Aspects of Two- and Three-Flavor Chiral Phase Transitions

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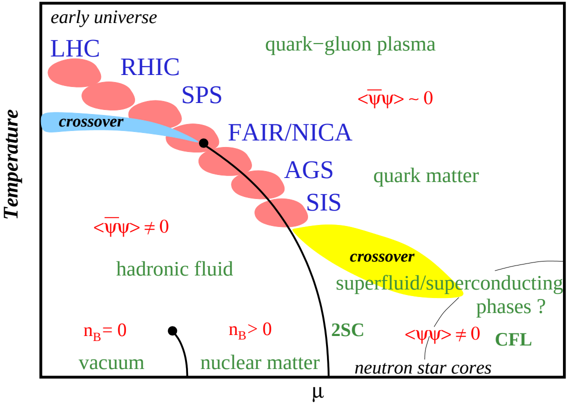
Conjecture for the QCD Phase Diagram



- (non-)standard scenario (philipsen, de forcrand last decade)
- weakening of crossover for small $\mu > 0$ (endrodi et al. 2011)
- influence of anomaly? (pisarski, wiczek 1984, chen et al. 2009)

How To Spoil Symmetries

- explicit symmetry breaking: symmetry only approximate
- anomalies: violation by quantum terms
  via regularization/renormalization procedure
- spontaneously broken symmetry:
  ground state/vacuum less symmetric than Lagrangian

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Chiral Symmetry

- $U_B(1) \times SU_L(N_f) \times SU_R(N_f) \times U_A(1)$
- $SU_L(N_f) \times SU_R(N_f) \times U_A(1)$ expl. broken by quark-masses
- $U_A(1)$ broken by axial anomaly
- $SU_L(N_f) \times SU_R(N_f)$ spontaneously broken to $SU_{R+L}(N_f)$

The Quark-Meson Model (QM-Model)

Mesons

- bound states of quark and antiquark:
  treated as separate dofs in QM model
- representation of $\chi-$sym
- coupled to quarks: Yukawa theory of quarks and mesons

$$
\begin{aligned}
\mathcal{L}_M &= \mathrm{tr}\left[\partial^\mu \Sigma \partial^\mu \Sigma^\dagger\right] + U\left(\{\rho_i | i \in \mathbb{N}\}, \xi\right) - h_b \sigma_b \\
\Sigma &= t_b\left(\sigma_b + i\pi_b\right) \;, \quad t_b \text{ generators of } U(N_f) \\
\rho_i &= \mathrm{tr}\left[\left(\Sigma\Sigma^\dagger\right)^i\right] \;, \quad \xi = \det\left(\Sigma\right) + \det\left(\Sigma^\dagger\right) \\
\mathcal{L}_{QM} &= \mathcal{L}_m + \bar{q}\left(\partial^\mu \gamma^\mu - i h t_b\left(\sigma_b + i\gamma_5 \pi_b\right)\right) q
\end{aligned}
$$

Order Parameter in the QM-Model

The Chiral Condensate as Order Parameter

- $\langle \bar{q} q \rangle > 0$: $SU_R(N_f) \times SU_L(N_f) \to SU_{R+L}(N_f)$
- $N_f = 2 : SO(4) \to SO(3)$

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- usual 2-flavor qm-model
  - only $\rho_1 = \text{tr}\left[\Sigma\Sigma^\dagger\right]$
  - $O(4)$-representation: $(\sigma, \vec{\pi})$ (without $\sigma_{1,2,3}, \pi_0$)
  - $\sigma_{cl} \leftrightarrow \langle \sigma \rangle \leftrightarrow \langle \bar{q}q \rangle$
  - rotations in $\mathbb{R}^4$ $\to$ rotations around σ_0

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In General

- one quark-condensate for every flavor

Wetterich RG for Effective Potential (FDE→PDE)

Derivative Expansion (↔ Low Momentum Expansion)

