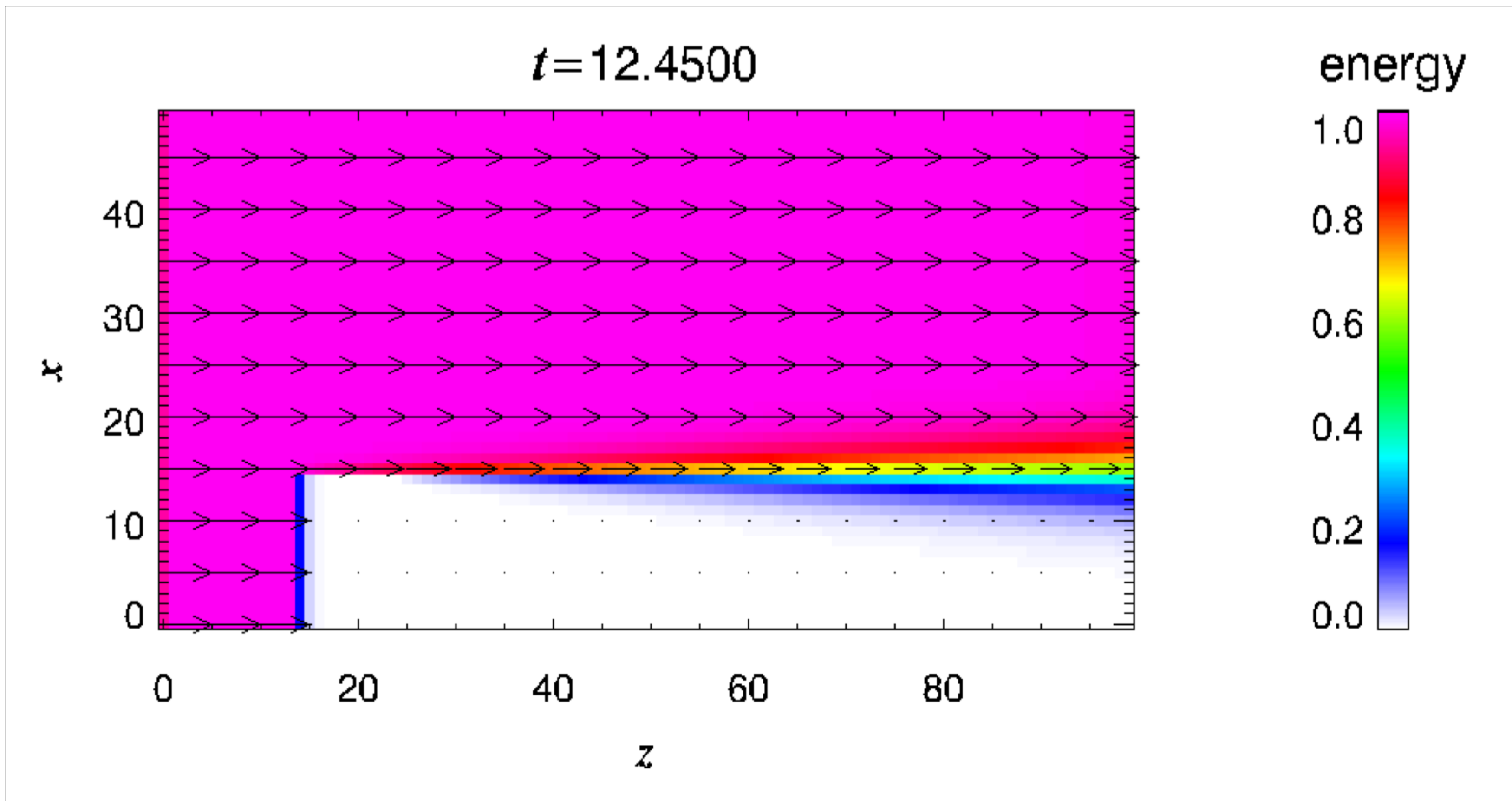
モーメント法によるニュートリノ輸送の数値解法



少ない計算量で
影、散乱、伝播を
扱える近似解法

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モーメント法: 拡散近似の改良

Intensity
輻射強

$$I_\nu\left(\boldsymbol{r},\ t,\ \boldsymbol{n}\right)$$

↓

Moment

0th: $$\boxed{E_\nu\left(\boldsymbol{x},\ t\right) \equiv \frac{1}{c}\oint I\left(\boldsymbol{x},\ t,\ \boldsymbol{n}\right) d\boldsymbol{n}}$$

1st: $$\boldsymbol{F}_\nu(\boldsymbol{x},\ t) \equiv \oint \boldsymbol{n}\, I(\boldsymbol{x},\ t,\ \boldsymbol{n})\, d\boldsymbol{n}$$

2nd $$\overleftrightarrow{\boldsymbol{P}}_\nu(\boldsymbol{x},\ t) \equiv \frac{1}{c}\oint \boldsymbol{n}\boldsymbol{n}\, I(\boldsymbol{x},\ t,\ \boldsymbol{n})\, d\boldsymbol{n}$$

拡散近似 *E*

*F*は推定

M1 model (*E*, ***F***)

Pは推定

輻射輸送方程式

輸送　　吸収

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \boldsymbol{n}\cdot\boldsymbol{\nabla}\right) I\left(\boldsymbol{r},\ t,\ \boldsymbol{n}\right) = -\sigma_\nu I_\nu(\boldsymbol{r},\ t,\ \boldsymbol{n})$$

厳密

$$+\sigma_\nu S_\nu\left(\boldsymbol{r},\ t,\ \boldsymbol{n}\right) + \sigma_{\nu,s}\oint g\left(\boldsymbol{n},\ \boldsymbol{n}'\right) I_\nu(\boldsymbol{r},\ t,\ \boldsymbol{n}')\, d\boldsymbol{n}'$$

放射　　散乱

$$\frac{\partial E_\nu}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{F}_\nu = \sigma_\nu\left(4\pi S_\nu - cE_\nu\right)$$

M1

$$\frac{\partial \boldsymbol{F}_\nu}{\partial t} + \boldsymbol{\nabla}\cdot\overleftrightarrow{\boldsymbol{P}}_\nu = -c\left(\sigma_\nu + \sigma_{\nu,s}\right)\boldsymbol{F}_\nu$$

Closure relation (終結条件)

$$\overleftrightarrow{\boldsymbol{P}}_\nu = \left(\frac{1-\chi}{2}\overleftrightarrow{\boldsymbol{I}} + \frac{3\chi-1}{2}\boldsymbol{n}\boldsymbol{n}\right)E_\nu,\quad \boldsymbol{n} = \frac{\boldsymbol{F}_\nu}{\boldsymbol{F}_\nu},\quad \chi = \frac{3+4f^2}{5+2\sqrt{4-3f^2}},\quad f = \frac{|\boldsymbol{F}_\nu|}{E_\nu}$$

風上差分 (特性曲線)

(simple) HLL

$$\boldsymbol{F}^{(\mathrm{HLL})}_{i+1/2,j,k} = \frac{\lambda_{\mathrm{R}} \boldsymbol{F}_{i,j,k} - \lambda_{\mathrm{L}} \boldsymbol{F}_{i+1,j,k} + \lambda_{\mathrm{R}} \lambda_{\mathrm{L}} \left(\boldsymbol{U}_{i+1,j,k} - \boldsymbol{U}_{i,j,k} \right)}{\lambda_{\mathrm{R}} - \lambda_{\mathrm{L}}}$$

$\lambda_{\mathrm{R}} = c, \quad \lambda_{\mathrm{L}} = -c$ 信号伝播の最大値，安全だが拡散大

Godunov (特性曲線と伝播モード)

