The phase diagram of QCD: Phase structure and Thermodynamics

&

Some remarks on Kugo-Ojima, and Dynamical Hadronisation

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Universität Heidelberg & ExtreMe Matter Institute

Kyoto, September 7th 2011

QCD | Cold quantum gases

Constants

$$\hbar = 1$$

$1.01 \times 10^{-34}\,\mathrm{J\,s}$

$$k_B = 1$$

$1.38 \times 10^{-23}\,\mathrm{m^2\,kg\,s^{-2}\,K^{-1}}$

$$c = 1$$

$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$

$$2m = 1$$

$$\mathbf{100}\mathrm{MeV} = \mathbf{1.16 \times 10^{12} K}$$

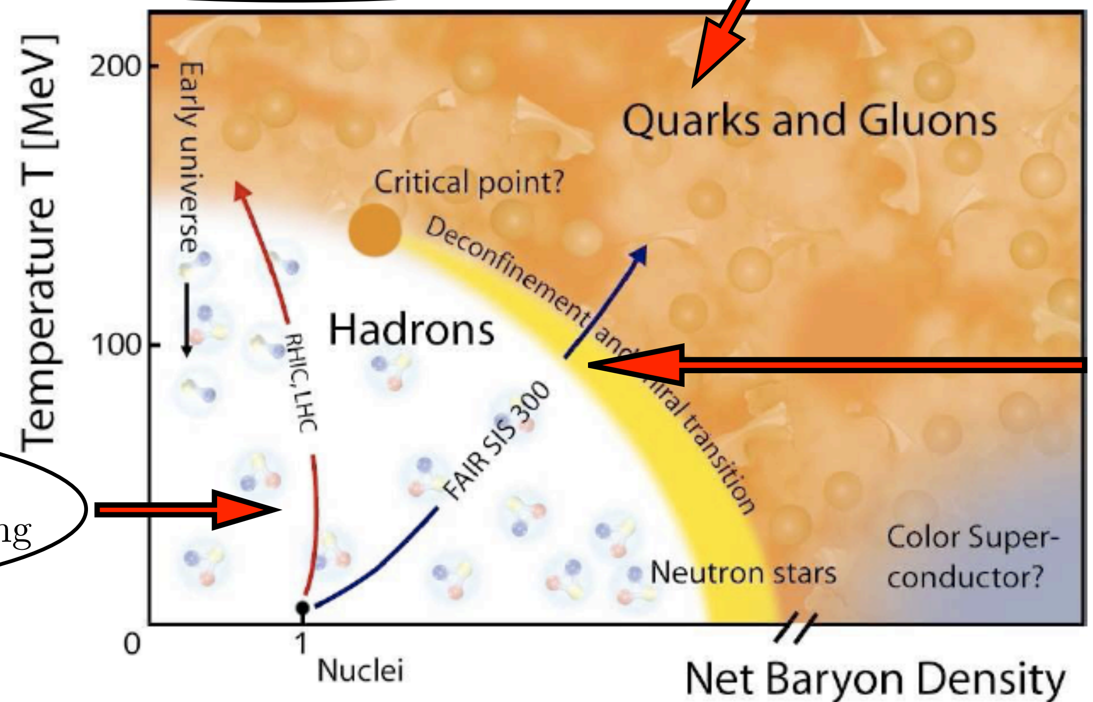
$$\mathbf{T_c \sim 10^{12} K}$$

$$\mathbf{T_c \sim 10^{-8} K}$$

Phase diagram of QCD

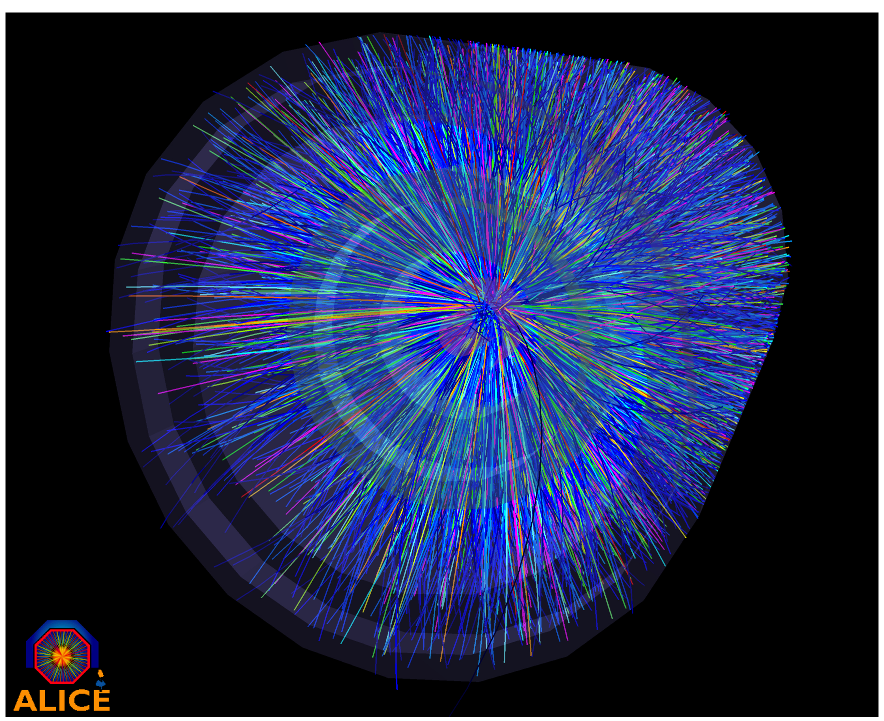
Strongly correlated quark-gluon-plasma
'RHIC serves the perfect fluid'

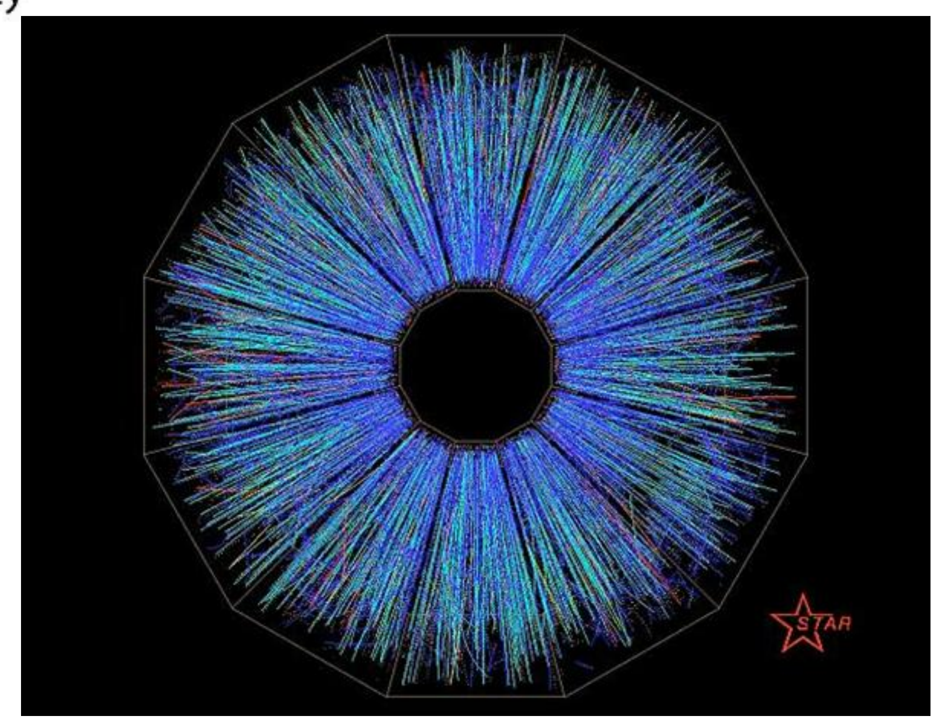
massless quarks (chiral symmetry)
deconfinement



quarkyonic:
confinement & chiral symmetry?

hadronic phase
confinement & chiral symmetry breaking





ALICE, LHC

UrQMD Frankfurt/M

Simulation of heavy ion collision

STAR, RHIC

Phase diagram of QCD

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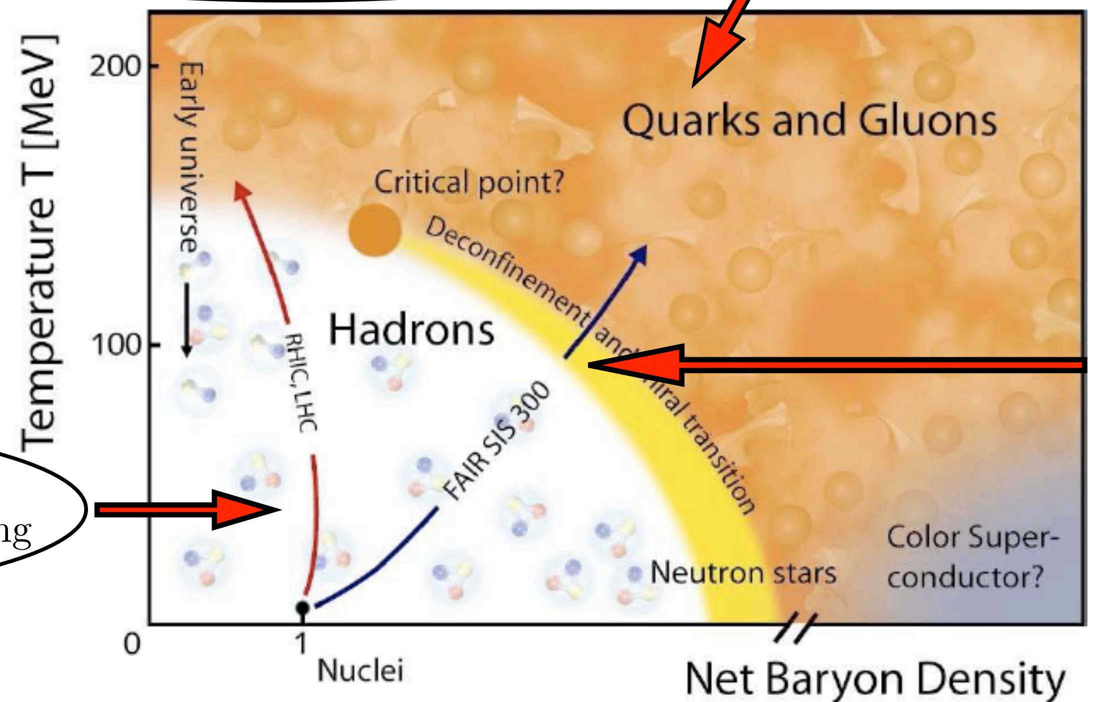

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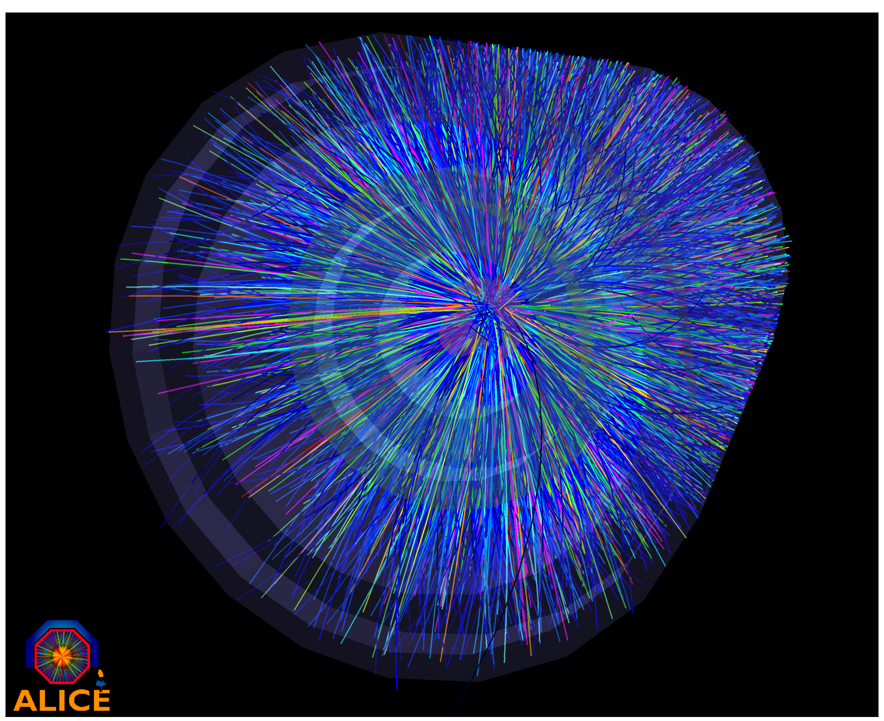
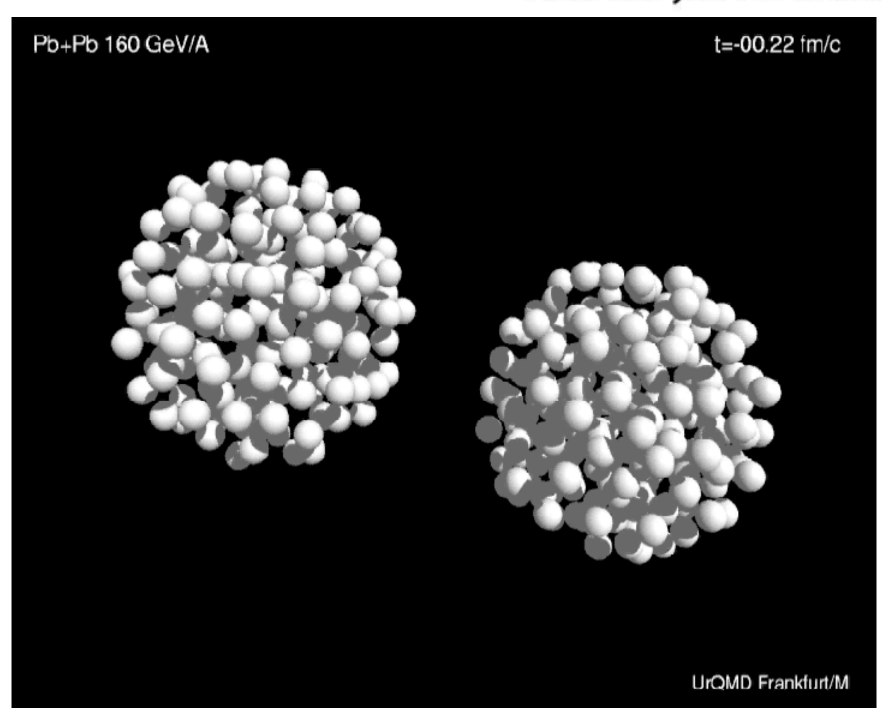
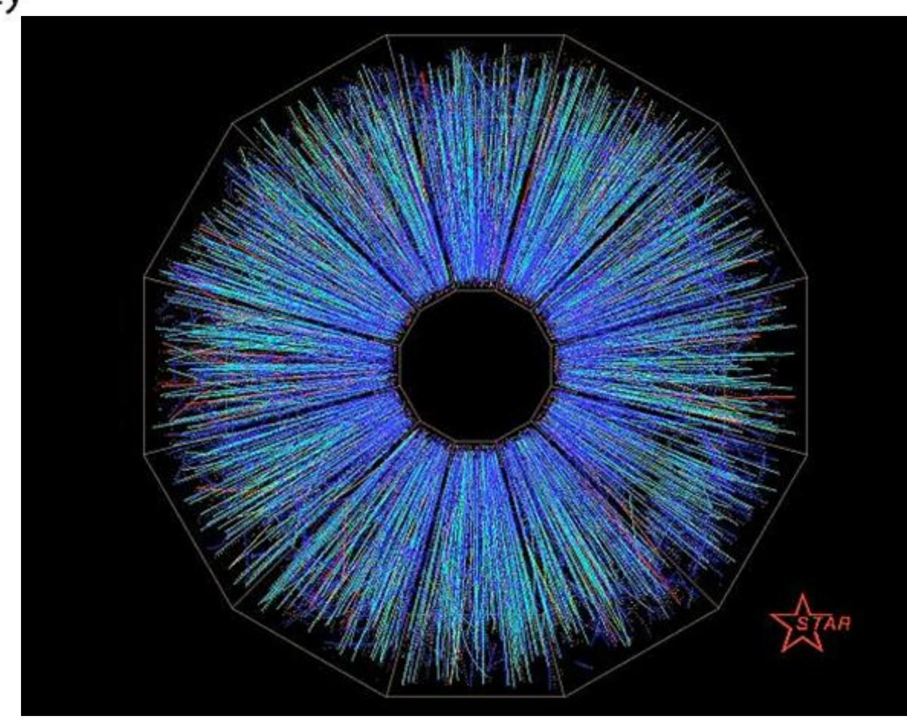






ALICE, LHC

UrQMD Frankfurt/M
Simulation of heavy ion collision

STAR, RHIC

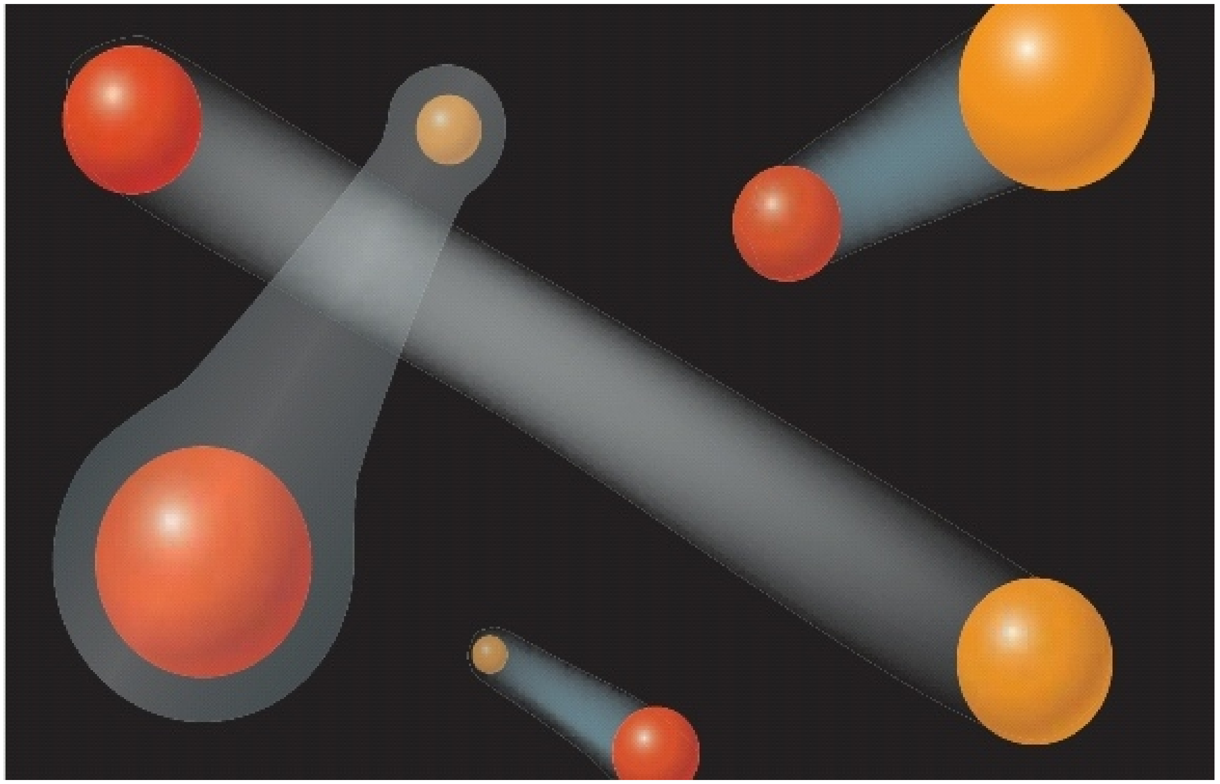
Phase diagram of cold quantum gases

BEC-BCS cross-over

Eagles ’69, Leggett ‘80

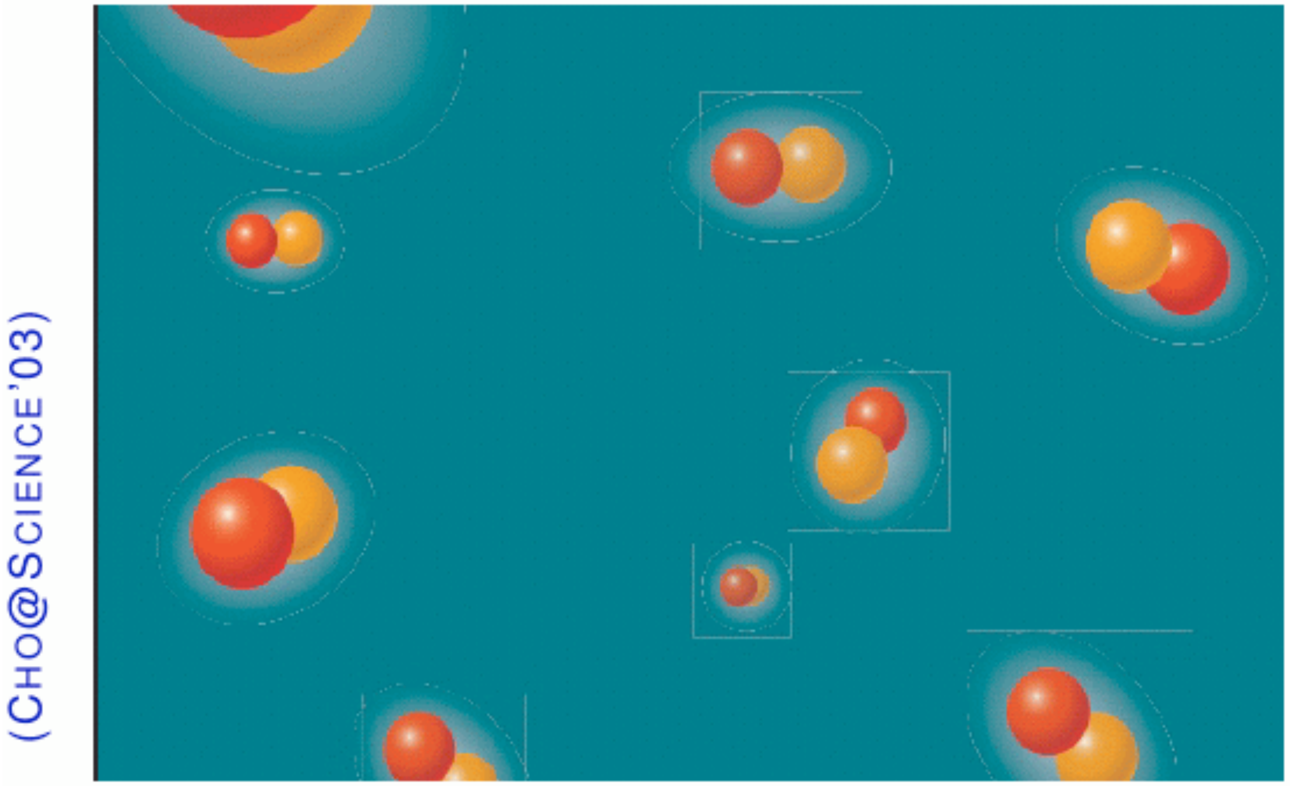
Fermions with attractive interactions

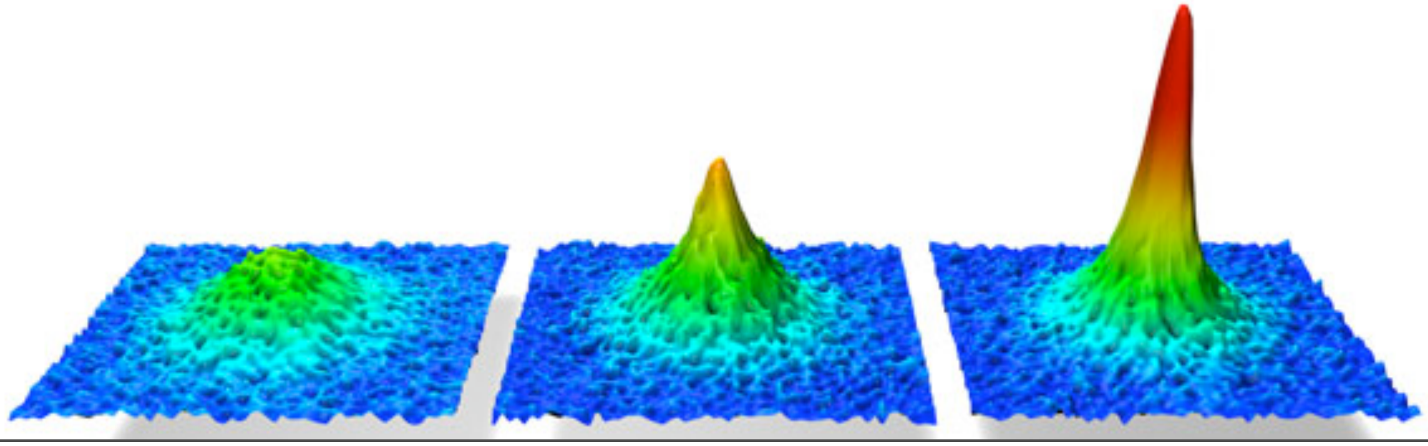
BCS superfluidity at low T $a < 0$



Bound molecules of two atoms on microscopic scale

BEC at low T $a > 0$



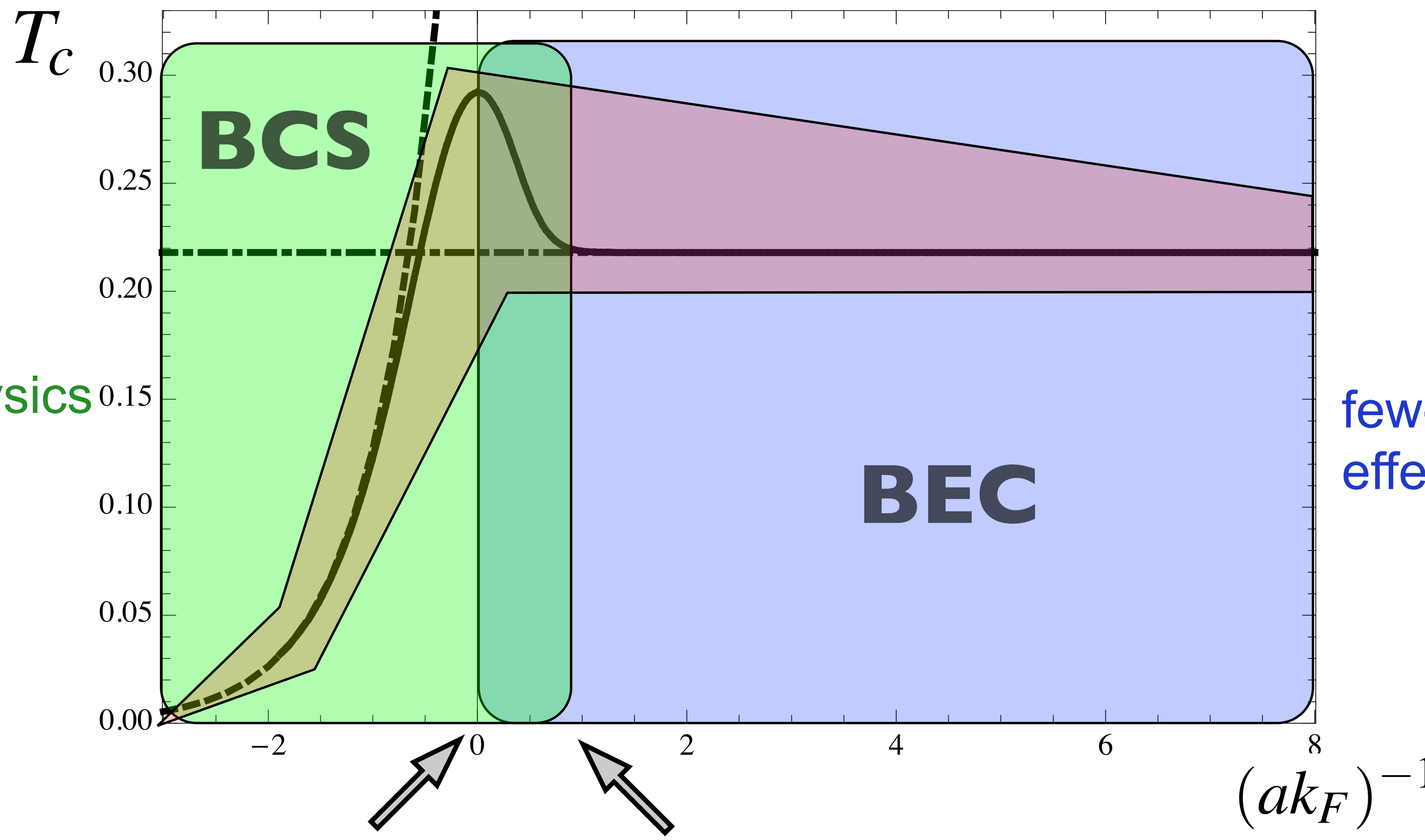


Regal et al ’04

Phase diagram of cold quantum gases

critical behavior: long distance scales

Many-Body fermion physics



few-body physics of effective dimers

two-body bound state

zero crossing of fermion chemical potential

$$a = a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

scattering length **external magnetic field**

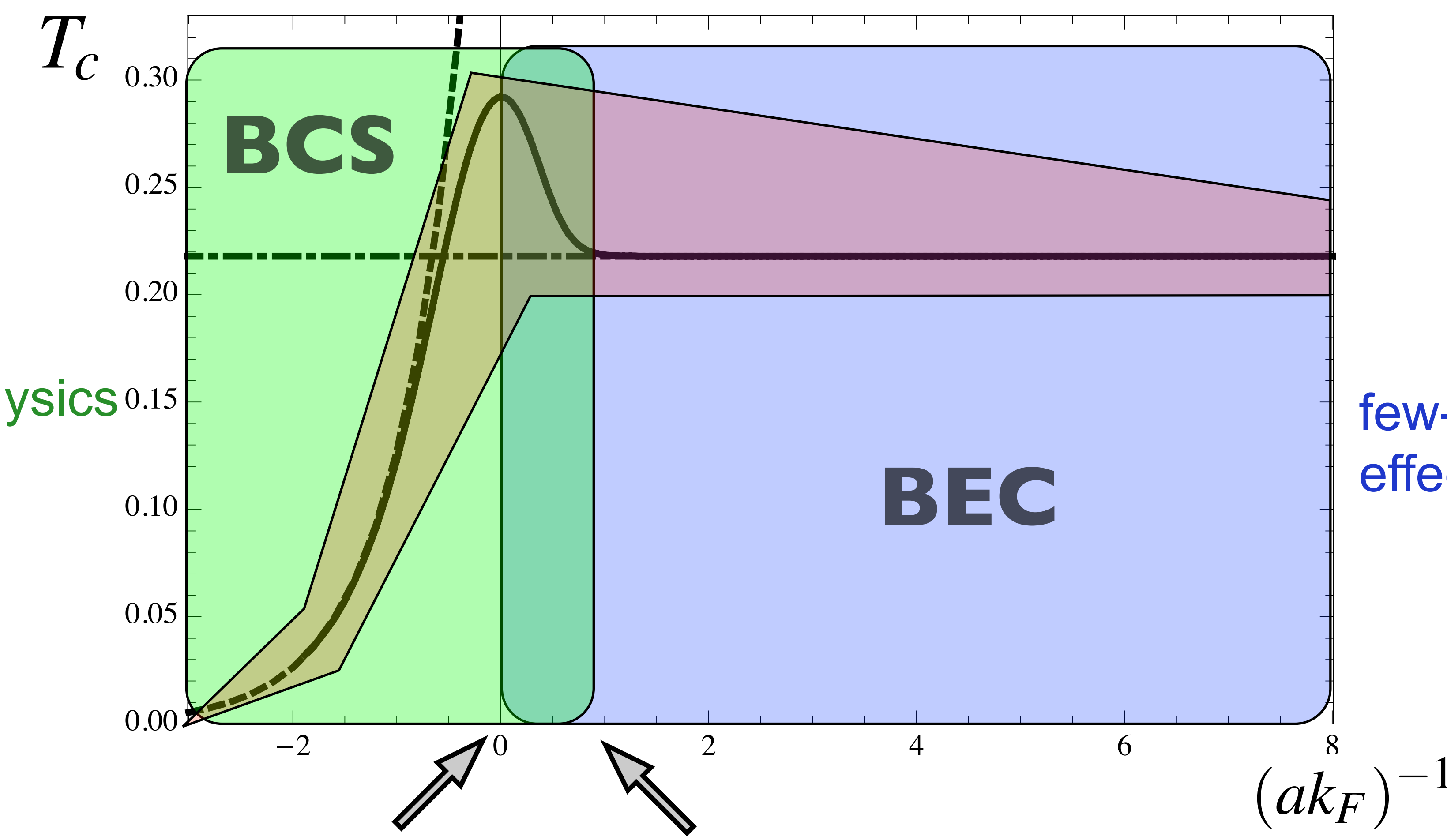
tunable fundamental interaction parameter

$$B_0|_{^6\rm Li} \approx 830 \text{ Gauß}$$

Phase diagram of cold quantum gases

critical behavior: long distance scales

Many-Body fermion physics



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QCD ⟺ CQG

EMMI

Outline

- QCD & cold quantum gases primer
- Confinement & chiral symmetry breaking in QCD
- Phase diagram of QCD
- Summary & outlook

QCD & cold quantum gases primer

QCD

QCD, asymptotic freedom and all that

Field content of QCD

- **Gluons: colour-charged gauge fields $\mathbf{A}_\mu^\mathbf{a}$**

$$\mathbf{F}^\mathbf{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Fieldstrength

- **Quarks: fermions $\psi^\mathbf{f}$ with flavours $\mathbf{f}$**

Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
Quark	u	c	t	$\frac{2}{3}$
Quark	d	s	b	$-\frac{1}{3}$
Mass [MeV]	4-8	80-130	$(4.1\text{-}4.4)\times10^3$	

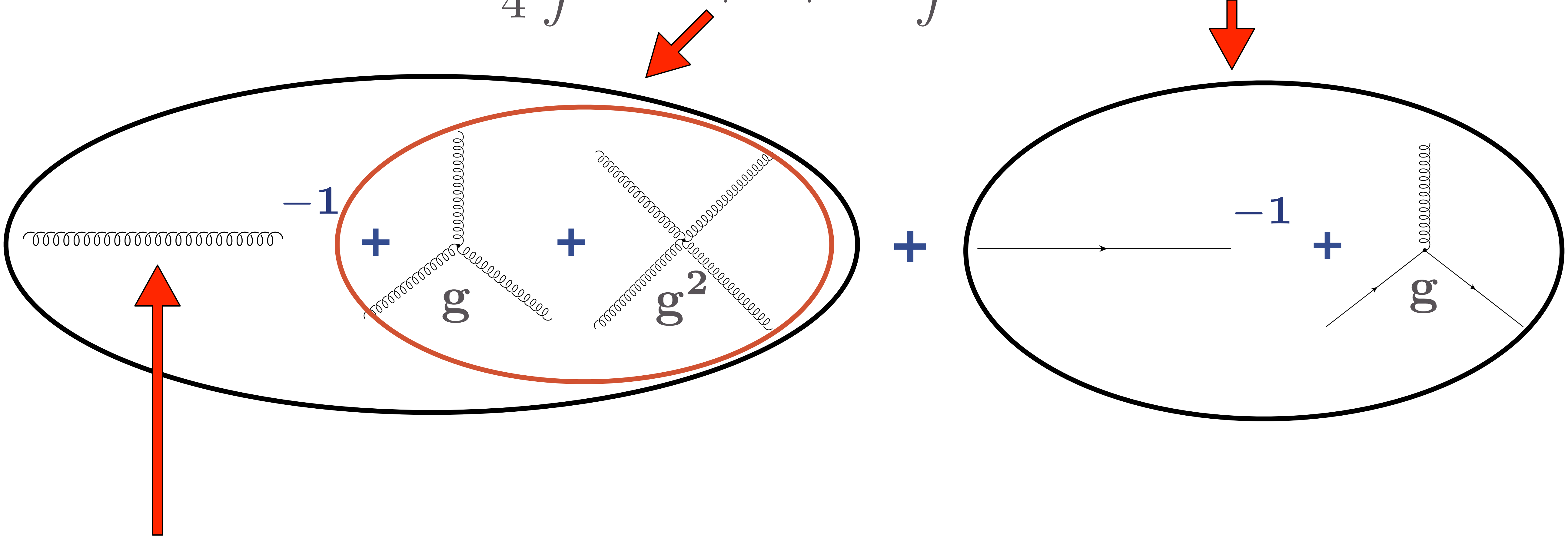
masses via Higgs mechanism

QCD, asymptotic freedom and all that

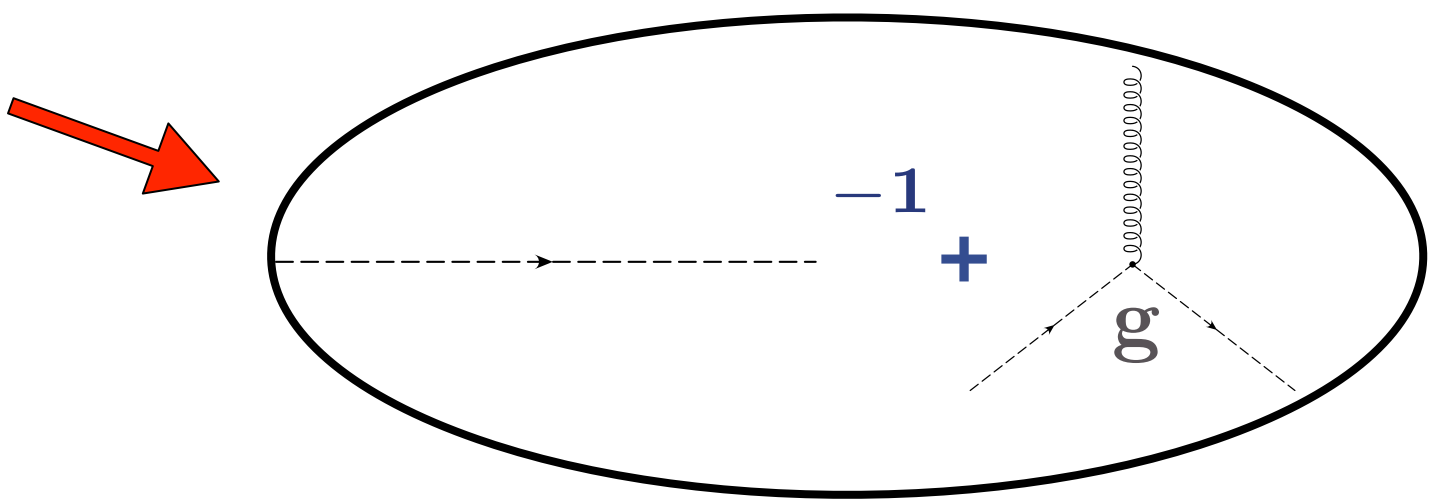
Action and interactions

- **Interactions**

$$S_{\rm QCD}[A,\psi,\bar\psi] \quad = \quad \frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^a_{\mu\nu} + \int d^4x\, \bar\psi_f(i\not{D} + i\, m_f)\psi_f$$



- **Gauge fixing**



QCD, asymptotic freedom and all that

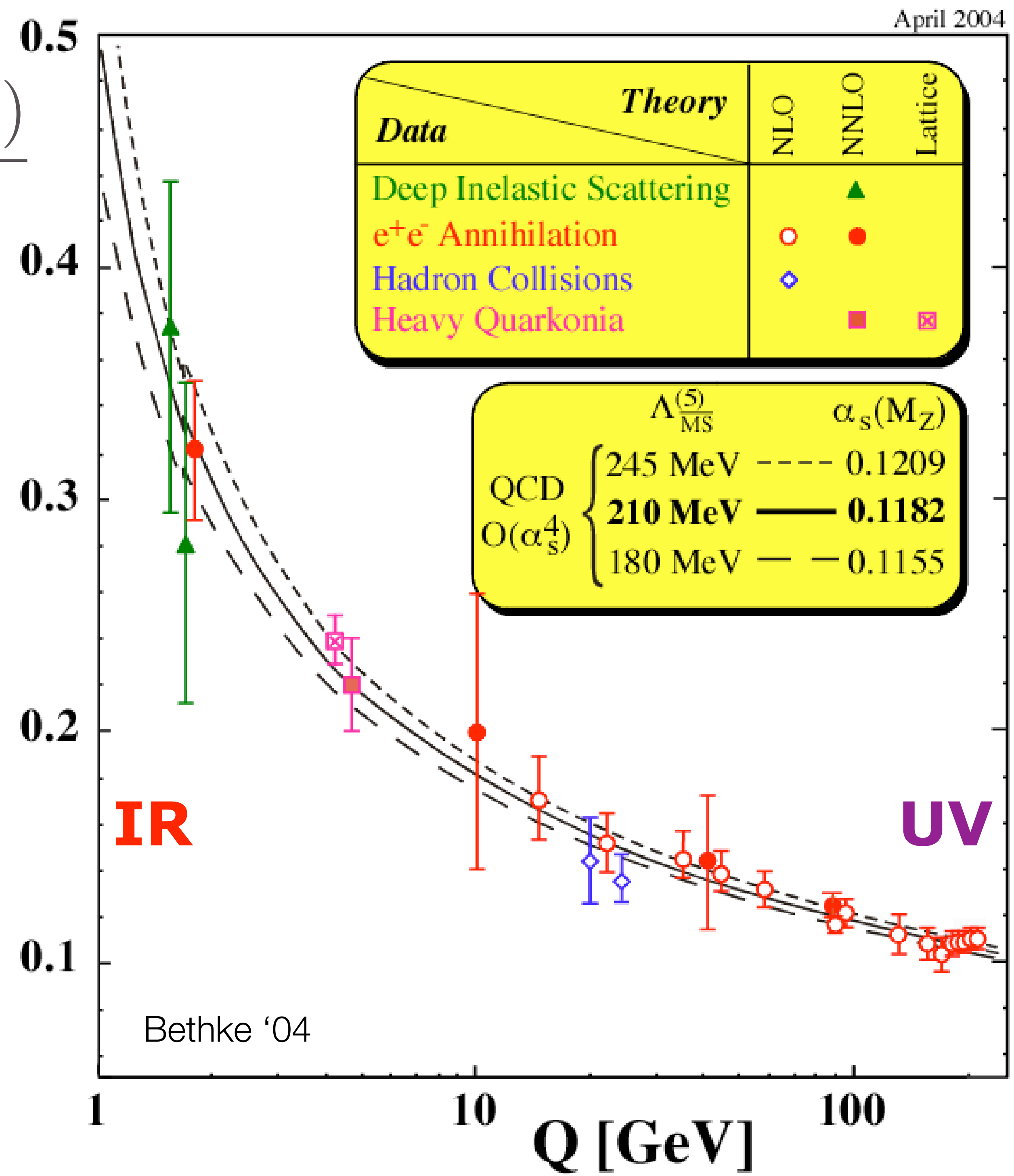
Running coupling at low and high energies

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$

- **UV: asymptotic freedom**

$$\alpha_s(Q \to \infty) = 0$$

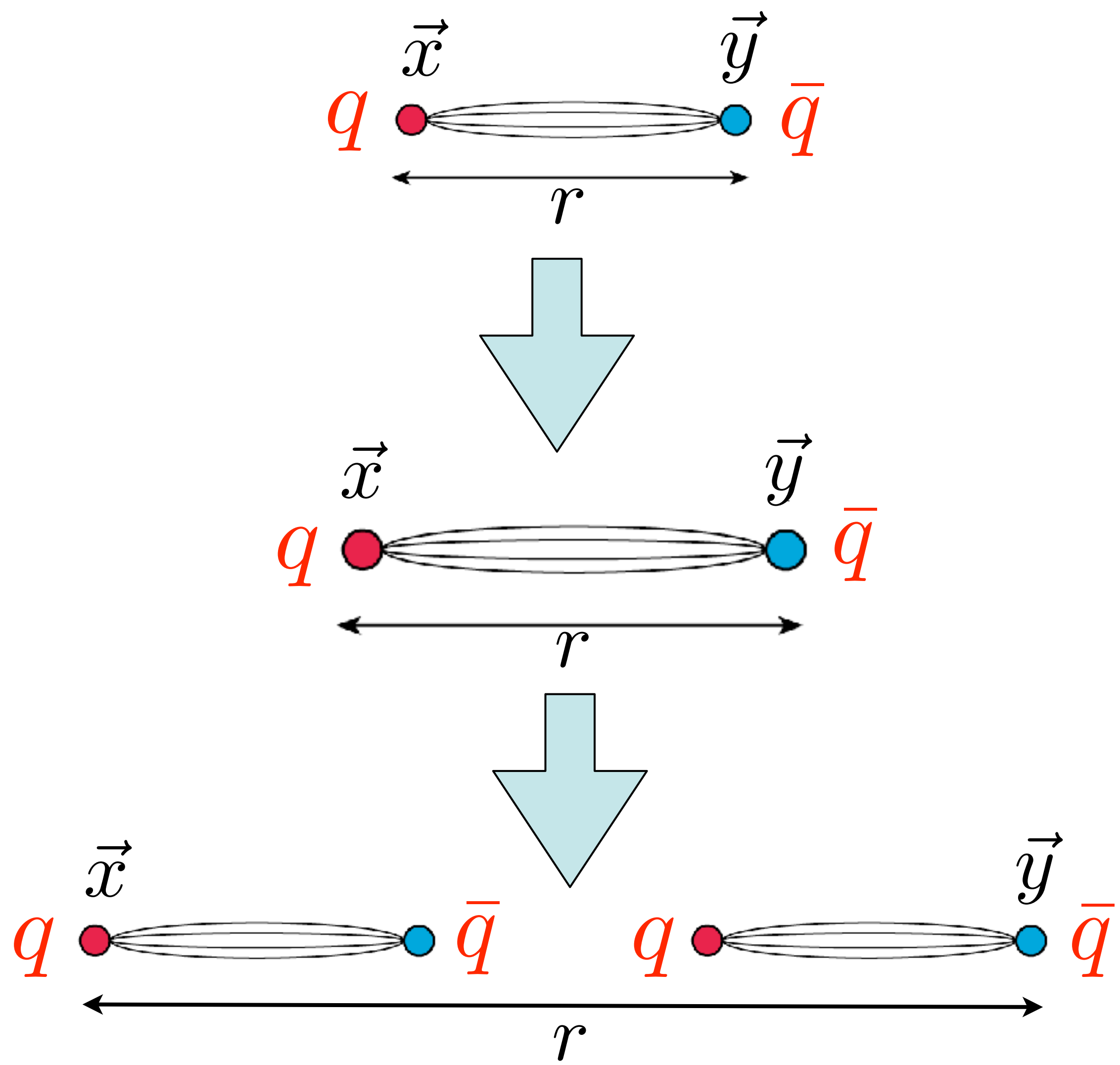
- **IR: failure of perturbation theory**



Confinement

$$r = |\vec{x} - \vec{y}|$$

$$\langle q(\vec{x})\bar{q}(\vec{y})\rangle \simeq e^{-\beta F_{q\bar{q}}(r)}$$



$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

$$F_{q\bar{q}} \simeq \sigma r$$

$$F_{q\bar{q}} \simeq \sigma r_0$$

string breaking at $\mathrm{r} \approx 1\mathrm{fm}$

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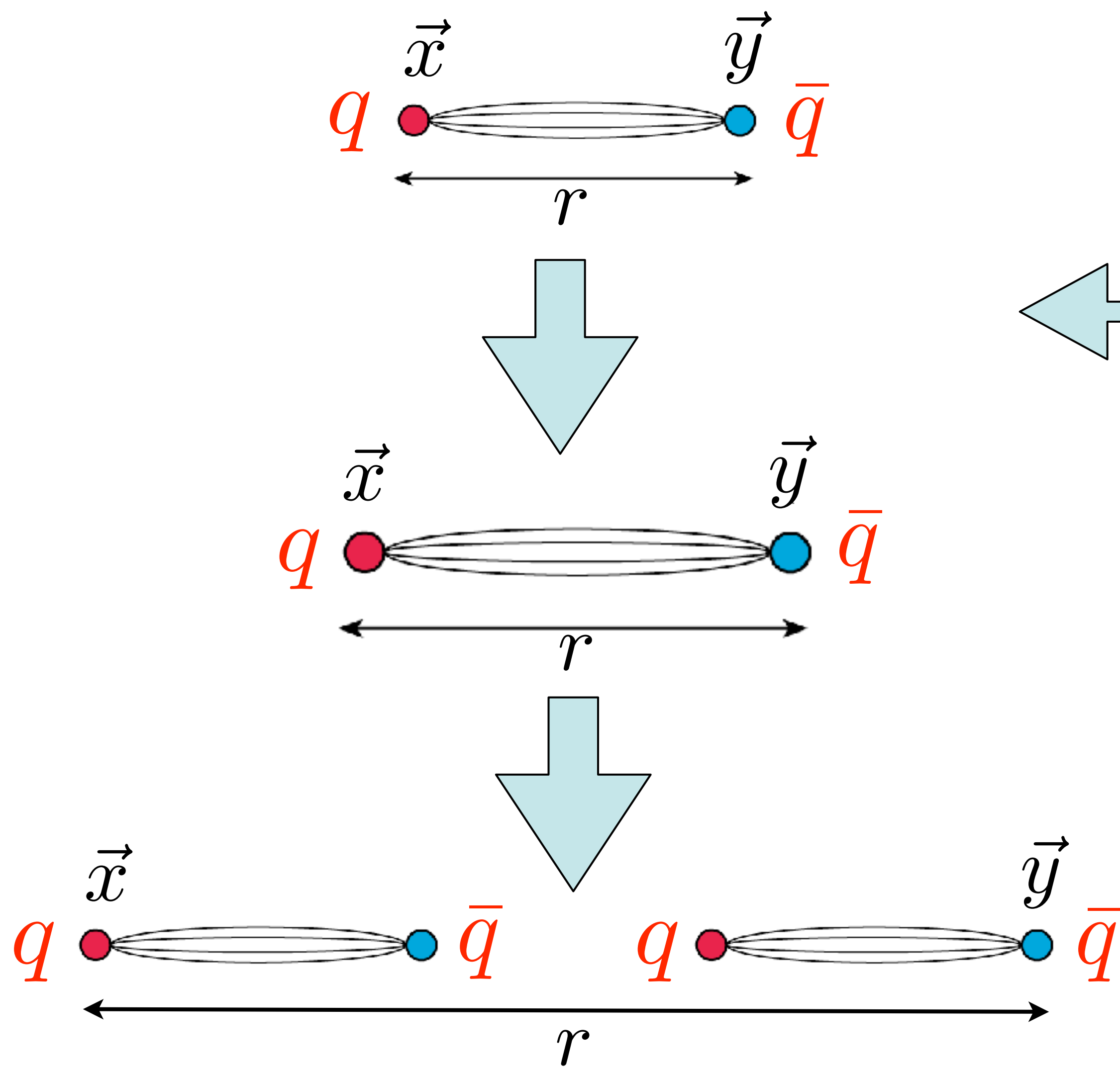

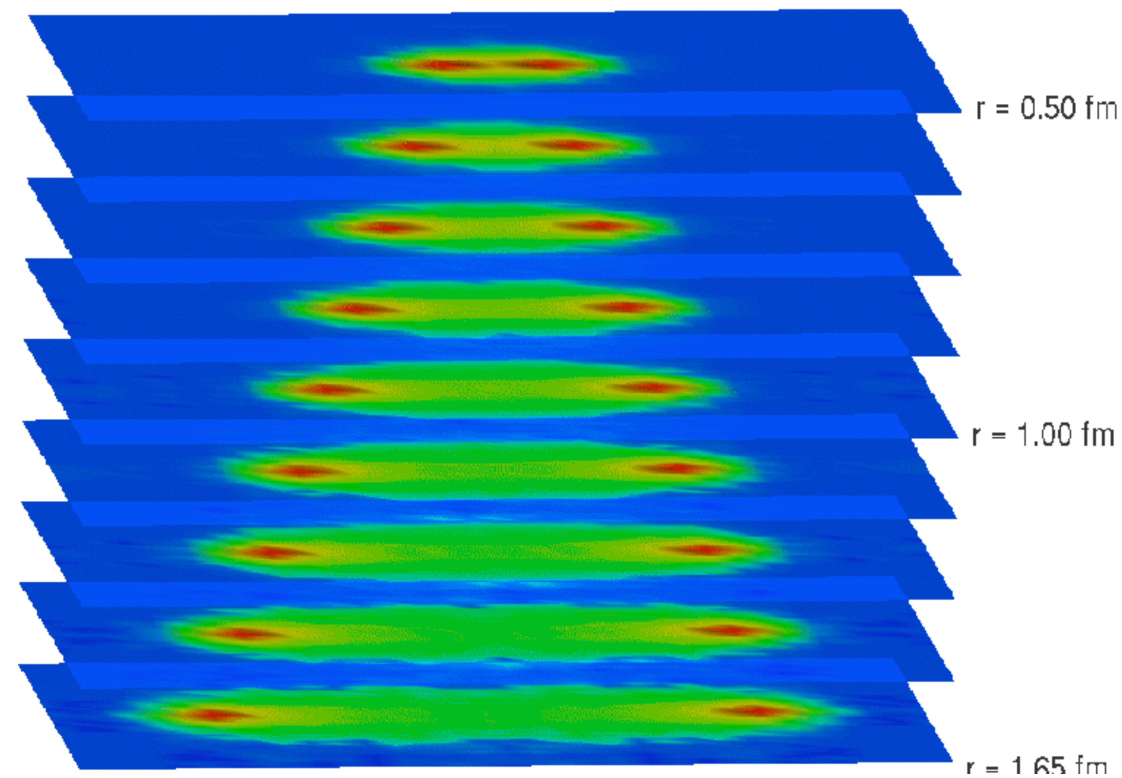

Bali et al. '94

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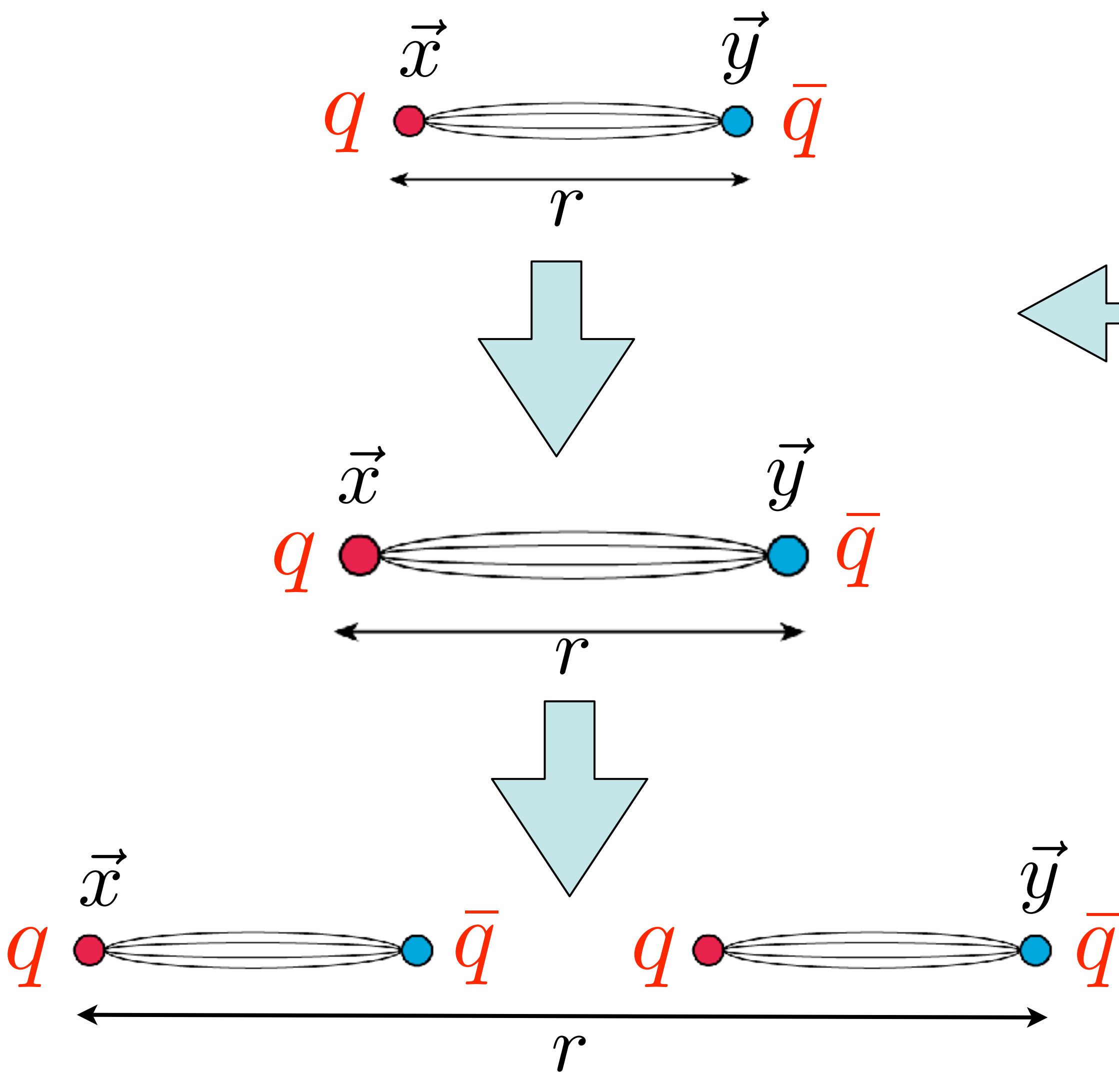
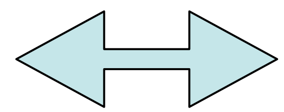
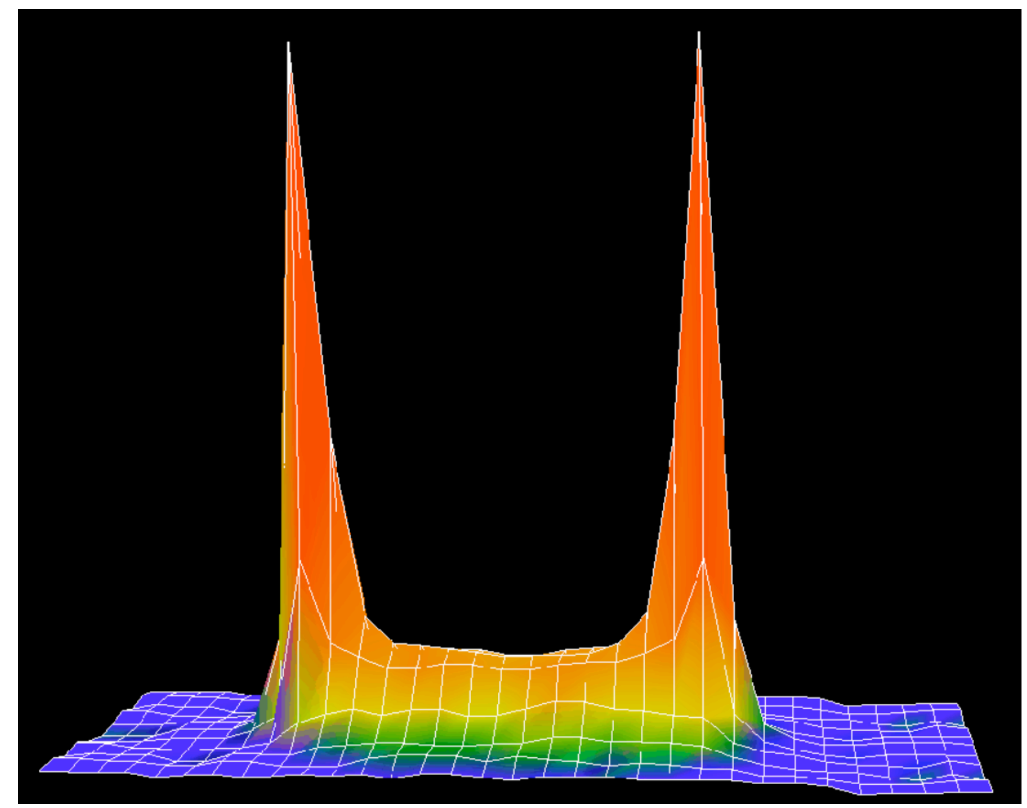




Energy density



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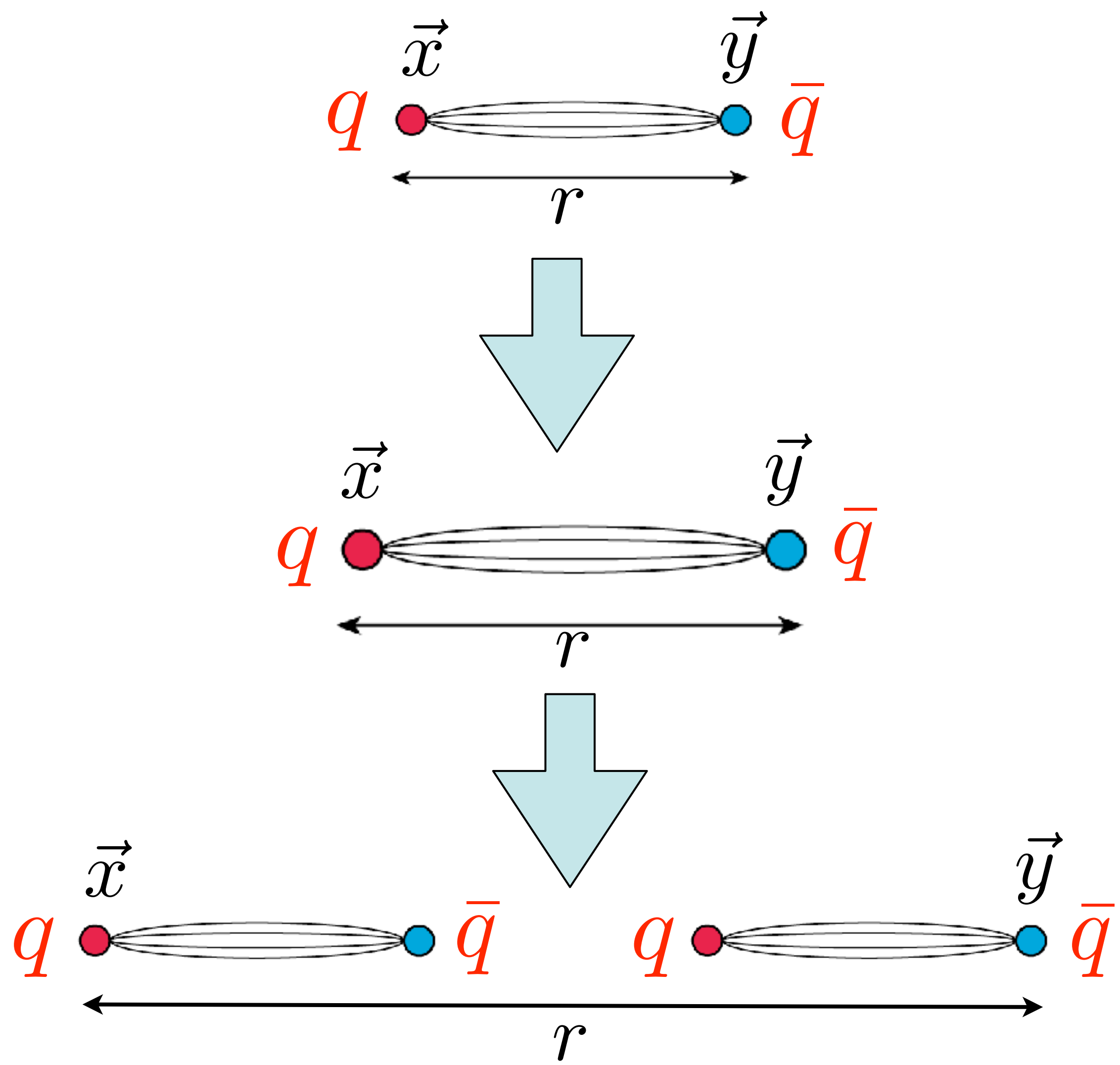

Order parameter $\sim \; '\langle q \rangle'$

$$\Phi = e^{-\frac{1}{2}\beta F_{q\bar{q}}(\infty)}$$

- **Confinement:** $\Phi = 0$
- **Deconfinement:** $\Phi \neq 0$

Mechanism?

- **not fully resolved**

Chiral symmetry breaking

chiral symmetry

Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
Quark	u	c	t	$\frac{2}{3}$
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chiral symmetry breaking: $\Delta m \approx 400\,\mathrm{MeV}$

chiral symmetry breaking

2 light flavours, one heavy flavour 2+1

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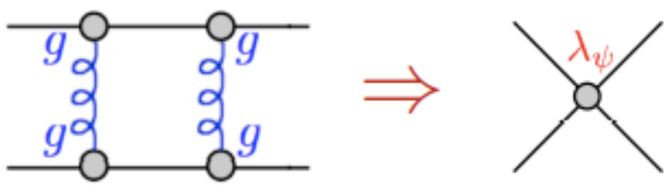

$\propto \alpha_s^2$

$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2\right]$$

$\langle \bar{q}q \rangle \neq 0$

mass term: $\langle \bar{q}q \rangle\, \bar{q}q$

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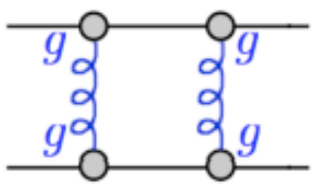
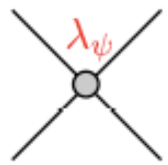
Chiral symmetry breaking



$\Rightarrow$



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$\langle \bar{q}q \rangle \neq 0$

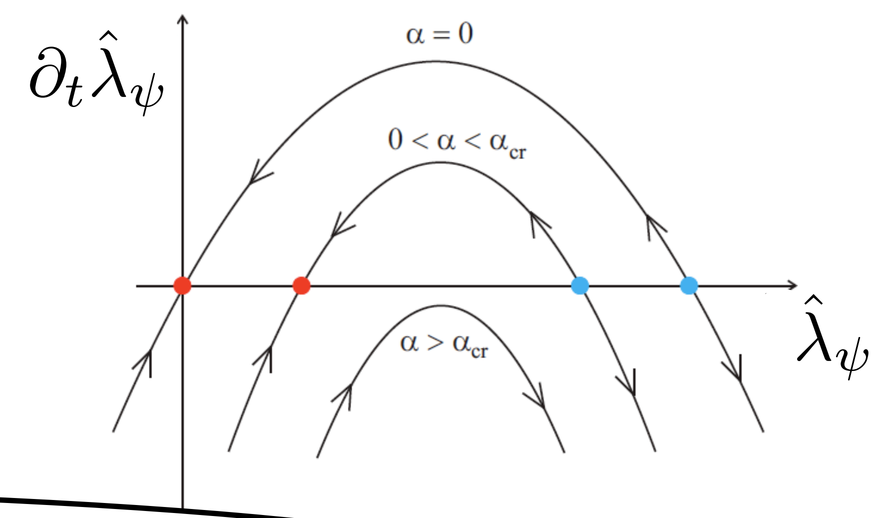
mass term: $\langle \bar{q}q \rangle \, \bar{q}q$

$\alpha_s > \alpha_{s,\mathrm{crit}}$

Order parameter

$$\sigma = \langle \bar{q}q \rangle$$ chiral condensate

- **chiral symmetry:** $\sigma = 0$
- **symmetry breaking:** $\sigma \neq 0$



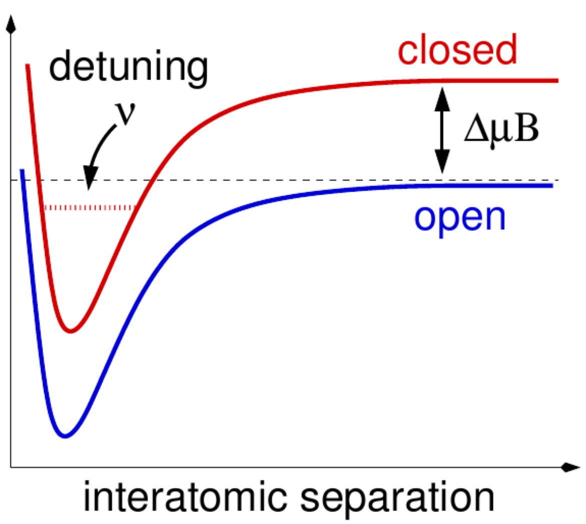
Chiral symmetry breaking directly sensitive to size of α_s

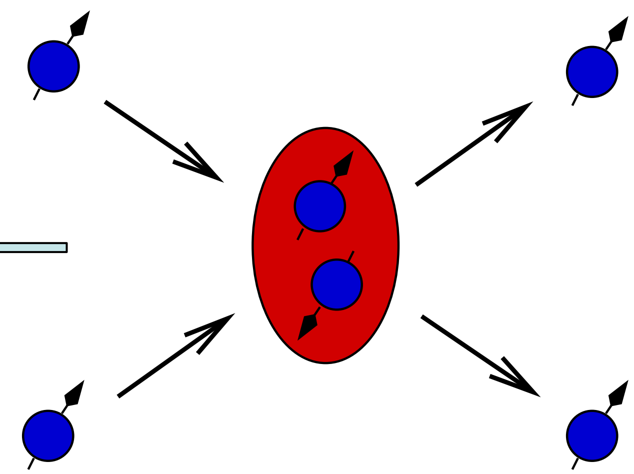
cool atoms

Phase diagram of cold quantum gases

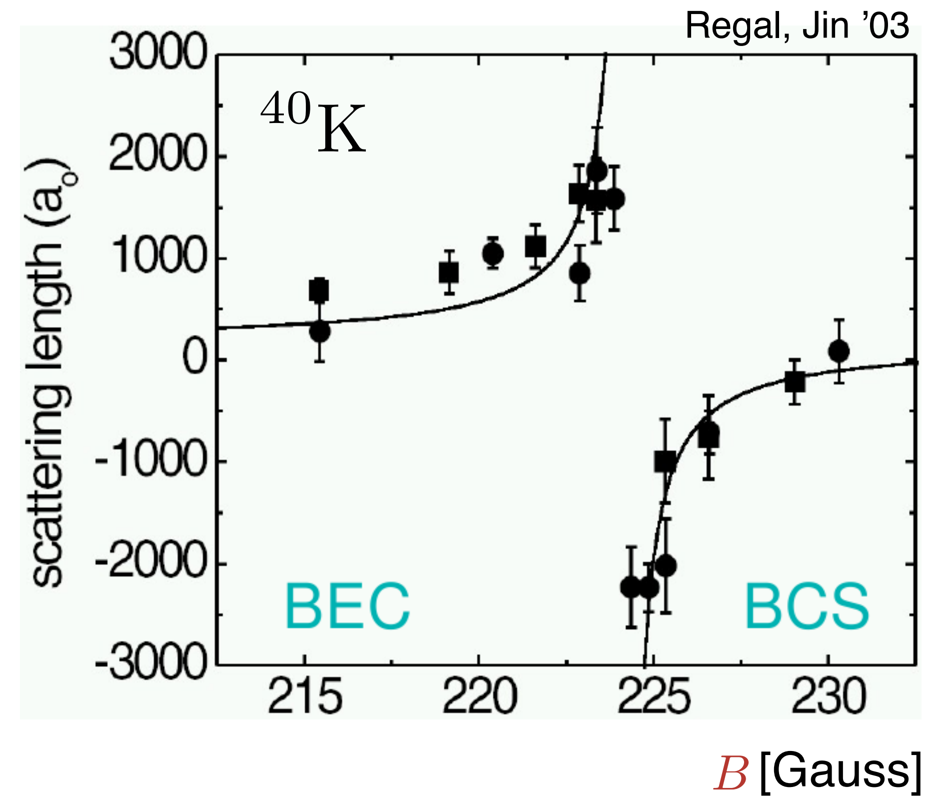
Feshbach resonance

2-atom scattering





e.g



hyperfine interaction between interaction channels ⇨ **s-wave scattering length**

$a(B)$

near Feshbach resonance

Non-perturbative methods

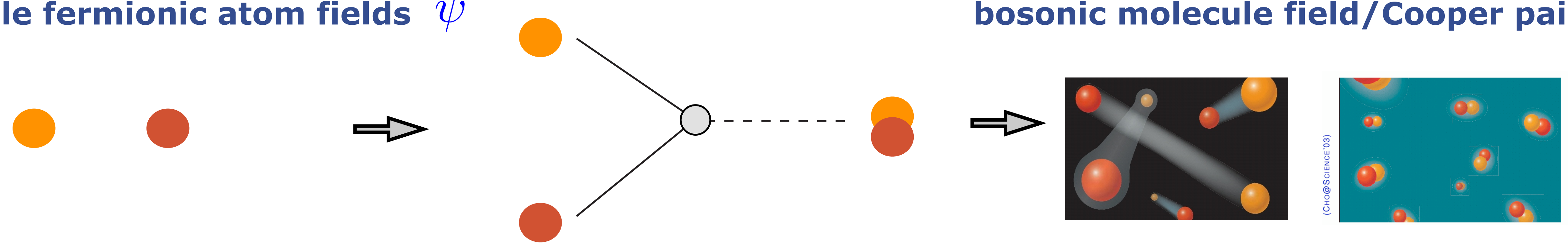
BEC/Superfluidity

$\mathrm{T} > \mathrm{T_C}$

stable fermionic atom fields ψ

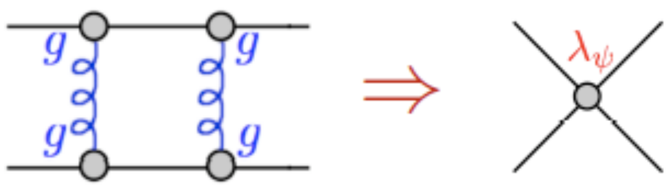
$\mathrm{T} < \mathrm{T_C}$

bosonic molecule field/Cooper pair ϕ



Condensation phenomena

chiral symmetry breaking



$\propto \alpha_s^2$

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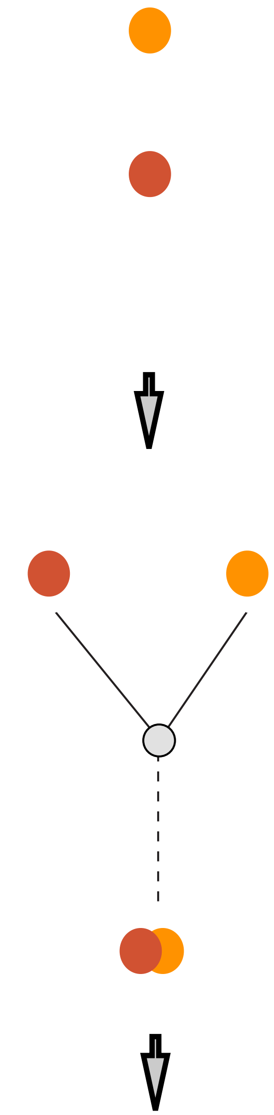
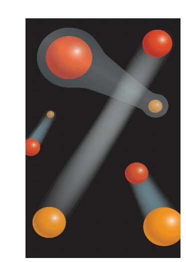
$\langle \bar{q}q \rangle \neq 0$

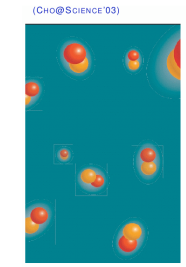
mass term: $\langle \bar{q}q \rangle \, \bar{q}q$

Bose-Einstein condensation





(Cho@Science'03)



Toolbox for strongly correlated systems

FunMethods: FRG-DSE-2PI-...

Lattice

Toolbox for strongly correlated systems

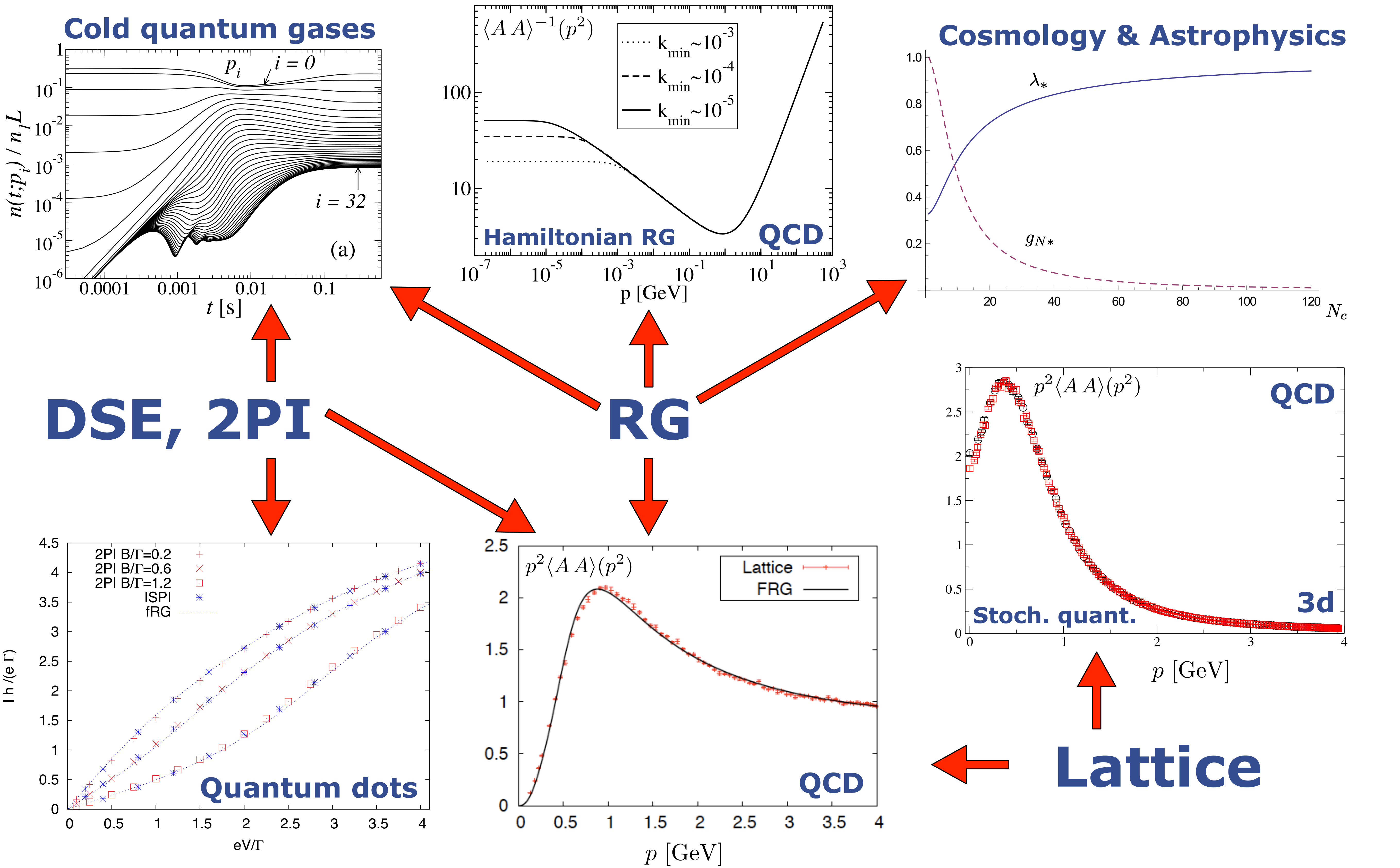
QCD

ab initio continuum FunMethods: FRG-DSE-2PI-...

Models

ab initio Lattice

Toolbox for strongly correlated systems



Functional RG in gauge theories

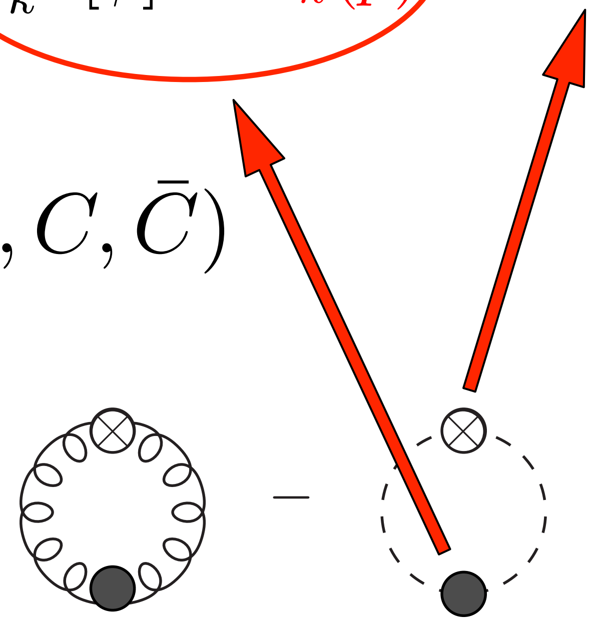
Wetterich '93

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k(p)}\,k\partial_k R_k(p)$$

- **Yang Mills Theory:** $\phi = (A, C, \bar{C})$

RG-scale k: $t = \ln k$

$$\partial_t \Gamma_k[\phi] \;=\; \tfrac{1}{2}$$



- **Fermions are straightforward** though 'physically' complicated
  - **no sign problem** numerics as in scalar theories
  - **chiral fermions** reminder: Ginsparg-Wilson fermions from RG arguments

FunMethods: FRG-DSE-2PI-...

- **bound states via dynamical hadronisation** effective field theory techniques

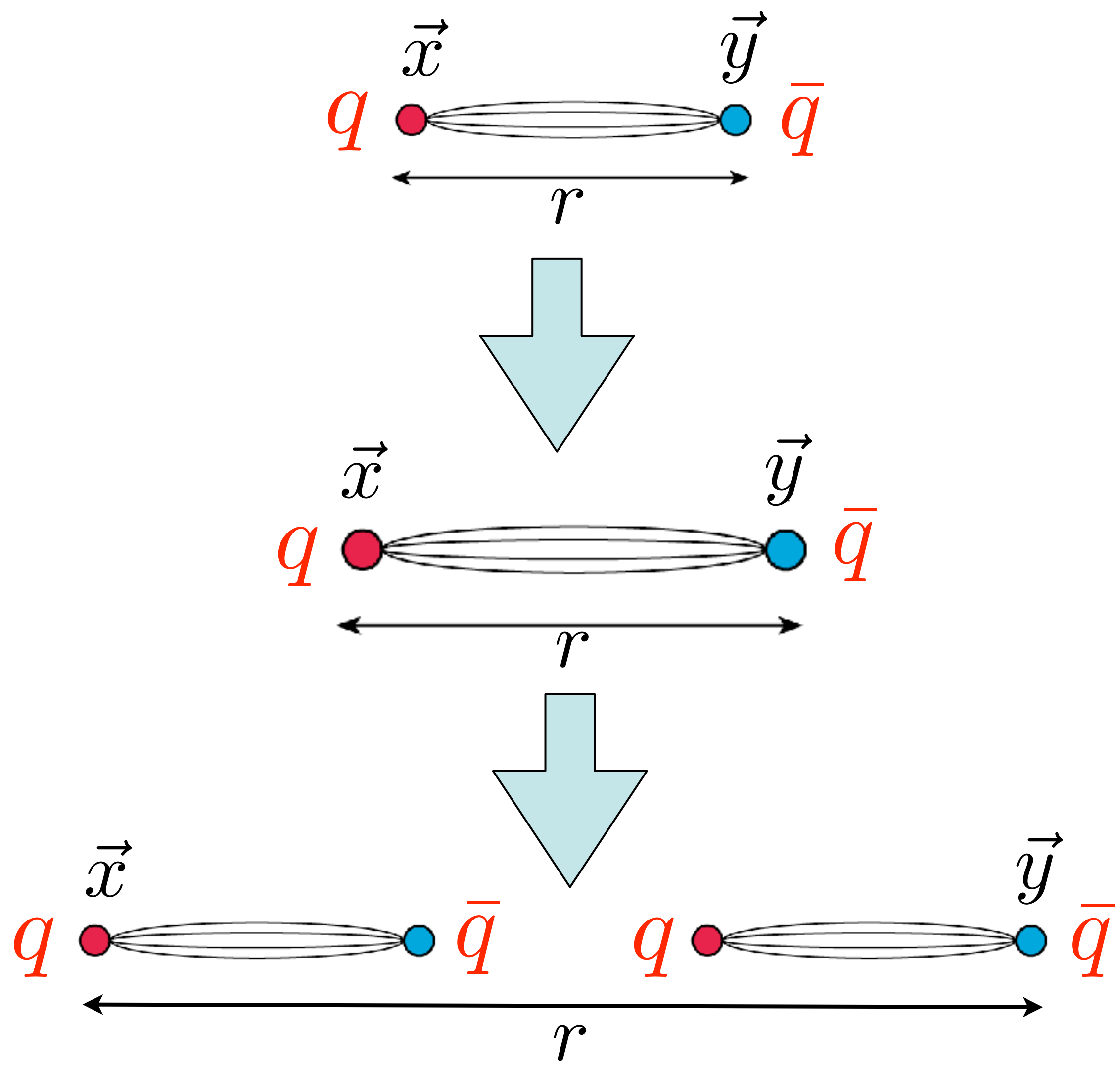
Complementary to lattice!

e.g. finite volume scaling: Braun, Klein, Piasecki '10

Confinement in Yang-Mills theory

Confinement

$$r = |\vec{x} - \vec{y}|$$



Order parameter $\sim \; '\langle q \rangle'$

$$\Phi \sim e^{-\frac{1}{2}\beta F_{q\bar{q}}(\infty)}$$

- **Confinement:** $\Phi = 0$
- **Deconfinement:** $\Phi \neq 0$

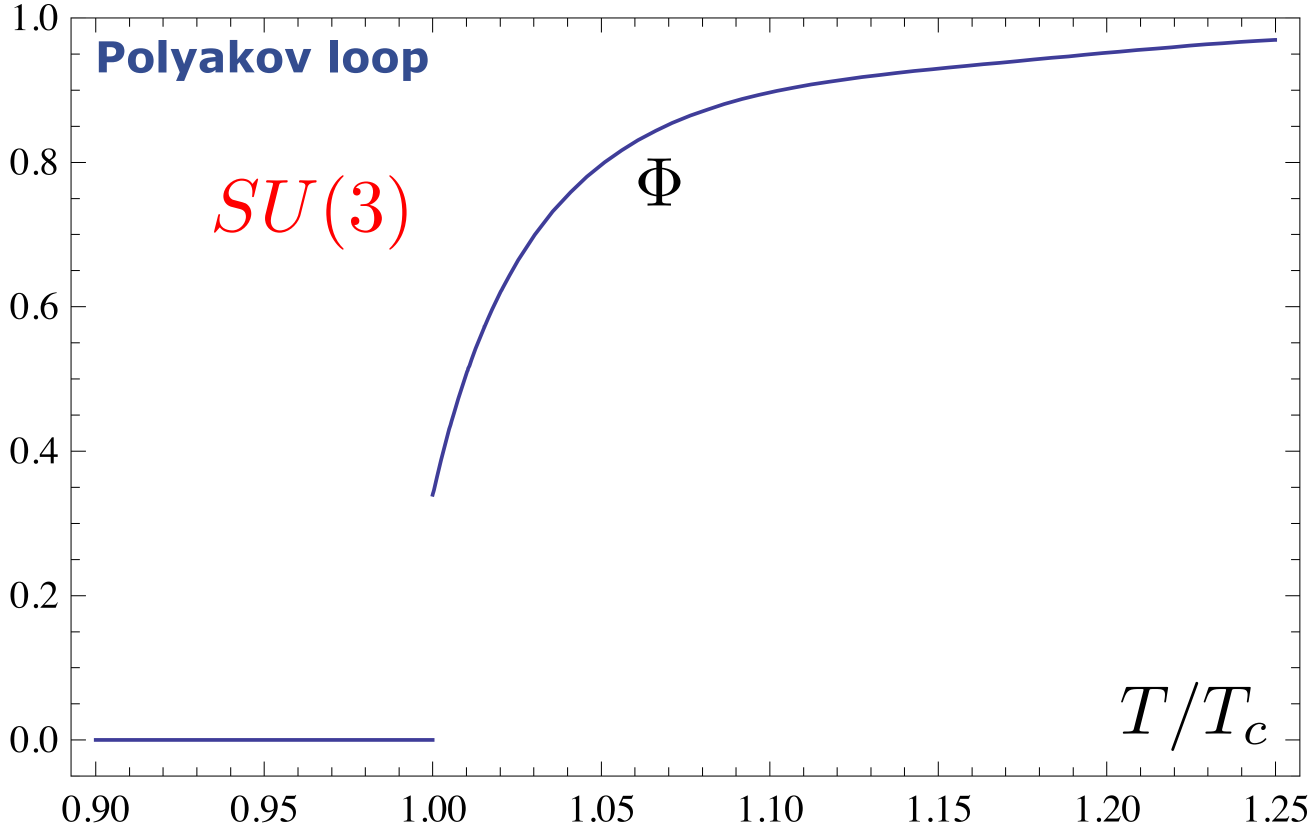
Polyakov loop

$$\Phi = \frac{1}{3}\langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\}\rangle$$

Confinement

FRG+DSE+2PI+lattice

Braun, Gies, JMP '07



$$T_c = 276 \pm 10\,\mathrm{MeV}$$

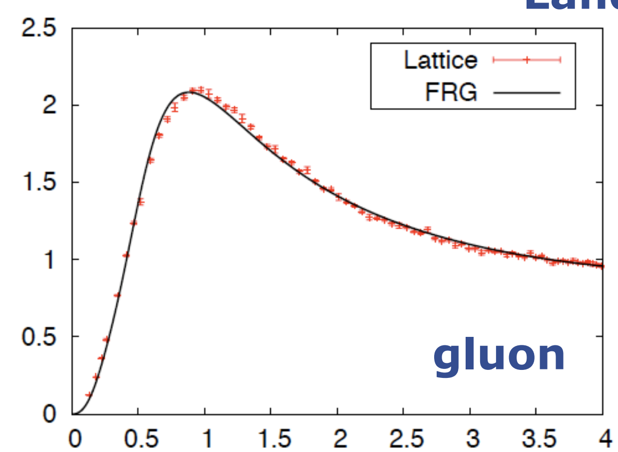
$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023 \qquad \mathrm{lattice}: T_c/\sqrt{\sigma} = 0.646$$

SU(N), Sp(2), E(7): Braun, Eichhorn, Gies, JMP '10

G2: work in progress

Landau gauge

computed from the full propagators:



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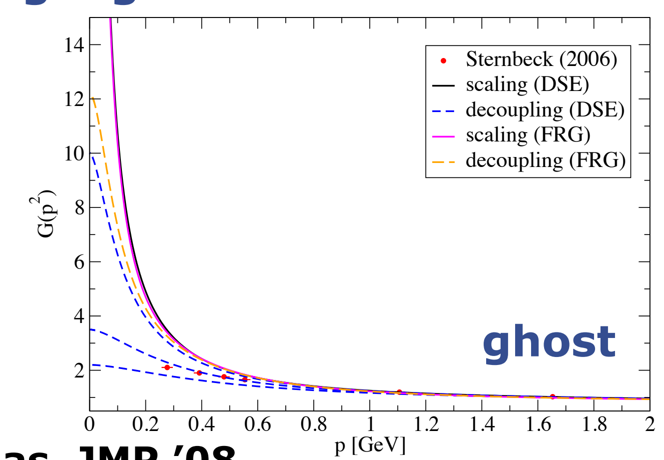


Fischer, Maas, JMP '08

gauge independence: **Marhauser, JMP '08**

confinement criteria from FRG: **Braun, Gies, JMP '07 + Eichhorn, Gies, JMP '10**

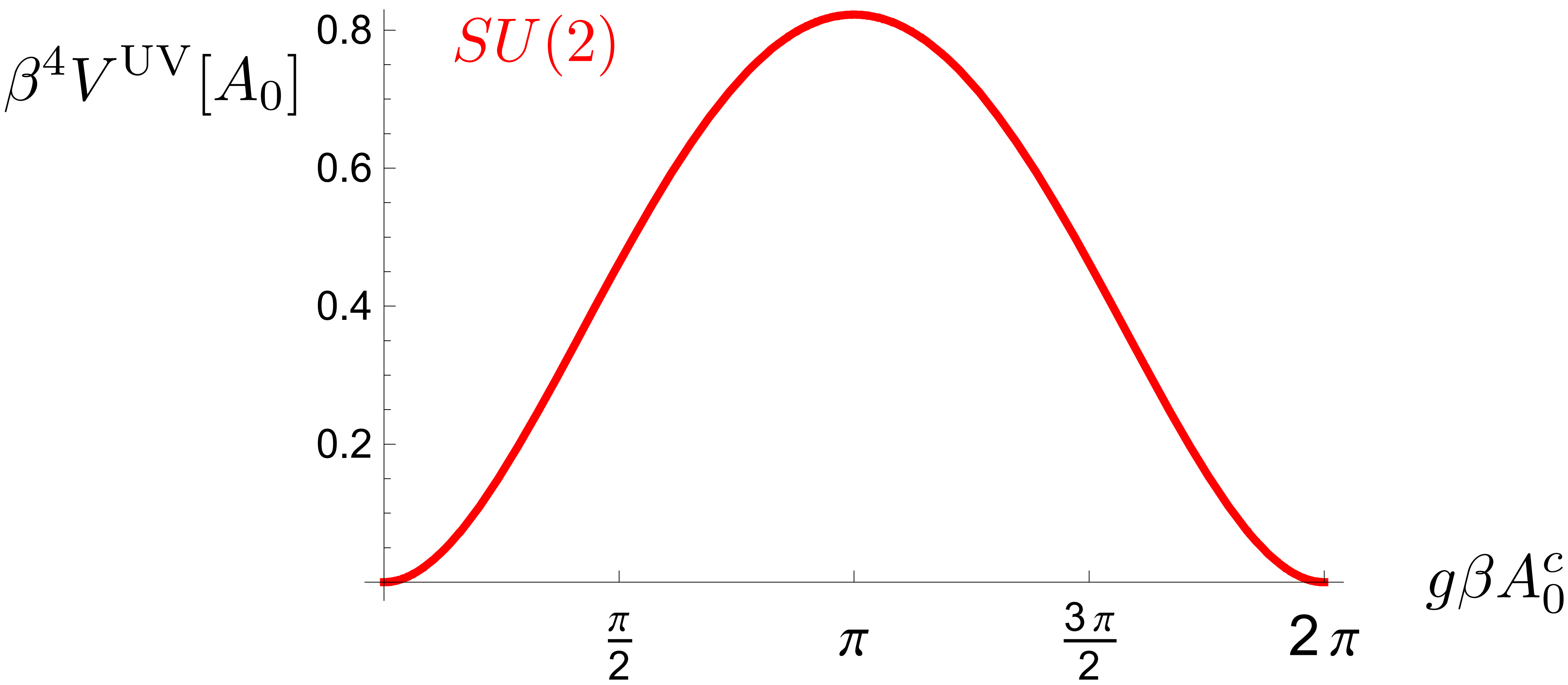
tightened confinement criterium from FRG & DSE: **Fister, JMP, in preparation**

Confinement

Perturbation theory

Gross, Pisarski, Yaffe '81
Weiss '81

$$V^{\rm UV}[A_0] = \frac{1}{2\Omega}\,{\rm Tr}\log S^{(2)}_{AA}[A_0] - \frac{1}{\Omega}{\rm Tr}\log S^{(2)}_{C\bar{C}}[A_0]$$



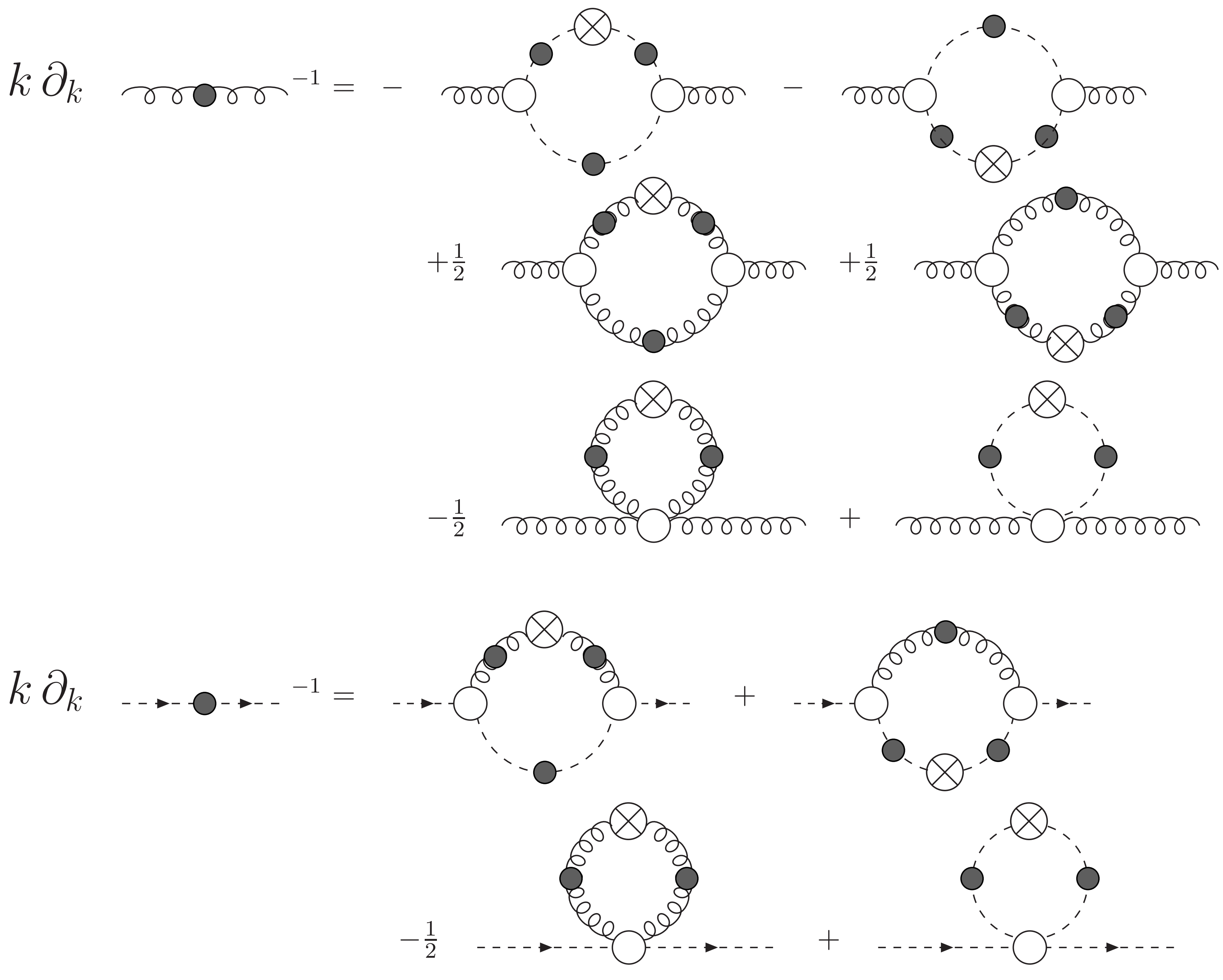
$$SU(2):\ \Phi[A_0] = \cos\frac{1}{2}\beta g A_0^c \qquad \text{with} \qquad A_0 = A_0^c\,\frac{\sigma_3}{2}$$

Confinement

FRG+DSE+2PI+lattice

Braun, Gies, JMP '07

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$



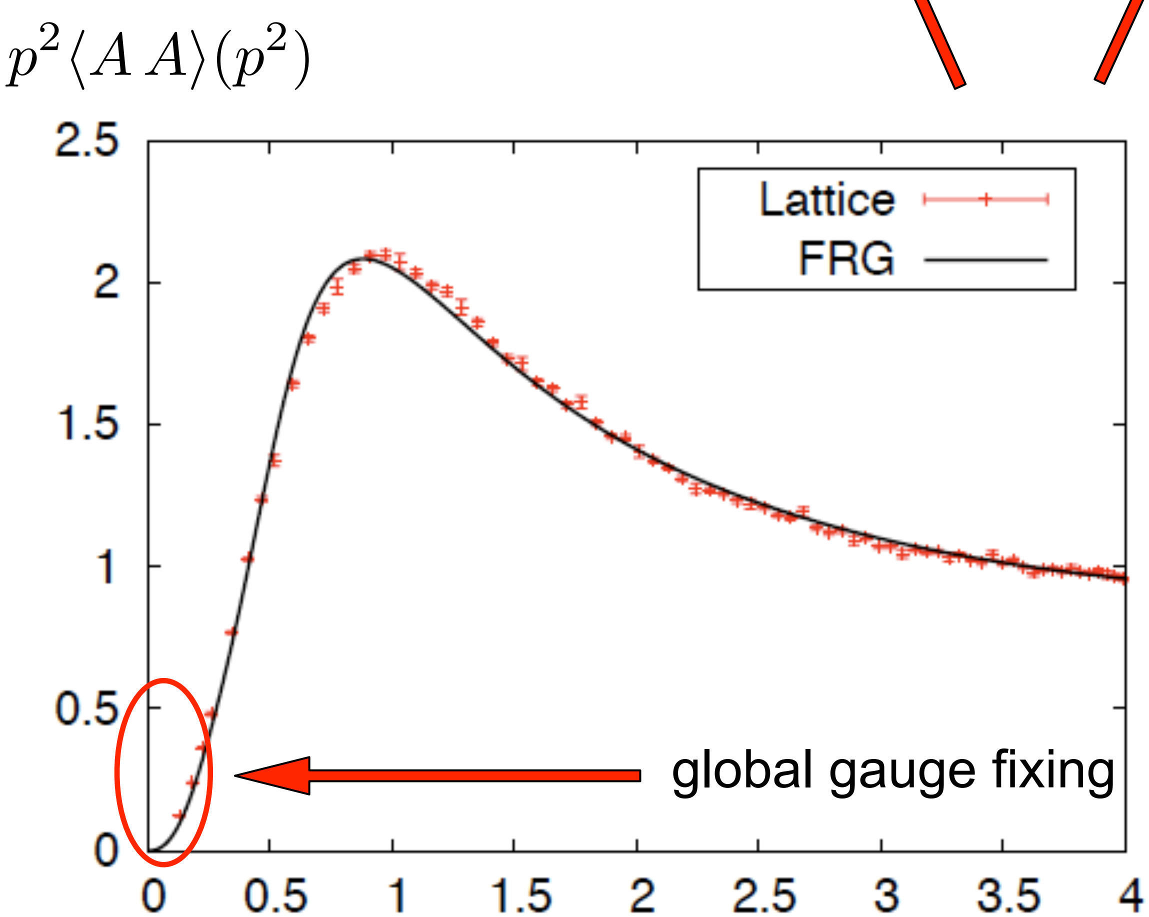
Confinement

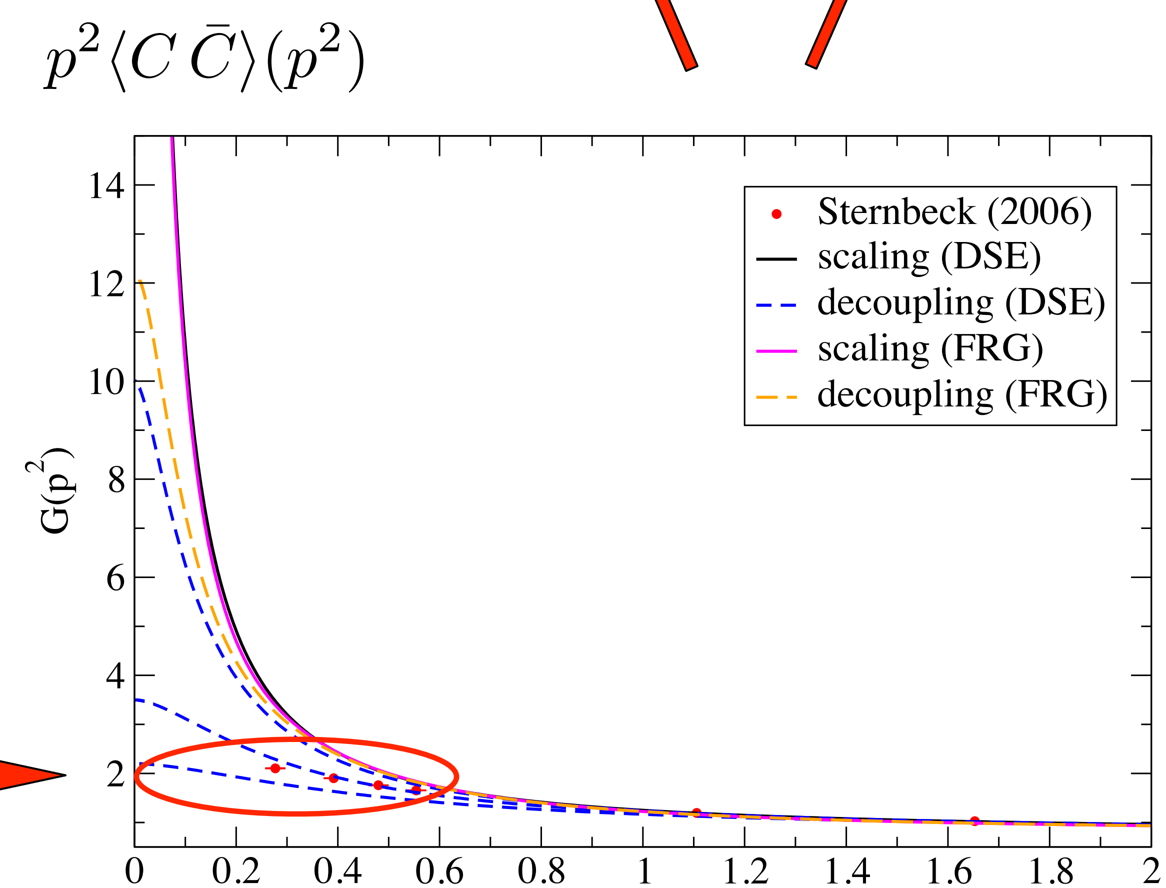
FRG+DSE+2PI+lattice

Braun, Gies, JMP '07

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

$$p_0 \to 2\pi T n - g A_0$$





p [GeV]

Lattice data: Sternbeck et al '07

Fischer, Maas, JMP '08

Confinement not directly sensitive to size of α_s

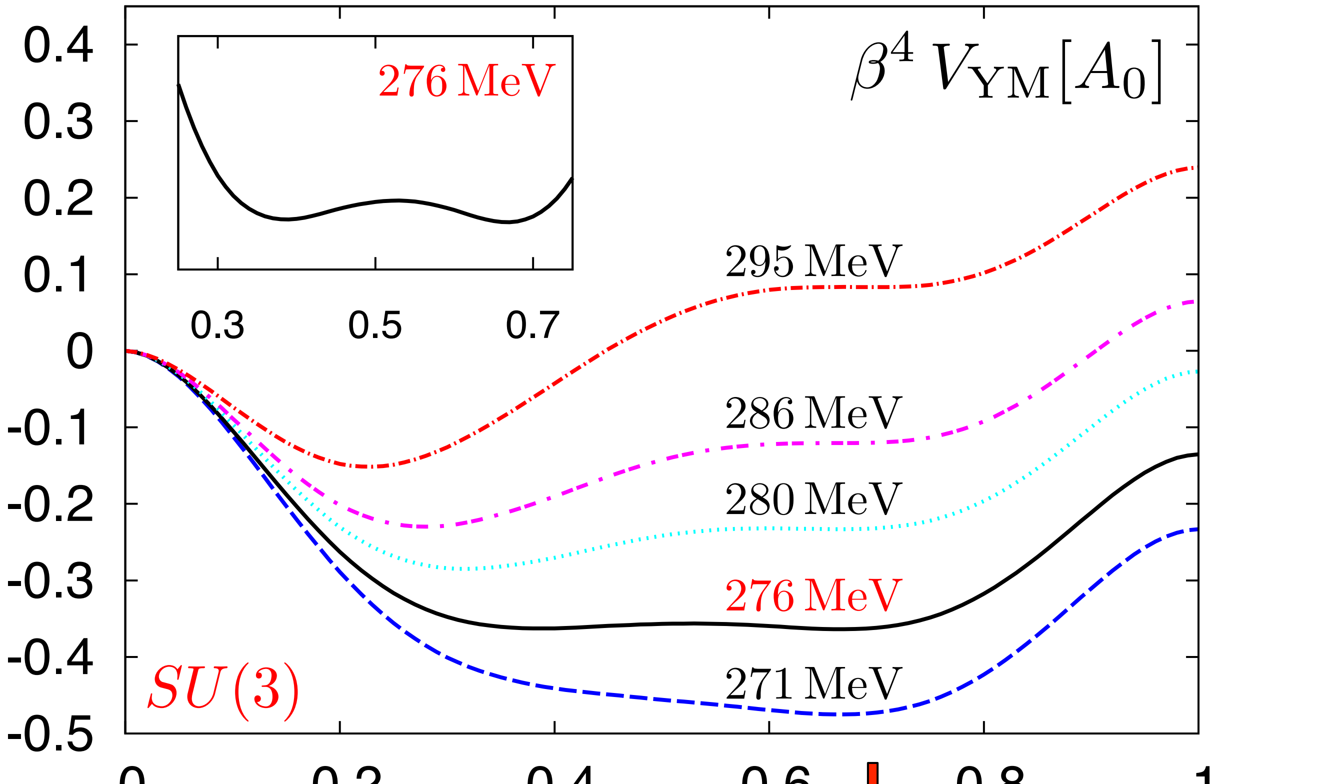
Confinement

Order parameter $SU(3)$

Braun, Gies, JMP '07

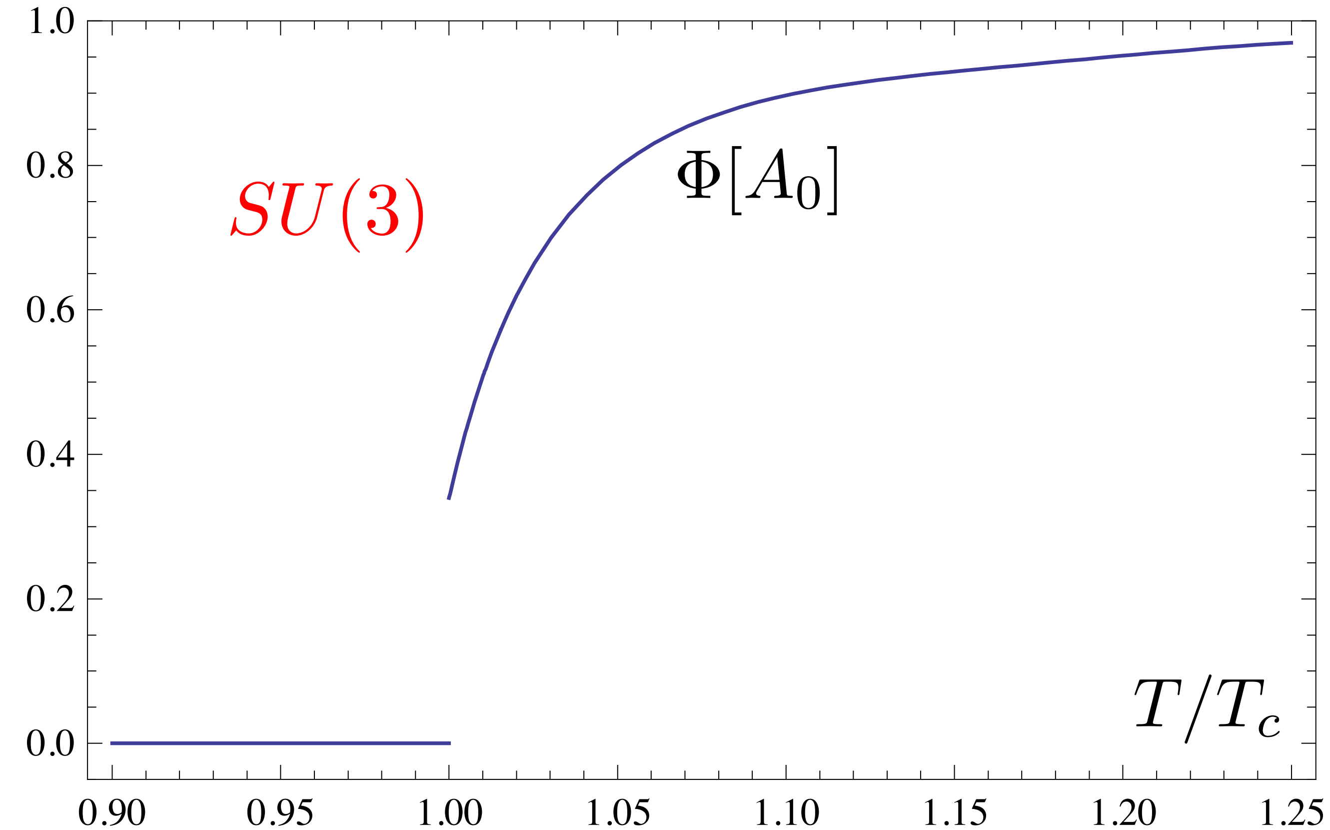
$$T_c = 276 \pm 10\,\text{MeV} \qquad T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

$\text{lattice} : T_c/\sqrt{\sigma} = 0.646$



$$\Phi[\frac{4}{3}\pi \frac{1}{\beta g}] = 0$$

$$\Phi[A_0] = \frac{1}{3}(1 + 2\cos\frac{1}{2}\beta g A_0)$$



SU(N), Sp(2), E(7): Braun, Eichhorn, Gies, JMP '10
G2: work in progress

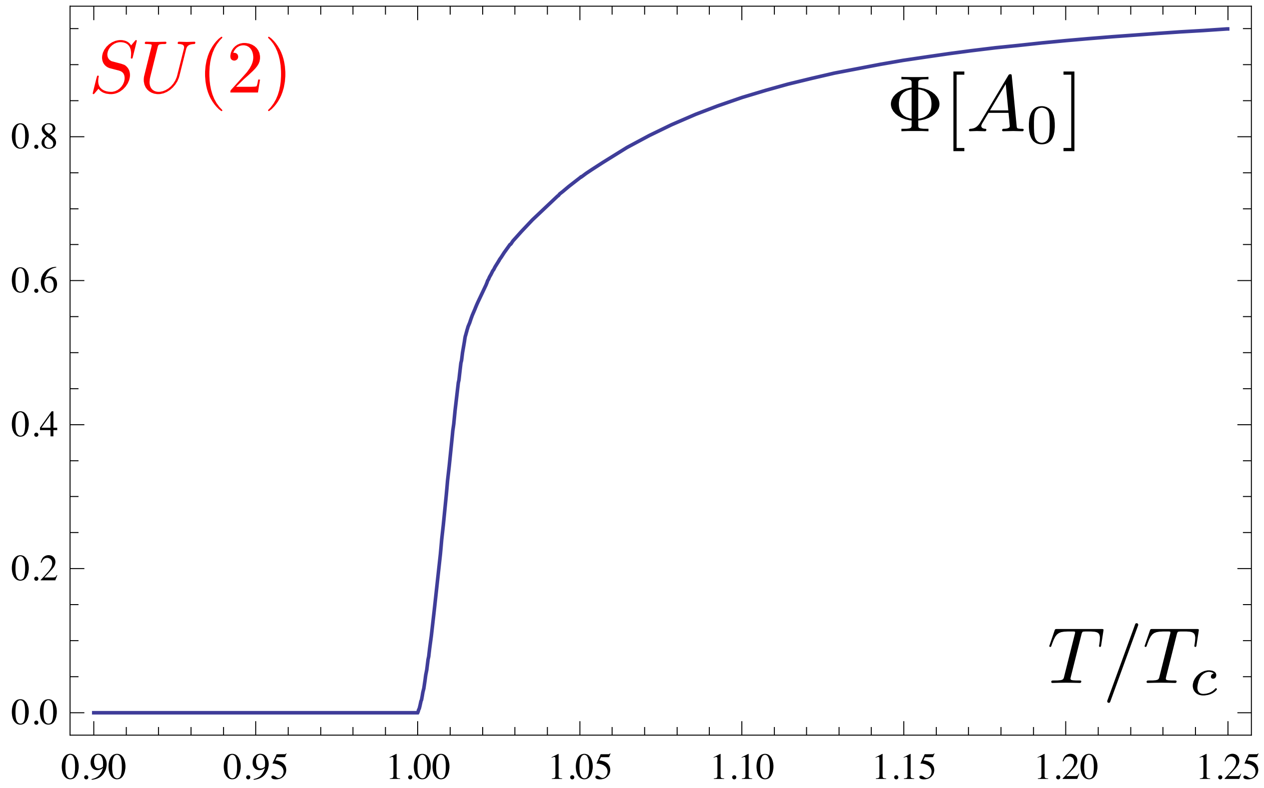
Confinement

Order parameter $SU(2)$

Braun, Gies, JMP '07

$T_c = 254 \pm 10\,\mathrm{MeV}$ $\qquad$ $T_c/\sqrt{\sigma} = 0.605 \pm 0.023$ ⟷ $\mathrm{lattice}: T_c/\sqrt{\sigma} = 0.709$

missing V'' − terms

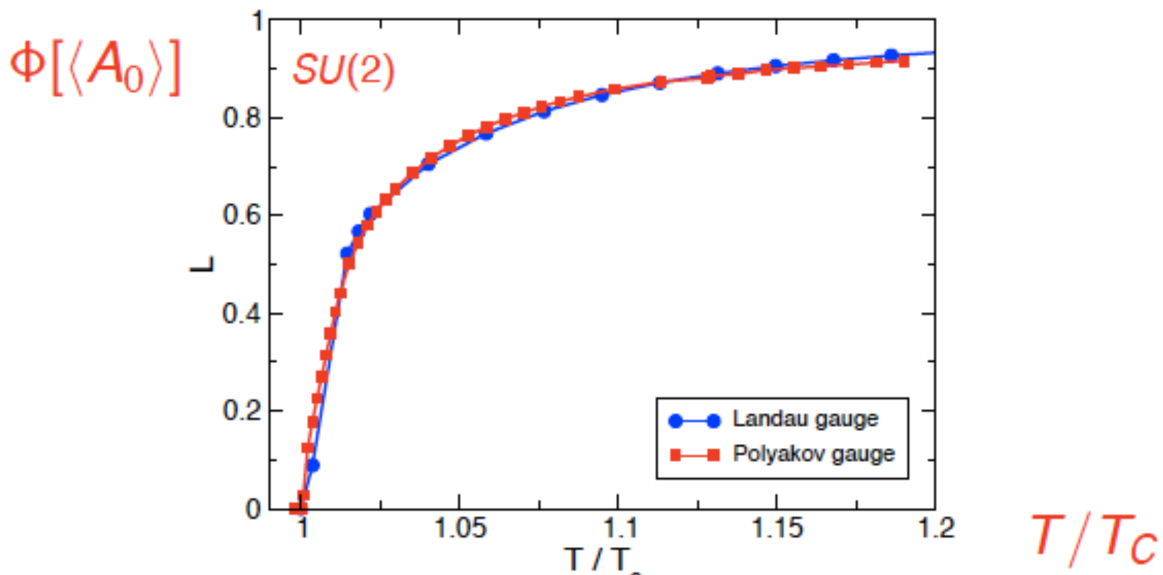


V'' -terms in Polyakov gauge

Marhauser, JMP '08

Polyakov gauge: $A_0 = A_0^c(\vec{x})\sigma_3$

$$\text{RG-flow}: \quad V[A_0] = -\int dt\, \mathrm{flow}[V''[A_0], \alpha_s]$$



- – –: Polyakov gauge: crit. exp. $\nu = 0.65$ $\qquad$ $\nu_{\mathrm{Ising}} = 0.63$
- – –: Landau gauge propagators

V'' -terms in Landau gauge

Braun, Gies, JMP, Spallek, in prep.

$$T_c = 299 \pm 20\,\mathrm{MeV}$$

$$T_c/\sqrt{\sigma} = 0.716 \pm 0.046$$

Confinement

Confinement criteria!

infrared behaviour of propagators & confinement

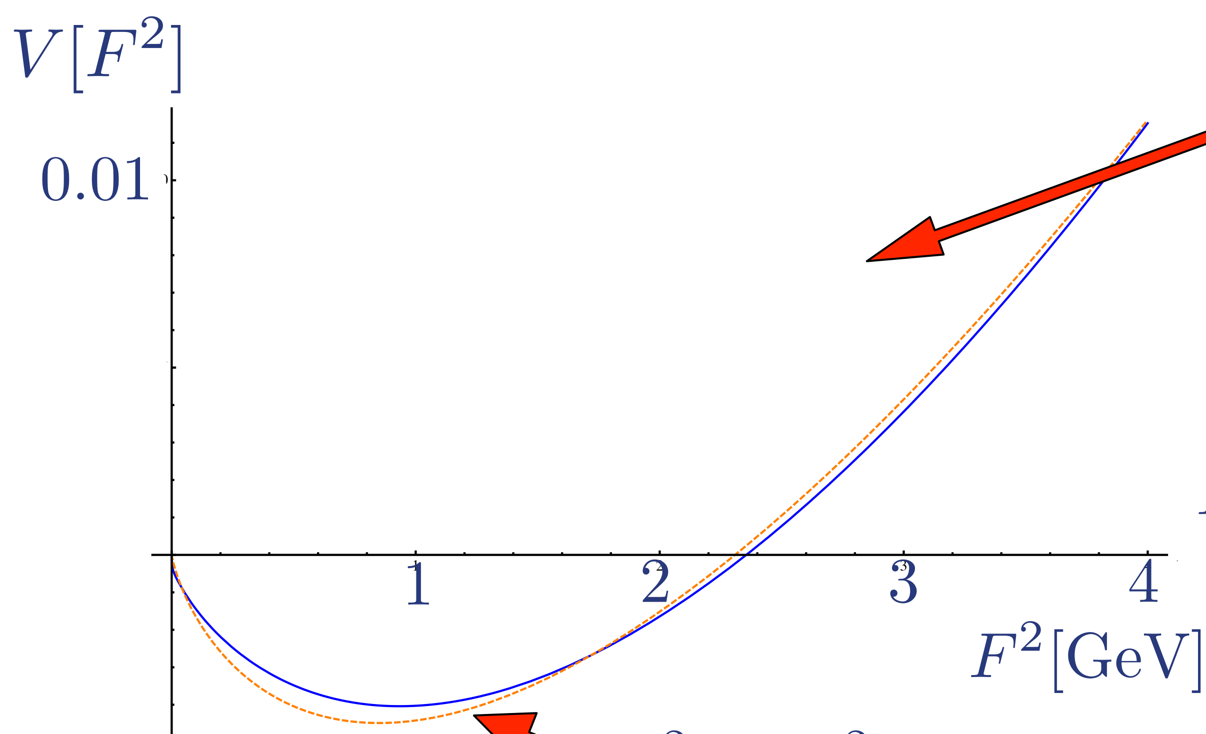
IR gluon

$$p^2\langle A\,A\rangle(p^2) \propto (p^2)^{-\kappa_A}$$

IR ghost

$$p^2\langle C\,\bar{C}\rangle(p^2) \propto (p^2)^{-\kappa_C}$$

scaling	$\kappa_A = -2\kappa_c : \kappa_C \simeq 0.595...$	$\kappa_C < 0.605$	$\kappa_C > 1/4$
decoupling	$\kappa_A = -1 \;\&\; \kappa_c = 0$	✓	✓
		infrared stability Eichhorn, Gies, JMP '10	**confinement** Braun, Gies, JMP '07



$F^2 \log F^2$ **- model**

$F^2 \log F^2$ **- model** → **String tension:** $\approx 747\mathrm{MeV}$

Confinement

Confinement criteria!

Braun, Gies, JMP '07

IR gluon

$$p^2\langle A\,A\rangle(p^2) \propto (p^2)^{-\kappa_A}$$

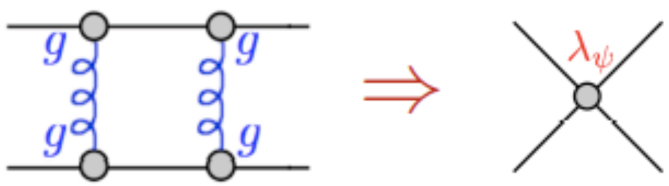
IR ghost

$$p^2\langle C\,\bar{C}\rangle(p^2) \propto (p^2)^{-\kappa_C}$$

scaling	$\kappa_A = -2\kappa_c: \;\; \kappa_C \simeq 0.595...$	$\kappa_C < 0.605$	$\kappa_C > 0$
decoupling	$\kappa_A = -1 \;\&\; \kappa_c = 0$	✓	✓
		infrared stability	**confinement**
		Eichhorn, Gies, JMP '10	Fister, JMP, in prep.

Chiral symmetry breaking in QCD

Chiral symmetry breaking



$\propto \alpha_s^2$

$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2\right]$$

$\langle \bar{q}q \rangle \neq 0$

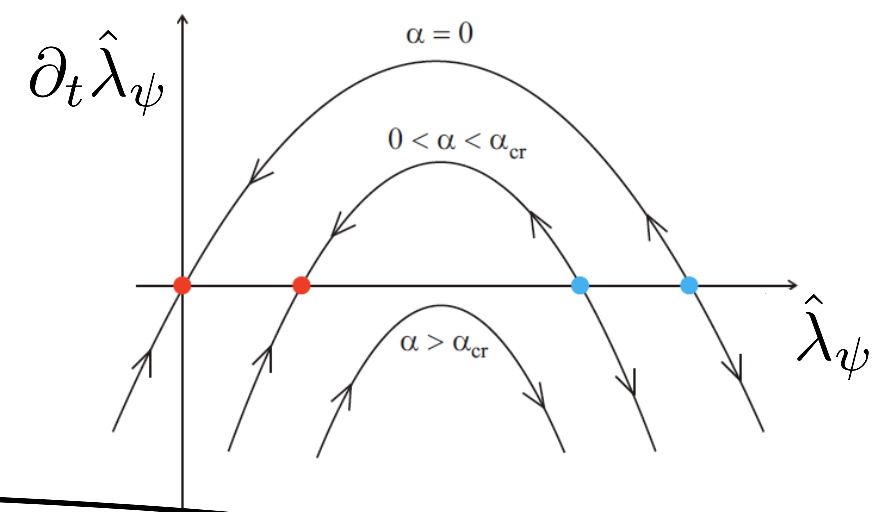
mass term: $\langle \bar{q}q \rangle\, \bar{q}q$

$\alpha_s > \alpha_{s,\mathrm{crit}}$

Order parameter

$$\sigma = \langle \bar{q}q \rangle$$ chiral condensate

- **chiral symmetry:** $\sigma = 0$
- **symmetry breaking:** $\sigma \neq 0$



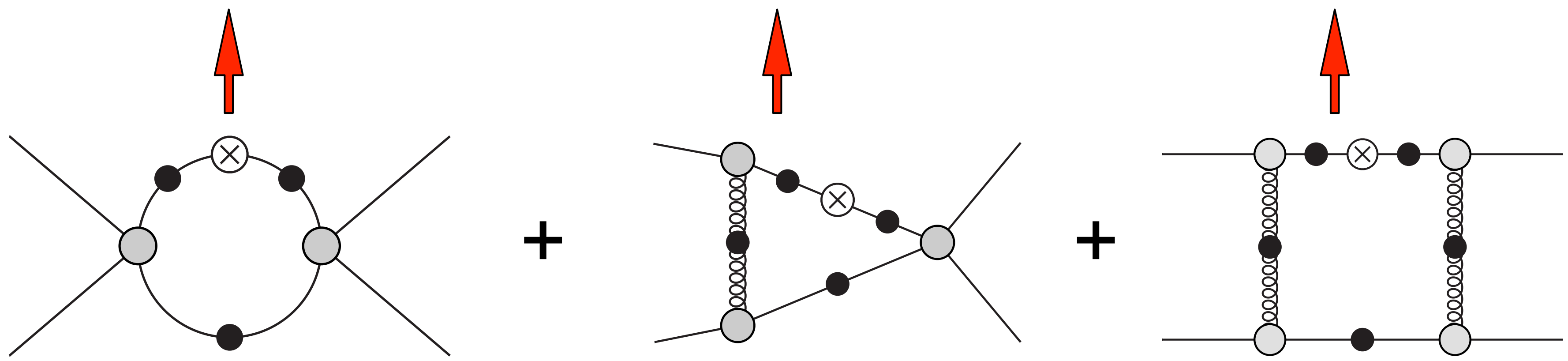
Chiral symmetry breaking directly sensitive to size of α_s

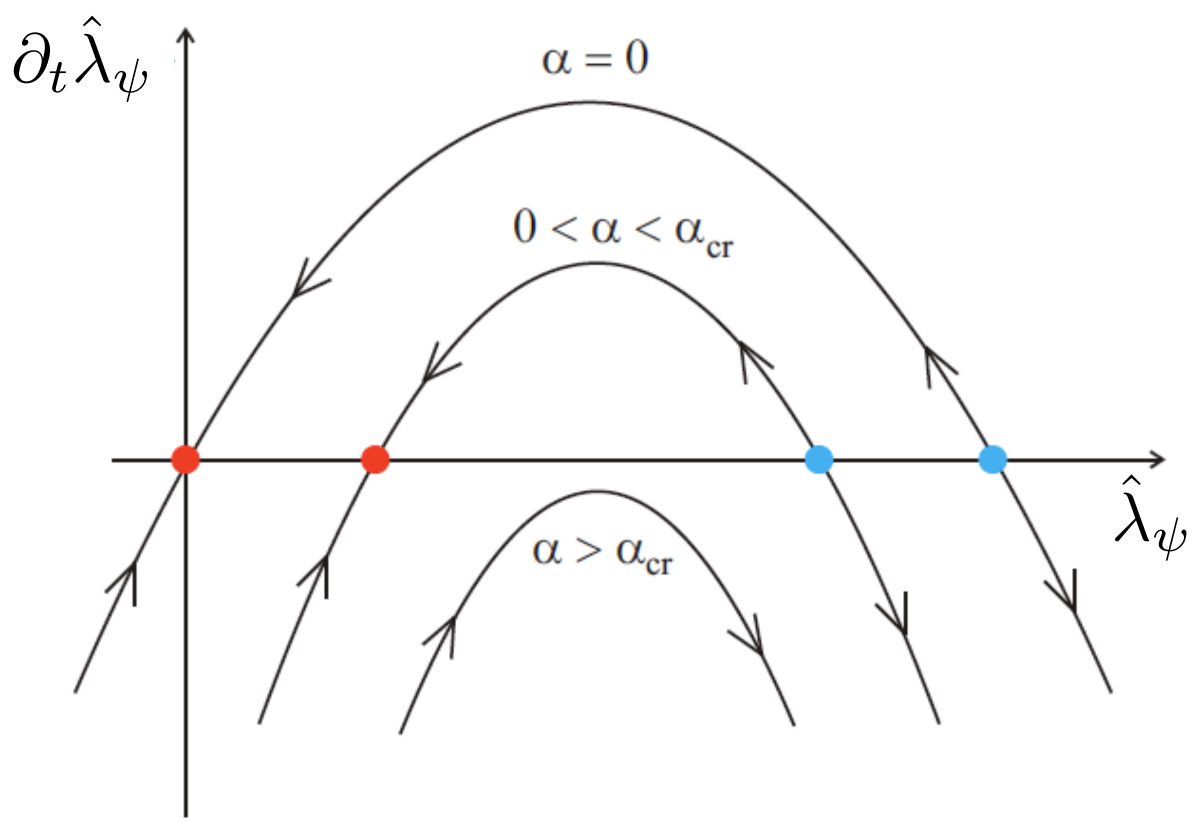
Chiral symmetry breaking

A glimpse at chiral symmetry breaking in QCD within the FRG

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

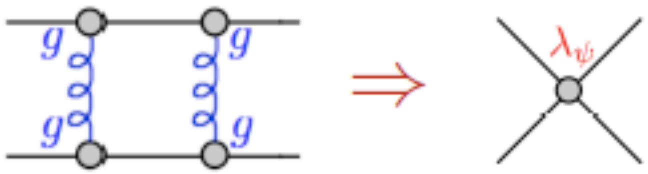
$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi + A\left(\frac{T}{k}\right)\hat{\lambda}_\psi^2 + B\left(\frac{T}{k}\right)\hat{\lambda}_\psi\,\alpha_s + C\left(\frac{T}{k}\right)\alpha_s^2 + \cdots$$





Dynamical hadronisation

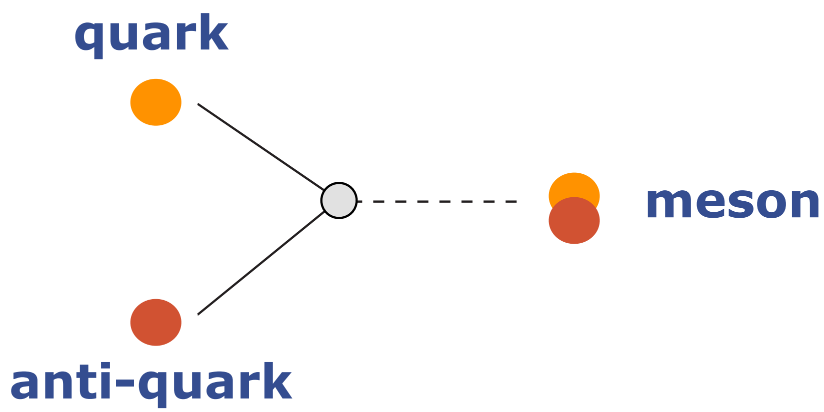
$g\,\bar\psi \not{A}\,\psi \quad \xrightarrow{\int_p}$



Hubbard-Stratonovich

$$\lambda_\psi\,(\bar\psi\psi)^2 = h\,\bar\psi\psi\,\sigma + \tfrac{1}{2}m^2\sigma^2$$

with $m^2 = \dfrac{h^2}{2\lambda_\psi}$ and EoM(σ) $\Longrightarrow$



+Baryons and Glueballs

+Baryonisation

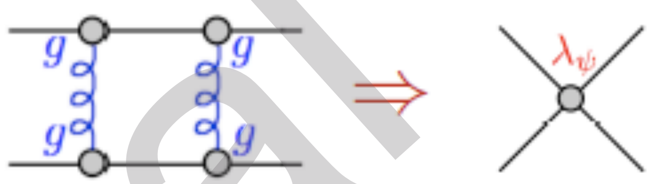
Dynamical degrees of freedom

Quarks, Gluons $\psi,\ A$ $\Longrightarrow$ $\psi,\ A$ + Mesons, Baryons $\phi \sim \bar\psi\psi,\ b \sim \psi^3$

Dynamical hadronisation

functional RG-flows

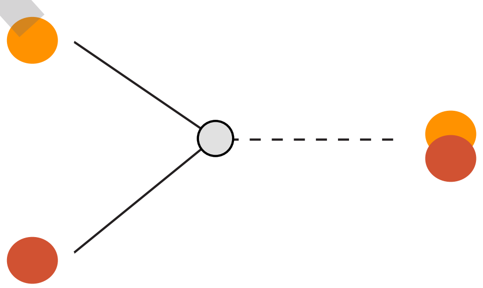
$$g\,\bar\psi \not{A}\,\psi \quad \xrightarrow{\int_{k-dk}^{k}}$$



Hubbard-Stratonovich

$$\lambda_\psi\,(\bar\psi\psi)^2 = h\,\bar\psi\psi\,\sigma + \tfrac{1}{2}m^2\sigma^2$$

with $m^2 = \dfrac{h^2}{2\lambda_\psi}$ and EoM(σ) $\Longrightarrow$



+Baryons and Glueballs

+Baryonisation

Dynamical degrees of freedom

Quarks, Gluons ψ, A $\Longrightarrow$ ψ, A + Mesons, Baryons $\phi \sim \bar\psi\psi$, $b \sim \psi^3$

also very successfully used in ultracold atoms!

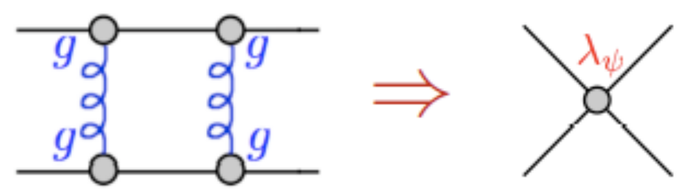
Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

QCD meets cold quantum gases

Condensation phenomena ← **functional RG-flows**

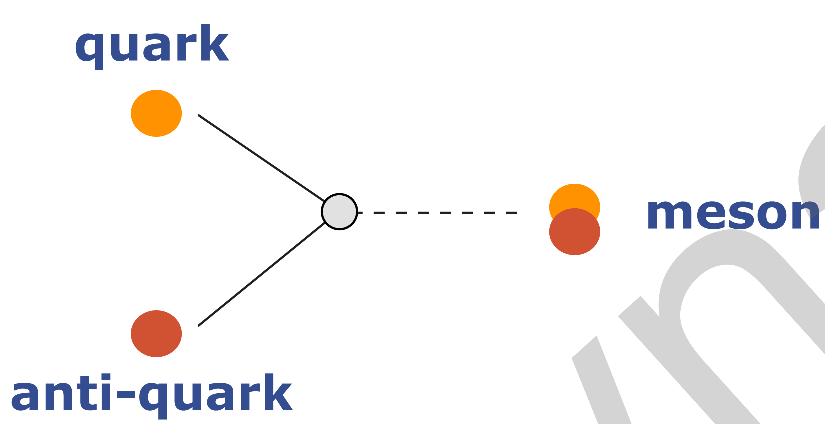
QCD

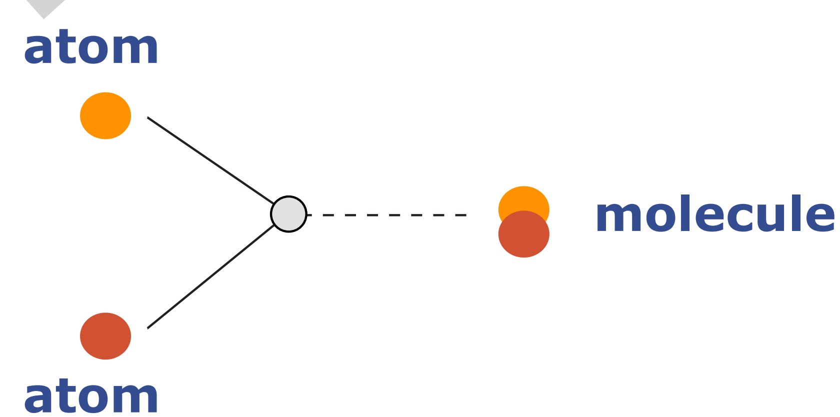
Cold Atoms



Hadronisation

Molecule formation





+Baryons and Glueballs

+Trions

Dynamical

Condensation

Gies, Wetterich '01

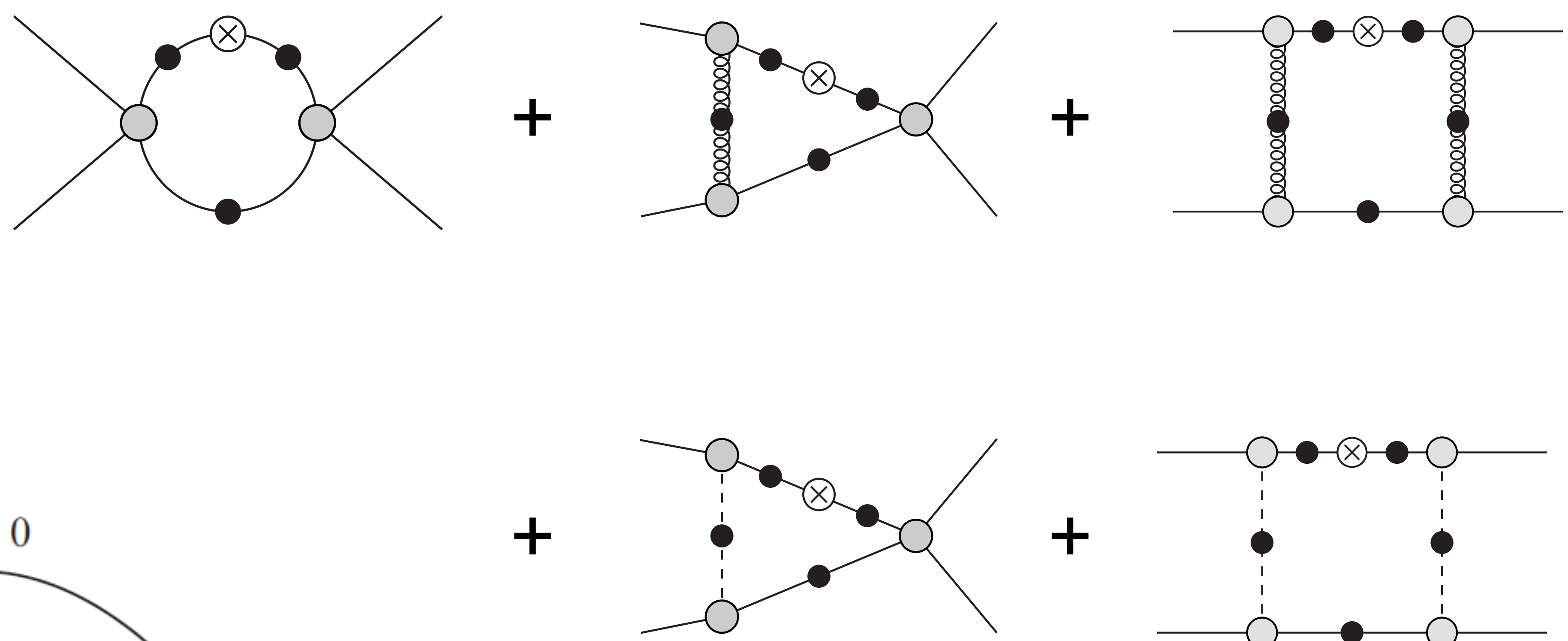
JMP '05

Flörchinger, Wetterich '09

Dynamical hadronisation

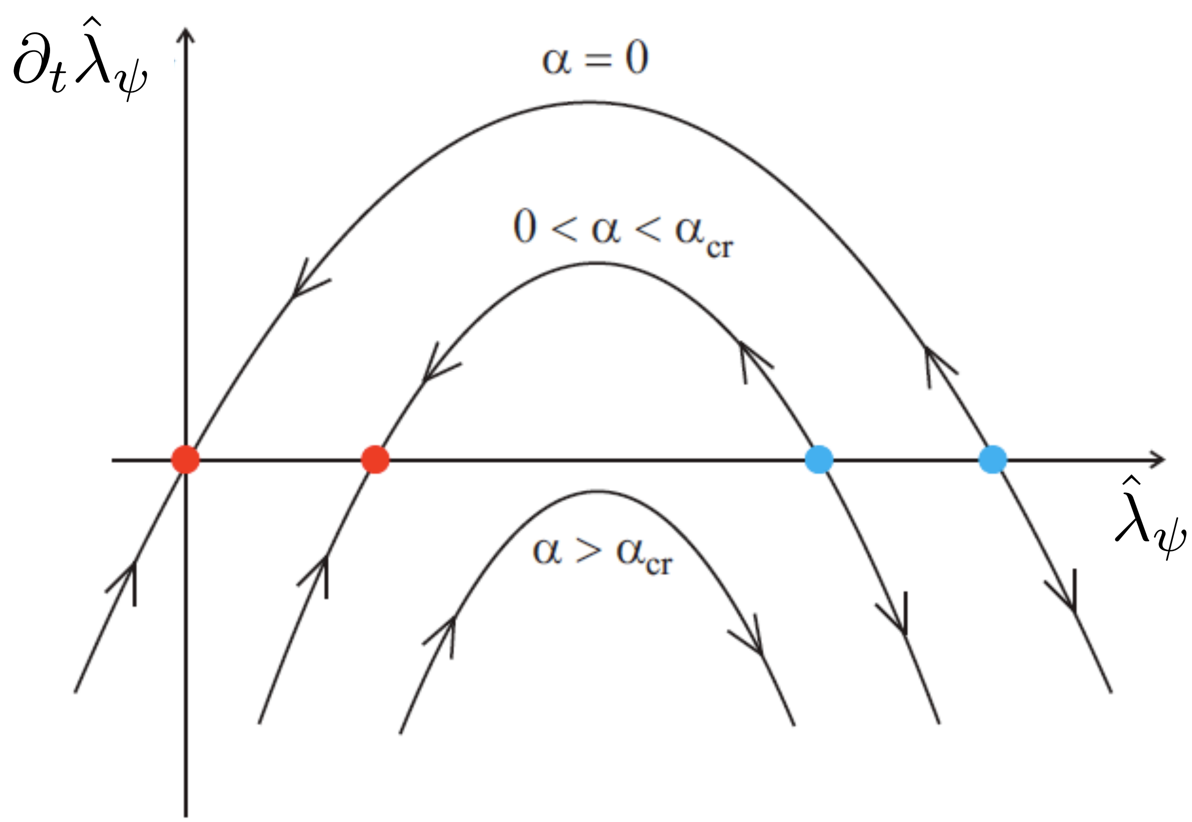
Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi +$$



$$+ \partial_t \frac{h^2}{m^2} \text{ - terms}$$

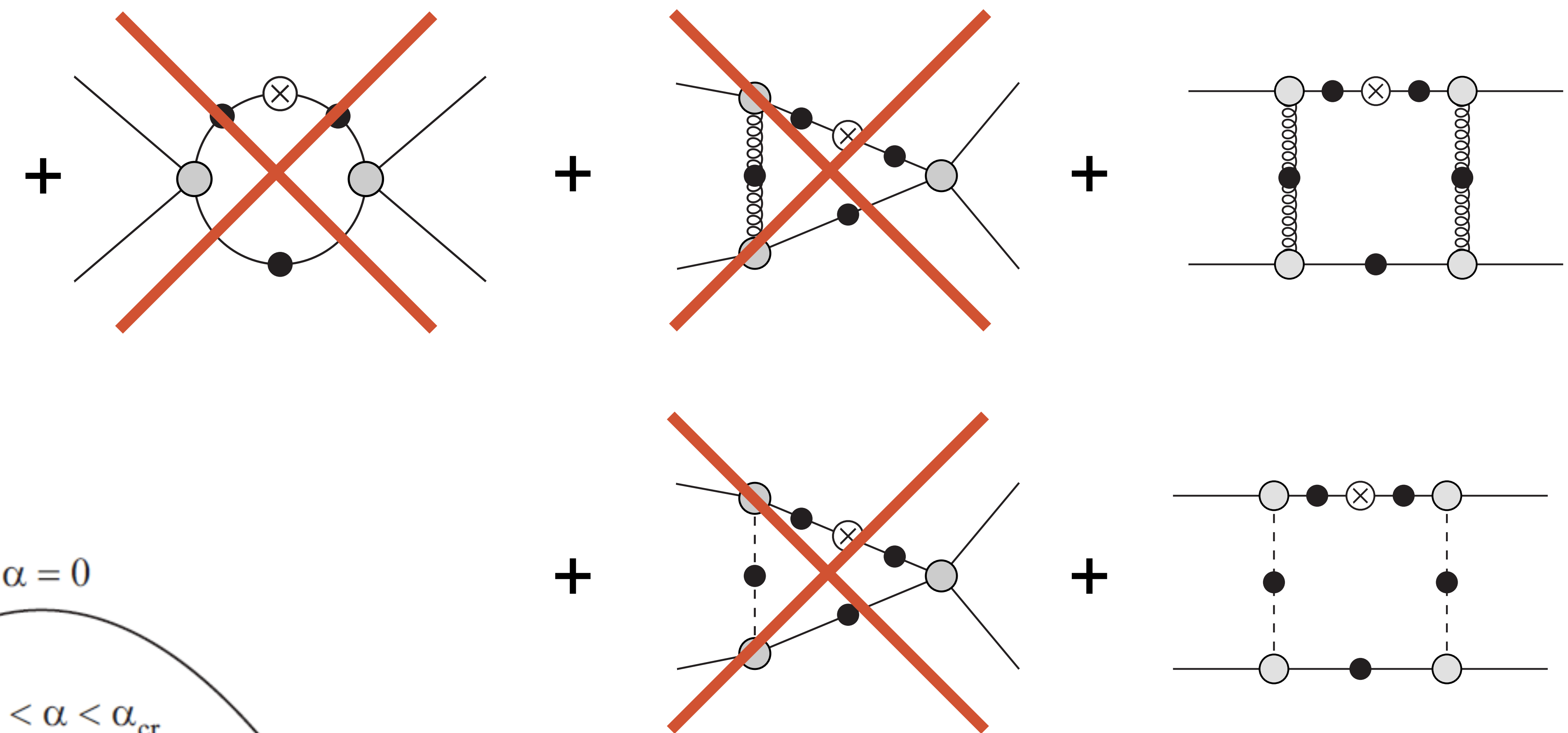
$$+ \cdots$$



Dynamical hadronisation

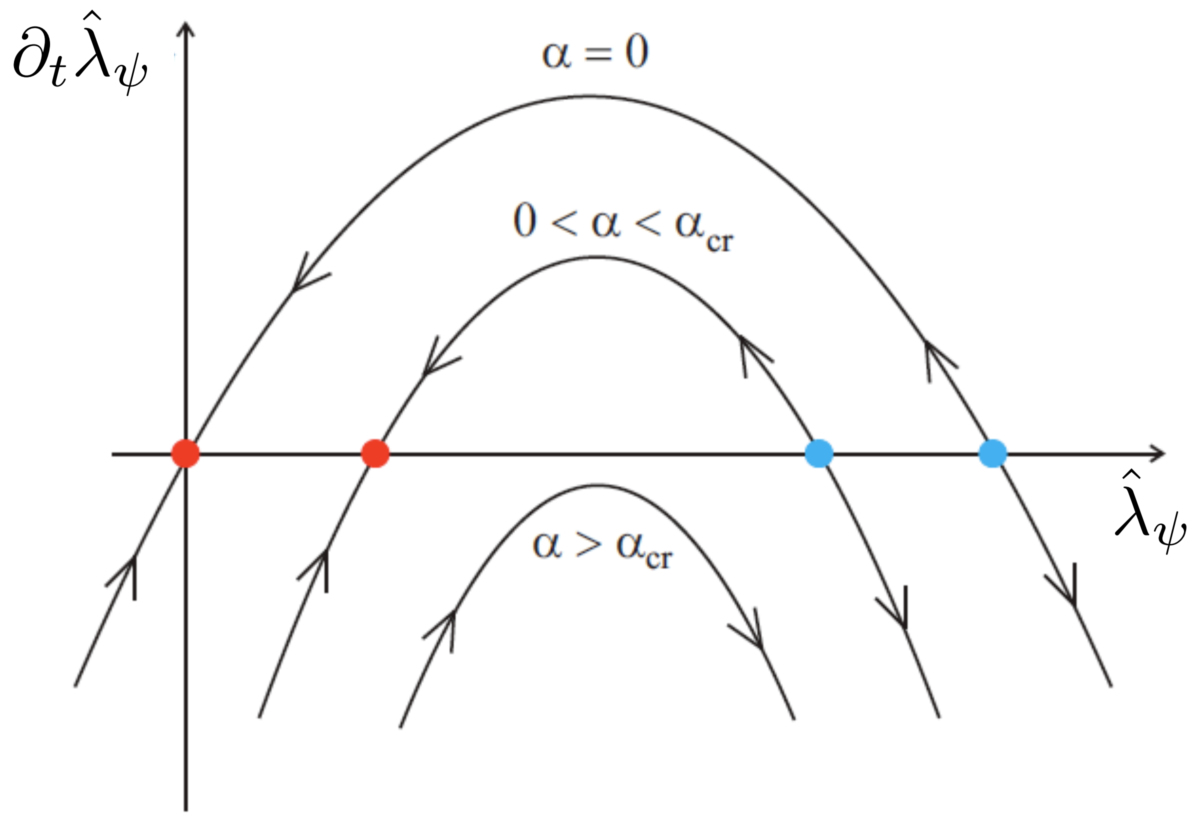
Full bosonisation $\hat{\lambda}_\psi = 0$

$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi$ +

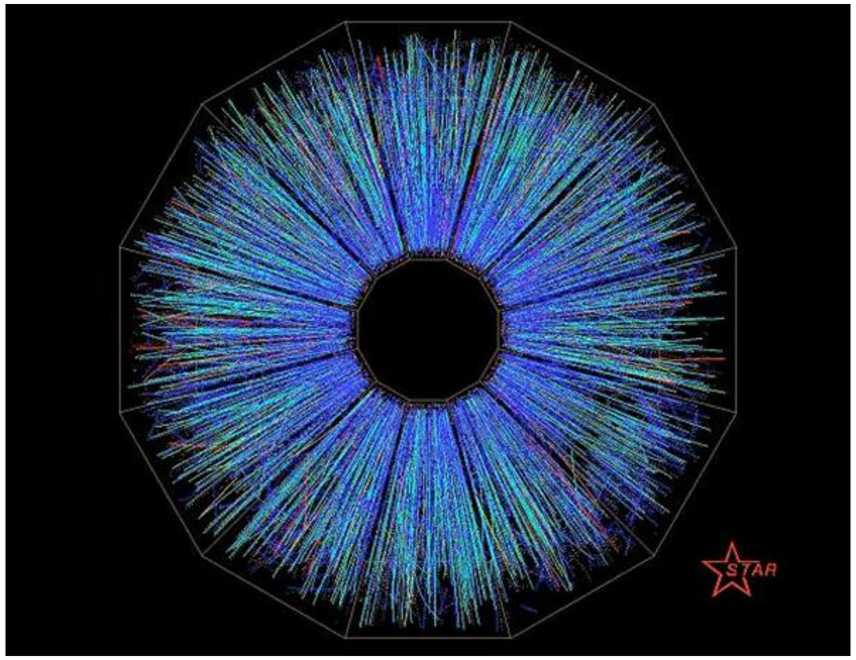


$+ \; \partial_t \dfrac{h^2}{m^2}$ **- terms**

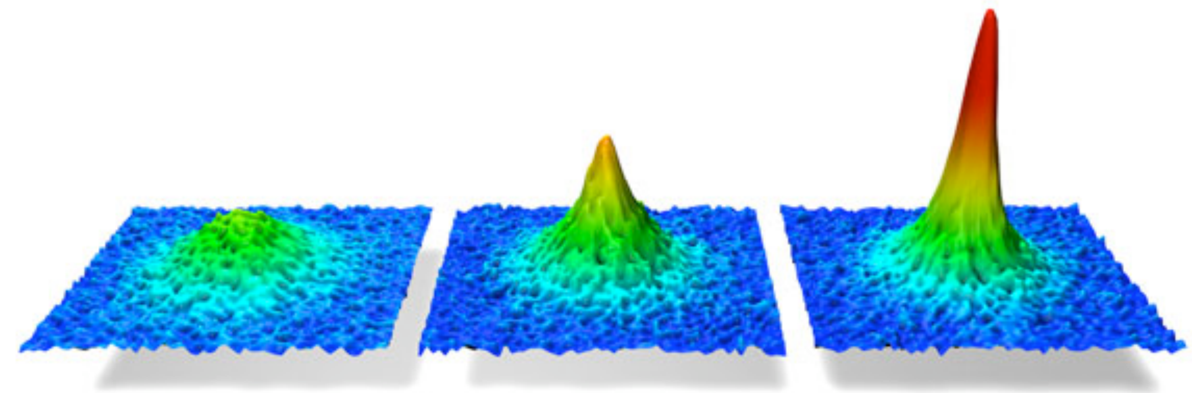
$+ \; \cdots$



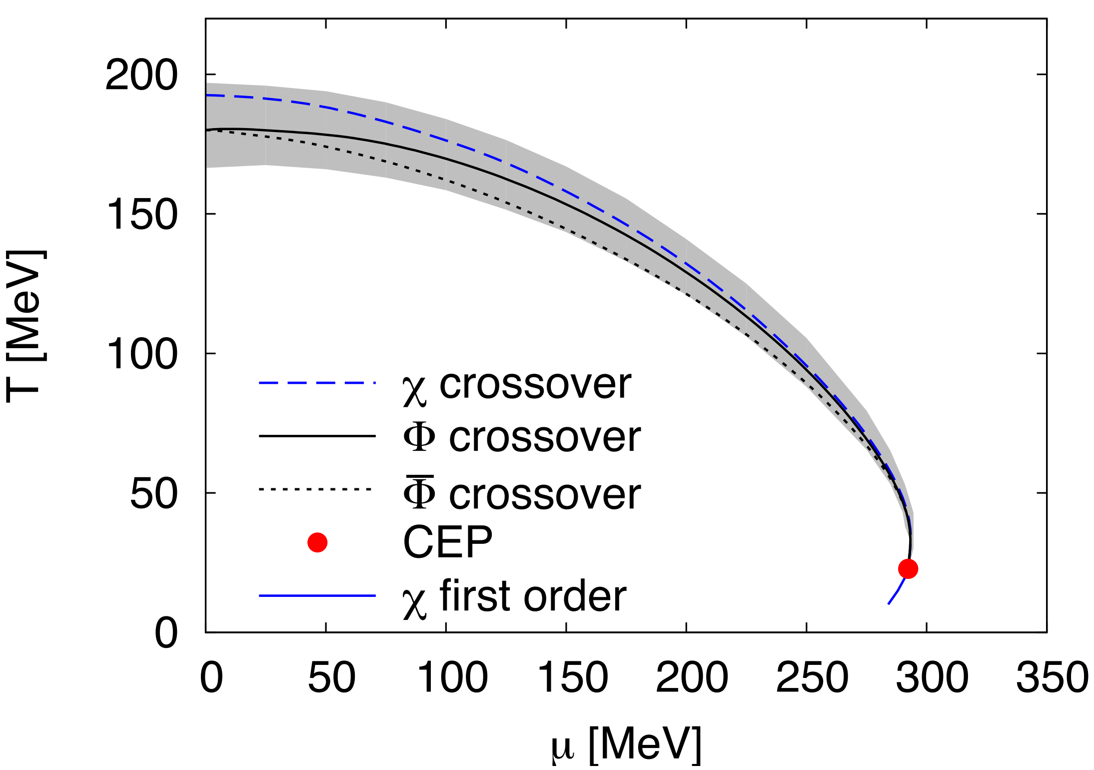
QCD

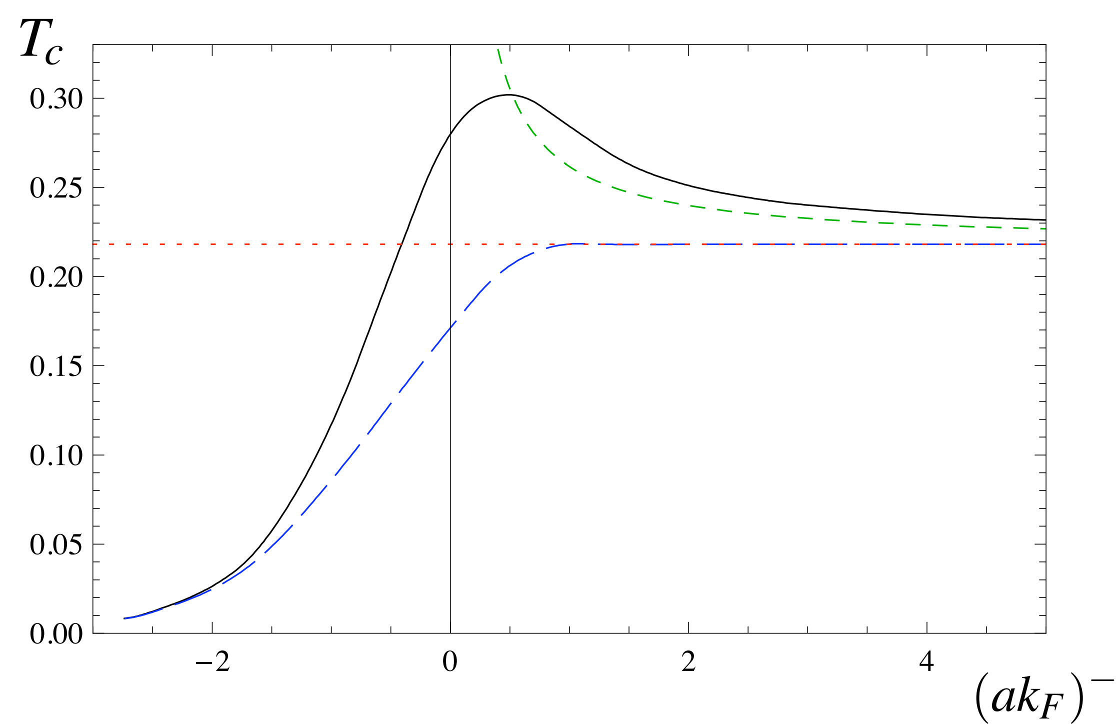


Cold quantum gases



Phase diagrams





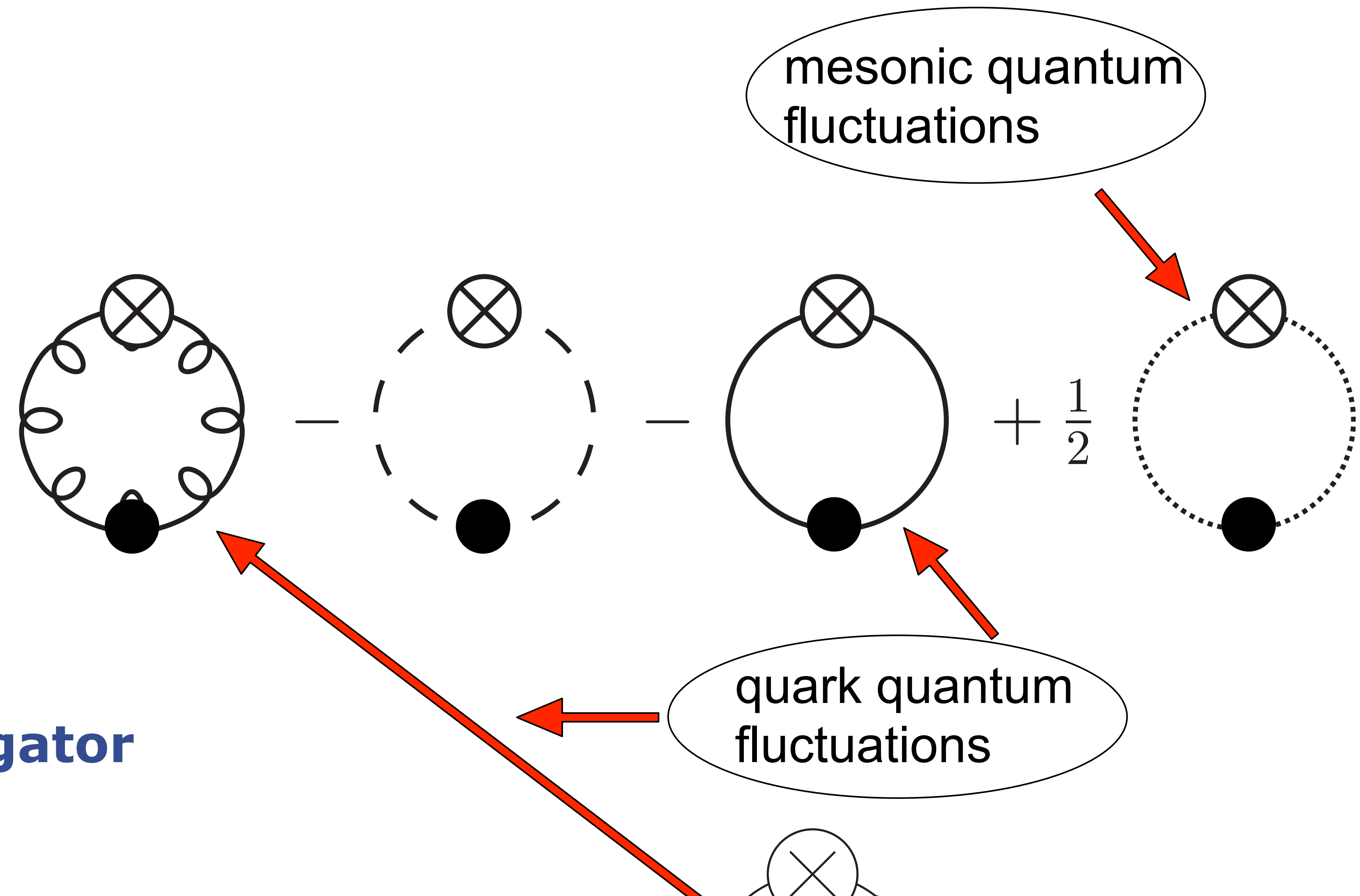
vanishing density

Full dynamical QCD: N_f = 2 & chiral limit

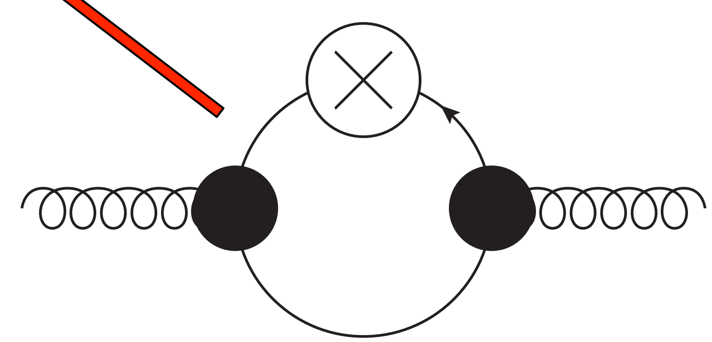
FRG+DSE+2PI+lattice

- **RG-flow of Effective Action**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



- **flow of gluon propagator**

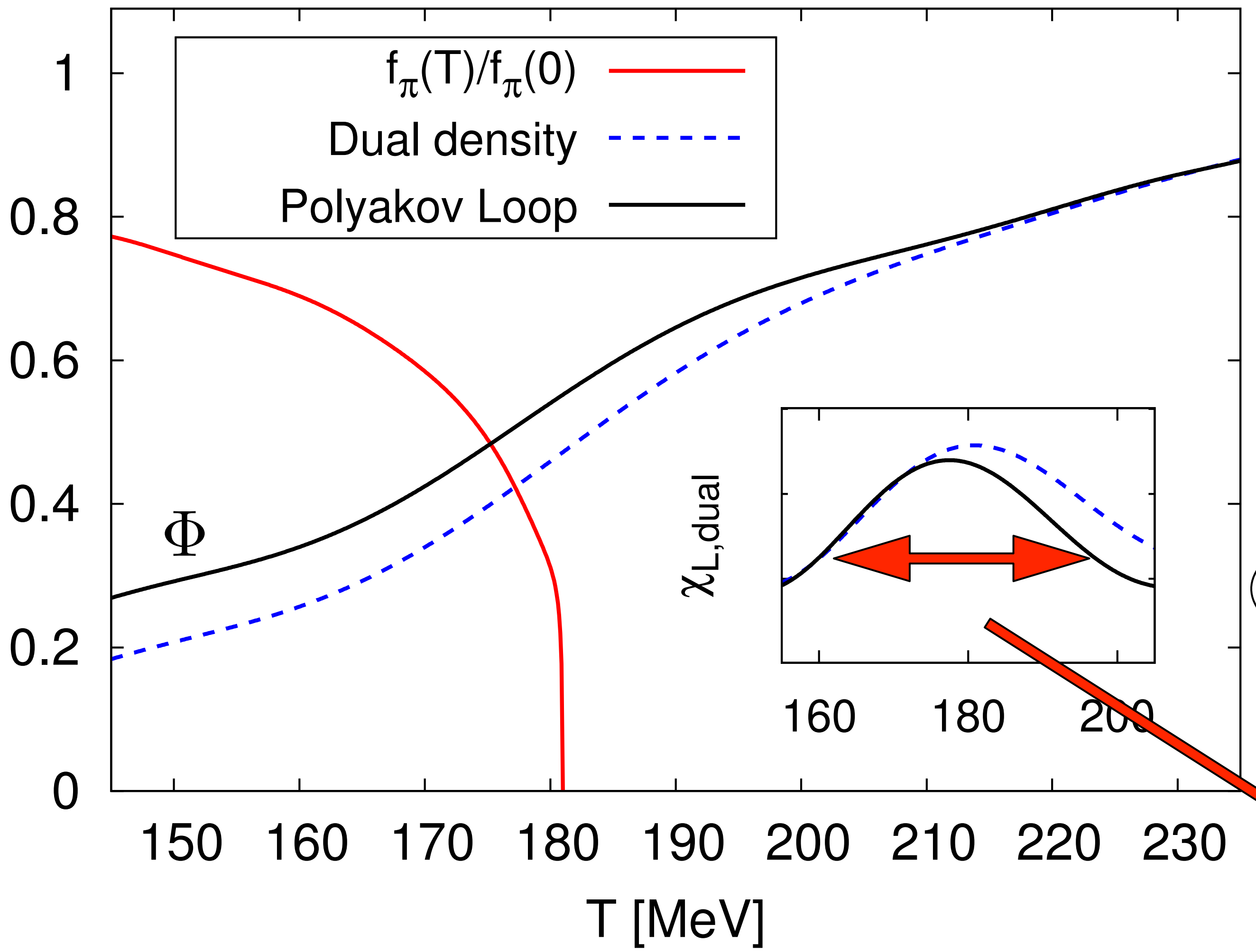
pure gauge theory flow **+**  **+ ...**

Naturally encorporates PQM/PNJL models as specific low order trunations

Full dynamical QCD: N_f = 2 & chiral limit

FRG+DSE+2PI+lattice

Braun, Haas, Marhauser, JMP '09



$$T_\chi \simeq T_{\rm conf} \simeq 180{\rm MeV}$$

$N_f = 2$

compatible with Budapest-Wuppertal

compatible with hotQCD

$N_f = 2+1$

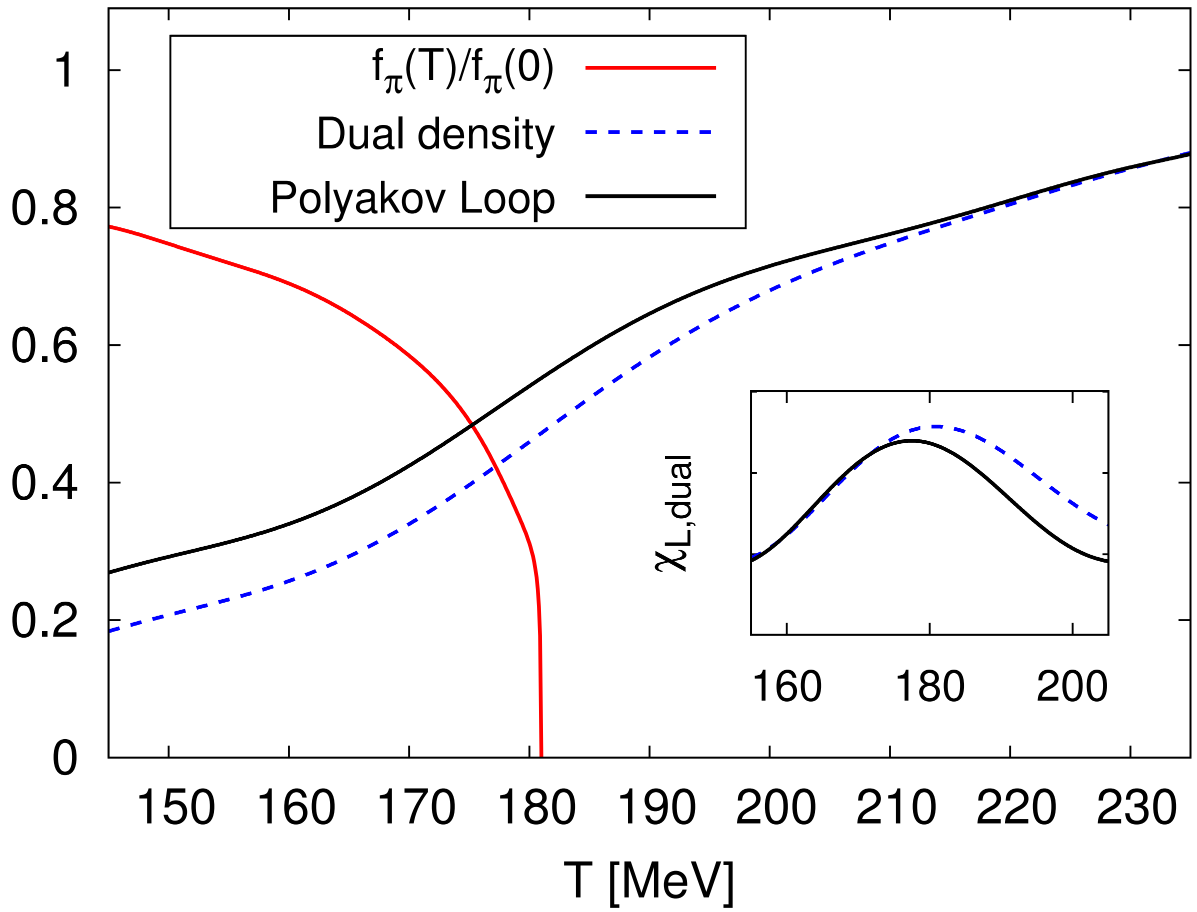
± 20 MeV

Beware: $\Phi_{\rm FRG} \neq \Phi_{\rm lattice}$ ⟷ $T_{\rm conf,FRG} \lesssim T_{\rm conf,lattice}$

Full dynamical QCD: N_f = 2 & chiral limit

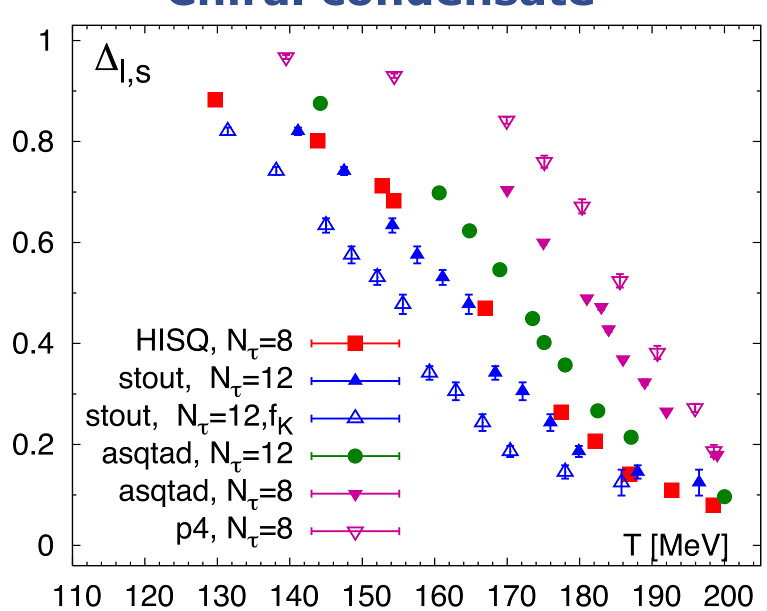
FRG+DSE+2PI+lattice

Braun, Haas, Marhauser, JMP '09

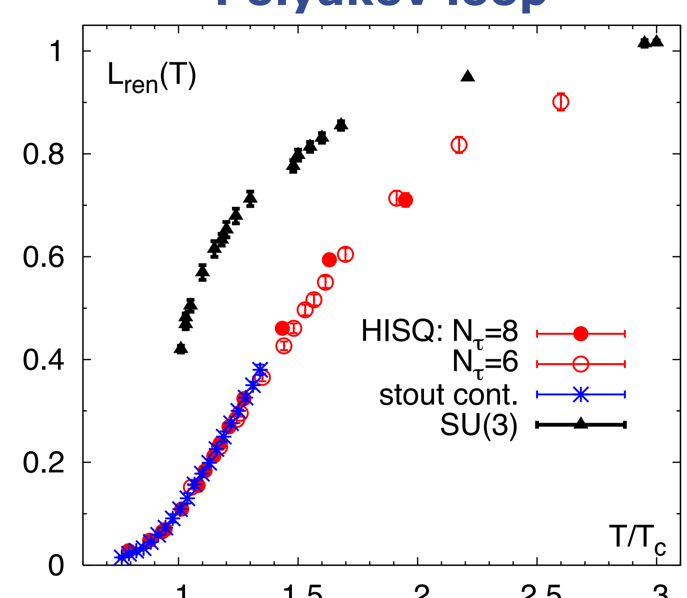


$$T_\chi \simeq T_{\text{conf}} \simeq 180\text{MeV}$$

Chiral condensate

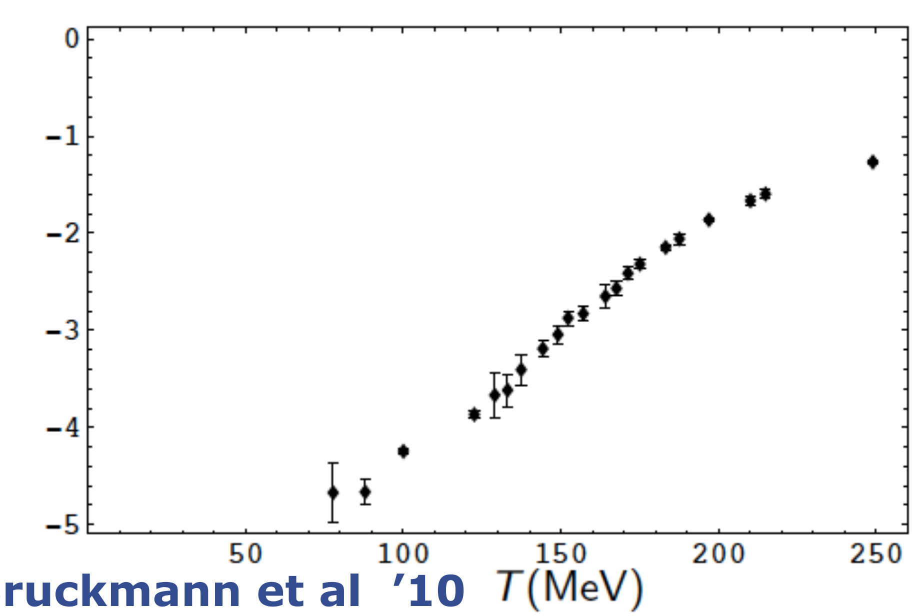


Polyakov loop



hotQCD '10

Log of dual condensate, m=60 MeV

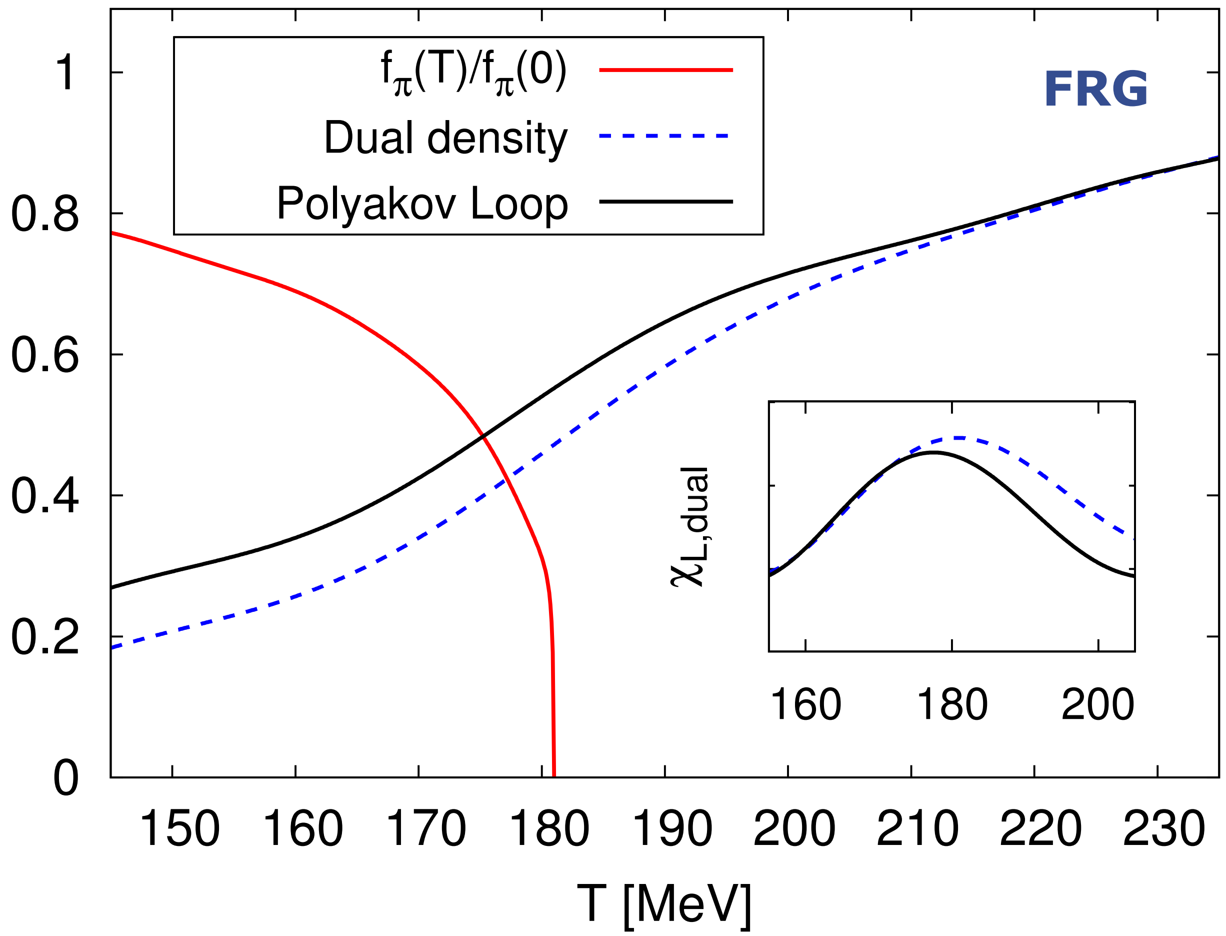


Bruckmann et al '10

Full dynamical QCD: N_f = 2

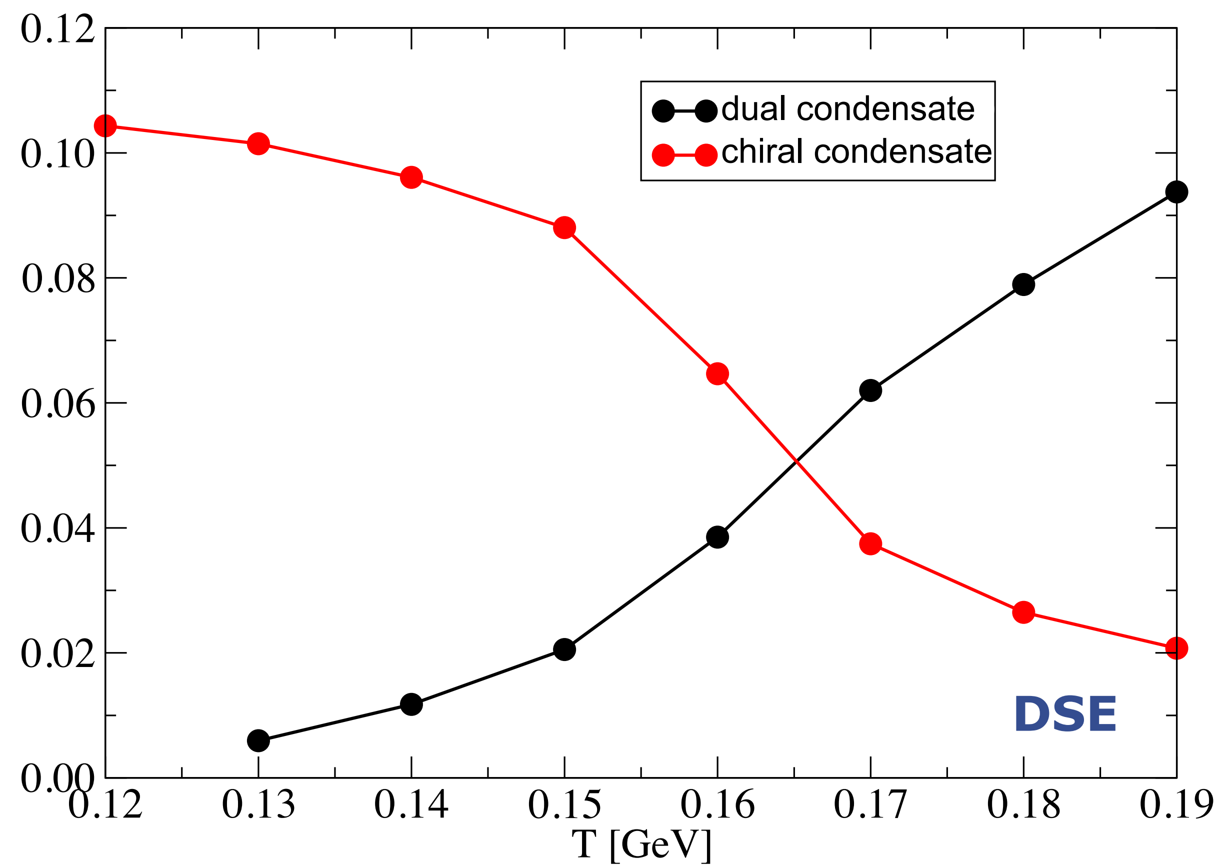
FRG+DSE+2PI+lattice

chiral limit



Braun, Haas, Marhauser, JMP '09

phys. quark masses

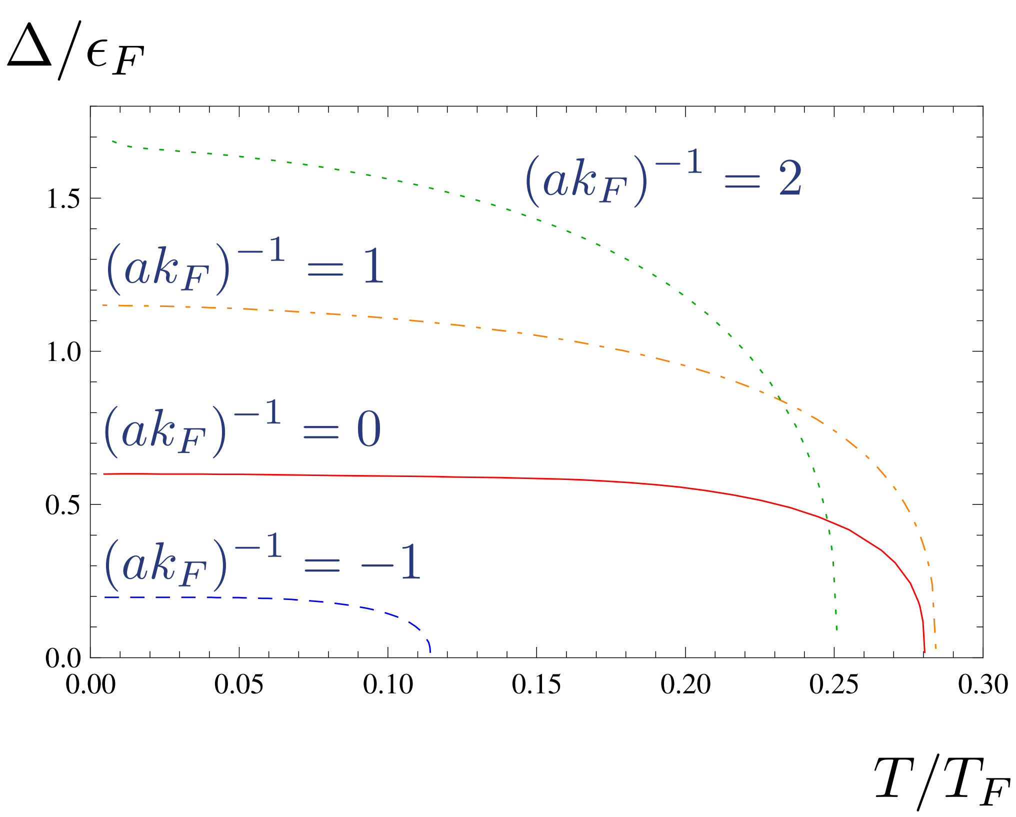


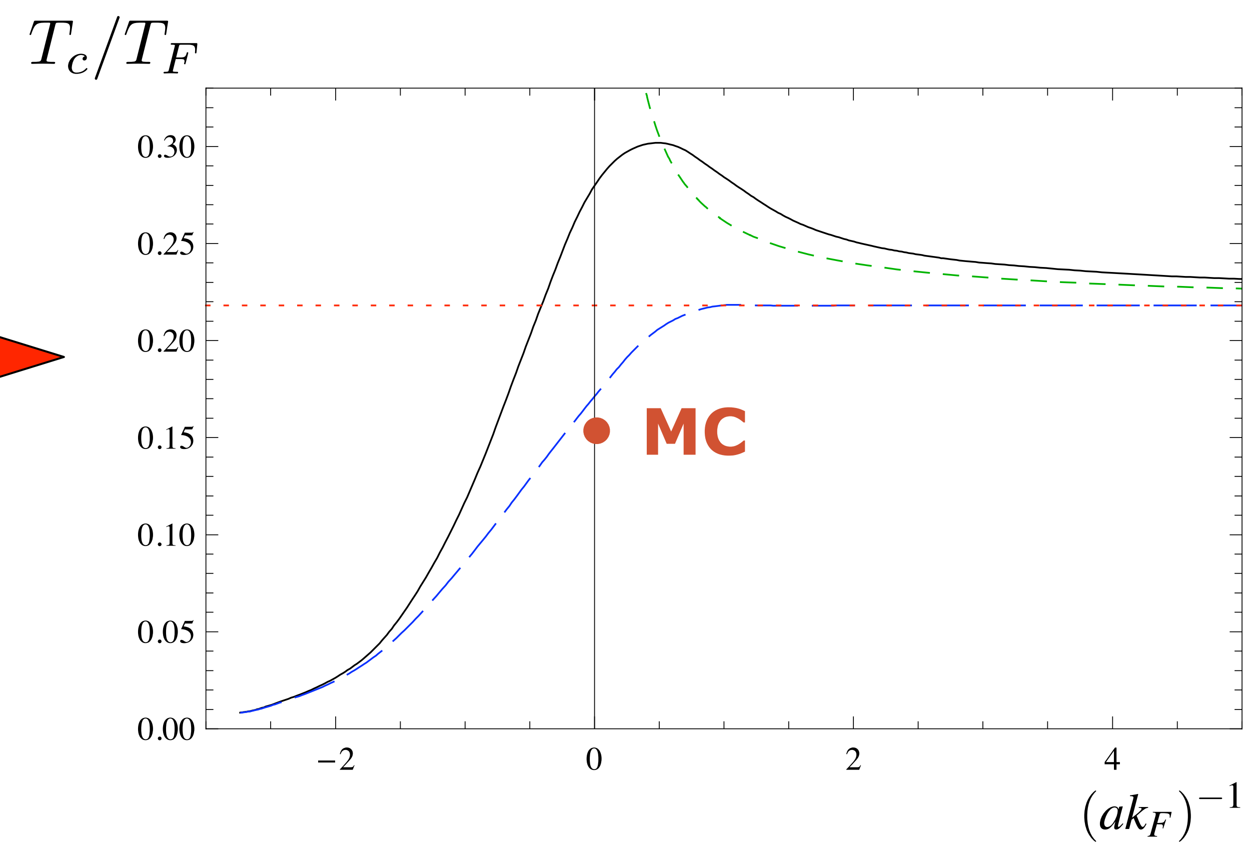
Fischer, Lücker, Mueller '11

Phase diagram of cold quantum gases

FRG + DSE

Diehl, Floerchinger, Gies, JMP, Scherer, Wetterich '07-'10





MC: Burovski, Prokof'ev, Svistunov, Troyer '06

Bulgac, Drut, Magierski '06

Improve approximations:

different channels/multi-scatterings

momentum/frequency-dependencies

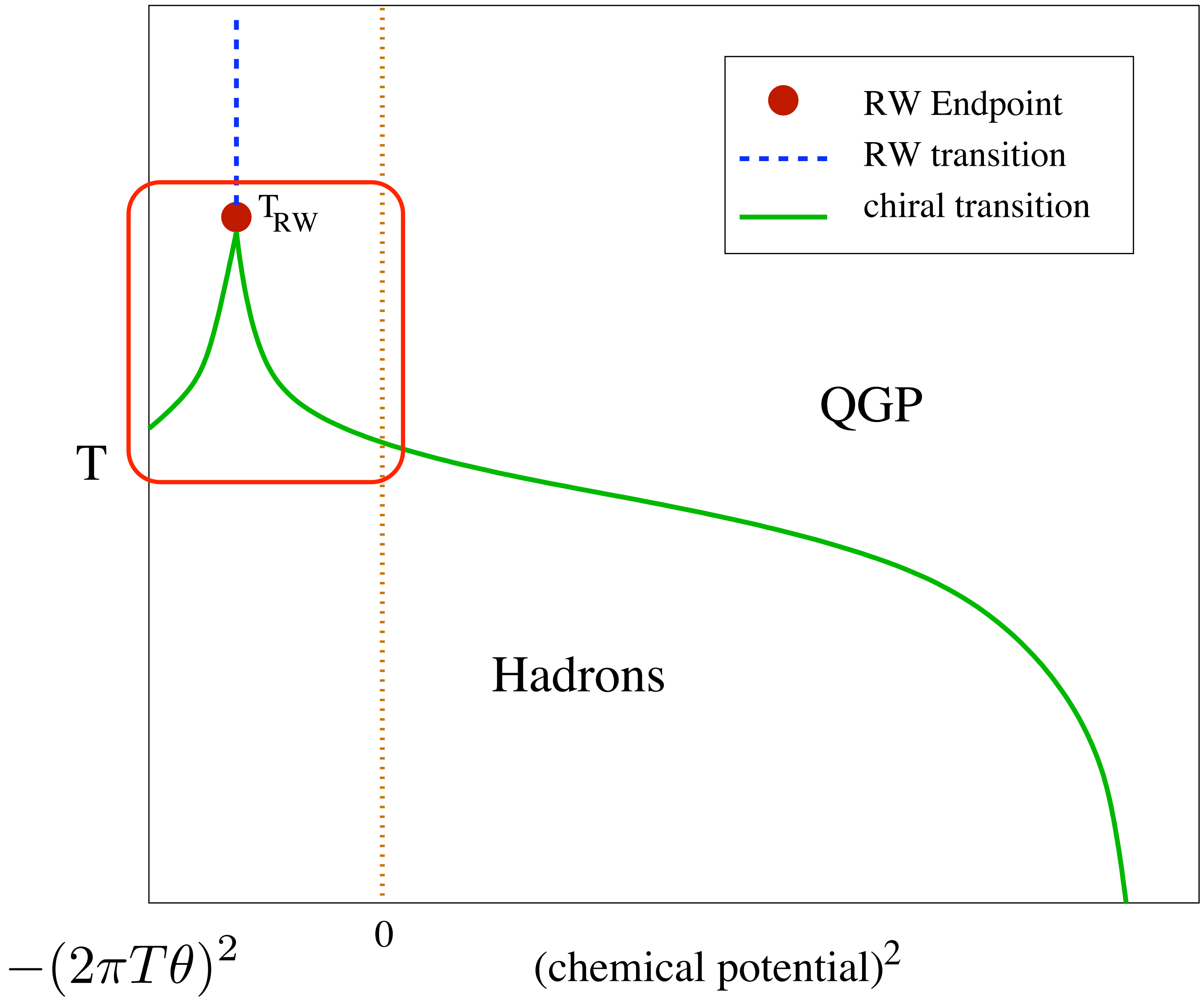
Physics differences

finite size-finite volume effects

finite chemical potential

Imaginary chemical potential

$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \text{with} \qquad \mu_I = 2\pi T\theta$$



- Roberge-Weiss symmetry: $\theta \rightarrow \theta + 1/3$

Imaginary chemical potential

$$\mathcal{O}_\theta = \langle O[e^{2\pi i\theta t/\beta}\psi]\rangle \qquad \text{with} \qquad \psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x)$$

imaginary chemical potential $\mu = 2\pi i\theta/\beta$ for $\psi_\theta = e^{2\pi i\theta t/\beta}\psi$

$$z = e^{2\pi i\theta_z} \longrightarrow \tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

order parameter for confinement

Dual order parameter

Gattringer '06
Synatschke, Wipf, Wozar '07
Bruckmann, Hagen, Bilgici, Gattringer '08

Fischer, '09; Fischer, Maas, Müller '10
Braun, Haas, Marhauser, JMP '09 ← imaginary chemical potential

necessary for dynamical quarks

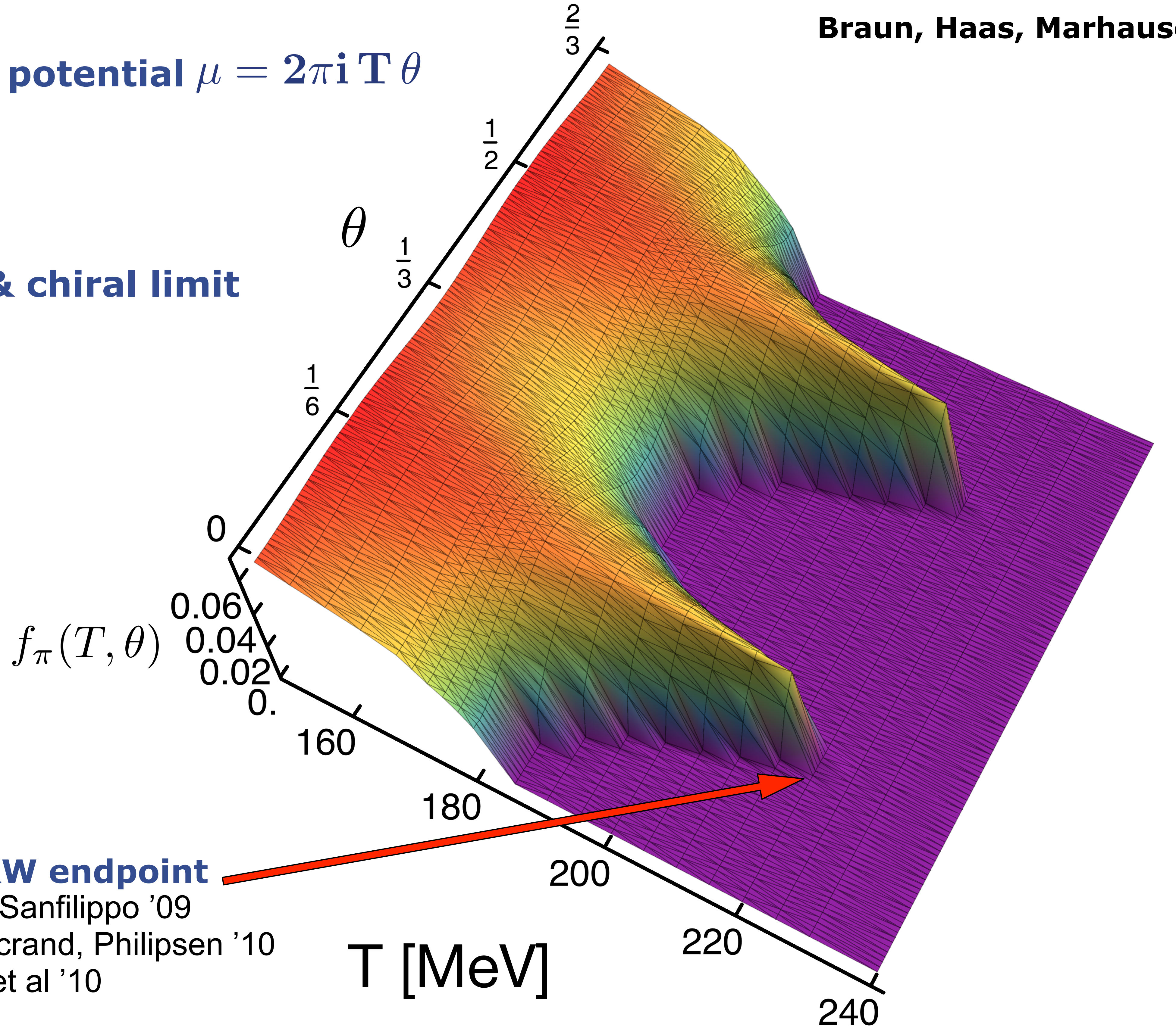
Imaginary chemical potential

FRG+DSE+2PI+lattice

Braun, Haas, Marhauser, JMP '09

chemical potential $\mu = 2\pi \mathrm{i}\, T\, \theta$

$N_f = 2$ **& chiral limit**



Nature of RW endpoint

lattice: D'Elia, Sanfilippo '09
de Forcrand, Philipsen '10

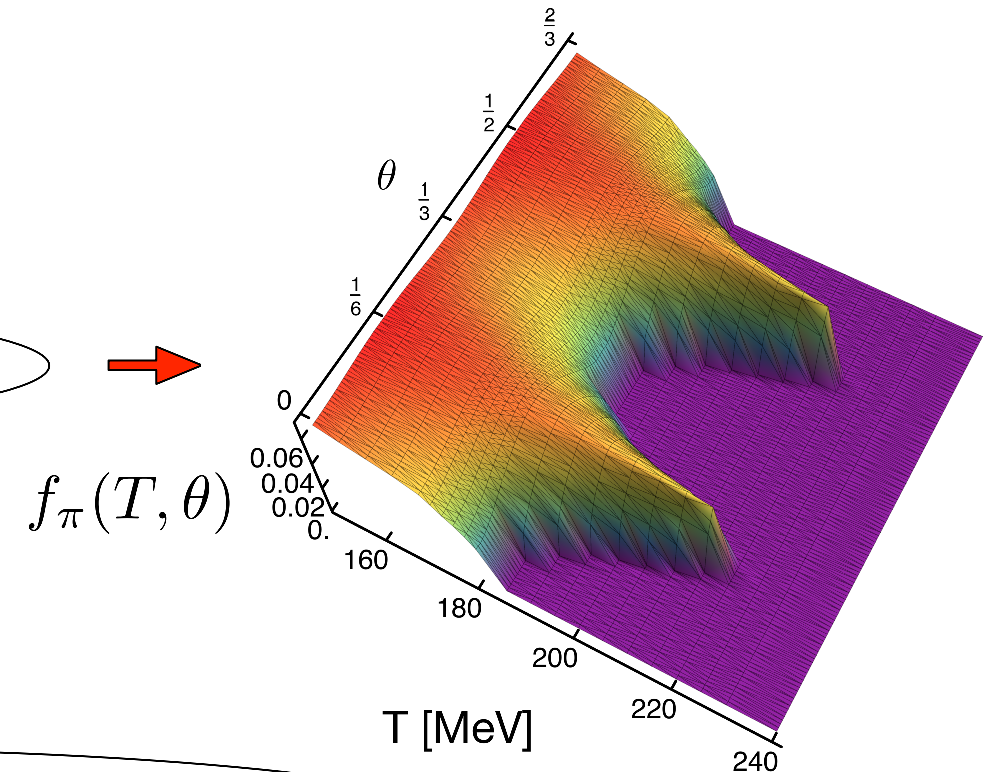
PNJL: Sakai et al '10

Imaginary chemical potential

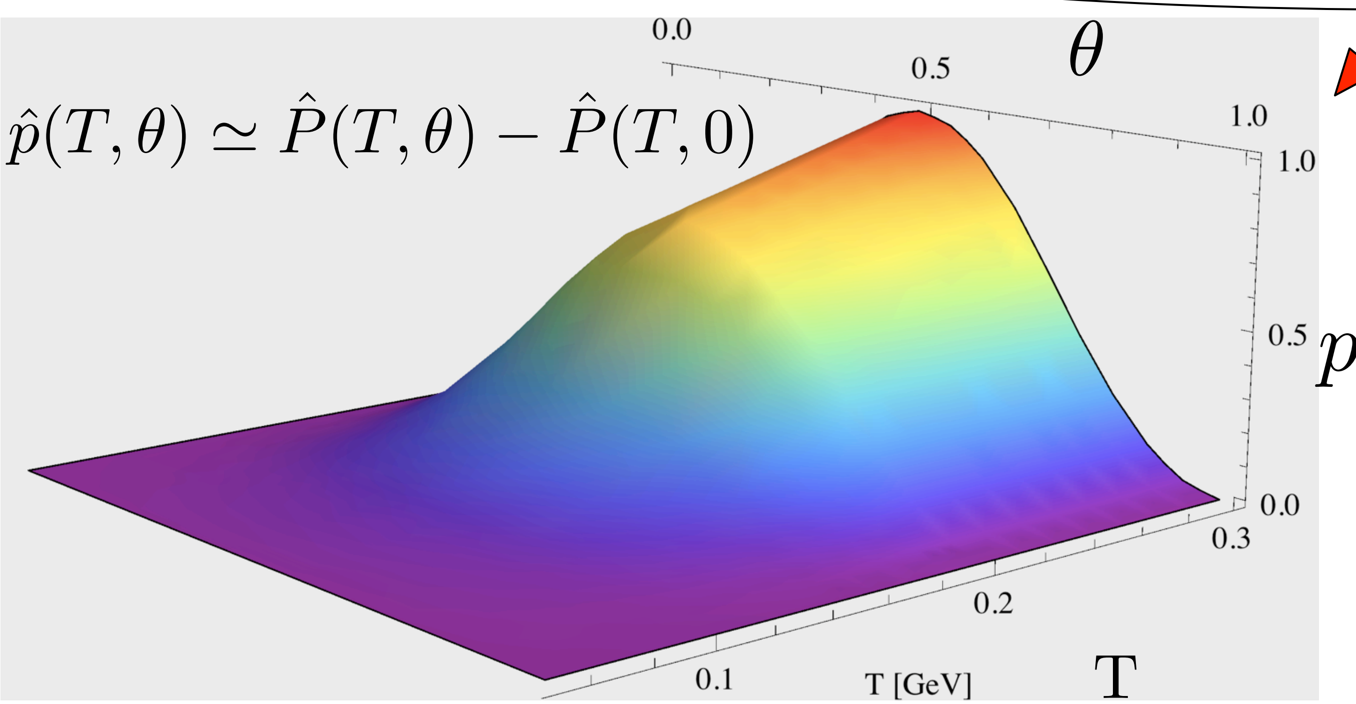
FRG+DSE+2PI+lattice

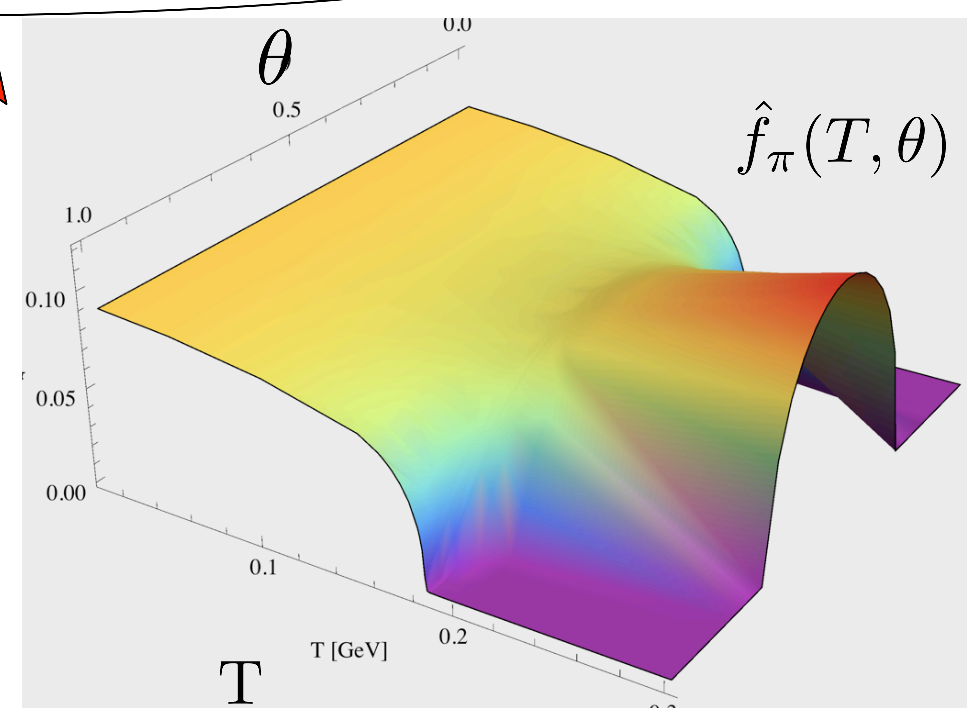
Braun, Haas, Marhauser, JMP '09

A_0 solution of EoM: Roberge-Weiss periodicity



fixed A_0: no Roberge-Weiss periodicity

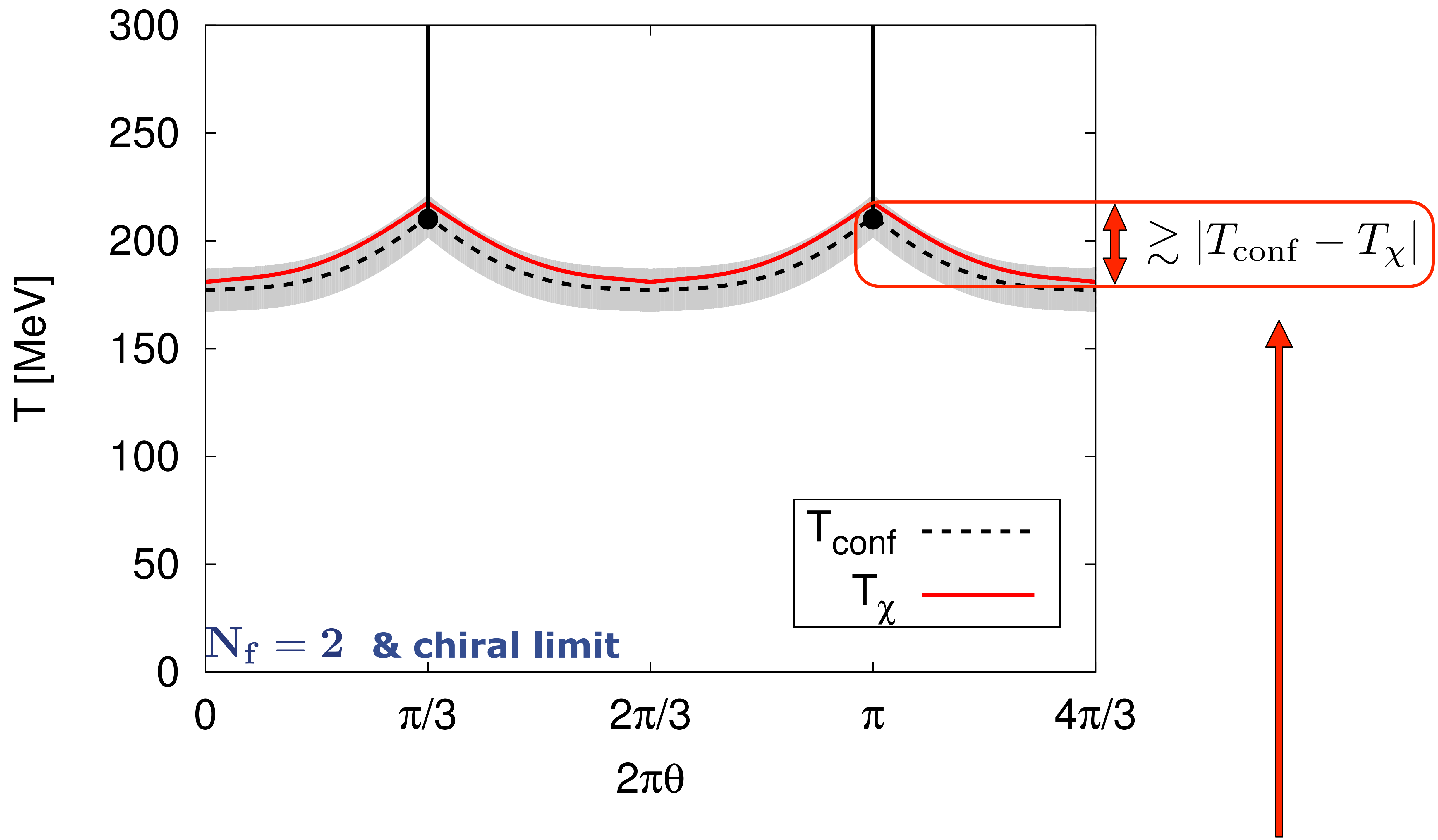




Imaginary chemical potential

FRG+DSE+2PI+lattice

Braun, Haas, Marhauser, JMP '09



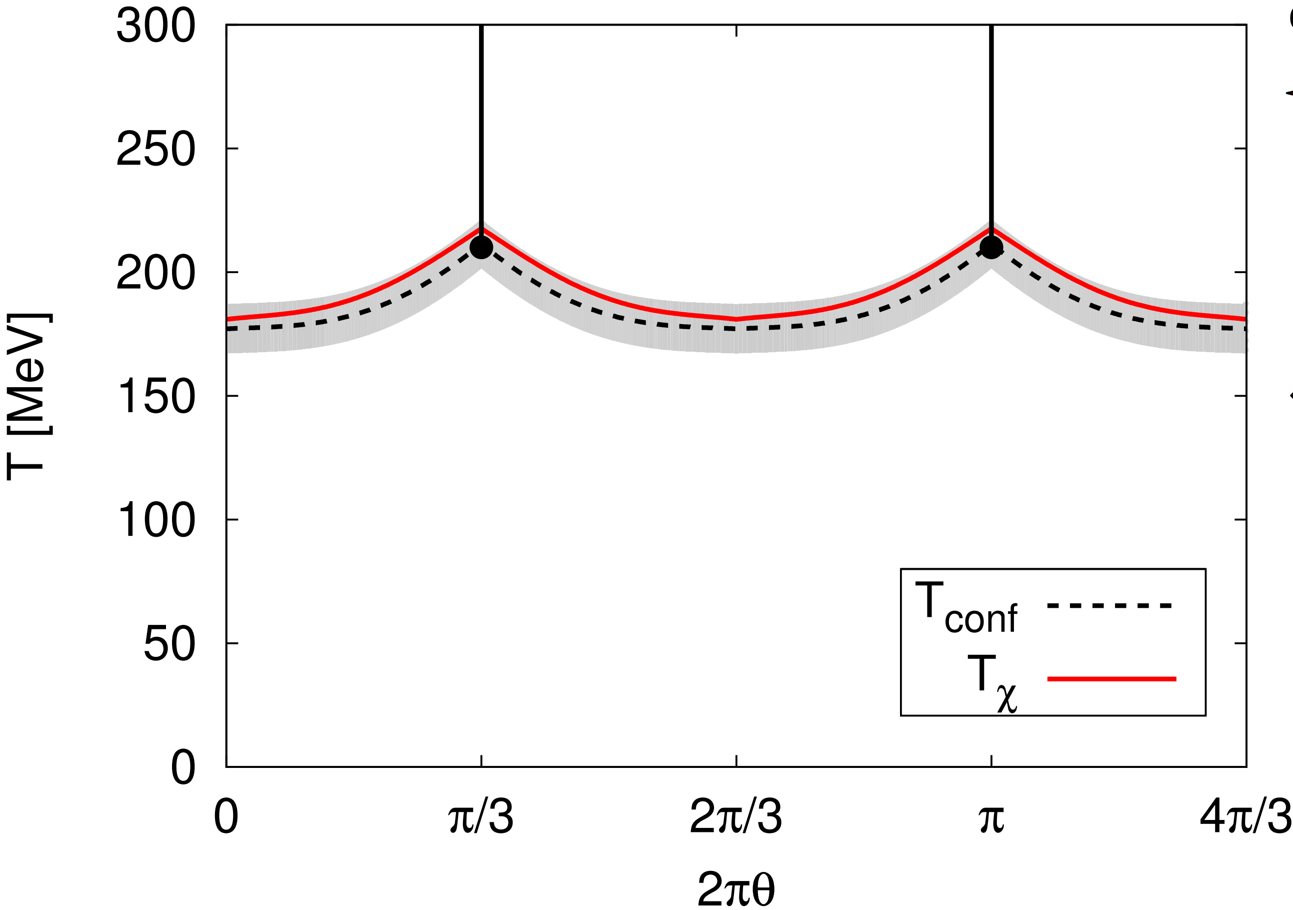
dual order parameters & imaginary chemical potential

Imaginary chemical potential

FRG+DSE+2PI+lattice

Braun, Haas, Marhauser, JMP '09

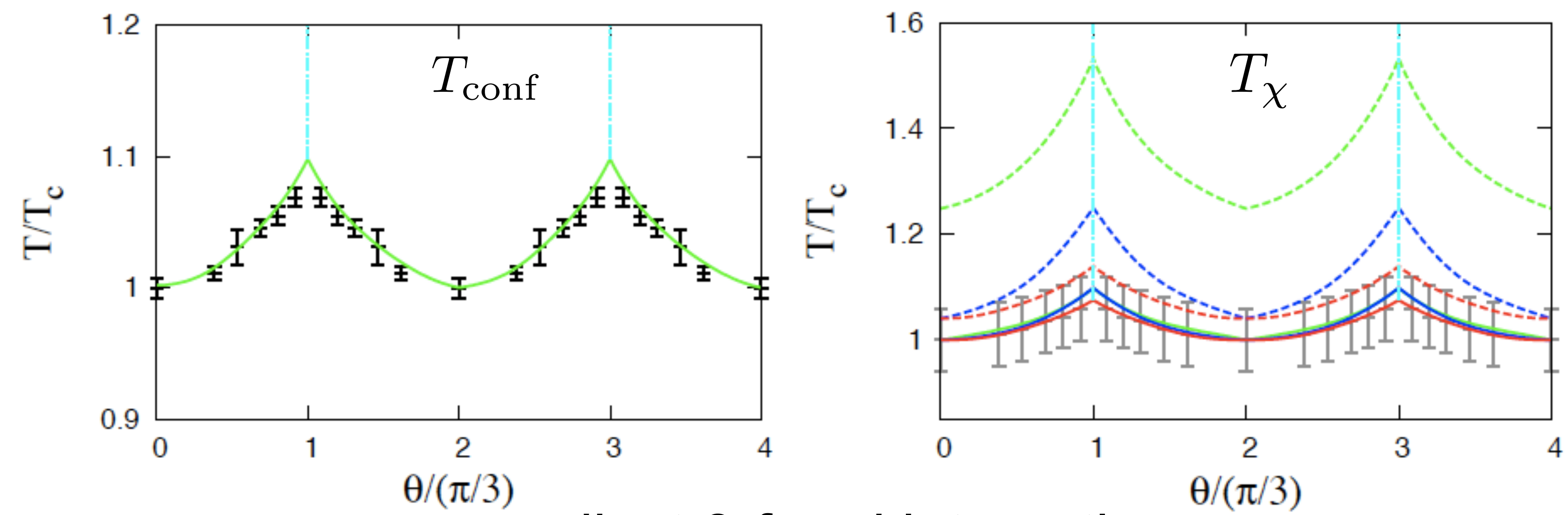
chemical potential $\mu = 2\pi \mathrm{i}\, T\, \theta$



compatibility

lattice results, e.g.
Kratochvila et al '06, Wu et al '06
& D'Elia et al '07

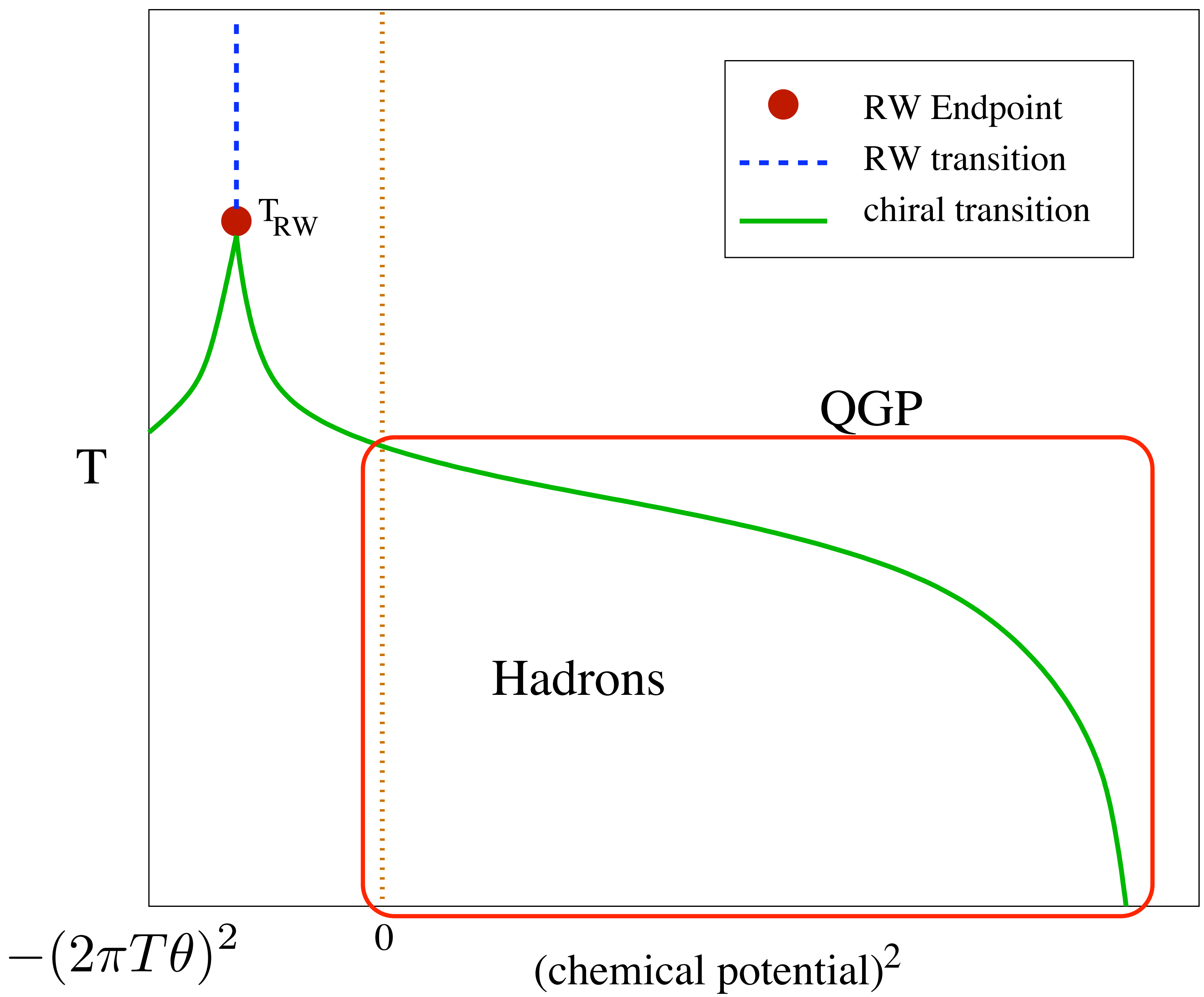
Polyakov-NJL model
Sakai et al '09



adjust 8-fermi interaction

Real chemical potential

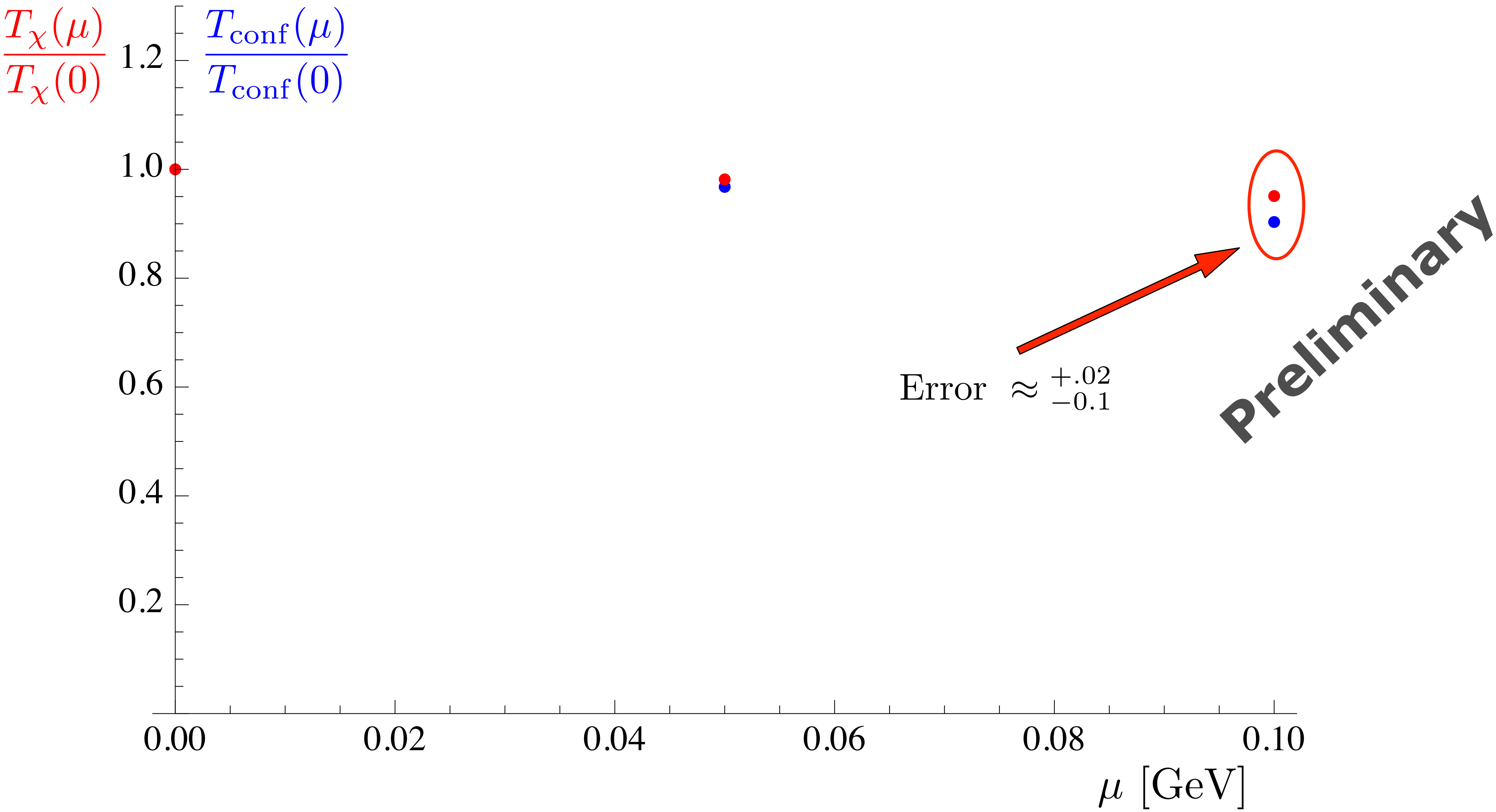
$$\psi_\theta(t+\beta,\vec{x}) = -\psi(t,x)$$



Real chemical potential

FRG+DSE+2PI+lattice

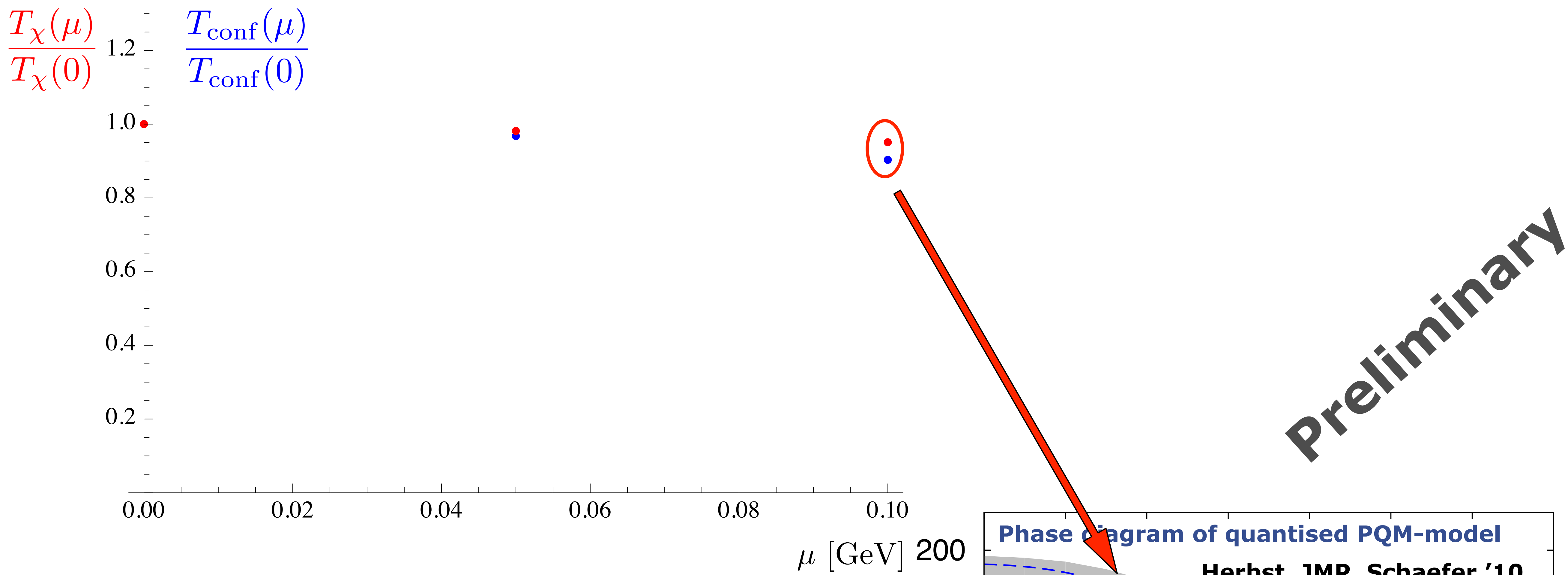
Braun, Haas, JMP, in prep.

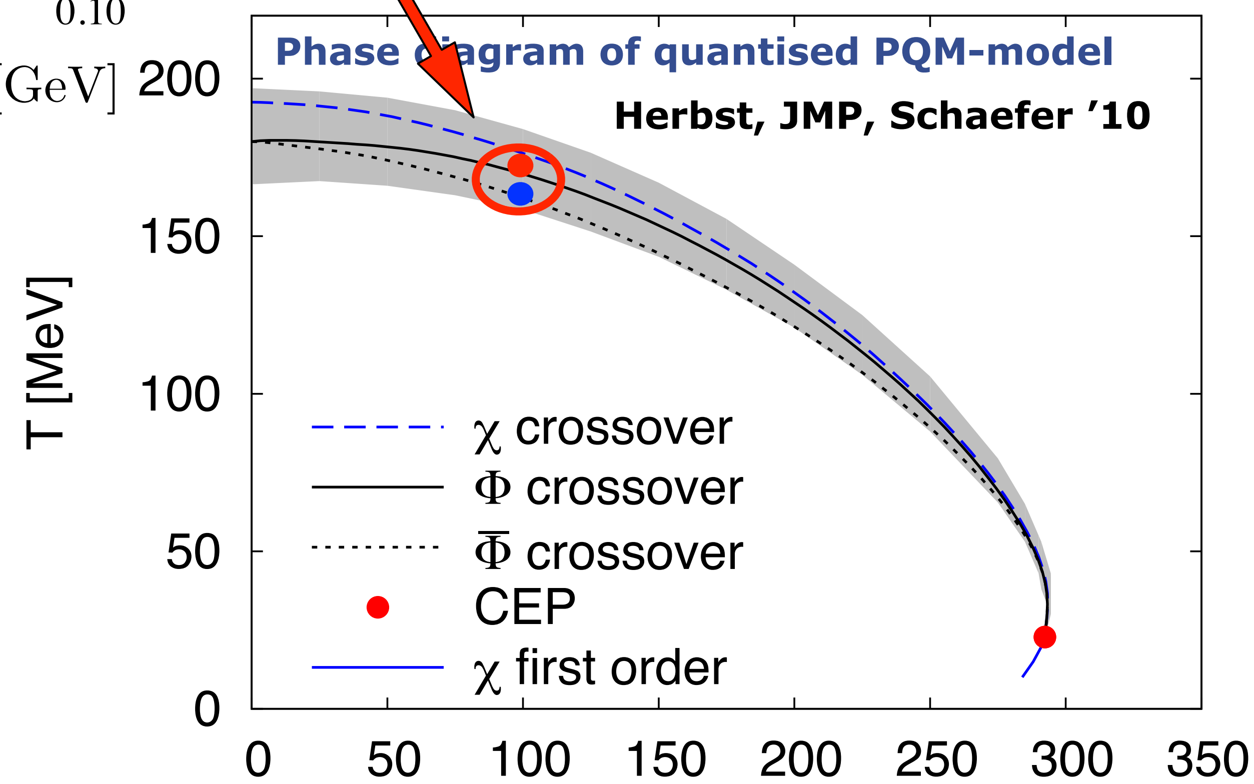


Real chemical potential

FRG+DSE+2PI+lattice

Braun, Haas, JMP, in prep.





Real chemical potential

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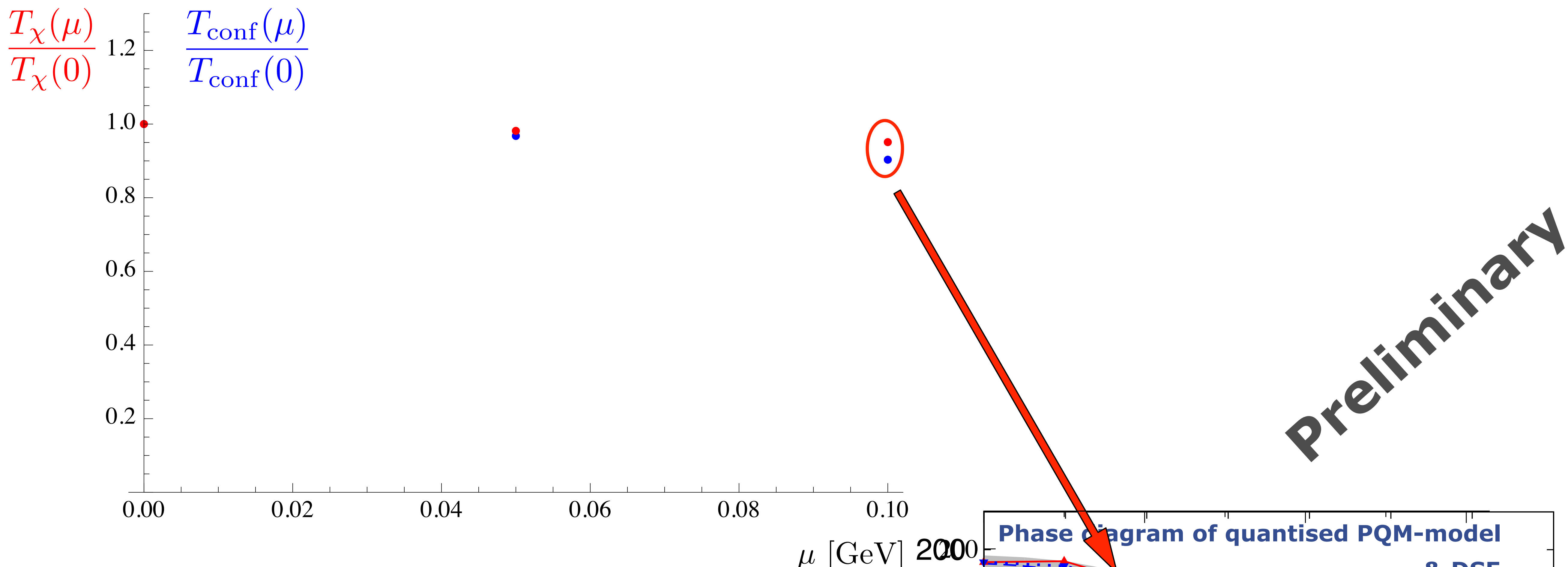
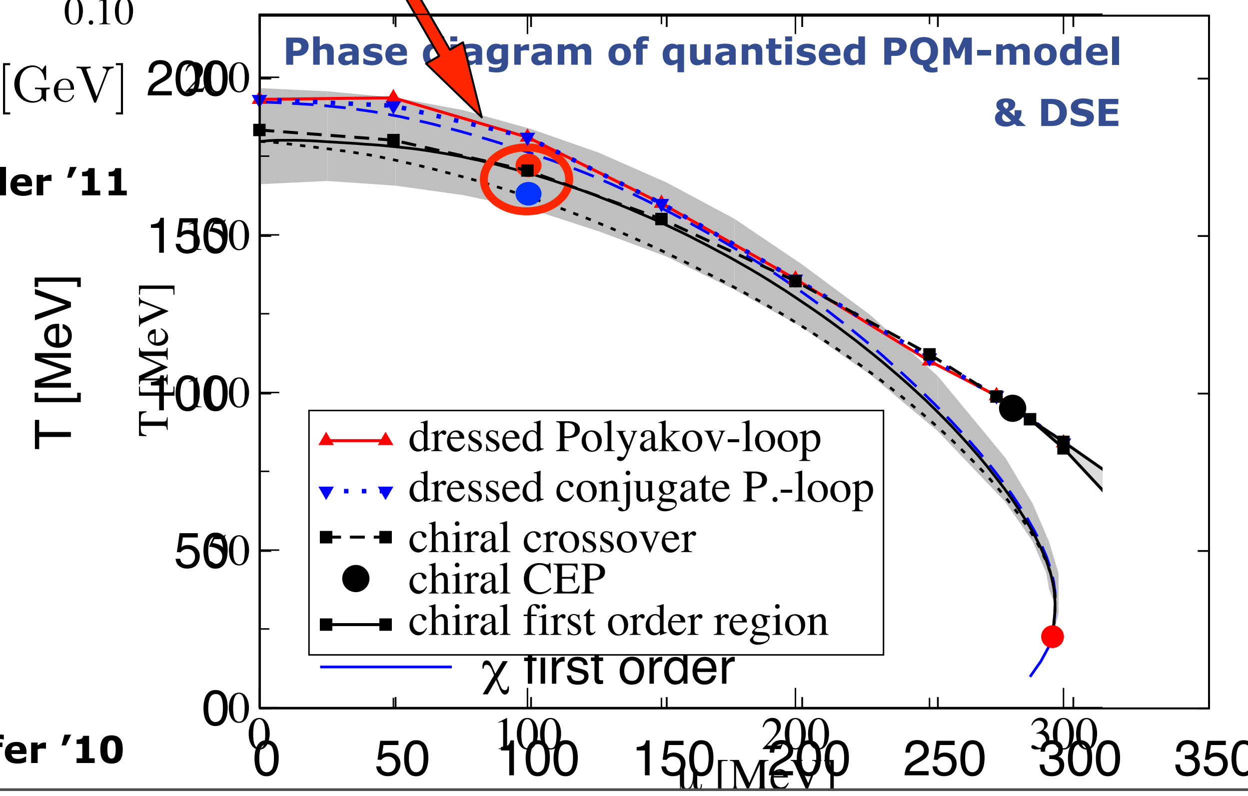


Fischer, Lücker, Mueller '11



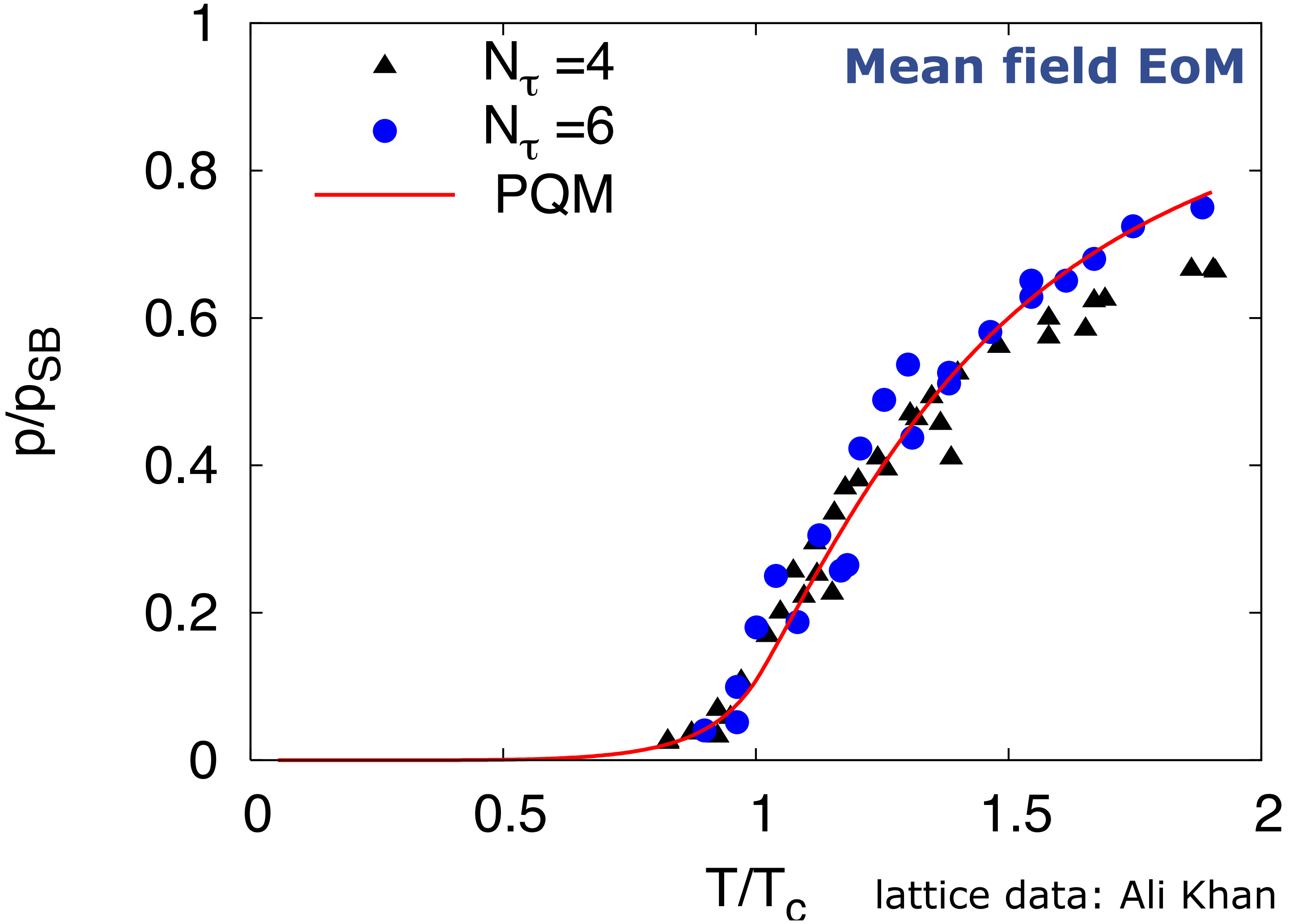
Herbst, JMP, Schaefer '10

Real chemical potential

Polyakov-extended Quark-Meson Models

Potential

Polyakov-loop Potential		Fermionic fluctuations		Mesonic potential
$U[\Phi, \bar{\Phi}]$	$+$	$\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$	$+$	$V[\sigma, \vec{\pi}]$
Fit to YM-thermodynamics		One loop computation		Fit of meson phenomenology



lattice data: Ali Khan et al '01

Meisinger, Ogilvie '96

Pisarski '00

⋮

PNJL

Fukushima '04

Ratti, Thaler, Weise '06

Megias, Ruiz Arriola, Salcedo '06

Ghosh, Mukherjee, Mustafa, Ray '06

C. Sasaki, B. Friman and K. Redlich '06

PQM → Schaefer, JMP, Wambach '07

⋮

Real chemical potential

dynamical Polyakov-extended models

Potential

Herbst, JMP, Schaefer '10

Polyakov-loop Potential

$$U[\Phi, \bar{\Phi}]$$

$+$

Fermionic fluctuations

$$\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$$

$+$

Mesonic potential

$$V[\sigma, \vec{\pi}]$$

Fit to YM-thermodynamics

fermionic fluctuations

mesonic fluctuations

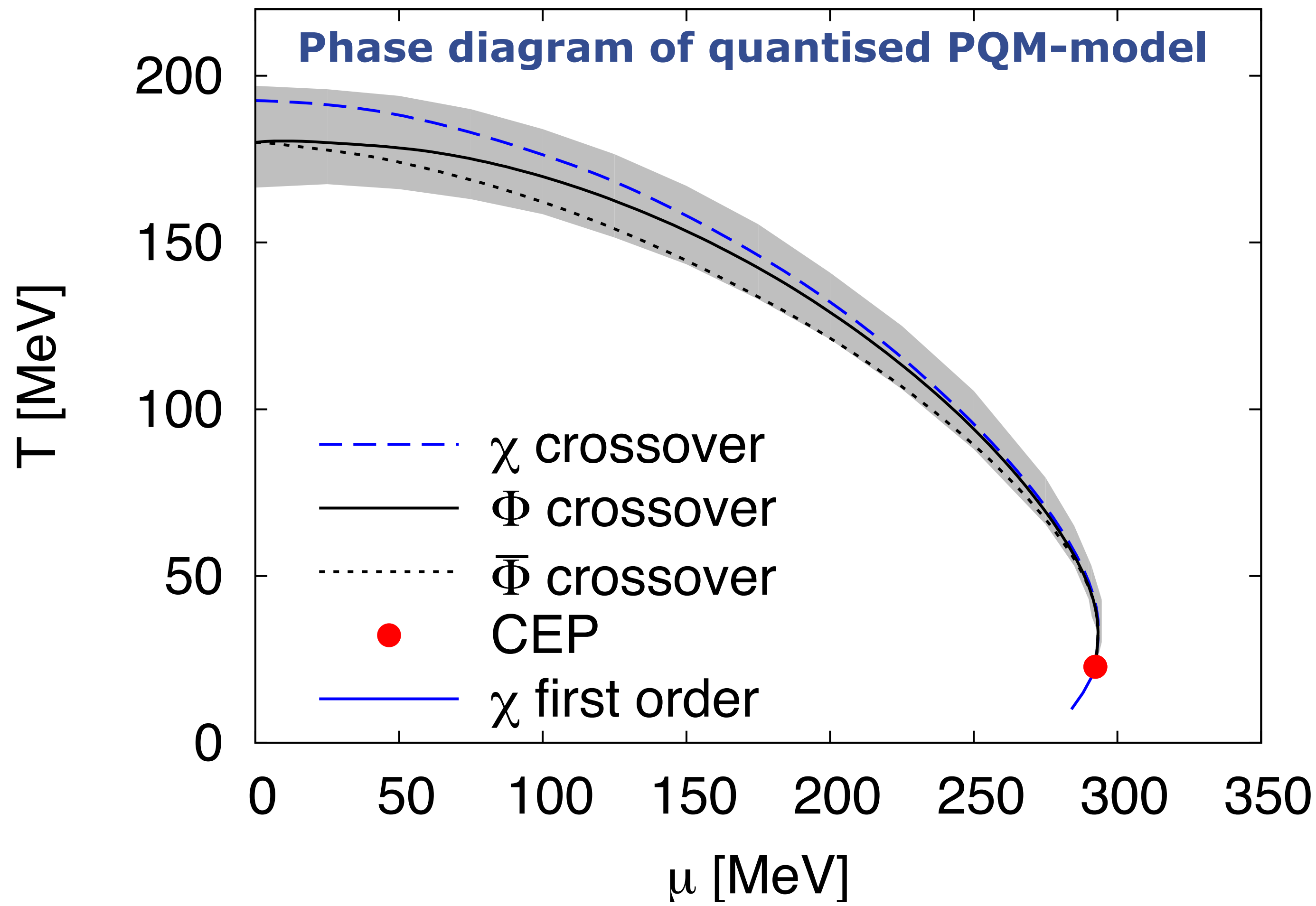
$$T_{0\mathrm{YM}} \rightarrow T_0(N_f, \mu; m_q)$$

quark fluctuations change glue dynamics

estimated via HTL/HDL computation

Schaefer, JMP, Wambach '07

FRG



see also Skokov, Friman, Redlich'10

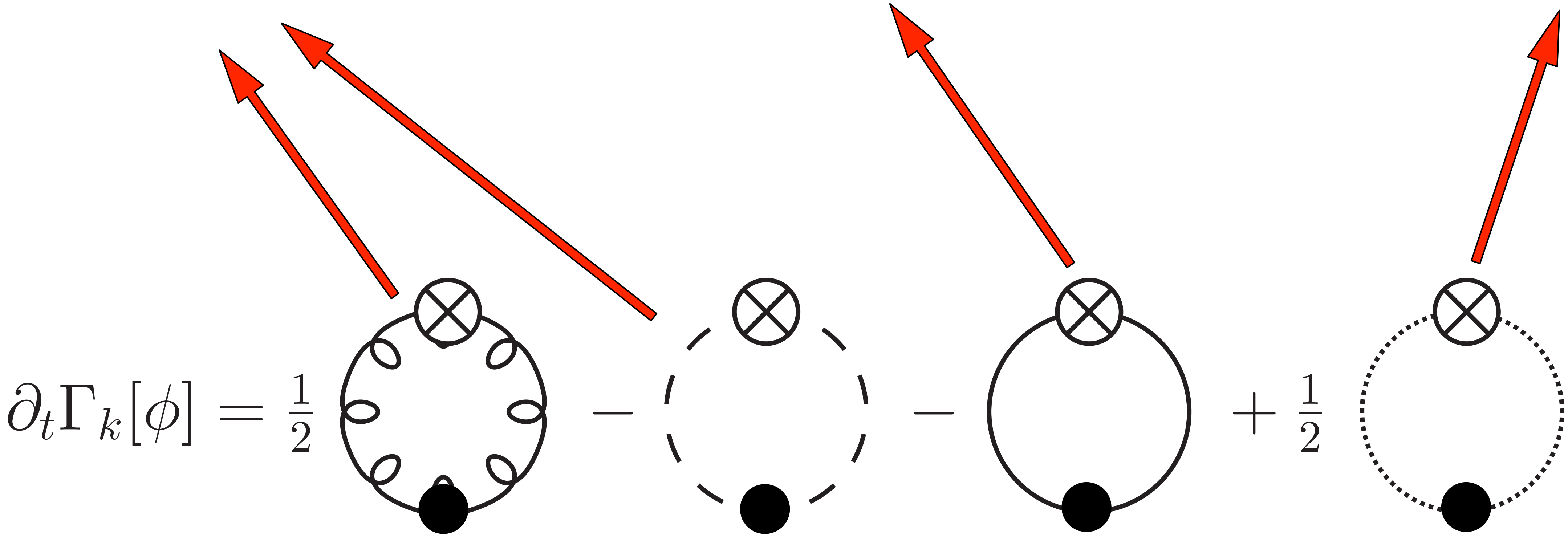
Real chemical potential

Polyakov-extended models as reduced QCD

Potential

Braun, Haas, JMP, in prep.

Polyakov-loop Potential		Fermionic fluctuations		Mesonic potential
$U[\Phi, \bar{\Phi}]$	$+$	$\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$	$+$	$V[\sigma, \vec{\pi}]$



Braun, Haas, Marhauser, JMP '09

Flow equation for QCD

Real chemical potential

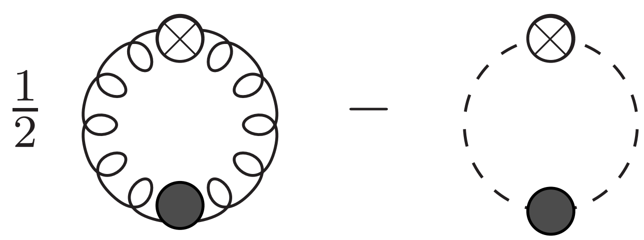
Polyakov-extended models as reduced QCD

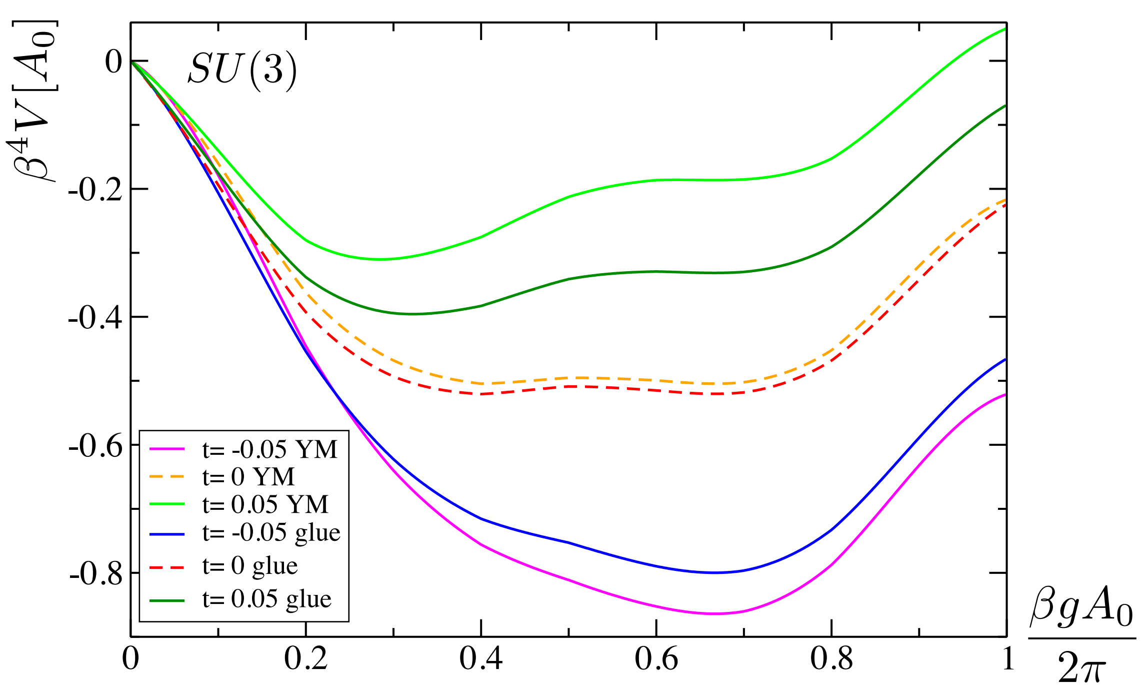
Improvement towards QCD

Braun, Haas, JMP, in prep.
JMP '10

Polyakov-loop Potential — **Fermionic fluctuations** — **Mesonic potential**

$$U[\Phi, \bar{\Phi}] \quad + \quad \Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}] \quad + \quad V[\sigma, \vec{\pi}]$$





QCD confirmation of

HTL/HDL quark fluctuations in YM sector

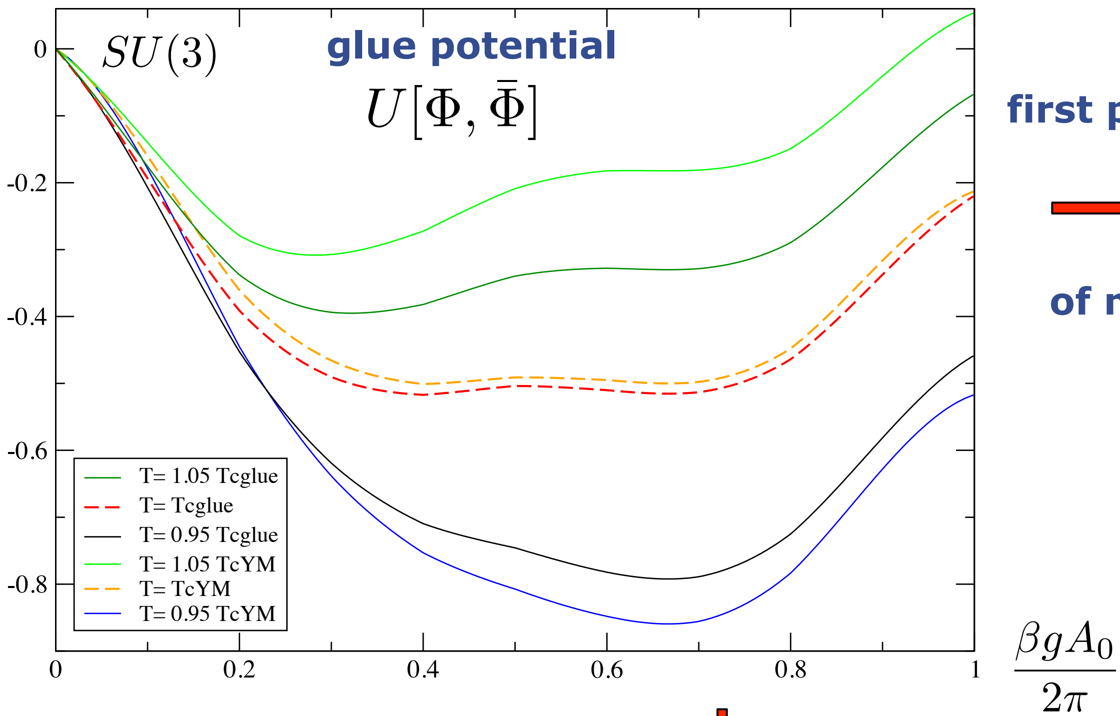
Schaefer, JMP, Wambach '07

$$(\beta^4 V)_{\rm glue}[t\,,\,A_0] \simeq (\beta^4 V)_{\rm YM}[t_{\rm YM}(t)\,,\,A_0]$$

Real chemical potential

FRG+DSE+2PI+lattice+model

full QCD

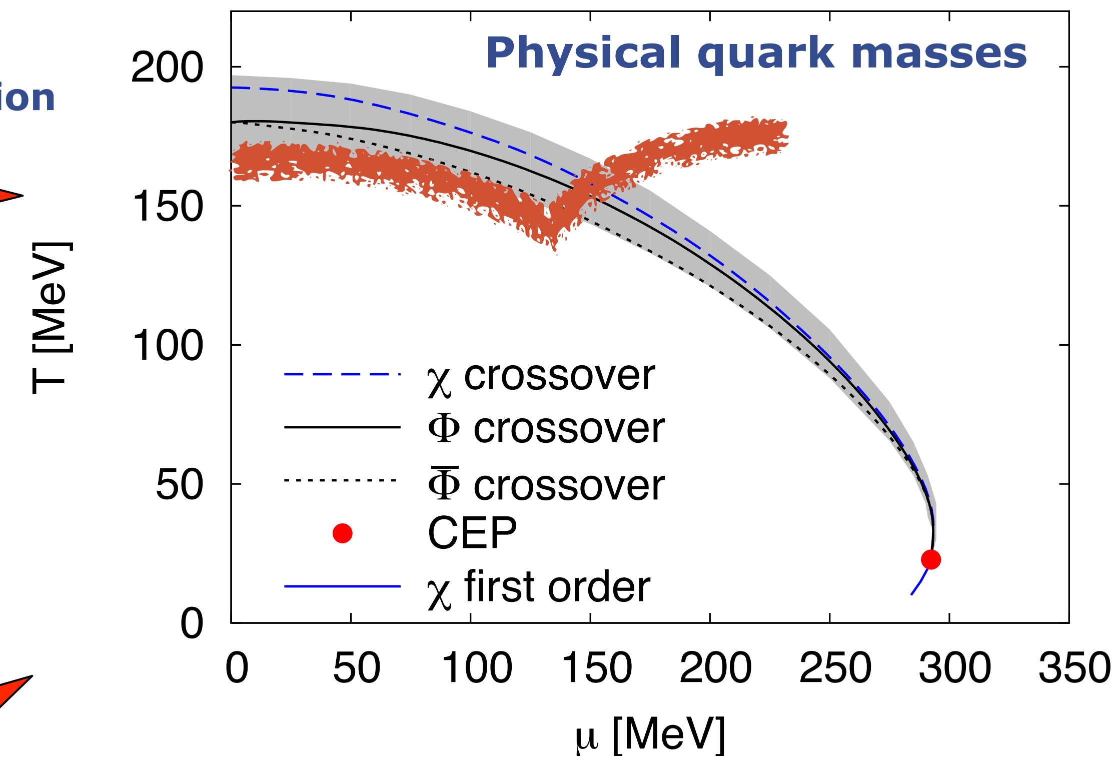


Braun, Haas, JMP, in prep.

first principle determination

of model parameters

QCD effective models



Herbst, JMP, Schaefer '10

QCD confirmation of

HTL/HDL quark fluctuations in YM sector

Schaefer, JMP, Wambach '07

Critical point unlikely for

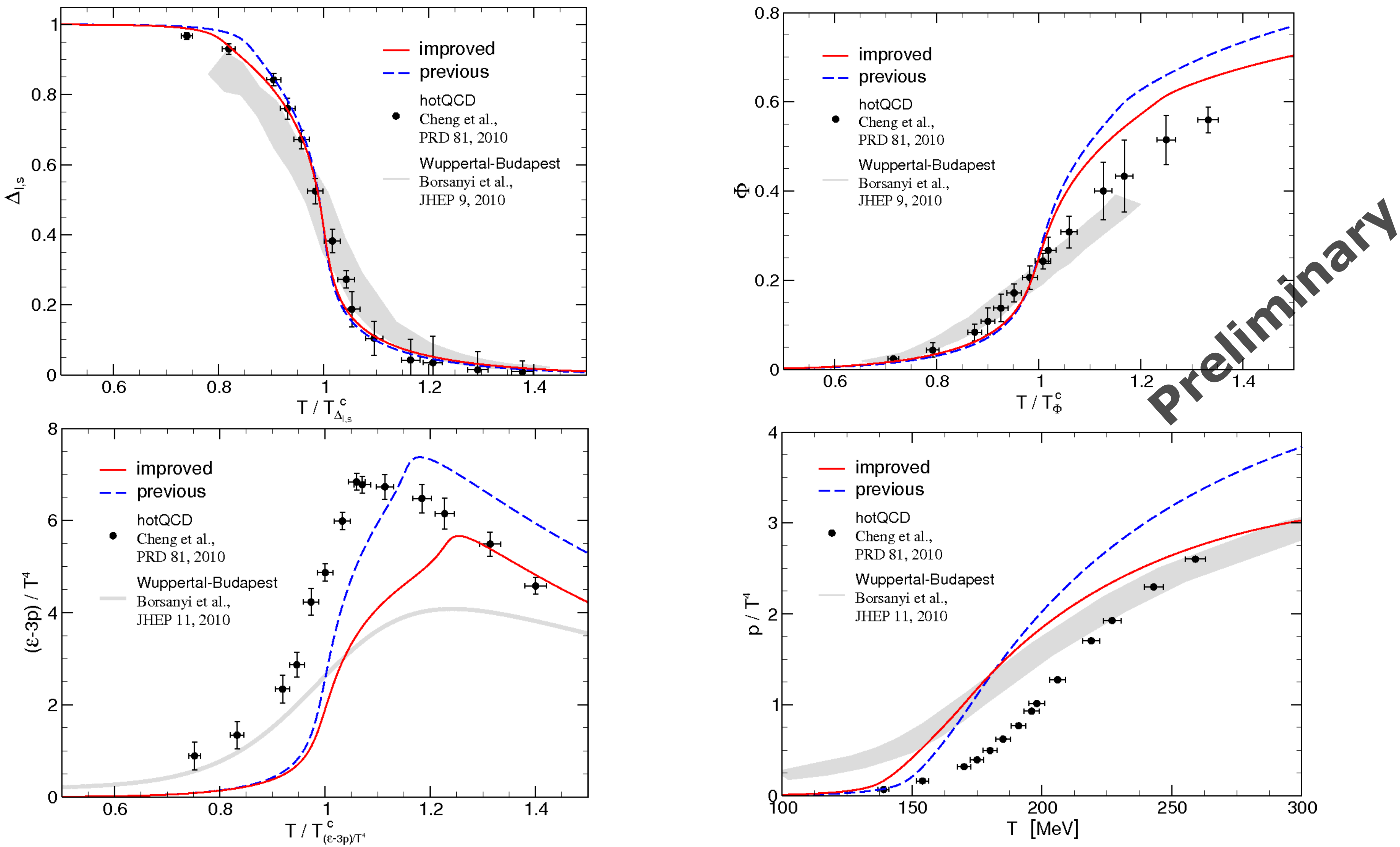
$$\frac{\mu_b}{T} < 2$$

thermodynamics

thermodynamics

PQM MF & full glue potential: 2+1 flavors

Haas, JMP, Schaffner-Bielich, Stiele, in prep.



beyond MF: JMP, Schaefer, work in progress

Summary & Outlook

Summary & outlook

- **Yang-Mills flows**
    - **propagators in quantitative agreement with lattice**
    - **Polyakov loop potential & conf-deconf phase transition**
- **QCD**
    - **conf-deconf & chiral phase transition at imaginary chemical potential**
    - **first steps at real chemical potential**
- **Outlook**
    - **2+1 flavours, baryons, phenomenology**
    - **two colour QCD, chiral magnetic effect**