- $\Sigma(x) \to \Sigma$
- only scale-dependency in mesonic potential U_k
- three dimensional version of optimized ("Litim") regulator
- flow for U_k with $\Gamma_{k=\Lambda}[\Sigma] = S[\Sigma, 0, 0]$:

$$\partial_k U_k = \frac{k^4}{12\pi^2} \left[\sum_{b=1}^{2N_f^2} \frac{1}{E_b} \coth\left(\frac{E_b}{2T}\right) - 2N_c \sum_{f=1}^{N_f} \frac{1}{E_f} \left\{ \tanh\left(\frac{E_f + \mu_f}{2T}\right) + \tanh\left(\frac{E_f - \mu_f}{2T}\right) \right\} \right]$$

Solution Techniques (PDE→ODE)

Grid Method

- discretize domain of effective potential: $D \subset \mathbb{R}^n \to \{x_i\}$
- coupled ODE for $\{U_k(x_i)\}$
  - non-local information available
  - chiral limit
  - boundary of domain

Taylor Method

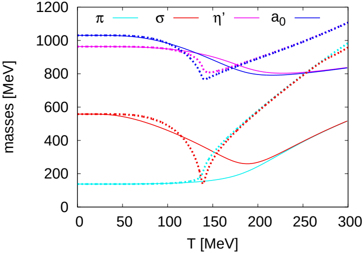
- Taylor expand potential (around minimum)
- coupled ODE for expansion coefficients (couplings)
  - numerically cheaper
  - critical exponents
  - interpretation in terms of couplings
  - chiral limit
  - first order transitions

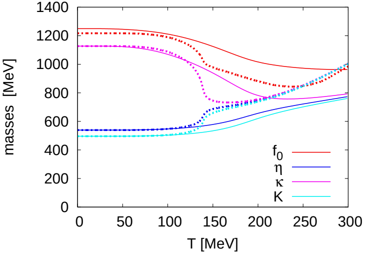
2+1 Flavor Quark Meson Model with FRG

- condensates rotated to non-strange/strange basis $(\sigma_0, \sigma_8) \rightarrow (\sigma_x, \sigma_y)$
- potential as function $U_k\,(\rho_1, \rho_2, \xi(\rho_1, \rho_2))$ with and without coupling of ξ: $c_k \rightarrow c$
- different realizations depending on $\langle \sigma_{x,y} \rangle$
- explicit symmetry breaking in light and strange direction $h_x < h_y$
- mesonic spectrum: $\sigma, a, \kappa, f, \eta, \pi, K, \eta'$
- degenerate chemical potential $\mu_x = \mu_y$
- Taylor expansion through $\mathcal{O}(\rho_{1,2}^3)$

Mesonic Masses With Anomaly: FRG vs. Mean-Field (MF)

fix initial potential U_Λ in vacuum ...



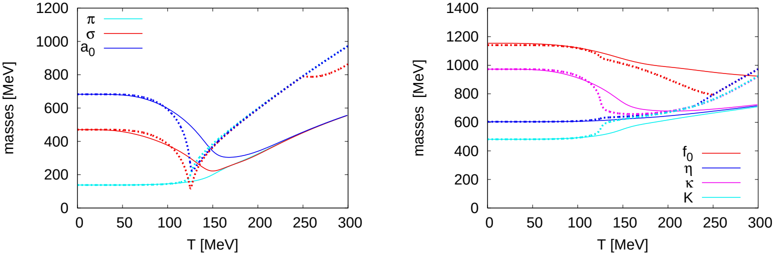


solid line: FRG, dashed line: MF

MF: Schaefer, Wagner 2009 FRG: MM, Schaefer in preparation 2011

Mesonic Masses Without Anomaly: FRG vs. MF

... to experimental values (physical point)



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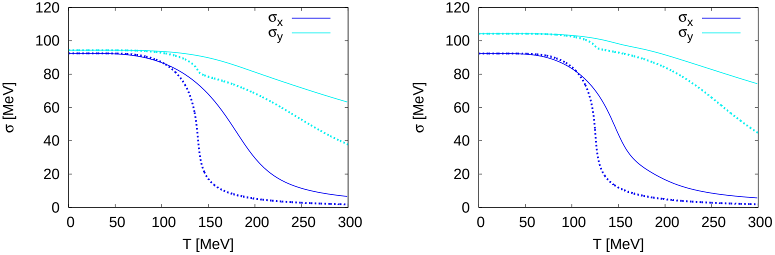
Condensates With and Without Anomaly: FRG vs. MF

