# Probing QCD phase diagram with charge fluctuations

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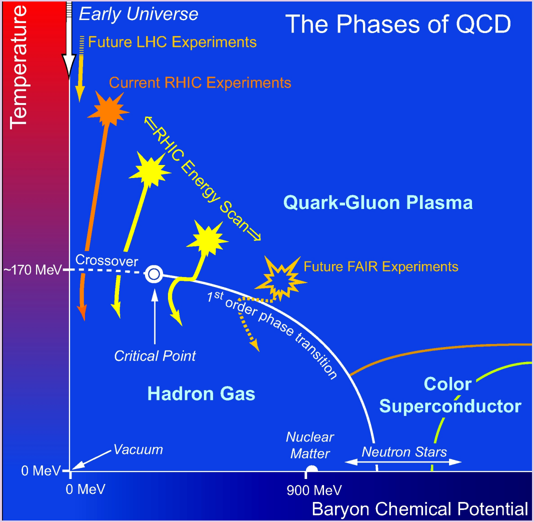
# Outline

- Introduction and motivation
- Fluctuations of conserved charges
- O(4) scaling and fluctuations
- Experimental measurement: ratios of cumulants
- Probabilities: comparison to experiment

# QCD PHASE DIAGRAM




## QCD phase diagram can be studied by

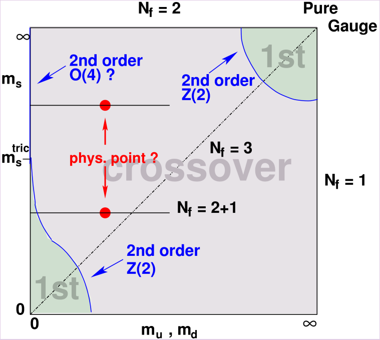
- first-principle QCD calculations
- experiment
- model calculations

## Phase transitions

- crossover at small $\mu$, $T \approx 165$ MeV, O(4) scaling
- critical end-point (CEP), 3d Z(2) universality class [*expected*]
- first-order phase transition [*expected*]

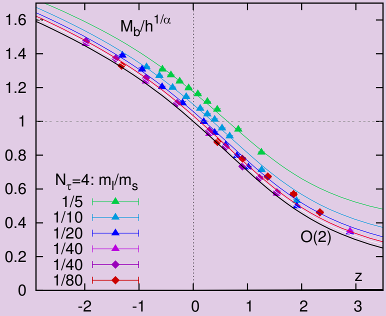
# QCD phase diagram: quark mass dependence

$\mu = 0$



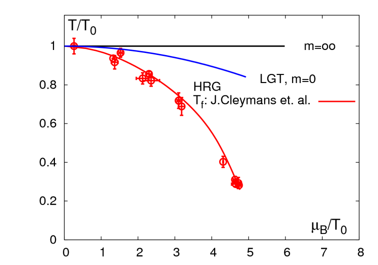

- QCD at physical $m_q$?
- $m_l \to 0$: O(4) or Z(2)?

F. Karsch et. al. LGT QCD:




- QCD at physical $m_\pi$ in O(4) scaling
- $N_\tau = 8$ supports these results

# QCD PHASE DIAGRAM: ADDITIONAL FACTS




F. Karsch '10

- Experiment + HRG model: freeze-out curve

$$\begin{aligned} T(\mu_B) &= a - b\mu_b^2 - c\mu_B^4 \\ \mu_B &= d/(1 + e\sqrt{s_{NN}}) \\ T/T_c &\approx 1 - 0.023(\mu_B/T)^2 \end{aligned}$$

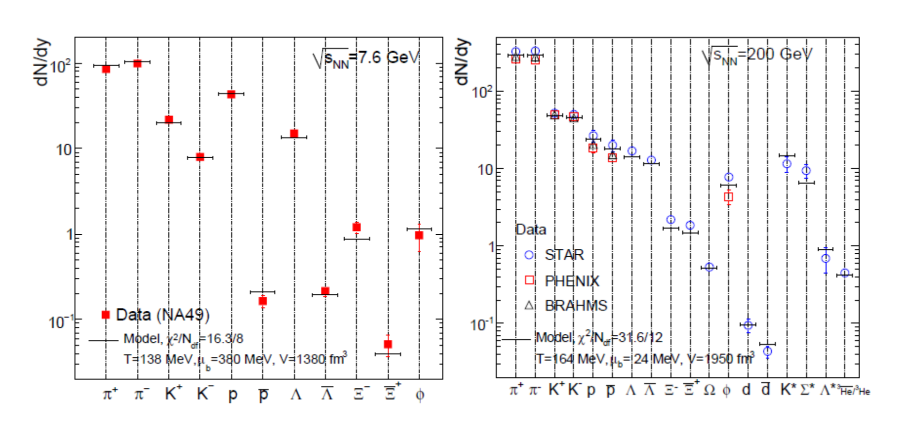
- LGT QCD: curvature of crossover line at small $\mu_B$ ($\kappa_q \approx 0.06$)

$$T/T_c = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2 - \mathcal{O}(\mu_q^4)$$

- Is there any relation between freeze-out and crossover lines?

