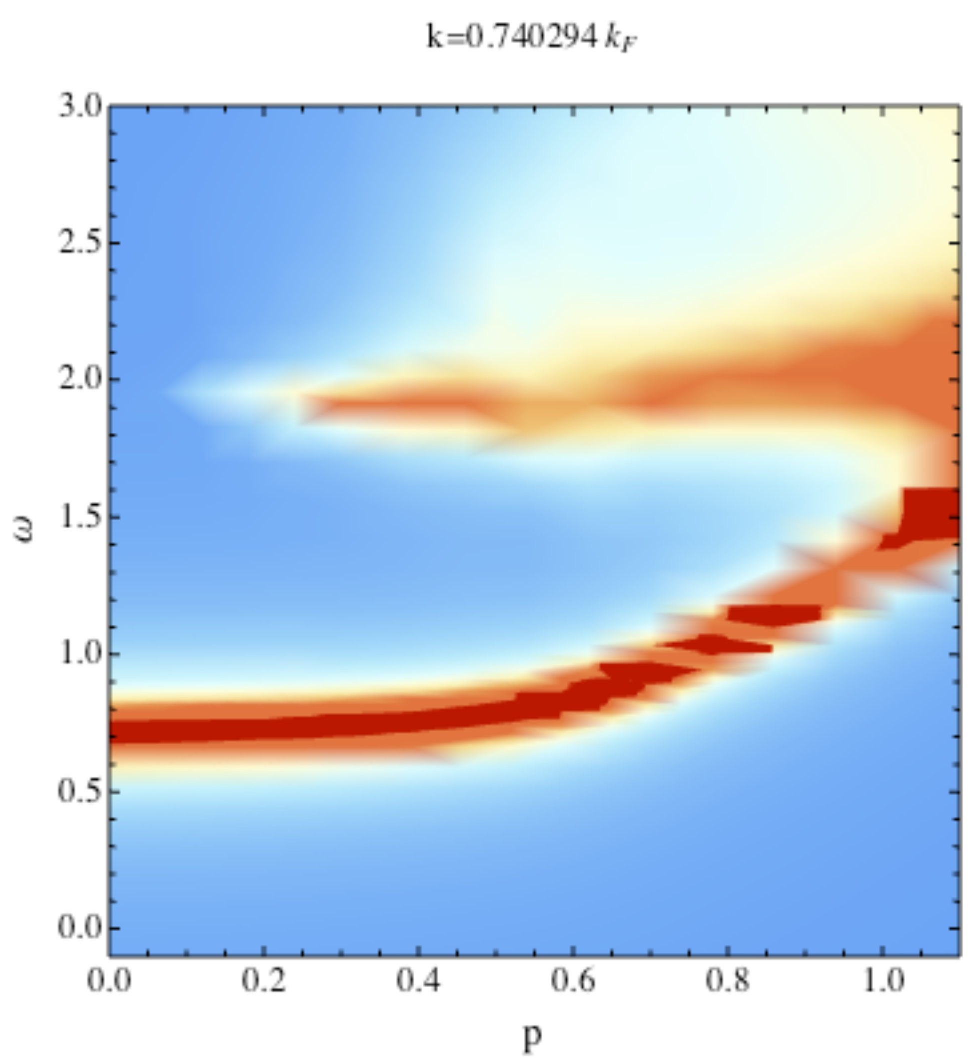
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

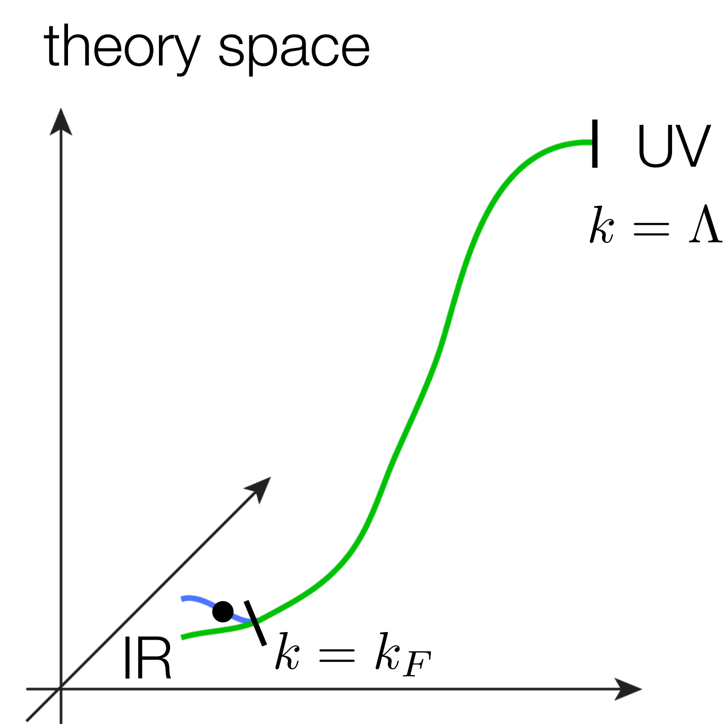
$\longrightarrow$ ground state energy

$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

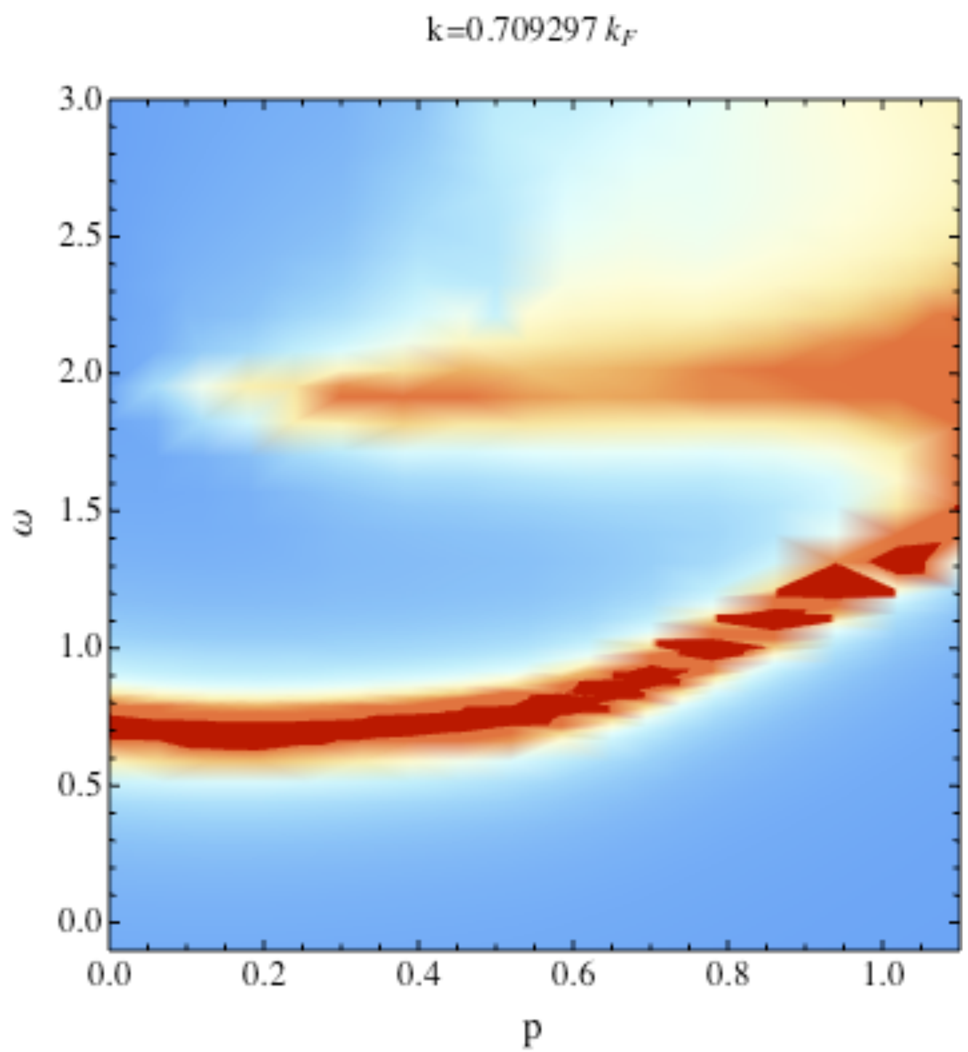
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

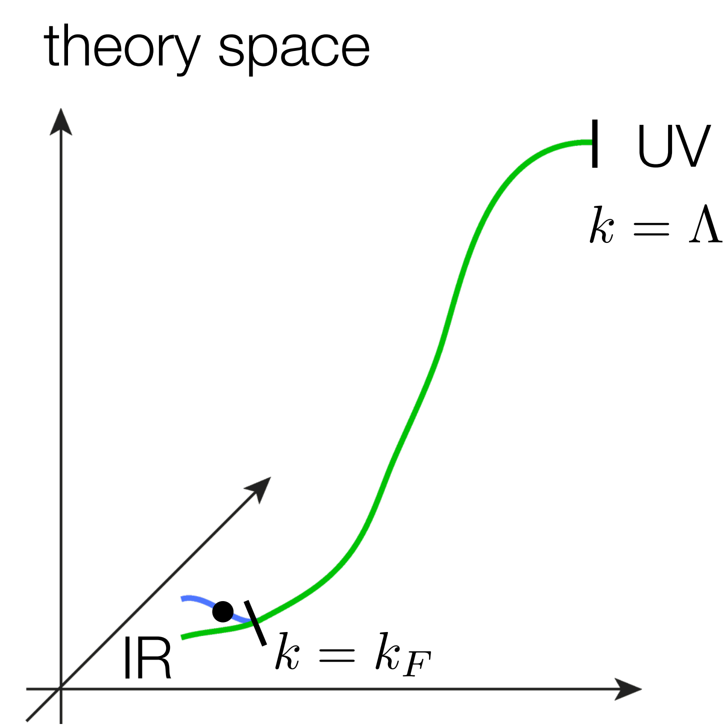
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

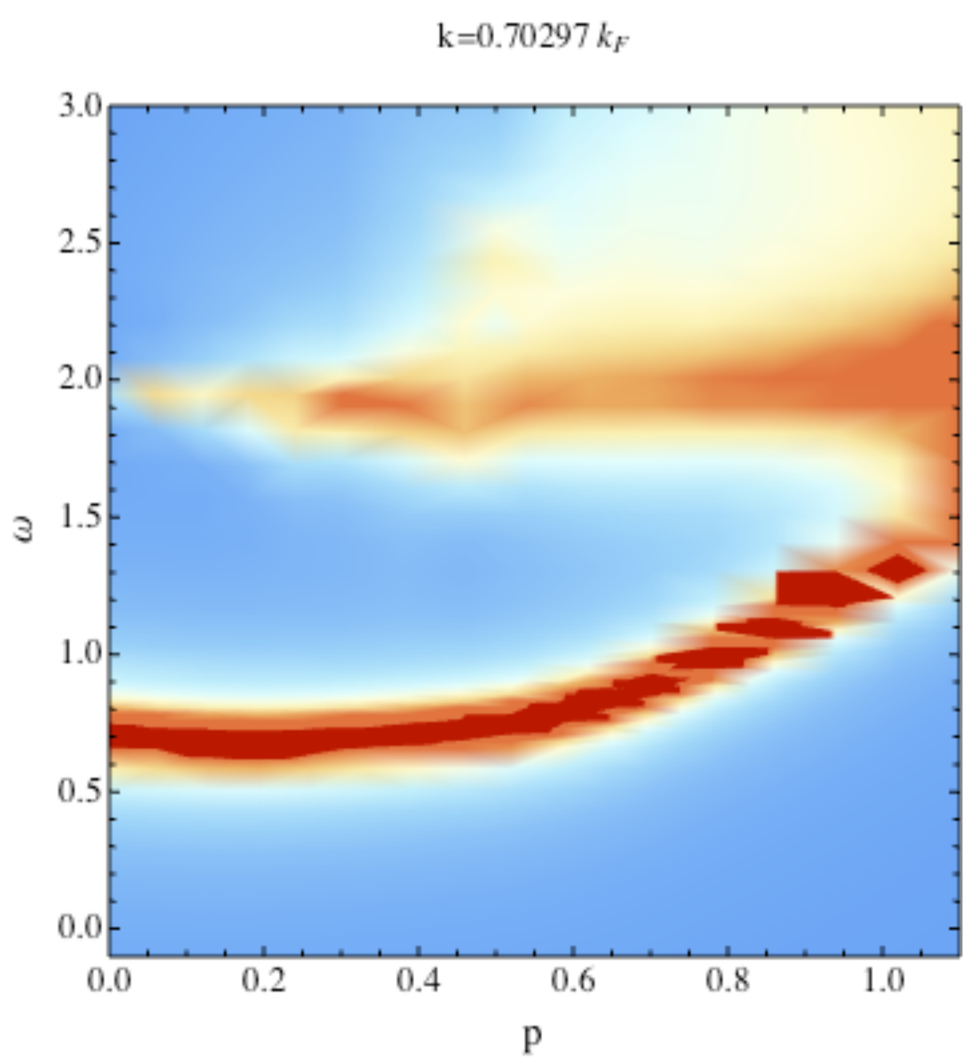
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

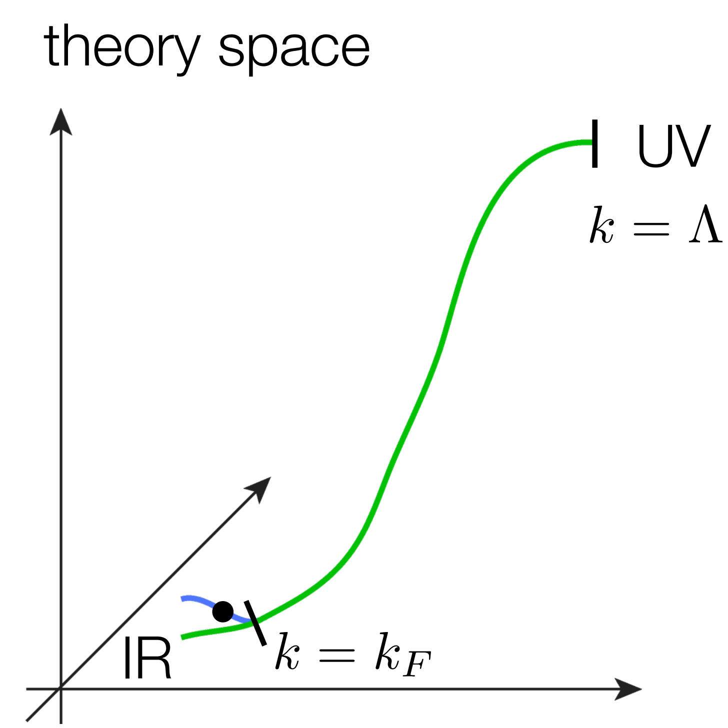
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

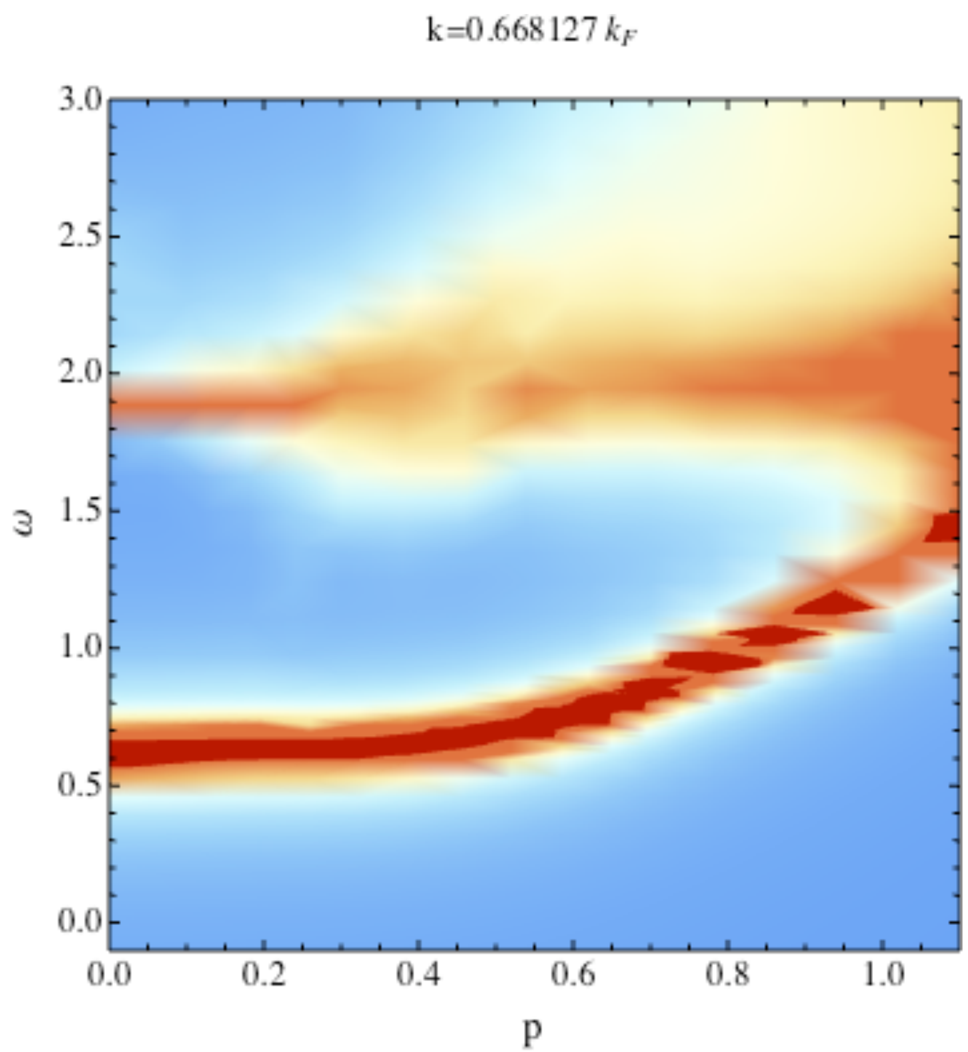
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

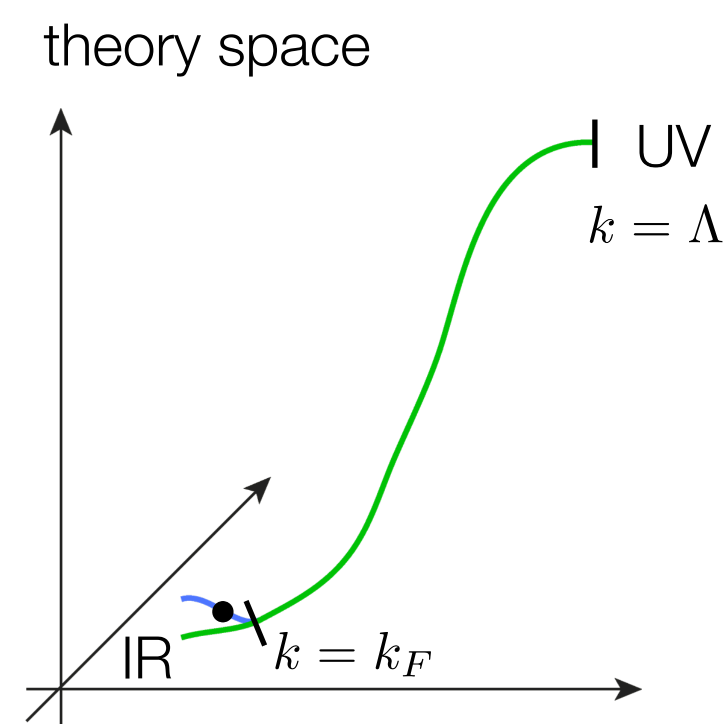
$\longrightarrow$ ground state energy

$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

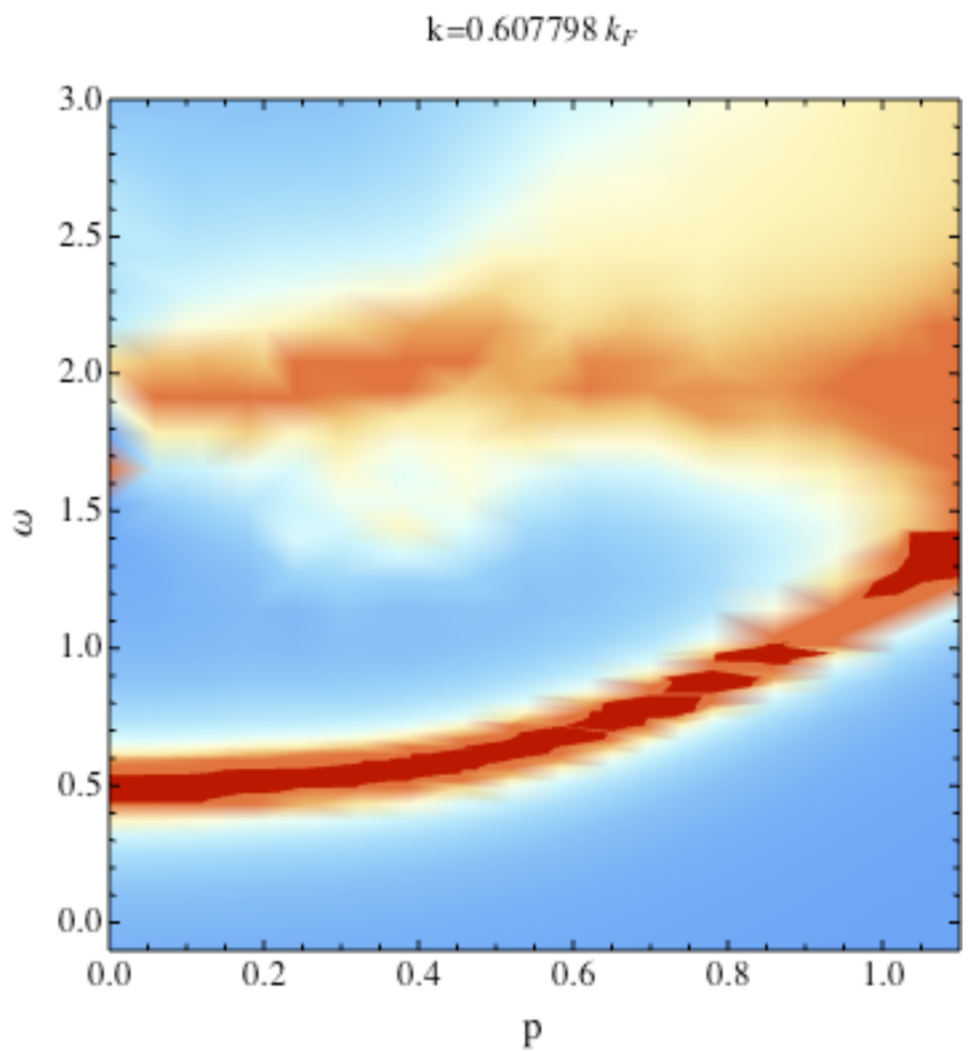
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

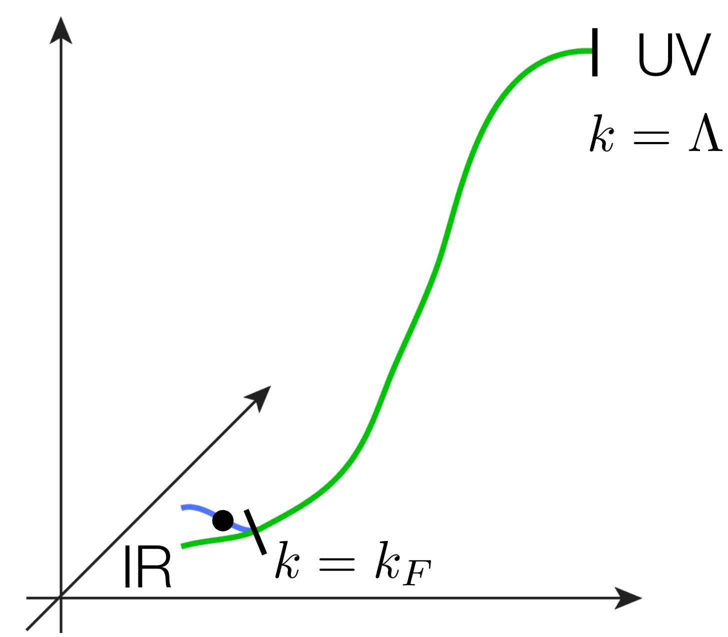
$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$



theory space



$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

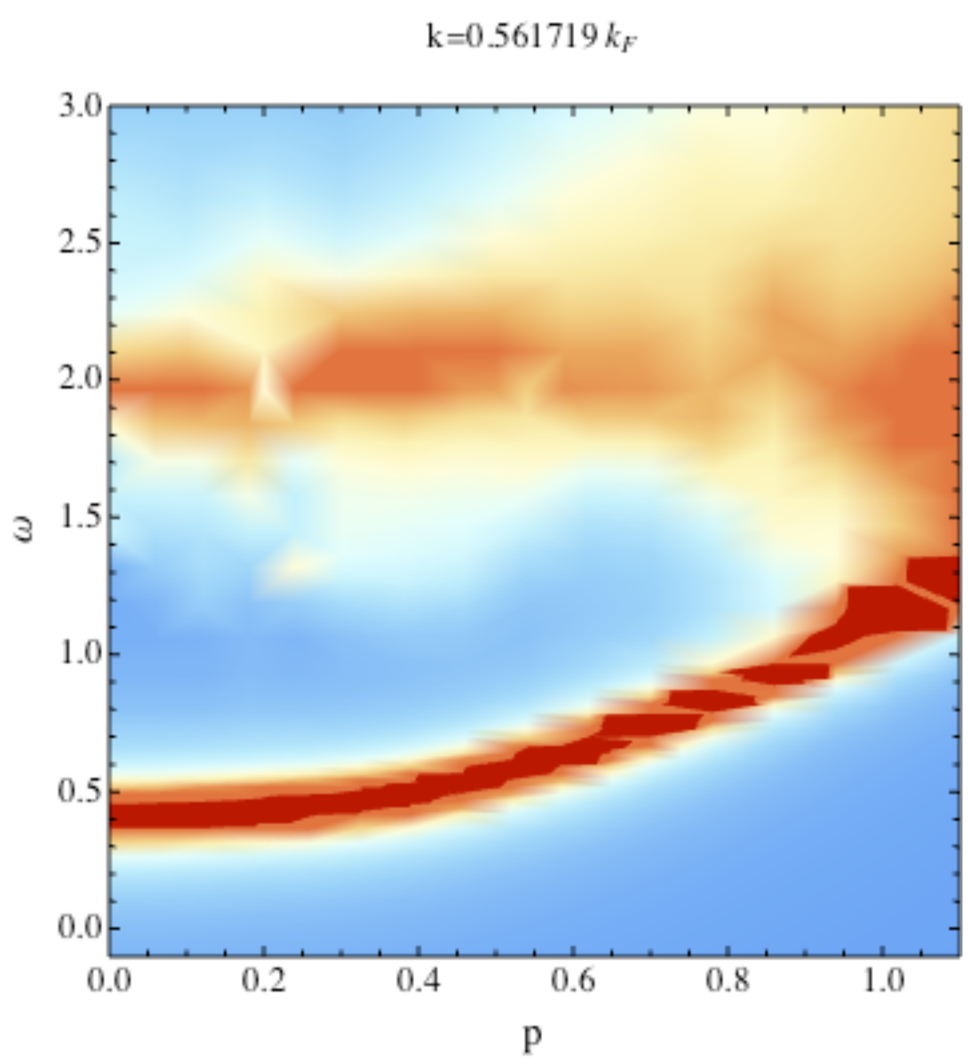
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

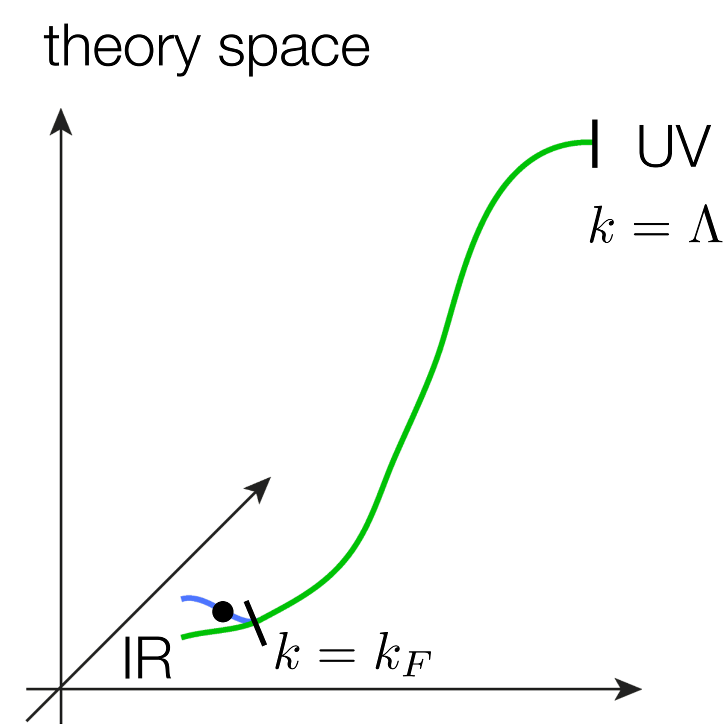
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

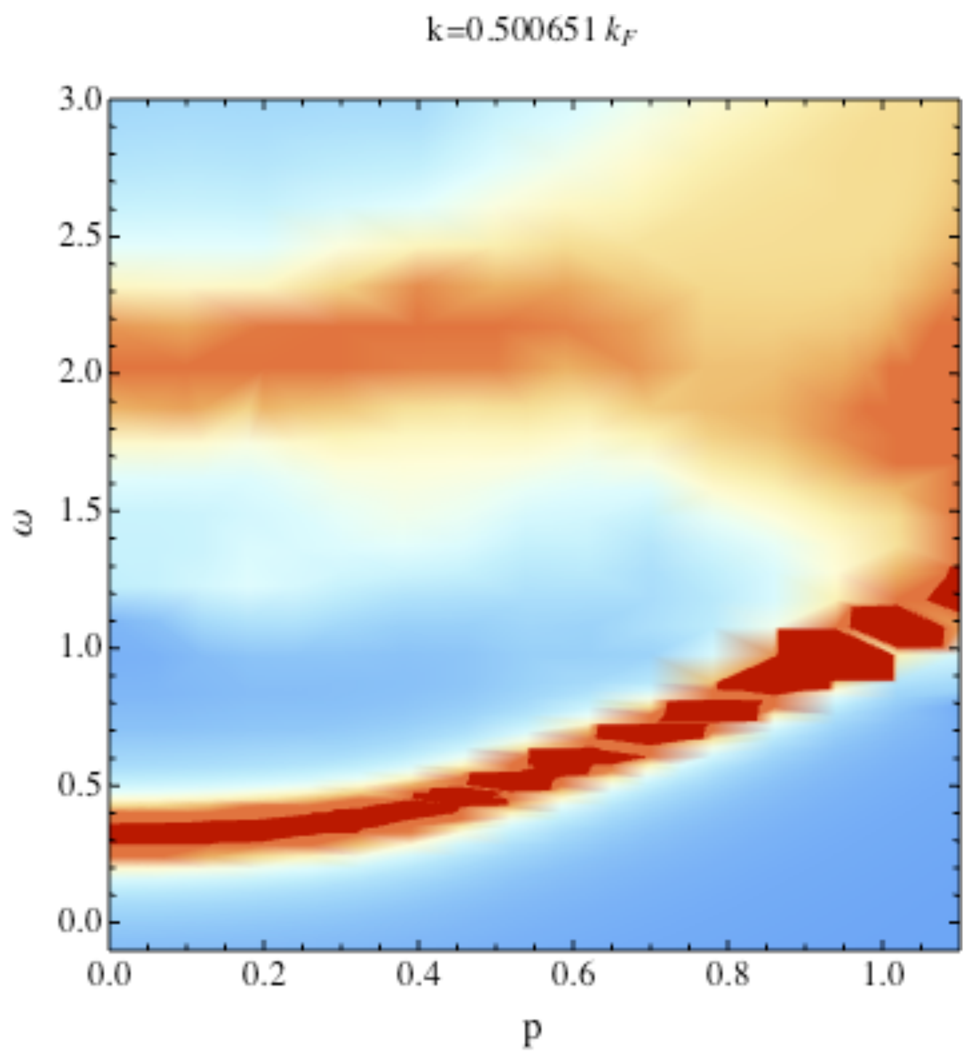
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

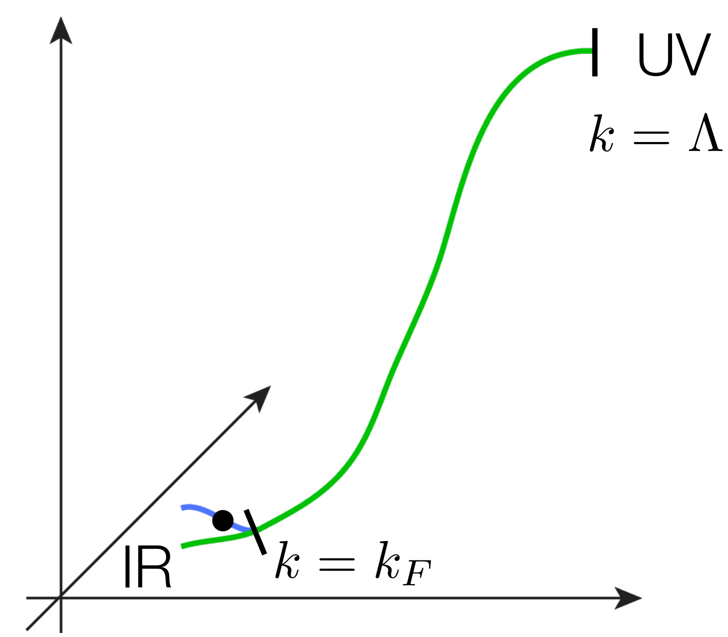
$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$



theory space



$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

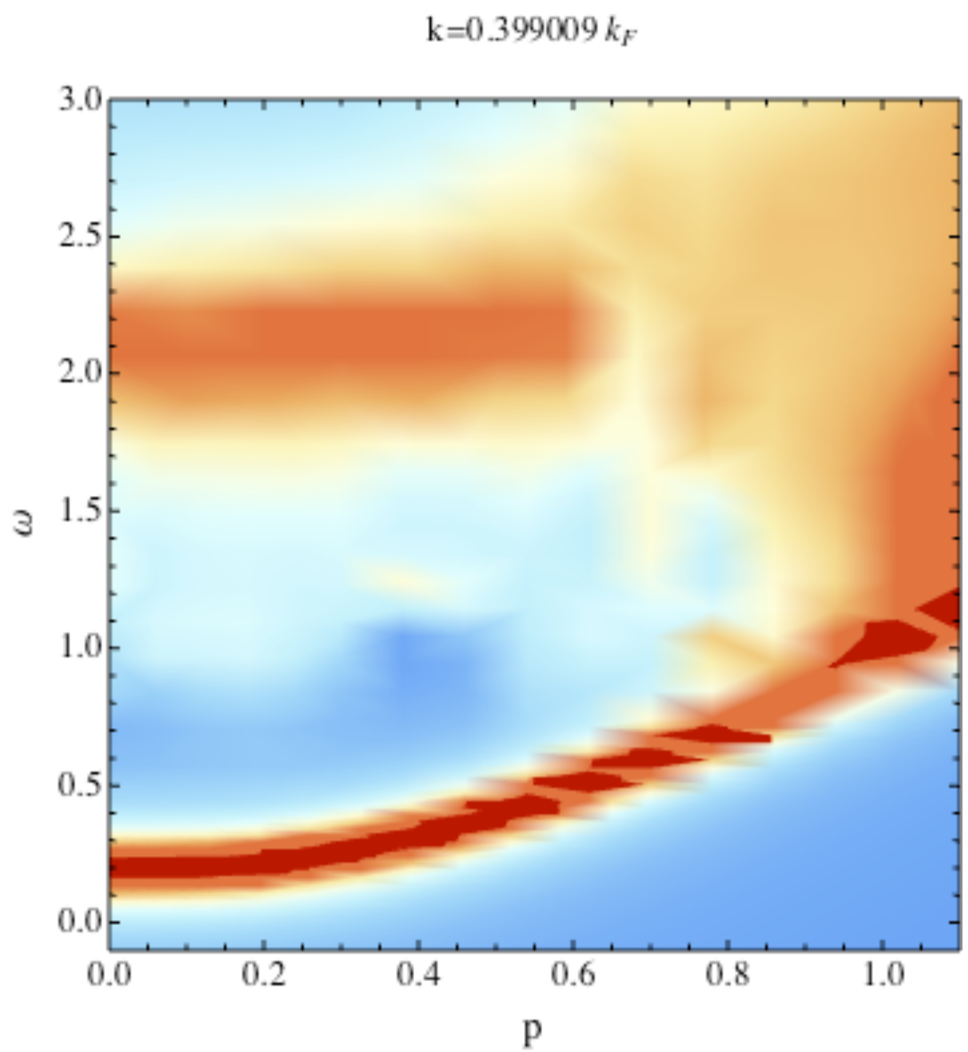
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