- fluctuations wash out chiral crossover in light sector
- effect on strange sector mitigated



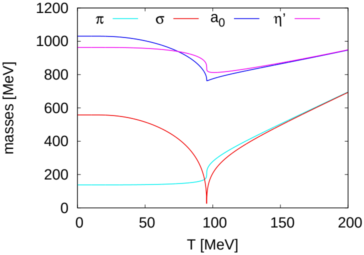
solid line: FRG, dashed line: MF

MF: Schaefer, Wagner 2009 FRG: MM, Schaefer in preparation 2011

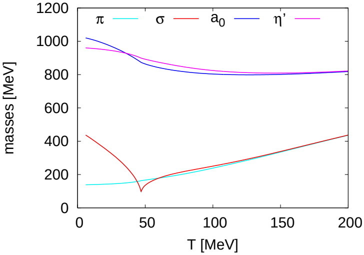
Critical Endpoint (CEP) With Anomaly

there exists a critical endpoint ...

- $\mu_c = 196$
  MF



- $\mu_c = 305$
  FRG

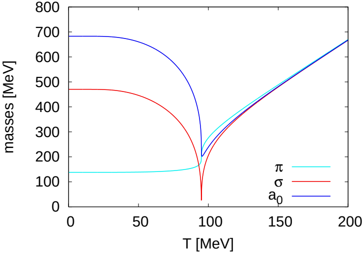


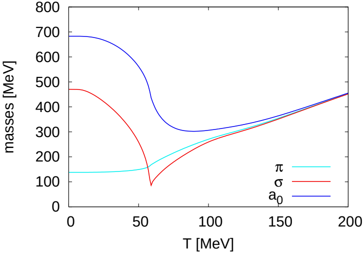
MF: Schaefer, Wagner 2009 FRG: MM, Schaefer in preparation 2011

CEP Without Anomaly

... driven towards higher μ/lower T by fluctuations

- $\mu_c = 157$
  MF
- $\mu_c = 262$
  FRG

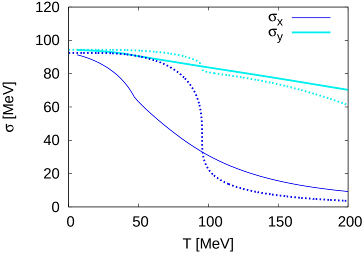




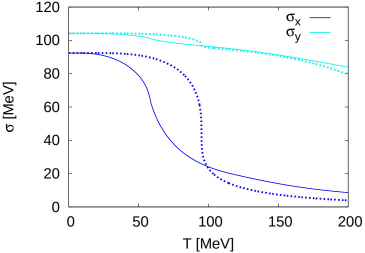
MF: Schaefer, Wagner 2009 FRG: MM, Schaefer in preparation 2011

Condensates at CEP: FRG vs. MF

- with anomaly



- without anomaly



solid line: FRG, dashed line: MF

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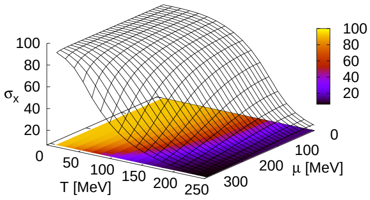
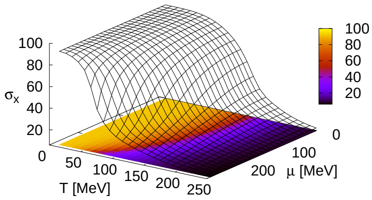
Light Condensate in Phase Diagram

- with anomaly



- without anomaly



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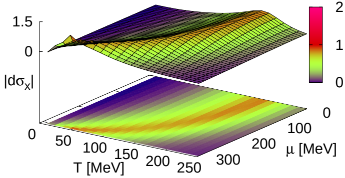
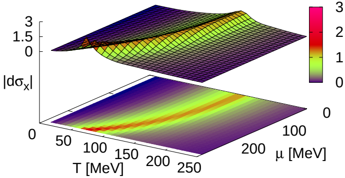
Gradient of Light Condensate in Phase Diagram