**Hadron Resonance Gas (HRG) model:**

$$p_{\mathrm{HRG}}(T,\mu_B,\mu_S,\mu_Q) = \sum_{\mathrm{mesons}} p_i + \sum_{\mathrm{baryons}} p_i, \quad m_i < 2.5\ \mathrm{GeV}$$

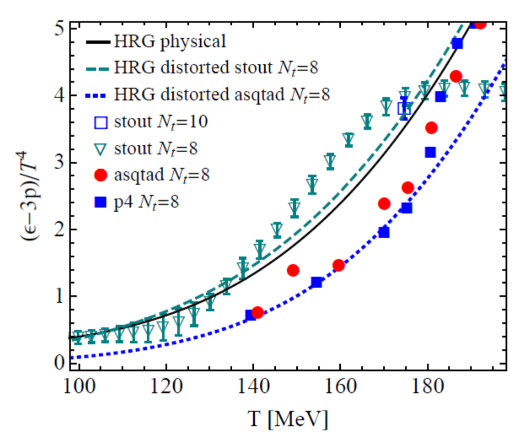



A. Andronic et al.

Particle yields are well described by HRG model

# Hadron resonance gas model vs LGT QCD

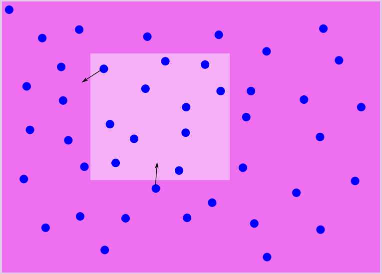
Low temperature lattice QCD results are in agreement with HRG




C. Ratti et. al. '09

# Fluctuations of conserved charges: definition

Assume system in equilibrium with external reservoir with respect to both particle and energy exchange.



Heavy-ion collisions:

- Conserved charges do not fluctuate
- Part of phase space ($p_t$-cuts, rapidity cuts, etc)

- Energy density and number of particles = random variables with a certain $\mathcal{P}$
- $\mathcal{P}(N)$ depends on $T$, $\mu$ and $V$.
- $\mathcal{P}(N)$ characterises by
  - mean $\bar{N} \equiv \langle N \rangle = \sum_N N\mathcal{P}(N)$
  - variance $\sigma^2 = \langle (N - \bar{N})^2 \rangle$
  - higher order moments and/or cumulants $\chi_n$

$$\mathrm{MGF}(y) = \langle e^{Ny} \rangle = 1 + \bar{N}y + \langle N^2 \rangle y^2/2 + \cdots$$

$$\mathrm{CGF}(y) = \ln(\langle e^{Ny} \rangle) = \sum_n \frac{c_n}{n!} y^n$$

$$c_1 = \bar{N}, c_2 = \sigma^2$$

$$c_4 = \langle (N - \bar{N})^4 \rangle - 3\langle (N - \bar{N})^2 \rangle^2$$

# Fluctuations of conserved charges: definition

Probability theory:

$$\mathrm{MGF}(y) = \langle e^{Ny} \rangle = 1+\bar{N}y+\langle N^2\rangle\frac{y^2}{2}+\cdots$$

$$\mathrm{CGF}(y) = \ln(\langle e^{Ny}\rangle) = \sum_n \frac{c_n}{n!}y^n$$

$$\mathcal{P}(N)$$

Thermodynamics, GCE:

$$\mathrm{MGF}(y) = Z_{\mathrm{GC}}(y \equiv \bar{\mu} = \mu/T) = \mathrm{Tr} e^{-\frac{\hat{H}}{T}+\bar{\mu}N}$$

$$\mathrm{CGF}(y) = \ln Z_{\mathrm{GC}}(\bar{\mu}) = \frac{V}{T}p(\bar{\mu}) = (VT^3)\cdot\frac{p(\bar{\mu})}{T^4}$$

$$Z_{GC} = \sum_{N=-\infty}^{\infty} Z_C(N)\exp(\mu N)$$

$$\mathcal{P}(N) = \mathrm{const}(T,\mu)\cdot Z_{\mathrm{C}}(N)e^{\hat{\mu}N}$$

**Experiment:**

$\mathcal{P}(N) \rightsquigarrow \langle N^k \rangle = \sum_N N^k P(N) \rightsquigarrow$ cumulants

**Theory:**

$p(T,\mu) \rightsquigarrow \partial^n/\partial\mu^n \rightsquigarrow \chi_i \cdot (VT^3) \equiv$ cumulants

# CUMULANTS $\propto$ SUSCEPTIBILITIES

- Fluctuations of net-quark number $\chi_n^q$ and net-baryon charge $\chi_n^B$

$$\chi_n^q = \frac{\partial^n (p/T^4)}{\partial(\mu_q/T)^n} \quad | \quad \chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n} = \left(\frac{1}{3}\right)^n \chi_n^q$$