拡散を抑えられるが、計算が面倒 (定義も曖昧)

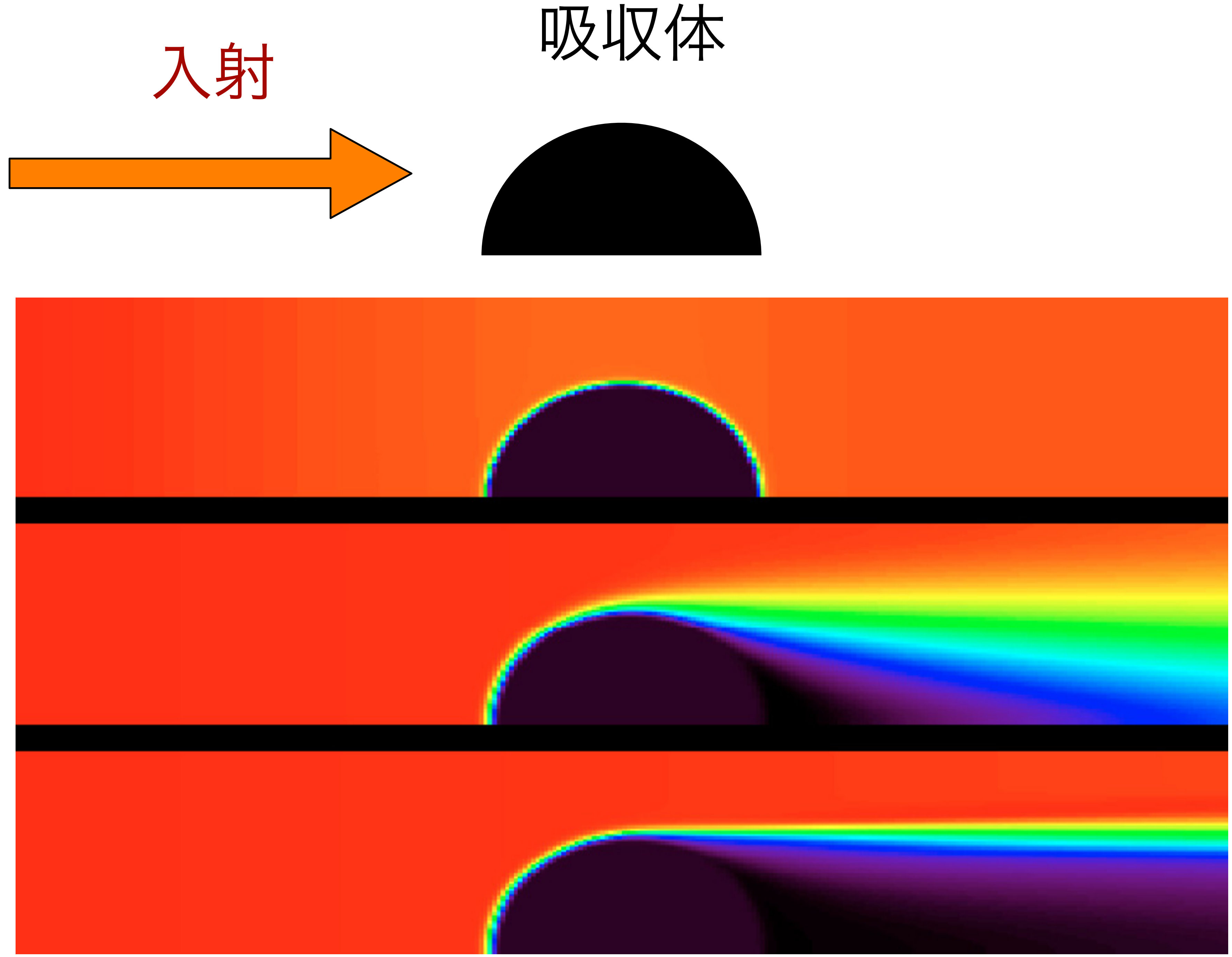
再構成 (本研究)

$$\boldsymbol{U} = \begin{pmatrix} E_\nu \\ F_{x,\nu} \\ F_{y,\nu} \\ F_{z,\nu} \end{pmatrix} \longrightarrow \quad I_\nu(\boldsymbol{n}) = \frac{3E_\nu}{8\pi} \frac{(1-\beta^2)^3}{3+\beta^2} (1 - \boldsymbol{\beta}\cdot\boldsymbol{n})^{-4}$$

$$\beta = \frac{3f}{2+\sqrt{4-3f^2}}, \quad \boldsymbol{\beta} = \beta \frac{\boldsymbol{F}}{|\boldsymbol{F}|}$$

Closure relation を満たす輻射

M1 モデルは影が扱える



Diffusion

M1 (HLL)

M1

(characteristics)

Gonzalez et al. ‘06

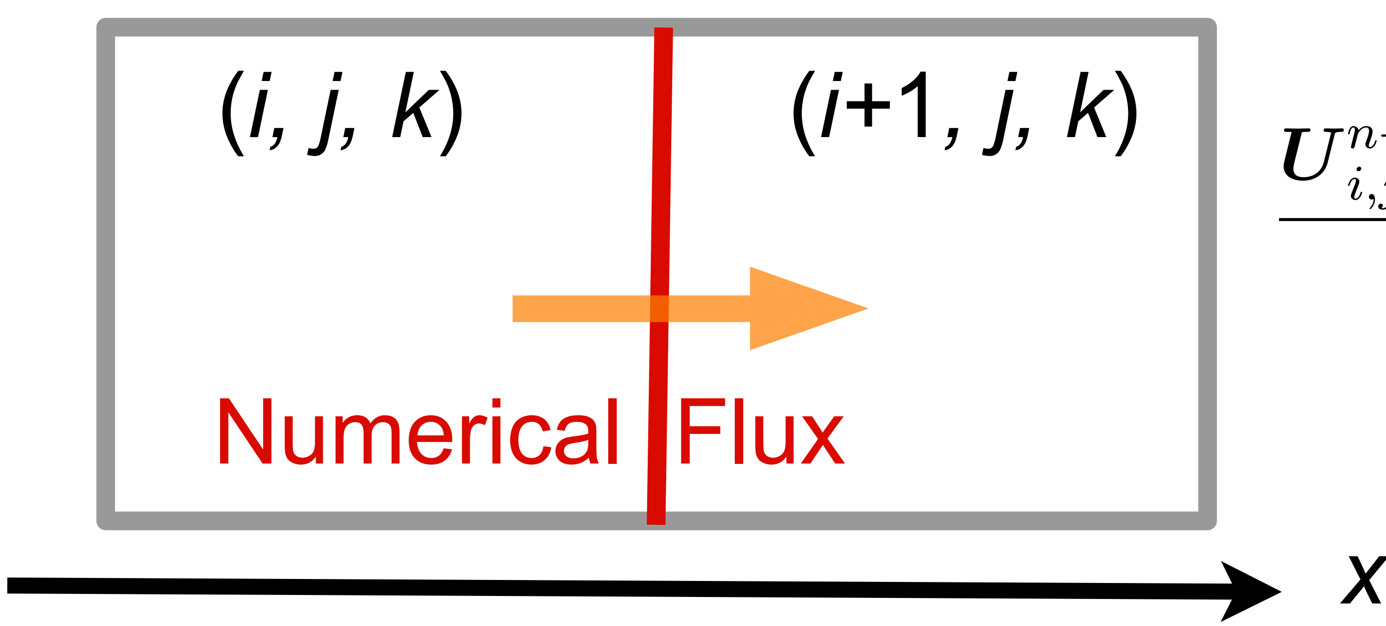
M1モデルの数値積分

保存形

$$\frac{\partial}{\partial t}\boldsymbol{U} + \frac{\partial}{\partial x}\boldsymbol{F}_x + \frac{\partial}{\partial y}\boldsymbol{F}_y + \frac{\partial}{\partial z}\boldsymbol{F}_z = \boldsymbol{S}$$