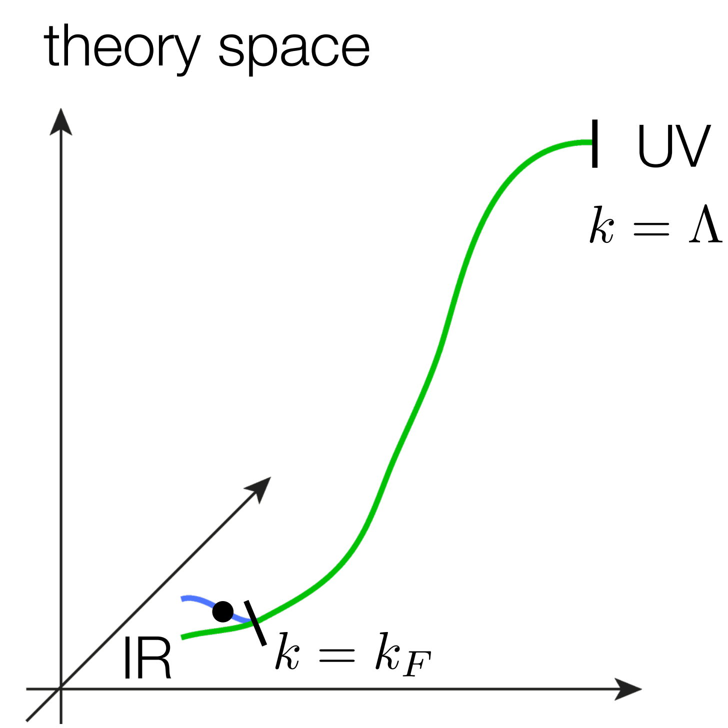
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

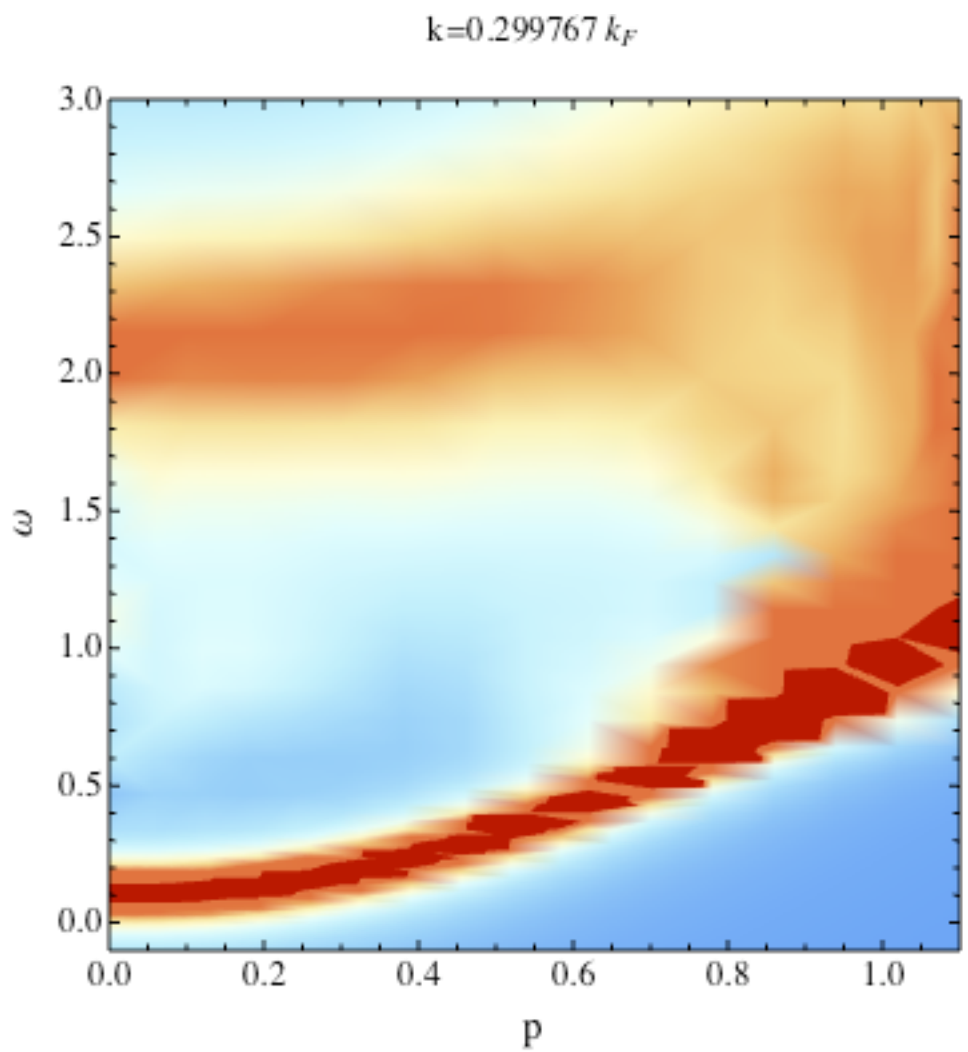
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

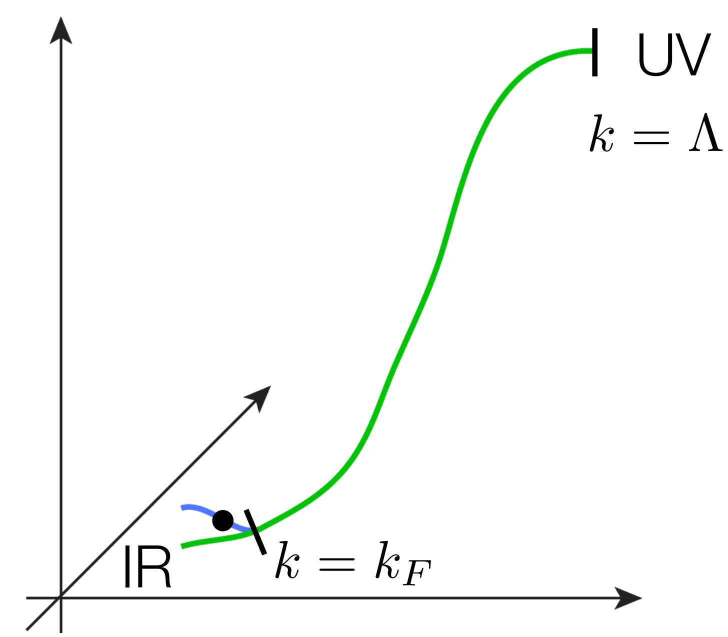
$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$



theory space



$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

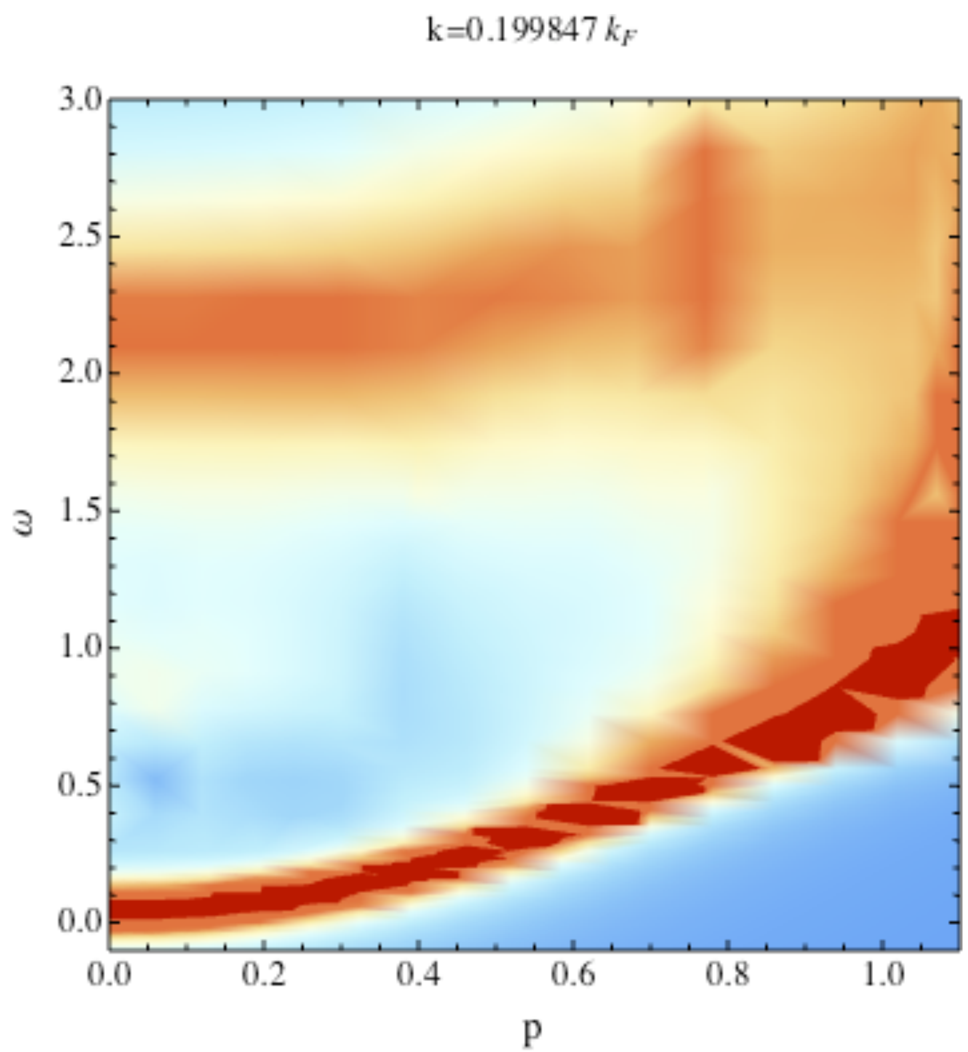
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

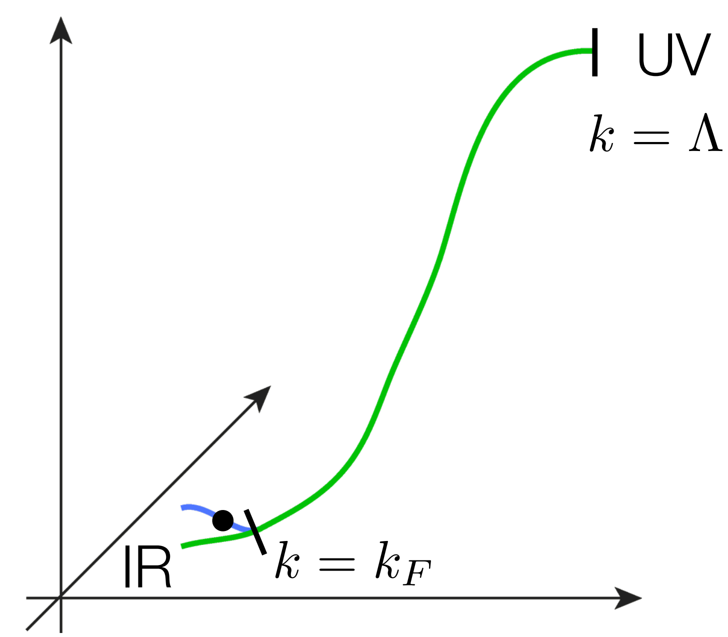
$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$



theory space



$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

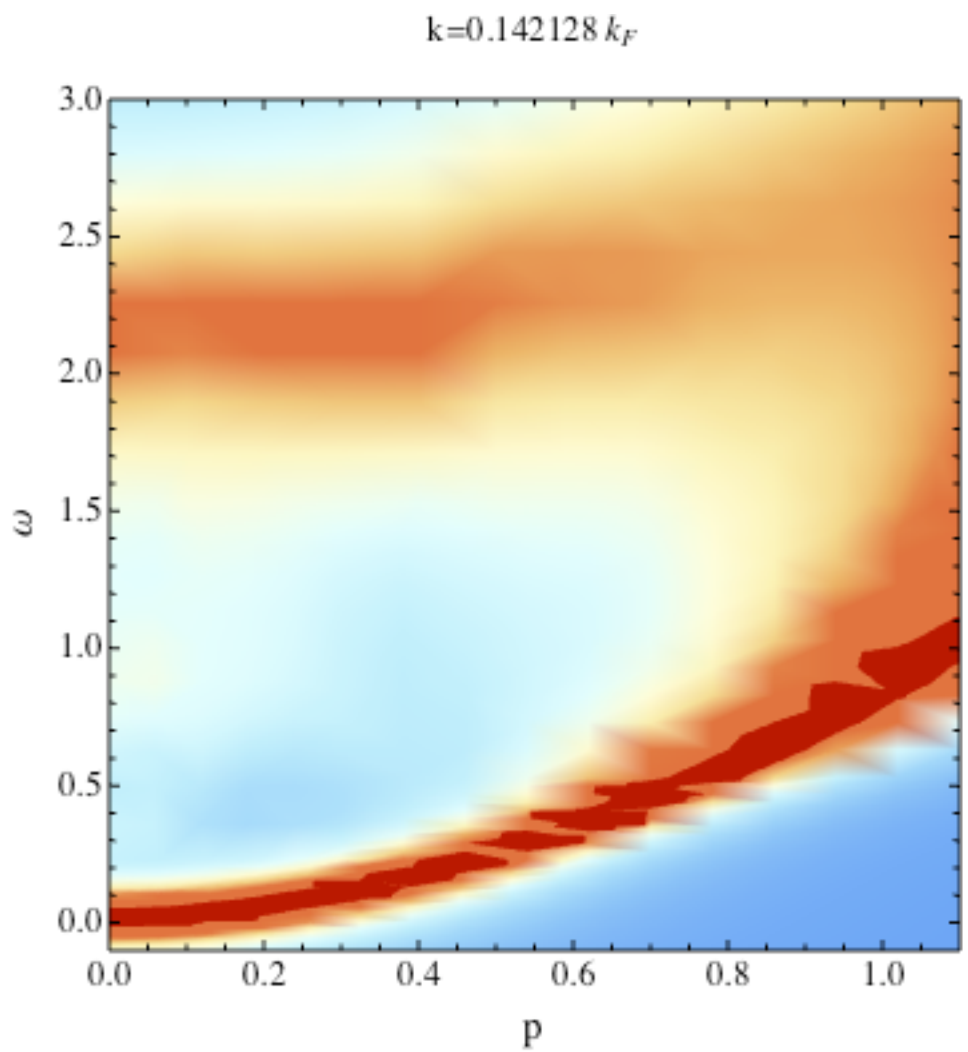
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

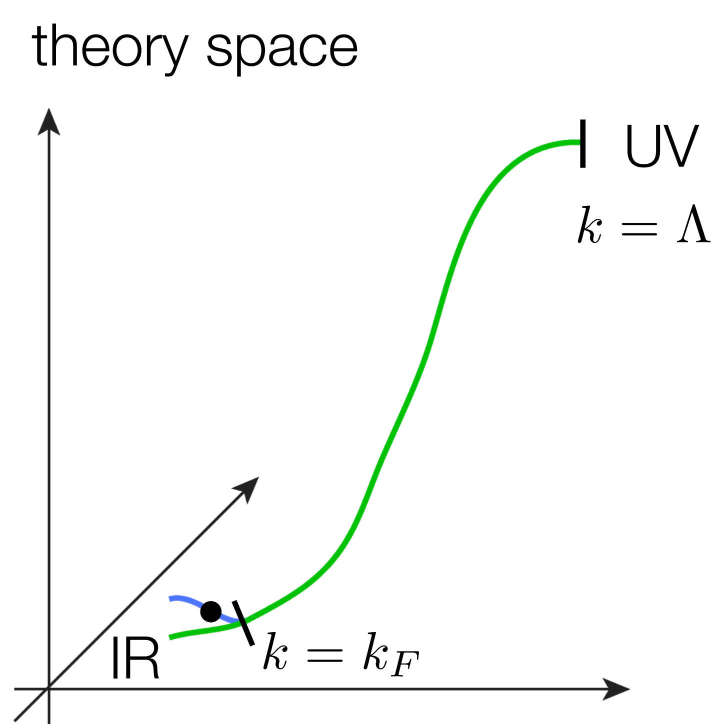
$\longrightarrow$ ground state energy

$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

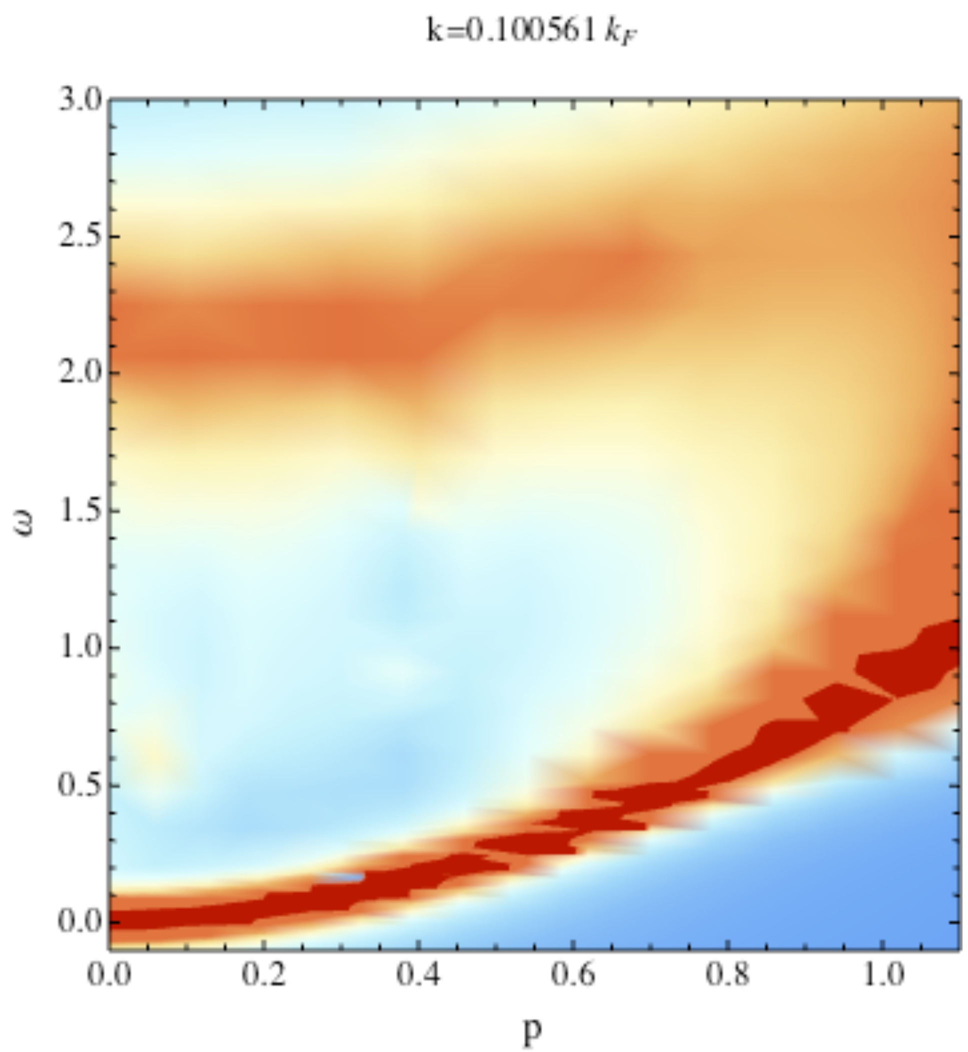
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

$\longrightarrow$ ground state energy

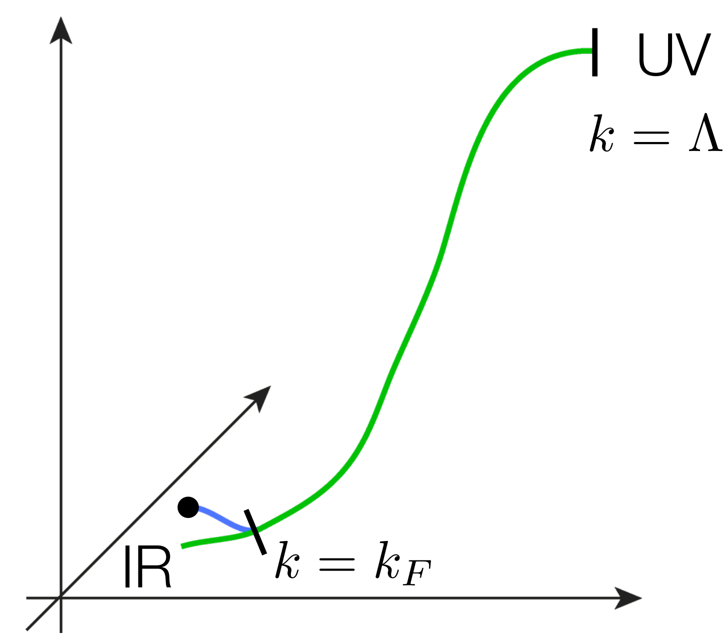
$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega,\mathbf{p})$



theory space



$$P_{\downarrow,k=\Lambda}(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

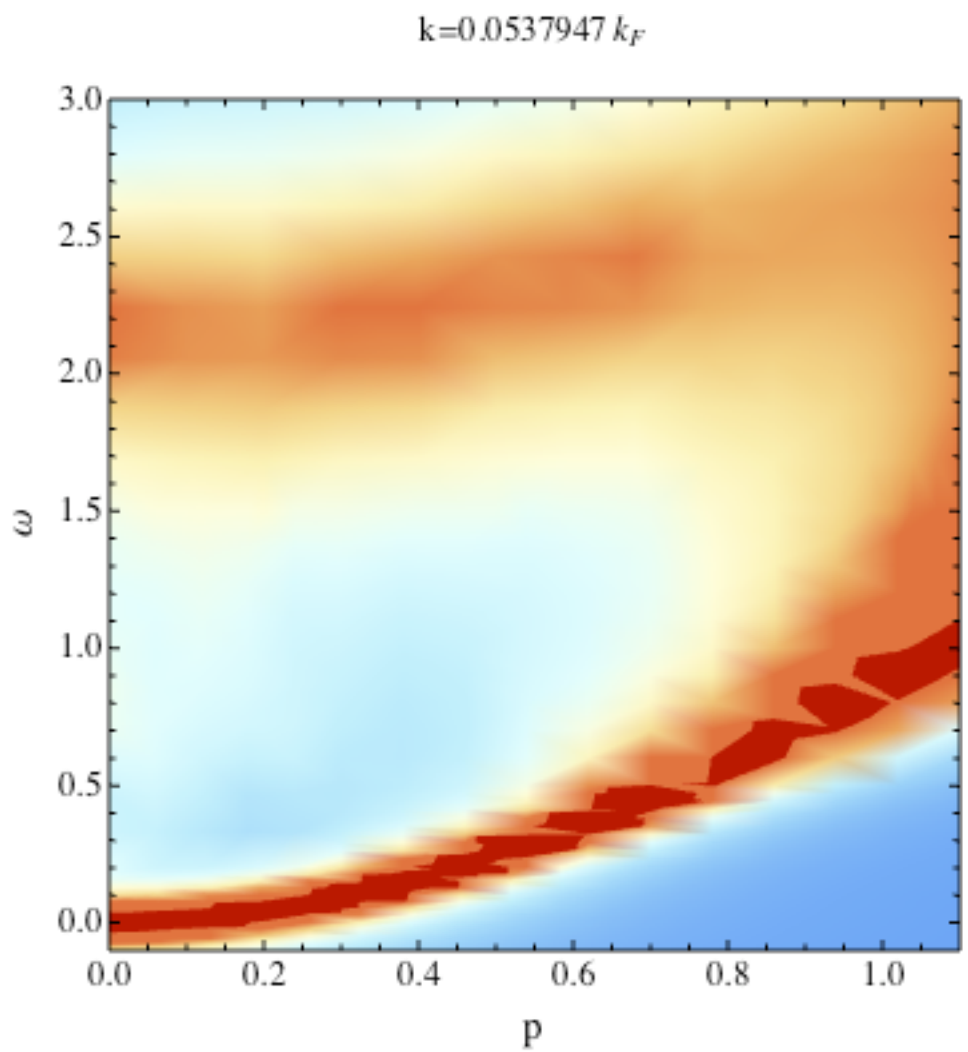
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

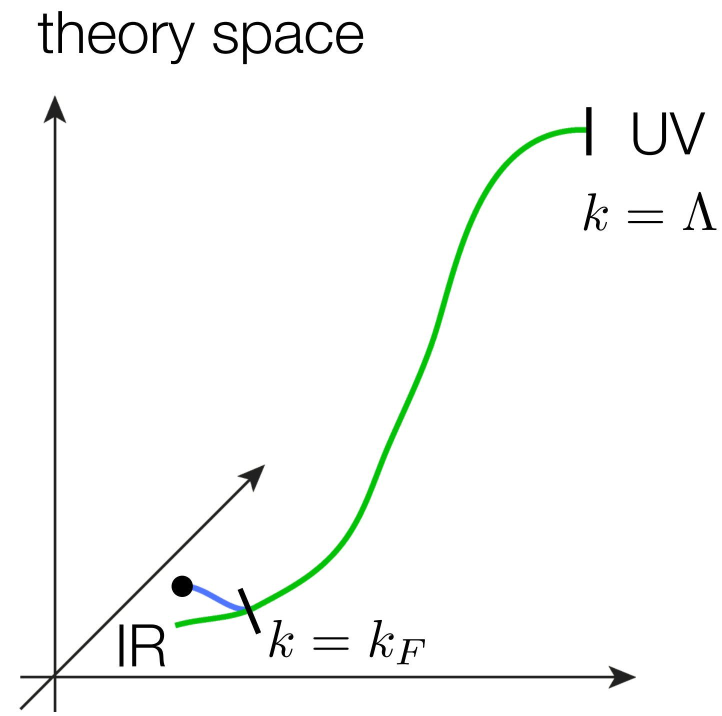
$\longrightarrow$ ground state energy

$$P_k(\omega,\mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega,\mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

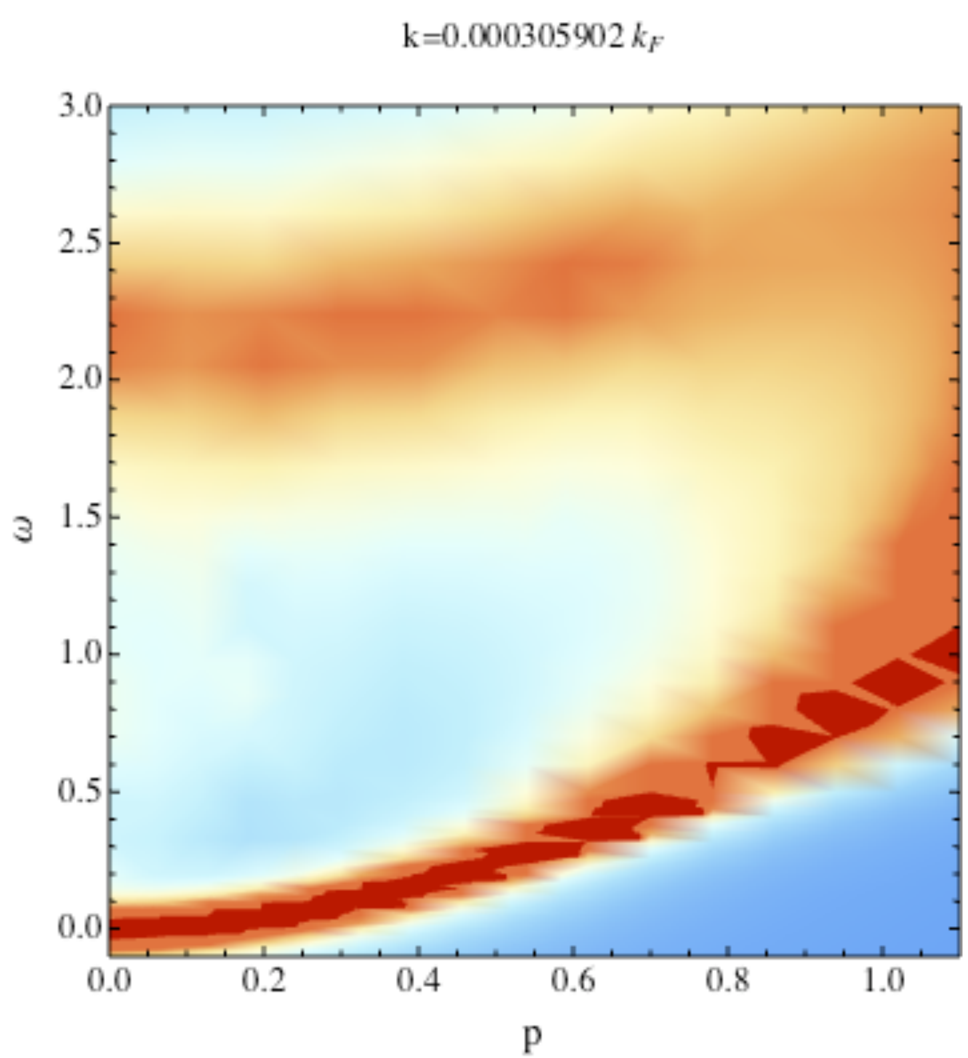
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

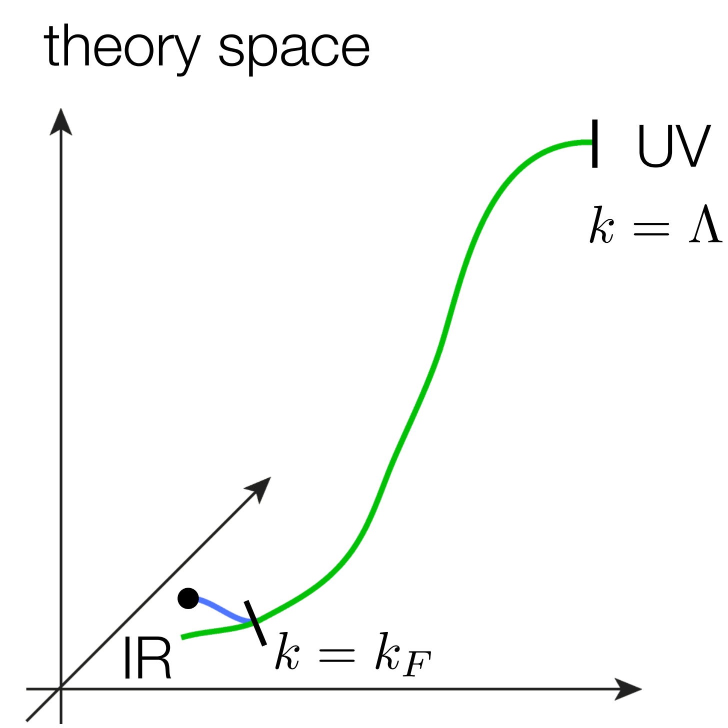
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

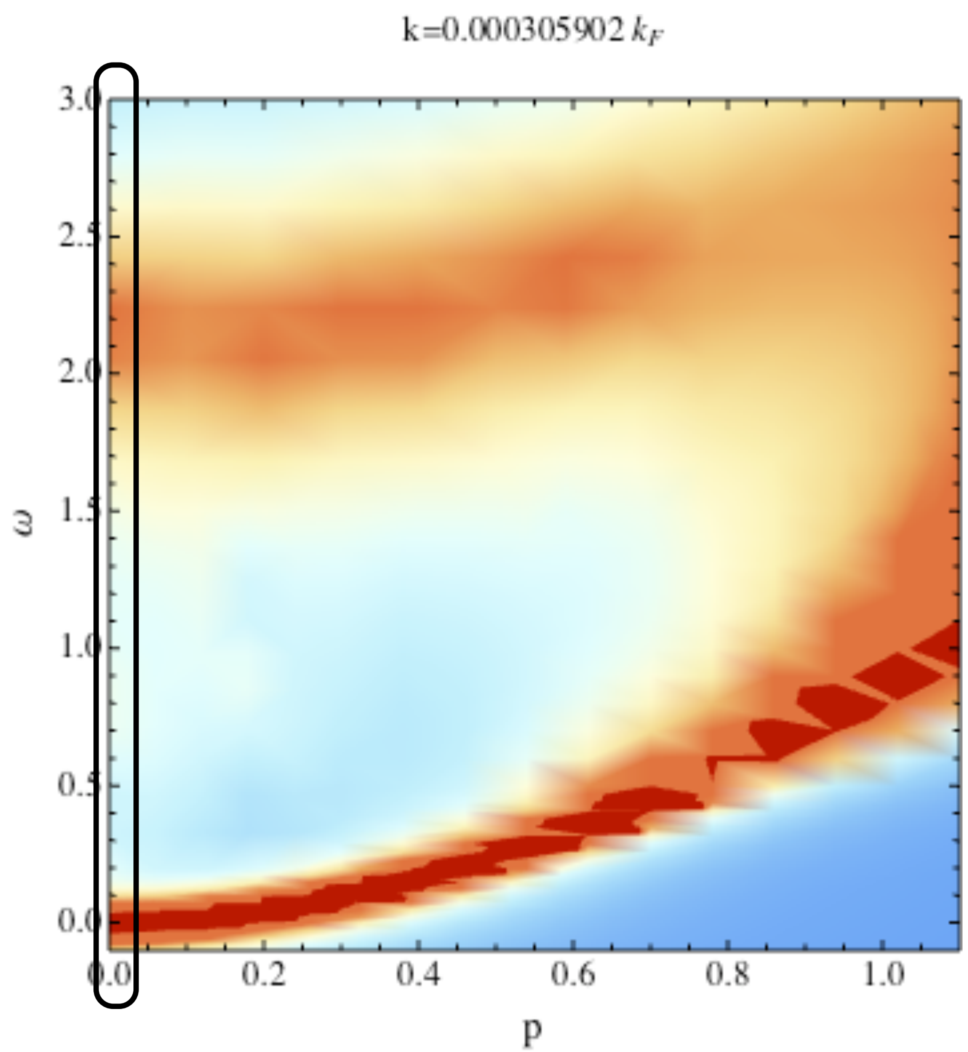
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