- Fluctuations of electric charge $\chi_n^Q$

$$\chi_n^Q = \frac{\partial^n (p/T^4)}{\partial(\mu_Q/T)^n}$$

- Fluctuations of net-strange number…

**HRG** ($\mu_S = \mu_Q = 0$):

- $p/T^4 = \sum_i f(m_i/T) \cosh(\mu_B/T) + g(T)$

- $\chi_{2n} \propto \cosh(\mu_B/T) \quad \chi_{2n+1} \propto \sinh(\mu_B/T)$

- $\chi_{2n}/\chi_2 = 1 \quad \chi_{2n+1}/\chi_1 = 1$

- $\chi_{2n} > 0$

**Properties:**

- At CEP: for $n \geq 2$, $\chi_n \propto \xi^{n\beta\delta/\nu-3} \approx \xi^{5n/2-3}$, e.g. $\chi_4 \sim \xi^7$ (M. Stephanov '09)
- Diverging $\chi_2$ can signal spinodal decomposition of a non-equilibrium 1st order transition (C. Sasaki et. al. '07)

# Fluctuations of conserved charges: why?

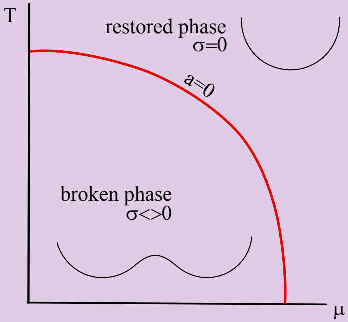
- $m_\pi = 0$ (critical line):
  at $\mu_B = 0$, $\chi_n \propto \xi^{(n+2\alpha-4)/(2\nu)}$ for even $n \geq 6$,
  e.g. $\chi_6 \sim \xi^{1.1}$
  at $\mu_B \neq 0$, $\chi_n \propto \xi^{(n+\alpha-2)/\nu}$ for $n \geq 3$,
  e.g. $\chi_3^B \sim \xi^{1.1}$
- $m_\pi \neq 0$: rapid change in crossover region
  (B. Friman et. al. '11)
- Negative values of high order cumulants close to crossover
  (B. Friman et. al. '11)

Landau theory for 2d-order phase transition ($m_\pi = 0$):

$$\Omega = \frac{a}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4$$

$\sigma$ - order parameter

$$a = \frac{1}{t_0}\left[\left(\frac{T}{T_c} - 1\right) + \kappa_q \left(\mu_q/T\right)^2\right]$$




Minimization of $\Omega$: $\partial\Omega/\partial\sigma$=0 leads to
$\sigma^2_{\min} = -\frac{a}{\lambda}$ for $a < 0$ and $\sigma^2_{\min} = 0$ for $a > 0$.

Pressure: $p = -\Omega(\sigma = \sigma_{\min}) = \frac{a^2}{4\lambda}$
**Second-order cumulant** $\mu = 0$**:** $\chi_2 \sim (T - T_c)\theta(T - T_c)$
**Higher order cumulants** $n > 4$ $\chi_n = 0$

Fluctuations of order parameter $\rightsquigarrow$ **non-trivial** exponents

Pressure: $p \sim a^2$ $\rightsquigarrow p \sim a^{2-\alpha}$

$a = \frac{1}{t_0}\left[\left(\frac{T}{T_c} - 1\right) + \kappa_q \left(\mu_q/T\right)^2\right]$

$\alpha$ is non-integer number.

3-dimensional O(4) universality class: $\alpha \approx -0.21$

$\mu = 0$: higher cumulants are non-trivial: $\chi_n \sim (T - T_c)^{-\frac{1}{2}(n-4+2\alpha)}$

$\chi_6 \sim 1/(T - T_c)^{1+\alpha}$ $\chi_8 \sim 1/(T - T_c)^{2+\alpha}$ divergent

$\mu \neq 0$: higher cumulants are non-trivial: $\chi_n \sim \left(\frac{\mu}{T}\right)^n a^{-(n-2+\alpha)}$

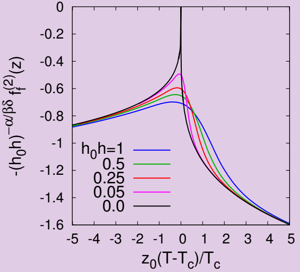
$\chi_4 \sim \left(\frac{\mu}{T}\right)^4 /a^{2+\alpha}$ $\cdots$ divergent

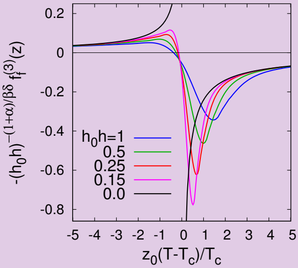
# BEYOND MEAN-FIELD: O(4) SCALING FUNCTIONS ON LATTICE

Based on: J. Engels, F. Karsch, arXiv:1105.0584 and
B. Friman et. al., arXiv:1103.3511