- with anomaly



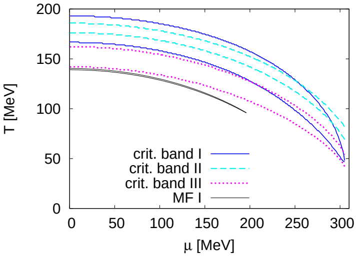
- without anomaly



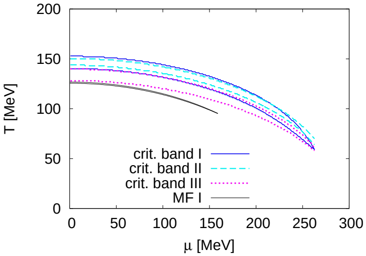
MM, Schaefer in preparation 2011

Critical Band

- $T_{max,\mu}$: maximizes $\left|\nabla\langle\sigma_x(T,\mu)\rangle\right|$
- I: $\dfrac{\left|\nabla\langle\sigma_x(T,\mu)\rangle\right|}{\left|\nabla\langle\sigma_x(T_{max,\mu},\mu)\rangle\right|}\geq 0.9$
- II: $\dfrac{\left|\langle\sigma_x(T,\mu)\rangle-\langle\sigma_x(T_{max,0},\mu)\rangle\right|}{\left|\langle\sigma_x(T_{max,0},\mu)\rangle\right|}\geq 0.9$
- III: the same as II with $T_{max,0}\rightarrow T_{max,\mu_c}$



with anomaly



without anomaly

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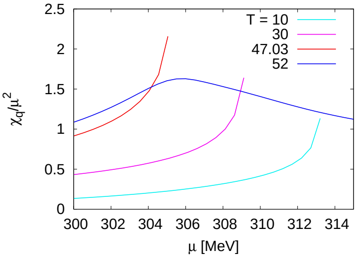
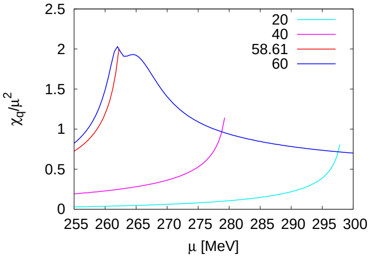
First Order Line