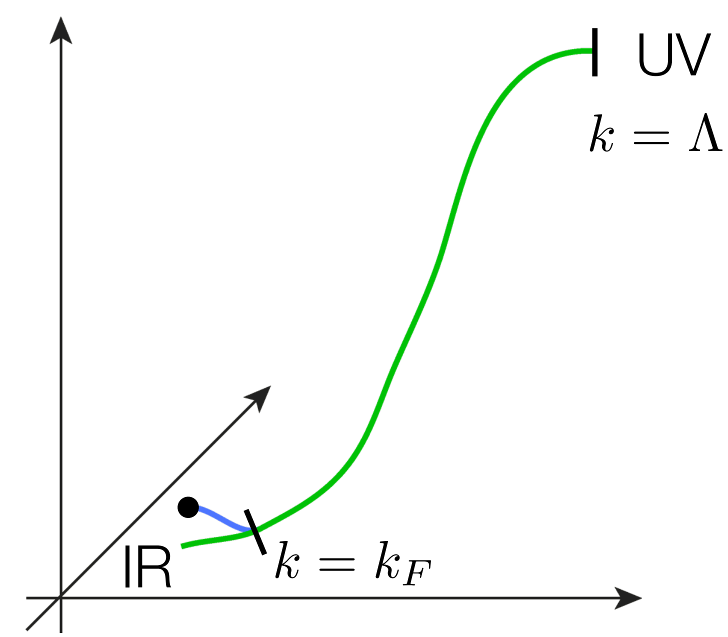
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

RG flow of polaron spectral function

RG Flow of $\mathcal{A}_{\downarrow,k}(\omega, \mathbf{p})$





$$P_{\downarrow,k=\Lambda}(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - i0$$

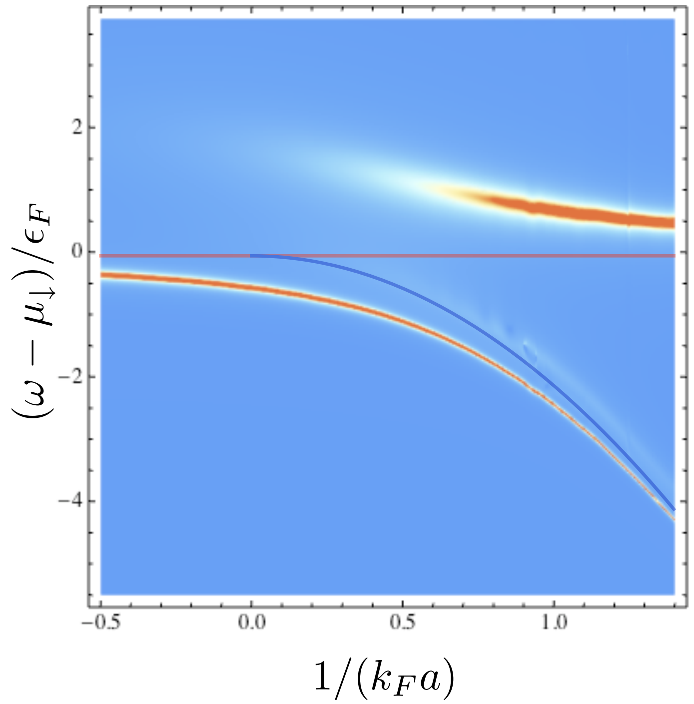
$$\mu_\downarrow = E(N_\downarrow) - E(N_\downarrow - 1)$$

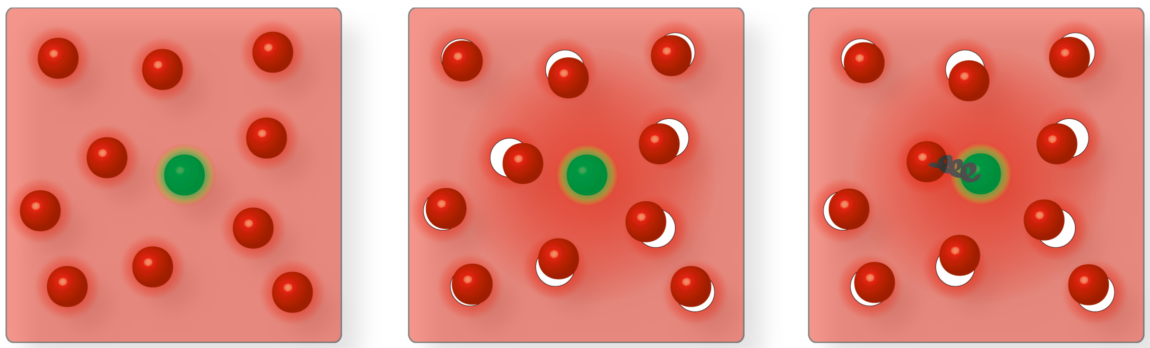
$\longrightarrow$ ground state energy

$$P_k(\omega, \mathbf{p}) = -\omega + \mathbf{p}^2 - \mu_\downarrow - \Sigma_k(\omega, \mathbf{p}) - i0$$

excitation spectrum

polaron spectral function $\mathcal{A}_\downarrow(\omega, \mathbf{p}=0)$





quasi-particle energy and decay width from

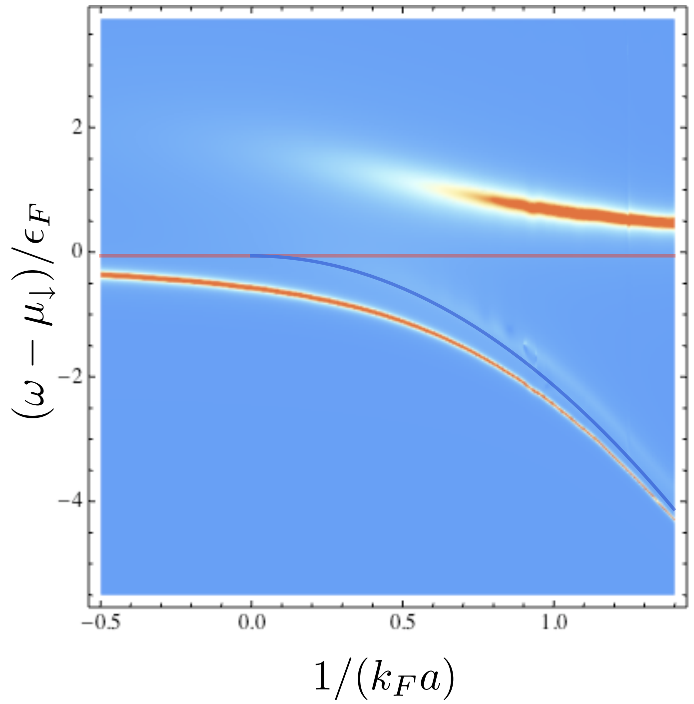
$$G_R^{-1}(\omega, \mathbf{p}=0)|_{\omega=\omega_{\rm qp}} = 0$$

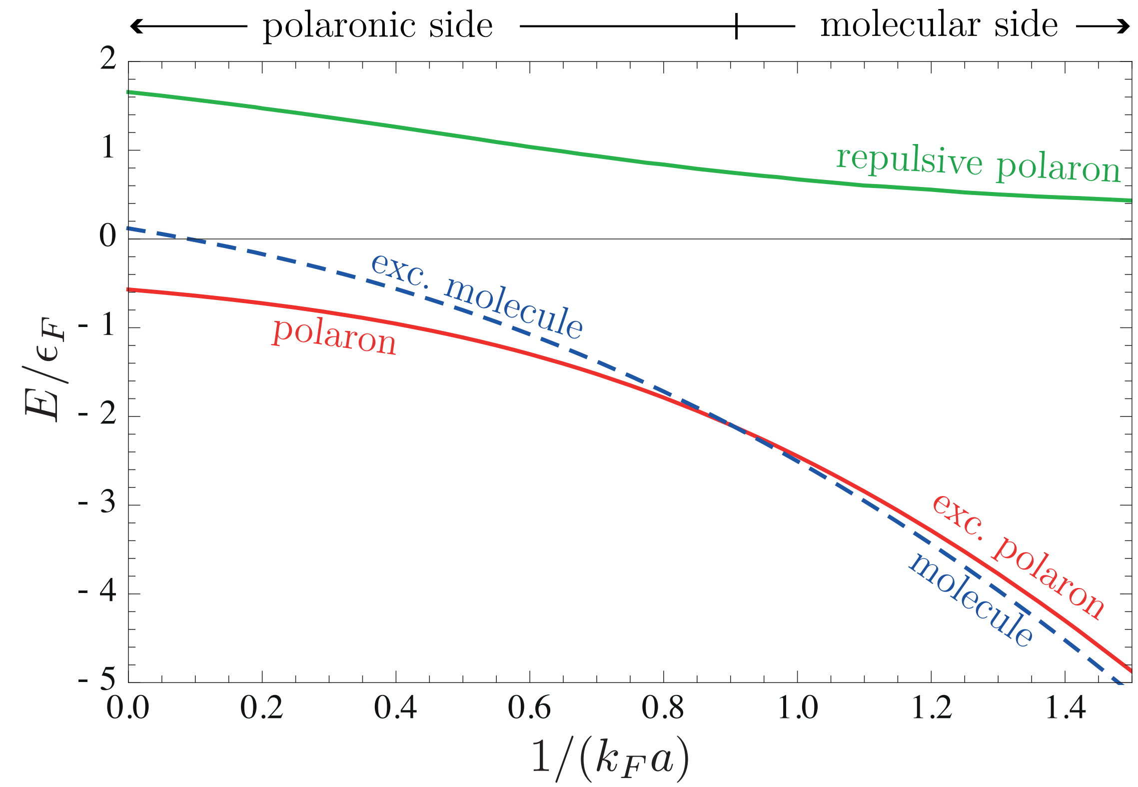
$$E_{\rm qp} = \mu_\downarrow + \mathrm{Re}[\omega_{\rm qp}]$$

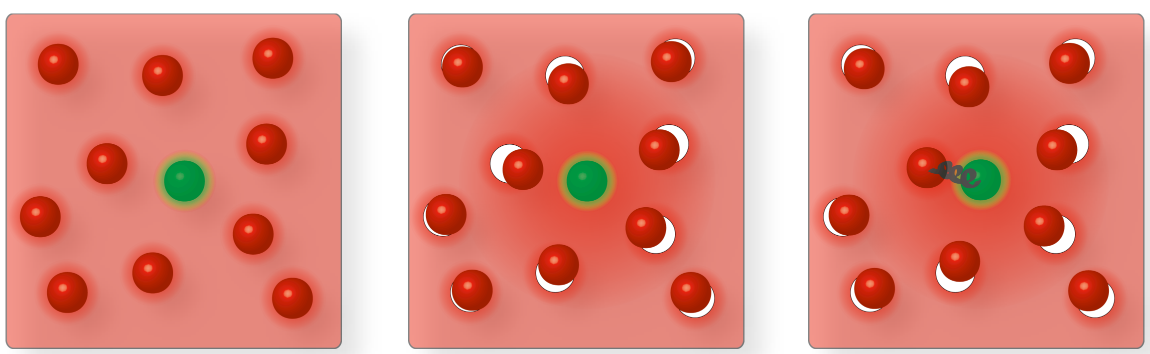
$$\Gamma_{\rm qp} = -\mathrm{Im}[\omega_{\rm qp}]$$

excitation spectrum

polaron spectral function $\mathcal{A}_\downarrow(\omega, \mathbf{p}=0)$







quasi-particle energy and decay width from

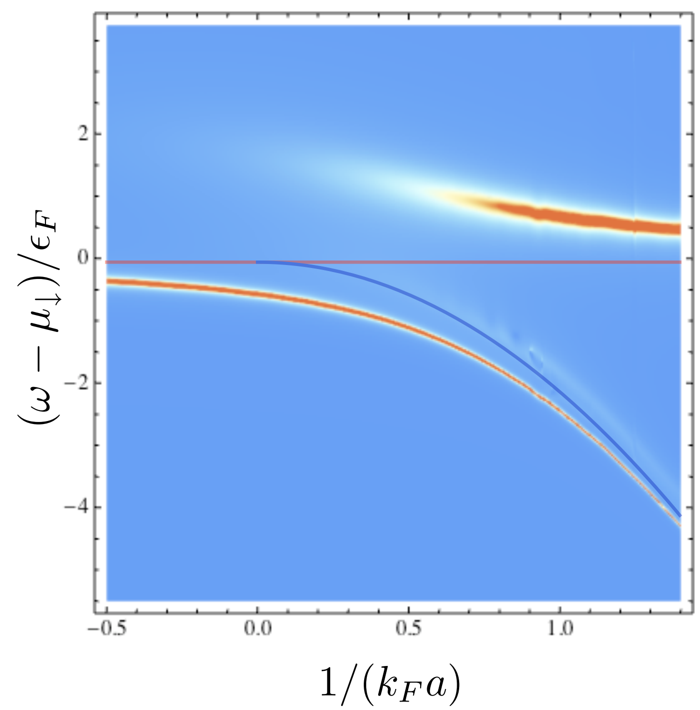
$$G_R^{-1}(\omega, \mathbf{p}=0)|_{\omega=\omega_{\mathrm{qp}}} = 0$$

$$E_{\mathrm{qp}} = \mu_\downarrow + \mathrm{Re}[\omega_{\mathrm{qp}}]$$

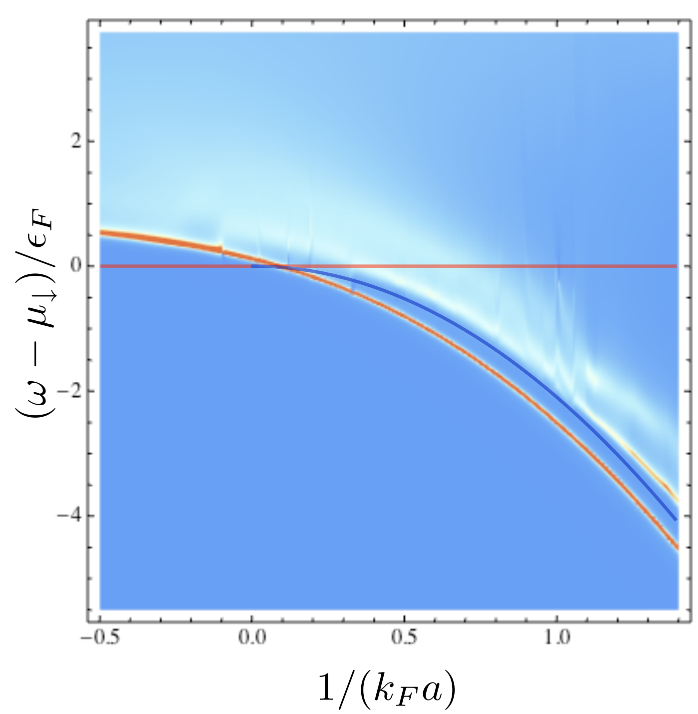
$$\Gamma_{\mathrm{qp}} = -\mathrm{Im}[\omega_{\mathrm{qp}}]$$

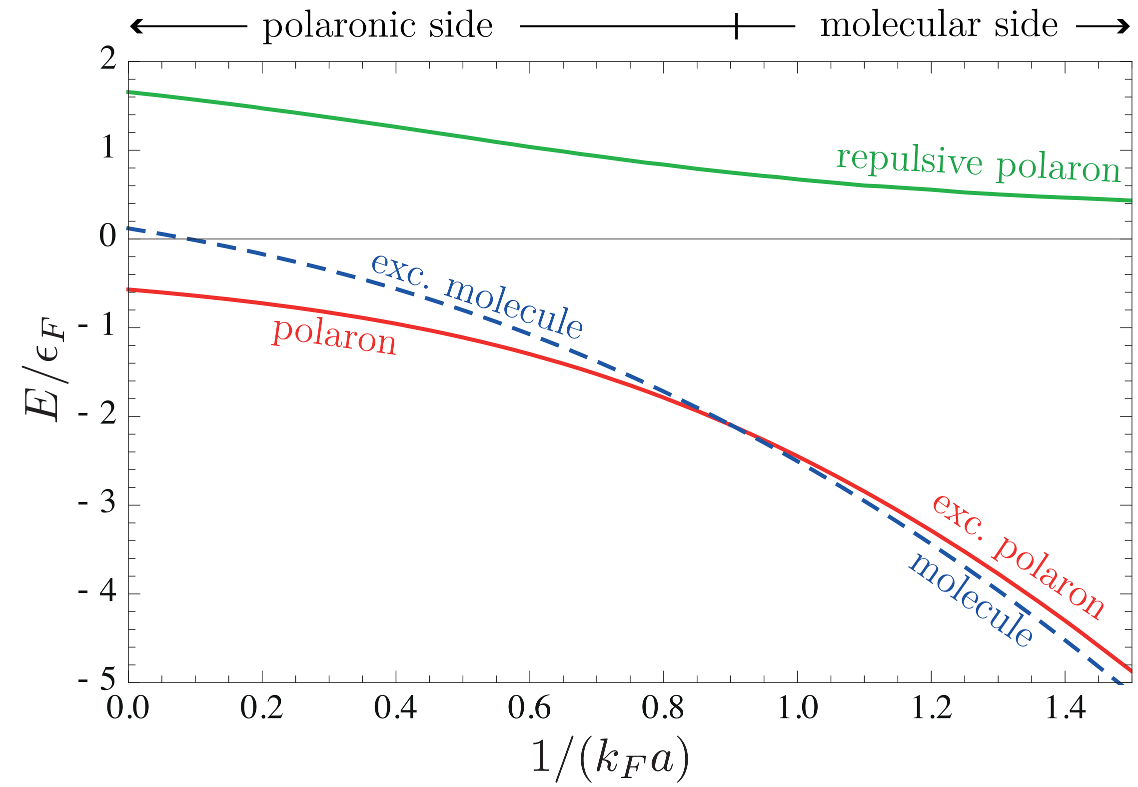
excitation spectrum

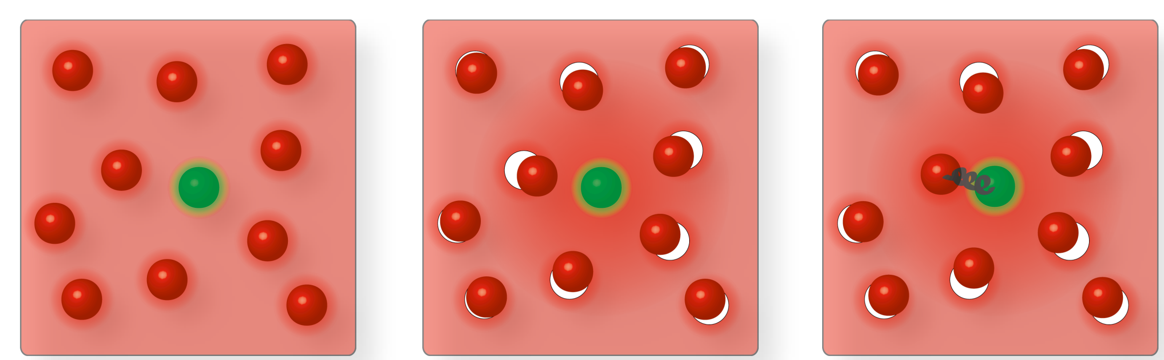
polaron spectral function $\mathcal{A}_{\downarrow}(\omega, \mathbf{p}=0)$



molecule spectral function $\mathcal{A}_{\phi}(\omega, \mathbf{p}=0)$







quasi-particle energy and decay width from

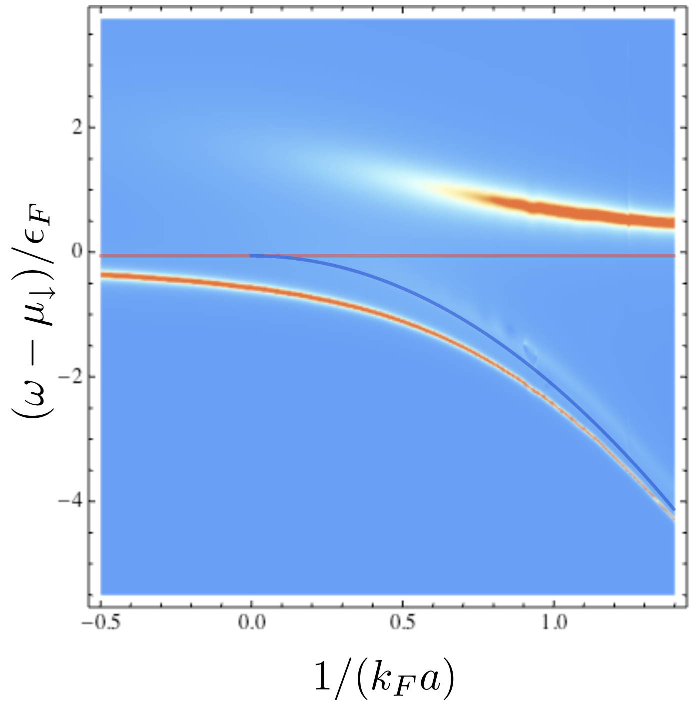
$$G_R^{-1}(\omega, \mathbf{p}=0)|_{\omega=\omega_{\rm qp}} = 0$$

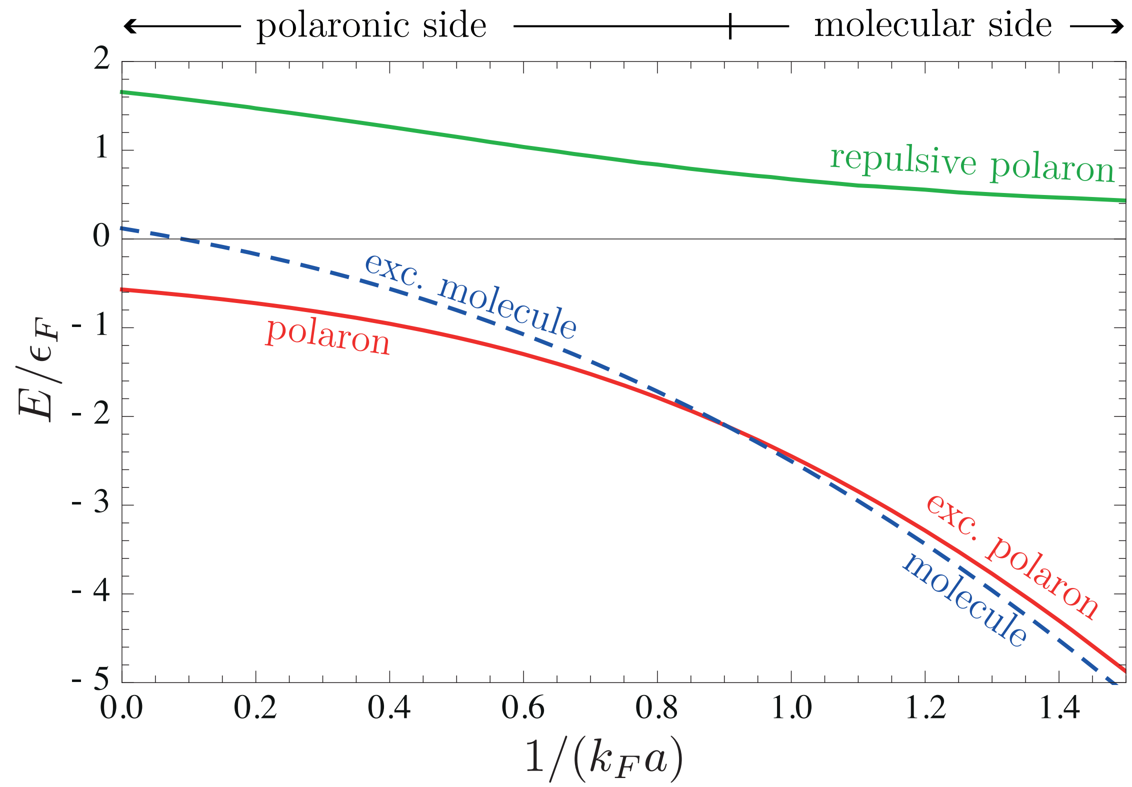
$$E_{\rm qp} = \mu_{\downarrow} + \mathrm{Re}[\omega_{\rm qp}]$$

$$\Gamma_{\rm qp} = -\mathrm{Im}[\omega_{\rm qp}]$$

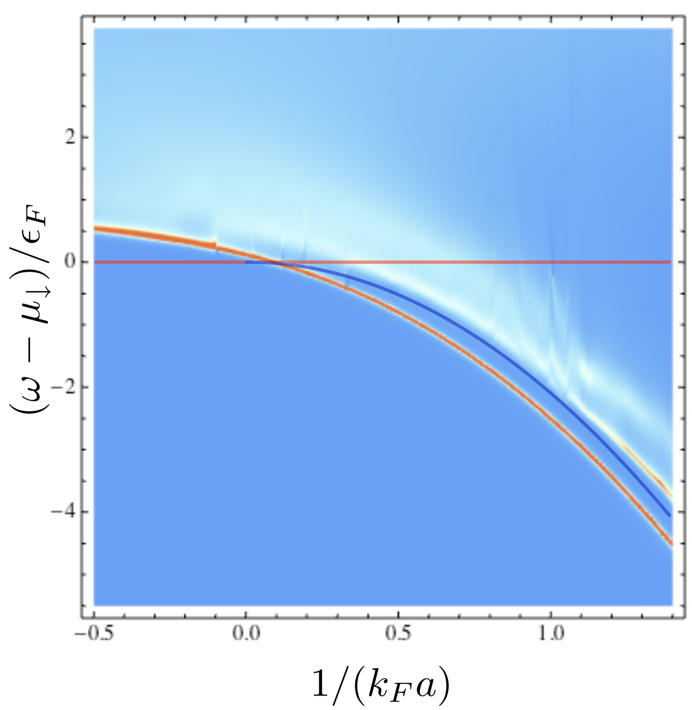
excitation spectrum

polaron spectral function $\mathcal{A}_\downarrow(\omega, \mathbf{p}=0)$





molecule spectral function $\mathcal{A}_\phi(\omega, \mathbf{p}=0)$



- polaron to molecule transition at $(k_F a_c)^{-1} = 0.904(5)$ fRG

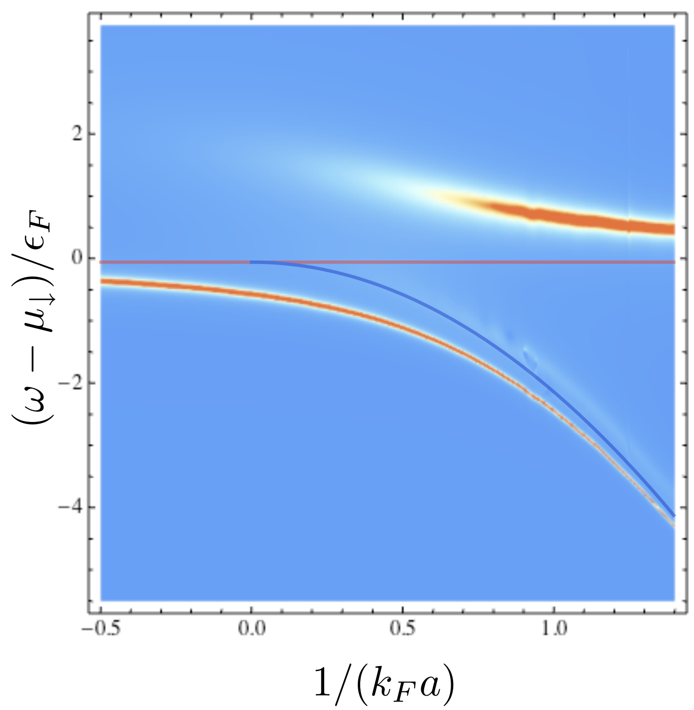
  $(k_F a_c)^{-1} = 0.90(2)$ diagMC

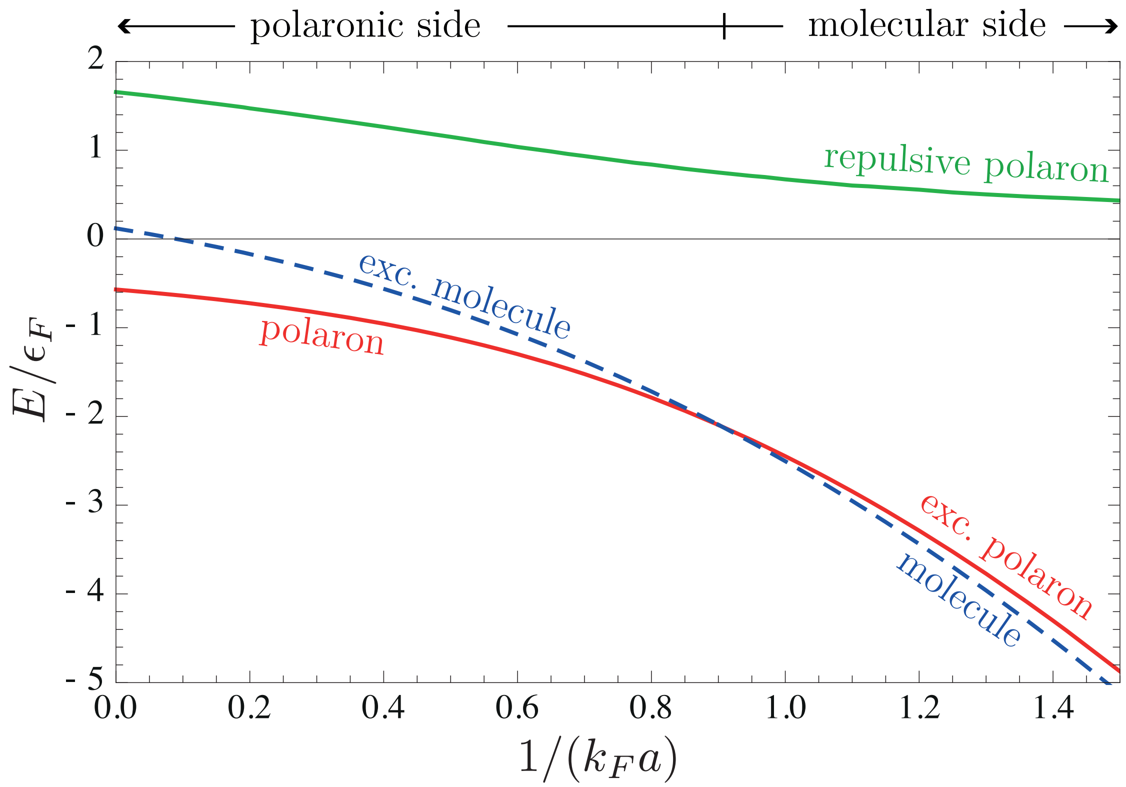
  Prokove'v, Svistonov (20009)

  $(k_F a_c)^{-1} = 1.27$ nsc T-Matrix, Nozieres-Schmitt-Rink

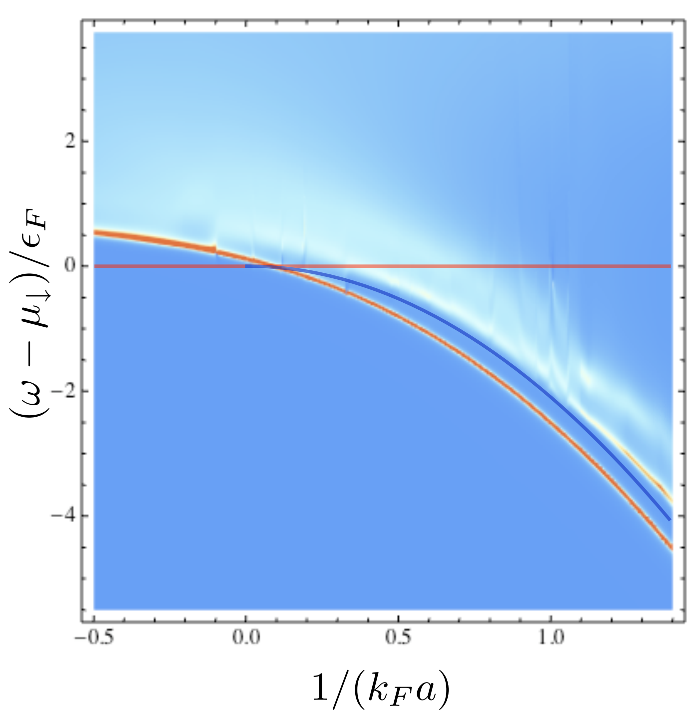
excitation spectrum

polaron spectral function $\mathcal{A}_\downarrow(\omega, \mathbf{p}=0)$





molecule spectral function $\mathcal{A}_\phi(\omega, \mathbf{p}=0)$



- polaron to molecule transition at $(k_F a_c)^{-1} = 0.904(5)$ fRG

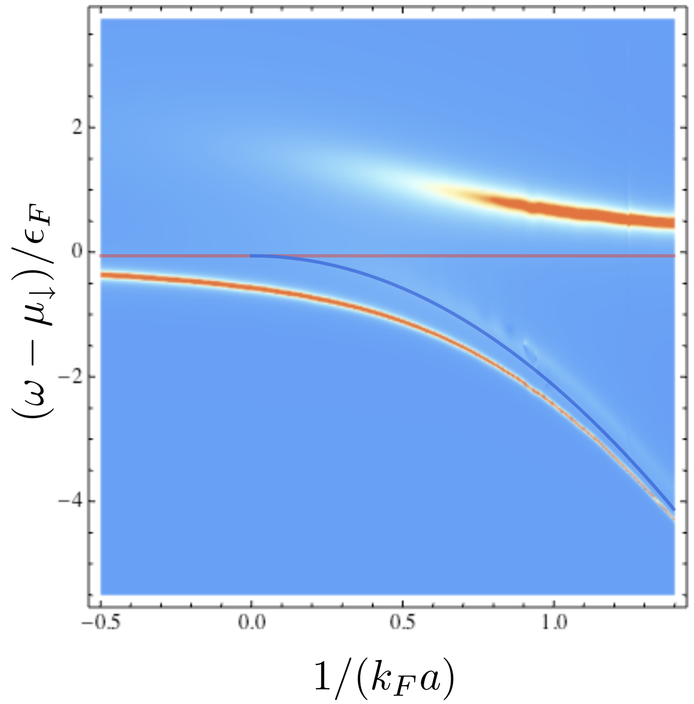
  $(k_F a_c)^{-1} = 0.90(2)$ diagMC
  Prokove'v, Svistonov (20009)