$$\boldsymbol{U} = \begin{pmatrix} E_\nu \\ F_{x,\nu} \\ F_{y,\nu} \\ F_{z,\nu} \end{pmatrix}, \quad \boldsymbol{F}_x = \begin{pmatrix} F_{x,\nu} \\ c^2 P_{xx,\nu} \\ c^2 P_{xy,\nu} \\ c^2 P_{xz,\nu} \end{pmatrix}, \quad \boldsymbol{S} = \begin{bmatrix} \sigma_\nu \left(4\pi S_\nu - cE_\nu\right) \\ -c\left(\sigma_\nu + \sigma_{\nu,s}\right)F_{x,\nu} \\ -c\left(\sigma_\nu + \sigma_{\nu,s}\right)F_{y,\nu} \\ -c\left(\sigma_\nu + \sigma_{\nu,s}\right)F_{z,\nu} \end{bmatrix}$$

$$\boldsymbol{F}_{x,i+1/2,j,k} = \boldsymbol{F}_{x,i+1/2,j,k}\left(\boldsymbol{U}_{x,i,j,k}, \boldsymbol{U}_{x,i+1,j,k}\right)$$



$$\frac{\boldsymbol{U}^{n+1}_{i,j,k} - \boldsymbol{U}^{n}_{i,j,k}}{\Delta t} + \frac{\boldsymbol{F}^{n}_{i+1/2,j,k} - \boldsymbol{F}^{n}_{i-1/2,j,k}}{\Delta x} = \boldsymbol{S}^{n}_{i,j,k}$$

「風上」性を考慮した再構成

$$I^*_{\nu,i+1/2,j,k}(\boldsymbol{n}) = \begin{cases} I_{\nu,i,j,k}(\boldsymbol{n}) & (n_x > 0) \\ I_{\nu,i+1,j,k}(\boldsymbol{n}) & (n_x < 0) \end{cases}$$

$I_{j+1}(\boldsymbol{n})$ ⟵ ⟶ $I_j(\boldsymbol{n})$

$$\begin{pmatrix} E_j \\ \boldsymbol{F}_j \end{pmatrix} = \begin{pmatrix} E_j \\ \boldsymbol{f}_j E_j \end{pmatrix} \qquad \begin{pmatrix} E_{j+1} \\ \boldsymbol{F}_{j+1} \end{pmatrix} = \begin{pmatrix} E_{j+1} \\ \boldsymbol{f}_{j+1} E_{j+1} \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{F}^*_{j+1/2} = \boldsymbol{F}^{(+)}_j + \boldsymbol{F}^{(-)}_{j+1} \\ \mathrm{P}^*_{j+1/2} = \mathrm{P}^{(+)}_j + \mathrm{P}^{(-)}_{j+1} \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{F}^*_{j+1/2} \\ \mathrm{P}^*_{j+1/2} \end{pmatrix}$$

解析関数

$$I_\nu(\boldsymbol{n}) = \frac{3E_\nu}{8\pi} \frac{(1-\beta^2)^3}{3+\beta^2} (1-\boldsymbol{\beta}\cdot\boldsymbol{n})^{-4}$$

$$\beta = \frac{3f}{2+\sqrt{4-3f^2}}, \qquad \boldsymbol{\beta} = \beta \frac{\boldsymbol{F}}{|\boldsymbol{F}|}$$

M1 closure と整合する輻射場

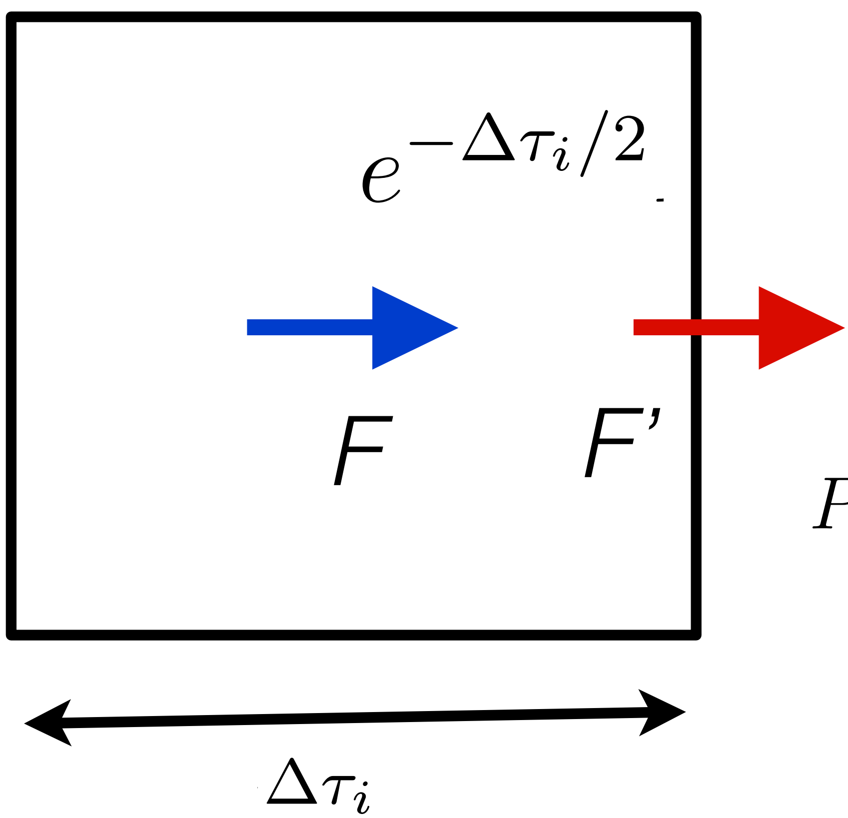
セル内での吸収・放射・散乱

境界のF　　減衰率　　中心のF　　放射

$$F'^{(+)}_{\nu,i+1/2,j,k} = \boxed{e^{-(\Delta\tau_{a,i}+\Delta\tau_{s,i})/2}} F^{(+)}_{\nu,i+1/2,j,k} + \boxed{\left(1 - e^{-\Delta\tau_{a,i,j,k}/2}\right) \pi B_{\nu,i,j,k}}$$

$$+ \boxed{e^{-(\Delta\tau_{a,i,j,k})/2} \left(1 - e^{-\Delta\tau_{s,i,j,k}/2}\right) \frac{cE_{\nu,i,j,k}}{4}}$$

散乱



光学的に厚い ($\Delta\tau_a, \Delta\tau_s \gg 1$) ところで良い近似

$$P'^{(+)}_{\nu,i+1/2,j,k} = e^{-(\Delta\tau_{a,i}+\Delta\tau_{s,i})/2} P^{(+)}_{\nu,i+1/2,j,k}$$
$$+ \left(1 - e^{-\Delta\tau_{a,i,j,k}/2}\right) \frac{2\pi B_{\nu,i,j,k}}{3c}$$
$$+ e^{-(\Delta\tau_{a,i,j,k})/2} \left(1 - e^{-\Delta\tau_{s,i,j,k}/2}\right) \frac{E_{\nu,i,j,k}}{6}$$

