Coarse-Graining and Resummation of Cosmological Perturbations

Massimo Pietroni - INFN Padova

In collaboration with...

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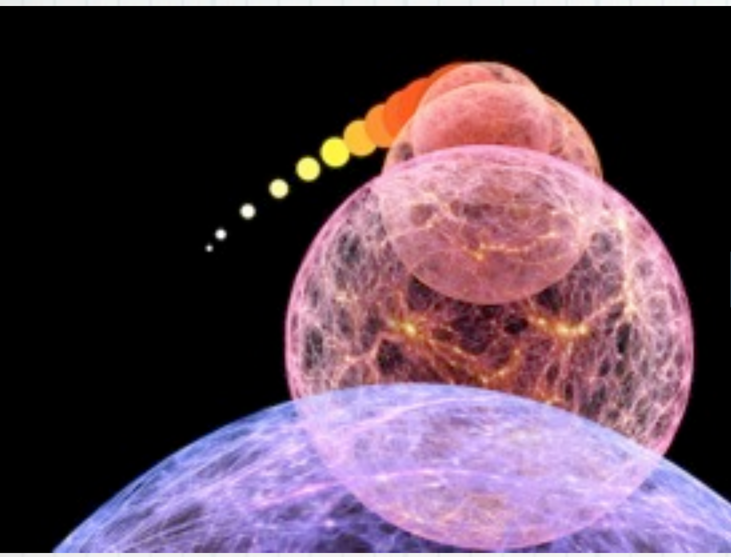
RG Workshop, Kyoto,
Sept 2nd, 2011

Understanding the LSS of the Universe

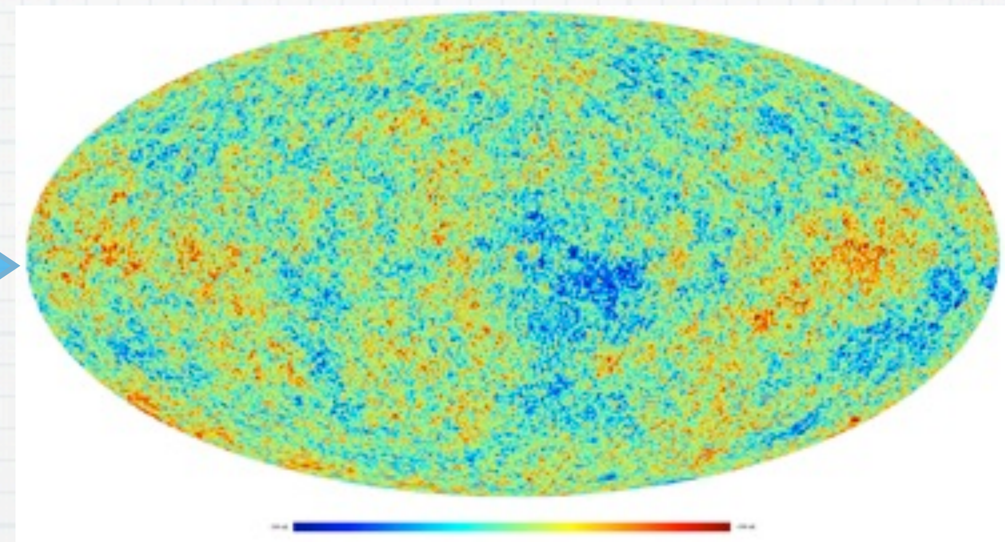
Inflation



Linear, Gaussian

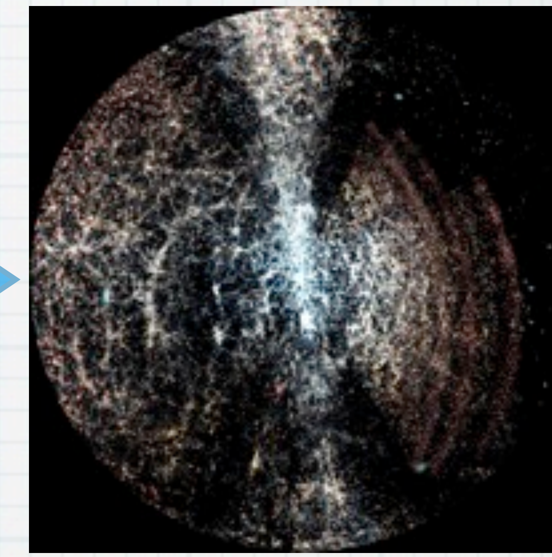
$$\left(\frac{\delta\rho}{\rho} \simeq 10^{-5}\right)$$

Decoupling



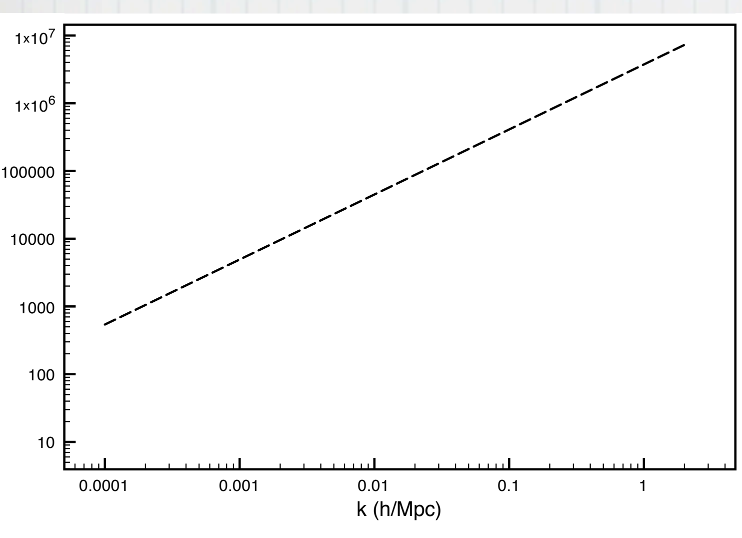
Linear, Gaussian

Today

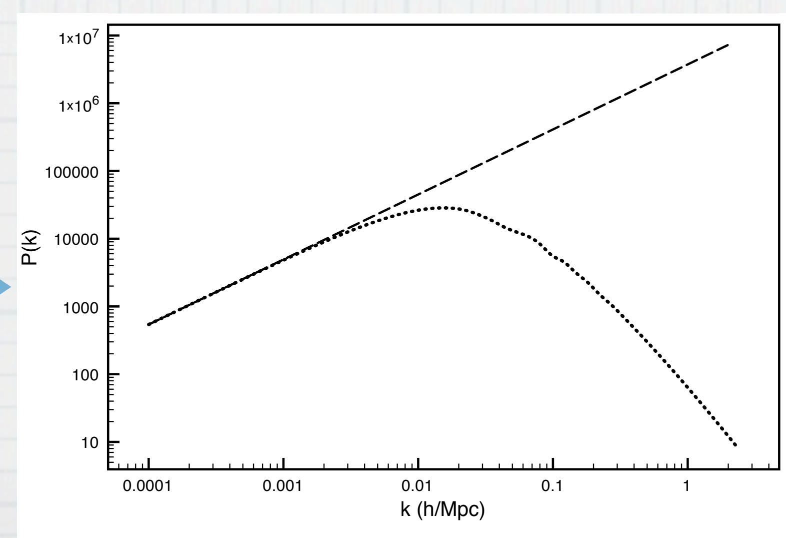


non-Linear, non-Gaussian

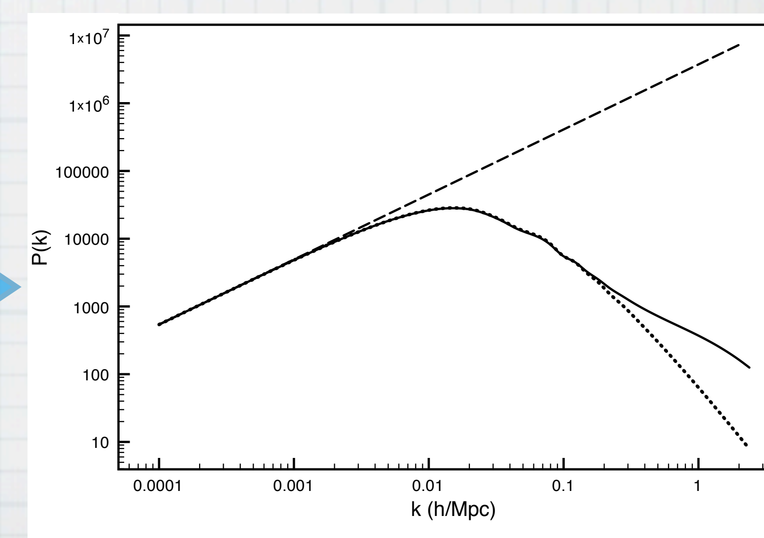
primordial density perturbations

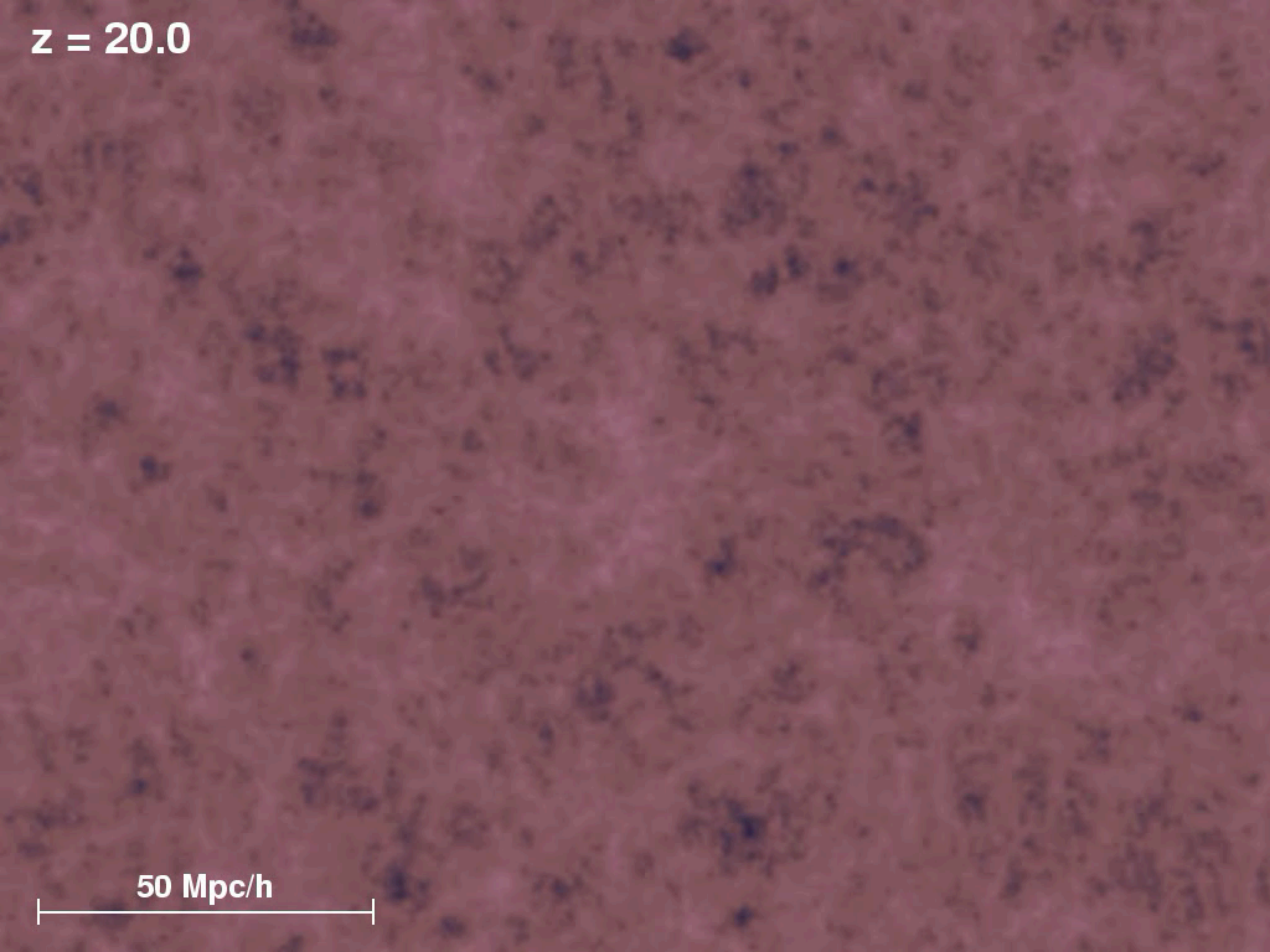


photon-baryon-DM-neutrino....fluid

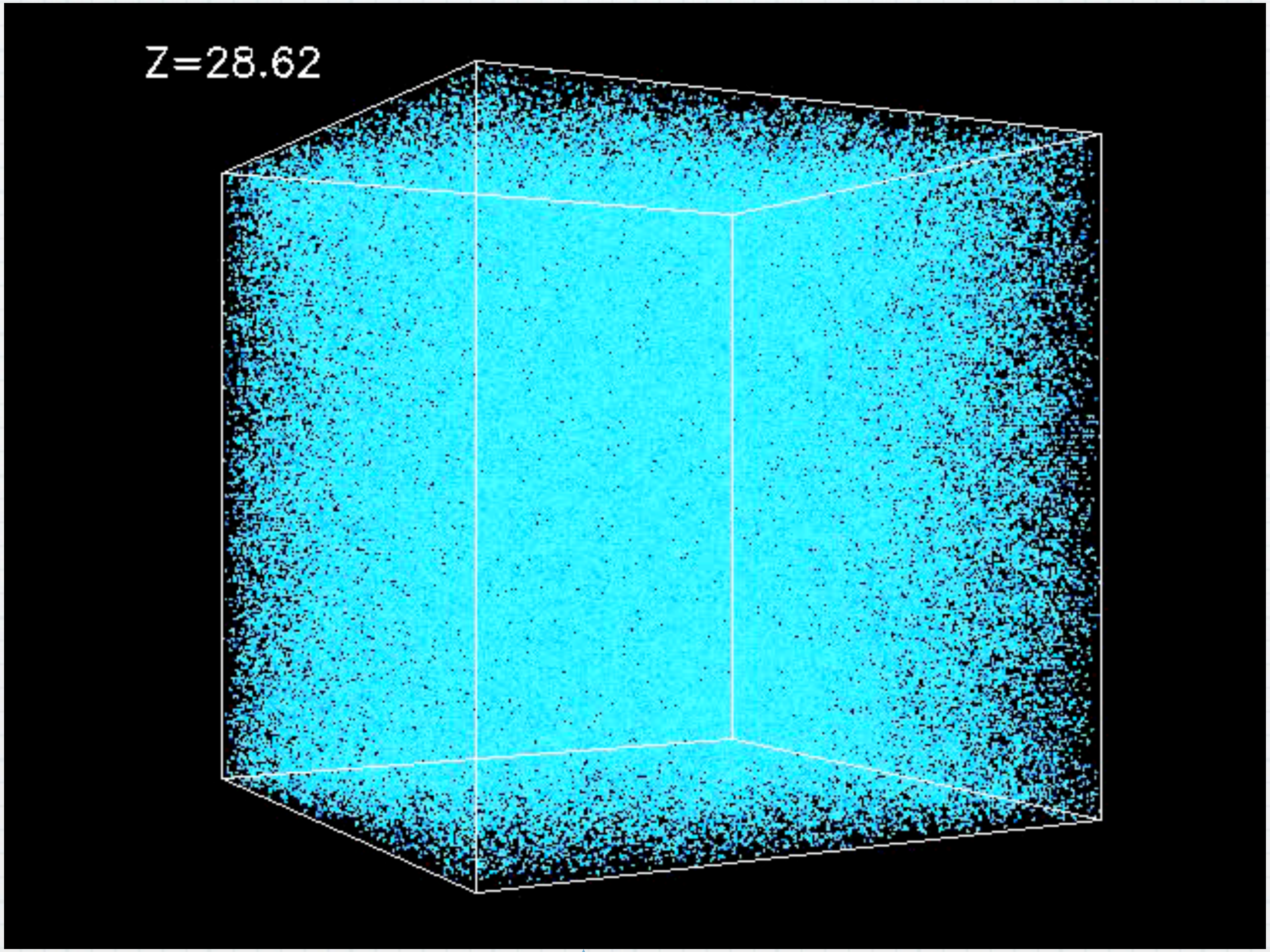


non-rel. matter




z = 20.0
50 Mpc/h

previous slide: Movie by Volker Springel
(Millennium Simulation)



Movie by Kravtsov, Klypin
(National Center for Supercomputer applications)

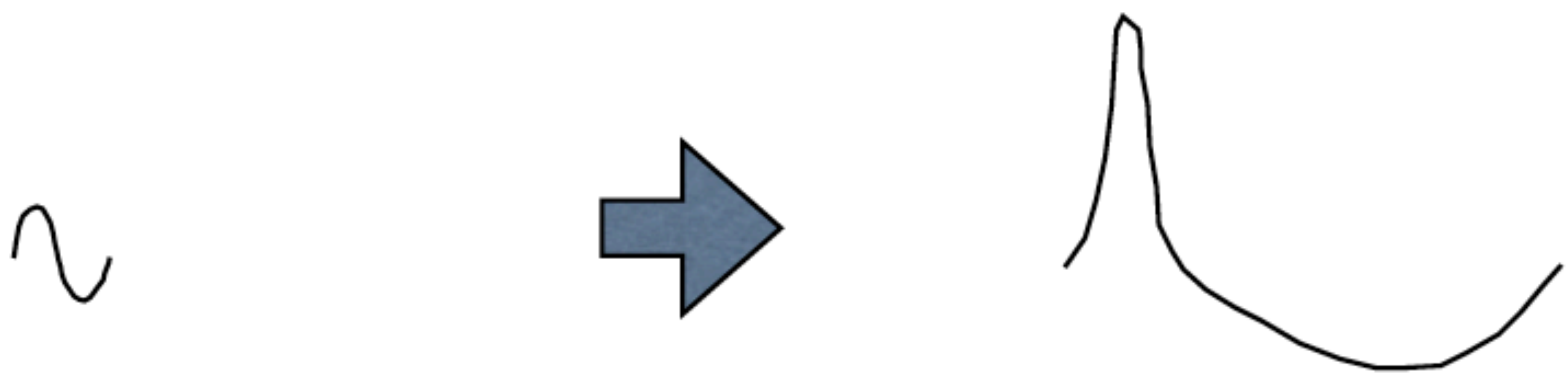
Nonlinear Evolution (Qualitative)

- Nonlinearity modifies the power spectrum

- Creates Non-Gaussianity

Growth of perturbations: gravity vs. the expansion of the universe

- underdense regions: expansion wins
- overdense regions: gravity wins



Small Gaussian Fluctuations

Non-Gaussian Fluctuations

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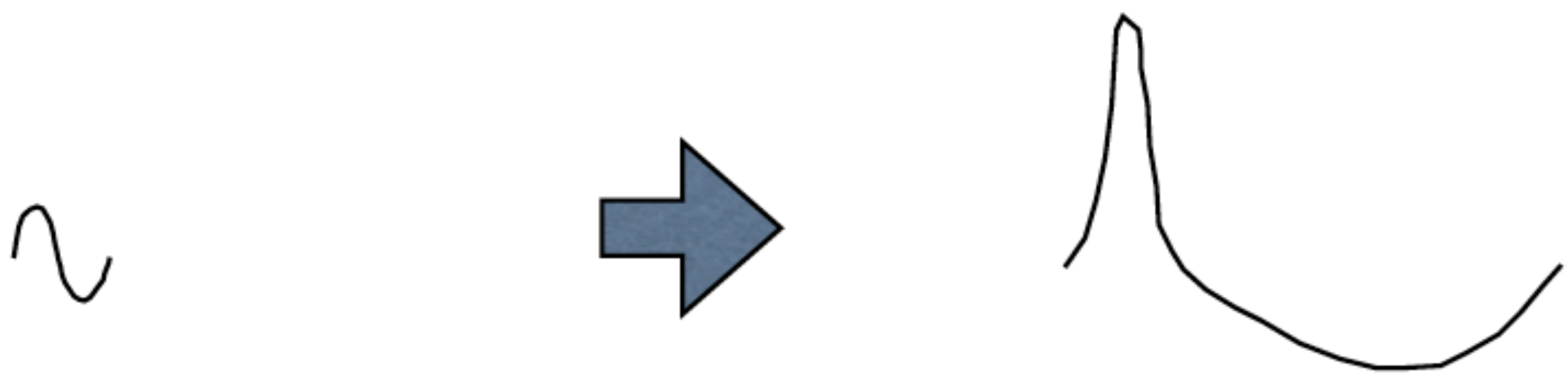


Small Gaussian Fluctuations

Non-Gaussian Fluctuations

Movie by R. Scoccimarro

Why do we need to study the late (and non-linear) evolution?

* Dark Energy (Baryonic Acoustic Oscillations)
* neutrino masses
* Non-Gaussianity
* Weak gravitational lensing
* ...