Lattice simulations of O(4) models $\rightsquigarrow$ singular part of

$$p/T^4 \propto -f(a,h)/T^4, \quad h \propto m_q$$



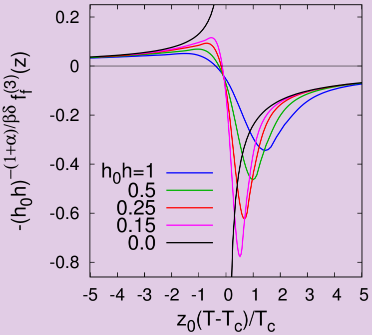




$\chi_4(\mu = 0)$ $\chi_6(\mu = 0)$

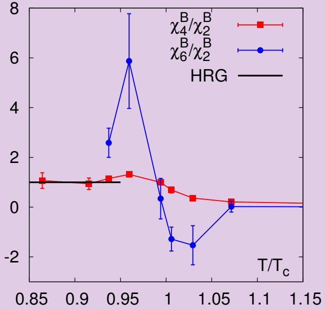
**Does singular part dominates in QCD?**

$\chi_6(\mu = 0)$

O(4) scaling:




Lattice QCD:




C. Schmidt, 2010

# Modeling QCD

Lattice QCD restrictions

- continuum limit for cumulants
- non-zero chemical potential

QCD inspired model

- O(4) symmetry in limit of vanishing mass for light quarks
- simulation of confinement properties (ratios of cumulants are sensitive to degrees of freedom)

Polyakov loop-extended NJL or QM model

- O(4) symmetry
- quark interaction with Polyakov loops $\rightsquigarrow$ statistical confinement
- in O(4) scaling regime for physical pion mass, as QCD

# Modeling QCD

**Mean-field** approximation ($\alpha = 0$)?

NO!

Importance of **non-trivial** critical exponents.

- Mean-field, $m_\pi = 0, \mu = 0$ $T = T_c$:

$$\chi_6^{sing} = 0$$

- Beyond mean-field:

$$\chi_6^{sing} = \infty$$

**Functional Renormalization Group**

- $p(T,\mu,k)$, $k$ defines IR cut off $\rightsquigarrow$
  $p(T,\mu,k)$ includes modes with momentum $> k$.
- Functional renormalization group equation (exact and general):

$$p(T,\mu,k-dk) = p(T,\mu,k) + \boxed{\text{Exact FRG flow}}$$

- Iterating towards $k \to 0$: $p(T,\mu,k=0)$ includes all momentum modes
- Exact FRG is useless, approximations (leading order in gradient expansion):

$$p(T,\mu,k-dk) = p(T,\mu,k) + \boxed{\text{Approximate FRG flow}}$$

FRG review: J. Berges, N. Tetradis & C. Wetterich, Phys.Rept.363:223-386, '02
FRG formulation of PQM model: V. S., B. Stokic, B. Friman & K. Redlich, PRC, '10

# Functional renormalization group

The general flow equation for the effective action

$$\partial_k \Gamma_k[\Phi, \psi] \quad = \quad \frac{1}{2} \mathrm{Tr} \left\{ \partial_k R_{kB} \left( \Gamma_k^{(2,0)}[\Phi, \psi] + R_{kB} \right)^{-1} \right\} - \mathrm{Tr} \left\{ \partial_k R_{kF} \left( \Gamma_k^{(0,2)}[\Phi, \psi] + R_{kF} \right)^{-1} \right\}$$

The flow equation for the PQM model

$$\partial_k \Omega(k, \rho \equiv \frac{1}{2}[\sigma^2 + \boldsymbol{\pi}^2]) = \frac{k^4}{12\pi^2} \left\{ \frac{3}{E_\pi} \left[ 1 + 2n_B(E_\pi; T) \right] + \frac{1}{E_\sigma} \left[ 1 + 2n_B(E_\sigma; T) \right] - \frac{4N_f N_c}{E_q} \left[ 1 - N(\ell, \ell^*; T, \mu_q) - \bar{N}(\ell, \ell^*; T, \mu_q) \right] \right\}$$

$n_B(E; T)$ is the boson distribution functions
$N(\ell, \ell^*; T, \mu_q)$ are fermion distribution function modified owing to coupling to gluons
$E_\sigma$ and $E_\pi$ are the functions of $k$, $\partial\Omega/\partial\rho$ and $\rho\partial^2\Omega/\partial\rho^2$
$E_q = \sqrt{k^2 + 2g\rho}$

FRG defines $\Omega(k, \rho; T, \mu_Q, \mu_B)$.
**Physically relevant quantity** is the thermodynamical potential
$\bar{\Omega}(T, \mu_Q, \mu_B) \equiv \Omega(k \to 0, \rho \to \rho_0; T, \mu_Q, \mu_B)$, where $\rho_0$ is the minimum of $\Omega$.