2次精度は不安定

時間進化

1次精度

移流は単純な 陽解法

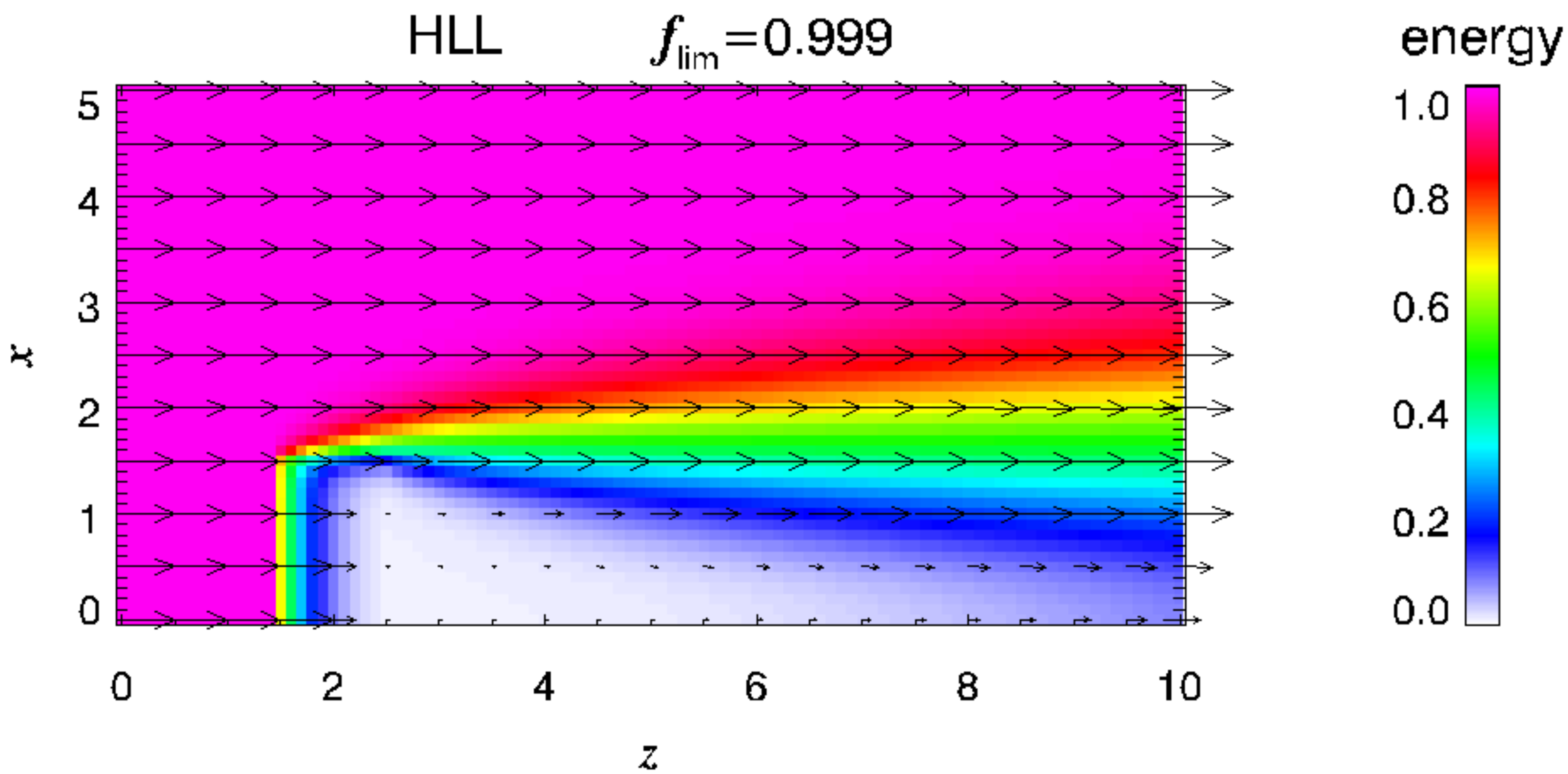
放射・吸収・散乱は「局所平衡」へ指数関数的に近づける (陽解法)