The future of precision cosmology: non-linear scales

matter density

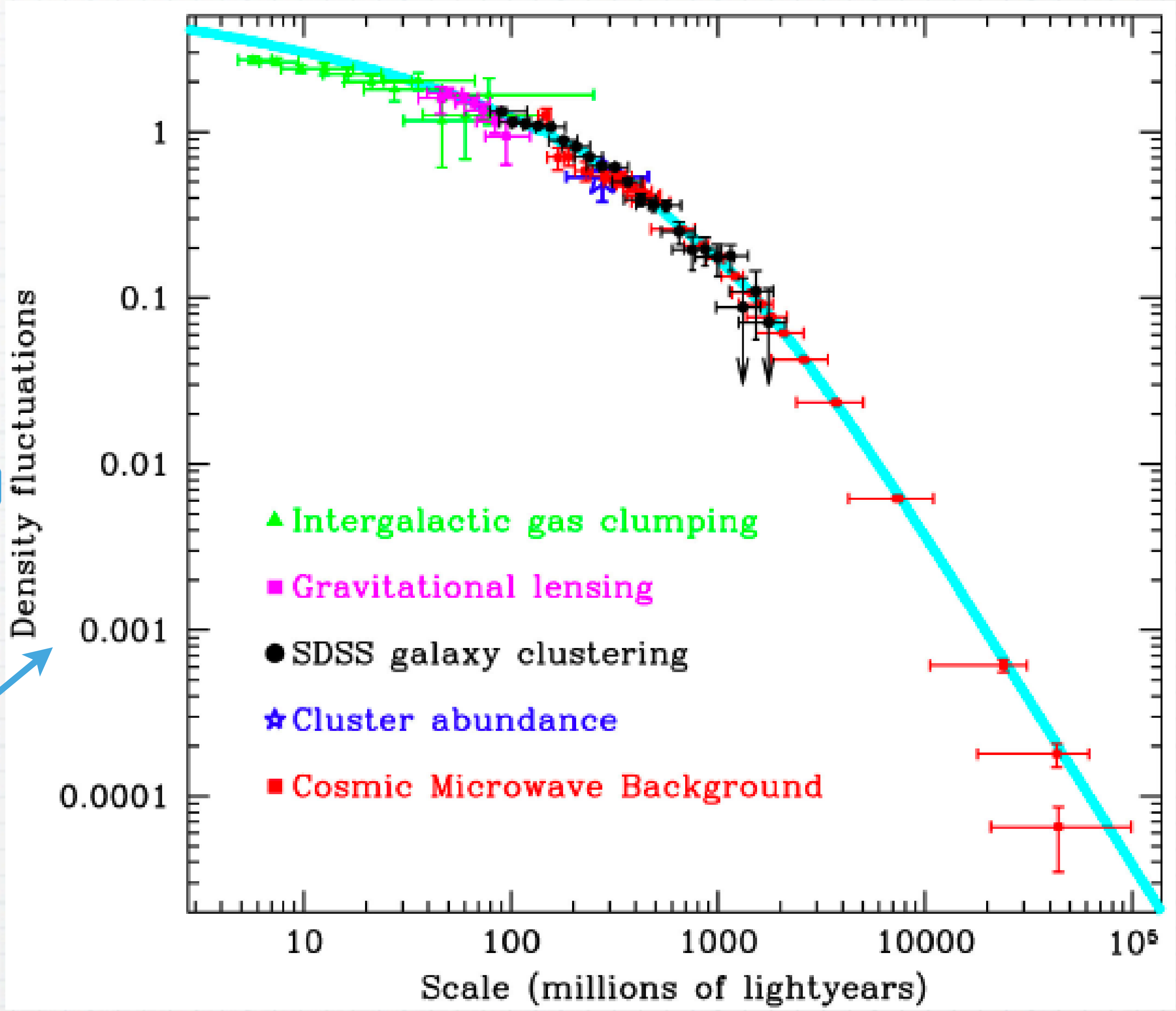
$$\rho(\mathbf{x}, \tau) \equiv \bar{\rho}(\tau)[1 + \delta(\mathbf{x}, \tau)]$$

power spectrum

$$\langle \delta(\mathbf{k}, \tau)\, \delta(\mathbf{k}', \tau) \rangle = P(k, \tau) \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

`size´ of the fluctuations at different scales/epochs:

$$\Delta^2(k, \tau) = 4\pi k^3 P(k, \tau)$$



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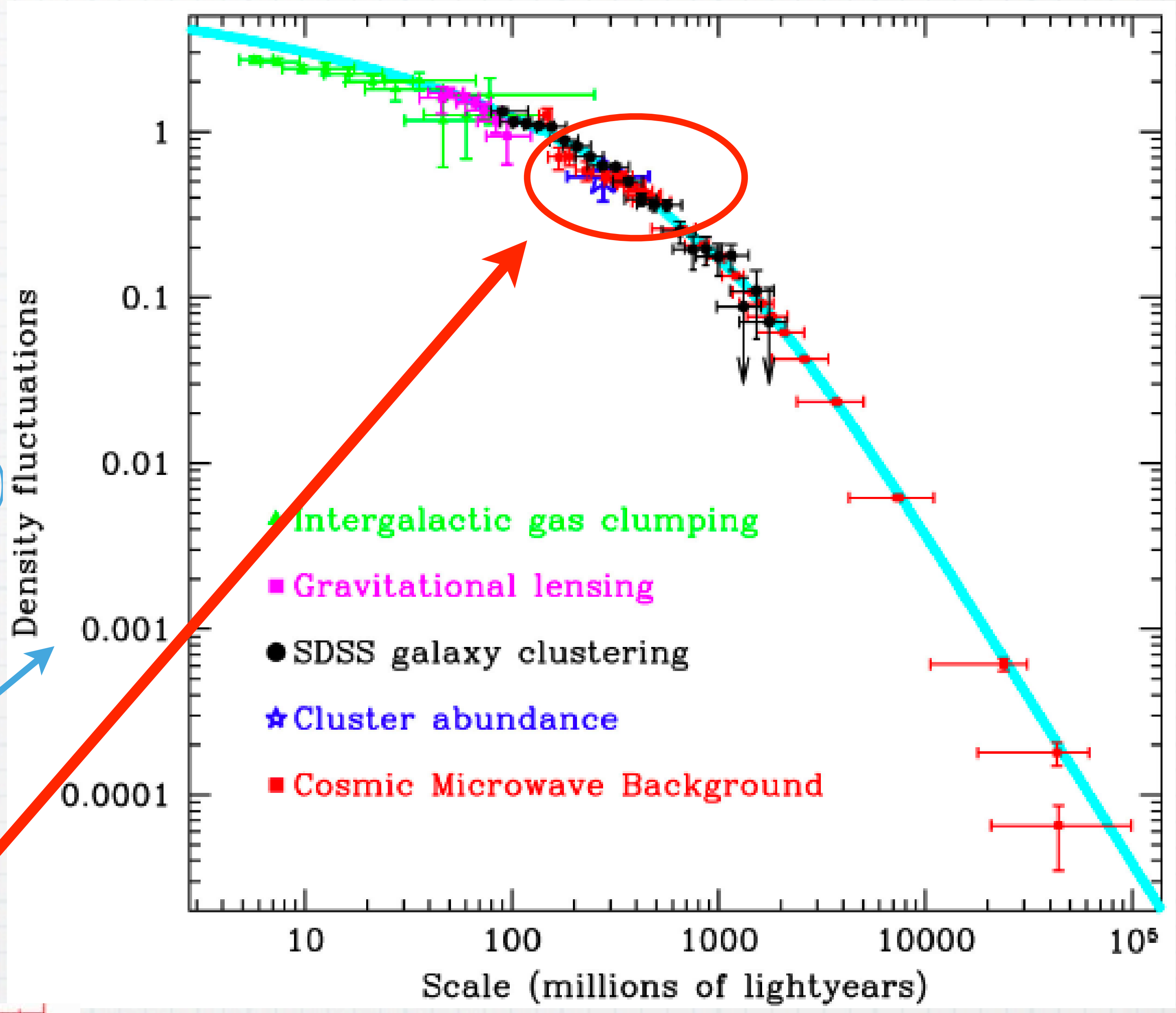
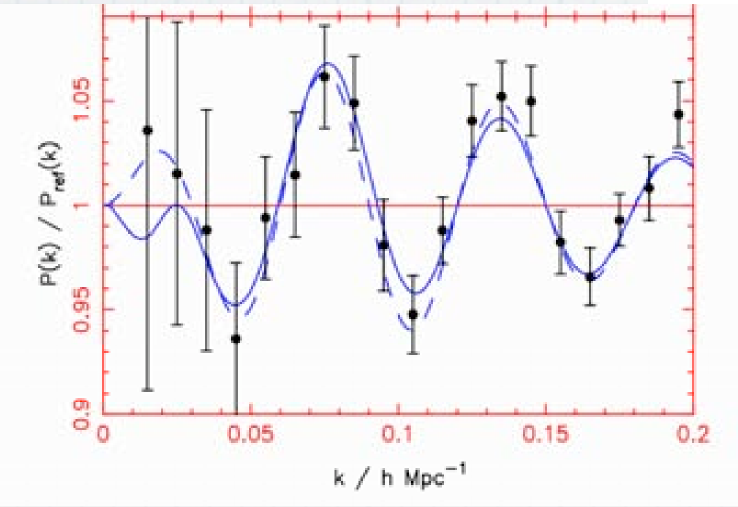


Baryonic Acoustic Oscillations (BAO)



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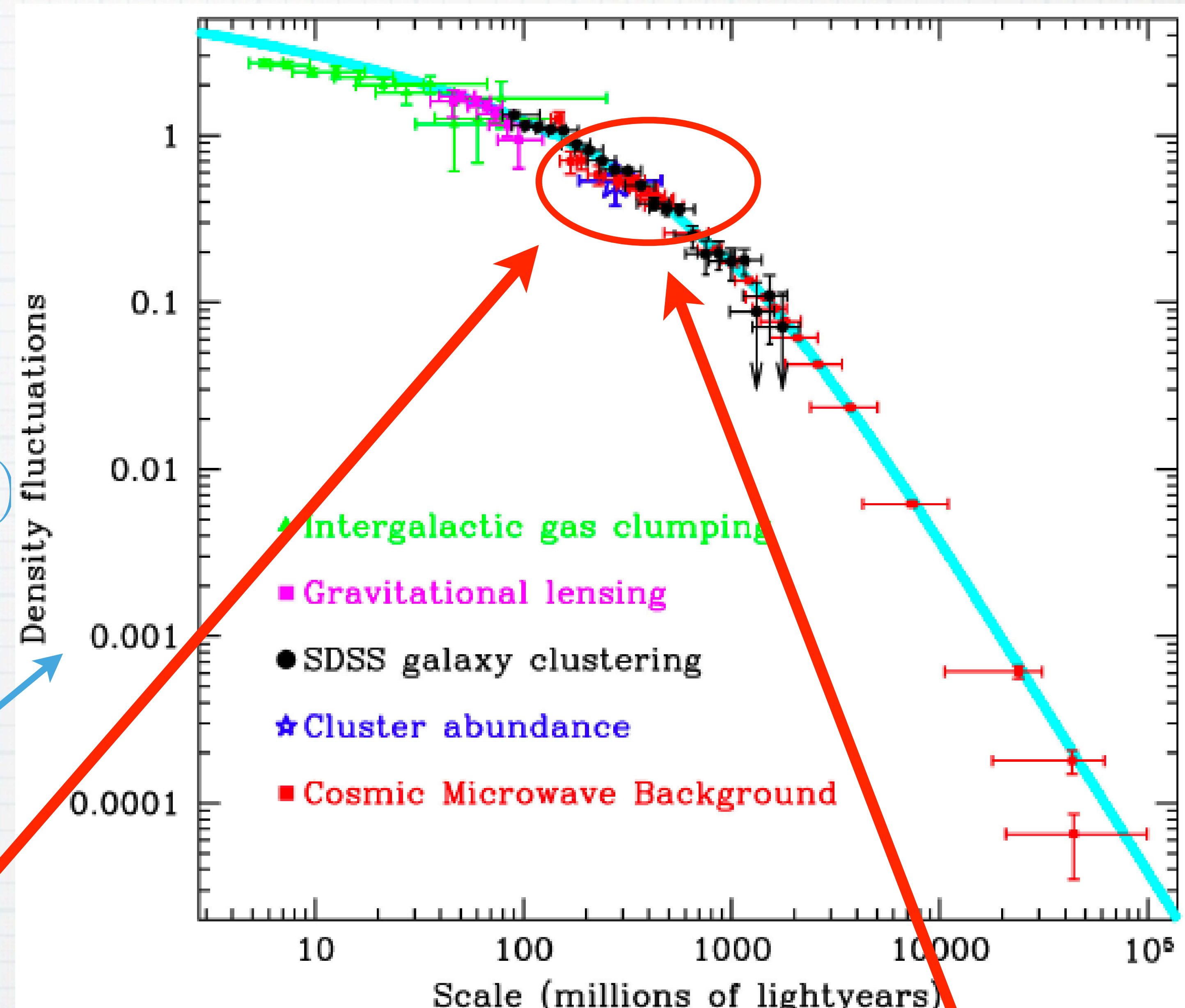

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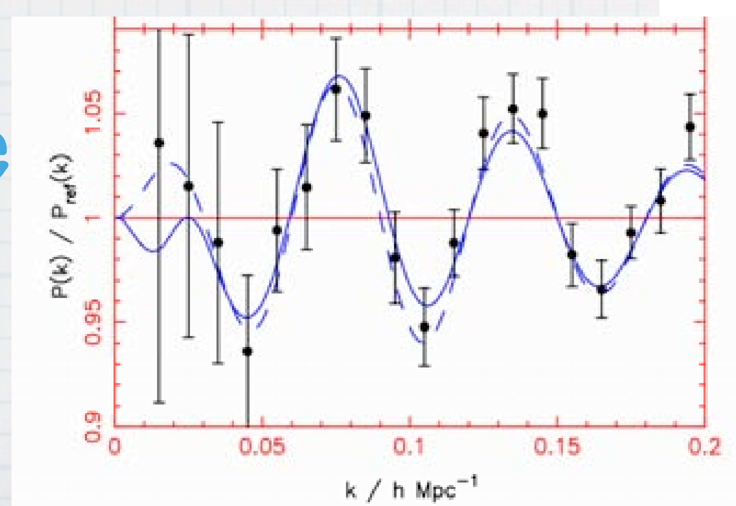
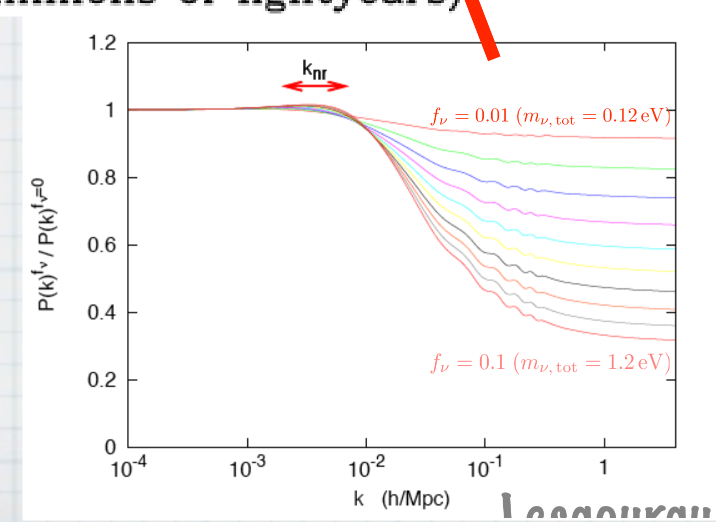
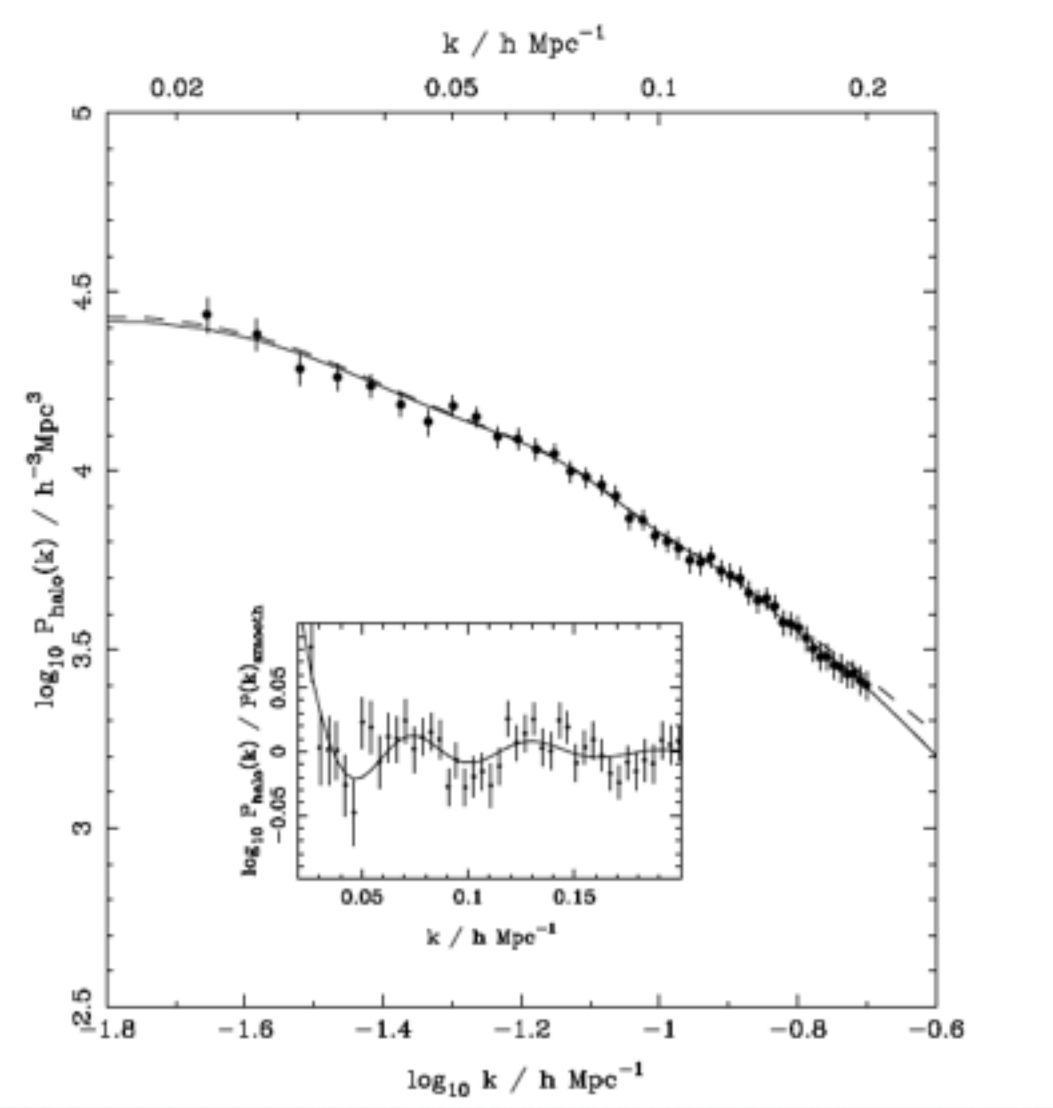
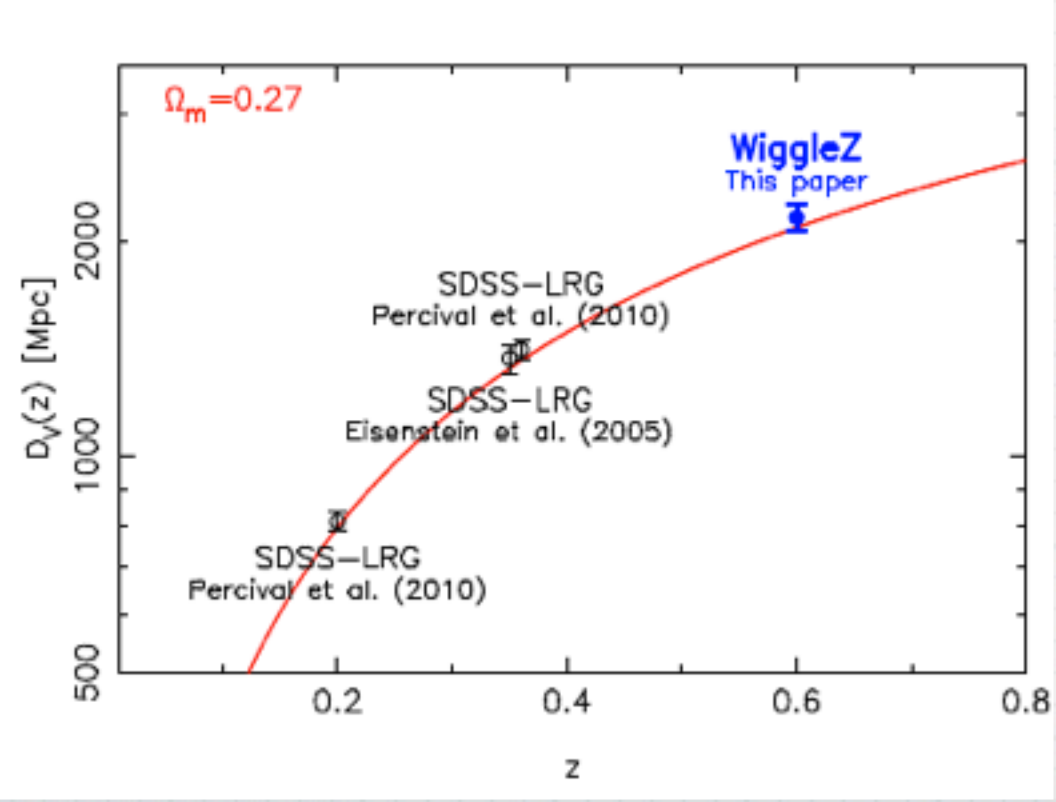


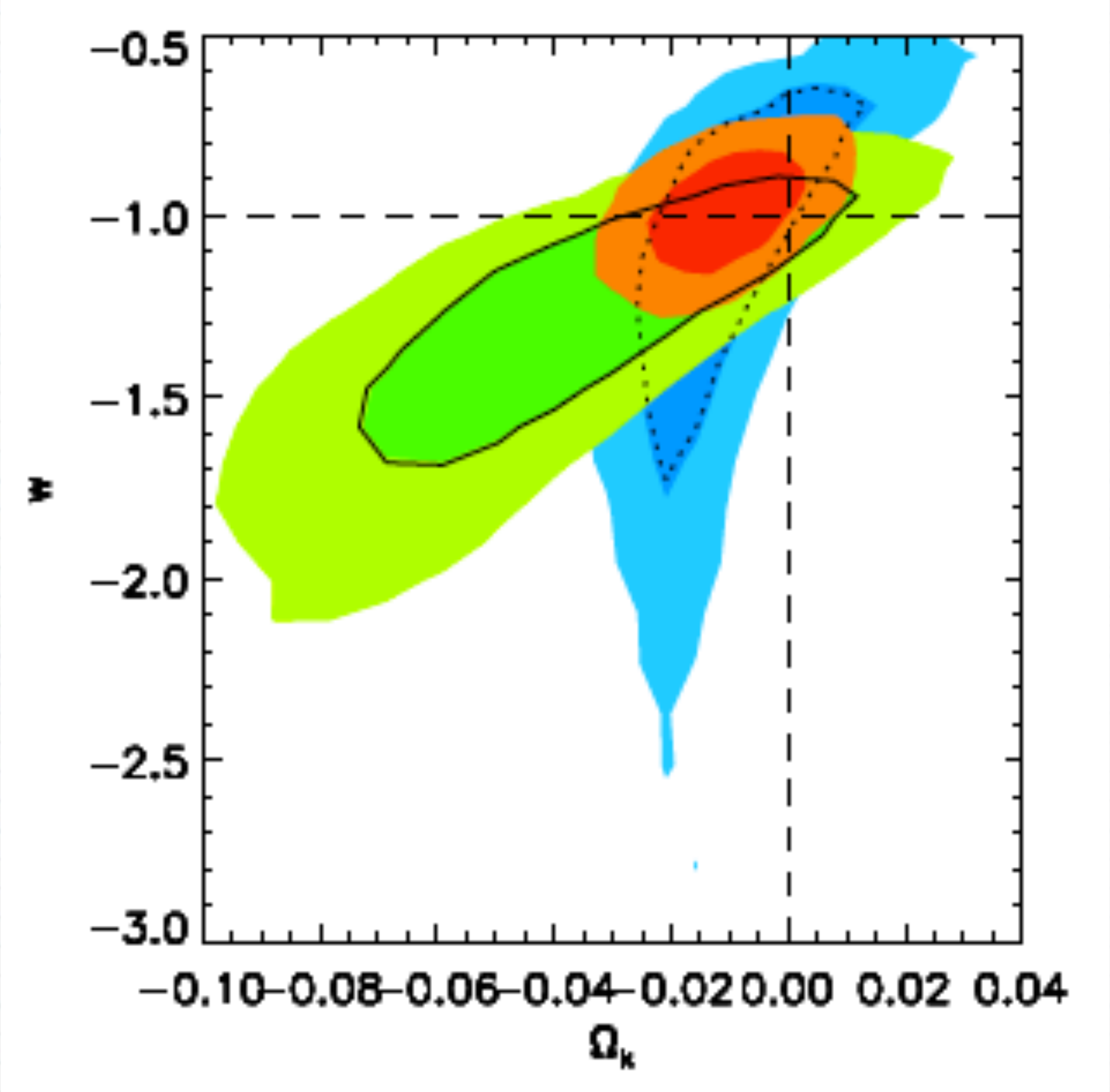
Neutrino mass bounds



the current situation...