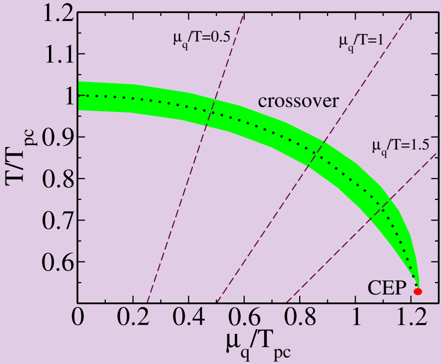
# Functional renormalization group: after approximations

- **accounts for universal critical behaviour near chiral transition**

- reproduces scaling properties and critical exponents
  (Berges '00, B. Stokic et. al. '10)

- respects symmetries
  (Goldstone theorem fulfilled, second-order phase transition in $O(4)$ model)

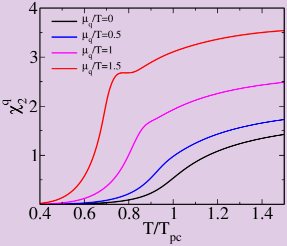
# Phase diagram in FRG PQM



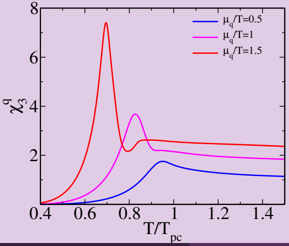

**Crossover**: $|\partial\sigma/\partial T| > 0.95 \cdot \max(|\partial\sigma/\partial T|)$

# Net-quark number density fluctuations $\delta N_q = N_q - \langle N_q \rangle$

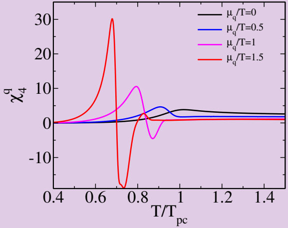
$$\chi_2^q = \frac{1}{VT^3}\langle(\delta N_q)^2\rangle \to \frac{N_c N_f}{3}\left[1 + \frac{3}{\pi^2}\left(\mu_q/T\right)^2\right]$$




$$\chi_3^q = \frac{1}{VT^3}\langle(\delta N_q)^3\rangle \to \frac{2N_c N_f}{\pi^2}\left(\mu_q/T\right):$$



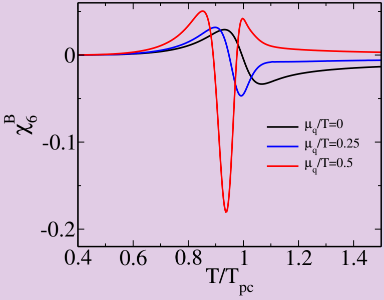

$$\chi_4^q = \frac{1}{VT^3}\left(\langle(\delta N_q)^4\rangle - 3\langle(\delta N_q)^2\rangle^2\right) \to \frac{2N_c N_f}{\pi^2}:$$

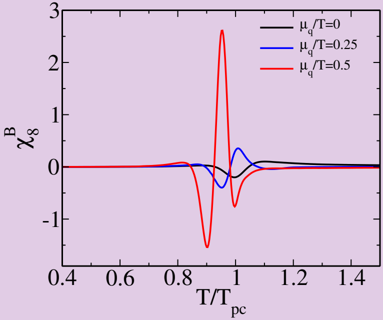



V.S., B. Friman and K. Redlich PRC'11

- $\chi_2^q$: non-monotonic structure (diverges at CEP)
- $\chi_4^q$: negative for nonzero $\mu_q$

# High-order cumulants of the baryon number density



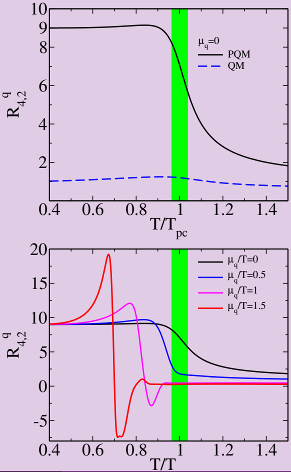

- Negative also at $\mu_q = 0$
- Temperature range of negative cumulants correlates with crossover temperature
- Many other constraints from O(4) scaling: B. Friman et. al. ’11

**Kurtosis** $R^q_{4,2} = \frac{\chi^q_4}{\chi^q_2} = \frac{\langle(\delta N_q)^4\rangle}{\langle(\delta N_q)^2\rangle} - 3\langle(\delta N_q)^2\rangle$

(S. Ejiri, F. Karsch and K. Redlich '05):

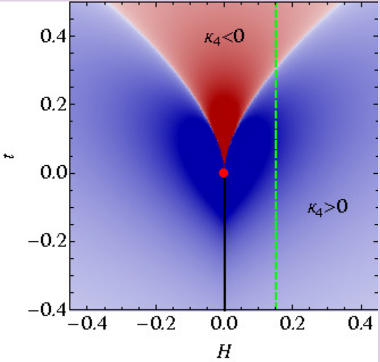
quark content of effective degrees of freedom that carry baryon number

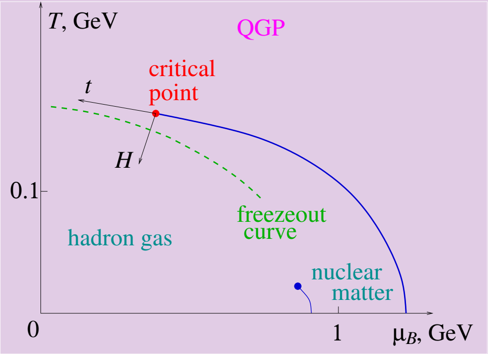
- **Low temperature phase**: dominance of effective three-quark states:
  $P_{\text{baryons}}/T^4 \approx \sum_i F(m_i/T)\cosh(3\mu_q/T)$
  $\rightsquigarrow R^q_{4,2} = 9$
- **High-temperature phase**:
  $P_{q\bar{q}}/T^4 \approx N_f N_c \left[\frac{1}{12\pi^2}\left(\frac{\mu_q}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu_q}{T}\right)^2 + \frac{7\pi^2}{180}\right]$
  $\rightsquigarrow R^q_{4,2} = (6/\pi^2) \approx 1$
- *PQM: statistical confinement*
- $m_\pi = 0, \mu_q \neq 0$: kurtosis diverges
  $R^q_{4,2} \sim \left(\frac{\mu_q}{T}\right)^4 / t^{2+\alpha}$ ($t \propto$ distance to chiral critical line)



# Sign of kurtosis

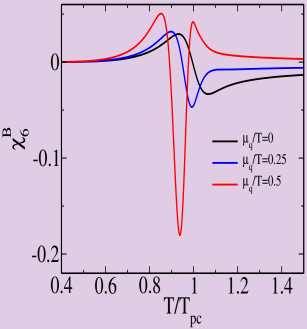
M. Stephanov '11: 3d Ising universality class $\rightsquigarrow$ kurtosis is negative close to CEP

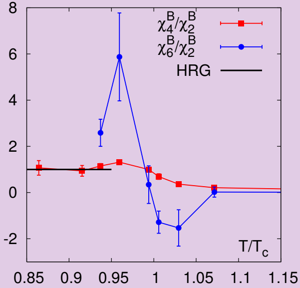


Negative kurtosis is necessary, but not sufficient condition of CEP.

$$\boxed{\text{CEP} \to R^B_{4,2} < 0, \text{ but } R^B_{4,2} < 0 \nrightarrow \text{CEP}}$$



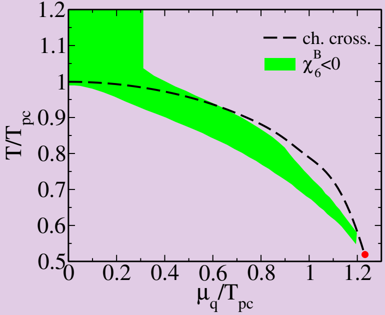



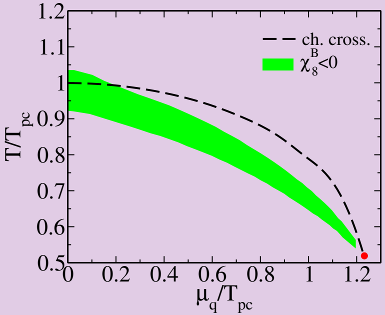

$$\text{sign}(R^B_{4,2}) = \text{sign}\chi^B_4, \quad \chi^B_4(\mu/T) \approx \chi^B_4(0) + \tfrac{1}{2}\chi^B_6(0)\cdot(\mu/T)^2 + \mathcal{O}((\mu/T)^4)$$

$$\boxed{R^B_{4,2} < 0 \to \text{non-trivial phase diagram}}$$

# High-order baryon cumulant

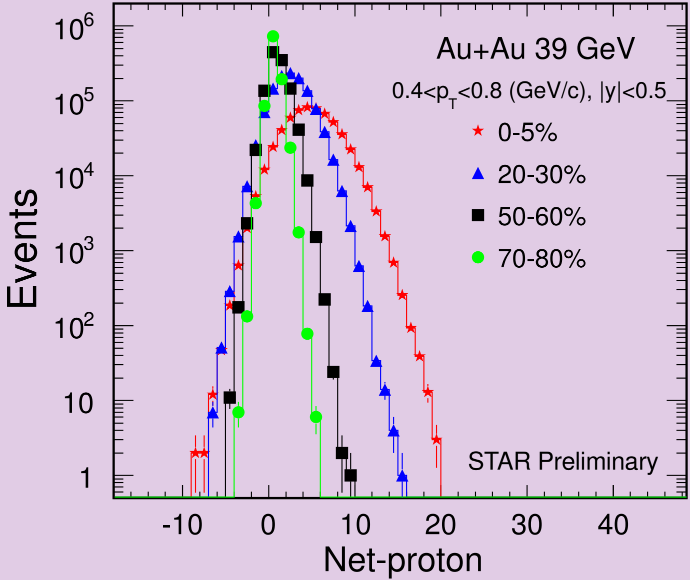
Temperature interval of negative cumulants closest to hadronic phase:



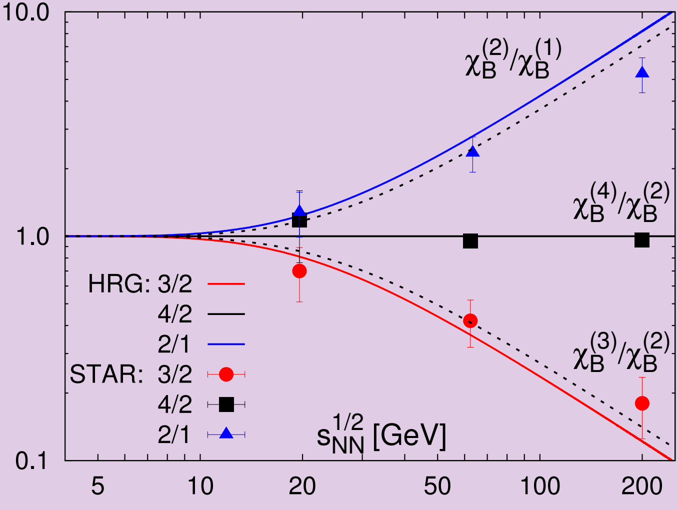

- Negative values (in broken phase!) of high-order cumulants: indicates proximity of freeze-out to crossover
- Accessible experimentally

# Experiment

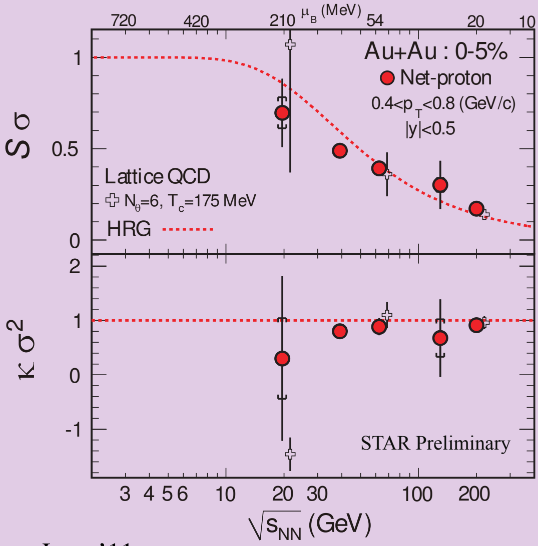



Xiaofeng Luo, ’11

# Comparison of the HRG model with experiment




F. Karsch and K. Redlich, '10




Xiaofeng Luo, '11

$$S\sigma = \tanh(\mu_B/T)$$

$$\kappa\sigma^2 = 1$$

Deviation from HRG:

- Critical properties?
- Conservation laws?

M. Nahrgang, QM'11
arXiv:0903.2911

- Statistics?

# HRG PROBABILITIES

Probability disctribution for HRG (Skellam distribution):

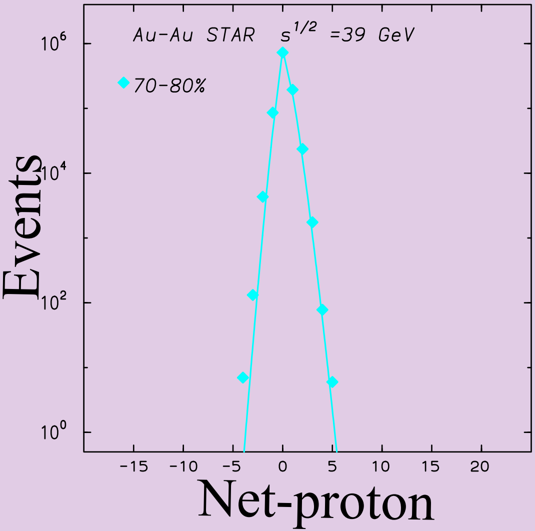
$$\mathcal{P}(N) = \left(\frac{N_b}{N_{\bar{b}}}\right)^{N/2} I_N\left(2\sqrt{N_b N_{\bar{b}}}\right) \exp\left[-(N_b + N_{\bar{b}})\right]$$

$N_b$ is mean number of baryons (protons)

$N_{\bar{b}}$ is mean number of anti-baryons (anti-protons)

For double charge particles and in case of quantum statistics $\mathcal{P}(N)$ is to be corrected