quark number susceptibility

- with anomaly
- without anomaly



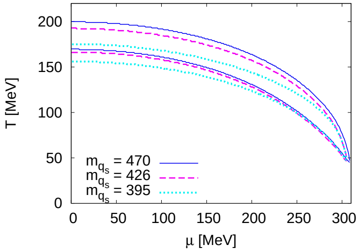


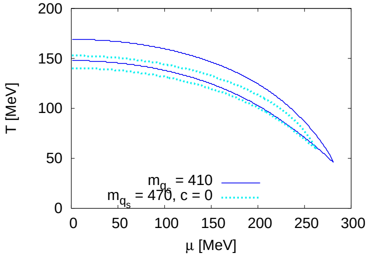
MM, Schaefer in preparation 2011

Influence of Anomaly

with anomaly
different strange quark masses

influence of anomaly

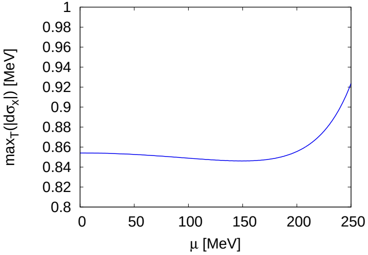




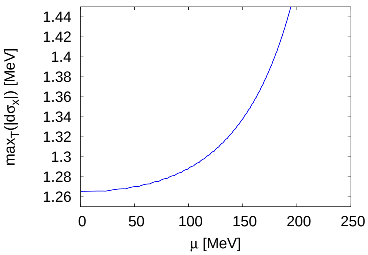
MM, Schaefer in preparation 2011

Sharpness of Crossover

with anomaly



without anomaly



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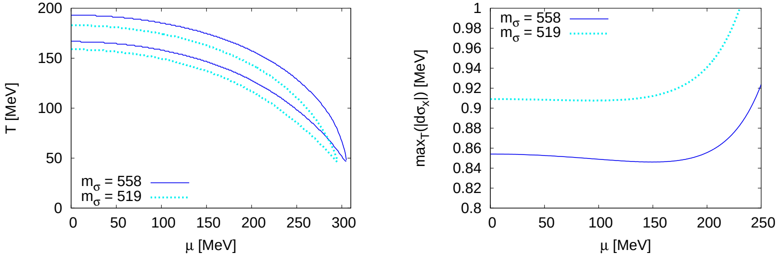
Influence of σ-Mass

with anomaly - all other masses fixed
$\leftrightarrow$ no weakening of crossover without anomaly?



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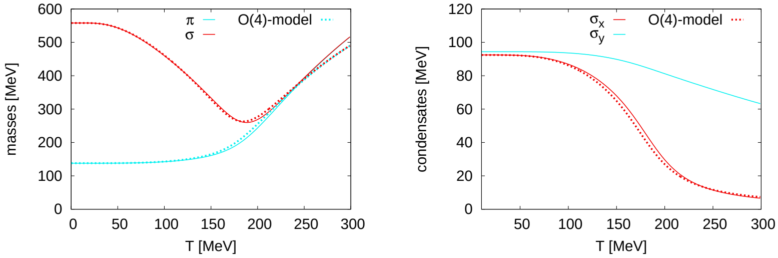
Comparison to Two Flavors I

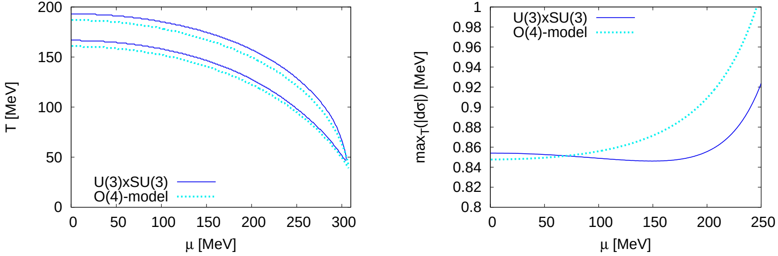
comparison to 2 flavor quark-meson model



MM, Schaefer in preparation 2011

Comparison to Two Flavors II

comparison to 2 flavor quark-meson model



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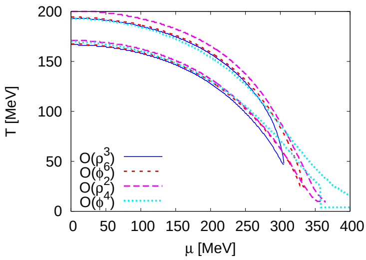
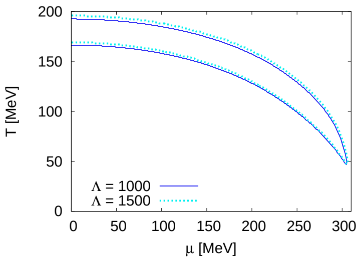
Effect of Taylor-Expansion Order and Cutoff Scale Λ

with anomaly





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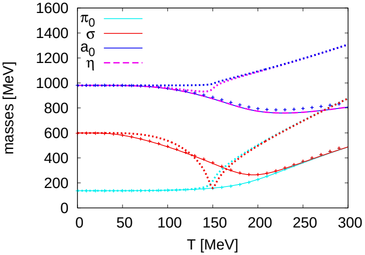
3 Flavor $\rightarrow$ 2 Flavor