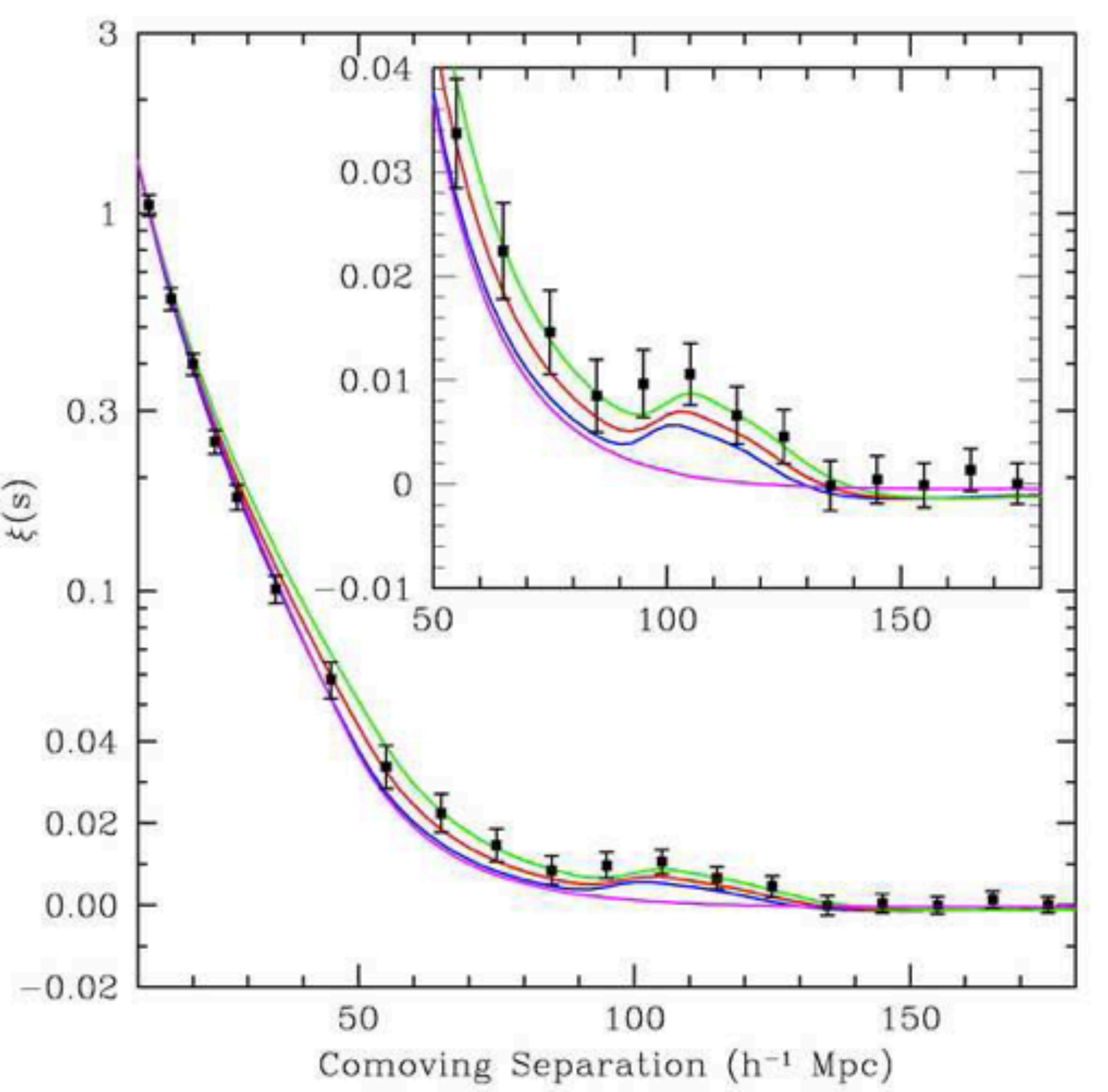


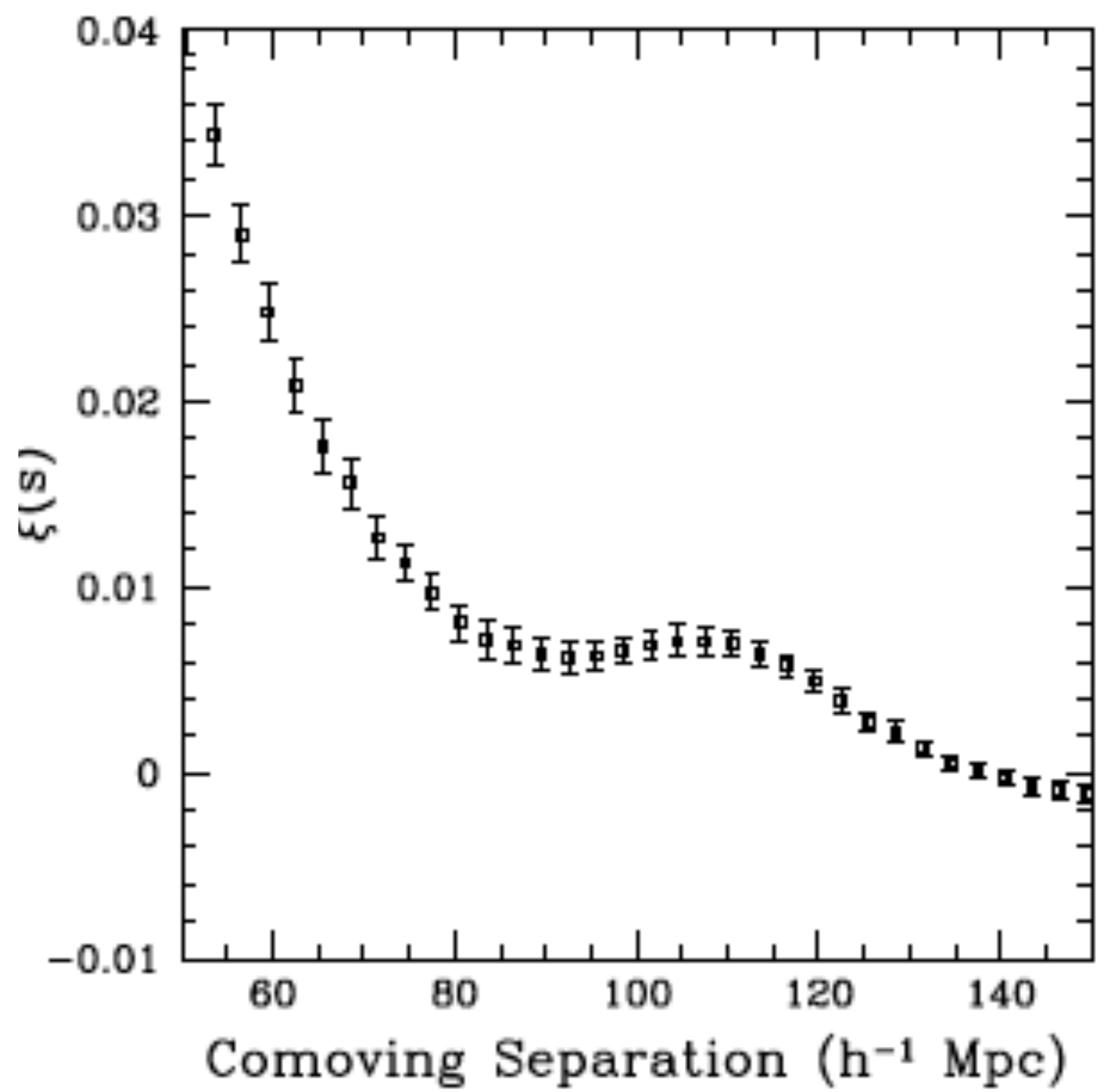


... and constraints on cosomological parameters

from SDSS, Reid et al. ('09)

future surveys



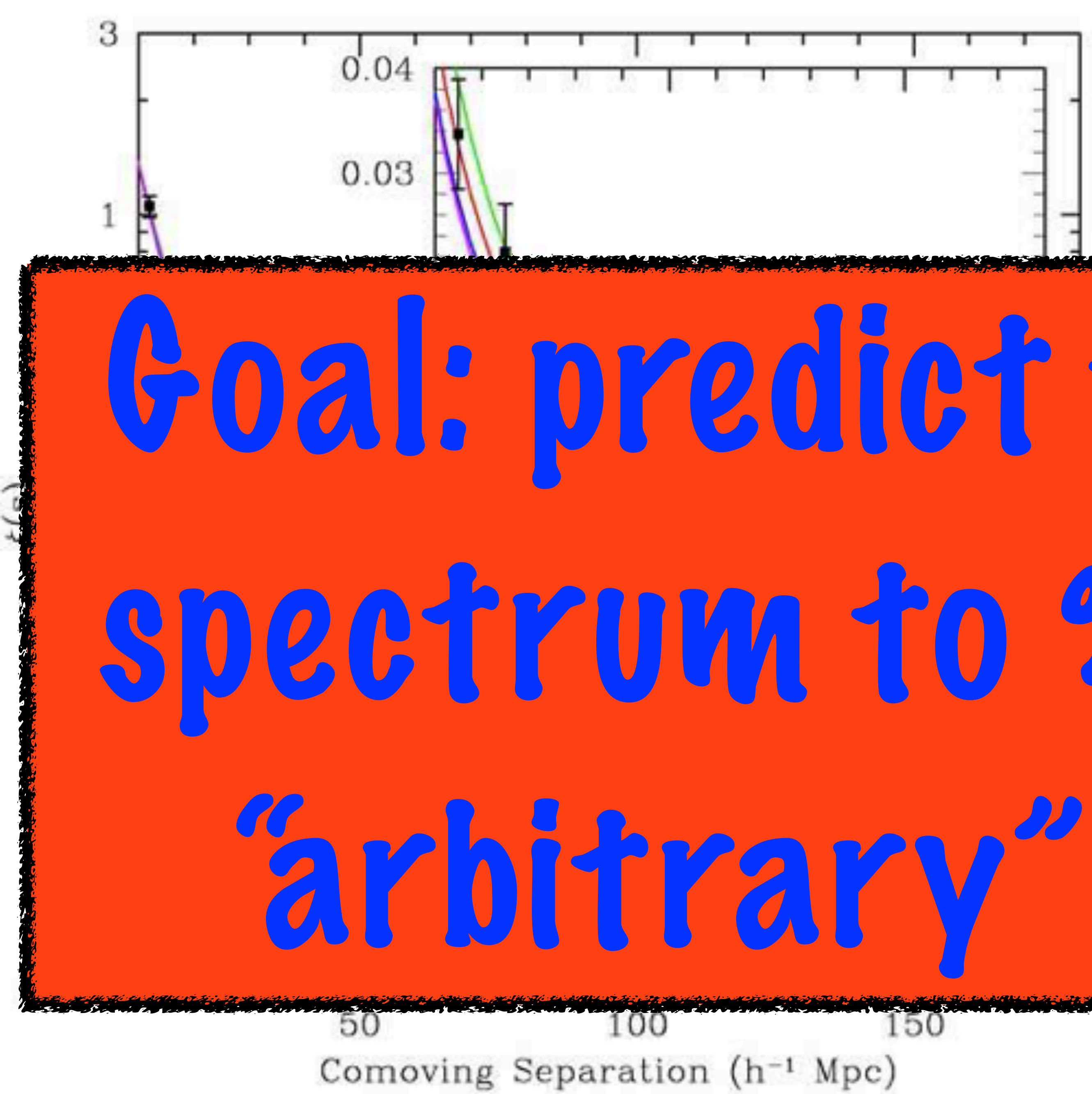


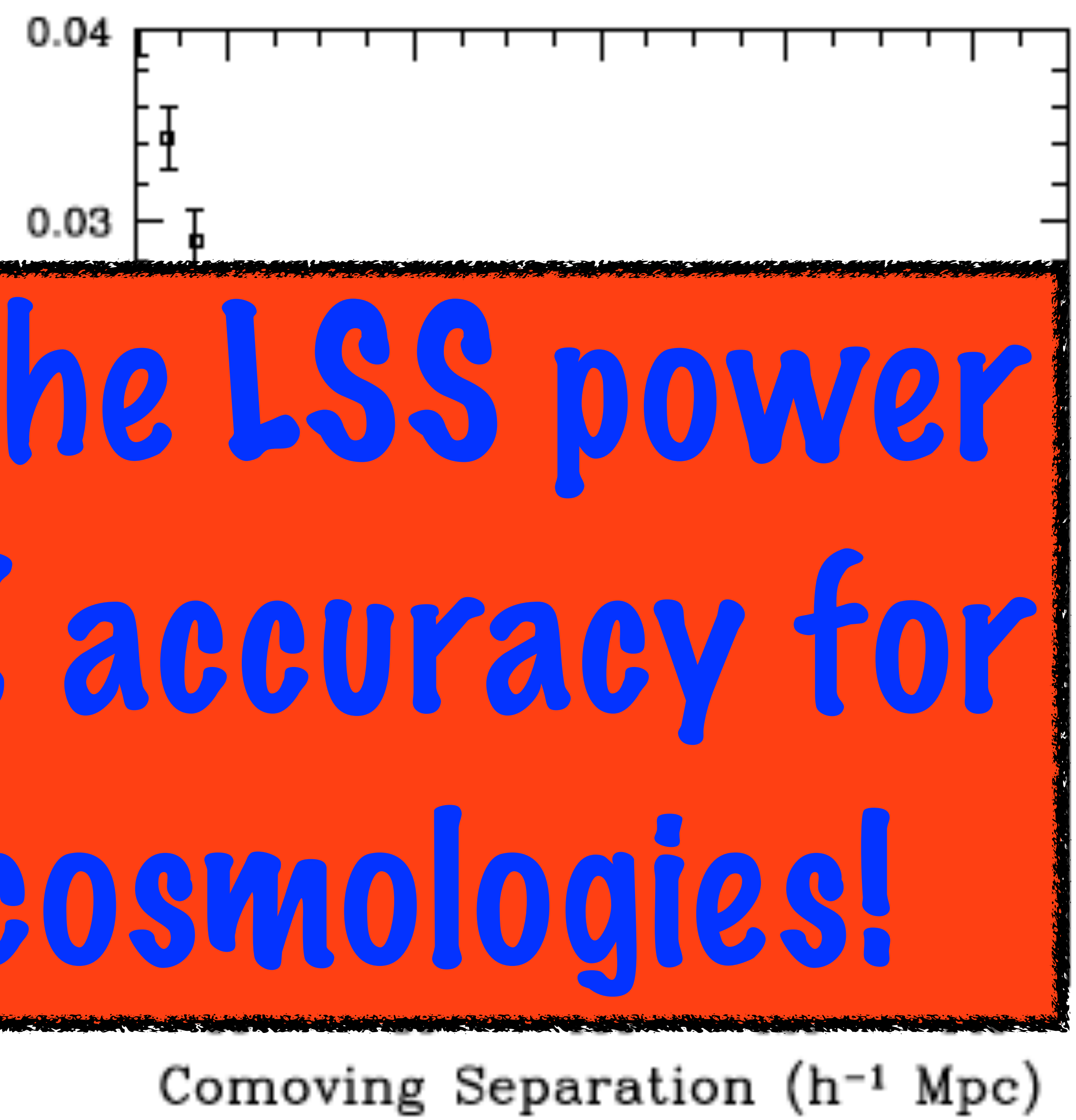
Now

Boss

1.5M of luminous
red galaxies up to $z < 0.8$

future surveys





Goal: predict the LSS power spectrum to % accuracy for "arbitrary" cosmologies!

Now

Boss

1.5M of luminous
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Why do we need new methods?

Linear perturbation theory

badly fails for $z<2$-3 and $k>0.05h/\mathrm{Mpc}$

N-body simulations

In principle ok, but need large volumes/resolutions:

- practically impossible to scan over cosmological models;
- non-standard but interesting scenarios are problematic: (massive neutrinos, non-gaussianity, DE-DM coupling...)

Outline

* From Vlasov to Feynman
* Perturbation Theory and the need for resummations
* Different schemes. Momentum RG, Time RG.
* Beyond Vlasov. Coarse-grained PT

Cosmo-notation

* scale factor: $a(\tau)$ (redshift: $z = a^{-1} - 1$)
* conformal time: $d\tau = dt/a$
* comoving momentum: $k = a\, k_{phys}$
* Hubble parameter: $\mathcal{H} = \frac{d \ln a}{d\tau}$
* critical density: $\rho_c = \frac{3\,\mathcal{H}^2}{8\pi a^2 G_N}$ $(\simeq 10^{-29} g\, cm^{-3}$ today)
* density parameter: $\Omega_i \equiv \frac{\rho_i}{\rho_c}$

Self-Gravitating Matter

The matter particle distribution function, $f(\mathbf{x}, \mathbf{p}, \tau)$, obeys the **Vlasov equation:**

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \nabla f - am\nabla\phi \cdot \nabla_{\mathbf{p}} f = 0$$

with $p = am\frac{d\mathbf{x}}{d\tau}$ and $\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$

sub-horizon scales, Newtonian gravity

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Taking moments,

$$\int d^3\mathbf{p}\, f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau) \equiv \bar{\rho}\,[1+\delta(\mathbf{x},\tau)]$$

$$\int d^3\mathbf{p}\,\frac{p_i}{am} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau)v_i(\mathbf{x},\tau)$$

$$\int d^3\mathbf{p}\,\frac{p_i\,p_j}{a^2m^2} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau)\,[v_i(\mathbf{x},\tau)v_j(\mathbf{x},\tau)+\sigma_{ij}(\mathbf{x},\tau)]$$

$$\ldots$$

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$$\cdots$$

and neglecting σ_{ij} and higher moments (single stream approximation), one gets...

Fluid equations for Cold Dark Matter

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$

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In Fourier space, (defining $\theta(\mathbf{x},\tau) \equiv \nabla\cdot\mathbf{v}(\mathbf{x},\tau)$),

$$\frac{\partial\,\delta(\mathbf{k},\tau)}{\partial\,\tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{k_1} d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\alpha(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\delta(\mathbf{k_2},\tau) = 0$$

$$\frac{\partial\,\theta(\mathbf{k},\tau)}{\partial\,\tau} + \mathcal{H}\,\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_M\mathcal{H}^2\delta(\mathbf{k},\tau) + \int d^3\mathbf{k_1} d^3\mathbf{k_2}\,\delta_D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2})\beta(\mathbf{k_1},\mathbf{k_2})\theta(\mathbf{k_1},\tau)\theta(\mathbf{k_2},\tau) = 0$$

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mode-mode coupling controlled by:

$$\alpha(\mathbf{k_1},\mathbf{k_2})\equiv\frac{(\mathbf{k_1}+\mathbf{k_2})\cdot\mathbf{k_1}}{k_1^2}$$

$$\beta(\mathbf{k_1},\mathbf{k_2})\equiv\frac{|\mathbf{k_1}+\mathbf{k_2}|^2(\mathbf{k_1}\cdot\mathbf{k_2})}{2k_1^2k_2^2}$$

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Notice: we Fourier transform only space, not time

linear approximation: $\alpha(\mathbf{k_1},\, \mathbf{k_2}) = \beta(\mathbf{k_1},\, \mathbf{k_2}) = 0$

no mode-mode coupling

$$\frac{\partial\, \delta(\mathbf{k}, \tau)}{\partial\, \tau} + \theta(\mathbf{k}, \tau) = 0$$

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$$\Omega_M = 1 \rightarrow \mathcal{H} \sim a^{-1/2}$$

$$\downarrow$$

$$\delta(\mathbf{k},\,\tau) = \delta(\mathbf{k},\,\tau_i)\left(\frac{a(\tau)}{a(\tau_i)}\right)^m$$

$$-\frac{\theta(\mathbf{k},\,\tau)}{\mathcal{H}} = m\,\delta(\mathbf{k},\,\tau)$$

$$m = \begin{cases} 1 & \text{growing mode} \\ -\frac{3}{2} & \text{decaying mode} \end{cases}$$

Compact Perturbation Theory

Crocce, Scoccimarro '05

Consider again the continuity and Euler equations

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0\,, \qquad \frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

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define $\begin{pmatrix} \varphi_1(\eta,\mathbf{k}) \\ \varphi_2(\eta,\mathbf{k}) \end{pmatrix} \equiv e^{-\eta} \begin{pmatrix} \delta(\eta\,,\mathbf{k}) \\ -\theta(\eta\,,\mathbf{k})/\mathcal{H} \end{pmatrix}$ with $\eta = \log\frac{a(\tau)}{a(\tau_i)}$ $\quad \mathbf{\Omega} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

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then we can write (we assume an EdS model):

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta,\mathbf{k}) = e^{\eta}\gamma_{abc}(\mathbf{k},\,-\mathbf{k_1},\,-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

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and the only non-zero components of the mode-mode coupling are

$$\gamma_{121}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \gamma_{112}(\mathbf{k_1},\,\mathbf{k_3},\,\mathbf{k_2}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\frac{\alpha(\mathbf{k_2},\,\mathbf{k_3})}{2}$$

$$\gamma_{222}(\mathbf{k_1},\,\mathbf{k_2},\,\mathbf{k_3}) = \delta_D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\,\beta(\mathbf{k_2},\,\mathbf{k_3})$$

An action principle

Matarrese, M.P., '07

An action principle

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The field equation can be derived by varying the **action**

$$S = \int d\eta_a d\eta_b \, \chi_a \, g^{-1}_{ab} \, \varphi_b - \int d\eta \, e^{\eta} \, \gamma_{abc} \, \chi_a \, \varphi_b \, \varphi_c$$

w.r.t. the auxiliary field $\chi_a(\eta, \mathbf{k})$

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$g_{ab}(\eta_1, \eta_2)$ is the retarded propagator: $(\delta_{ab}\partial_\eta + \Omega_{ab}) \, g_{bc}(\eta, \eta') = \delta_{ac} \, \delta_D(\eta - \eta')$

so that $\varphi^0_a(\eta, \mathbf{k}) = g_{ab}(\eta, \eta')\varphi^0_b(\eta', \mathbf{k})$ is the solution of the ***linear*** equation

An action principle

Matarrese, M.P., '07

The field equation can be derived by varying the **action**

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Explicitly, one finds: $\mathbf{g}(\eta_1, \eta_2) = \begin{cases} \mathbf{B} + \mathbf{A} \, e^{-5/2(\eta_1 - \eta_2)} & \eta_1 > \eta_2 \\ 0 & \eta_1 < \eta_2 \end{cases}$

$$\mathbf{B} = \frac{1}{5}\begin{pmatrix} 3 & 2 \\ 3 & 2 \end{pmatrix}$$

$$\mathbf{A} = \frac{1}{5}\begin{pmatrix} 2 & -2 \\ -3 & 3 \end{pmatrix}$$

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growing mode

Initial conditions: $\varphi^0_b(\eta', \mathbf{k}) \propto u_b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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growing mode

decaying mode

Initial conditions: $\varphi^0_b(\eta', \mathbf{k}) \propto u_b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad \begin{pmatrix} 1 \\ -3/2 \end{pmatrix}$

A generating functional

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$$Z[\mathbf{J},\,\mathbf{\Lambda}] \;=\; \int \mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int d\eta_1 d\eta_2\left[-\frac{1}{2}\,\chi\,\mathbf{g}^{-1}\mathbf{P^L}\mathbf{g^T}^{-1}\chi + i\,\chi\,\mathbf{g}^{-1}\,\varphi\right]\right.$$
$$\left. -i\int d\eta\,[\mathbf{e}^{\eta}\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi]\right\}$$

the initial conditions are encoded in the linear power spectrum:

$$P^L_{ab}(\eta,\,\eta';\,\mathbf{k}) \equiv \left(\mathbf{g}(\eta)\mathbf{P^0}(\mathbf{k})\mathbf{g}^T(\eta')\right)_{ab}$$

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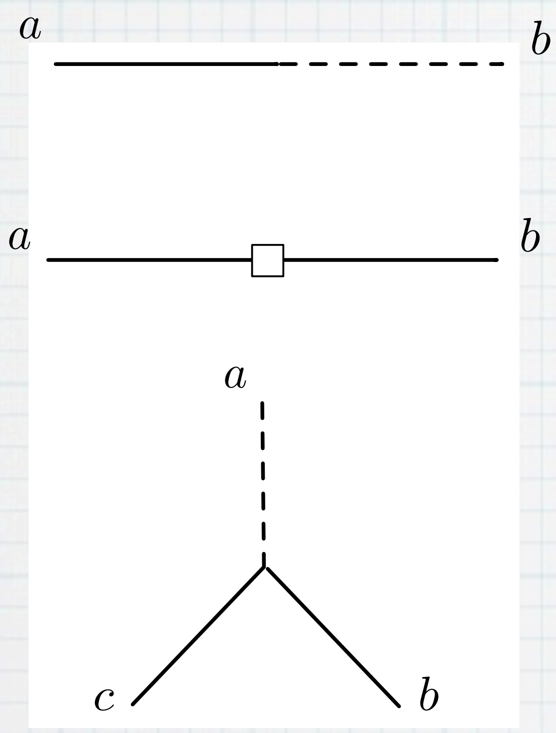
Derivatives of Z w.r.t. the sources J and Λ give all the n-point correlation functions (power spectrum, bispectrum, ...) and the full non-linear propagator

Perturbation Theory: Feynman Rules

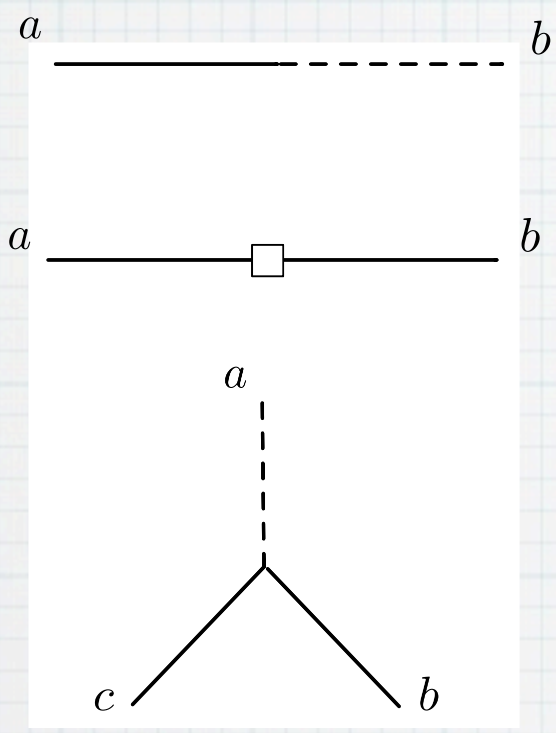


propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Perturbation Theory: Feynman Rules



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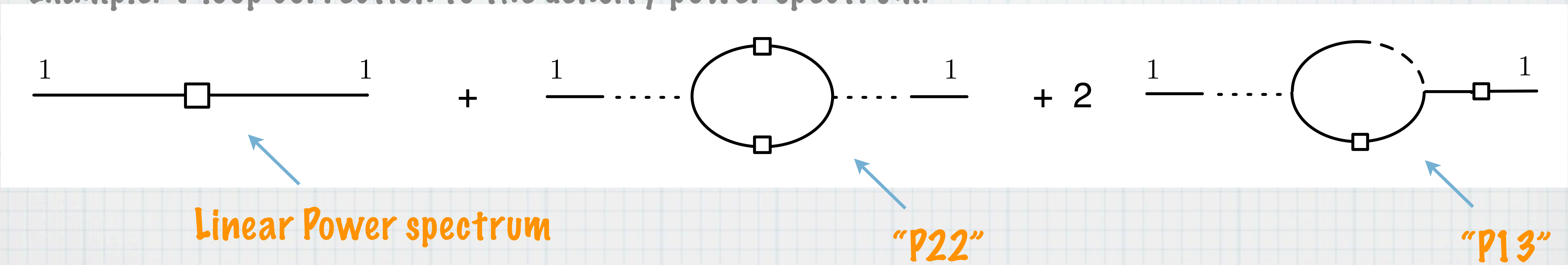
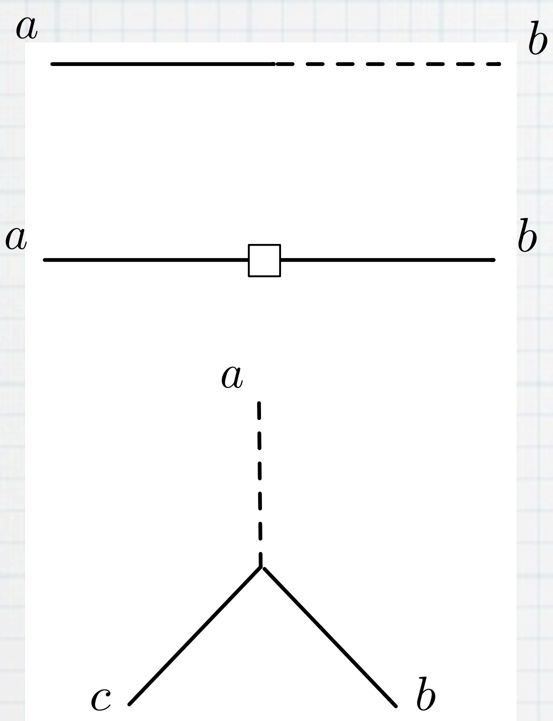
Example: 1-loop correction to the density power spectrum:



Perturbation Theory: Feynman Rules



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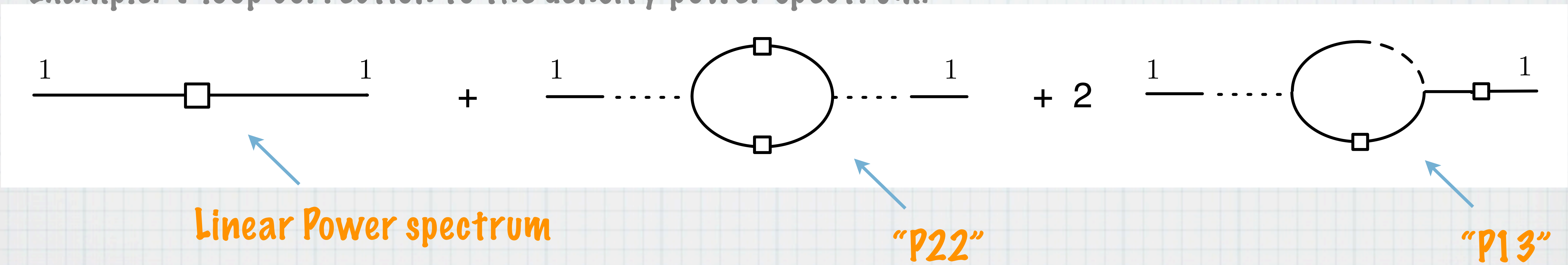
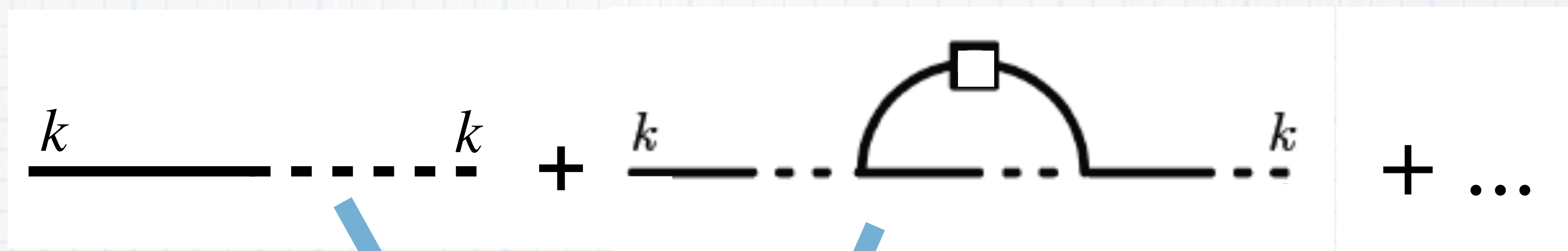


All known results in cosmological perturbation theory are expressible in terms of diagrams in which <u>only a trilinear fundamental interaction</u> appears

PT in the BAO range

1-loop propagator
@ large k:



$$G_{ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\left[1 - k^2\sigma^2\frac{(e^{\eta_a} - e^{\eta_b})^2}{2}\right] + O(k^4\sigma^4)$$

$$\left(\sigma^2 \equiv \frac{1}{3}\int d^3q\,\frac{P^0(q)}{q^2}\right)$$

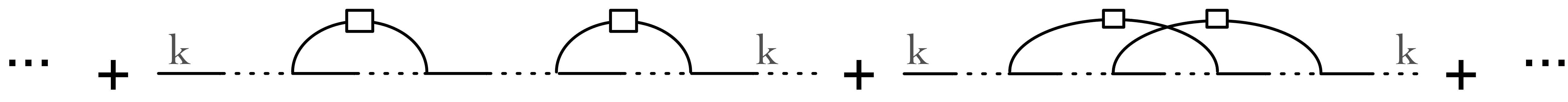
$(\sigma\, e^{\eta_a})^{-1} \simeq 0.15\,\mathrm{h\,Mpc^{-1}}$
in the BAO range!

2-loop

the PT series blows up in the BAO range

But it can be resummed!!

(Crocce-Scoccimarro '06)



$$G(k;\eta,\eta_{in}) = \frac{\langle \delta(k,\eta)\delta(k,\eta_{in})\rangle}{\langle \delta(k,\eta_{in})\delta(k,\eta_{in})\rangle} \sim e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$$

physically, it represents the effect of multiple interactions of the k-mode with the surrounding modes: memory loss

`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

damping in the BAO range!

RPT: use G, and not g, as the linear propagator

Partial (!) list of contributors to the field

* "traditional" P.T.: see Bernardeau et al, Phys. Rep. 367, 1, (2002), and refs. therein; Jeong-Komatsu; Saito et al; Sefusatti;...
* resummation methods: Valageas; Crocce-Scoccimarro; McDonald; Matarrese-M.P.; Matsubara; M.P.; Taruya-Hiratamatsu; Bernardeau-Valageas; Bernardeau-Crocce-Scoccimarro;...

The exact Renormalization Group S. Matarrese, M.P. '07

Inspired by applications of Wilsonian RG to field theory: the RG parameter is momentum

Modify the primordial ($z=z_{in}$) power spectrum as: $P^0_\lambda(k) = P^0(k)\,\Theta(\lambda - k)$ (step function)

then, plug it into the generating functional: $Z[\mathbf{J},\,\mathbf{\Lambda}] \longrightarrow Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}]$

$$Z_\lambda[\mathbf{J},\,\mathbf{\Lambda}] = \int \mathcal{D}\varphi\,\mathcal{D}\chi\,\exp\left\{\int\!\!\int d\eta_1 d\eta_2\left[-\frac{1}{2}\chi\,\mathbf{g}^{-1}\mathbf{P}^{\mathbf{L}}_\lambda\mathbf{g}^{\mathbf{T}^{-1}}\chi + i\,\chi\,\mathbf{g}^{-1}\varphi\right] - i\int d\eta\,[\mathbf{e}^\eta\gamma\,\chi\varphi\varphi - \mathbf{J}\varphi - \mathbf{\Lambda}\chi]\right\}$$

The evolution from $\lambda = 0$ to $\lambda = \infty$ can be described non-perturbatively by RG equations:

$$\frac{\partial}{\partial\lambda}Z_\lambda = \frac{1}{2}\int d^3q\,\delta(\lambda - q)\,P^0_{ab}(q)\,\frac{\delta^2 Z_\lambda}{\delta\Lambda_b(q)\delta\Lambda_a(-q)}$$

with $Z_{\lambda=0} = Z_{\mathrm{lin.\,th.}}$

The propagator

$$G_{\lambda,ab}(k;\,\eta_a,\,\eta_b) = -\frac{\delta^2 W_\lambda[J,\,\Lambda]}{\delta J_a(\mathbf{k},\,\eta_a)\delta\Lambda_b(-\mathbf{k},\,\eta_b)}$$

$$(W_\lambda = -i\log Z_\lambda)$$

$$\frac{\partial}{\partial\lambda}\,\frac{\delta^2 W_\lambda}{\delta J_a\,\delta\Lambda_b} = \frac{1}{2}\int d^3q\;\delta(\lambda-q)\,P^0_{cd}(q)\;\frac{\delta^4 W_\lambda}{\delta J_a\,\delta\Lambda_b\,\delta\Lambda_c\,\delta\Lambda_d}$$

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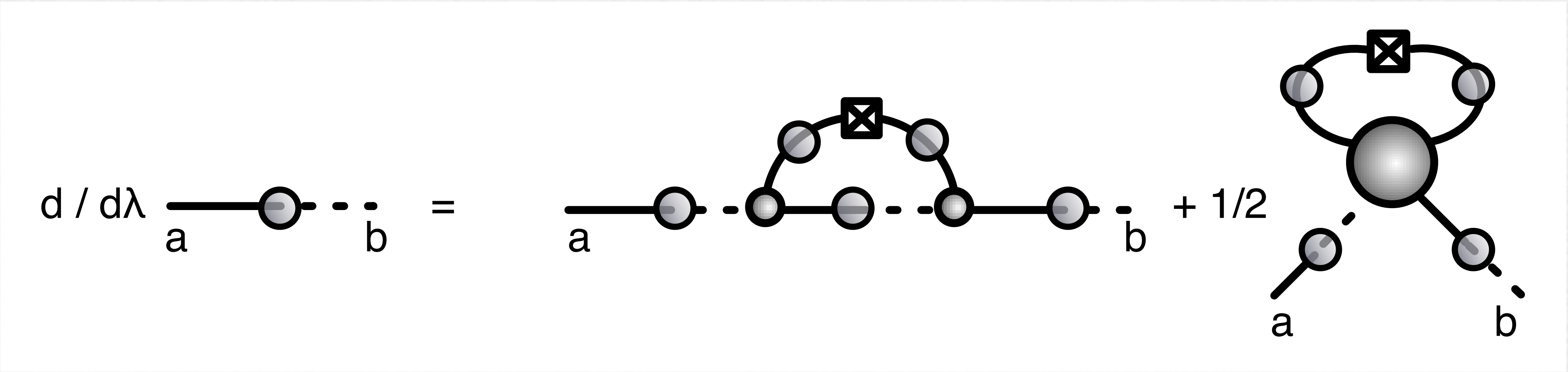
in pictures...



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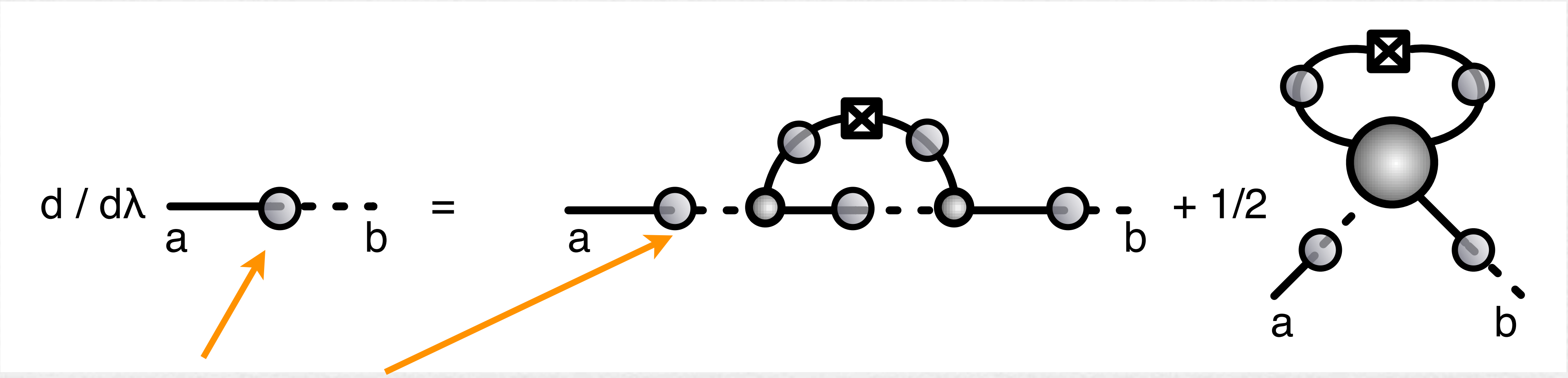

full non-linear propagators

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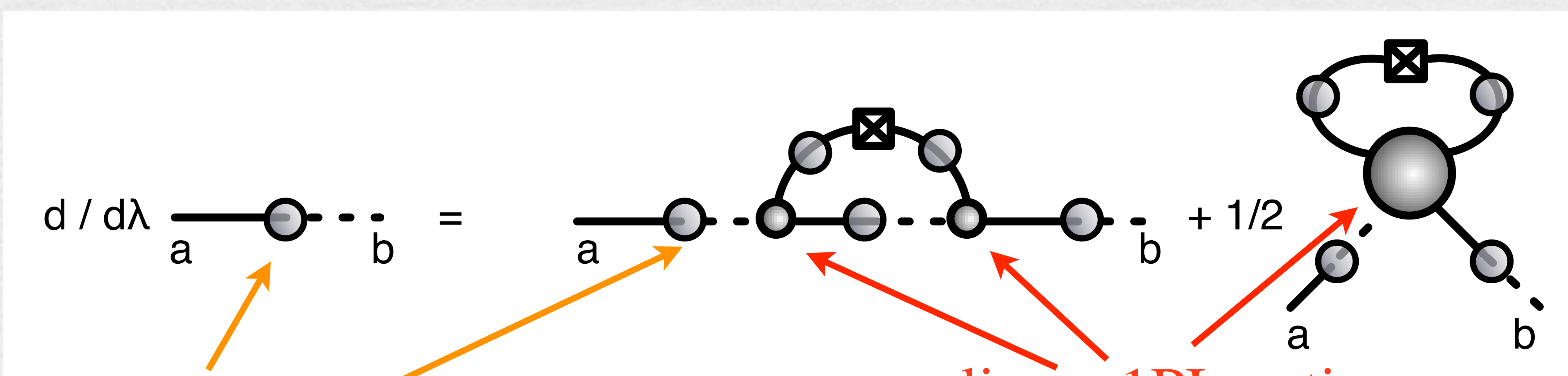
in pictures...



full non-linear propagators

non-linear 1PI vertices

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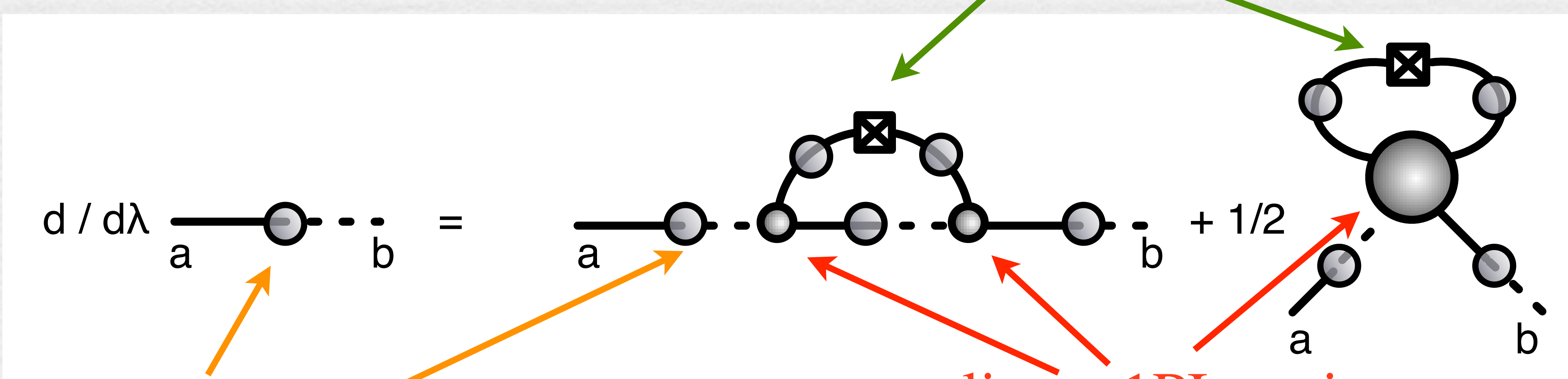

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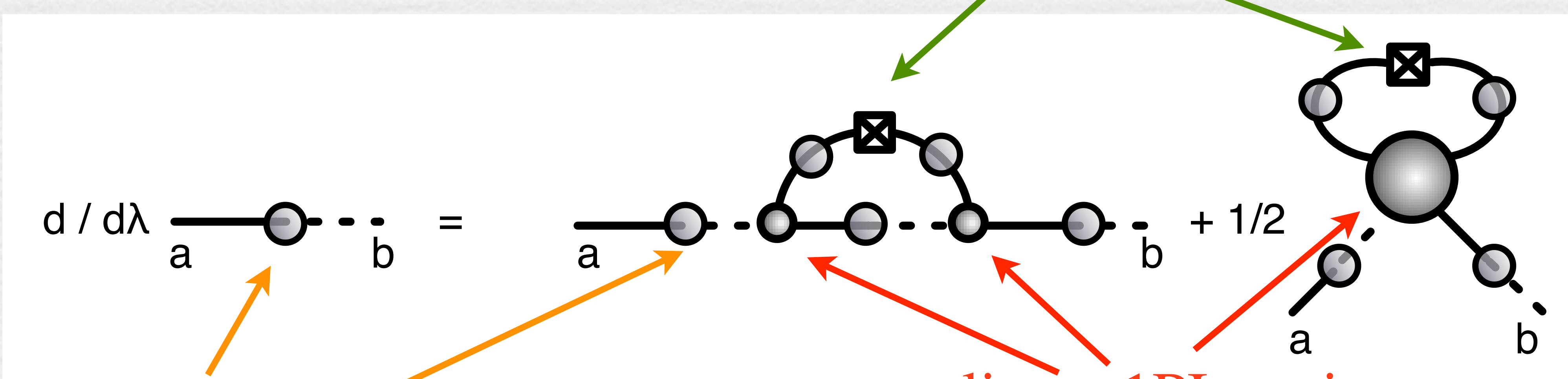
RG Kernel



full non-linear propagators

non-linear 1PI vertices

formally 1-loop, but exact

infinite tower of RGE's

Time-RG

M.P. 0806.0971

(also for cosmologies with $D^{\pm} = D^{\pm}(k,\ z)$)

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\varphi_b(\eta, \mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k},\ -\mathbf{k_1},\ -\mathbf{k_2})\,\varphi_b(\eta, \mathbf{k_1})\,\varphi_c(\eta, \mathbf{k_2})$$

$$\partial_\eta \varphi = -\Omega\,\varphi + e^\eta \gamma\,\varphi\,\varphi$$

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$$\cdots$$

infinite tower of equations

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infinite tower of equations

can be obtained from the RG-like physical requirement

$$\frac{d}{\eta_{in}} Z[J, \Lambda; \eta_{in}] = 0$$

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$\cdots$

infinite tower of equations

can be obtained from the RG-like physical requirement

$$\frac{d}{\eta_{in}} Z[J,\Lambda;\eta_{in}] = 0$$

Resonates with Gasenzer Pawlowski '07 !!

Approximation

$$\langle \varphi_a(\mathbf{k},\,\eta)\varphi_b(\mathbf{q},\,\eta)\rangle \equiv \delta_D(\mathbf{k}+\mathbf{q})P_{ab}(\mathbf{k}\,,\eta)\;,$$

$$\langle \varphi_a(\mathbf{k},\,\eta)\varphi_b(\mathbf{q},\,\eta)\varphi_c(\mathbf{p},\,\eta)\rangle \equiv \delta_D(\mathbf{k}+\mathbf{q}+\mathbf{p})B_{abc}(\mathbf{k},\,\mathbf{q},\,\mathbf{p};\,\eta)\;,$$

$$\begin{aligned}\langle \varphi_a(\mathbf{k},\,\eta)\varphi_b(\mathbf{q},\,\eta)\varphi_c(\mathbf{p},\,\eta)\varphi_d(\mathbf{r},\,\eta)\rangle \equiv\\ &[\delta_D(\mathbf{k}+\mathbf{q})\,\delta_D(\mathbf{p}+\mathbf{r})P_{ab}(\mathbf{k}\,,\eta)P_{cd}(\mathbf{p}\,,\eta)\\ &+\delta_D(\mathbf{k}+\mathbf{p})\,\delta_D(\mathbf{q}+\mathbf{r})P_{ac}(\mathbf{k}\,,\eta)P_{bd}(\mathbf{q}\,,\eta)\\ &+\delta_D(\mathbf{k}+\mathbf{r})\,\delta_D(\mathbf{q}+\mathbf{p})P_{ad}(\mathbf{k}\,,\eta)P_{bc}(\mathbf{q}\,,\eta)\\ &+\delta_D(\mathbf{k}+\mathbf{p}+\mathbf{q}+\mathbf{r})\,T_{abcd}(\mathbf{k}\,,\mathbf{q}\,,\mathbf{p}\,,\mathbf{r}\,,\eta)]\;,\end{aligned}$$

Only approximation: $T_{abcd} = 0$

Equations to solve:

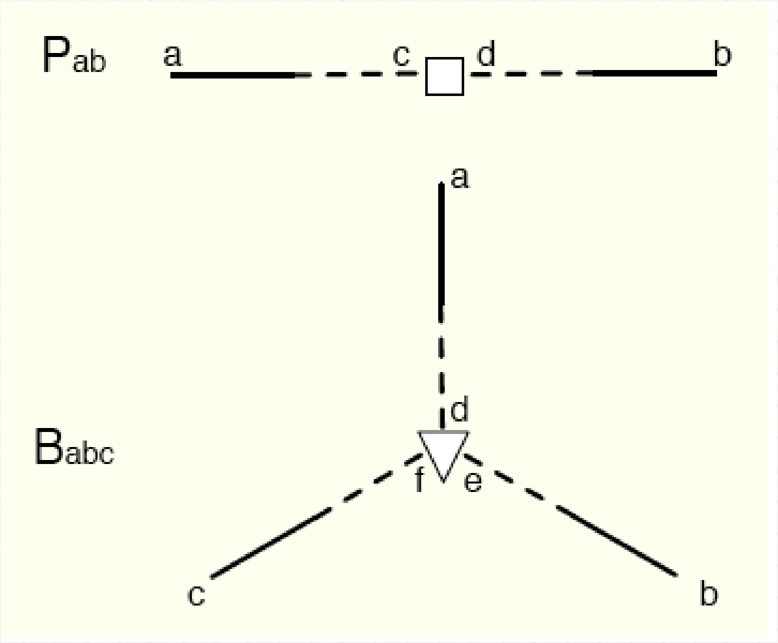
$$\partial_\eta P_{ab}(\mathbf{k},\eta) = -\Omega_{ac}(\mathbf{k},\eta)P_{cb}(\mathbf{k},\eta) - \Omega_{bc}(\mathbf{k},\eta)P_{ac}(\mathbf{k},\eta)$$
$$+ e^\eta \int d^3q \, [\gamma_{acd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k})\, B_{bcd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$+ B_{acd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)\, \gamma_{bcd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k})] \, ,$$

$$\partial_\eta B_{abc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta) = -\Omega_{ad}(\mathbf{k},\eta)B_{dbc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$- \Omega_{bd}(-\mathbf{q},\eta)B_{adc}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$- \Omega_{cd}(\mathbf{q}-\mathbf{k},\eta)B_{abd}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k}; \eta)$$
$$+ 2e^\eta \, [\gamma_{ade}(\mathbf{k}, -\mathbf{q}, \mathbf{q}-\mathbf{k})P_{db}(\mathbf{q},\eta)P_{ec}(\mathbf{k}-\mathbf{q},\eta)$$
$$+ \gamma_{bde}(-\mathbf{q}, \mathbf{q}-\mathbf{k}, \mathbf{k})P_{dc}(\mathbf{k}-\mathbf{q},\eta)P_{ea}(\mathbf{k},\eta)$$
$$+ \gamma_{cde}(\mathbf{q}-\mathbf{k}, \mathbf{k}, -\mathbf{q})P_{da}(\mathbf{k},\eta)P_{eb}(\mathbf{q},\eta)] \, .$$

initial conditions given at $\eta = 0$, corresponding to $z = z_{in}$

Iterative solution: step 1

$O(\gamma^0)$: linear PT



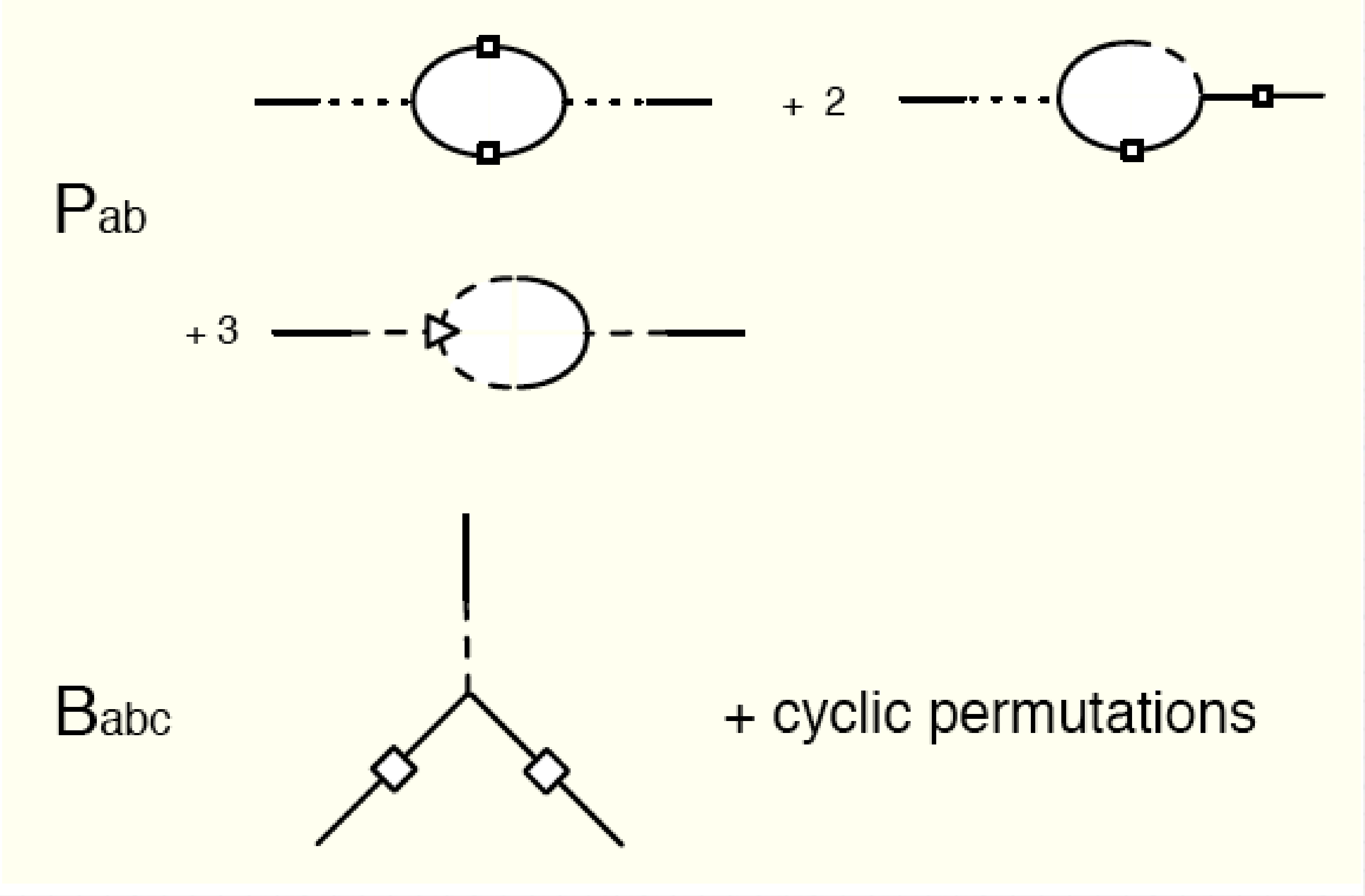
$$P^L_{ab}(\mathbf{k}\,,\eta) = g_{ac}(\mathbf{k}\,,\eta,0)\,g_{bd}(\mathbf{k}\,,\eta,0)P_{cd}(\mathbf{k}\,,\eta=0)\,,$$

$$B^L_{abc}(\mathbf{k},\;-\mathbf{q},\;\mathbf{q}-\mathbf{k};\,\eta) =$$
$$g_{ad}(\mathbf{k}\,,\eta,0)g_{be}(-\mathbf{q}\,,\eta,0)g_{cf}(\mathbf{q}-\mathbf{k}\,,\eta,0)B_{def}(\mathbf{k},\;-\mathbf{q},\;\mathbf{q}-\mathbf{k};\,\eta=0)$$

Iterative solution: step 2

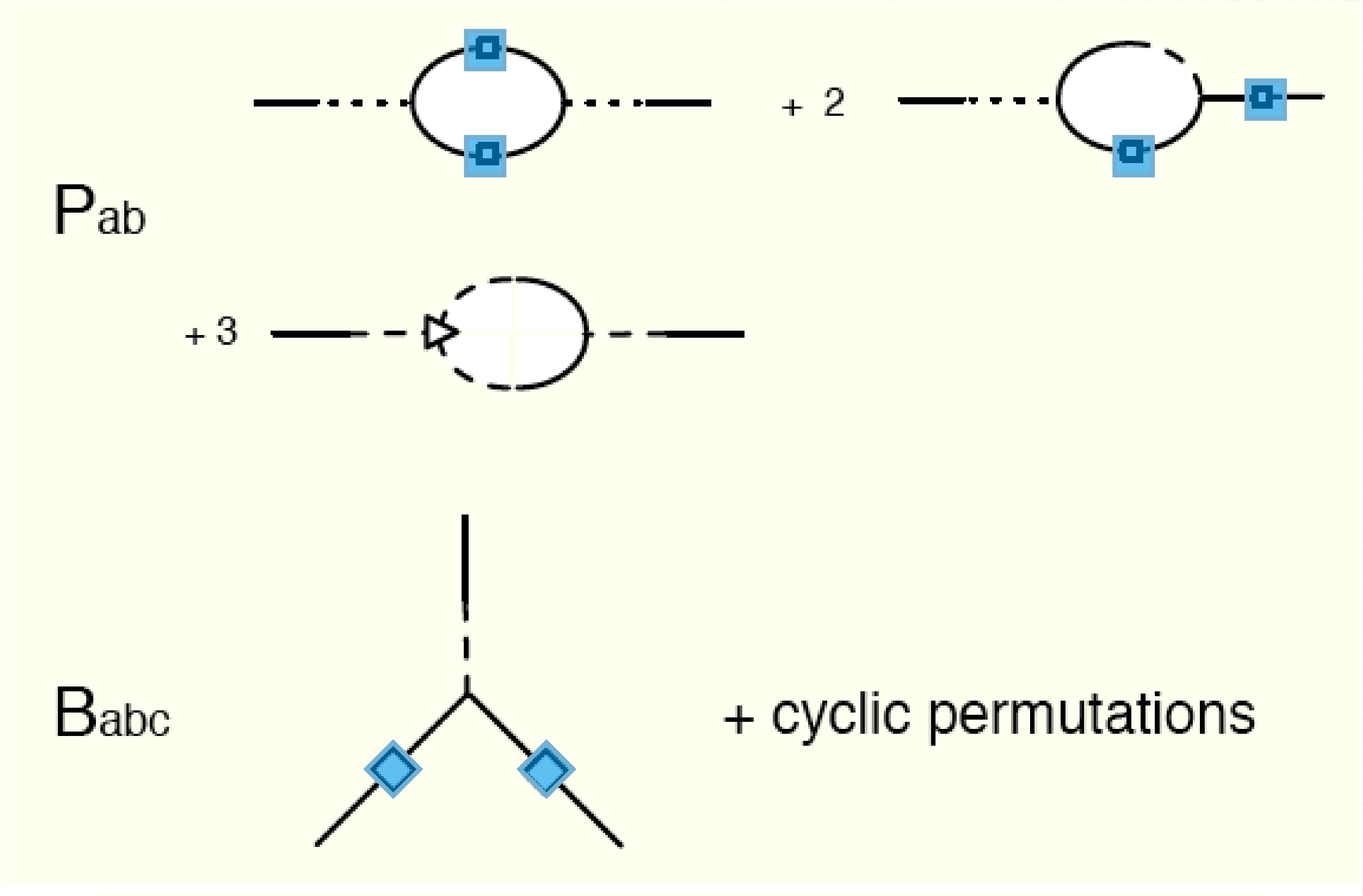
1-loop corrections for P_{ab} :

$O(\gamma)$ corrections for B_{abc} :



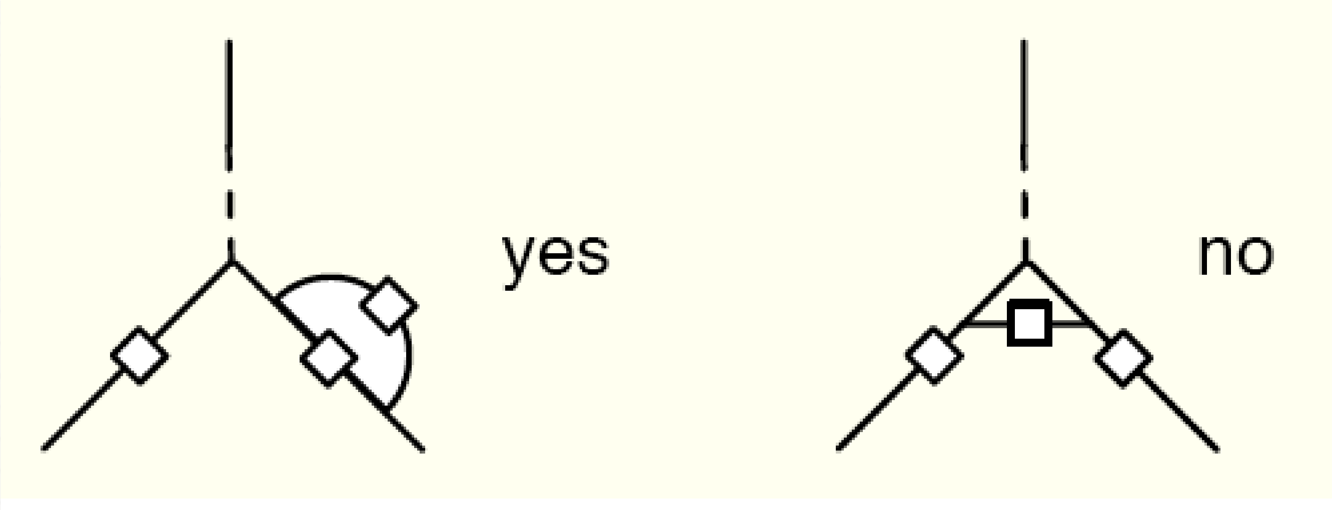
Iterative solution: step n>2

n-1-loop corrections for P_{ab}, <u>but not all!!</u>

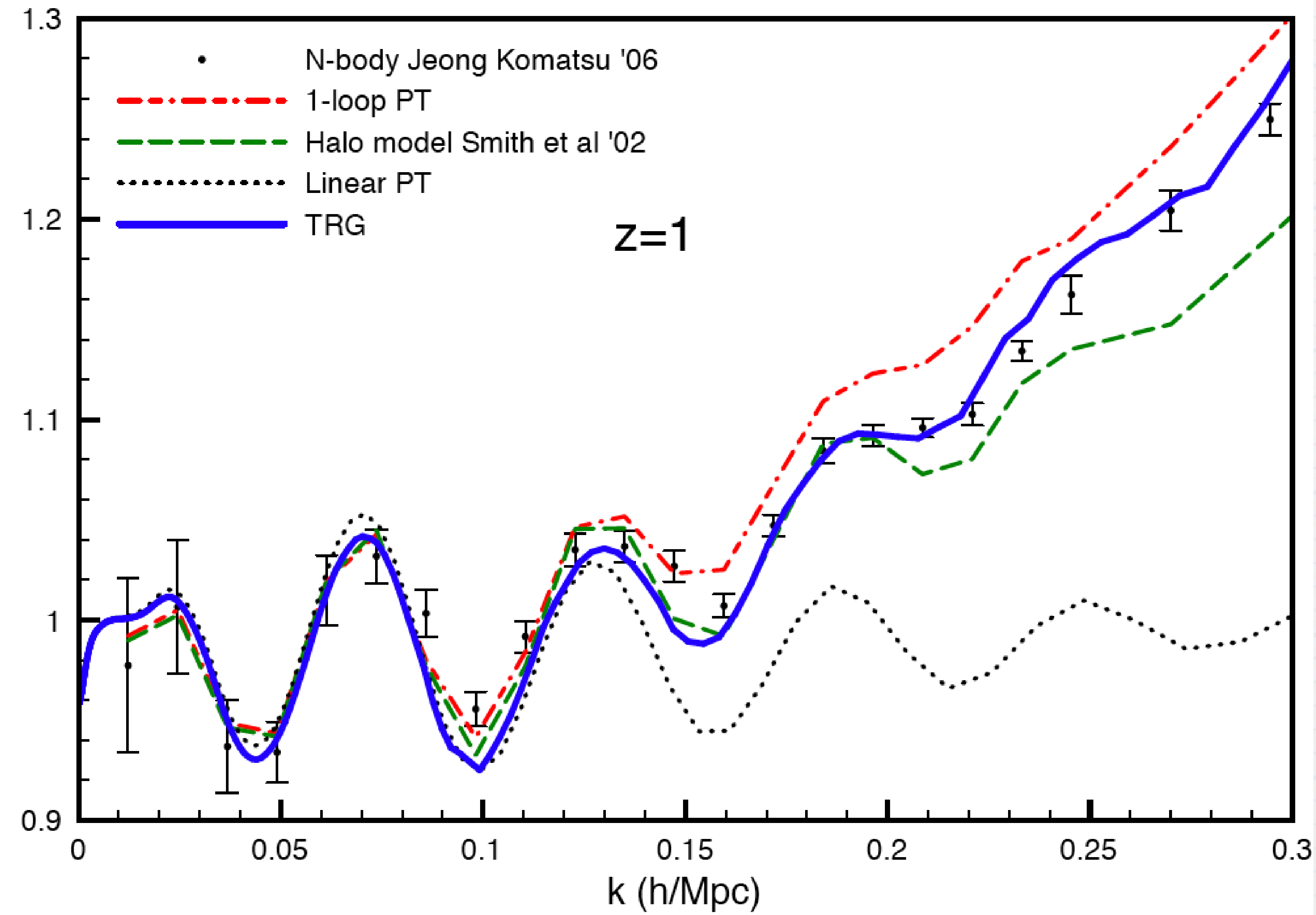


$O(\gamma^{2n-3})$ corrections for B_{abc} <u>but not all!</u>

e.g.: for n=3



Full equation: numerical results



M. Pietroni 0806.0971 (JCAP)

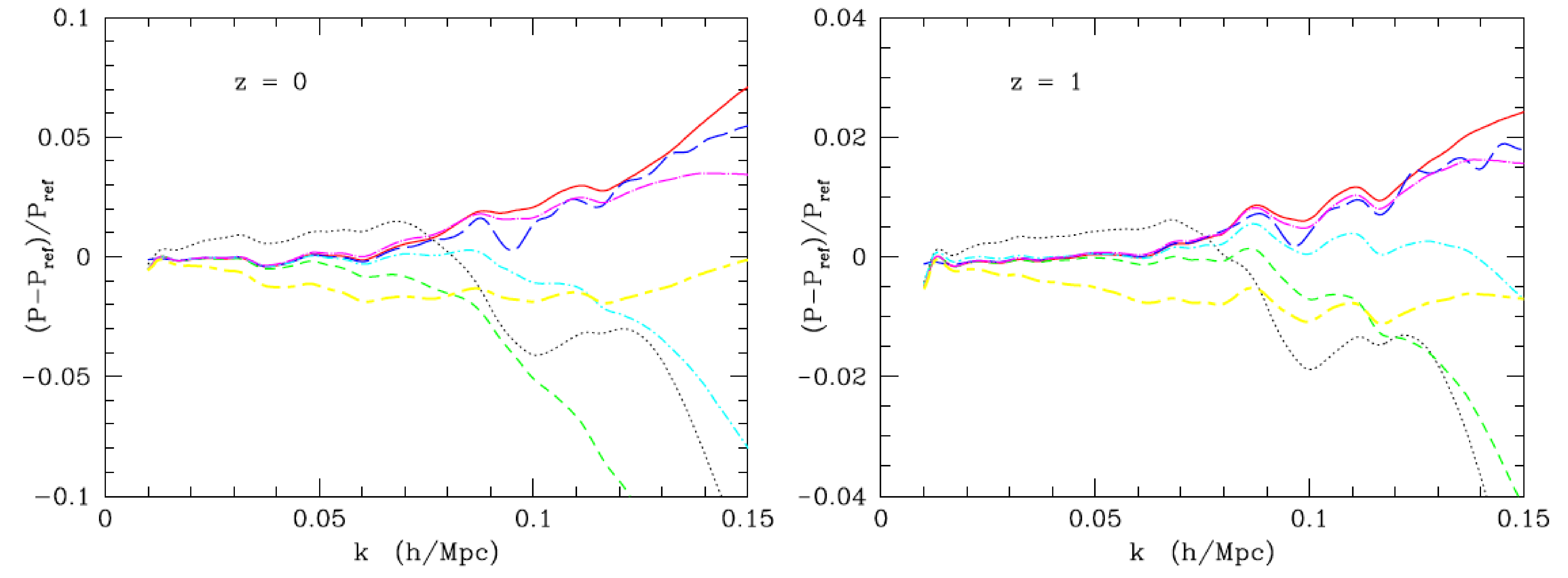
initial conditions:

$$P_{ab}(\mathbf{k},0) = P_{\rm Lin}(\mathbf{k},z_{in})u_a u_b$$

$$B_{abc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k},0) = 0$$

Comparison with other methods

Carlson, White,
Padmanabhan, '09



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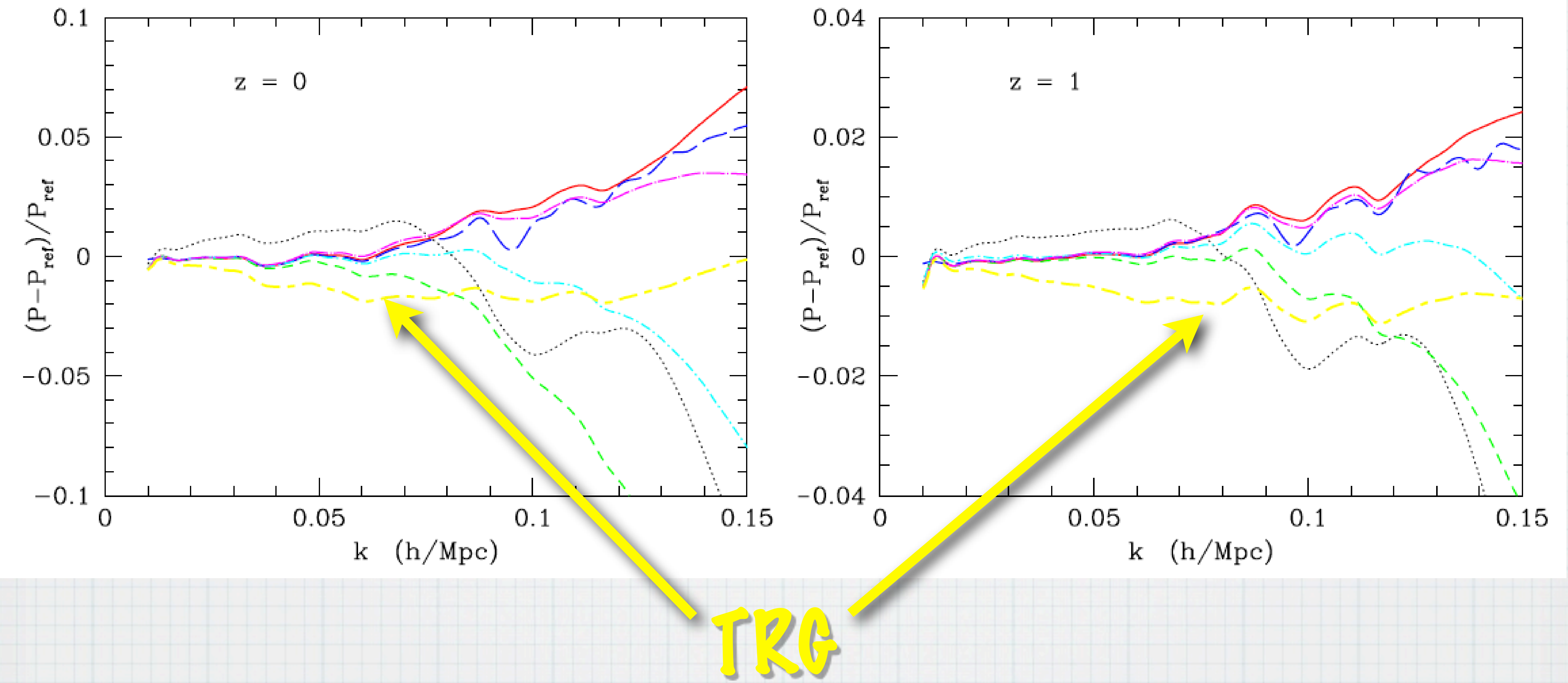

Fractional difference w.r.t. high resolution N-body below 2% in the BAO range down to z=0!

How far can (resummed) PT go on its own?

The DM particle distribution function, $f(\mathbf{x}, \mathbf{p}, \tau)$, obeys the Vlasov equation:

$$\frac{\partial f}{\partial \tau} + \frac{\mathbf{p}}{am} \cdot \nabla f - am \nabla \phi \cdot \nabla_{\mathbf{p}} f = 0$$

with $p = am \frac{d\mathbf{x}}{d\tau}$ and $\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta$ sub-horizon scales, Newtonian gravity

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Taking moments,

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Taking moments,

$$\int d^3\mathbf{p}\, f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau) \equiv \overline{\rho}\,[1+\delta(\mathbf{x},\tau)]$$

$$\int d^3\mathbf{p}\, \frac{p_i}{am} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau) v_i(\mathbf{x},\tau)$$

$$\int d^3\mathbf{p}\, \frac{p_i\, p_j}{a^2 m^2} f(\mathbf{x},\mathbf{p},\tau) \equiv \rho(\mathbf{x},\tau)\,[v_i(\mathbf{x},\tau) v_j(\mathbf{x},\tau) + \sigma_{ij}(\mathbf{x},\tau)]$$

$$\ldots$$

$$\frac{\partial\, n}{\partial \tau} + \frac{\partial}{\partial x^i}(n\, v^i) = 0$$

source term

$$\frac{\partial\, v^i}{\partial\, \tau} + \mathcal{H} v^i + v^k \frac{\partial}{\partial x^k} v^i + \frac{1}{n}\frac{\partial}{\partial x^k}(n\sigma^{ki}) = -\frac{\partial}{\partial x^i}\phi$$

$$\frac{\partial\, \sigma^{ij}}{\partial\, \tau} + 2\mathcal{H}\sigma^{ij} + v^k \frac{\partial}{\partial x^k}\sigma^{ij} + \sigma^{ik}\frac{\partial}{\partial x^k} v^j + \sigma^{jk}\frac{\partial}{\partial x^k} v^i + \frac{1}{n}\frac{\partial}{\partial x^k}(n\omega^{ijk}) = 0$$

$$\frac{\partial\, \omega^{ijk}}{\partial\, \tau} + \cdots = 0$$

$\cdots$

$$\nabla^2 \phi = \frac{3}{2}\, \Omega_M\, \mathcal{H}^2\, \delta$$

No sources for $\sigma^{ij},\ \omega^{ijk}, \ldots, \vec{\nabla} \times \vec{v}, \ldots$

$\sigma^{ij} = \omega^{ijk} = \cdots = \vec{\nabla} \times \vec{v} = 0$ is a fixed point

neglecting σ_{ij} and higher moments...

$$\frac{\partial n}{\partial \tau} + \frac{\partial}{\partial x^i}(n\, v^i) = 0$$ continuity

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi$$ Euler

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$ Poisson

$$[n = n_0(1+\delta)]$$

(RESUMMED) PT IS BASED ON THE "SINGLE STREAM APPROXIMATION"

$$\sigma_{ij} = 0 \leftrightarrow f(\vec{x},\vec{p},\tau) = g(\vec{x},\tau)\delta_D(\vec{p} - am\vec{v}(\vec{x},\tau))$$

self-consistent, but wrong!

Large scale impact of velocity dispersion

$$q_i(\mathbf{x},\tau) \equiv \frac{1}{\rho}\nabla_j(\rho\sigma_{ij}).$$

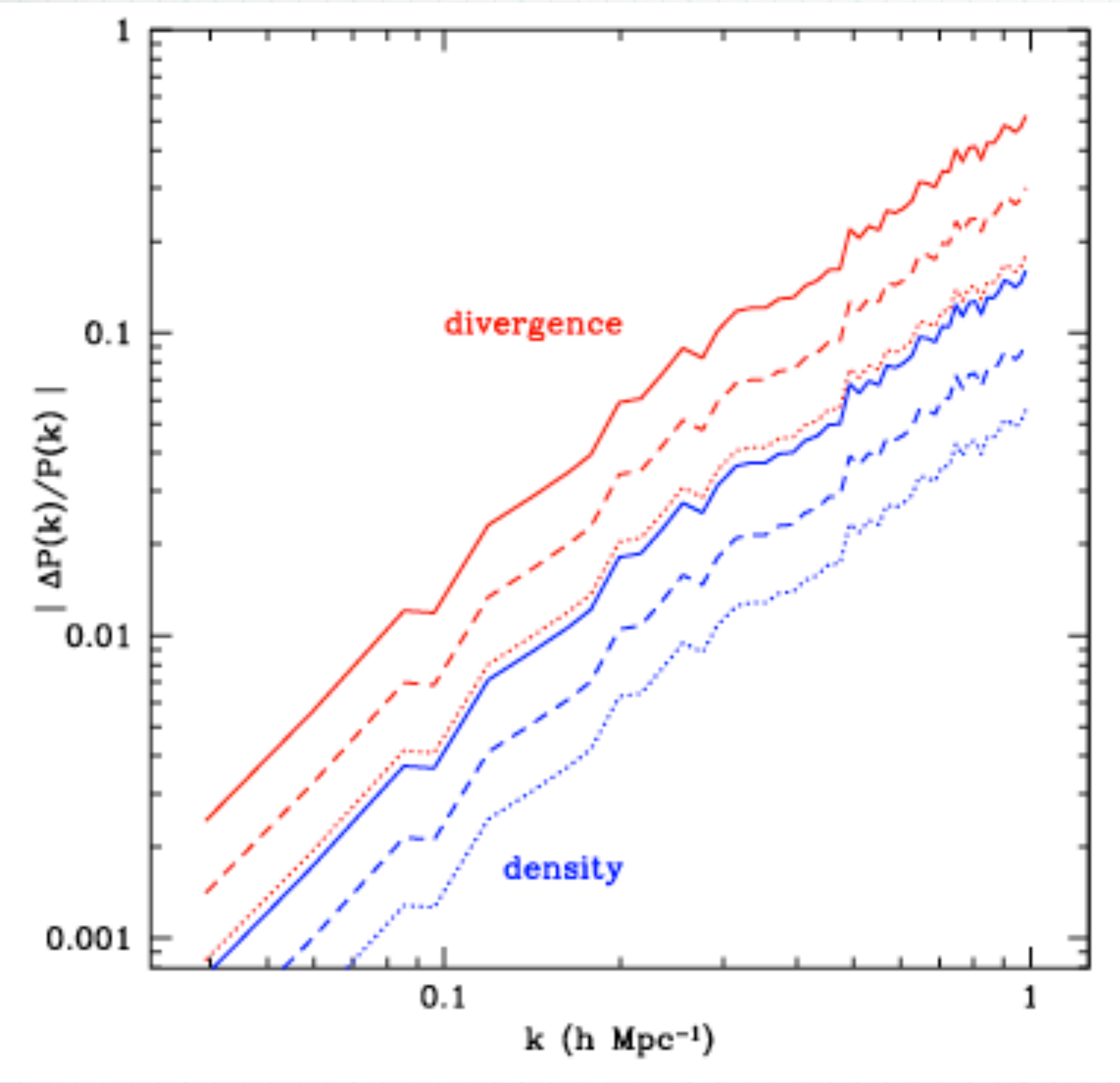
$$q_\theta \equiv \nabla \cdot \mathbf{q}, \qquad \mathbf{q}_w \equiv \nabla \times \mathbf{q},$$

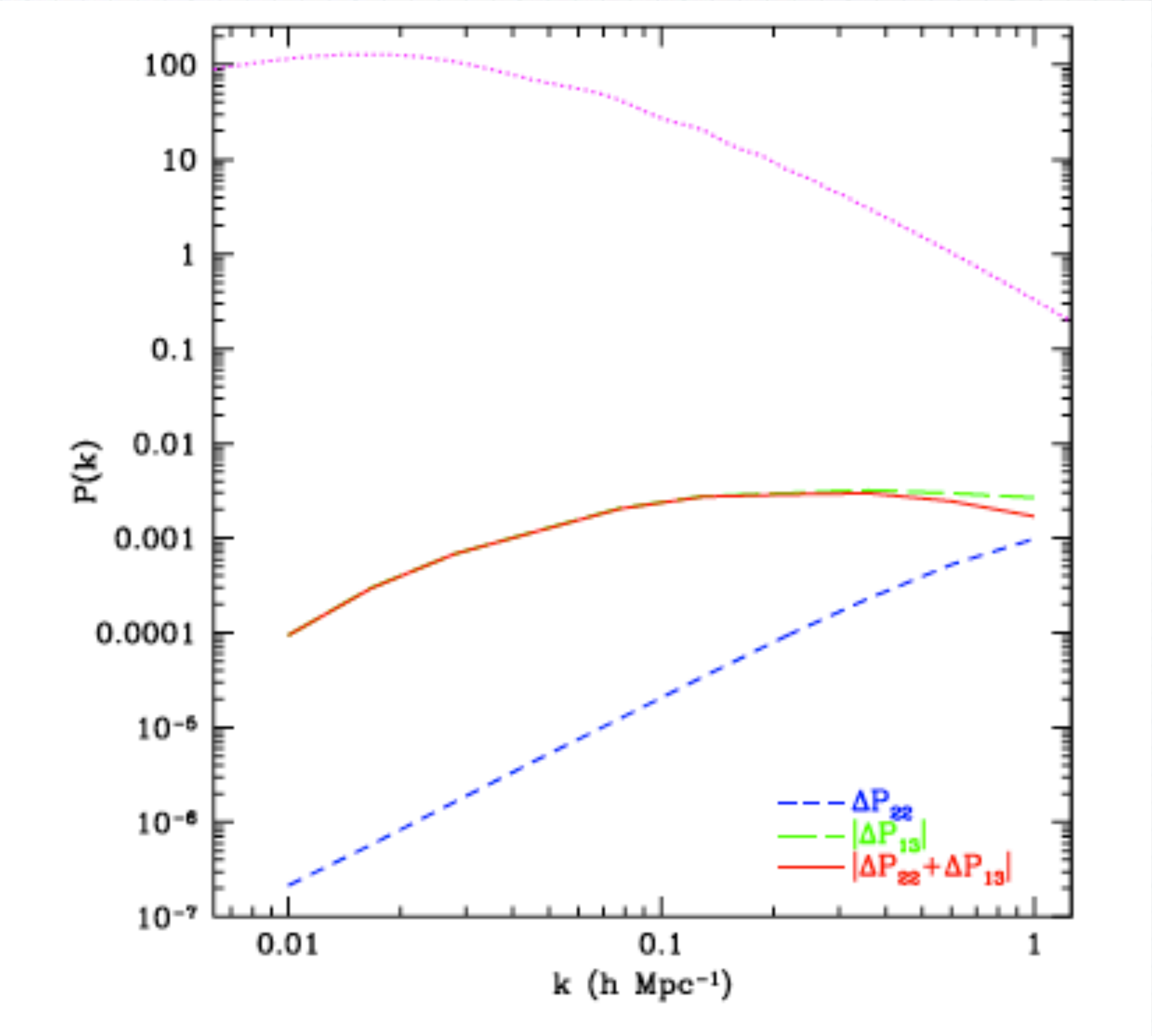
measure the q's from simulations

$$\begin{aligned} \partial_\eta \delta - \theta &= 0, \\ \partial_\eta \theta + \frac{\theta}{2} - \frac{3\delta}{2} &= \mathrm{q}_\theta, \\ \partial_\eta \mathbf{w} + \frac{\mathbf{w}}{2} &= \mathbf{q}_w. \end{aligned}$$

Pueblas Scoccimarro 2009

Effect on the power spectrum





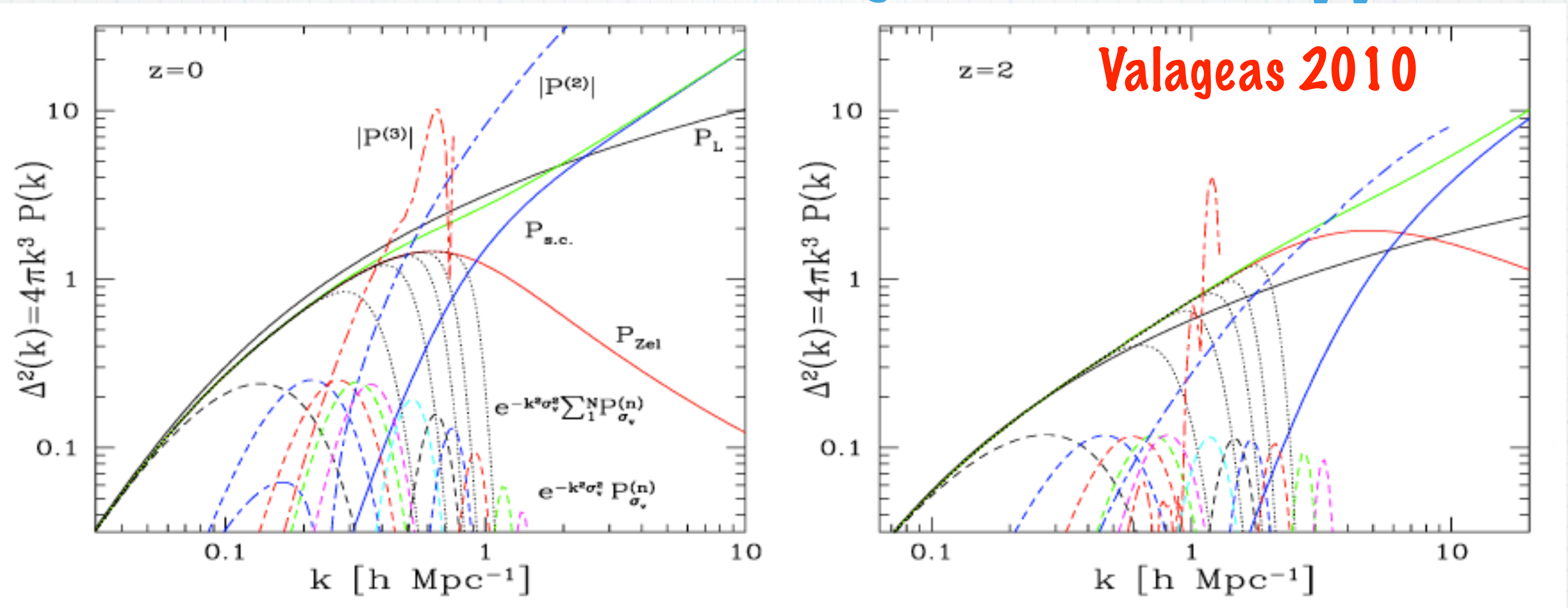
$q_\theta + q_w$

q_w only

around % at z=0 in the BAO range

Pueblas Scoccimarro 2009

Intrinsic limit: the single stream approx.



z	%	k_{1loop} $[h\text{Mpc}^{-1}]$	$k_{s.c.}$ $[h\text{Mpc}^{-1}]$	$n_{s.c.}$
0	1%	0.033	0.23	9
	10%	0.082	0.45	
	50%		0.9	
1	1%	0.043	0.44	18
	10%	0.11	1.1	
	50%		2.2	
2	1%	0.057	1.2	37
	10%	0.14	2.3	
	50%		6.4	
3	1%	0.07	2.2	66
	10%	0.18	5.2	
	50%		10.4	

The fluid picture starts to break down for k> k_sc = 0.23 h/Mpc at z=0

The higher z the higher the k_sc

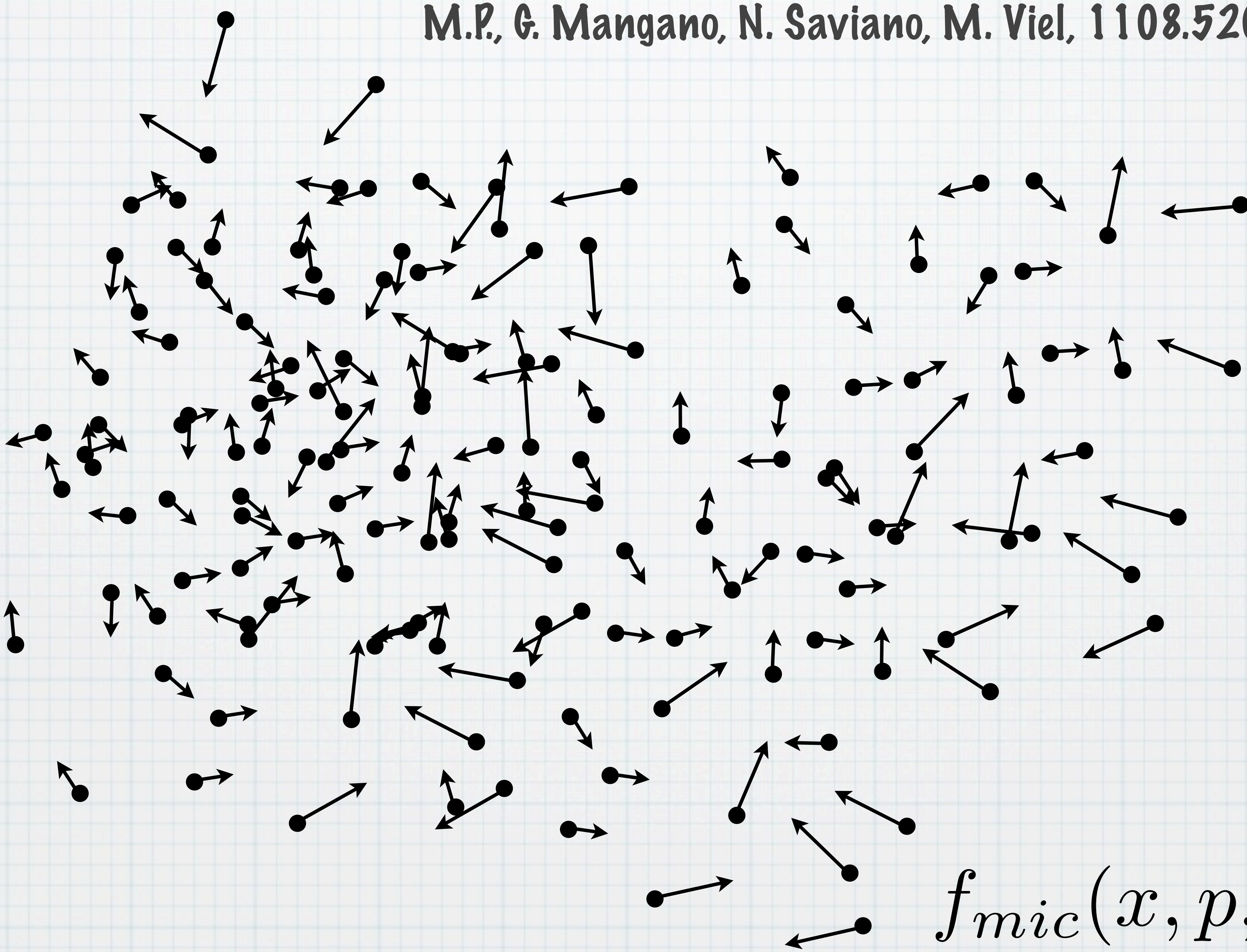
Resummation methods can bring from k_1loop to k_sc

How to go beyond k_sc?

Rederiving the fluid equations

Buchert, Dominguez, '05

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203



$$f_{mic}(x, p, \tau)$$

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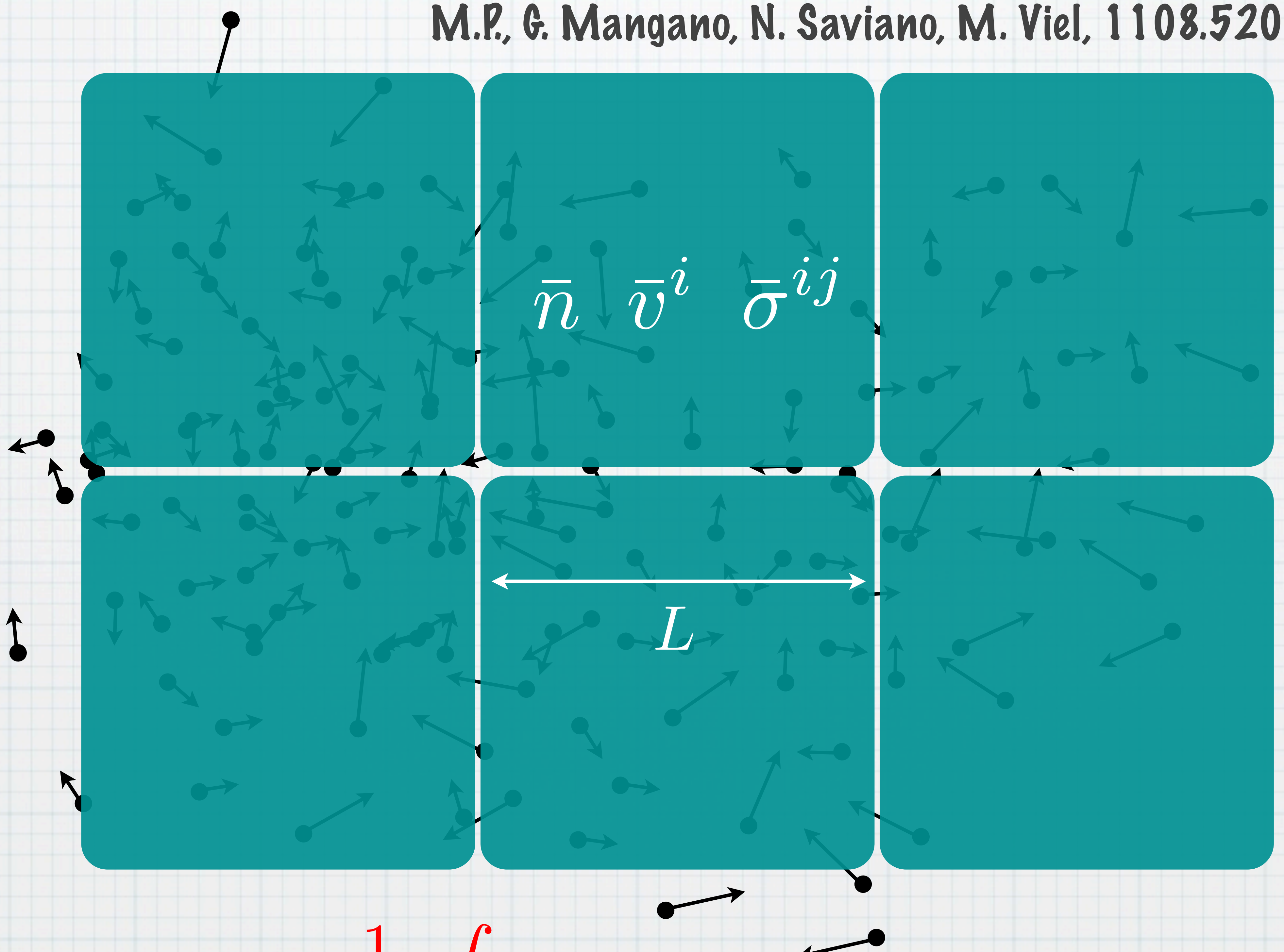


$$\bar{f}(x,p,\tau) \equiv \frac{1}{V}\int d^3y \mathcal{W}(y/L) f_{mic}(x+y,p,\tau)$$

Coarse-Grained Vlasov eq.

large scales

$$\left[\frac{\partial}{\partial\tau} + \frac{p^i}{ma}\frac{\partial}{\partial x^i} - am\nabla^i_x\bar{\phi}(\mathbf{x},\tau)\frac{\partial}{\partial p^i}\right]\bar{f}(\mathbf{x},\mathbf{p},\tau)$$

$$= \frac{am}{V}\int d^3y\,\mathcal{W}\left(\left|\frac{\mathbf{y}}{L}\right|\right)\nabla^i_{x+y}\delta\phi(\mathbf{x}+\mathbf{y},\tau)\,\frac{\partial}{\partial p^i}\delta f(\mathbf{x}+\mathbf{y},\mathbf{p},\tau)$$

short scales

Short-distance sources

$q_\theta + q_w$

$$\frac{\partial}{\partial\tau}\bar{n}(\mathbf{x}) + \frac{\partial}{\partial x^i}\Big(\bar{n}(\mathbf{x})\bar{v}^i(\mathbf{x})\Big) = 0\,.$$

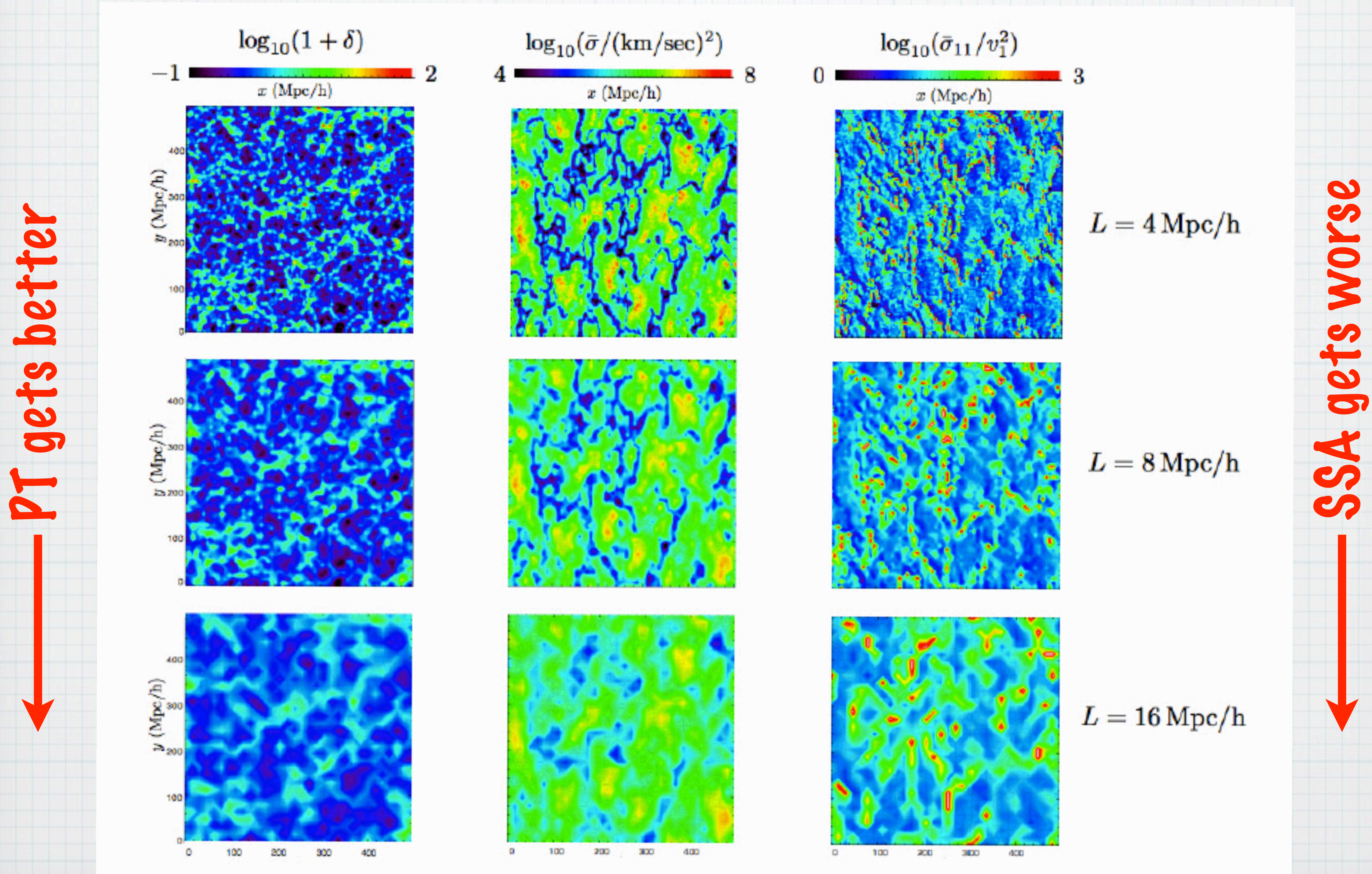
$$\frac{\partial}{\partial\tau}\bar{v}^i(\mathbf{x}) + \mathcal{H}\bar{v}^i(\mathbf{x}) + \bar{v}^k(\mathbf{x})\frac{\partial}{\partial x^k}\bar{v}^i(\mathbf{x}) + \frac{1}{\bar{n}(\mathbf{x})}\frac{\partial}{\partial x^k}(\bar{n}(\mathbf{x})\bar{\sigma}^{ki}(\mathbf{x}))$$
$$= -\nabla^i_x\bar{\phi}(\mathbf{x}) - \frac{1}{V}\int d^3y\,\mathcal{W}\left(\left|\frac{\mathbf{y}}{L}\right|\right)\frac{n(\mathbf{x}+\mathbf{y})}{\bar{n}(\mathbf{x})}\nabla^i_{x+y}\delta\phi(\mathbf{x}+\mathbf{y})\,,$$

$$\frac{\partial}{\partial\tau}\bar{\sigma}^{ij} + 2\mathcal{H}\bar{\sigma}^{ij} + \bar{v}^k\frac{\partial}{\partial x^k}\bar{\sigma}^{ij} + \bar{\sigma}^{ik}\frac{\partial}{\partial x^k}\bar{v}^j + \bar{\sigma}^{jk}\frac{\partial}{\partial x^k}\bar{v}^i + \frac{1}{\bar{n}}\frac{\partial}{\partial x^k}\left(\bar{n}\,\bar{\omega}^{ijk}\right)$$
$$= -\frac{1}{V}\int d^3y\;\mathcal{W}\left(\left|\frac{\mathbf{y}}{L}\right|\right)\frac{n(\mathbf{x}+\mathbf{y})}{\bar{n}(\mathbf{x})}$$
$$\times\left[\delta v^j(\mathbf{x}+\mathbf{y})\nabla^i_{x+y} + \delta v^i(\mathbf{x}+\mathbf{y})\nabla^j_{x+y}\right]\delta\phi(\mathbf{x}+\mathbf{y})\,.$$

Short-distance sources

$\bar{\sigma}^{ij}$ and all higher-order moments are dynamically generated by coarse-graining!

Coarse-Graining vs. Single-Stream



Compact form

$$\bar{\varphi}_a(\mathbf{k},\eta) = e^{-\eta}\begin{pmatrix} \bar{\delta} \\ -\frac{\bar{\theta}}{\mathcal{H}f} \\ \frac{k^2}{\mathcal{H}^2 f^2}\bar{\sigma} \\ \frac{k^2}{\mathcal{H}^2 f^2}\bar{\Sigma} \end{pmatrix}$$

$$\bar{\sigma}(\mathbf{k}) = \bar{\sigma}^{ii}(\mathbf{k}), \quad \bar{\Sigma}(\mathbf{k}) = \frac{k^i k^j}{k^2}\bar{\sigma}^{ij}(\mathbf{k})$$

$$(\delta_{ab}\partial_\eta + \Omega_{ab})\,\bar{\varphi}^b(\mathbf{k},\eta) =$$
$$e^{\eta}\int d^3q_1 d^3q_2 \delta_D(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)\gamma_{abc}(k,q_1,q_2)\bar{\varphi}_b(\mathbf{q}_1,\eta)\bar{\varphi}_c(\mathbf{q}_2,\eta) - h_a(\mathbf{k},\eta)$$

(resummed) PT expansion in γ_{abc}

short-distance sources: measure from simulations

$$0 \leq k \leq k_{(R)PT} \simeq \frac{2\pi}{L}$$

cosmology up to mildly non linear scales

$$k > \frac{2\pi}{L}$$

cosmology-independent?

perturbative solution for the large scales

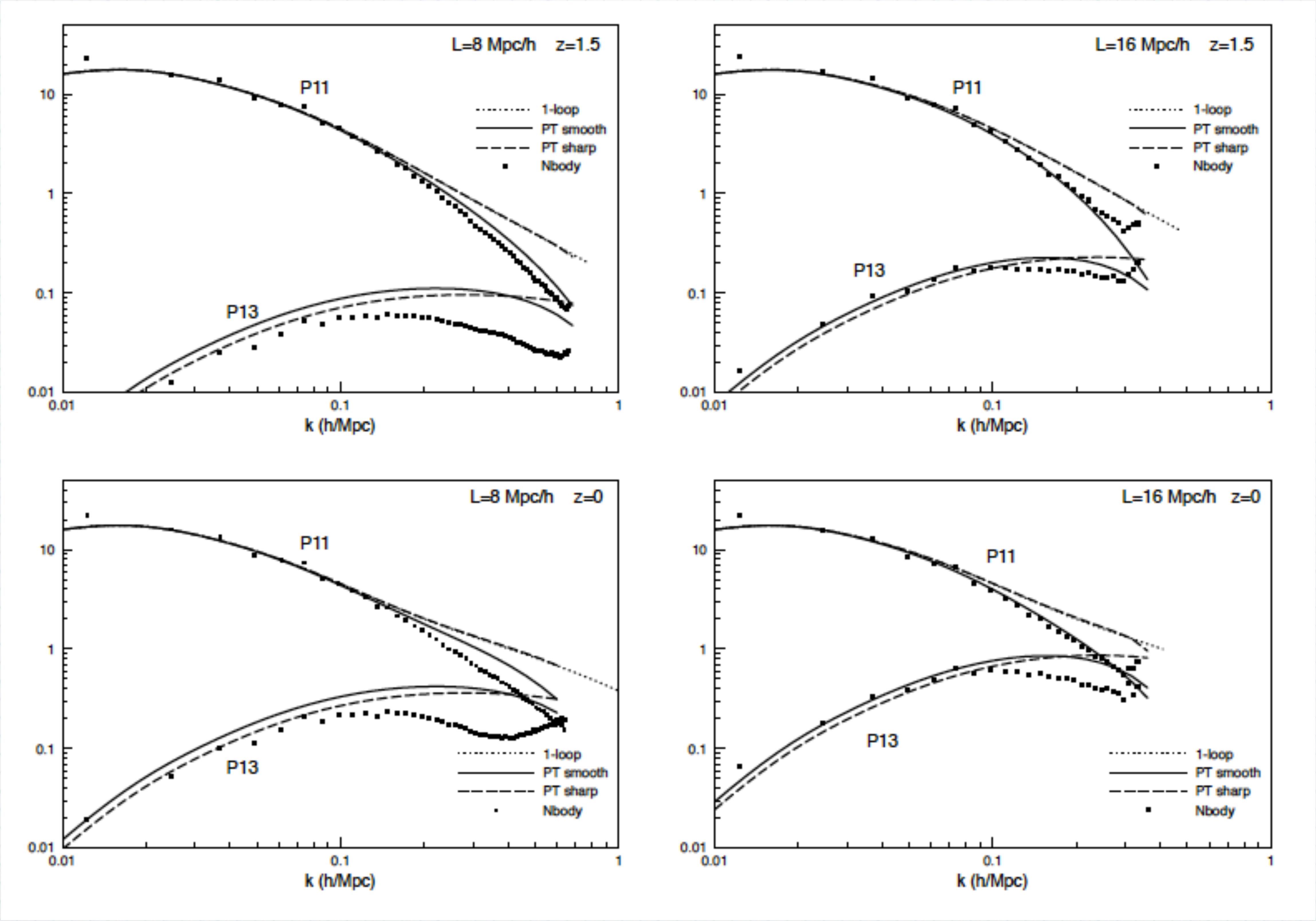
$$\bar{\varphi}_a^{(0)}(\mathbf{k},\eta) = g_{ab}(\eta)\bar{\varphi}_b^{in}(\mathbf{k}) - \int_0^\eta ds\, g_{ab}(\eta - s)\, h_b(\mathbf{k}, s)\,.$$

$$\bar{\varphi}_a^{(1)}(\mathbf{k},\eta) = \int_0^\eta ds\, g_{ab}(\eta - s)\, e^s\, \gamma_{bcd}(k, q_1, q_2)\bar{\varphi}_c^{(0)}(\mathbf{q}_1, s)\bar{\varphi}_d^{(0)}(\mathbf{q}_2, s)$$

$$\bar{\varphi}_a^{(2)}(\mathbf{k},\eta) = \int_0^\eta ds\, g_{ab}(\eta - s)\, e^s\, \gamma_{bcd}(k, q_1, q_2) \times \left(\bar{\varphi}_c^{(1)}(\mathbf{q}_1, s)\bar{\varphi}_d^{(0)}(\mathbf{q}_2, s) + \bar{\varphi}_c^{(0)}(\mathbf{q}_1, s)\bar{\varphi}_d^{(1)}(\mathbf{q}_2, s)\right)\,,$$

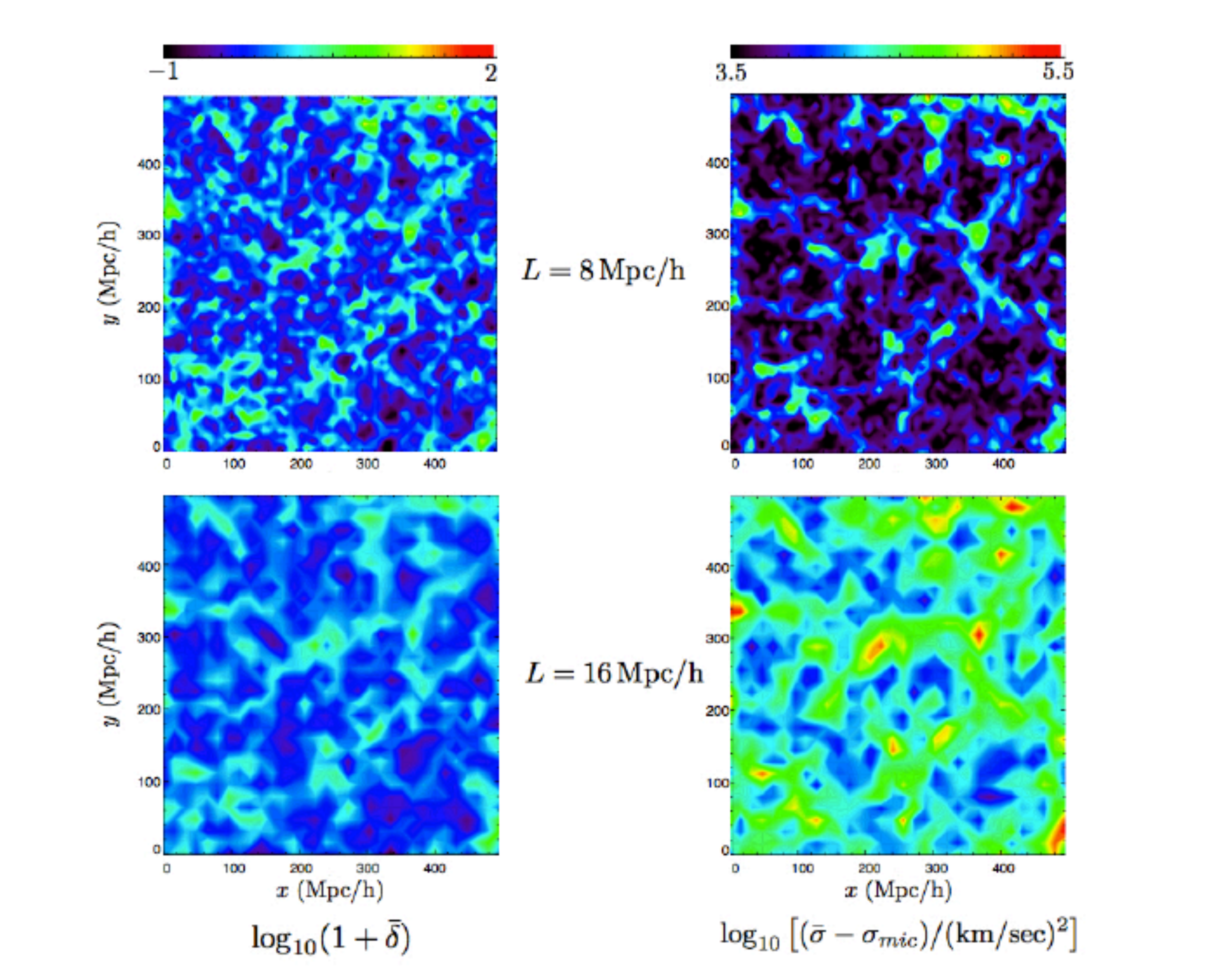
need $\langle \varphi_{a_1}^{(0)} \cdots \varphi_{a_n}^{(0)} h_{b_1} \cdots h_{b_m} \rangle$ **correlators !**

test: compute the sources in 1-loop PT

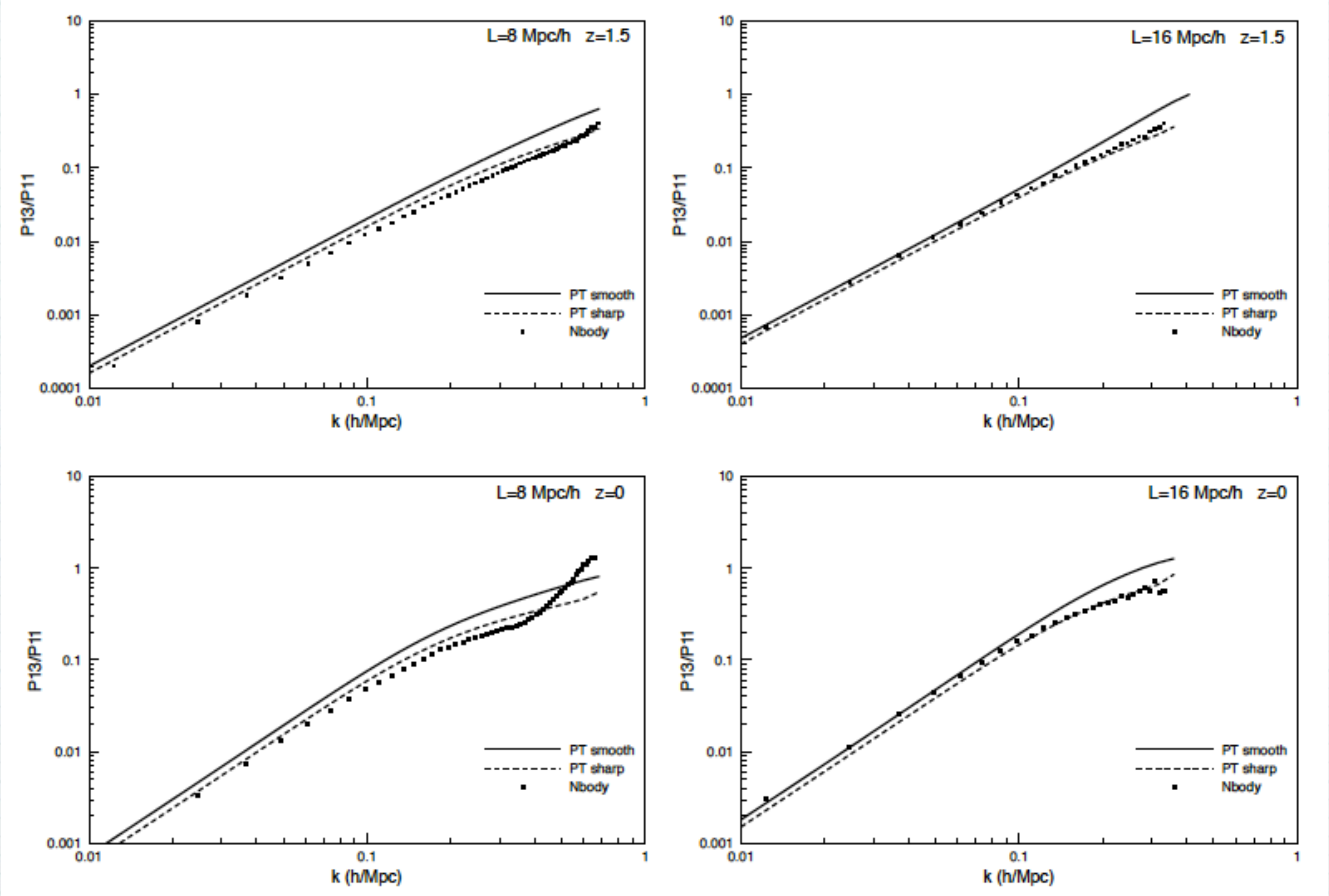


$\delta - \sigma^{ij}$ correlation seen in 1-loop PT!

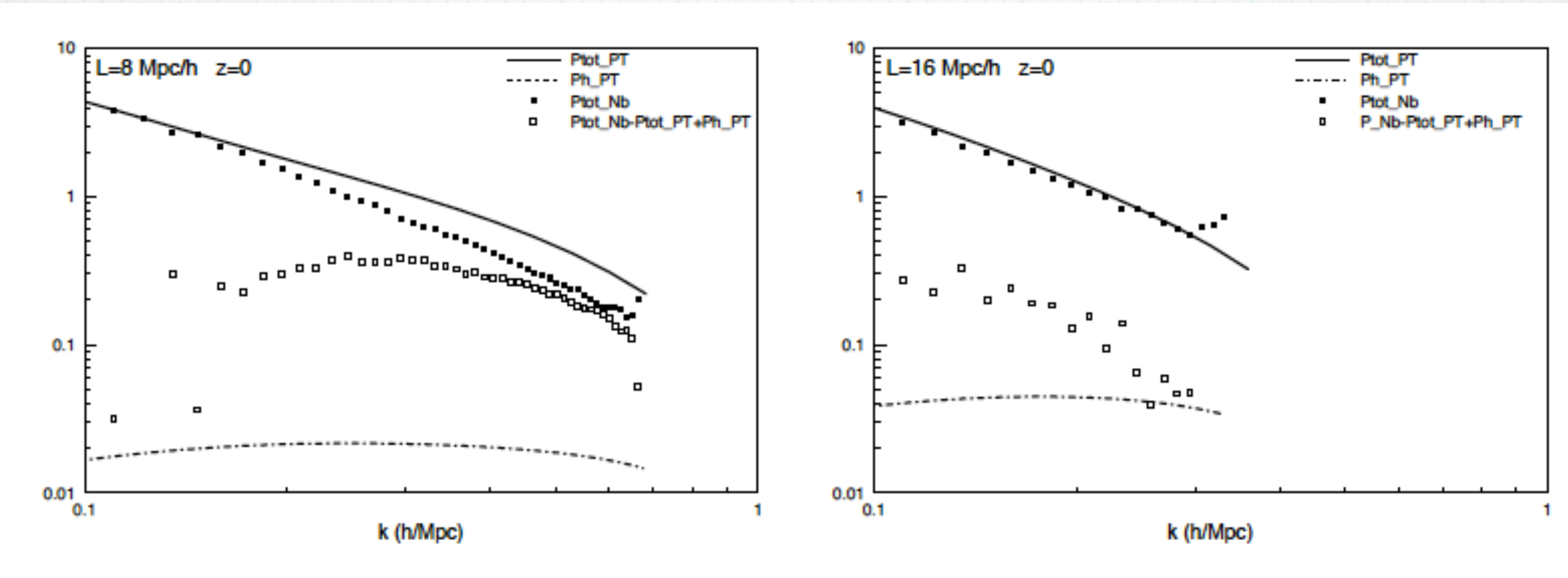
density CG-induced vel. dispersion



cut-off dependence alleviated in ratios



non-perturbative content of sources is leading



next: read the sources from N-body simulations.
How cosmology-independent is short distance physics?

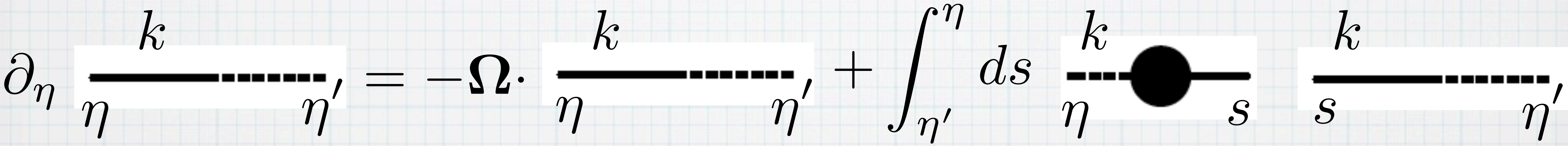
Summary

* (resummed) PT is an essential tool to explore cosmological models in the mildly non-linear regime
* the % level accuracy (for the DM PS!) is at reach in the BAO region
* to progress, PT must take conscience of its limits: the coarse graining scale can be exploited as a computational tool
* next: be Wilsonian, let L flow!

Time as the flow parameter

$$\partial_\eta G(k;\eta,\eta') = -\Omega \cdot G(k;\eta,\eta') + \int_{\eta'}^{\eta} ds\, \Sigma(k;\eta,s)\cdot G(k;s,\eta')$$

exact evolution equation for the propagator



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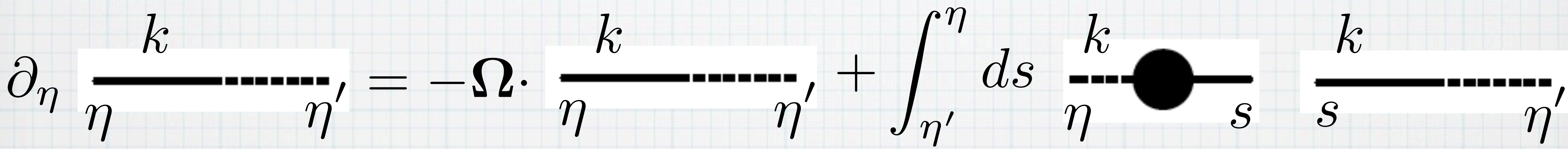
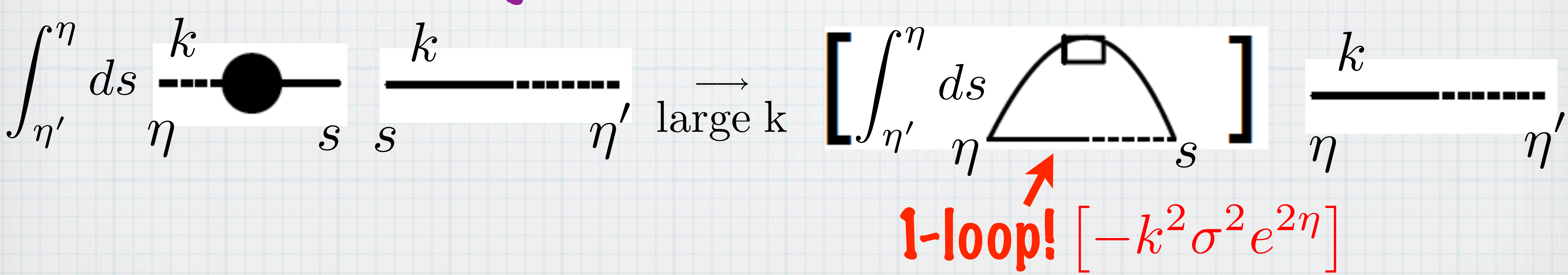


large-momentum factorization



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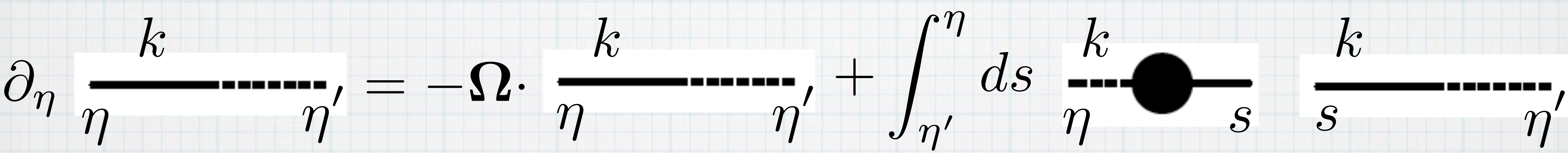

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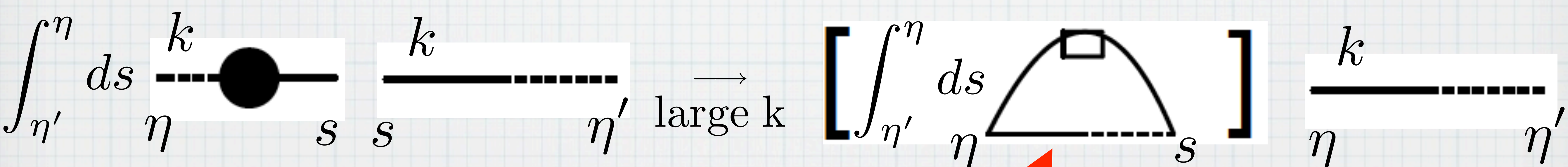

1-loop! $[-k^2\sigma^2 e^{2\eta}]$

reproduce the Crocce-Scoccimarro resummation: $G = e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$

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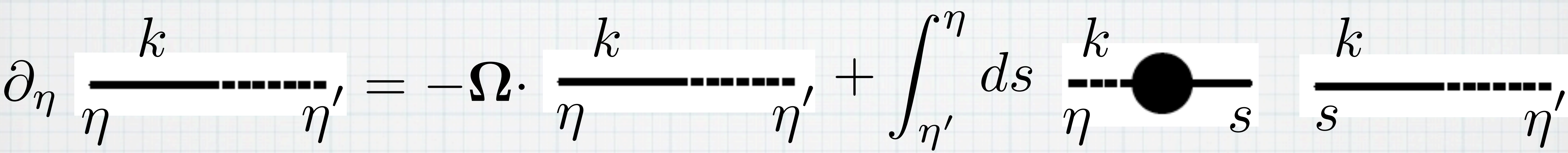

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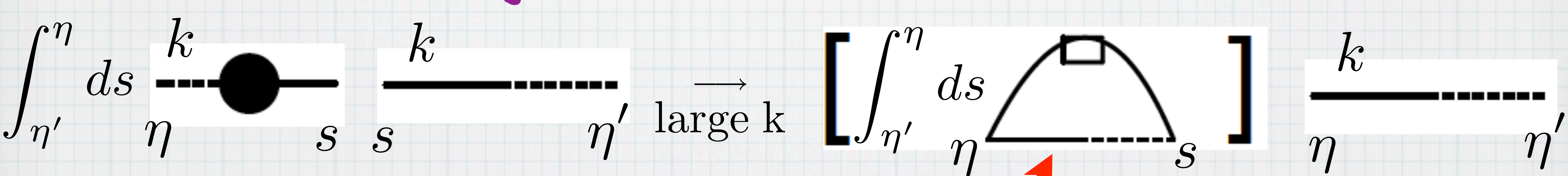

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Beyond CS: Anselmi, Matarrese, MP. 1011.4477

Advantages

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Works also for cosmologies with $\Omega_{ab} = \Omega_{ab}(k,\, \eta)$

not only for $\Omega_{ab} = \begin{pmatrix} 1 & -1 \\ -3/2 & 3/2 \end{pmatrix}$

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Power spectrum ($\langle \varphi \varphi \rangle$) and bispectrum ($\langle \varphi \varphi \varphi \rangle$) from a single run!

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Systematic approximation scheme straightforward

More General Cosmologies

$$\frac{\partial\,\delta}{\partial\,\tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0\,,$$

$$\frac{\partial\,\mathbf{v}}{\partial\,\tau} + \mathcal{H}(1+A(\vec{x},\,\tau))\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\phi\,,$$

$$\nabla^2\phi = 4\pi G\,(1+B(\vec{x},\,\tau))\,\rho\,a^2\,\delta$$

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deviation from geodesic
(e.g. DM-scalar field interaction)

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(e.g. scale-dep. growth factor)

$$(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta,\mathbf{k}))\,\varphi_b(\eta,\mathbf{k}) = e^\eta \gamma_{abc}(\mathbf{k},-\mathbf{k_1},-\mathbf{k_2})\,\varphi_b(\eta,\mathbf{k_1})\,\varphi_c(\eta,\mathbf{k_2})$$

$$(\eta = \log a)$$

$$\mathbf{\Omega}_{ab} = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(1+B(\eta,\mathbf{k})) & 2 + \frac{\mathcal{H}'}{\mathcal{H}} + A(\eta,\mathbf{k}) \end{pmatrix}$$

Ex: Scalar-Tensor: $\mathcal{A} = \alpha\, d\varphi/d\log a$ $\quad \mathcal{B} = 2\alpha^2$ $\quad \alpha^2 = 1/(2\omega+3)$

see Saracco, MP, Tetradis, Pettorino, Robbers '09