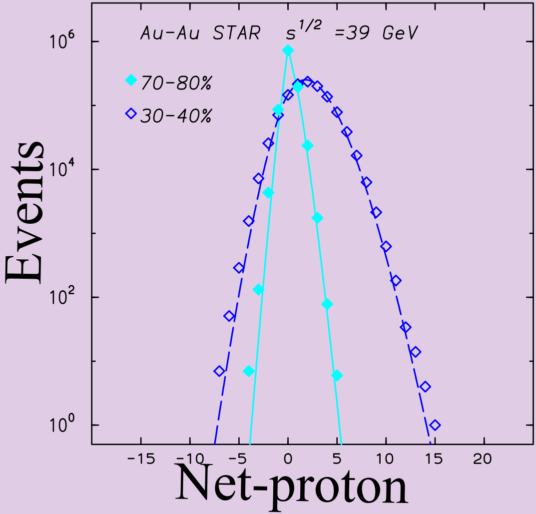
# HRG PROBABILITIES




Experimental data: STAR **Preliminary**, Xiaofeng Luo, '11

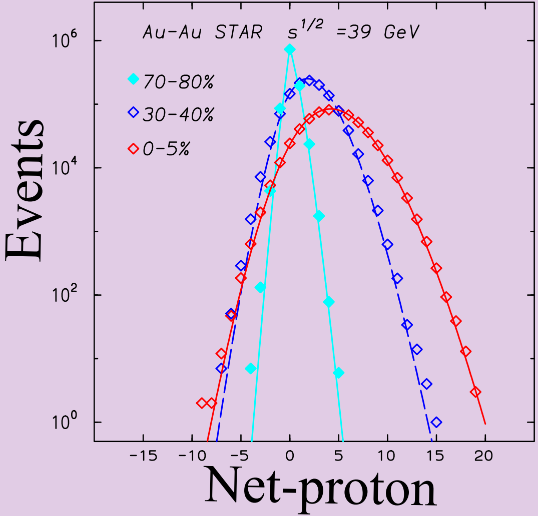
HRG line: $N_p$ and $N_{\bar{p}}$ are corrected for efficiency

# HRG PROBABILITIES



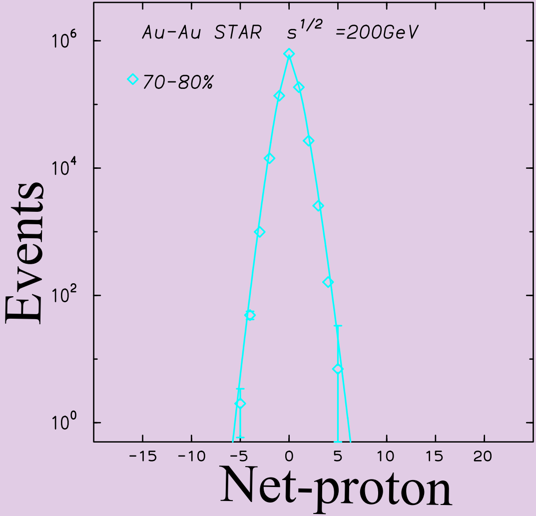

Experimental data: STAR **Preliminary**, Xiaofeng Luo, ’11

HRG line: $N_p$ and $N_{\bar{p}}$ are corrected for efficiency




Experimental data: STAR **Preliminary**, Xiaofeng Luo, '11

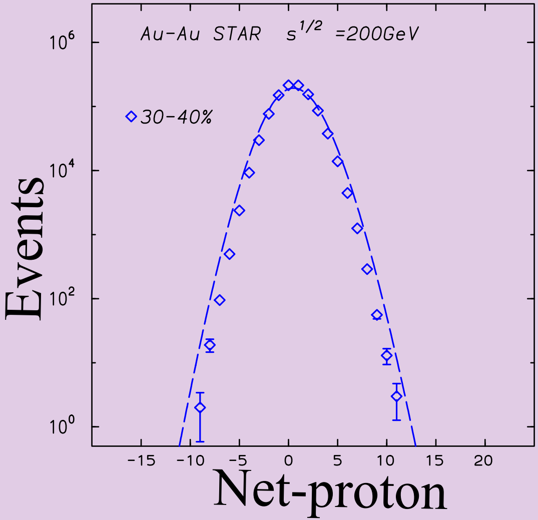
HRG line: $N_p$ and $N_{\bar{p}}$ are corrected for efficiency




arXiv:1107.4267

HRG line: $N_p$ and $N_{\bar{p}}$ are corrected for efficiency

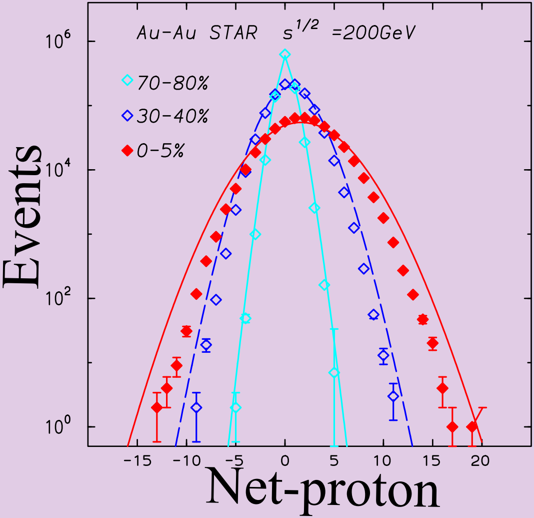
# HRG probabilities




arXiv:1107.4267

HRG line: $N_p$ and $N_{\bar{p}}$ are corrected for efficiency

# HRG probabilities




**Conclusions are premature $\rightsquigarrow$ efficiency**

# Conclusions

- **Fluctuations** of conserved charges are **sensitive** probes of the phase structure. They carry information about deconfinement and chiral phase transitions. Fluctuations can be used to identify order and universality class of a phase transition.
- The **negative** values of $\chi_6^B$ may indicate the proximity of the chemical freeze-out to the crossover line.
- Experimentally, deviations from HRG values of the cumulants or the corresponding probabilities may signal remnants of the transition.
- At the moment, the published data is not conclusive.

# Thank you for attention

Collaborators: B. Friman, F. Karsch, K. Morita and K. Redlich