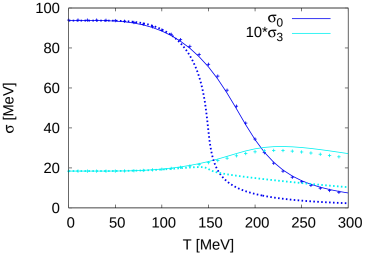
$$U(2) \times SU(2)$$

Taylor-RG vs. Grid-RG vs. MF

fixed initial anomaly coupling such that $m_\eta(k=0) = 980$ MeV

- solid line: Taylor-RG
- crosses: grid-RG
- dashed: MF

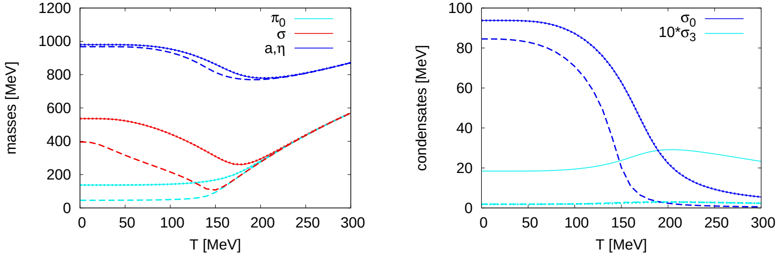




[MM,Schaefer,Strodthoff,von Smekal in preparation 2011]

Sensitivity to Variation of $h_{0,3}$

- solid line: $h_0 = h_3 = h_{phys}$
- short dashed: $h_0 = h_{phys}$, $h_3 = 0.1 * h_{phys}$
- long dashed: $h_0 = h_3 = 0.1 * h_{phys}$



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$U_A(1)$ Violating Couplings

- gauge theories with fermions: not $U_V(1) \wedge U_A(1)$

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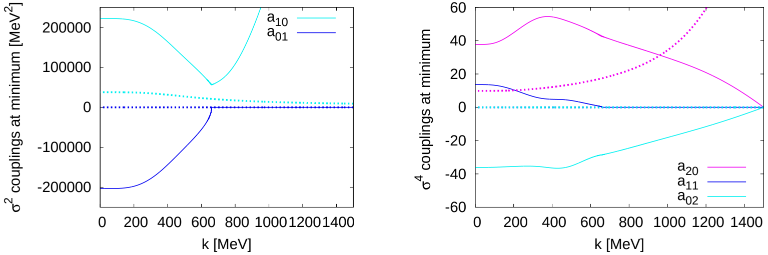
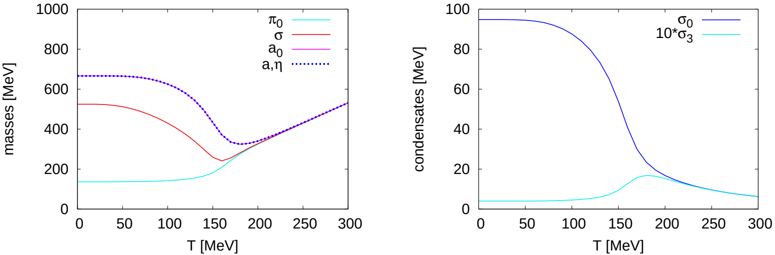
- gauge theories with fermions: not $U_V(1) \wedge U_A(1)$
- $a_{ij}(k) = (\partial_{\rho_1})^i (\partial_\xi)^j U_k(\rho_1, \xi)\big|_{min}$
- no $U_A(1)$ violating couplings in initial action
- solid: with quarks, dashed: without quarks



[MM,Schaefer,Strodthoff,von Smekal in preparation 2011]

Restoration of Mesonic Mass Spectrum Degeneracy

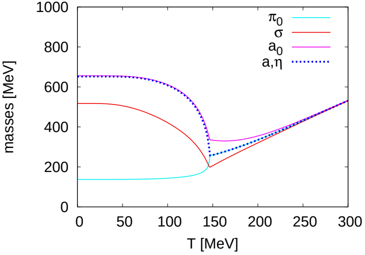
- no $U_A(1)$ violating couplings at $k = \Lambda$
- $h_0 = h_{phys}, \;\; h_3 = 0.1 * h_{phys}$

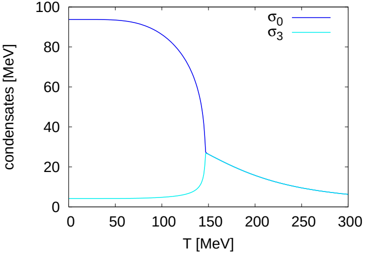


[MM,Schaefer,Strodthoff,von Smekal in preparation 2011]

