coarse-graining, thermodynamics and quantum gravitation

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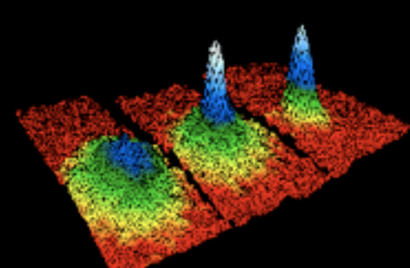
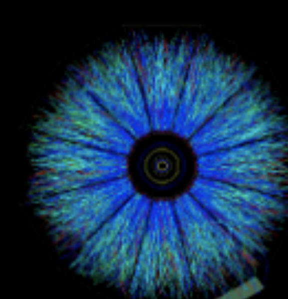
Renormalization Group Approach from Ultra Cold Atoms to the Hot QGP

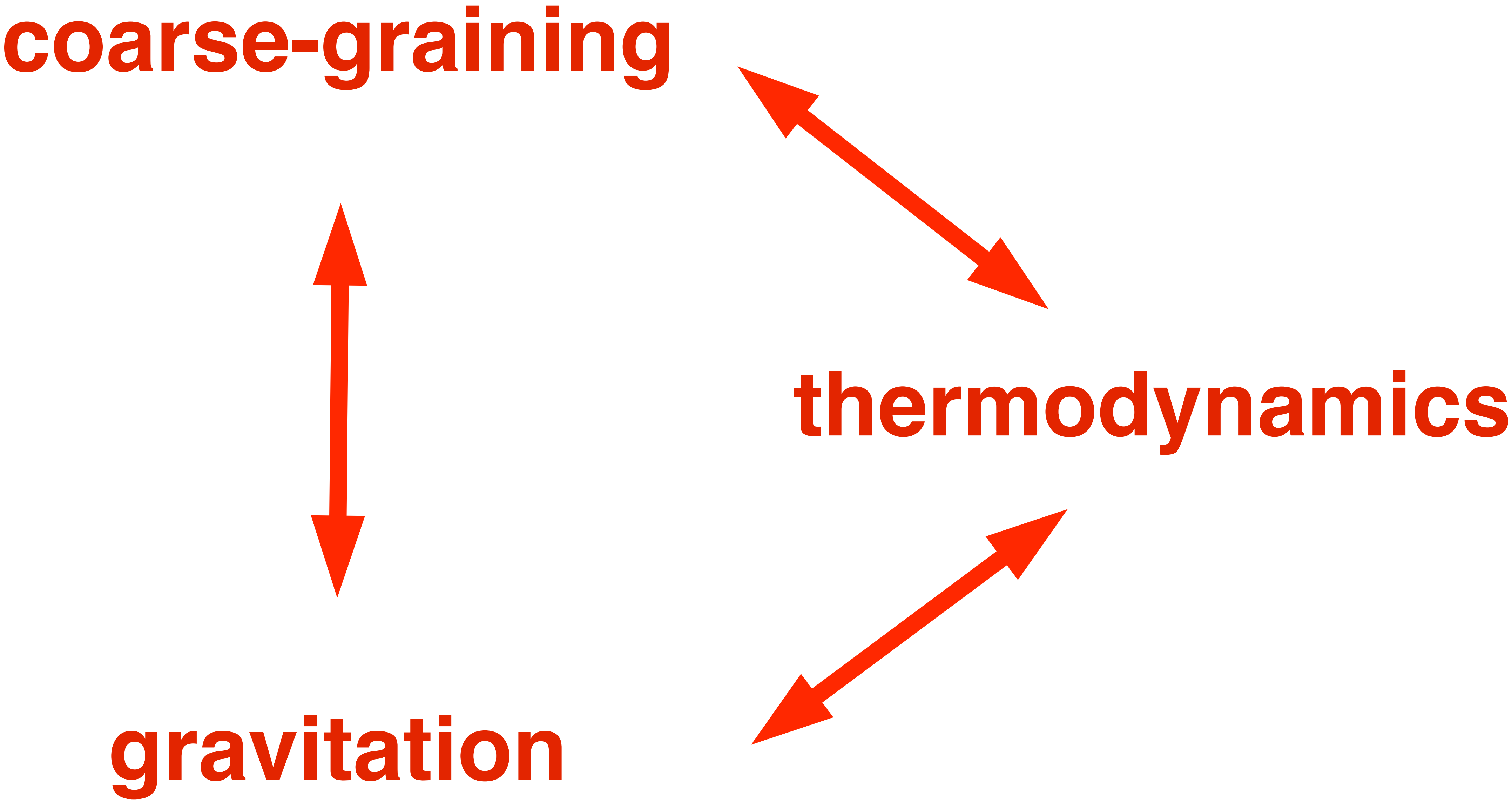


BEC picture from NIST

August 22 – September 9, 2011
Yukawa Institute for Theoretical Physics



HIC picture from BNL


coarse-graining
thermodynamics
gravitation

black hole thermodynamics

black holes in general relativity

solutions to Einstein's classical equations
event horizon with area **A**

most general static BH solution for long-ranged forces:
Kerr Newman black holes

uniqueness theorem:

$$A = A(M, J, q)$$

singularities / break-down of predictivity
cosmic censorship ?

black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking '73)

infinitesimal amount of matter crossing the horizon

$$\delta M = \frac{\kappa}{8\pi\, G_N}\,\delta A + \Omega\,\delta J + \Phi\,\delta q$$

first law of thermodynamics

$$\delta U = \delta Q + \sum_i \mu_i\,\delta N_i$$

with

$$U \leftrightarrow M \qquad \mu_i \leftrightarrow \{\Omega, \Phi\}$$

$$\delta Q \leftrightarrow \frac{\kappa}{8\pi G_N}\,\delta A \qquad N_i \leftrightarrow \{J, q\}$$

black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking '73)

infinitesimal amount of matter crossing the horizon

$$\delta M = \frac{\kappa}{8\pi\, G_N}\, \delta A + \Omega\, \delta J + \Phi\, \delta q$$

reversible thermodynamical process at temperature **T**

$$\frac{\delta Q}{T} = \delta S$$

second law of thermodynamics

$$S \propto A$$

(Bekenstein '73)

$$T = \frac{\kappa}{2\pi}$$

(Hawking '75)

black hole thermodynamics

laws of black hole mechanics

(Bardeen, Carter, Hawking '73)

infinitesimal amount of matter crossing the horizon