  $(k_F a_c)^{-1} = 1.27$ nsc T-Matrix, Nozieres-Schmitt-Rink

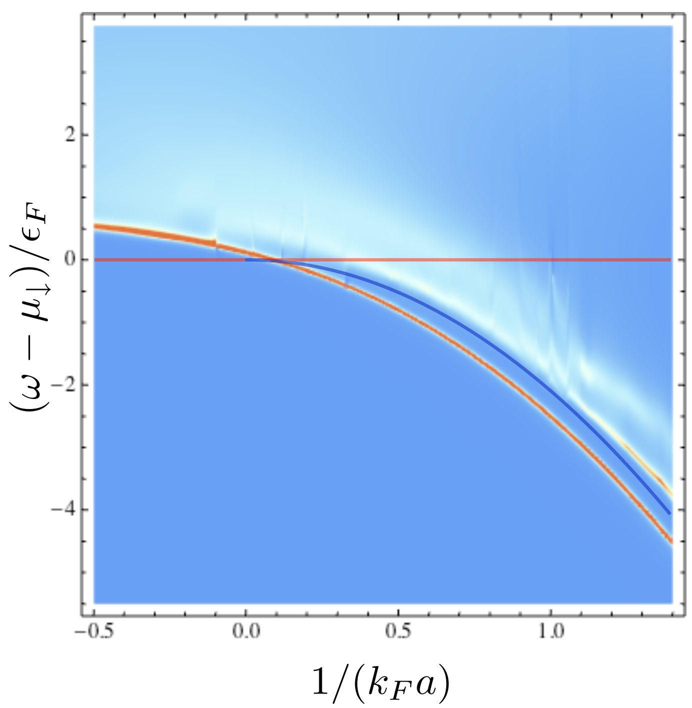
- emergence of excited repulsive polaron branch

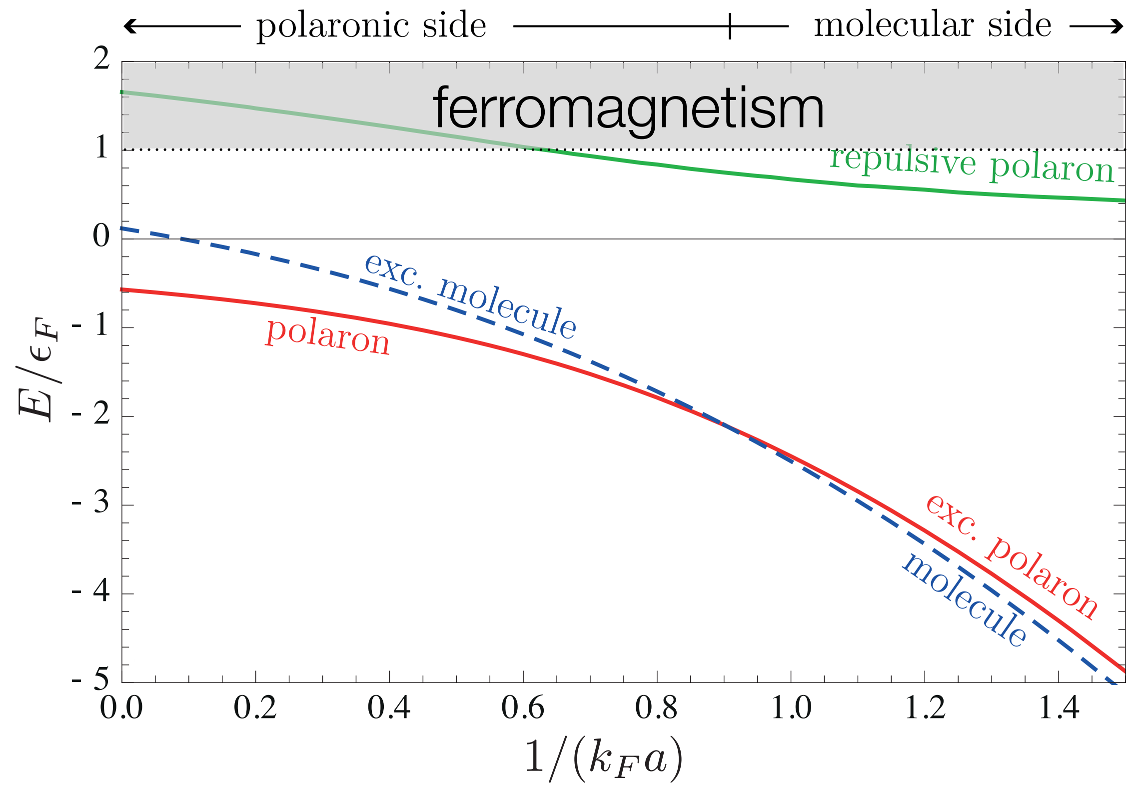
excitation spectrum

polaron spectral function $\mathcal{A}_\downarrow(\omega, \mathbf{p}=0)$



molecule spectral function $\mathcal{A}_\phi(\omega, \mathbf{p}=0)$





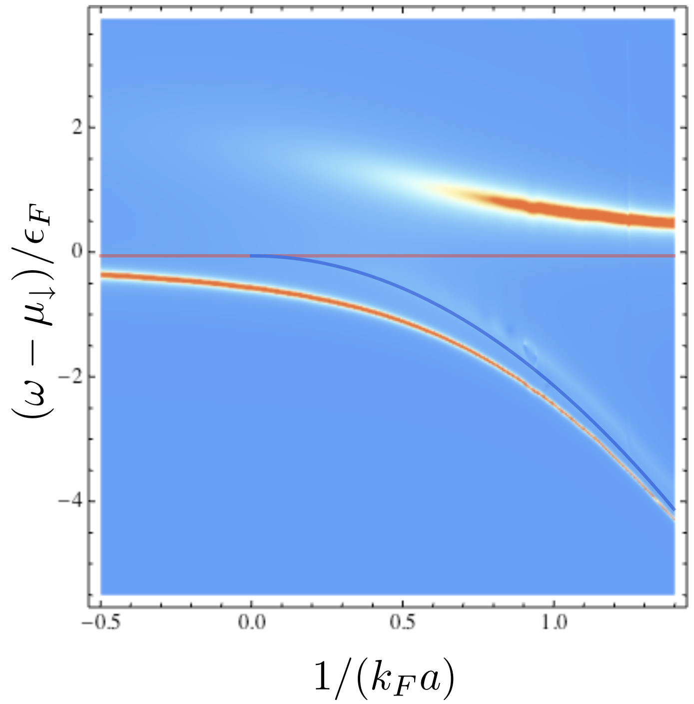
- polaron to molecule transition at $(k_F a_c)^{-1} = 0.904(5)$ fRG
  $(k_F a_c)^{-1} = 0.90(2)$ diagMC
  Prokove'v, Svistonov (20009)
  $(k_F a_c)^{-1} = 1.27$ nsc T-Matrix, Nozieres-Schmitt-Rink
- emergence of excited repulsive polaron branch
- energy E_{rep} exceeds ϵ_F for $(k_F a)^{-1} < 0.6$

  $\rightarrow$ onset of saturated ferromagnetism!

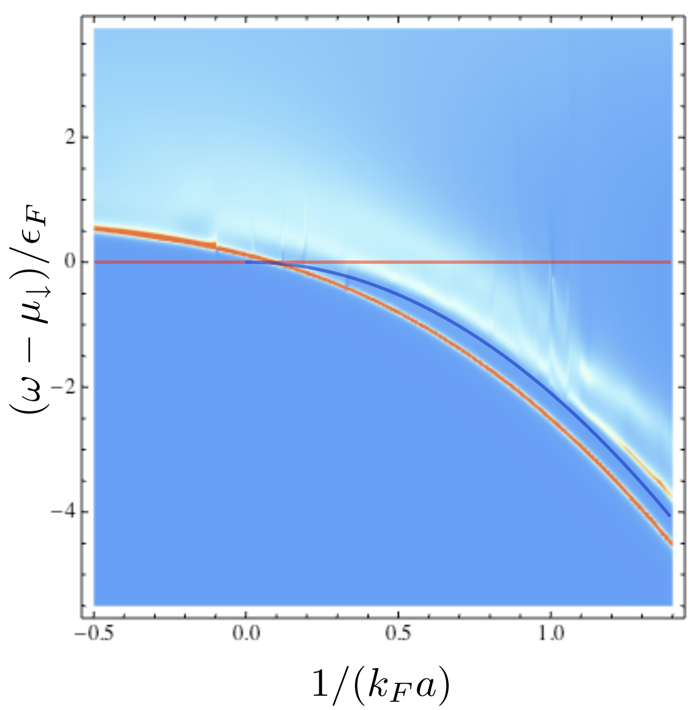
  cf. Bruun, Massignan (2011)

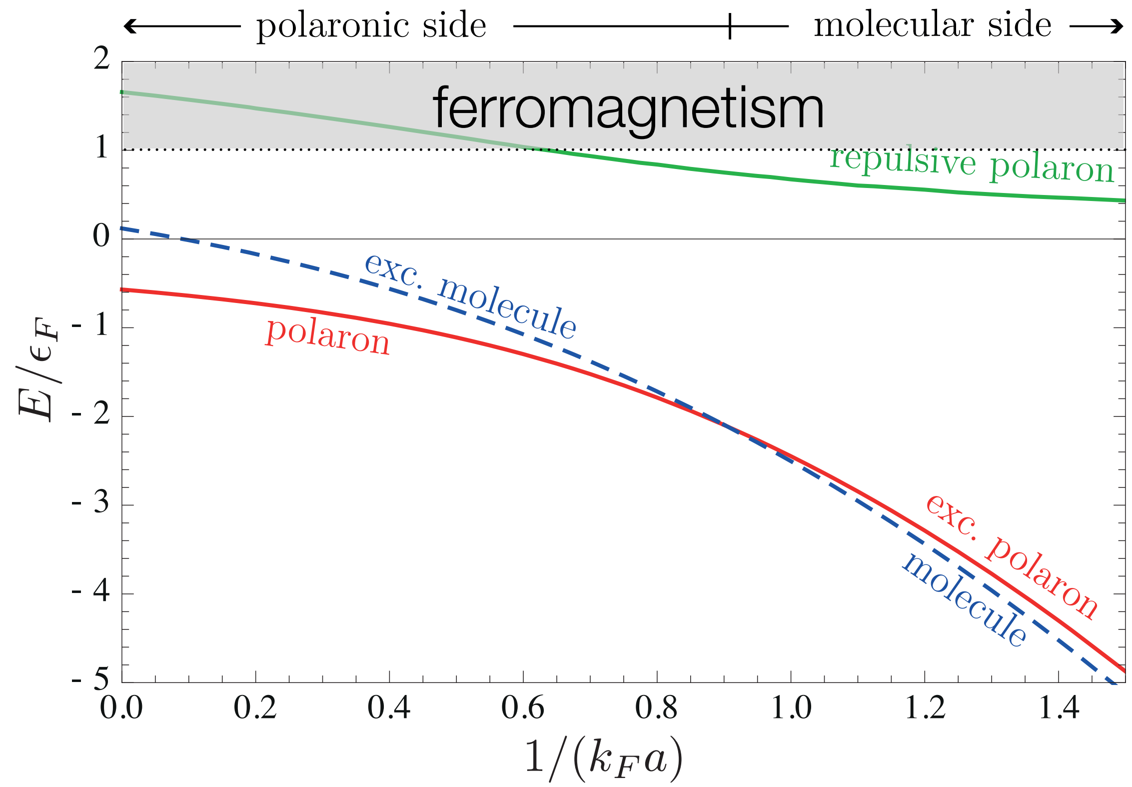
excitation spectrum

polaron spectral function $\mathcal{A}_{\downarrow}(\omega, \mathbf{p} = 0)$



molecule spectral function $\mathcal{A}_{\phi}(\omega, \mathbf{p} = 0)$





- polaron to molecule transition at $(k_F a_c)^{-1} = 0.904(5)$ fRG

  $(k_F a_c)^{-1} = 0.90(2)$ diagMC

  Prokove'v, Svistonov (20009)

  $(k_F a_c)^{-1} = 1.27$ nsc T-Matrix, Nozieres-Schmitt-Rink

- emergence of excited repulsive polaron branch
- energy E_{rep} exceeds ϵ_F for $(k_F a)^{-1} < 0.6$

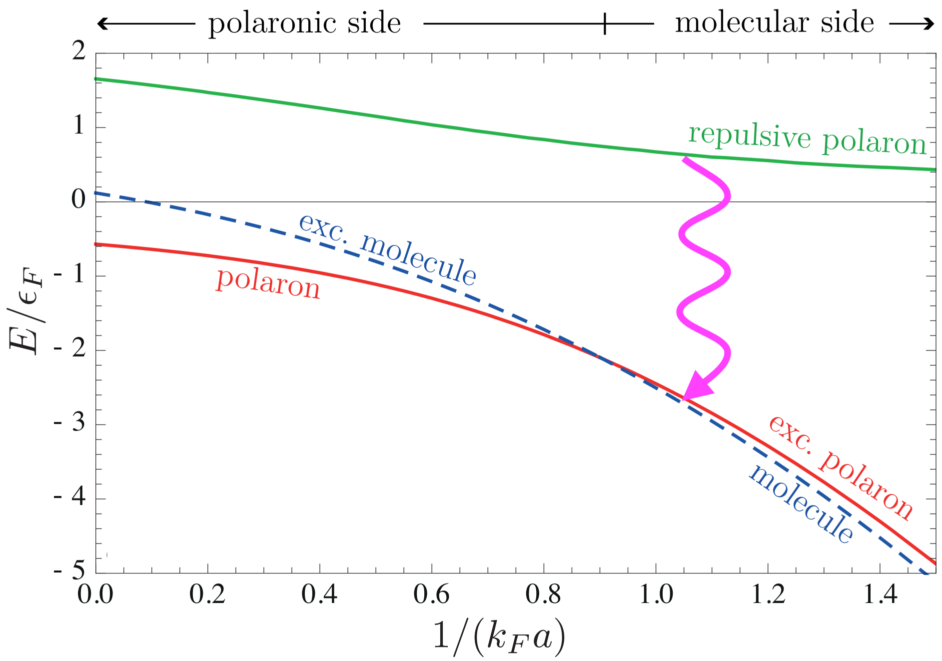
  → onset of saturated ferromagnetism!

  cf. Bruun, Massignan (2011)

can not be addressed within a simple derivative expansion!

decay widths

repulsive polaron



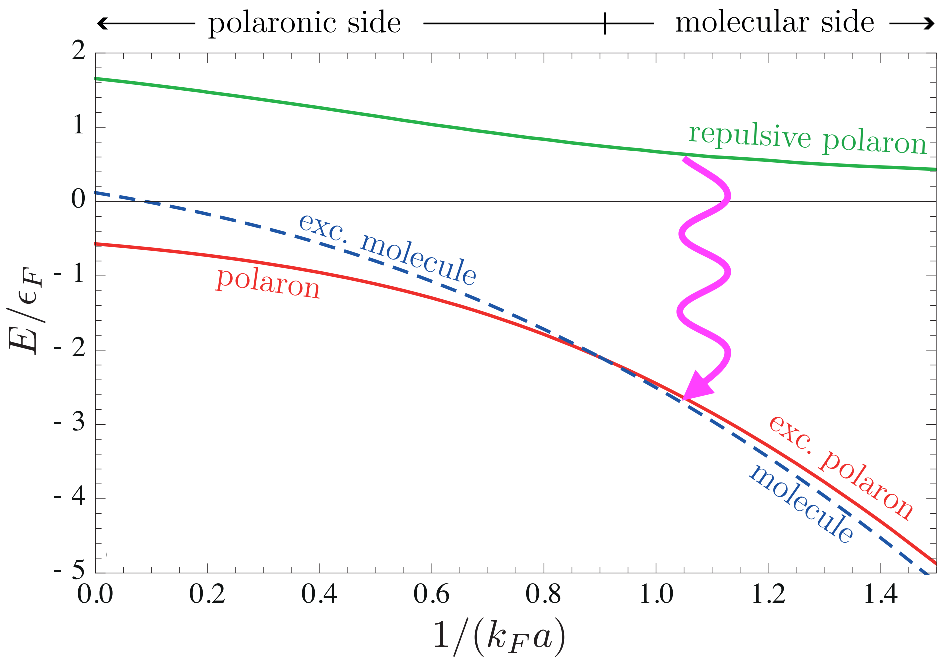
$$G_R^{-1}(\omega, \mathbf{p}=0)|_{\omega=\omega_{\rm qp}} = 0$$

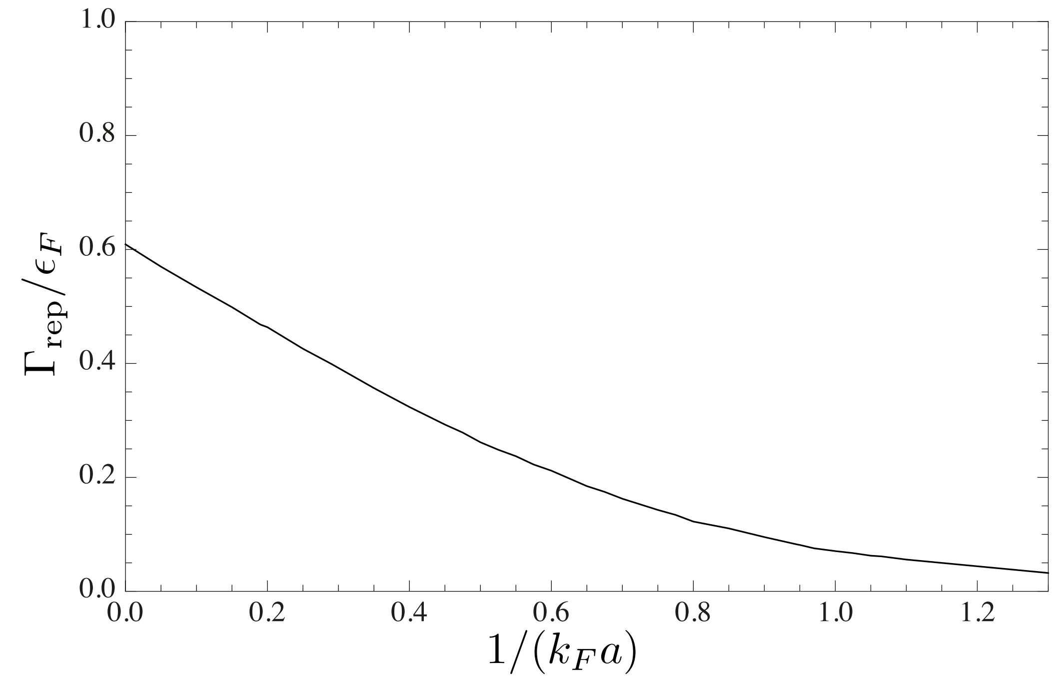
$$E_{\rm qp} = \mu_\downarrow + \mathrm{Re}[\omega_{\rm qp}]$$

$$\Gamma_{\rm qp} = -\mathrm{Im}[\omega_{\rm qp}]$$

decay widths

repulsive polaron



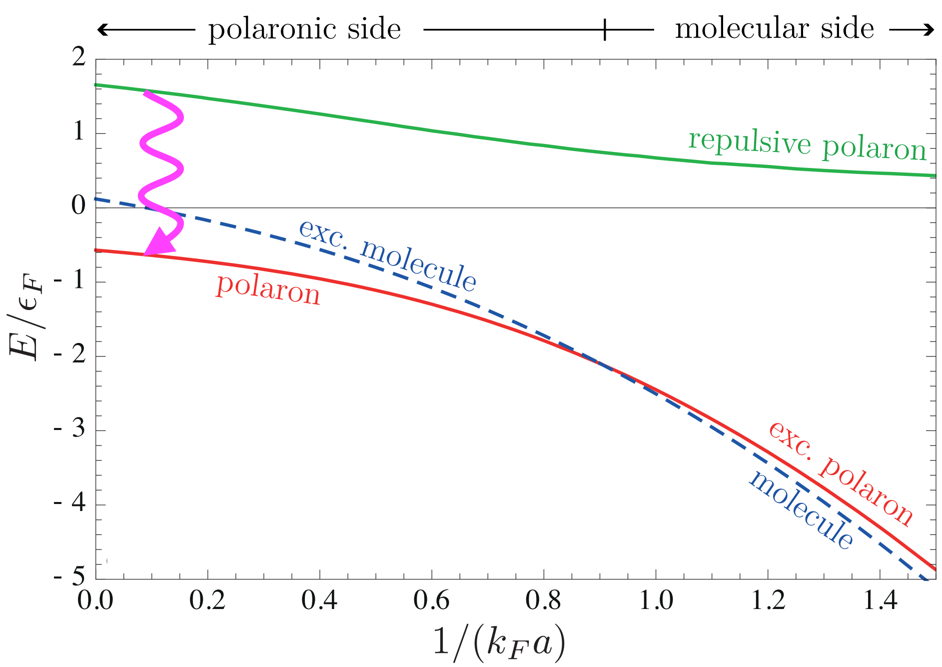


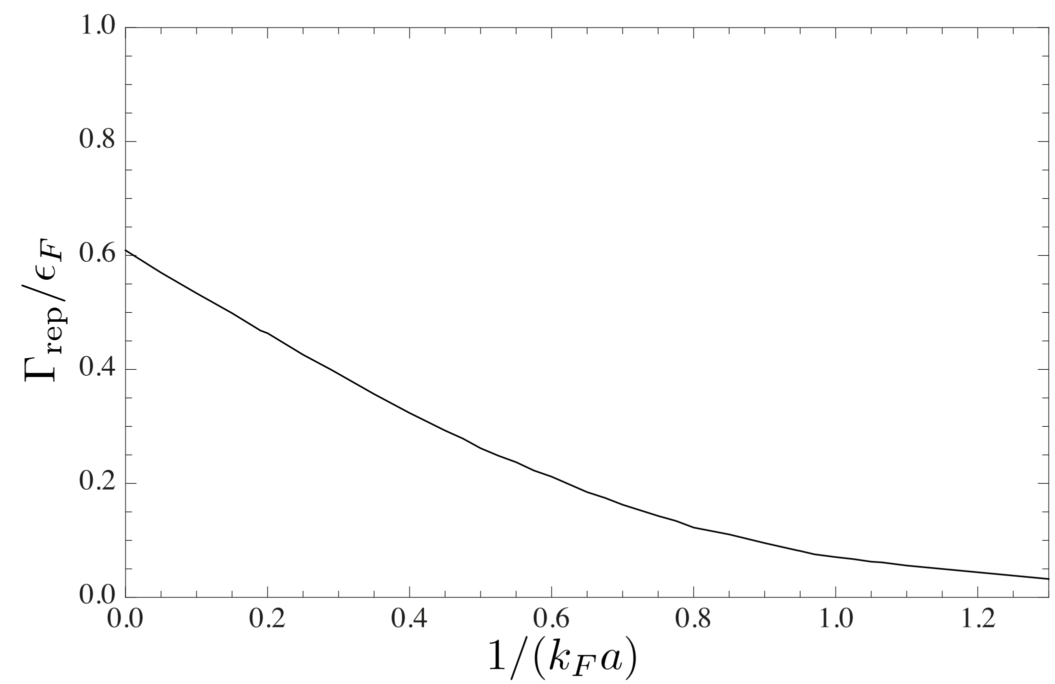
- weak coupling: excitation becomes sharp

→ stable repulsive branch!

decay widths

repulsive polaron





- weak coupling: excitation becomes sharp

  → stable repulsive branch!

- strong coupling, $(k_F a)^{-1} < 0.6$:

  $E_{\mathrm{rep}} > \epsilon_F$ → onset of ferromagnetism

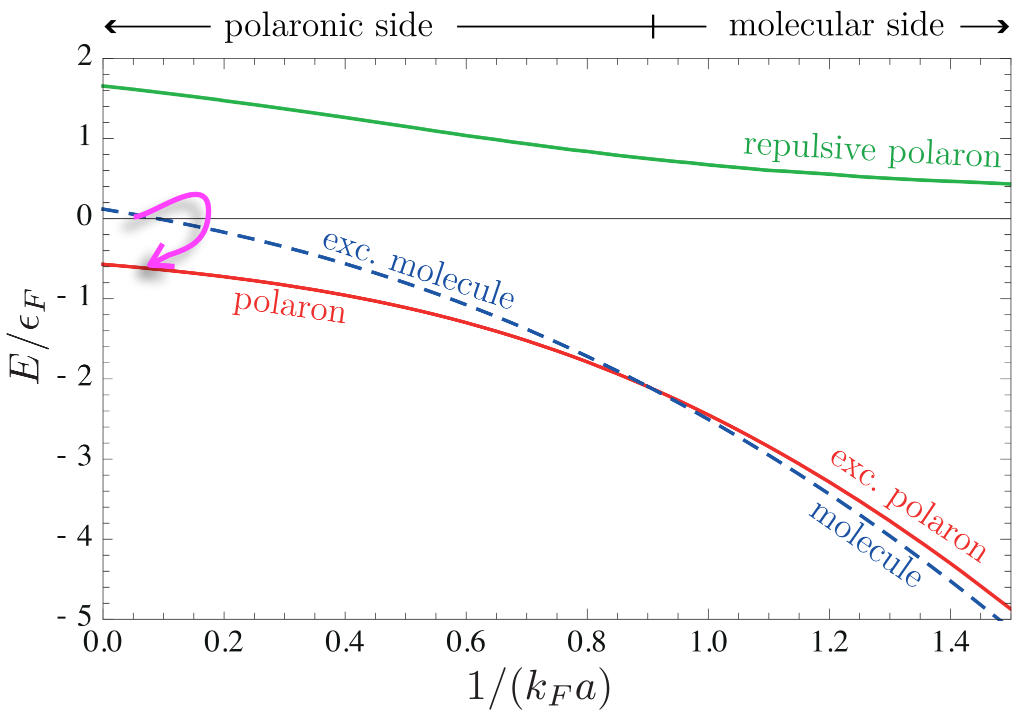
  $\Gamma_{\mathrm{rep}} > 0.2\epsilon_F$ → destabilization of FM phase, molecule formation

Competition of dynamical phenomena!

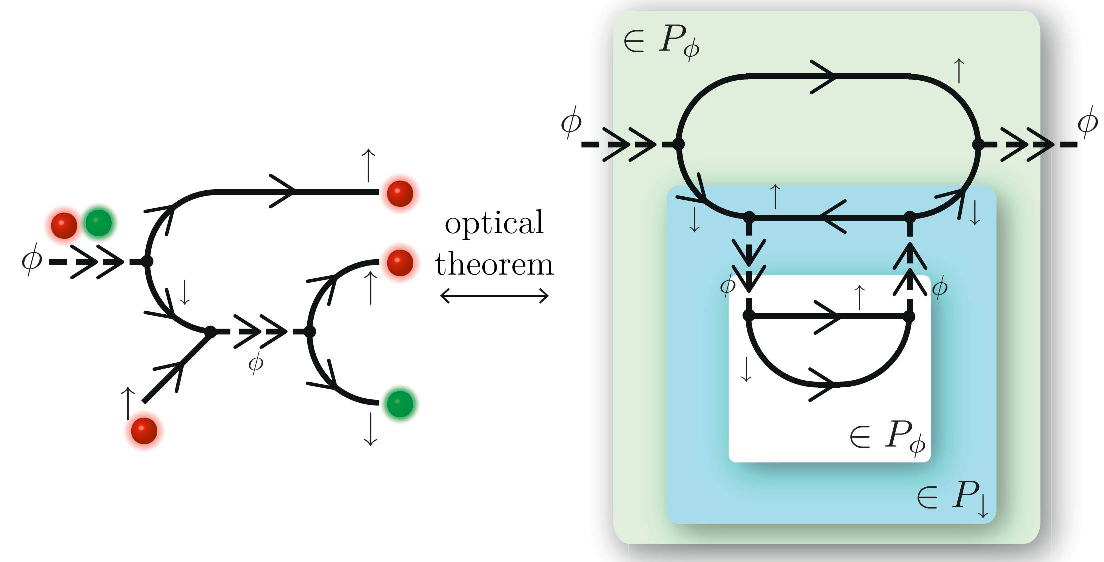
C.F. PEKKER ET AL. (2011)

decay widths

molecule

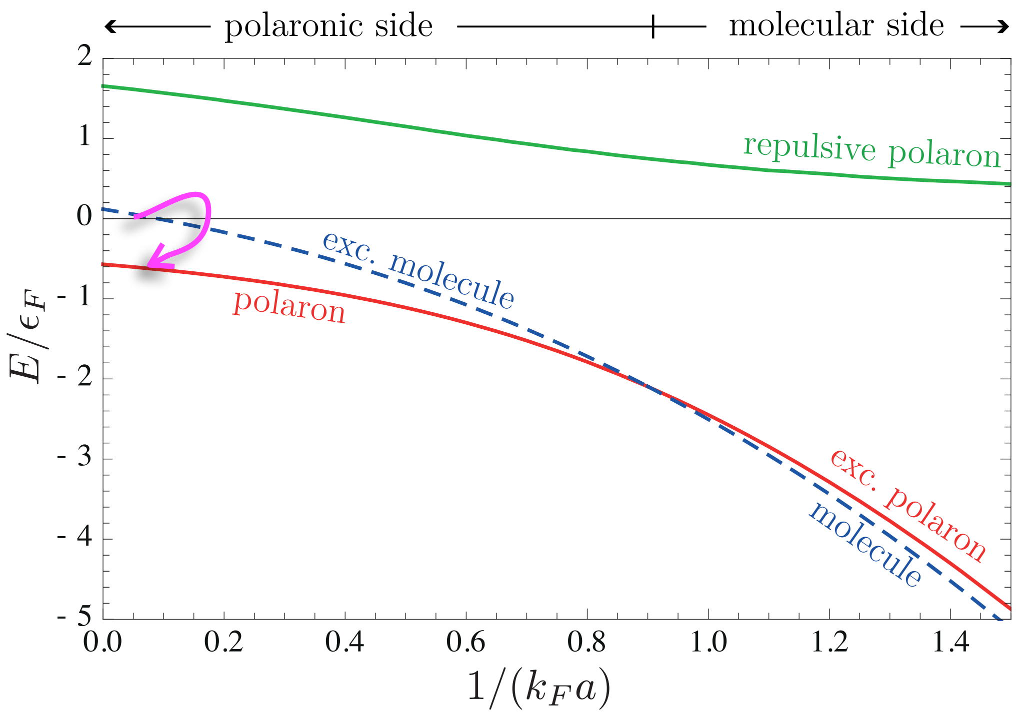


- dominant decay process: three-body decay

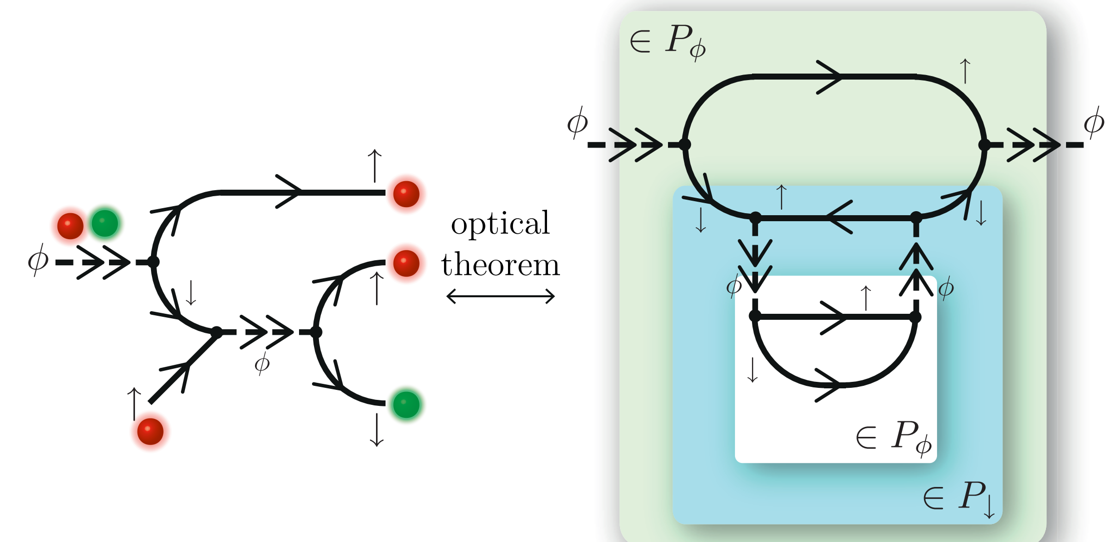


decay widths

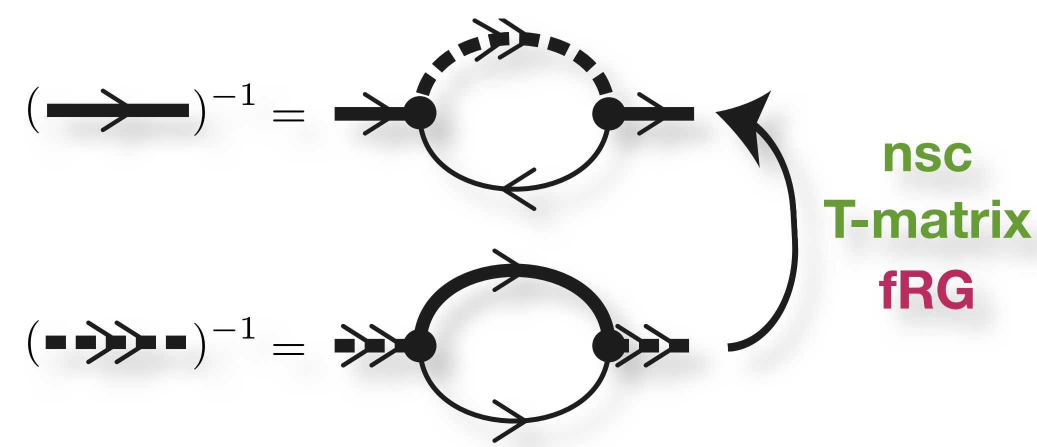
molecule



- dominant decay process: three-body decay

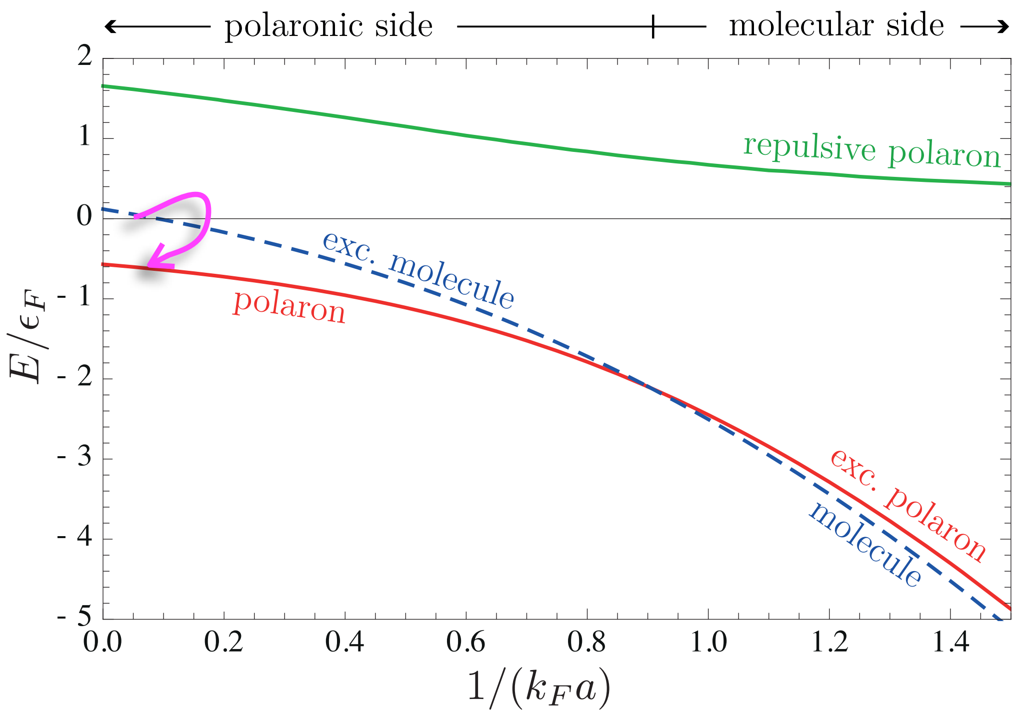


[not included in Nozieres-Schmitt-Rink approach!]