Partial Chiral Limit

- no $U_A(1)$ violating couplings at $k = \Lambda$
- $h_0 = h_3 = h_{phys}$
- additional (exact!) symmetry above chiral T_c

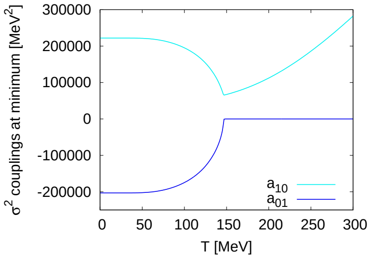


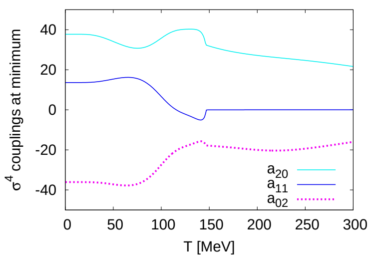


[MM,Schaefer,Strodthoff,von Smekal in preparation 2011]

Couplings in Partial Chiral Limit

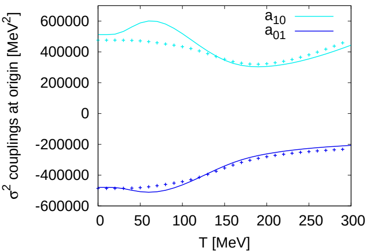
- additional symmetry above T_c is $\sigma_0 \leftrightarrow \sigma_3$ (Z_2)
- calculated order parameter critical exponent: $\beta = 0.366$ $(0.32 - 0.33)$

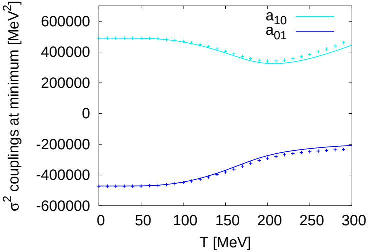




[MM,Schaefer,Strodthoff,von Smekal in preparation 2011]

Taylor vs. Grid Revisited





[MM,Schaefer,Strodthoff,von Smekal in preparation 2011]

Summary and Outlook

- 2+1 flavor Quark-Meson Model
  - weakening of crossover for small μ with CEP
  - anomaly seems to weaken transition
  - possibly no "back-bending" of first order line
- $U(2) \times SU(2)$ Quark-Meson Model
  - partial restoration of mass $U_A(1)$ at chiral T_c
  - spontanous generation of anomalous mass by quarks
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- $2+1$ flavor
  - grid technique for Columbia plot
  - scale dependency of anomalous coupling
- $U(2) \times SU(2)$:
  - $\mu > 0$
  - include $\rho_2 \Rightarrow$ first order transition? compare next talk

$U(2) \times SU(2)$ Quark Meson Model with FRG

- condensates (σ_0, σ_3): $\langle\sigma_3\rangle \leftrightarrow$ quark-mass splitting
- potential as function $U_k(\rho_1, \xi)$: scale dependent anomaly
- explicit symmetry breakings h_0 and h_3 (quark splitting)
- mesonic spectrum: σ, a, η, π
- grid and Taylor technique

Realizations

- $\langle\sigma_0\rangle = \langle\sigma_3\rangle = 0$ $(m_\pi \geq 0,\ m_{\eta,a_{1/2}} \geq 0)$:
- $\langle\sigma_0\rangle > 0,\ \langle\sigma_3\rangle = 0$ $(m_\pi = 0,\ m_{\eta,a_{1/2}} \geq 0)$:
- $0 \neq \langle\sigma_0\rangle \neq \langle\sigma_3\rangle \neq 0$ $(m_\pi = 0,\ m_{\eta,a_{1/2}} = 0)$:
- $|\langle\sigma_0\rangle| = |\langle\sigma_3\rangle| \neq 0$ $(m_\pi = 0,\ m_{\eta,a_{1/2}} = 0)$: