Asymptotically Safe Gravity
From Euclidean to Lorentzian Signature

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Renormalization Group Approach – from Ultra Cold Atoms to the Hot QGP

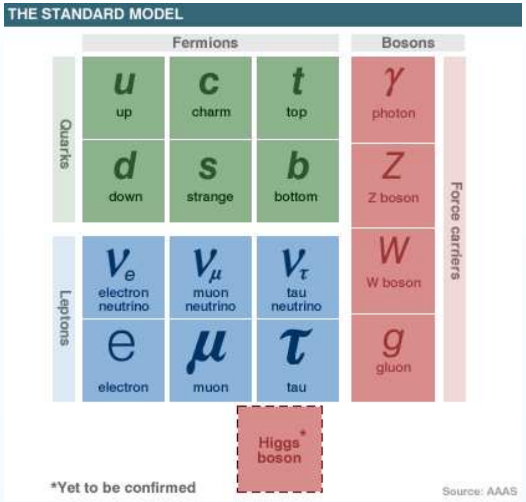
Kyoto, September 2nd, 2011

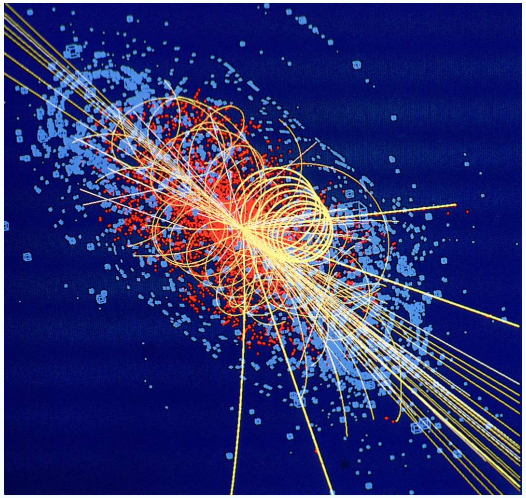
Outline

- Motivation for Quantum Gravity
- Foundations of Asymptotic Safety
- Functional Renormalization Group Equations: Part I
    - covariant construction
    - Einstein-Hilbert results
    - Higher-derivative summary
- Functional Renormalization Group Equations: Part II
    - causal construction
    - Einstein-Hilbert results
- Conclusion and perspectives

Introduction

standard model of particle physics:





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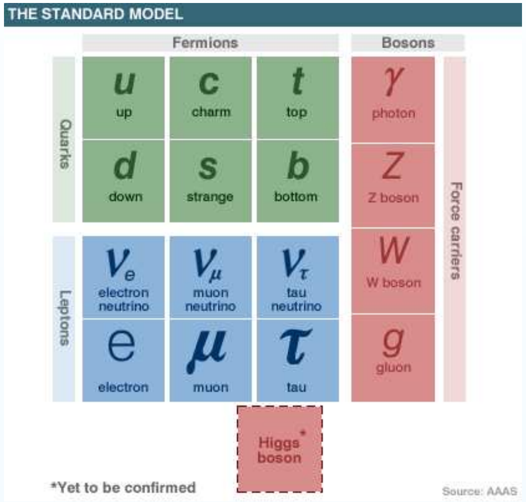

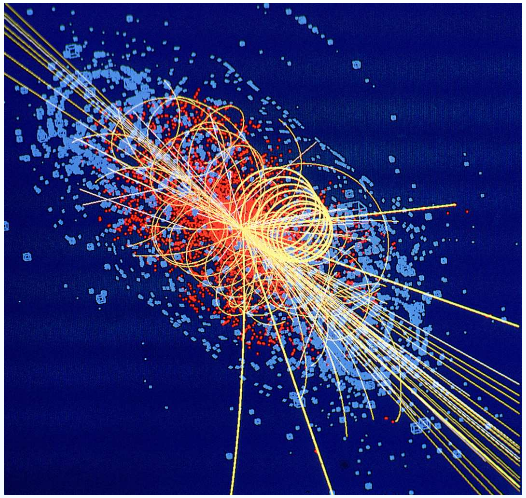

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theoretical basis: quantum field theory in flat, non-dynamical space-time

- includes only relevant and marginal couplings

  $\Longrightarrow$ renormalizable quantum field theory

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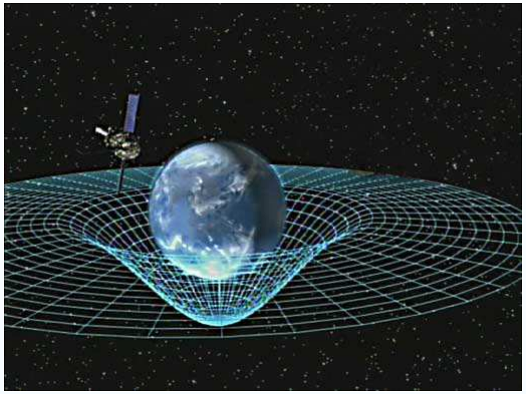

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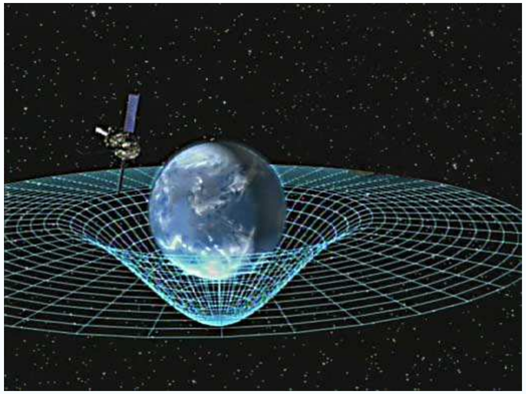

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theoretical basis: classical theory in curved, dynamical space-time

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{space-time curvature}} = \underbrace{-\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}}_{\text{matter content}}$$

- Newton constant G_N has negative mass-dimension

Gravity: Perturbative quantization

Length scale for Quantum Gravity Effects:

$$\text{Planck scale:} \quad \ell_{\rm Pl} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}{\rm cm}\,;\; m_{\rm Pl} = 10^{19}{\rm GeV}$$

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$$S^{\rm EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,\{-R+2\Lambda\}$$

- G_N has negative mass-dimension $\Leftrightarrow$ infinite number of counterterms

gravity + scalar: $\Delta S^{1-{\rm loop}} \propto \int d^4x\sqrt{g}\left\{C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu}\right\}$

pure gravity: $\Delta S^{2-{\rm loop}} \propto \int d^4x\sqrt{g}\left\{C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\alpha\beta}C_{\alpha\beta}{}^{\mu\nu}\right\}$

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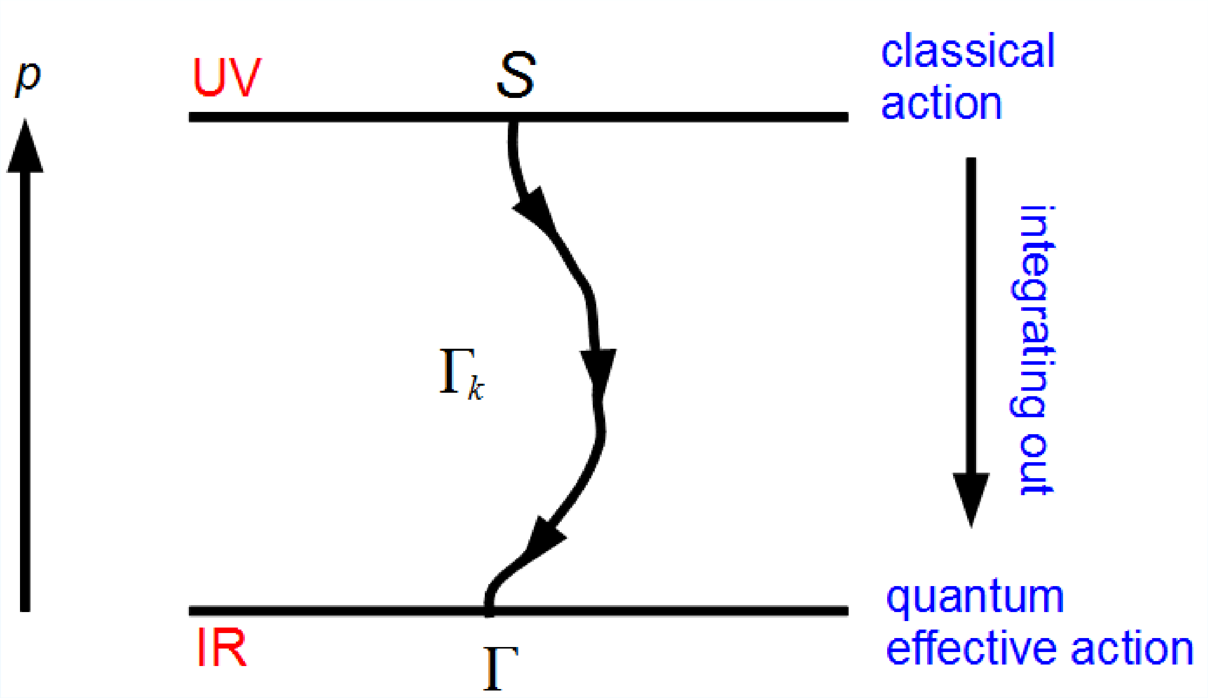
Renormalizing

the Non-Renormalizable

Wilson's modern picture of renormalization

central idea: integrate out quantum fluctuations shell-by-shell in momentum-space



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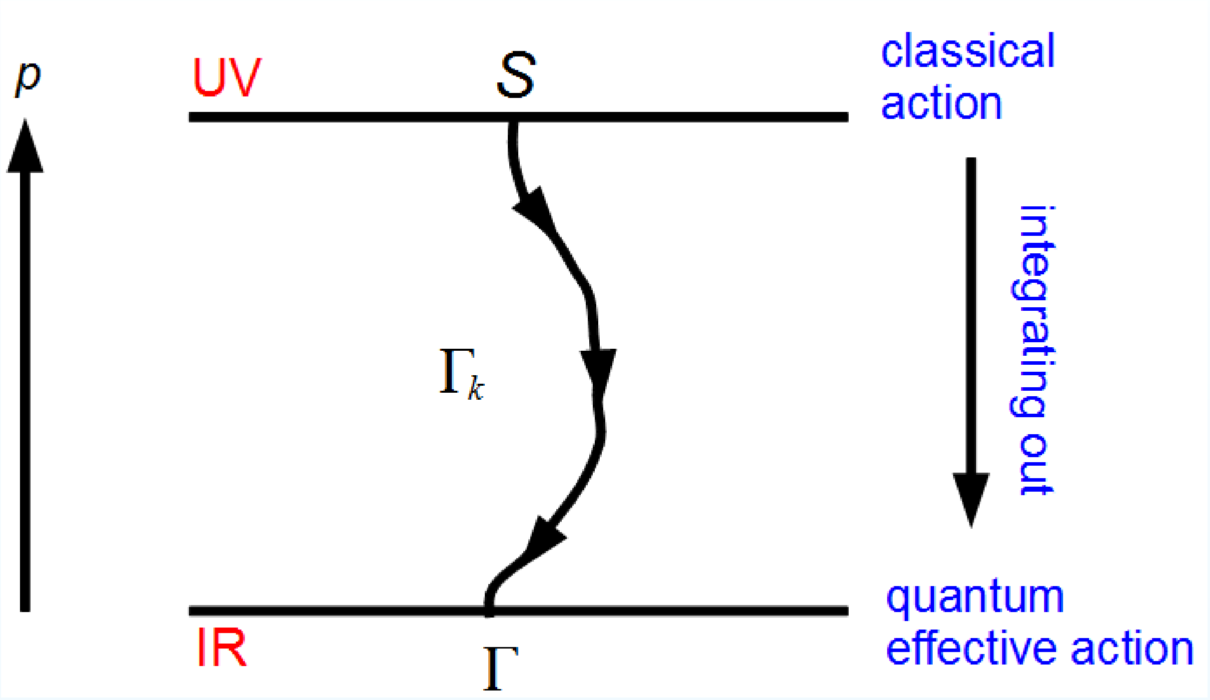

implementation:

- action with scale-dependent couplings $(G_N, \Lambda, \ldots)$: $g_i(k)$
- scale-dependence governed by β-functions: $k\partial_k g_i = \beta_{g_i}(\{g_i\})$

Ensuring good UV-behavior: fixed points of the RG-flow

amplitudes depend on dimensionless couplings only

- RG-flow for dimensionless running couplings: $g_i(k)$

Fixed points g_i^*:

- β-functions vanish: $\beta_{g_i}(\{g_i^*\}) \stackrel{!}{=} 0$

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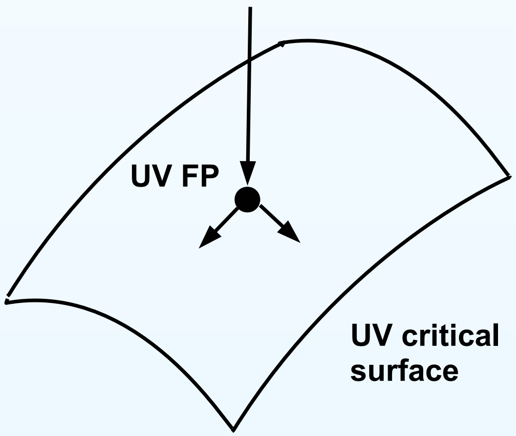
Concepts associated with UV-fixed points:

- trajectories emanating from fixed point in UV
  $\equiv$ span UV critical surface
- predictivity:
  $\equiv$ UV critical surface has finite dimension



Renormalization: asymptotic freedom and asymptotic safety

Wilsonian formulation:

- UV fixed points allow two classes of renormalizable Quantum Field Theories
- Gaussian Fixed Point (GFP):
  - perturbatively renormalizable field theories
  - UV-limit: free theory
  - asymptotic freedom (example: QCD)

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> Wilsonian picture: generalization of perturbative renormalization
>
> asymptotic safety as predictive as asymptotic freedom

Renormalizing gravity

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Gravity

> Weinberg's asymptotic safety conjecture (1979):
>
> gravity in $d = 4$ has non-Gaussian UV fixed point

Functional Renormalization Group Equations I

covariant construction

Covariant functional RG equation for gravity

Theory: degrees of freedom: metric field $g_{\mu\nu}$

symmetries: coordinate transformations (diffeomorphisms)

Functional Renormalization group equation:

- Wetterich equation for effective average action Γ_k

  [C. Wetterich, Phys. Lett. **B301** (1993) 90]

- adapted to gravity

  [M. Reuter, Phys. Rev. D **57** (1998) 971, hep-th/9605030]

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- starting point: generic diffeomorphism invariant action $S^{\rm grav}[g_{\mu\nu}]$
- background covariance $\Longleftrightarrow$ background field formalism
  - quantum field is split into (fixed) background value + arbitrary fluctuation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

  - implement gauge-fixing term:

$$S^{\rm gf} = \frac{1}{2\alpha} \int d^4x \sqrt{\bar{g}} F_\mu \bar{g}^{\mu\nu} F_\nu \, , \quad F_\mu = \bar{D}^\mu h_{\mu\nu} - \beta \bar{D}_\mu h$$

  - and corresponding ghost action

$$S^{\rm gh}[h, C, \bar{C}; \bar{g}] = -\sqrt{2} \int d^4x \sqrt{\bar{g}} \, \bar{C}_\mu \, \mathcal{M}^\mu{}_\nu \, C^\nu$$

$$\mathcal{M}^\mu{}_\nu = \bar{g}^{\mu\rho} \bar{g}^{\sigma\lambda} \bar{D}_\lambda (g_{\rho\nu} D_\sigma + g_{\sigma\nu} D_\rho) - \bar{g}^{\rho\sigma} \bar{g}^{\mu\lambda} \bar{D}_\lambda (g_{\sigma\nu} D_\rho)$$

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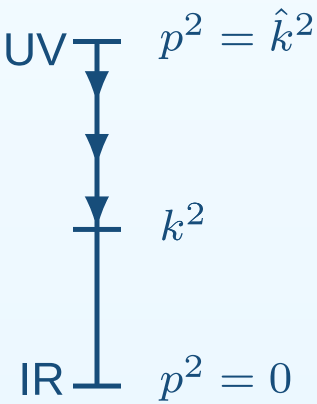
- add: k-dependent IR cutoff:

$$\Delta_k S[h;\bar{g}] = \int d^4x \sqrt{\bar{g}}\, \{h_{\mu\nu} \mathcal{R}_k[\bar{g}]^{\mu\nu\rho\sigma} h_{\rho\sigma} + \ldots\}$$

  - $\mathcal{R}_k[\bar{g}] \propto \mathcal{Z}_k k^2 R^{(0)}$ = k-dependent mass term
  - discriminates low/high- $\bar{D}^2$-eigenmodes

$$R^{(0)}(p^2/k^2) = \begin{cases} 0 & p^2 \gg k^2 \\ 1 & p^2 \ll k^2 \end{cases}$$



- high momentum modes: integrated out
- low momentum modes: suppressed by mass term

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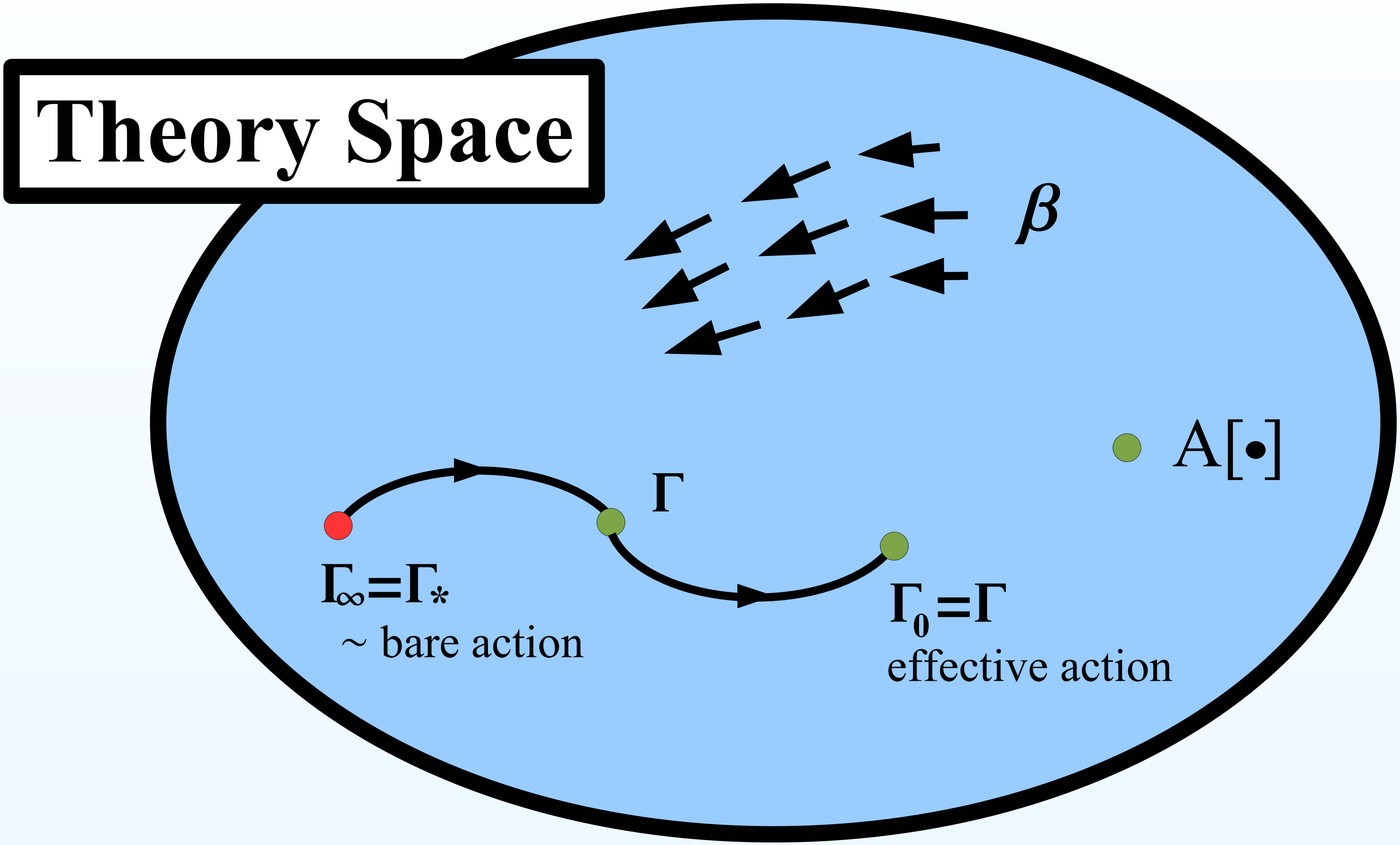
- exact RG equation for Γ_k:

$$k\partial_k \Gamma_k[h;\bar{g}] = \tfrac{1}{2} {\rm STr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right]$$

  - $\Gamma_k^{(2)}$ = Hessian with respect to fluctuation fields
  - “extra” $\bar{g}$-dependence necessary for formulating exact equation

Theory space underlying the Functional Renormalization Group



Non-perturbative approximation: derivative expansion of Γ_k

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- expand Γ_k in derivatives and truncate series:

$$\Gamma_k[\Phi] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[\Phi]$$

$\Longrightarrow$ substitute into FRGE

$\Longrightarrow$ projection of flow gives β-functions for running couplings

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

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- testing the reliability:
  - within a given truncation:
    cutoff-scheme dependence of physical quantities (= vary $\mathcal{R}_k$)
  - stability of results within extended truncations

Letting things flow

The Einstein-Hilbert truncation

The Einstein-Hilbert truncation: setup

Einstein-Hilbert truncation: two running couplings: $G(k), \Lambda(k)$

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^4x\sqrt{g}\,[-R + 2\Lambda(k)] + S^{\rm gf} + S^{\rm gh}$$

- project flow onto G-$\Lambda-$plane

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explicit β-functions for dimensionless couplings $g_k := k^2 G(k)\,,\ \lambda_k := \Lambda(k) k^{-2}$

- Particular choice of $\mathcal{R}_k$ (optimized cutoff)

$$k\partial_k g_k = (\eta_N + 2) g_k\,,$$

$$k\partial_k \lambda_k = -\,(2 - \eta_N)\,\lambda_k - \frac{g_k}{2\pi}\left[5\,\frac{1}{1-2\,\lambda_k} - 4 - \frac{5}{6}\,\frac{1}{1-2\lambda_k}\eta_N\right]$$

- anomalous dimension of Newton's constant:

$$\eta_N = \frac{gB_1}{1 - gB_2}$$

$$B_1 = \frac{1}{3\pi}\left[5\,\frac{1}{1-2\lambda} - 9\,\frac{1}{(1-2\lambda)^2} - 7\right]\,,\ B_2 = -\frac{1}{12\pi}\left[5\,\frac{1}{1-2\lambda} + 6\,\frac{1}{(1-2\lambda)^2}\right]$$

Einstein-Hilbert truncation: Fixed Point structure

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  - UV attractive in g_k, λ_k

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Asymptotic safety: non-Gaussian Fixed Point is UV completion for gravity

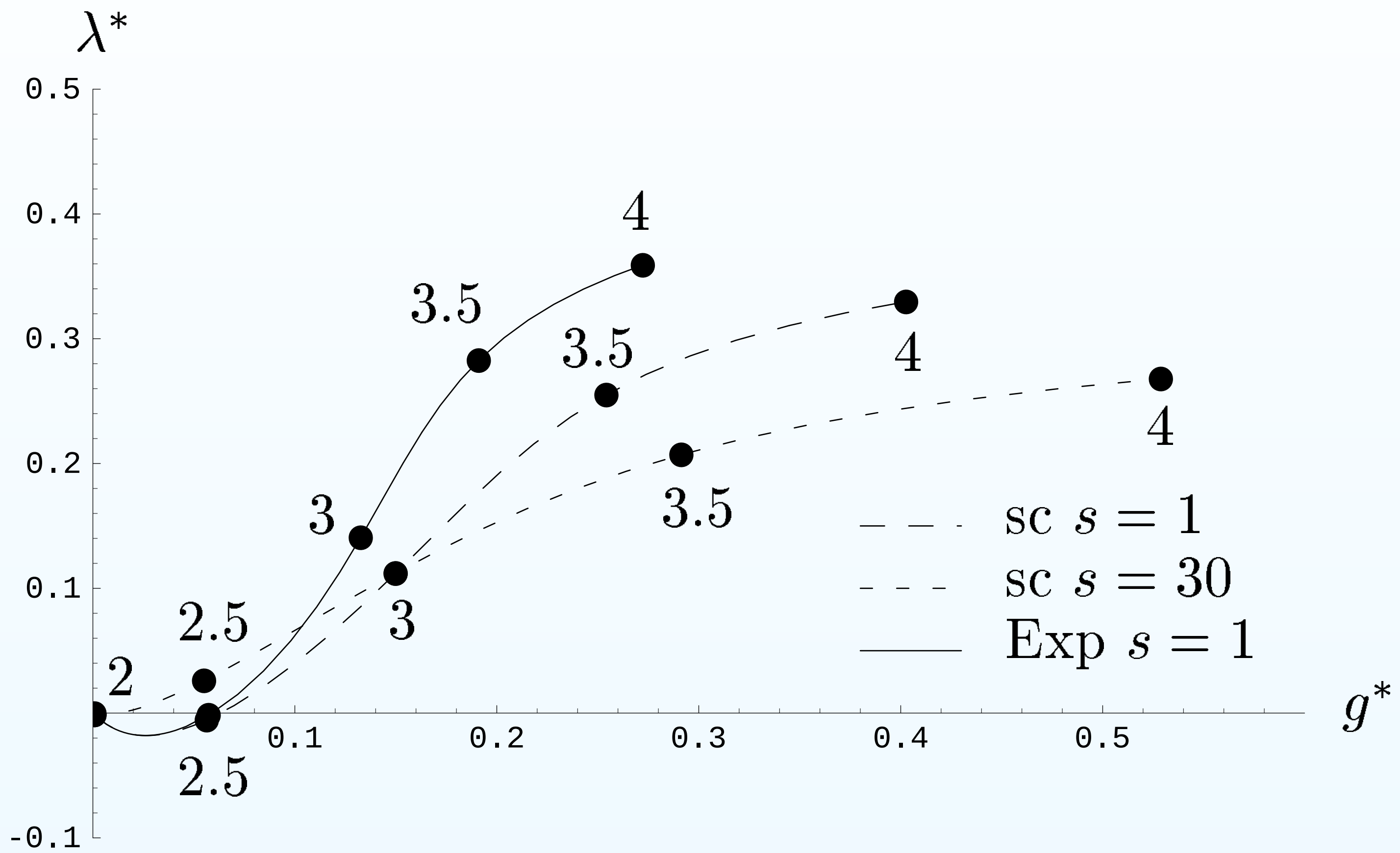
Einstein-Hilbert truncation: Stability properties

Ref.	g^*	λ^*	$g^*\lambda^*$	$\theta' \pm i\theta''$	gauge	$\mathcal{R}_k$
BMS	0.902	0.109	0.099	$2.52 \pm 1.78i$	geometric	II, opt
RS	0.403	0.330	0.133	$1.94 \pm 3.15i$	harmonic	I, sharp
LR	0.272	0.348	0.095	$1.55 \pm 3.84i$	harmonic	I, exp
	0.344	0.339	0.117	$1.86 \pm 4.08i$	Landau	I, exp
L	1.17	0.25	0.295	$1.67 \pm 4.31i$	Landau	I, opt
CPR	0.707	0.193	0.137	$1.48 \pm 3.04i$	harmonic	I, opt
	0.556	0.092	0.051	$2.43 \pm 1.27i$	harmonic	II, opt
	0.332	0.274	0.091	$1.75 \pm 2.07i$	harmonic	III, opt

BMS: Benedetti, Machado, Saueressig, 2009.
RS: Reuter, Saueressig, 2002.
LR: Lauscher, Reuter, 2002.
L: Litim, 2004.
CPR: Codello, Percacci, Rahmede, 2009.

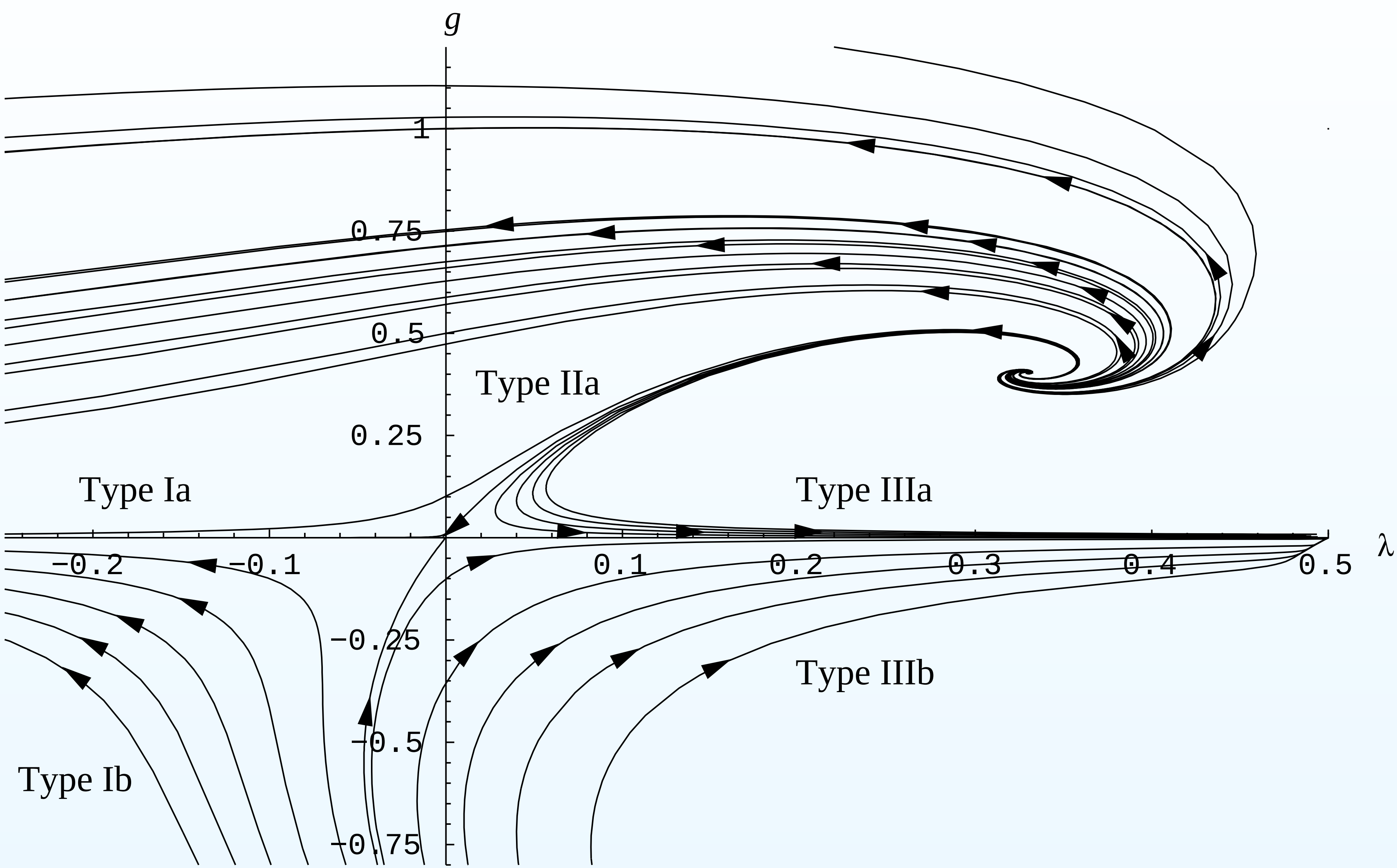
Einstein-Hilbert truncation: NGFP in $d = 2 + \epsilon$

β-functions continuous in $d \Longleftrightarrow$ reproduce perturbative fix point in $d = 2 + \epsilon$



NGFP in $d = 4 \Longleftrightarrow$ analytic continuation of NGFP in $d = 2 + \epsilon$

Einstein-Hilbert-truncation: the phase diagram



Letting things flow

Higher-derivative terms

Charting the theory space of gravity

Einstein-Hilbert truncation

$\vdots$			
R^8	$\dots$		
R^7	$\dots$		
R^6	$\dots$		
R^5	$\dots$		
R^4	$\dots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
$\mathbb{1}$			

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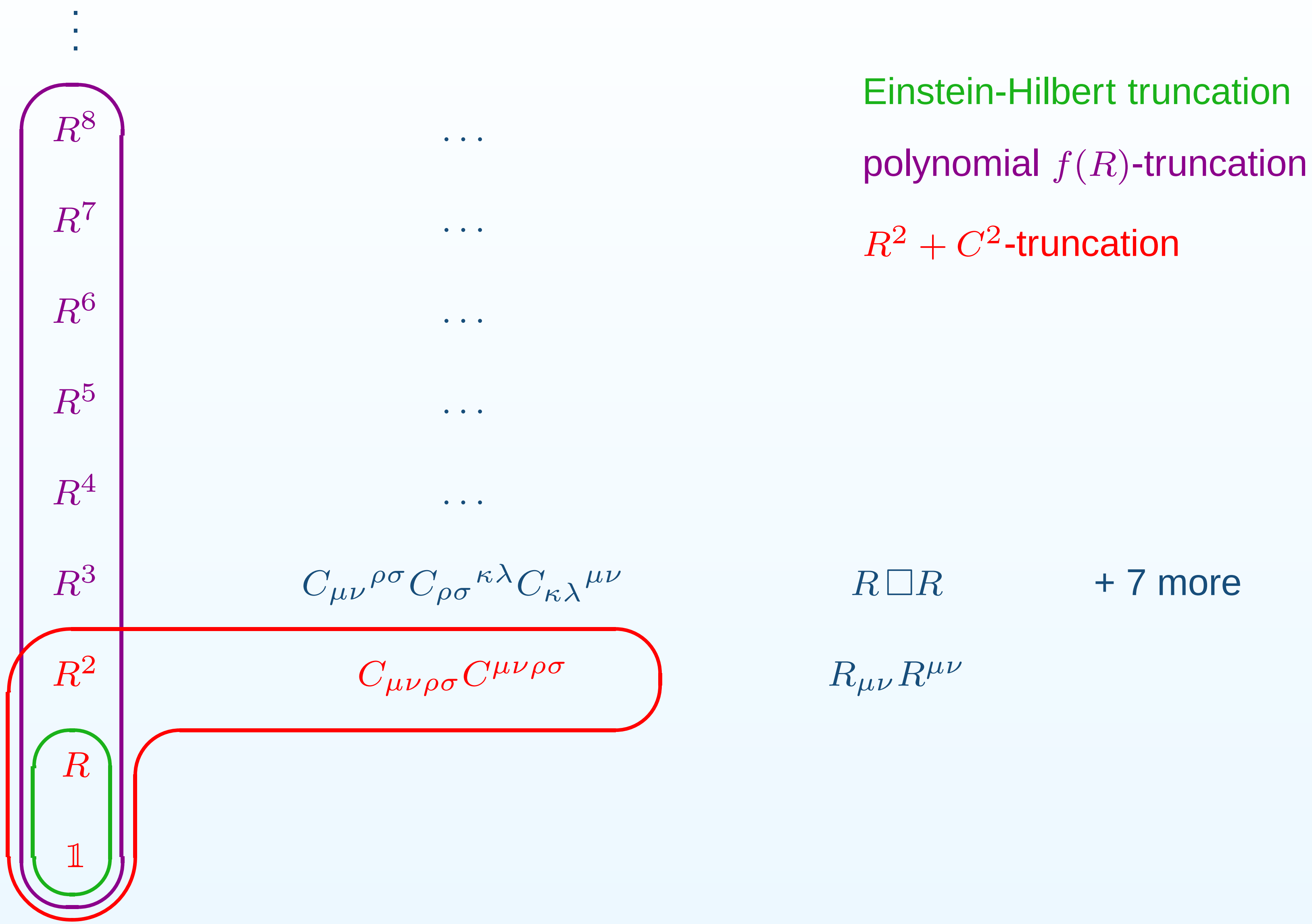

Exploring the gravitational theory space

Some key results . . .

- evidence for asymptotic safety
  - ⇒ non-Gaussian fixed point provides UV completion of gravity
- finite dimensional UV-critical surface
  - ⇒ possibly: 3 relevant parameters
- perturbative counterterms:
  - gravity + matter: asymptotic safety survives 1-loop counterterm

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. . . and open questions:

- Existence of NGFP in extended truncations?
- Dimension of its UV-critical surface?

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Some key results . . .

- evidence for asymptotic safety
  - ⇒ non-Gaussian fixed point provides UV completion of gravity
- finite dimensional UV-critical surface
  - ⇒ possibly: 3 relevant parameters
- perturbative counterterms:
  - gravity + matter: asymptotic safety survives 1-loop counterterm

. . . and open questions:

- Existence of NGFP in extended truncations?
- Dimension of its UV-critical surface?

How does the signature of space-time

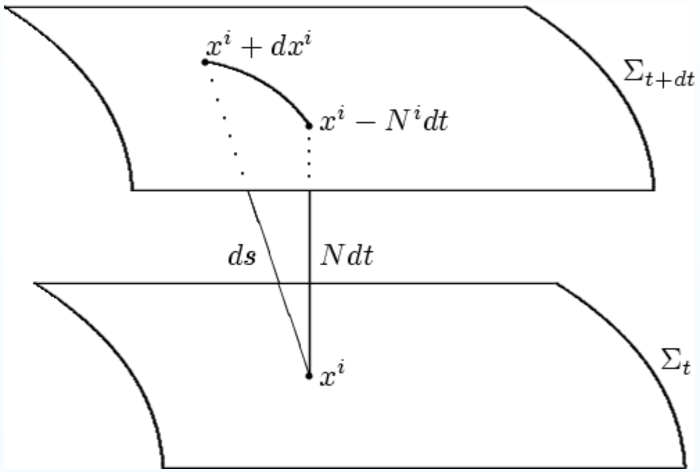
affect asymptotic safety?

Functional Renormalization Group Equations II

foliated space-times

Foliation structure via ADM-decomposition

Preferred “time”-direction via foliation of space-time



- foliation structure $\mathcal{M}^{d+1} = S^1 \times \mathcal{M}^d$ with $y^\mu \mapsto (\tau, x^a)$:

$$ds^2 = N^2 dt^2 + \sigma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

- fundamental fields: $g_{\mu\nu} \mapsto (N, N_i, \sigma_{ij})$

$$g_{\mu\nu} = \begin{pmatrix} N^2 + N_i N^i & N_j \\ N_i & \sigma_{ij} \end{pmatrix}$$

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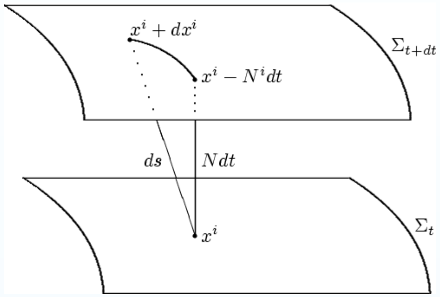

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$$ds^2 = \epsilon N^2 dt^2 + \sigma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

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Allows to include signature parameter $\epsilon = \pm 1$

Foliated functional renormalization group equation

Theory: degrees of freedom: component fields N, N_i, σ_{ij}

symmetries: diffeomorphisms (full or foliation preserving)

Construction of the flow equation:

- starting point: generic diffeomorphism invariant action $S^{\mathrm{grav}}[N, N_i, \sigma_{ij}]$
  - diffeomorphism invariant or Horava-type

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gauge-fixing: Background field formalism for (N, N_i, σ_{ij}):

$$N = \bar{N} + h \, , \qquad N_i = \bar{N}_i + h_i \, , \qquad \sigma_{ij} = \bar{\sigma}_{ij} + h_{ij}$$

- choice of backgrounds: $\bar{N} = 1, \bar{N}_i = 0$

$\Longrightarrow$ admit temporal gauge: $h = 0, h_i = 0$

$$S^{\rm gf} = \frac{1}{2\alpha} \sqrt{\epsilon} \int d\tau \int d^3x \sqrt{\bar{\sigma}} \left\{ h^2 + \bar{\sigma}^{ij} h_i h_j \right\}$$

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ghost action:

$$S^{\rm gh} = \sqrt{\epsilon} \int d\tau \int d^3x \sqrt{\bar{\sigma}} \left\{ \bar{C} \partial_\tau C + \bar{C}_i \partial_\tau C^i \right\}$$

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k-dependent IR-cutoff $\Delta_k S$

$$\Delta_k S[h; \bar{\sigma}] = \sqrt{\epsilon} \int d\tau \int d^3x \sqrt{\bar{\sigma}} \; \{ h_{ij} \mathcal{R}_k[\bar{\sigma}] h^{ij} + \ldots \}$$

- $\mathcal{R}_k$: depends on spatial Laplacian Δ only!
  - $\Delta = -\bar{\sigma}^{ij} \bar{D}_i \bar{D}_j$ is positive definite
  - Time-like fluctuations: regulated by circle
  - cutoff respects foliation-preserving diffeomorphisms only

    $\Rightarrow$ explore RG-flows in Horava gravity

Foliated functional renormalization group equation

Flow equation: formally the same as in covariant construction

$$k\partial_k \Gamma_k[h, h_i, h_{ij}; \bar{\sigma}_{ij}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

- covariant: $\mathcal{M}^4$

$$\mathrm{STr} \approx \sum_{\text{fields}} \int d^4y \sqrt{\bar{g}}$$

- foliated: $S^1 \times \mathcal{M}^3$

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Advantages of the foliated flow equation:

- limits: same as covariant equation
- ϵ-dependence: keep track of signature effects
- structure: same as lattice approach of CDT

signature-dependent renormalization group flows

Einstein-Hilbert truncation

ADM-decomposed Einstein-Hilbert truncation: setup

ADM-decomposed Einstein-Hilbert truncation: running couplings: G_k, Λ_k

$$\Gamma_k^{\rm ADM} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int d\tau d^3x N\sqrt{\sigma} \left\{ \epsilon^{-1} K_{ij} \left[\sigma^{ik}\sigma^{jl} - \sigma^{ij}\sigma^{kl}\right] K_{kl} - R^{(3)} + 2\Lambda_k \right\} + S^{\rm gf} + S^{\rm gh}$$

- K_{ij}: extrinsic curvature
- $R^{(d)}$: intrinsic curvature
- $\epsilon = \pm 1$: signature parameter

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Structure of the flow equation

$$k\partial_k \Gamma_k = T^{\rm TT} + T^0$$

$$T^{\rm TT} = \frac{\sqrt{\epsilon} k^3 d_{\rm 2T}}{(4\pi)^{3/2}} \sum_n \int d^3x \sqrt{\bar{\sigma}} \left[q^{1,0}_{3/2}(w_{\rm 2T}) + \frac{\bar{R}}{k^2} \left(\frac{1}{6} q^{1,0}_{1/2}(w_{\rm 2T}) - \frac{2}{3} q^{2,0}_{3/2}(w_{\rm 2T}) \right) \right]$$

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β-functions:

$$k\partial_k g_k = \beta_g(g, \lambda; m), \qquad k\partial_k \lambda_k = \beta_\lambda(g, \lambda; m)$$

- depend parametrically on dimensionless Kaluza-Klein-mass $m = \frac{2\pi}{Tk}$

Analyticity properties of β-functions

Kaluza-Klein sums: carry out analytically:

$$\sum_n q^{1,0}_{d/2}(w_{2\mathrm{T}}) \propto \sum_n \frac{1}{1+\frac{1}{2\epsilon}m^2 n^2 - 2\lambda_k}$$

Summation: depends on signature ϵ:

$$\sum_n \frac{1}{n^2+x^2} = \frac{\pi}{x\,\tanh(\pi x)}, \quad x^2 > 0 \quad \text{(hyperbolic functions)}$$

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analytic structure of β-functions: determined by ϵ, λ:

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No analogue of the Non-Gaussian fixed Point!

NGFP (Part II): running Kaluza-Klein mass

Scenario II: $T \propto k^{-1} \quad \Longleftrightarrow \quad \lim_{k\to\infty} m_k = m^* \neq 0 \qquad (\text{sequel} : m = 2\pi)$

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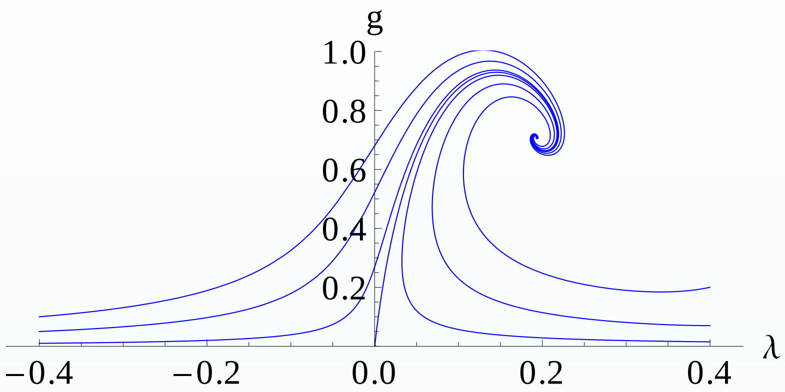
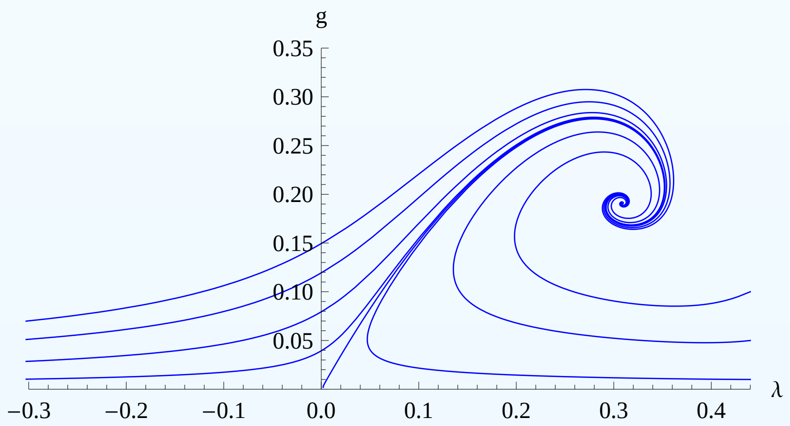
stability coefficients: almost the same!

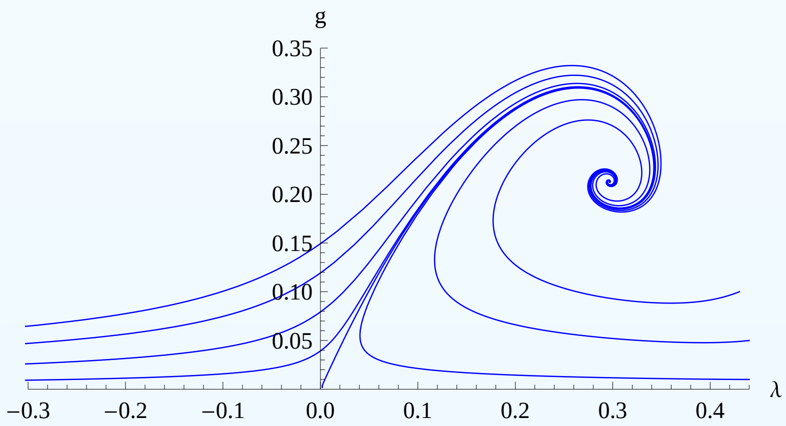
Comparison: phase diagrams



covariant computation



Euclidean



Lorentzian

Conclusion and Perspectives

novel causal functional renormalization group equation

- symmetries: foliation-preserving diffeomorphism
- applications:
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gravity in UV

signature does not affect asymptotic safety