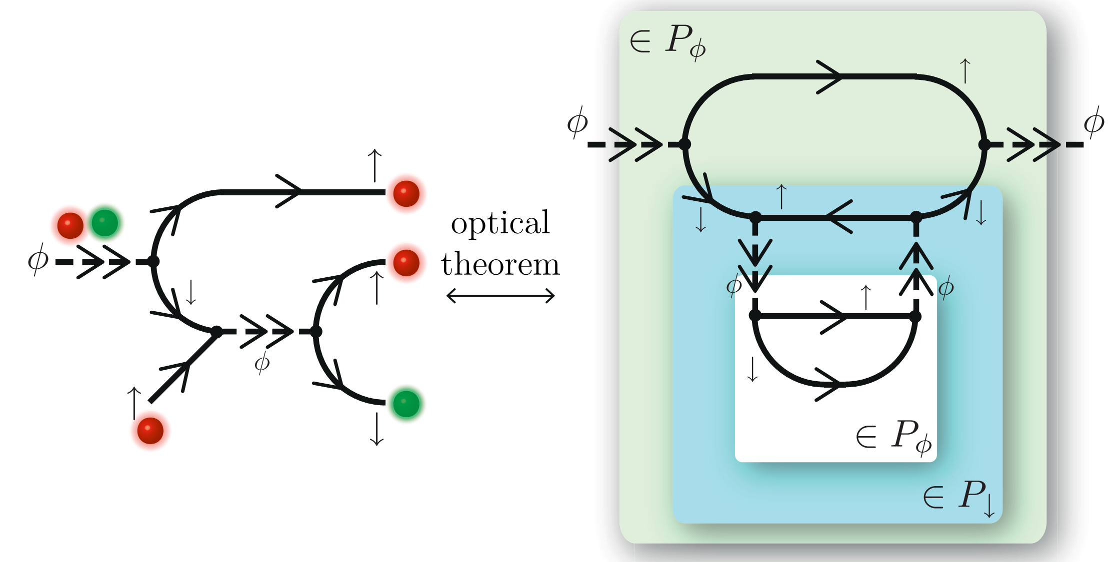


decay widths

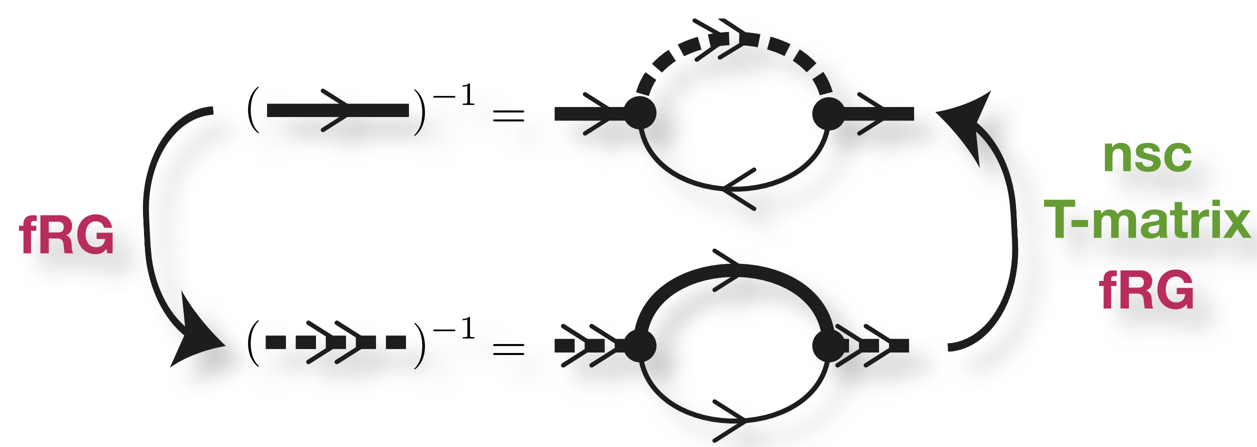
molecule



- dominant decay process: three-body decay

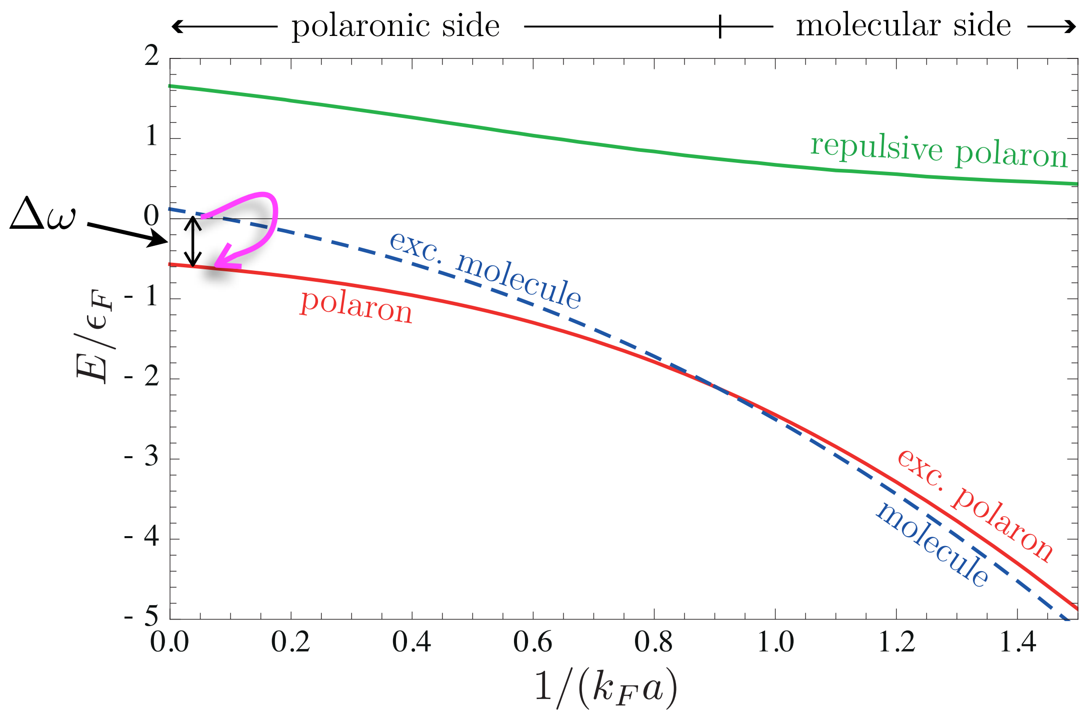


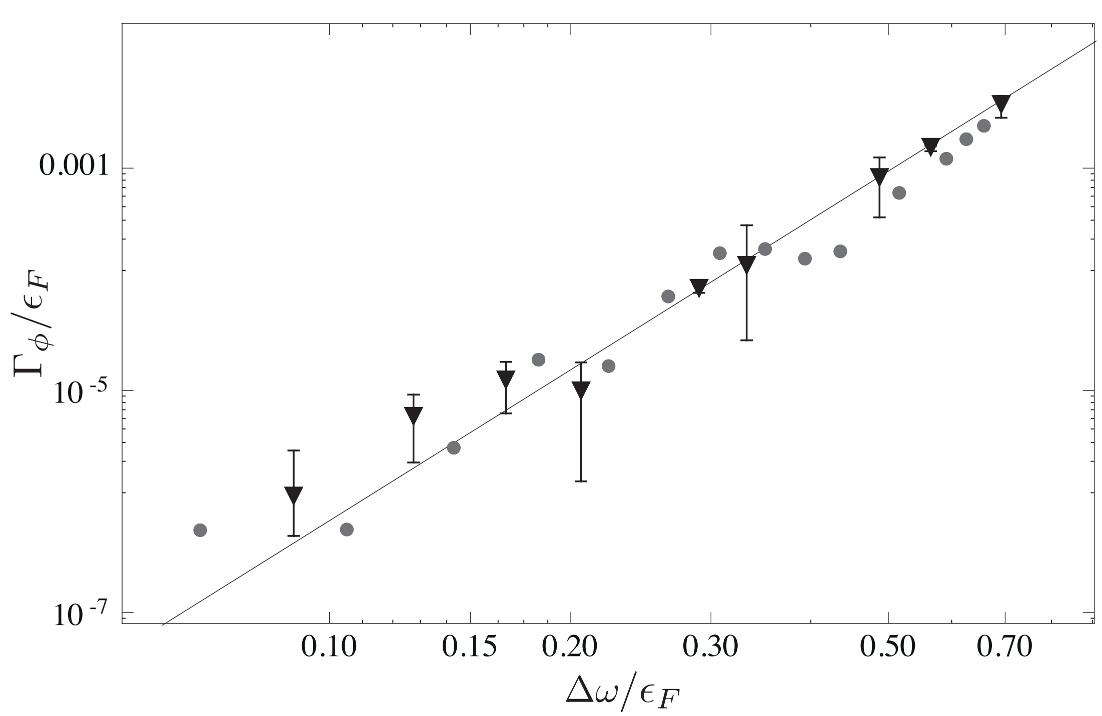
[not included in Nozieres-Schmitt-Rink approach!]



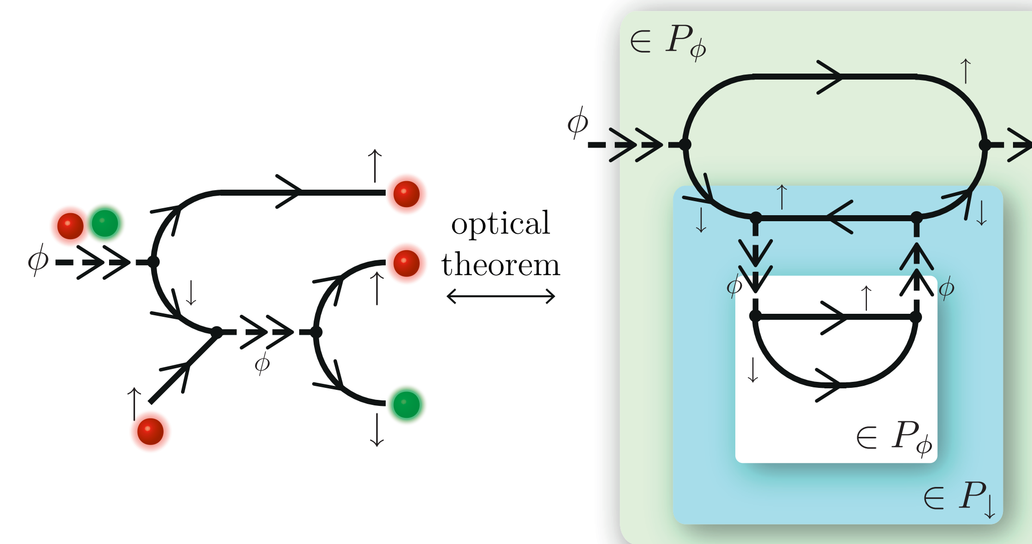
decay widths

molecule

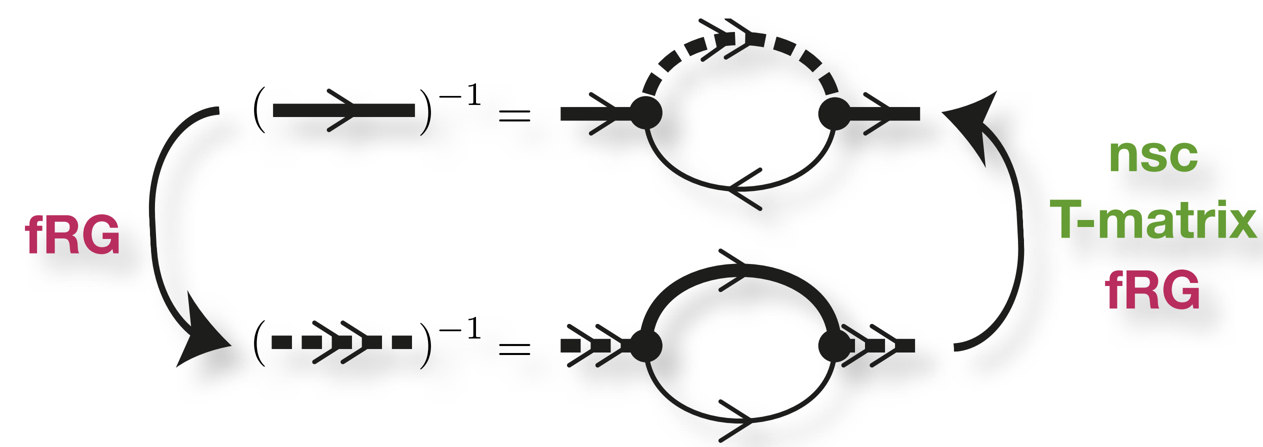




- dominant decay process: three-body decay



[not included in Nozieres-Schmitt-Rink approach!]



- power-law behavior of decay width

$$\Gamma_\phi \propto \Delta\omega^{9/2} \qquad \Delta\omega = E_\phi - E_{\downarrow,\mathrm{att}}$$

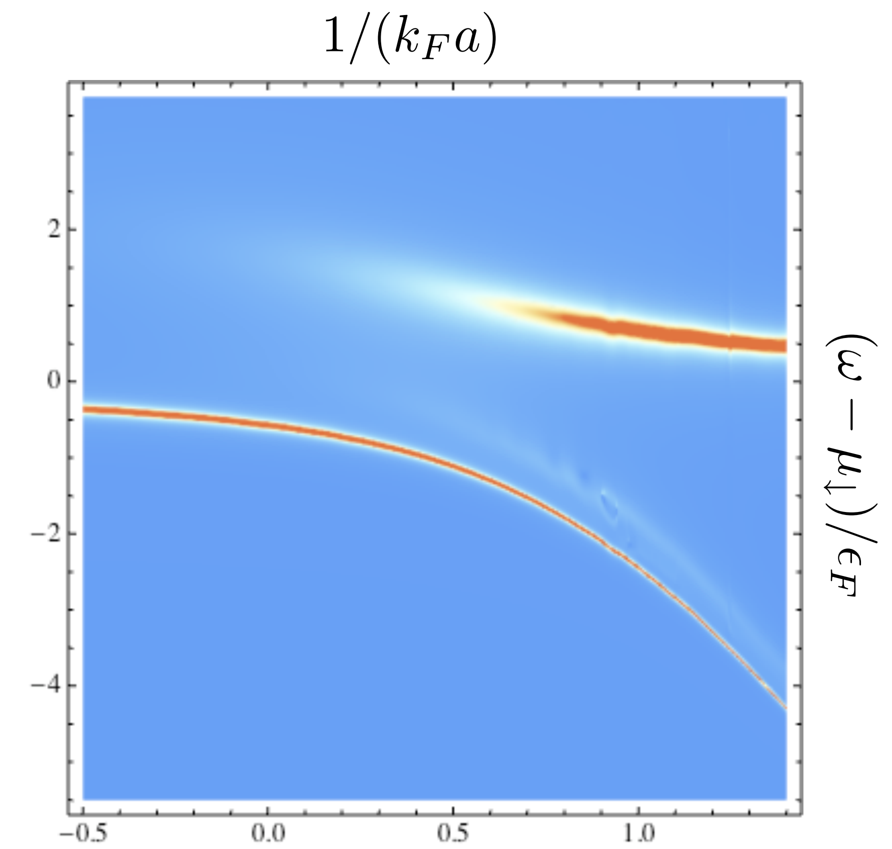
Bruun, Massignan (2010)

quasi-particle weight

polaron

- quasi-particle weight given by

$$Z_{\downarrow/\phi}^{-1} = -\frac{\partial}{\partial\omega} G_{\downarrow/\phi,\mathrm{R}}^{-1}(\omega, \mathbf{p}=0)\Big|_{\omega=\omega_{\mathrm{qp}}}$$

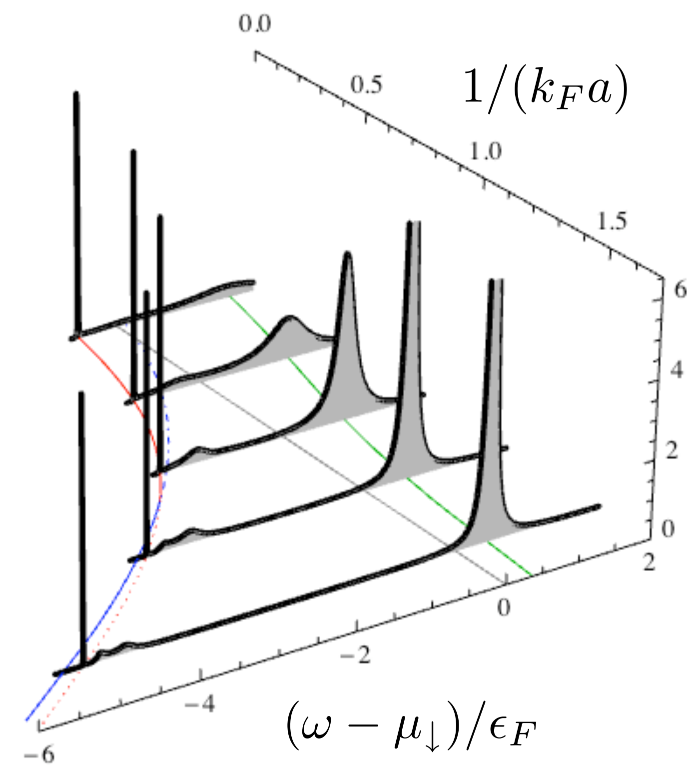


quasi-particle weight

polaron

- quasi-particle weight given by

$$Z_{\downarrow/\phi}^{-1} = -\frac{\partial}{\partial\omega} G_{\downarrow/\phi,\mathrm{R}}^{-1}(\omega, \mathbf{p}=0)\Big|_{\omega=\omega_{\mathrm{qp}}}$$

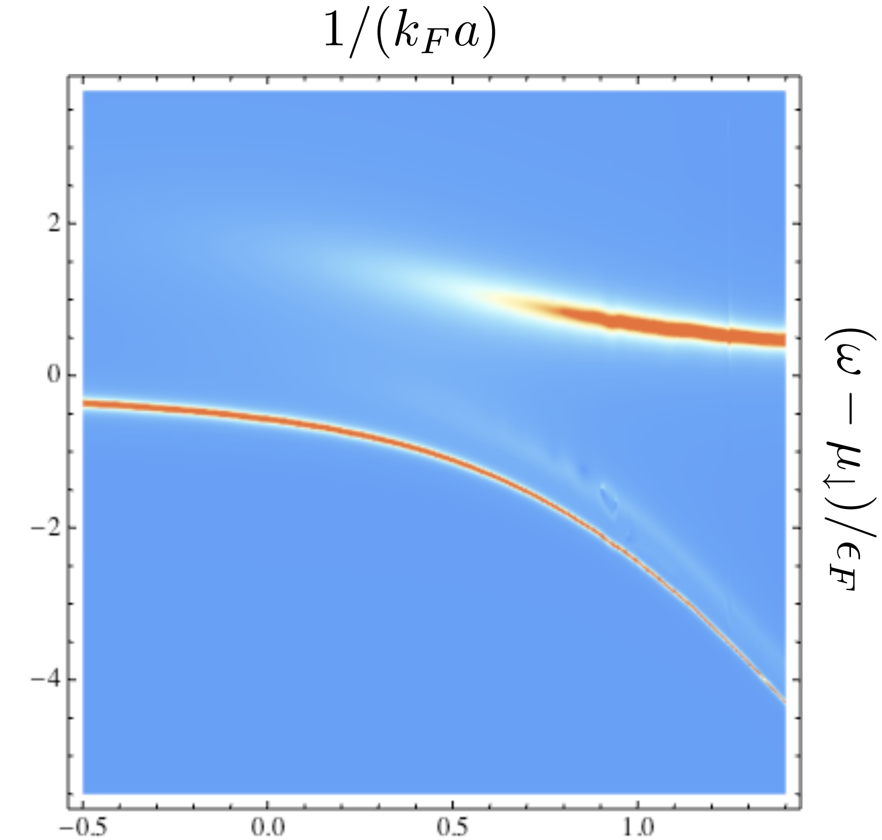


quasi-particle weight

polaron

- quasi-particle weight given by

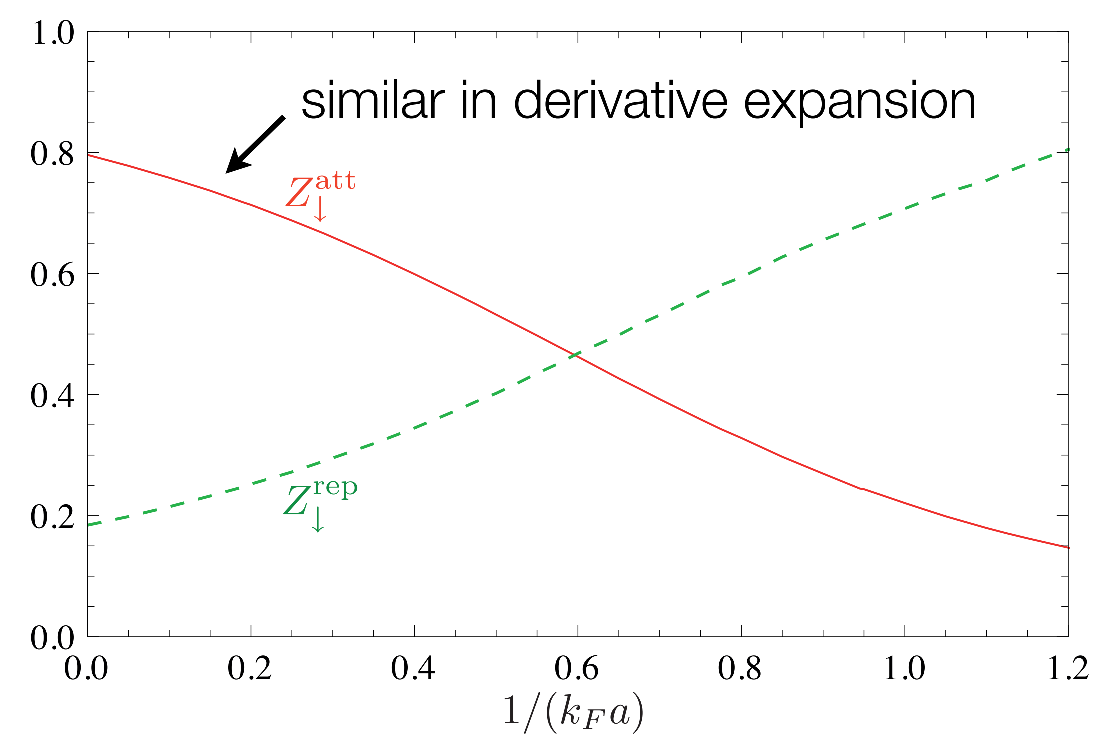
$$Z_{\downarrow/\phi}^{-1} = -\frac{\partial}{\partial\omega} G_{\downarrow/\phi,\mathrm{R}}^{-1}(\omega, \mathbf{p}=0)\Big|_{\omega=\omega_{\mathrm{qp}}}$$



- weight shifts towards repulsive polaron for weak coupling

- very small incoherent background!

$$Z_\downarrow^{\mathrm{att}} + Z_\downarrow^{\mathrm{rep}} > 0.98$$



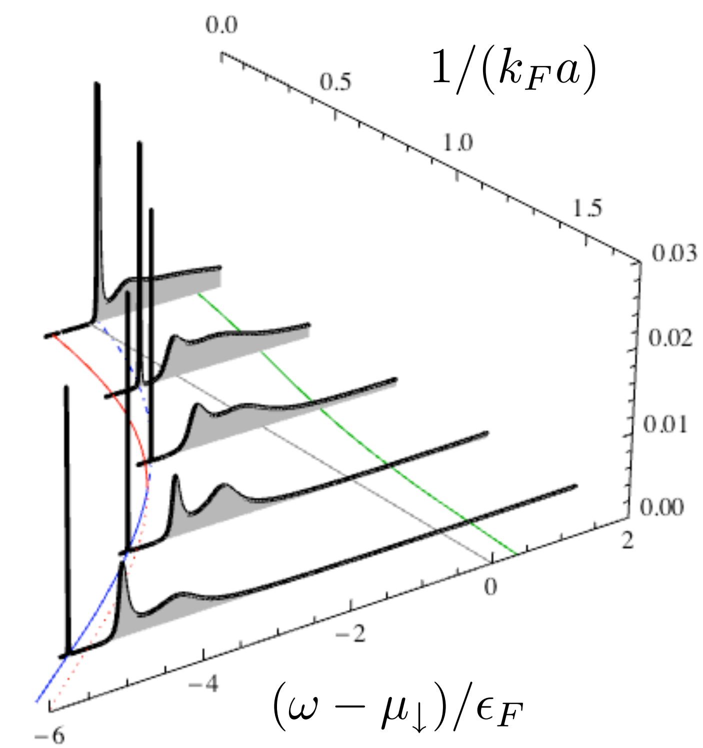
quasi-particle weight

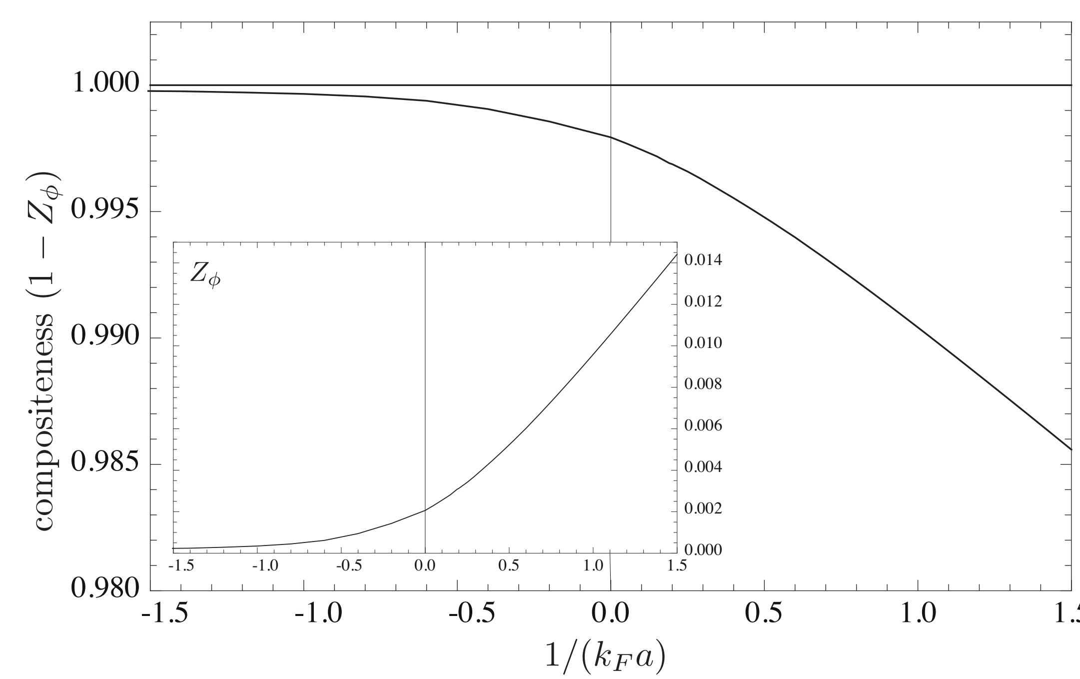
<u>molecule</u>

- weight of bound state extremely small

$$Z_\phi \approx 0.002 \qquad \text{(unitarity)}$$

why?





quasi-particle weight

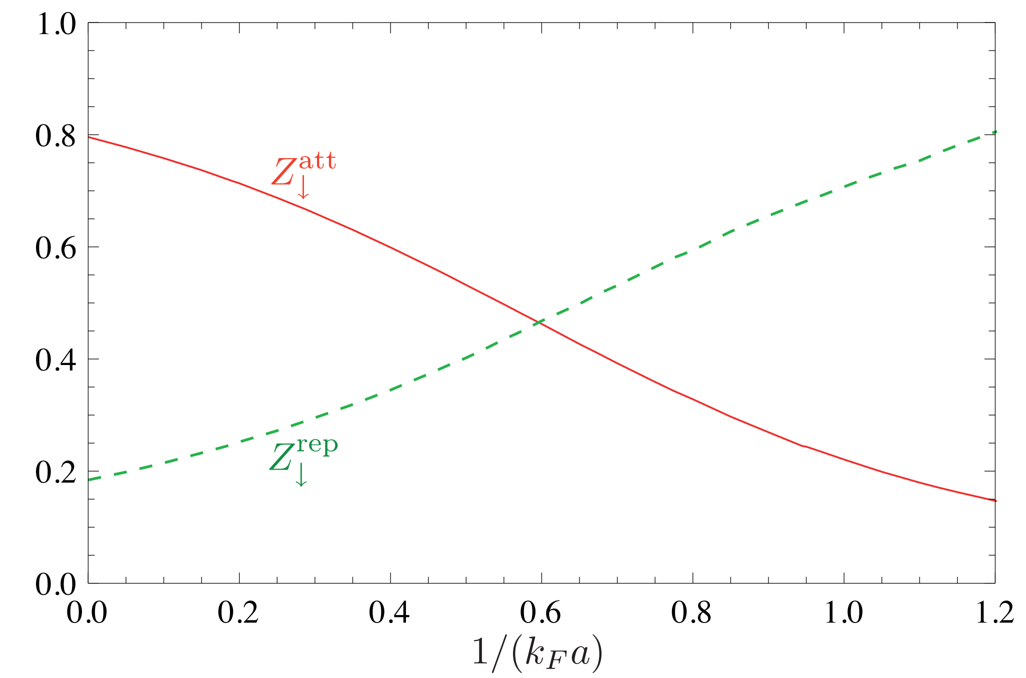
molecule

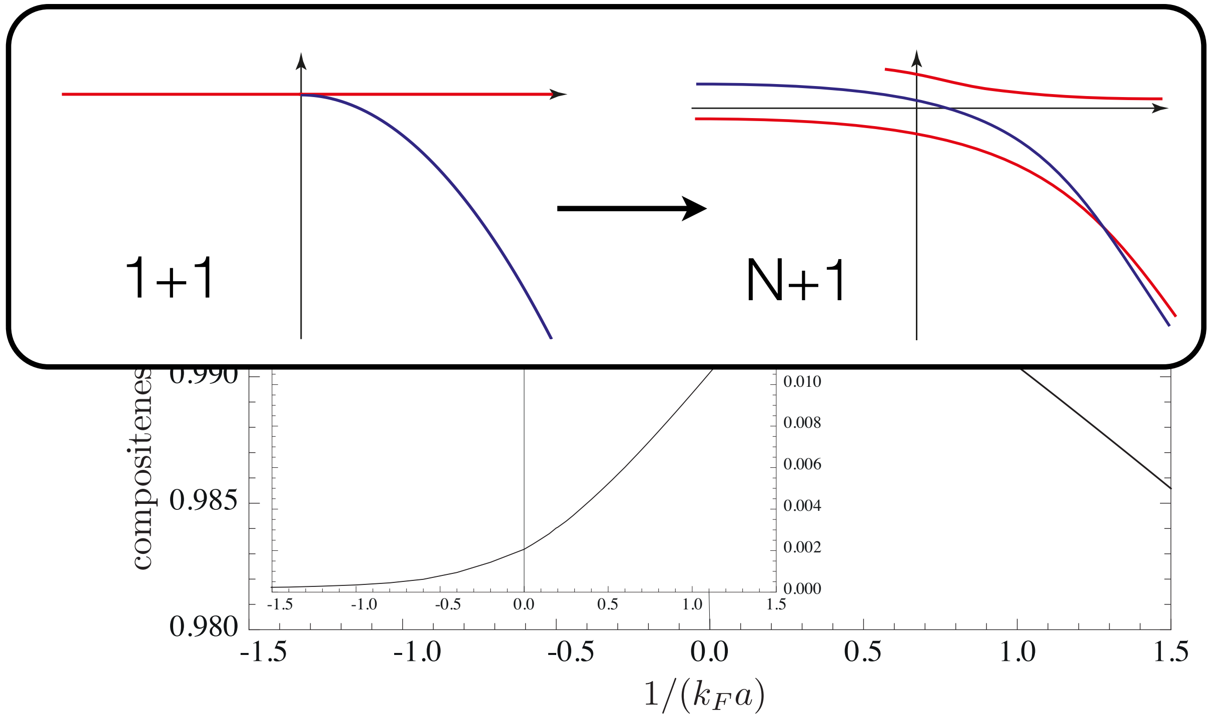
- weight of bound state extremely small

$$Z_\phi \approx 0.002 \qquad \text{(unitarity)}$$

why?

- Z can be interpreted as overlap of effective with elementary particles (in classical action)
  - attr. polaron elementary on BCS side
  - rep. polaron elementary on BEC side





quasi-particle weight

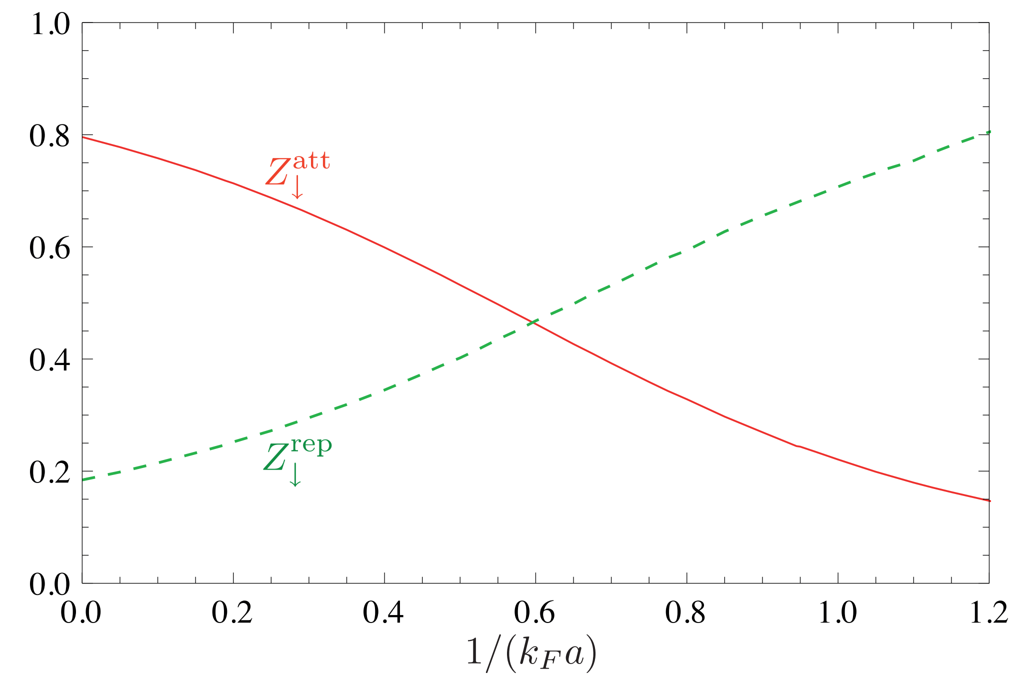
<u>molecule</u>

- weight of bound state extremely small

  $$Z_\phi \approx 0.002 \qquad \text{(unitarity)}$$

  why?

- Z can be interpreted as overlap of effective with elementary particles (in classical action)
  - attr. polaron elementary on BCS side
  - rep. polaron elementary on BEC side

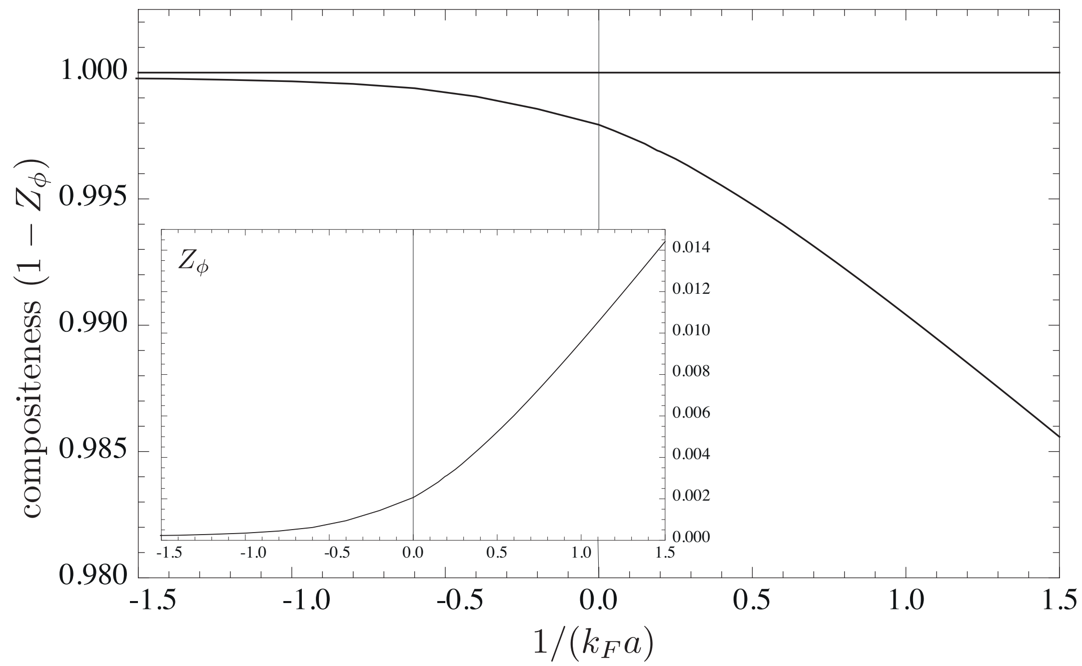


- idea: molecule not elementary

  → Weinberg's idea of *compositeness C*

  $$C = 1 - Z \qquad\qquad Z_\phi \sim a^{-1}/h^2$$

  WEINBERG (1965)



quasi-particle weight

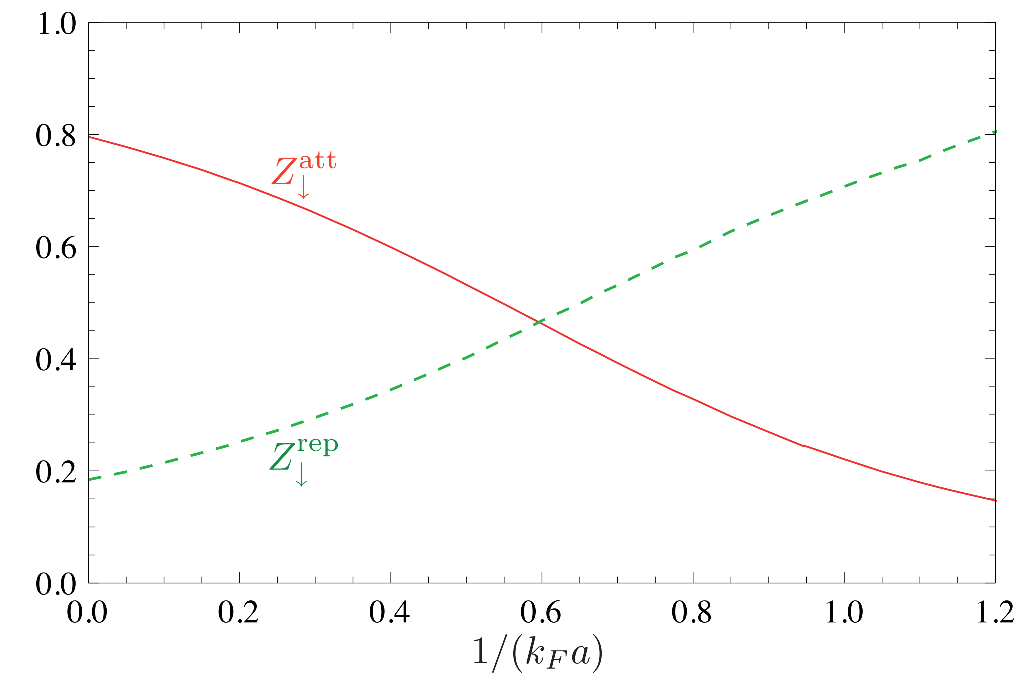
<u>molecule</u>

- weight of bound state extremely small

  $Z_\phi \approx 0.002$ (unitarity)

  why?

- Z can be interpreted as overlap of effective with elementary particles (in classical action)
  - attr. polaron elementary on BCS side
  - rep. polaron elementary on BEC side

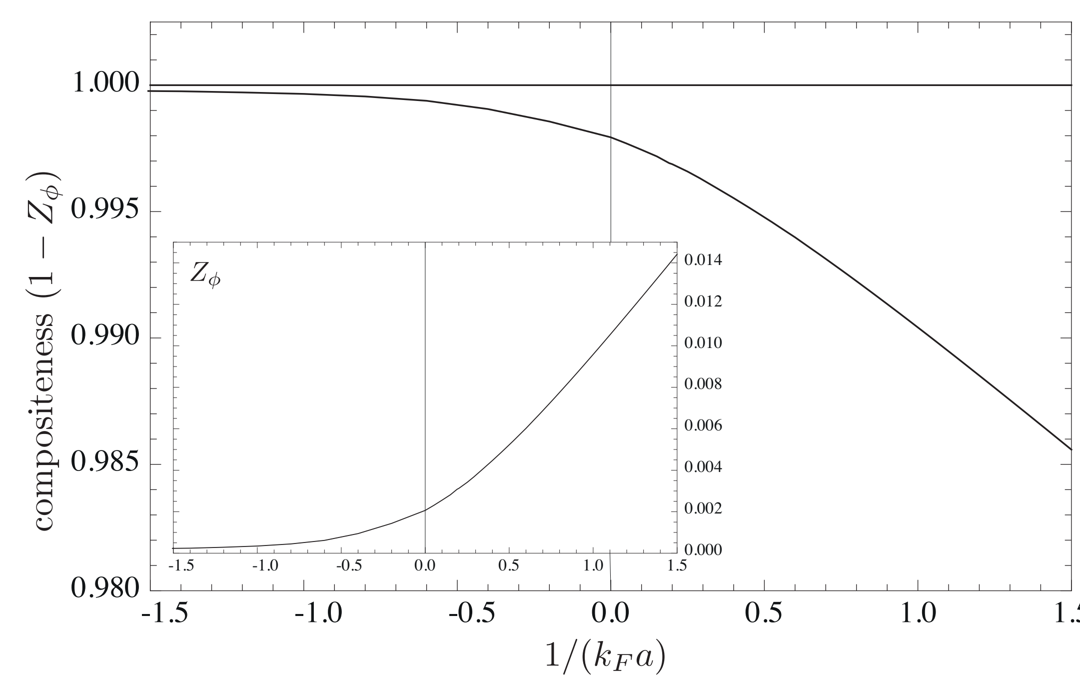


- idea: molecule not elementary

  ⟶ Weinberg‘s idea of *compositeness C*

  $$C = 1 - Z \qquad\qquad Z_\phi \sim a^{-1}/h^2$$

  WEINBERG (1965)



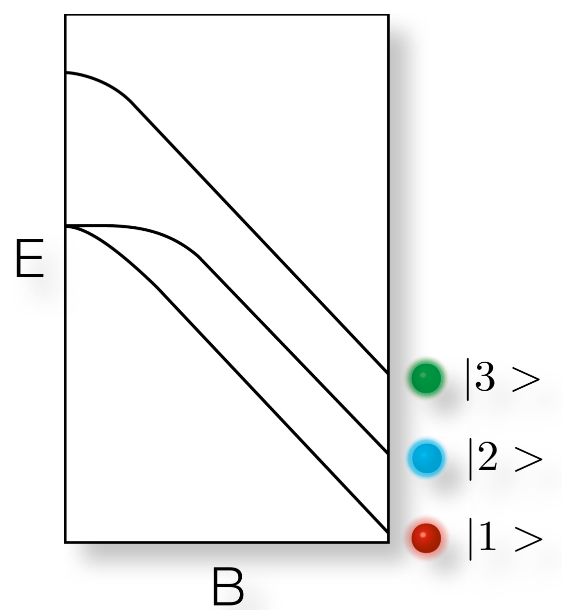
- fRG describes both situations:

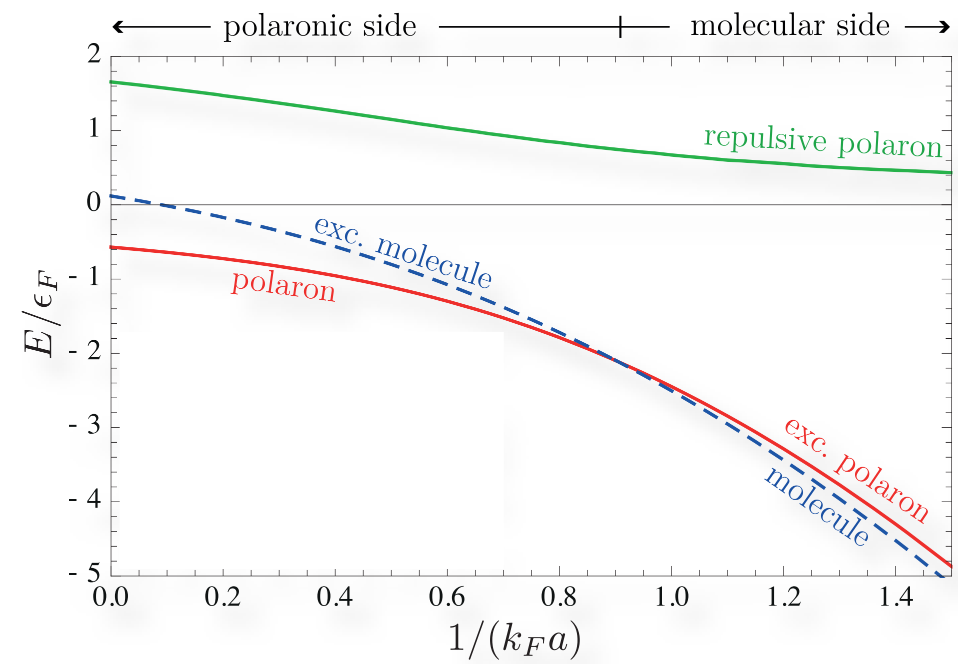
  $h \to \infty$: single channel model, molecule composite

  $h \to 1$: two-channel model, molecule elementary rf spectroscopy and experiment

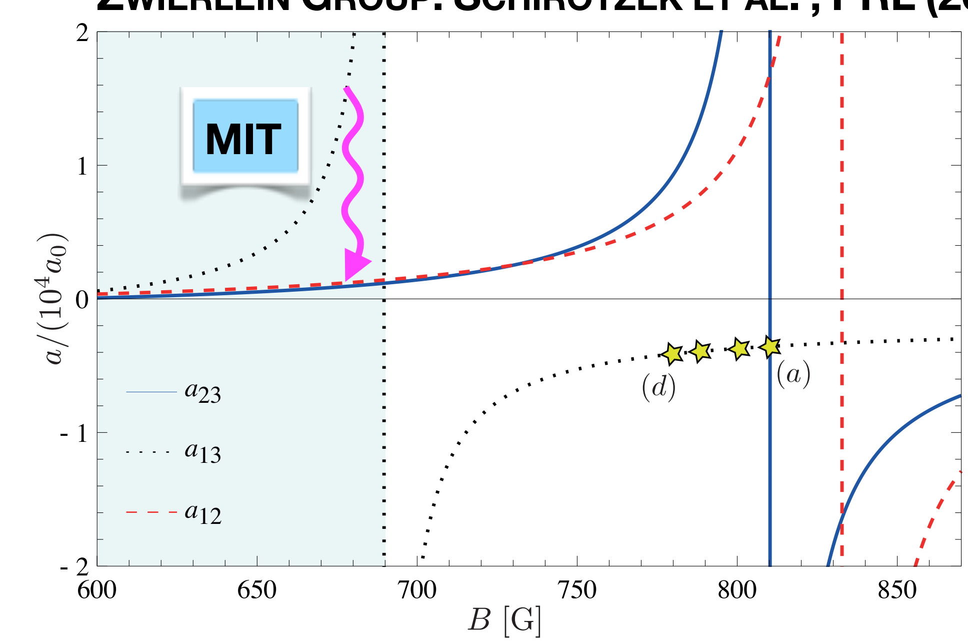
rf spectroscopy of the attractive polaron

- **ground state** observed experimentally by rf spectroscopy with imbalanced mixture of ^{6}Li atoms



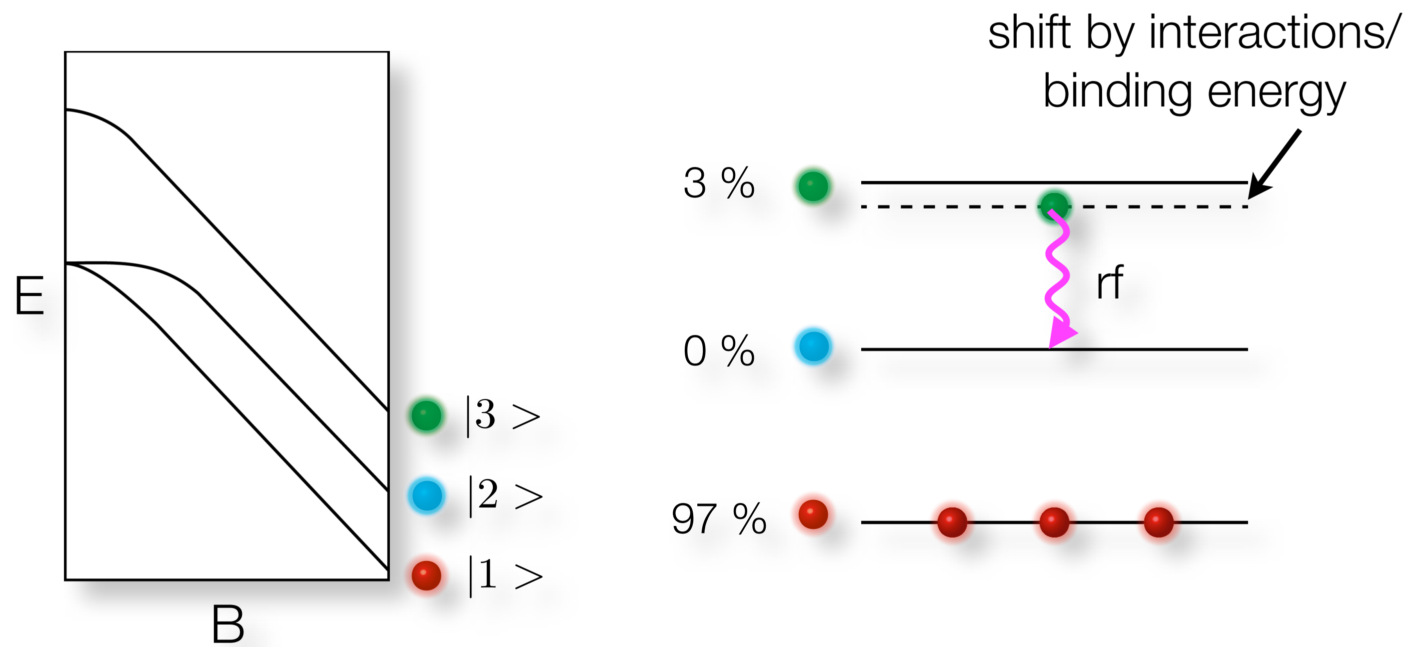


ZWIERLEIN GROUP: SCHIROTZEK ET AL., PRL (2010)

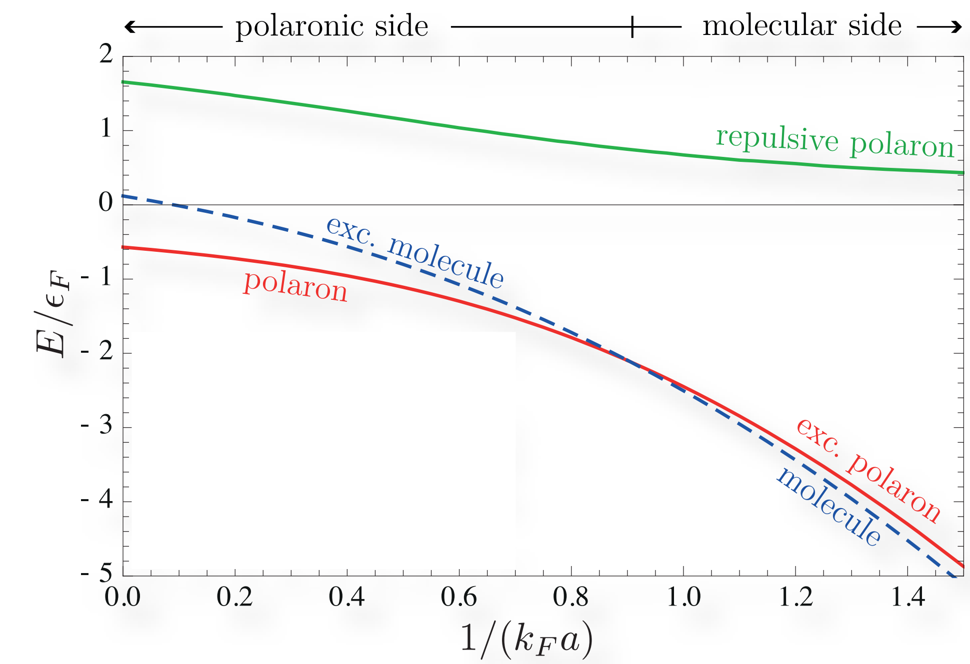


rf spectroscopy of the attractive polaron

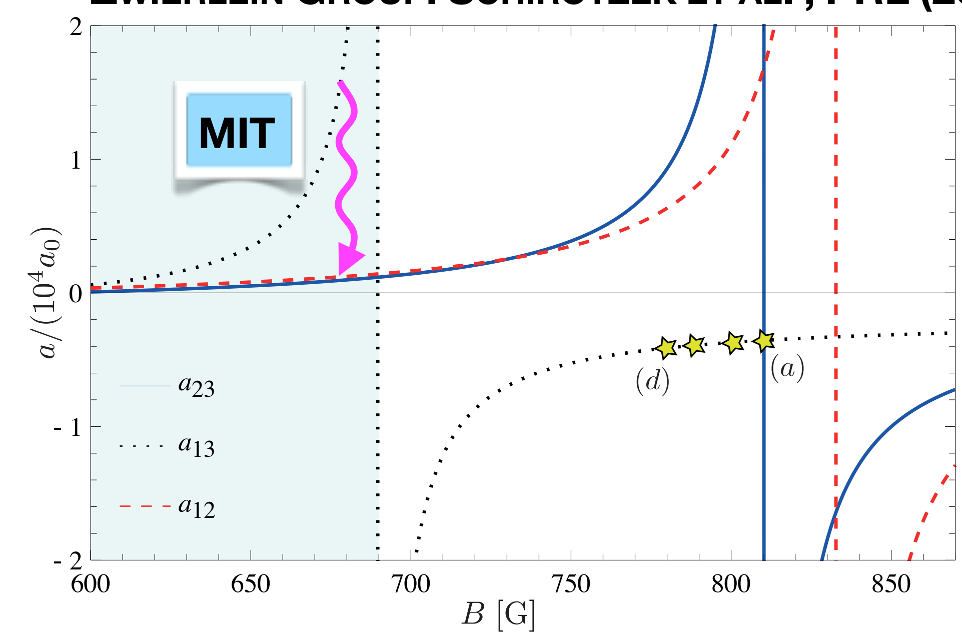
- **ground state** observed experimentally by rf spectroscopy with imbalanced mixture of ^{6}Li atoms



- initial: strongly interacting state of interest
- final: weakly interacting state (trivial spectral function)

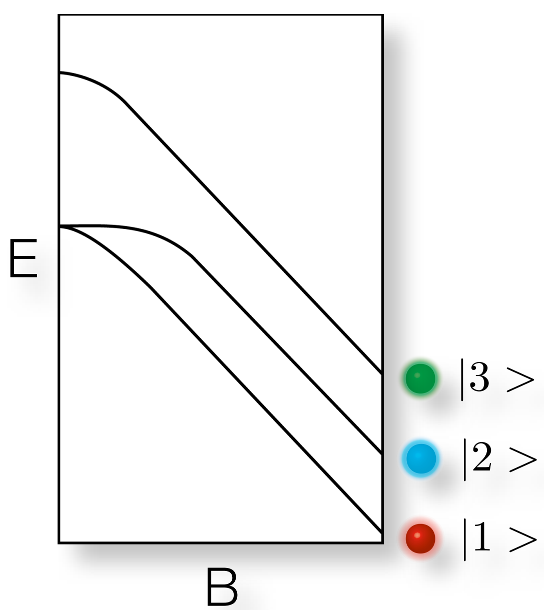


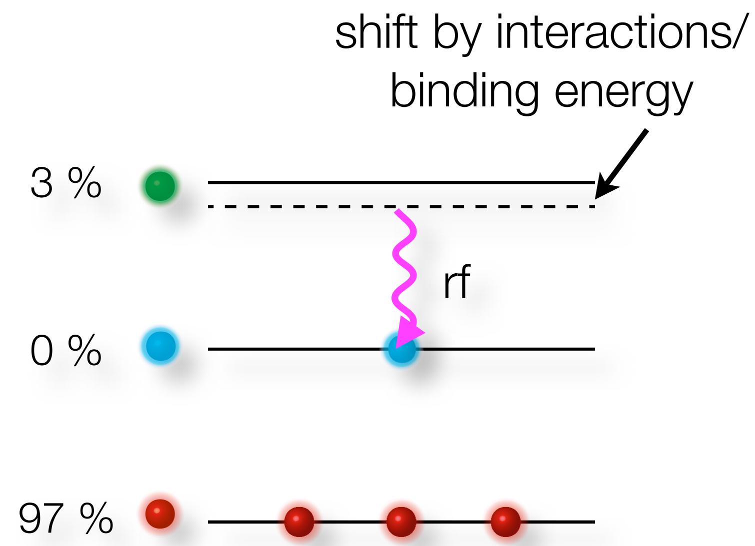
Zwierlein Group: Schirotzek et al. , PRL (2010)

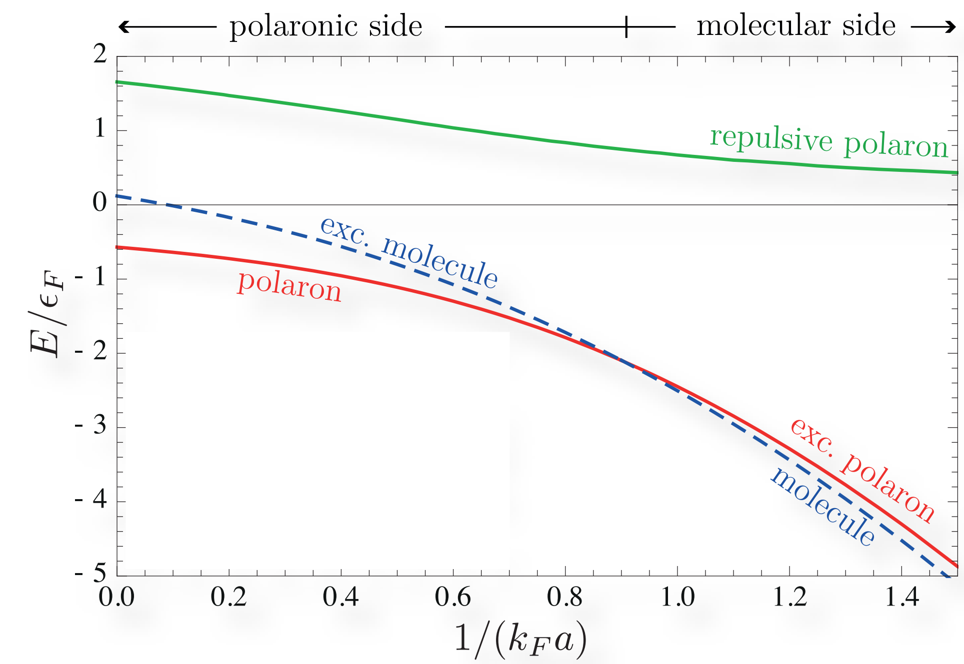


rf spectroscopy of the attractive polaron

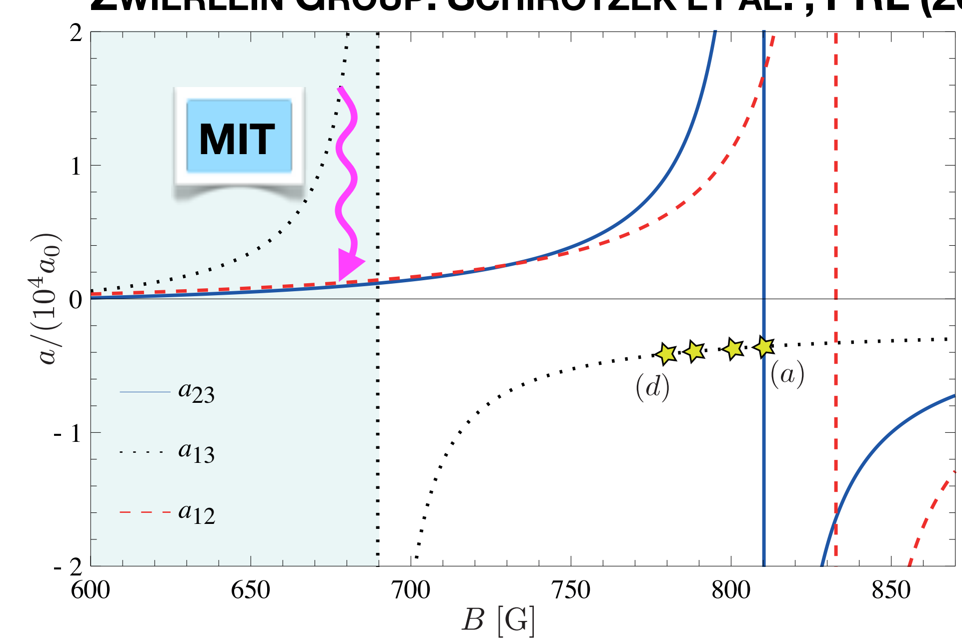
- **ground state** observed experimentally by rf spectroscopy with imbalanced mixture of ^{6}Li atoms



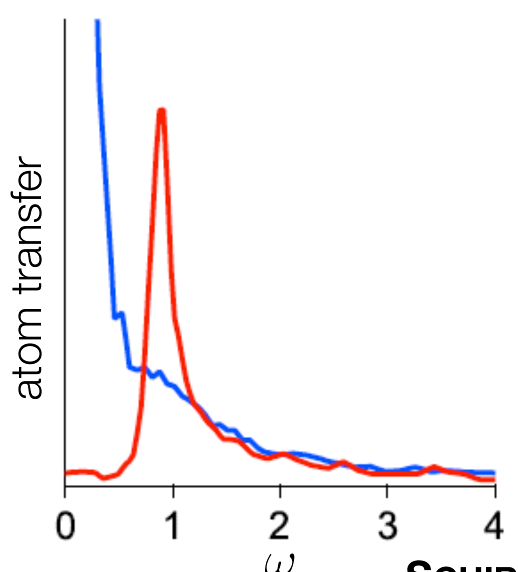




ZWIERLEIN GROUP: SCHIROTZEK ET AL. , PRL (2010)



- initial: strongly interacting state of interest
- final: weakly interacting state (trivial spectral function)

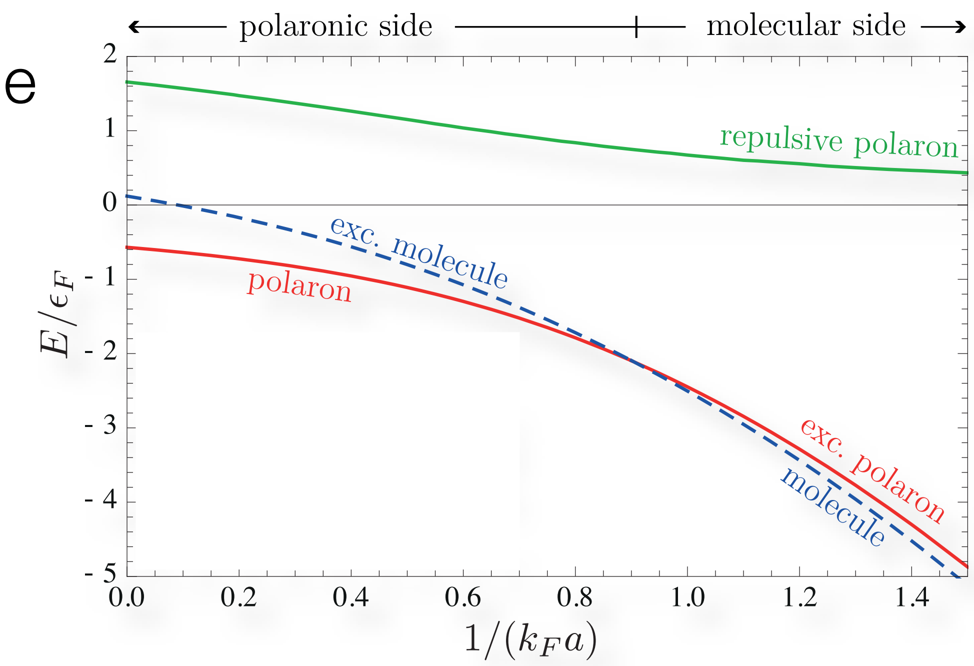


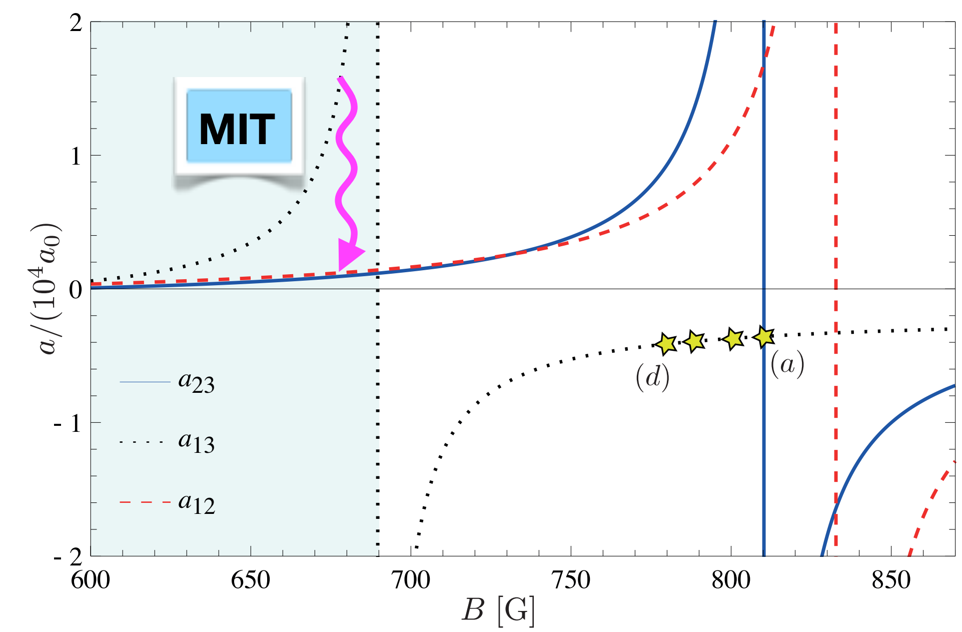
SCHIROTZEK ET AL. , PRL (2010)

inverse rf spectroscopy of the repulsive polaron

- **repulsive polaron** short lived

→ macroscopic population (MIT approach) impossible





inverse rf spectroscopy of the repulsive polaron

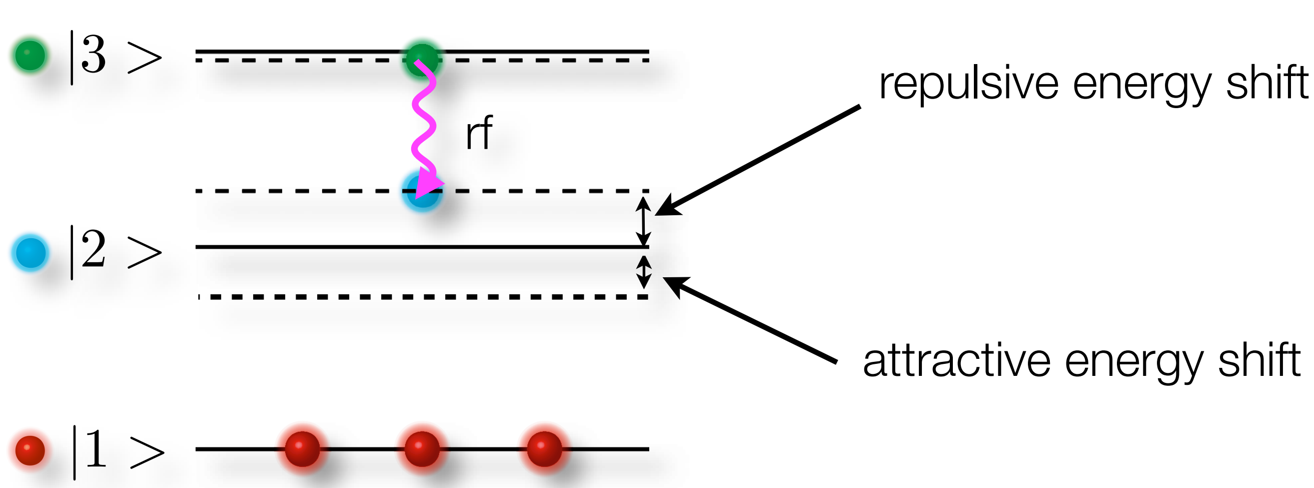
- **repulsive polaron** short lived

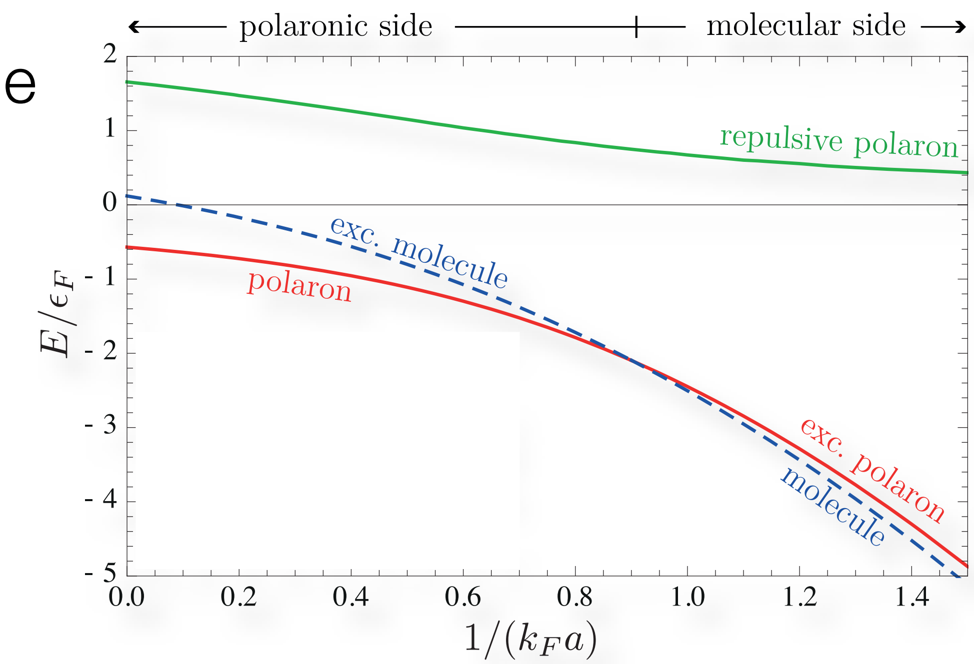
  ⟶ macroscopic population (MIT approach) impossible

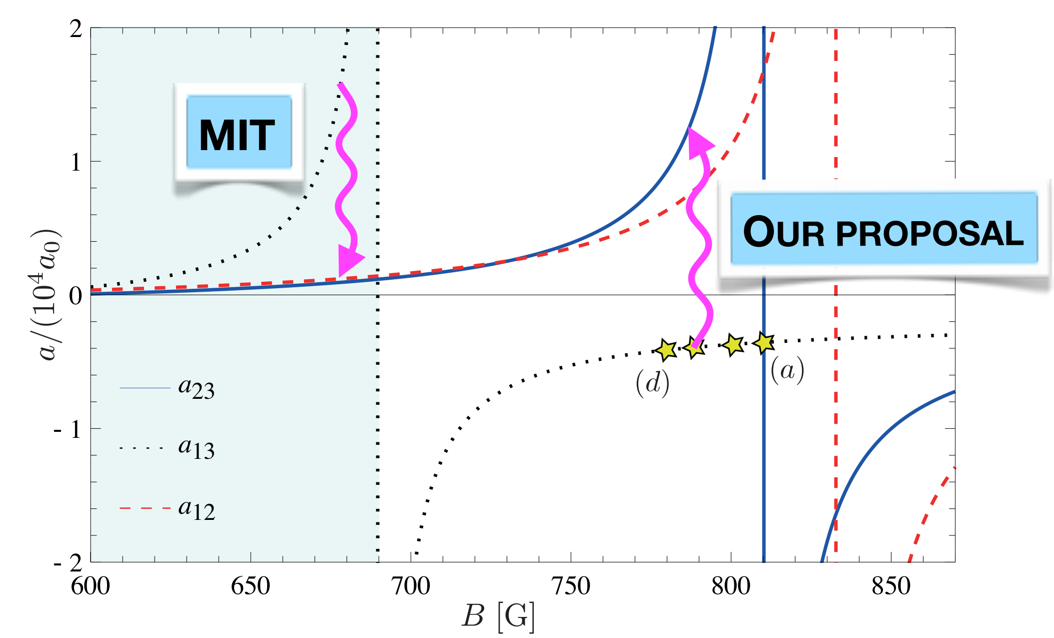
- our proposal:

  initial: state of with well-known spectral function (attractive polaron branch)

  final: strongly interacting state



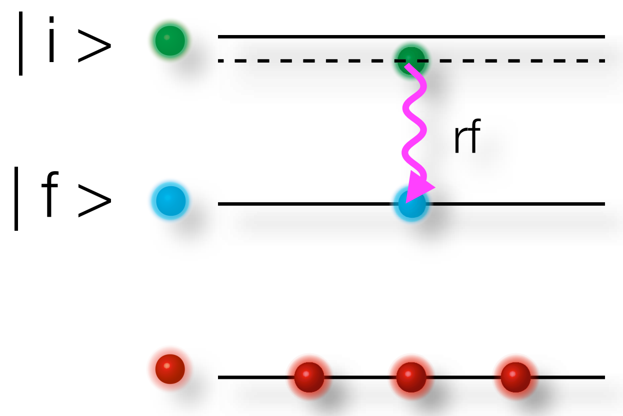




linear response theory

- rf photon gives perturbation

$$H_{\mathrm{rf}} \sim \psi_f \psi_i^*$$



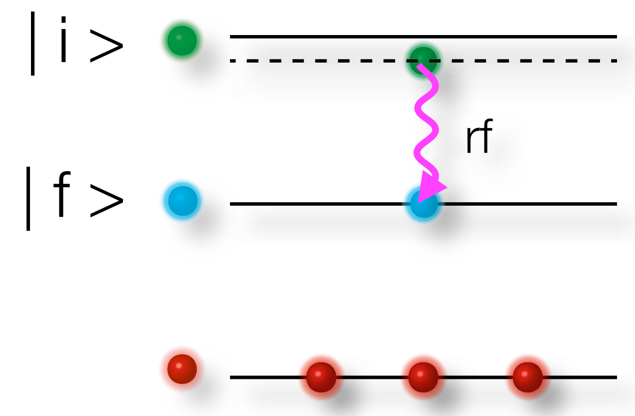
yields rf suszeptiblity

$$\chi(\Omega) = -\int_{\mathbf{r}}\int_{\mathbf{r}'}\int_{\tau} e^{i\Omega\tau}\langle T_\tau \psi_f^\dagger(\mathbf{r},\tau)\psi_i(\mathbf{r},\tau) \times \psi_i^\dagger(\mathbf{r}',0)\psi_f(\mathbf{r}',0)\rangle$$

linear response theory

- rf photon gives perturbation

$$H_{\rm rf} \sim \psi_f \psi_i^*$$

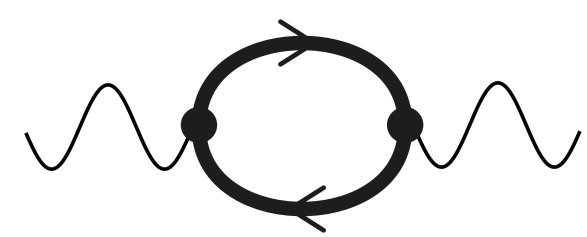


yields rf suszeptiblity

$$\chi(\Omega) = -\int_{\mathbf{r}}\int_{\mathbf{r}'}\int_{\tau} e^{i\Omega\tau}\langle T_\tau \psi_f^\dagger(\mathbf{r},\tau)\psi_i(\mathbf{r},\tau) \times \psi_i^\dagger(\mathbf{r}',0)\psi_f(\mathbf{r}',0)\rangle$$

- diagrammatic expansion

$$\chi(\Omega) = \int_{\mathbf{k},\omega} G_i(\mathbf{k},\omega)\, G_f(\mathbf{k},\omega+\Omega) \quad =$$

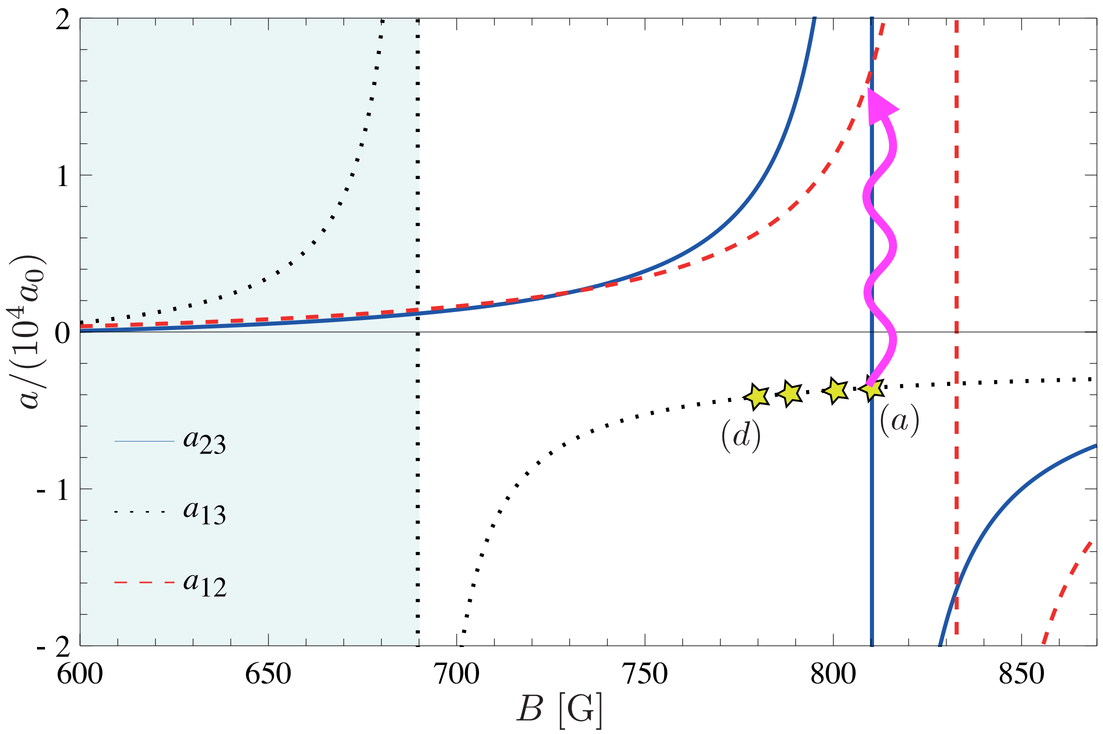


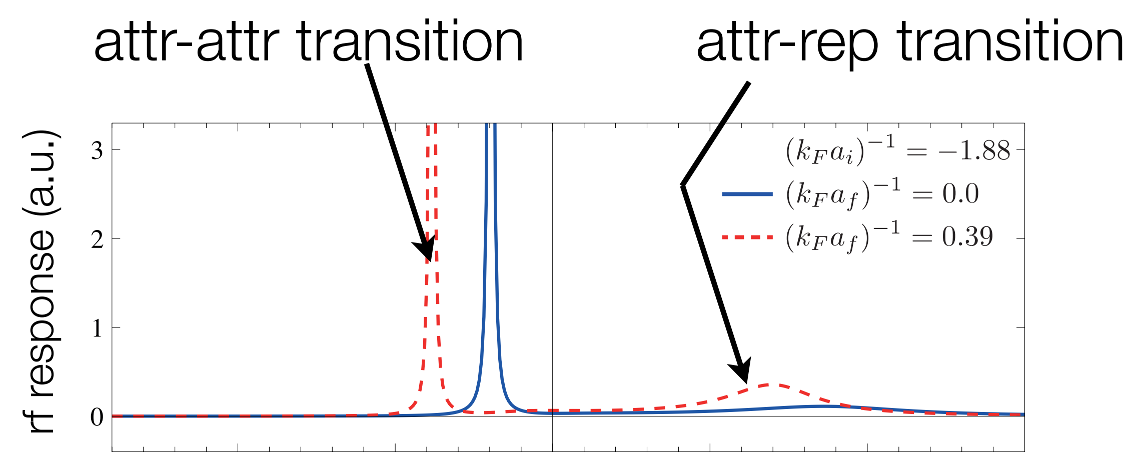
gives rf response

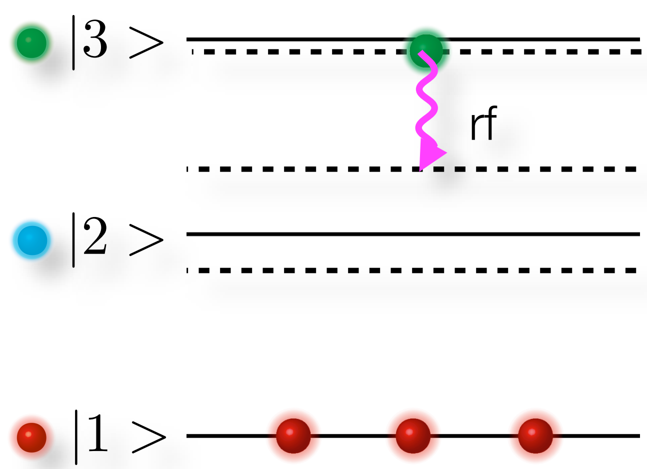
$$I(\omega) = 2\Omega_R^2 \mathrm{Im}\chi_{\rm R}(\omega = \mu_f - \mu_i - \omega_L)$$

$$I(\omega_L) = \Omega_R^2 \int_{\mathbf{k}} \int_0^{E_f - E_i + \omega_L} \frac{d\Omega}{2\pi}\, A_f(\mathbf{k},\Omega)\, A_i(\mathbf{k}, \Omega - (E_f - E_i + \omega_L))$$