$$\delta M = \frac{\kappa}{8\pi\, G_N}\, \delta A + \Omega\, \delta J + \Phi\, \delta q$$

identify the heat flow at temperature **T** as

$$\delta Q = \delta M - \Omega\, \delta J - \Phi\, \delta q$$

to find that the **first law of thermodynamics** leads to

$$S_{\rm BH} = \frac{A}{4\, G_N}$$

derivation from euclidean path integral

statistical entropy

(Gibbons, Hawking '77)

coarse-graining

(Falls, Litim '11)

quantum gravitation

challenge for any UV completion of gravity:
identify the underlying coarse-grained degrees of freedom

flowing effective action

(Wetterich '93)

$$\Gamma_k = \int d^4x \sqrt{-g} \left[\frac{1}{8\pi G_k} R + \frac{1}{4\alpha_k} F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{\rm m} \right] .$$

IR limit $\quad G_0 \approx 6.674 \times 10^{-11}\ \mathrm{N}\,(\mathrm{m}/\mathrm{kg})^2 \qquad \alpha_0 \approx 1/137$

family of Kerr-Newman BH solutions

$$A = A(M, J, q; k) \qquad\qquad S = \frac{A}{4G_k}$$

coarse-graining

(Falls, Litim '11)

choice of scale

"not too coarse- and not too fine-grained" $$k = k_{\rm opt}(M, J, q)$$

RG thermodynamics

infinitesimal amount of matter crossing the horizon, with heat flow

$$\frac{\delta Q}{T} = \frac{\delta A}{4G_k}$$

BH settles in a new state

$$M \to M + \delta M \quad J \to J + \delta J$$
$$q \to q + \delta q \quad k_{\rm opt} \to k_{\rm opt} + \delta k_{\rm opt}$$

total change of horizon area

$$\delta A = \frac{2\pi}{\kappa} T\,\delta A + \left.\frac{\partial A(M, J, q; k)}{\partial k}\right|_{k=k_{\rm opt}} \delta k_{\rm opt}$$

coarse-graining

(Falls, Litim '11)

choice of scale

"not too coarse- and not too fine-grained" $k = k_{\rm opt}(M, J, q)$

results

optimal scale $$k^2_{\rm opt}(M, J, q) \equiv k^2_{\rm opt}(A) = \frac{4\pi\xi^2}{A}$$

mass function $$M^2 \equiv \frac{4\pi}{A}\left[\left(\frac{A + 4\pi G(A)e^2(A)q^2}{8\pi G(A)}\right)^2 + J^2\right]$$

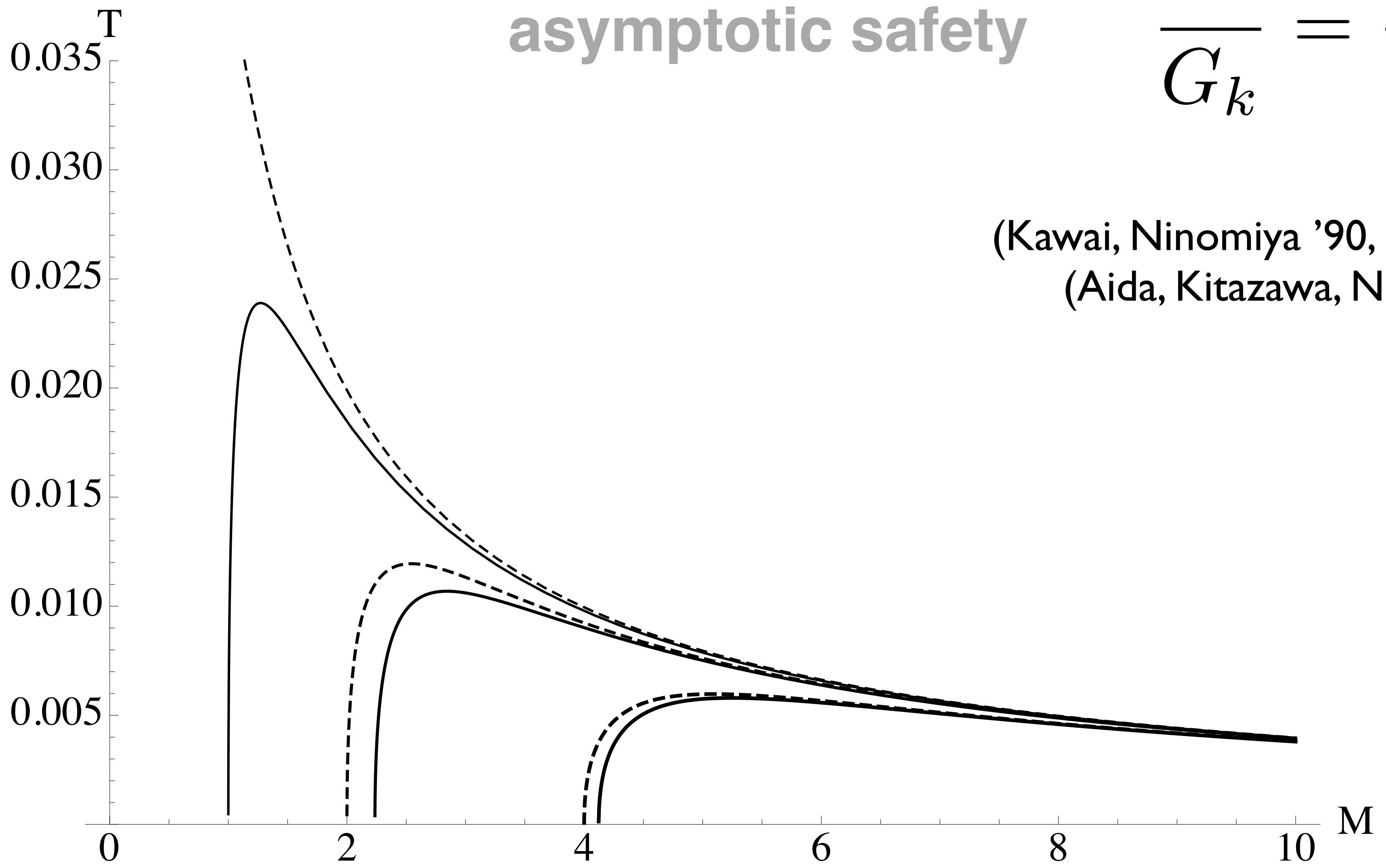
temperature $$T = 4G(A)\,\frac{\partial M}{\partial A}$$

entropy $$S = \frac{A}{4G_k}$$ with $k = k_{\rm opt}$

asymptotically safe black holes

(Falls, Litim '11)

RG thermodynamics



asymptotic safety

$$\frac{1}{G_k} = \frac{1}{G_N} + \frac{k^2}{g_*}$$

(Weinberg '79)

(Kawai, Ninomiya '90, Kawai, Kitazawa,Ninomiya '92)

(Aida, Kitazawa, Ninomiya '94, Aida, Kitazawa,'96)

(Reuter '96)

(Souma '99)

outer/inner horizon

$$A_\pm = 4\pi G_N \left[2G_N M^2 - \frac{1}{g_* \xi^2} \pm 2\sqrt{G_N^2 M^4 - J^2 - \frac{G_N M^2}{g_* \xi^2}} \right]$$

RG improved metrics

basic idea

replace $G_N \to G(r, \cdots)$ using $k \to k(r, \cdots)$ (Bonanno, Reuter '01)

(Reuter, Tiuran '10)

(Litim, Nikolakopoulos, ERG '10)

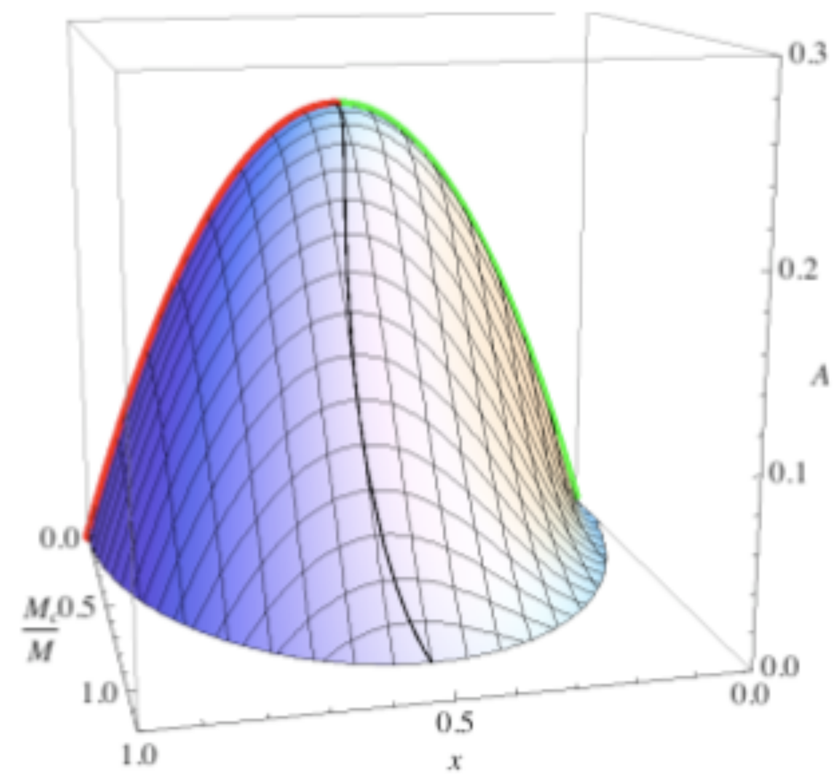
Kerr space-time

$$ds^2 = -\left(1 - \frac{2G(r)Mr}{\rho^2}\right) dt^2 - \frac{G(r)Mar\, s^2}{\rho^2} dtd\phi$$
$$+\frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{s^2}{\rho^2}\left[(r^2+a^2)^2 - a^2 \Delta\, s^2\right] d\phi^2$$

with
$$\Delta(r) = r^2 - 2G(r)Mr + a^2$$
$$\rho^2 = r^2 + a^2 \cos^2\theta$$

horizons

solve $\Delta(r_h) = 0$



RG improved metrics

thermodynamical equivalence

(Falls, Litim '11)

provided if

$$k^2(r) = \frac{\xi^2}{r^2 + (J/M)^2}$$

statistical entropy

"off-shell" conical singularity method (Soludkhin '96)

Schwarzschild black hole, free energy

improved metric

$$F = \frac{r_+}{2G_N} - \frac{A}{4G_N} T$$

(Bonanno, Reuter '00)

here

$$F = M - S\,T$$

conclusion

black hole thermodynamics

RG puts black holes under a microscope
BH thermodynamics consistent and meaningful
beyond the semi-classical approximation

predictions

thermodynamical consistency + asymptotic safety implies
smallest black holes
maximal temperature

extensions + challenges

high-energy degrees of freedom?
cosmological constant, higher curvature invariants
non-equilibrium thermodynamics