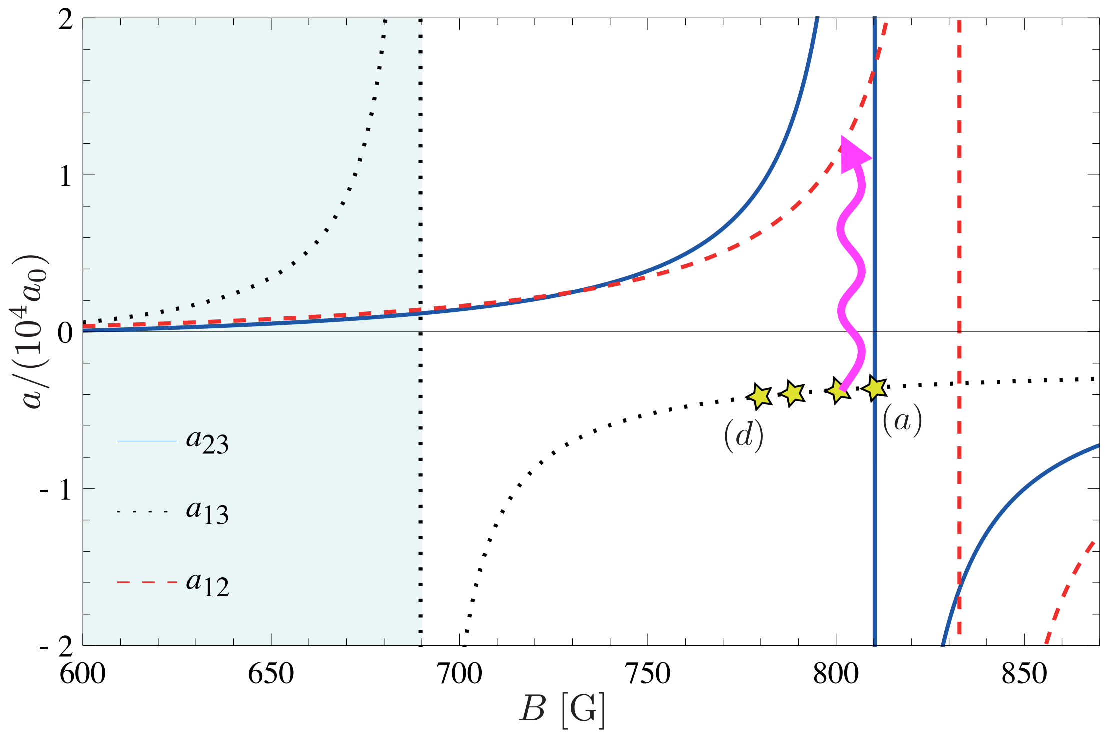
rf spectra from fRG in linear response

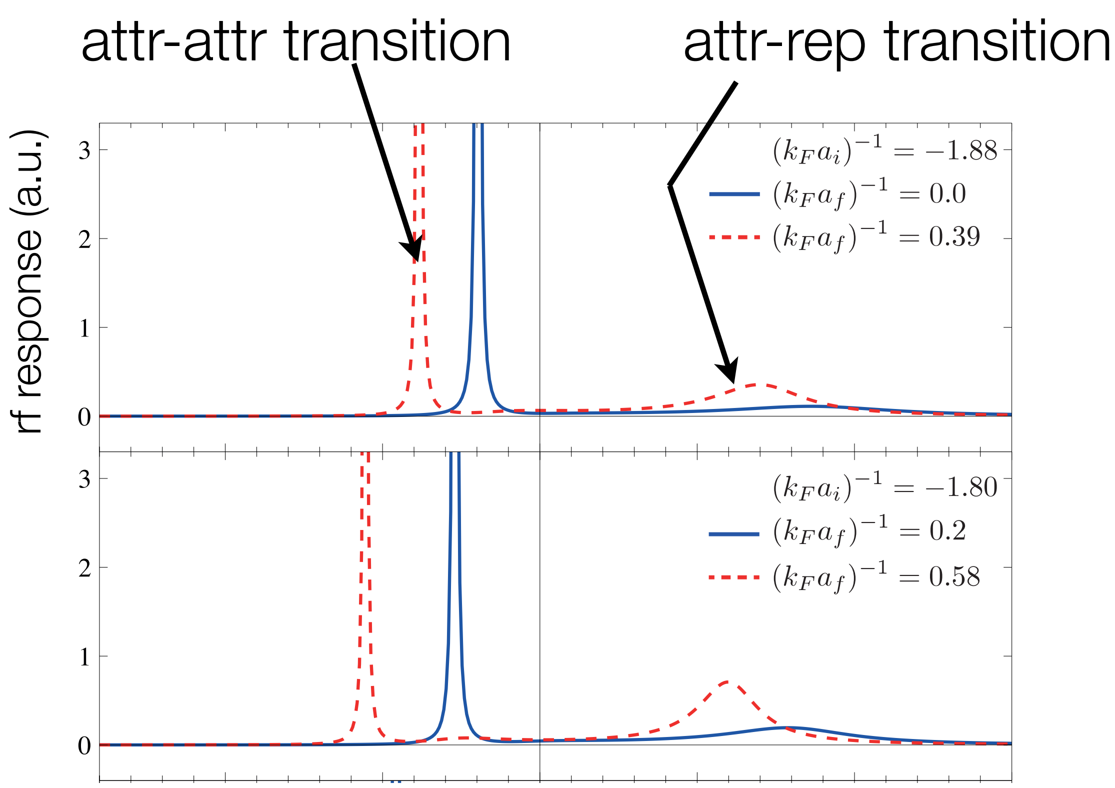


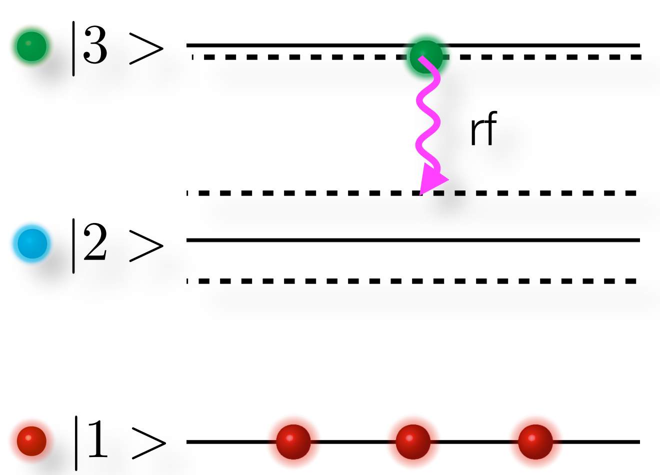




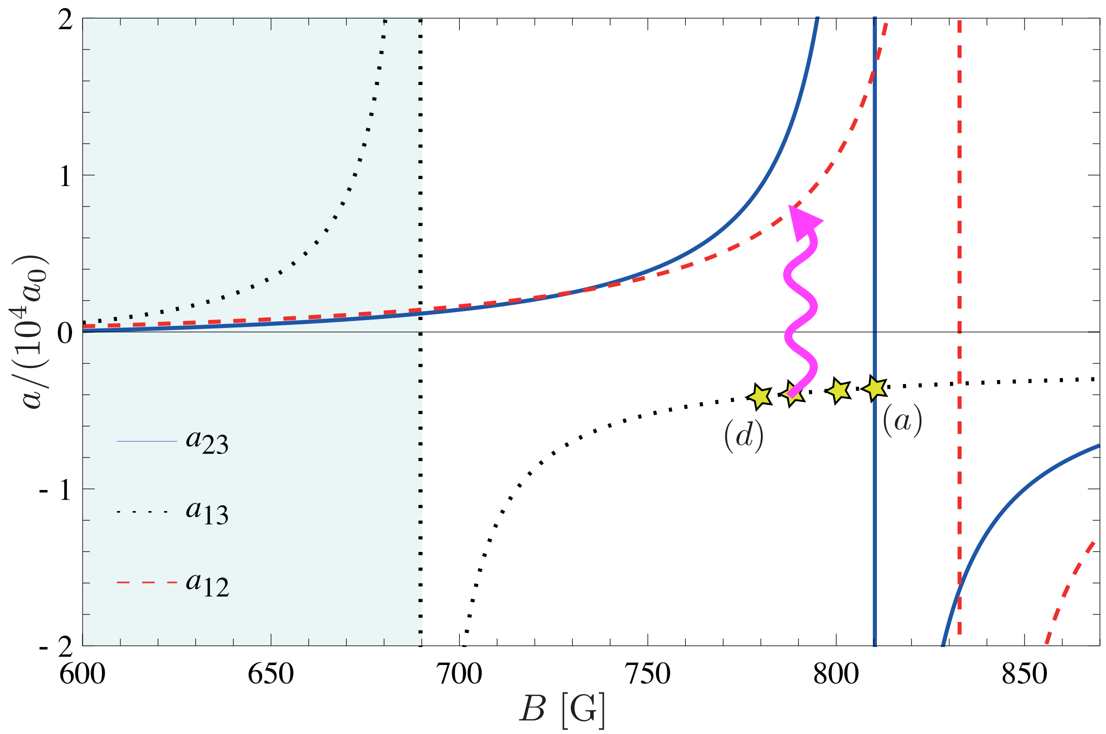
rf spectra from fRG in linear response

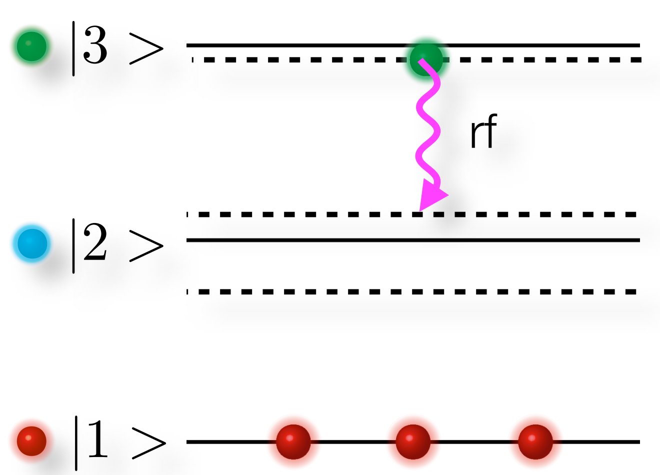


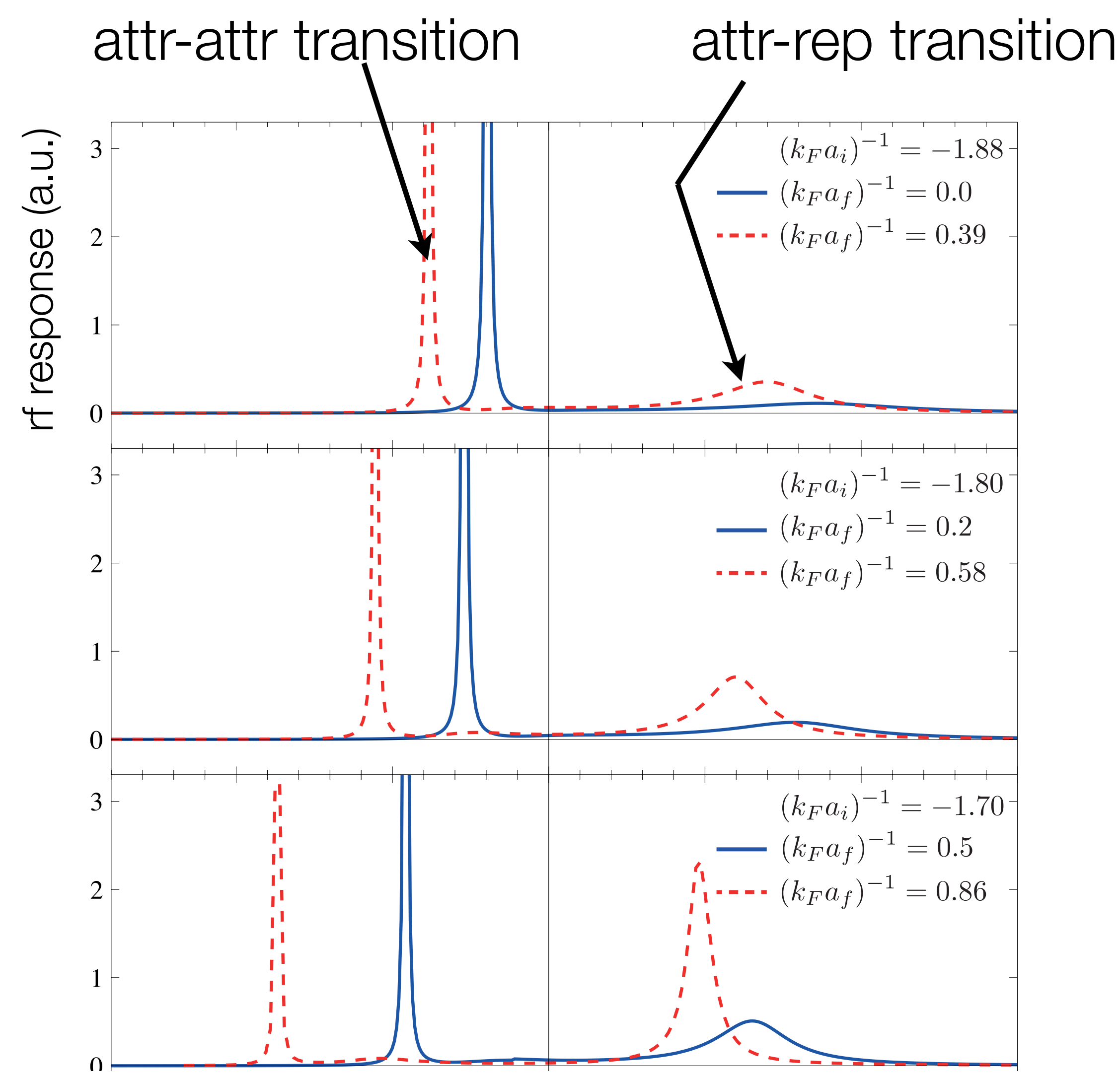




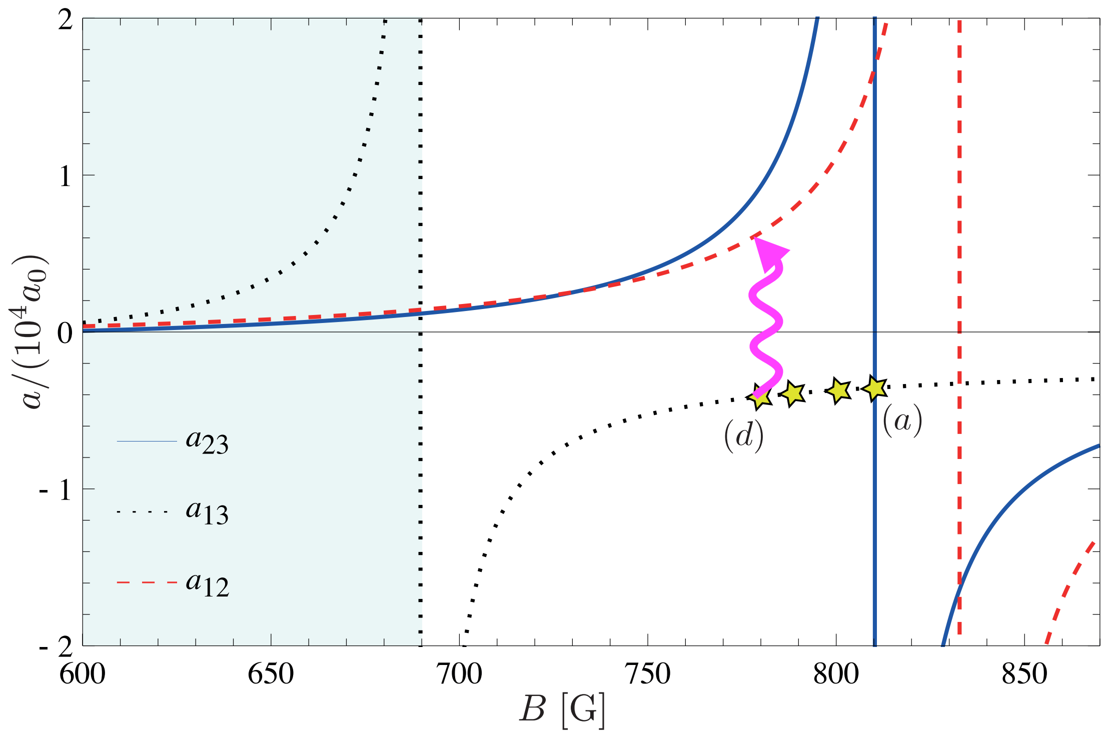
rf spectra from fRG in linear response

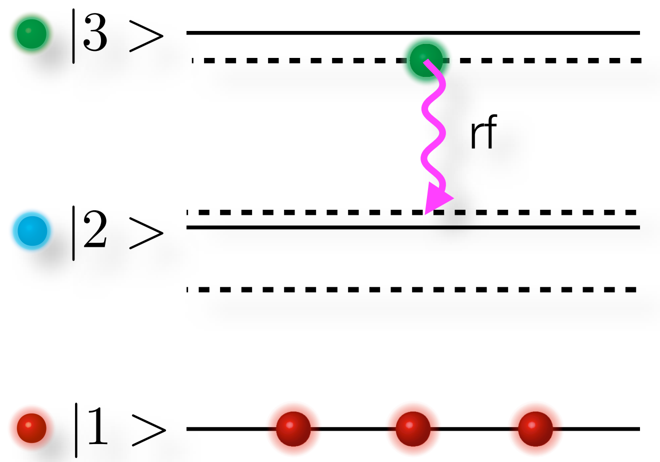


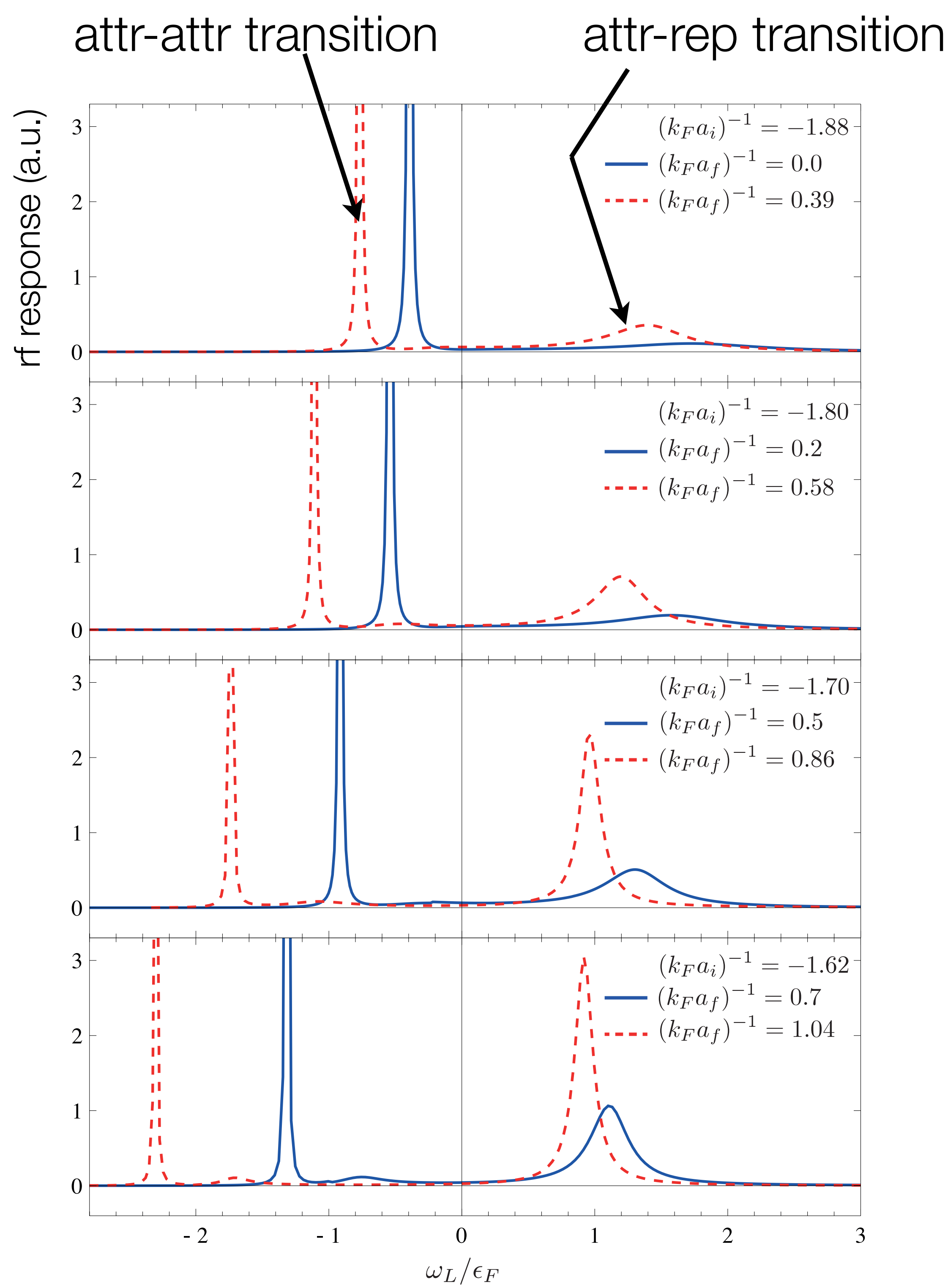




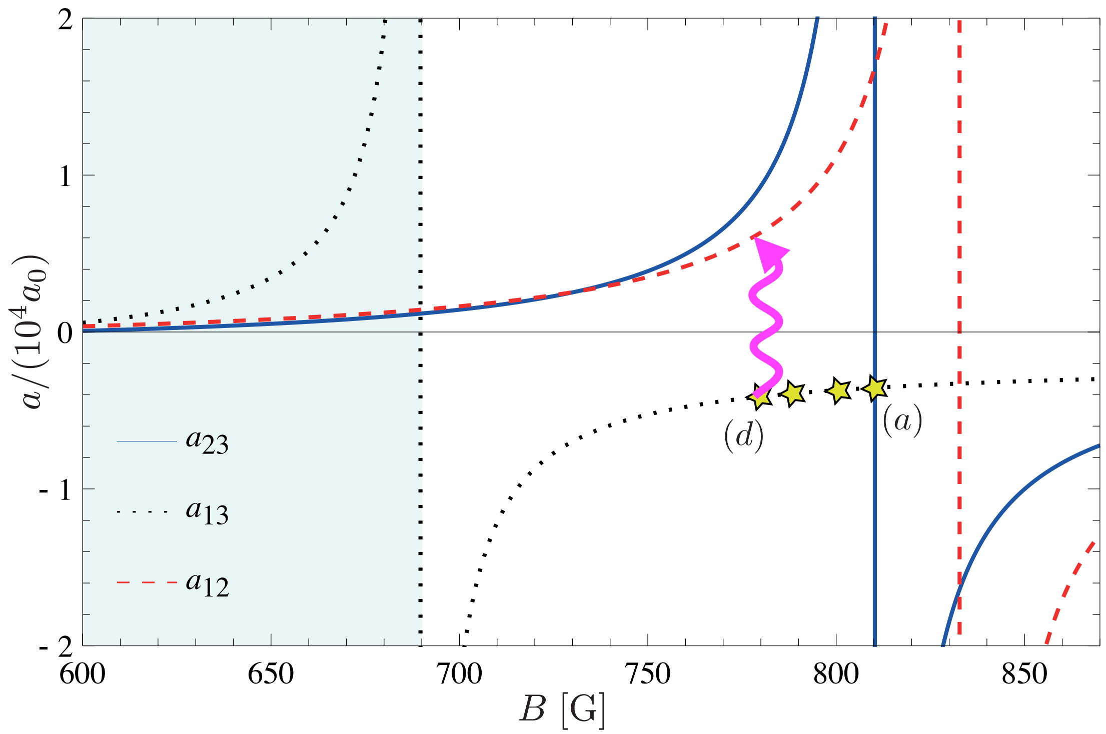
rf spectra from fRG in linear response

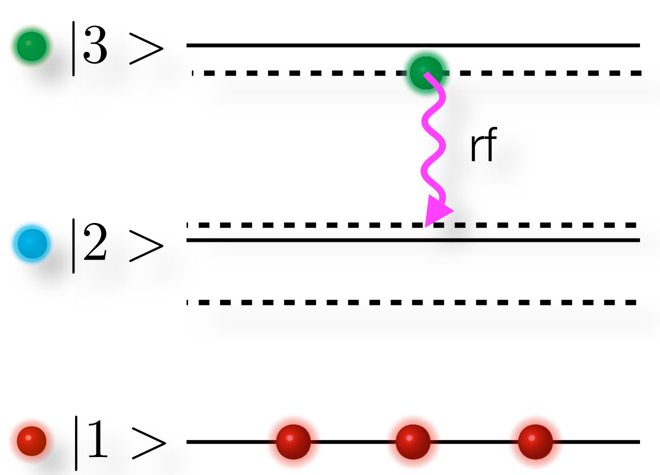


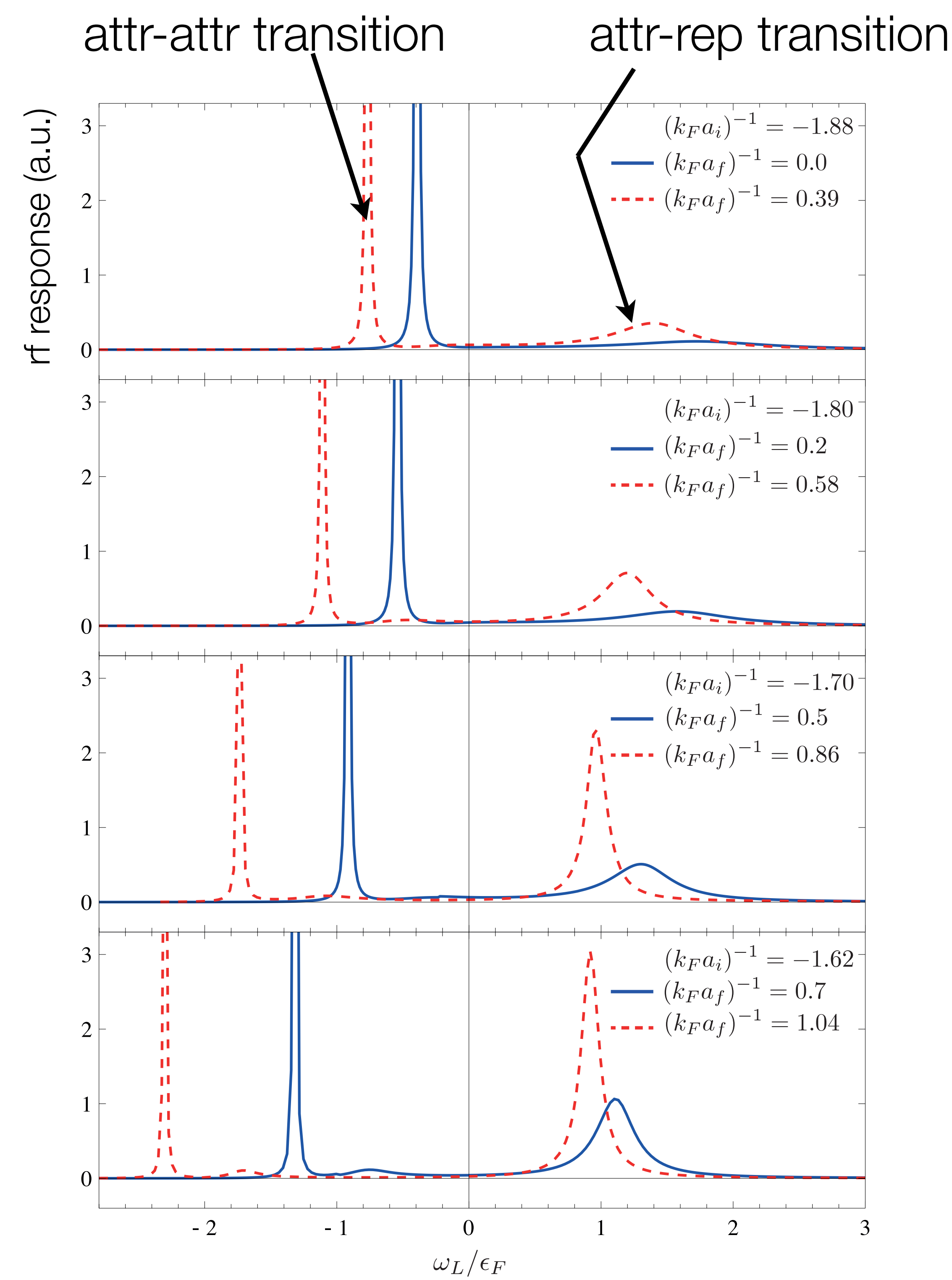




rf spectra from fRG in linear response







$\longrightarrow$ Experimentally realized and verified by Grimm group for Li-K mixture (Innsbruck)

C. Kohstall et al (in prep)

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi_\uparrow^*(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi_\downarrow^*(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$
$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

: scale dependent

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi_\uparrow^*(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi_\downarrow^*(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$
$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

: scale dependent

- observable: polaron energy $\mu_\downarrow$ at unitarity

approximation	der. exp.					diagMC	vMC	exp.
regulator	k[2]					-	-	-
$\mu_\downarrow$	-0.352					-0.615	-0.58	-0.58

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi_\uparrow^*(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi_\downarrow^*(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$

$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

$$+\lambda_{\phi\uparrow}\int_X \phi^*\psi_\uparrow^*\phi\psi_\uparrow + \lambda_{\phi\downarrow}\int_X \phi^*\psi_\downarrow^*\phi\psi_\downarrow$$

: scale dependent

- observable: polaron energy $\mu_\downarrow$ at unitarity

approximation	der. exp.					diagMC	vMC	exp.
regulator	k^2					-	-	-
$\mu_\downarrow$	-0.352					-0.615	-0.58	-0.58

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi_\uparrow^*(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi_\downarrow^*(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$

$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

$$\boxed{+\lambda_{\phi\uparrow}\int_X \phi^*\psi_\uparrow^*\phi\psi_\uparrow + \lambda_{\phi\downarrow}\int_X \phi^*\psi_\downarrow^*\phi\psi_\downarrow}$$

: scale dependent

- observable: polaron energy $\mu_\downarrow$ at unitarity

approximation	der. exp.	ext. der. exp				diagMC	vMC	exp.
regulator	k^2	k^2				-	-	-
$\mu_\downarrow$	-0.352	-0.357				-0.615	-0.58	-0.58

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi_\uparrow^*(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi_\downarrow^*(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$

$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

$$+\lambda_{\phi\uparrow}\int_X \phi^*\psi_\uparrow^*\phi\psi_\uparrow + \lambda_{\phi\downarrow}\int_X \phi^*\psi_\downarrow^*\phi\psi_\downarrow$$

: scale dependent

- observable: polaron energy $\mu_\downarrow$ at unitarity

approximation	der. exp.	ext. der. exp			full w/q	diagMC	vMC	exp.
regulator	k^2	k^2			sharp	-	-	-
$\mu_\downarrow$	-0.352	-0.357			-0.571	-0.615	-0.58	-0.58

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi^*_\uparrow(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi^*_\downarrow(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$

$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

$$+\lambda_{\phi\uparrow}\int_X \phi^*\psi^*_\uparrow\phi\psi_\uparrow + \lambda_{\phi\downarrow}\int_X \phi^*\psi^*_\downarrow\phi\psi_\downarrow$$

: scale dependent

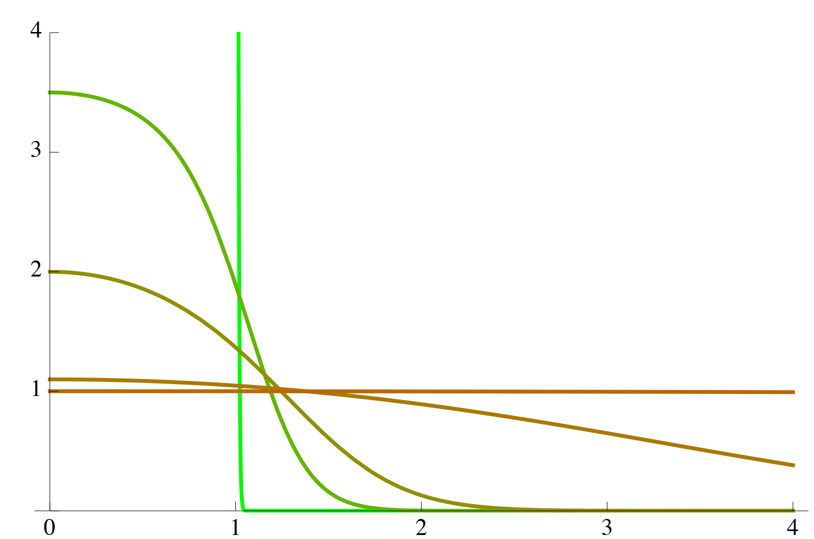
- observable: polaron energy $\mu_\downarrow$ at unitarity

approximation	der. exp.	ext. der. exp			full w/q	diagMC	vMC	exp.
regulator	k^2	k^2			sharp	-	-	-
$\mu_\downarrow$	-0.352	-0.357			-0.571	-0.615	-0.58	-0.58

<u>regulator dependency</u> $\longrightarrow$ TALK BY **B. DELAMOTTE** IN THE LUNCH BREAK!

full w/q-dep. vertex function calculation with class of regulator functions

$$R_k(p;T,c) = A\,k^2\left(1 + \frac{c}{T^2}\right)\frac{e^{-1/T}+1}{\exp\frac{q^2-k^2}{k^2T}+1}$$



also tested: exponential cutoff $\longrightarrow$ numerically very tedious!

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi_\uparrow^*(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi_\downarrow^*(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$

$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

$$\boxed{+\lambda_{\phi\uparrow}\int_X \phi^*\psi_\uparrow^*\phi\psi_\uparrow + \lambda_{\phi\downarrow}\int_X \phi^*\psi_\downarrow^*\phi\psi_\downarrow}$$

: scale dependent

- observable: polaron energy $\mu_\downarrow$ at unitarity

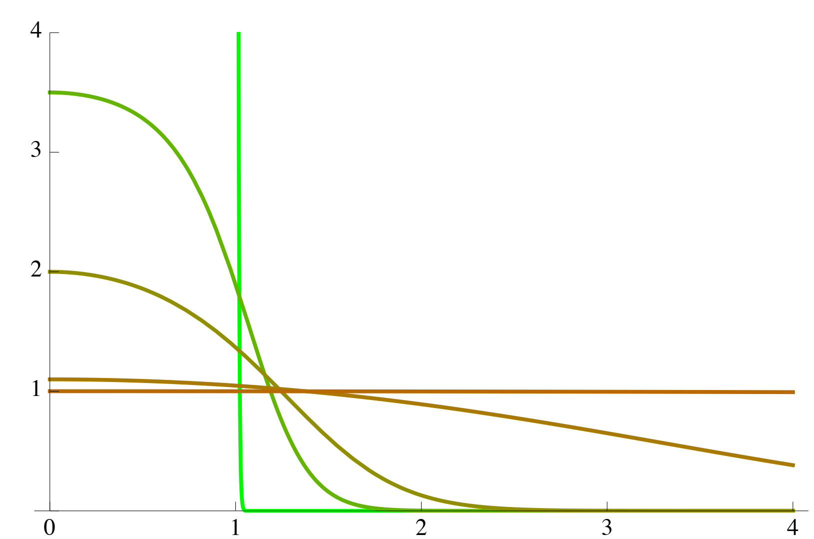
monotonous interpolation →

approximation	der. exp.	ext. der. exp	full w/q	full w/q	diagMC	vMC	exp.
regulator	k^2	k^2	k^2	sharp	-	-	-
$\mu_\downarrow$	-0.352	-0.357	-0.40	-0.571	-0.615	-0.58	-0.58

regulator dependency → TALK BY **B. DELAMOTTE** IN THE LUNCH BREAK!

full w/q-dep. vertex function calculation with class of regulator functions

$$R_k(p;T,c) = A\,k^2\left(1 + \frac{c}{T^2}\right)\frac{e^{-1/T}+1}{\exp\frac{q^2-k^2}{k^2T}+1}$$



also tested: exponential cutoff → numerically very tedious!

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi_\uparrow^*(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi_\downarrow^*(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$

$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

$$\boxed{+\lambda_{\phi\uparrow}\int_X \phi^*\psi_\uparrow^*\phi\psi_\uparrow + \lambda_{\phi\downarrow}\int_X \phi^*\psi_\downarrow^*\phi\psi_\downarrow}$$

: scale dependent

- observable: polaron energy $\mu_\downarrow$ at unitarity

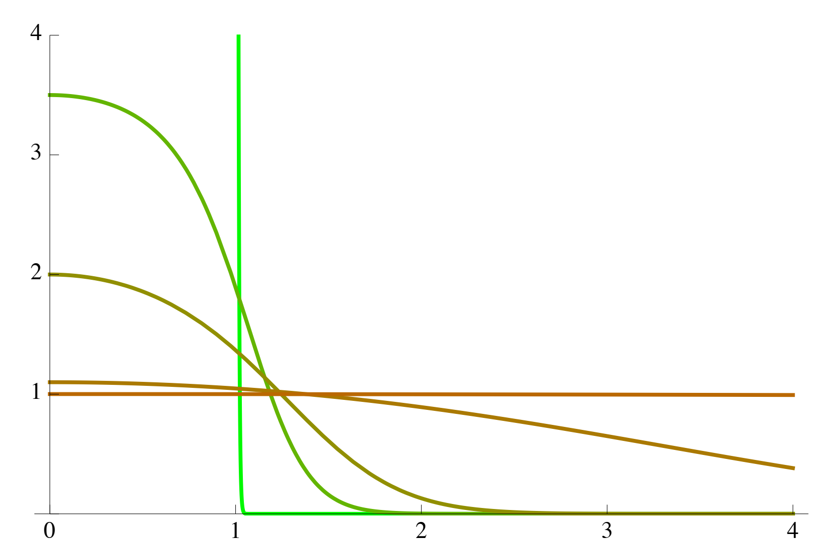
monotonous interpolation →

approximation	der. exp.	ext. der. exp	der. exp.	full w/q	full w/q	diagMC	vMC	exp.
regulator	k^2	k^2	sharp	k^2	sharp	-	-	-
$\mu_\downarrow$	-0.352	-0.357	-0.55	-0.40	-0.571	-0.615	-0.58	-0.58

<u>regulator dependency</u> → TALK BY **B. DELAMOTTE** IN THE LUNCH BREAK!

full w/q-dep. vertex function calculation with class of regulator functions

$$R_k(p;T,c) = A\,k^2\left(1 + \frac{c}{T^2}\right)\frac{e^{-1/T}+1}{\exp\frac{q^2-k^2}{k^2T}+1}$$



also tested: exponential cutoff → numerically very tedious!

Comments on derivative expansion and regulator dependence

- we also studied a derivative expansion, including higher order vertex (atom-dimer)

$$\Gamma_k = \int_P \psi^*_\uparrow(P)[i\omega + \vec{p}^2 - \mu_\uparrow]\psi_\uparrow(P) + \psi^*_\downarrow(P)[A_\downarrow(i\omega + \vec{p}^2) - \mu_\downarrow]\psi_\downarrow(P)$$
$$+ \int_P \phi^*(P)[A_\phi(i\omega + \vec{p}^2/2) + m_\phi]\phi(P) + h\int_X (\phi^*(X)\psi_\uparrow(X)\psi_\downarrow(X) + h.c)$$

IN SPIRIT OF ERG 2010

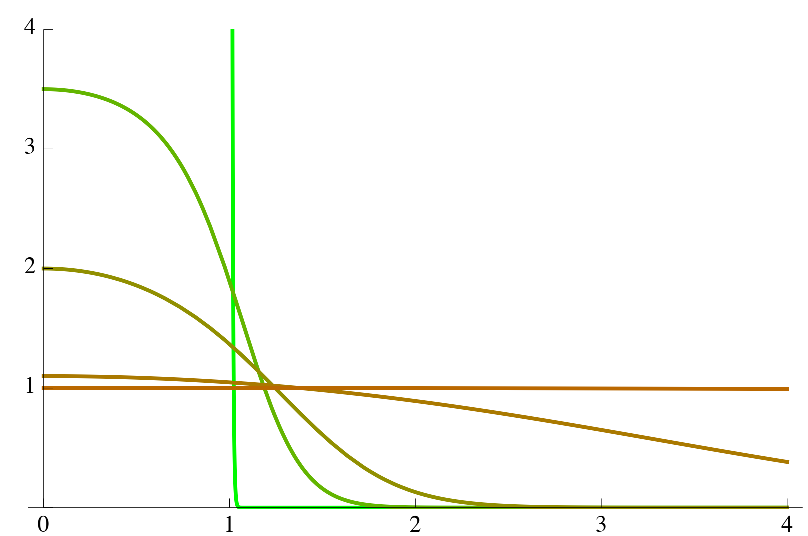
Can we reasonably estimate our error?

$\mu_\downarrow$	-0.352	-0.357	-0.55	-0.40	-0.571	-0.615	-0.58	-0.58

regulator dependency → TALK BY **B. DELAMOTTE** IN THE LUNCH BREAK!

full w/q-dep. vertex function calculation with class of regulator functions

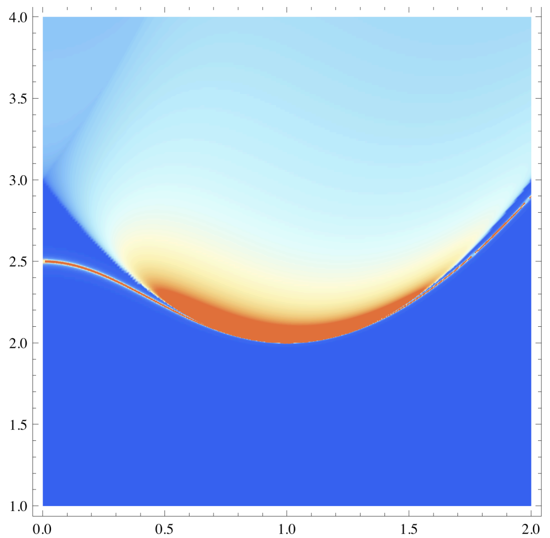
$$R_k(p;T,c) = A\,k^2\left(1 + \frac{c}{T^2}\right)\frac{e^{-1/T}+1}{\exp\frac{q^2-k^2}{k^2T}+1}$$

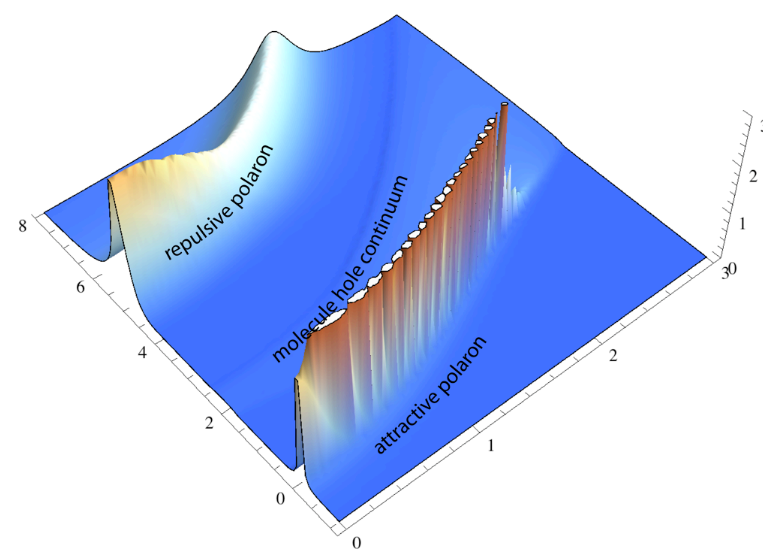


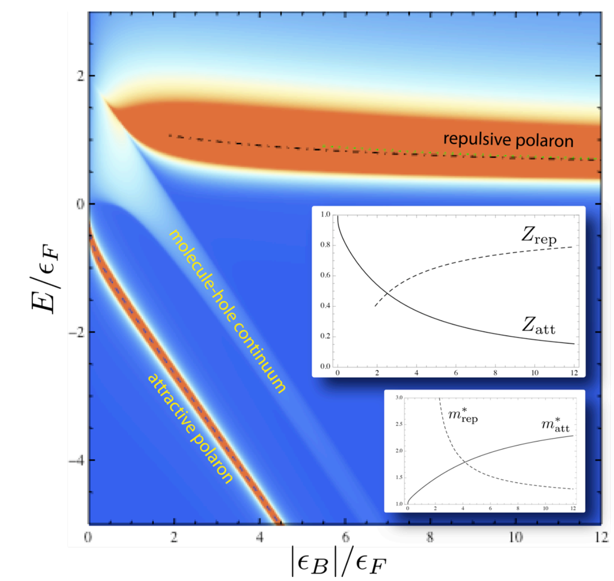
also tested: exponential cutoff → numerically very tedious!

outlook

- Fermi polaron in two dimensions (experiments: Cambrige & MIT)



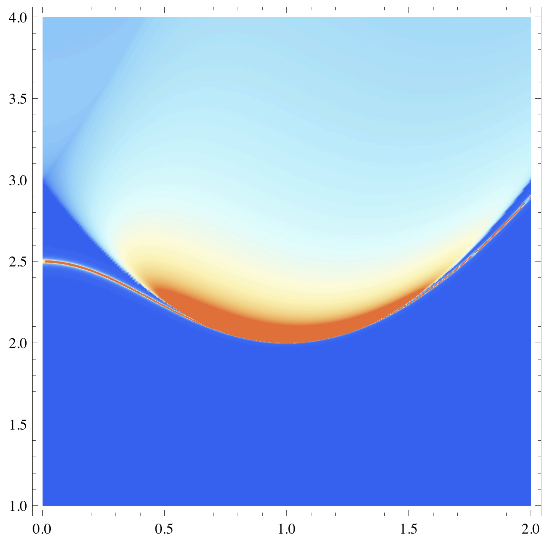


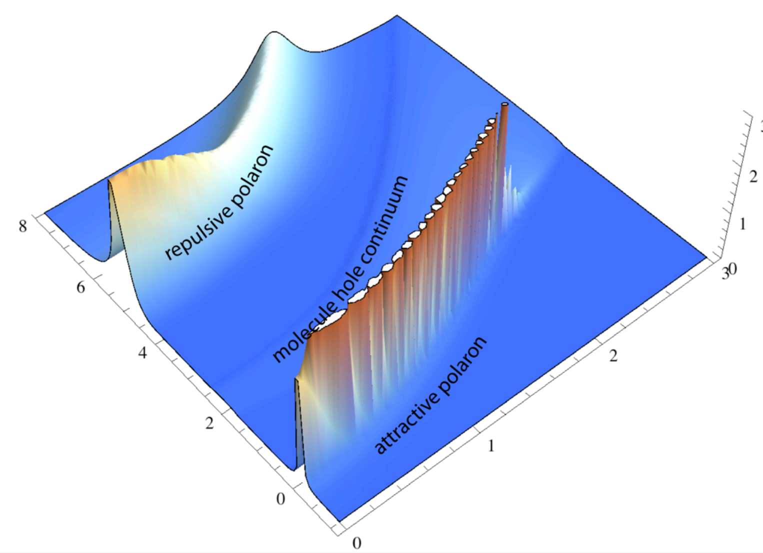


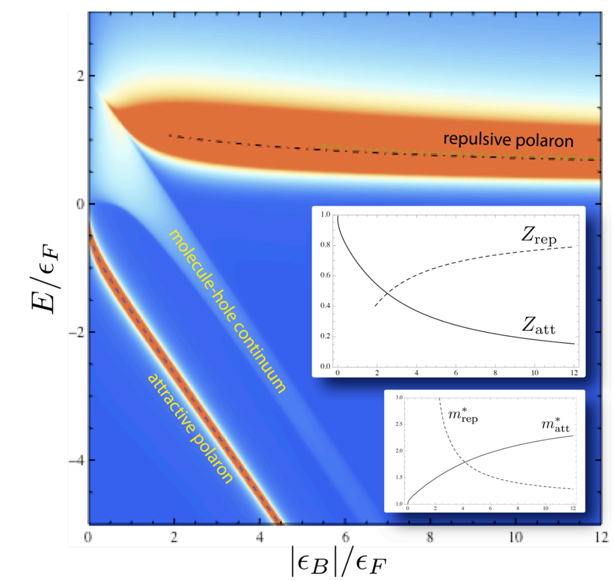
Schmidt, Enss & Pietilä, Demler; in prep.

outlook

- Fermi polaron in two dimensions (experiments: Cambrige & MIT)

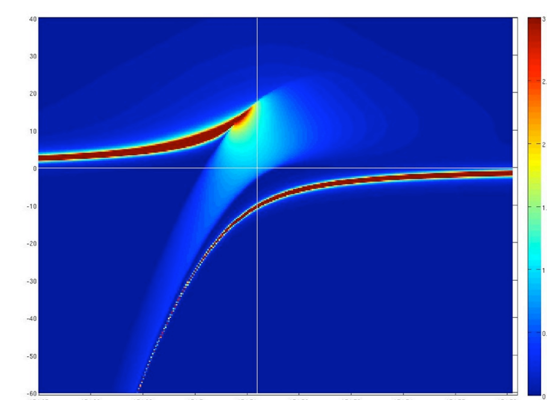






Schmidt, Enss & Pietilä, Demler; in prep.

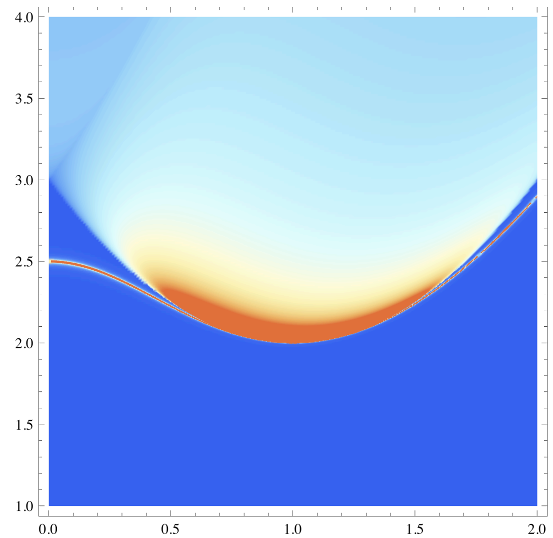
- rf spectroscopy in Li-K system (Innsbruck)
  effective range corrections

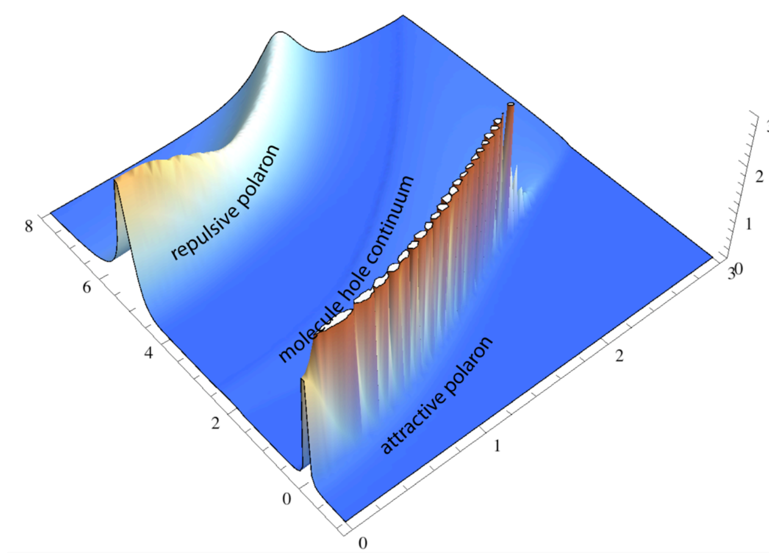


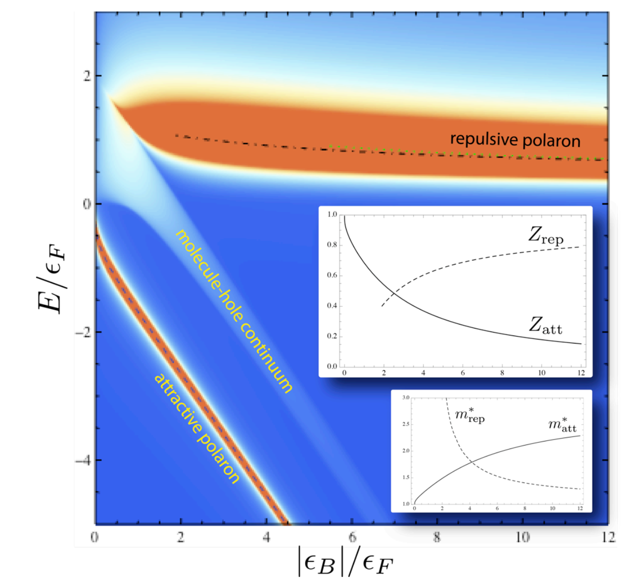
Schmidt (unpubl.)

outlook

- Fermi polaron in two dimensions (experiments: Cambrige & MIT)

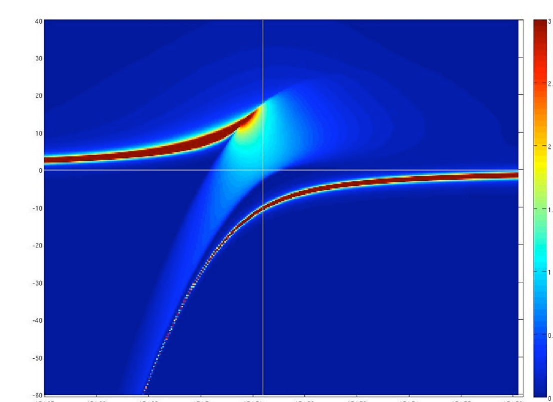






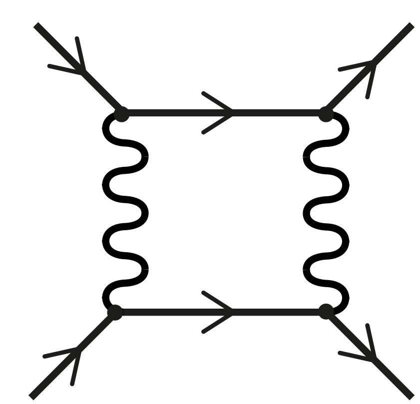
Schmidt, Enss & Pietilä, Demler; in prep.

- rf spectroscopy in Li-K system (Innsbruck)
  effective range corrections



Schmidt (unpubl.)

- competition between ferromagnetism and p-wave superfluidity



The End