$$E(t+\Delta t) = \eta E(t) + (1-\eta) E_{eq}$$

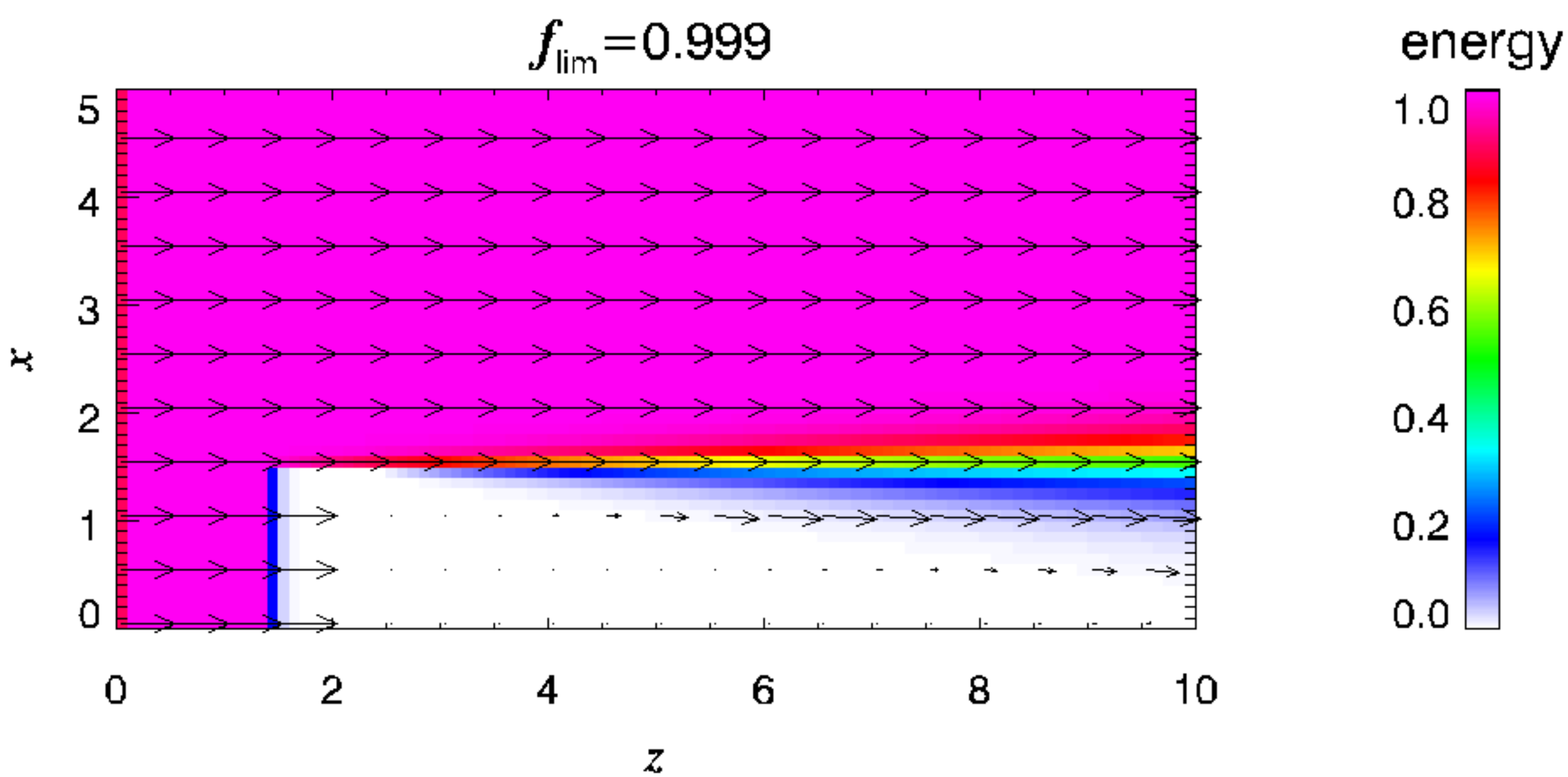
$$\eta = \exp(-\Delta t / \tau)$$

影

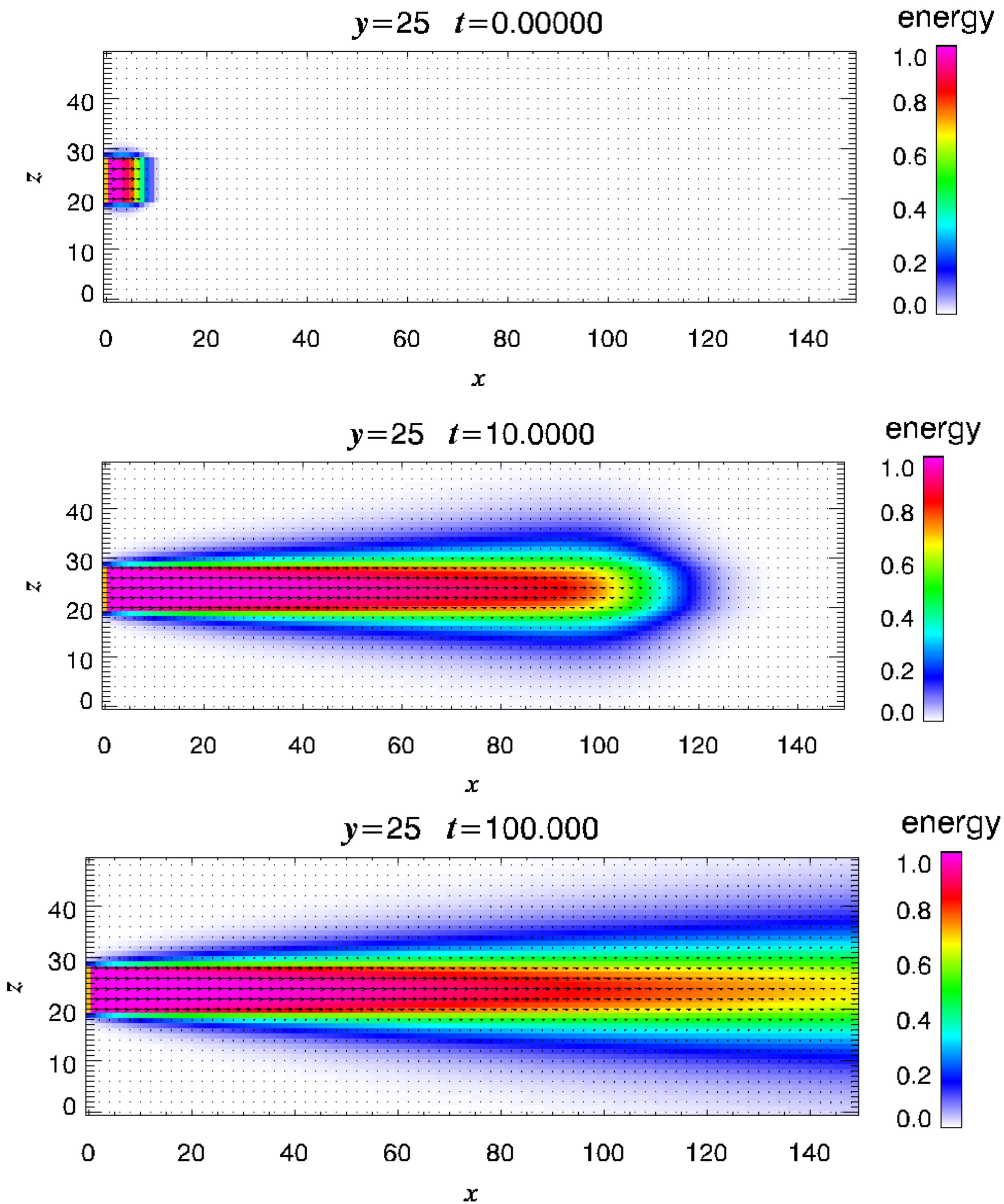
simple HLL



this work



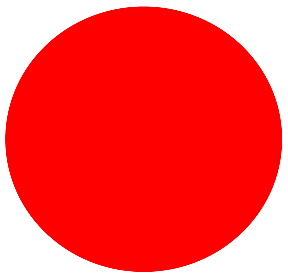
ビーム テスト (1)
光の伝播



1次精度

$\Delta x = 0.1$

閃光テスト (1)



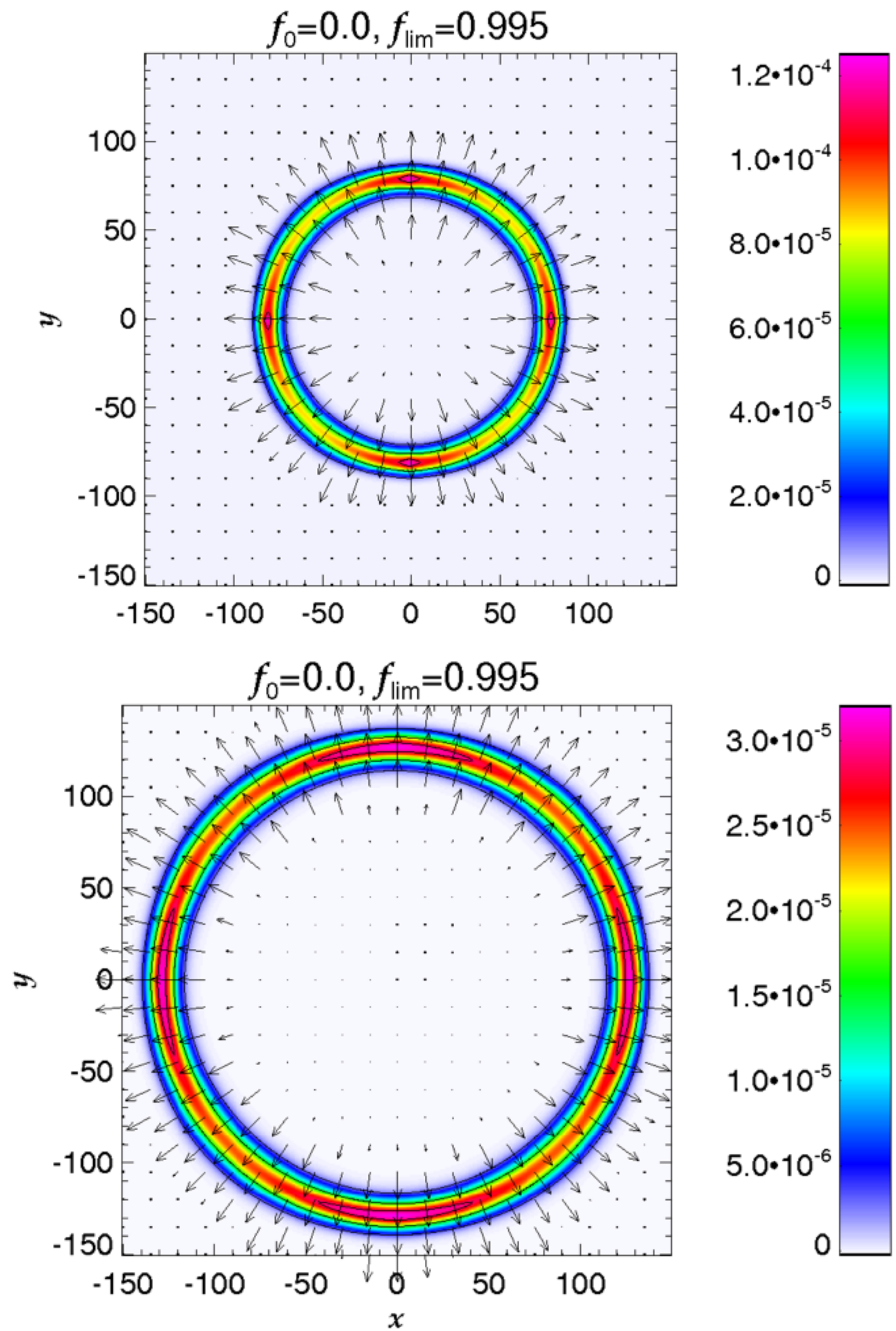
E = const.
F = 0

$$|\boldsymbol{F}| \; > \; f_{\rm lim} E$$

なら

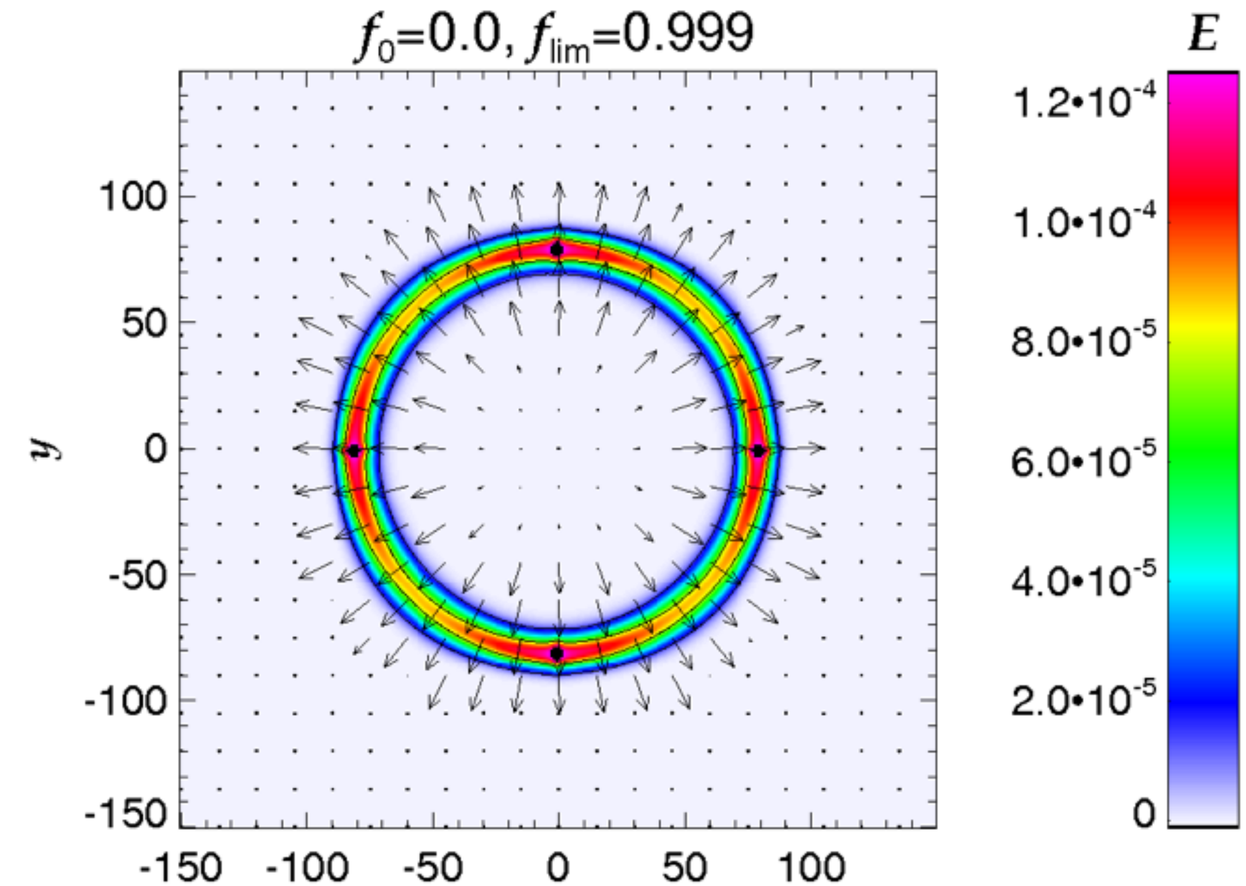
$$\boldsymbol{F} \; \rightarrow \; \frac{f_{\rm lim}\, E\, \boldsymbol{F}}{|\boldsymbol{F}|}$$

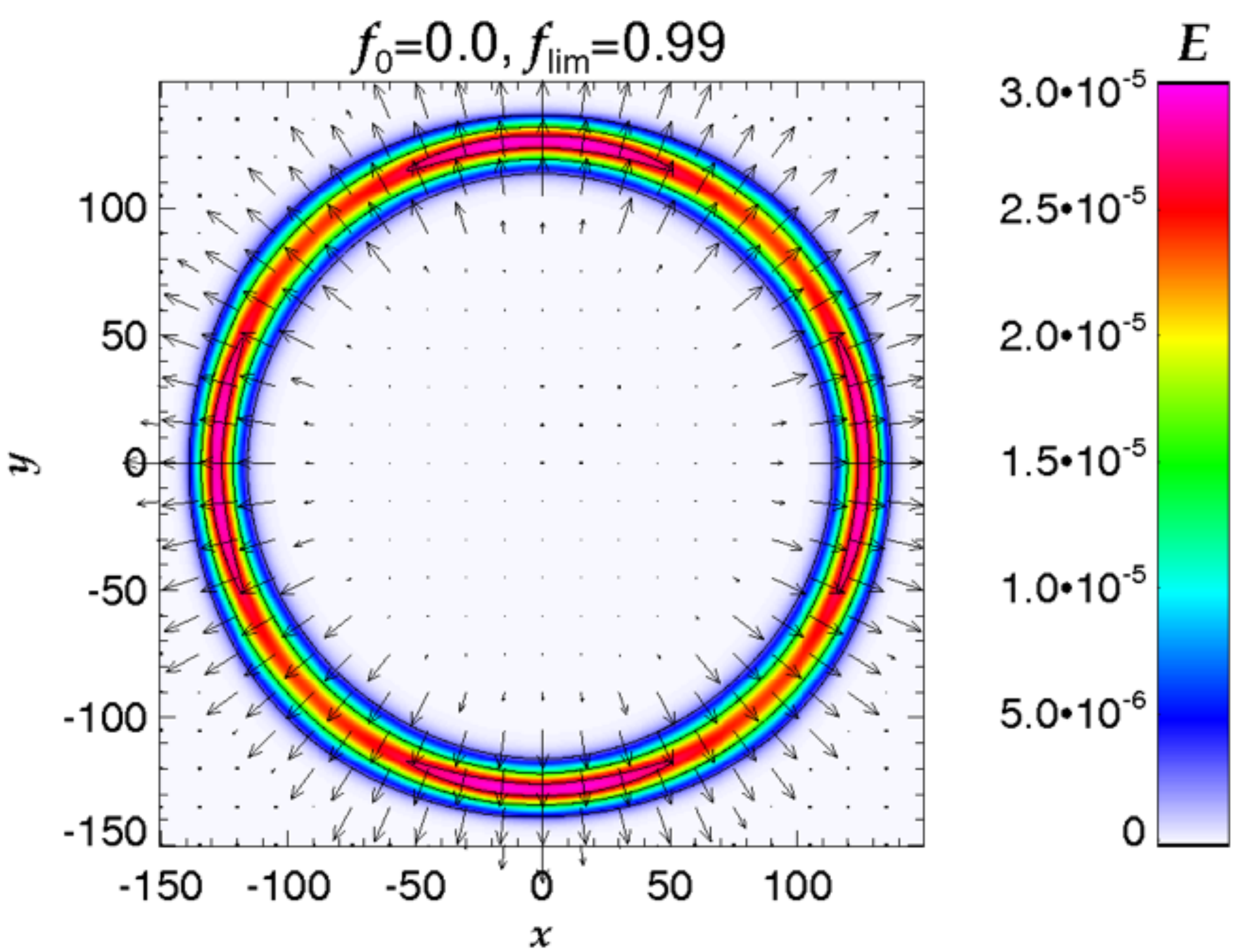
極端に細いビーム
は避ける

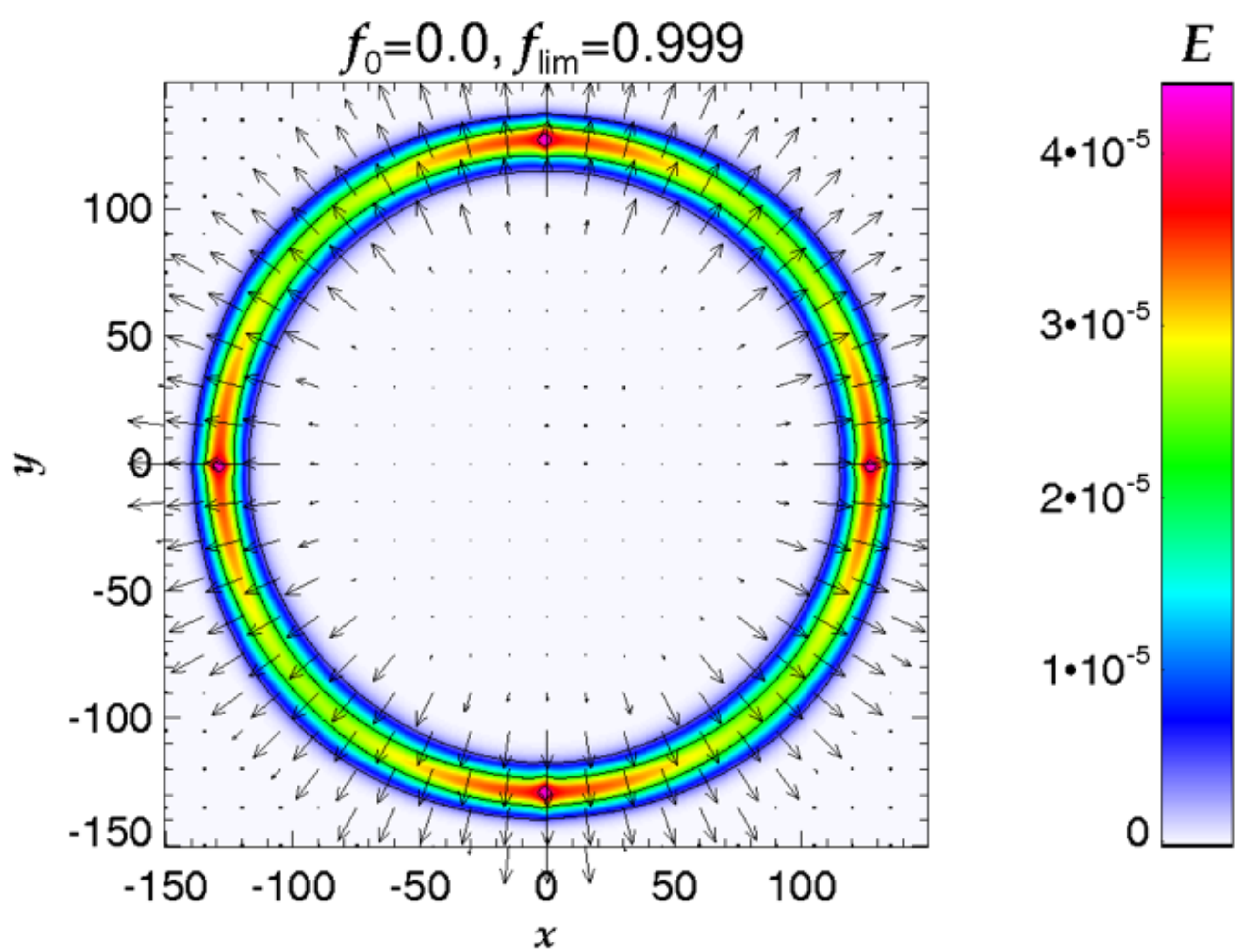


閃光(2)

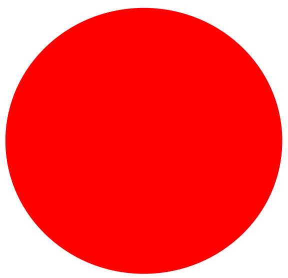
等方性とビームの鋭角性
は二律背反







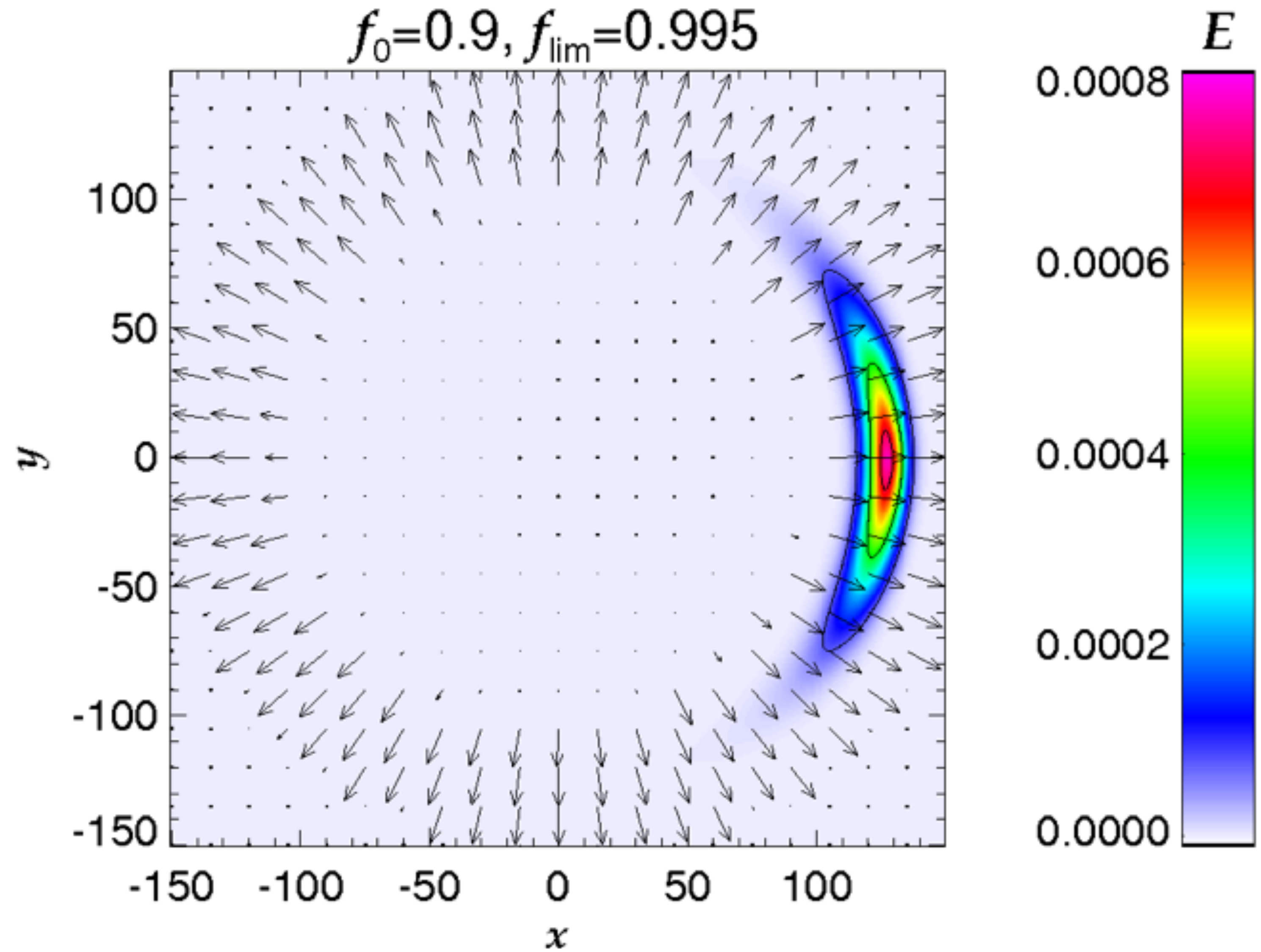
閃光 (3)

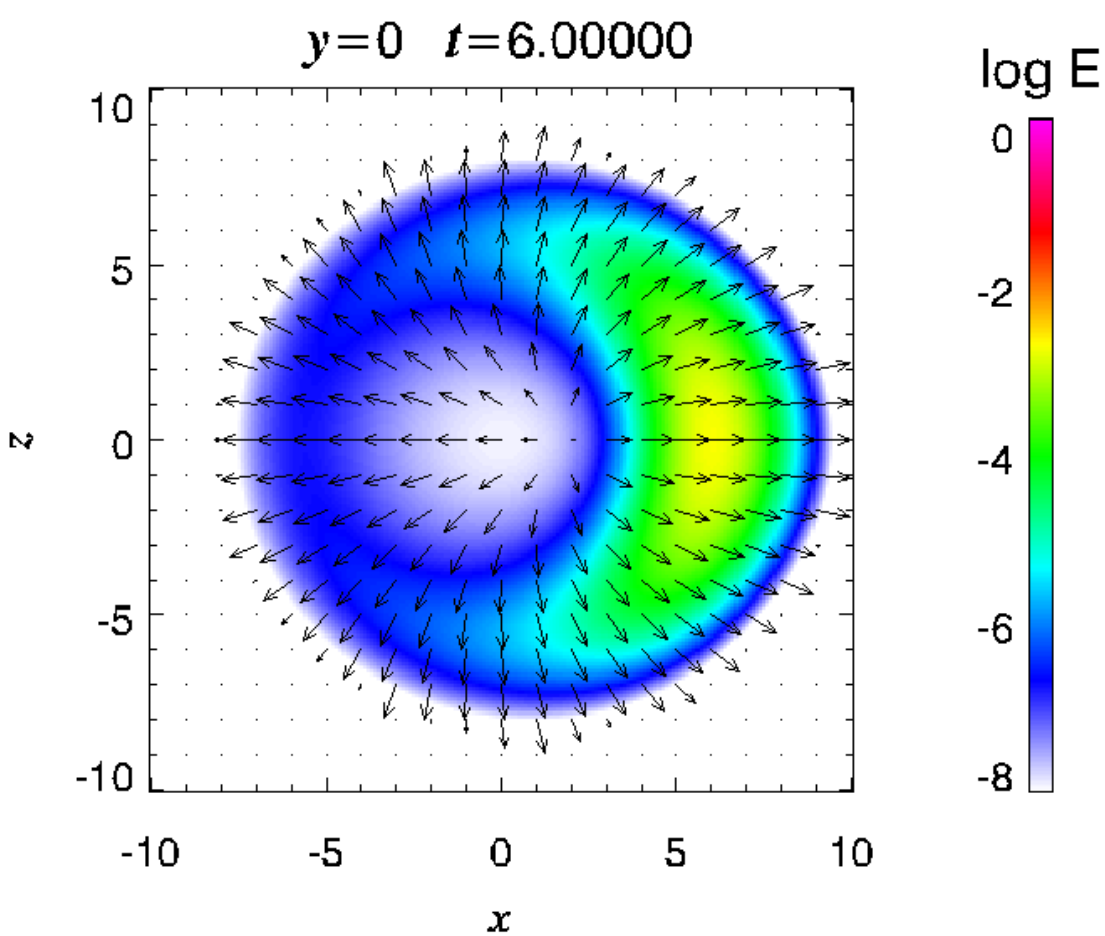


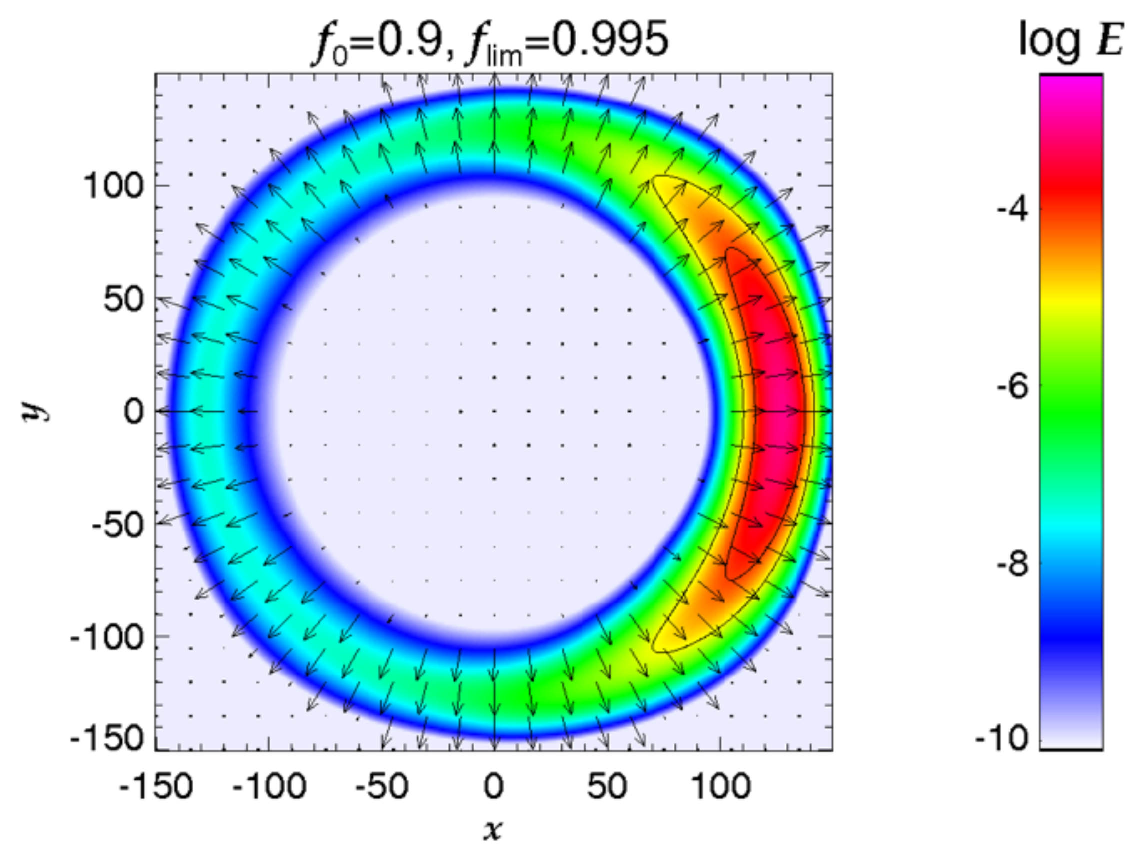
E = const.
F = (0.9, 0, 0)

矢印: $\boldsymbol{f} = \boldsymbol{F}/$

HLLでは「拡散」「偏心」

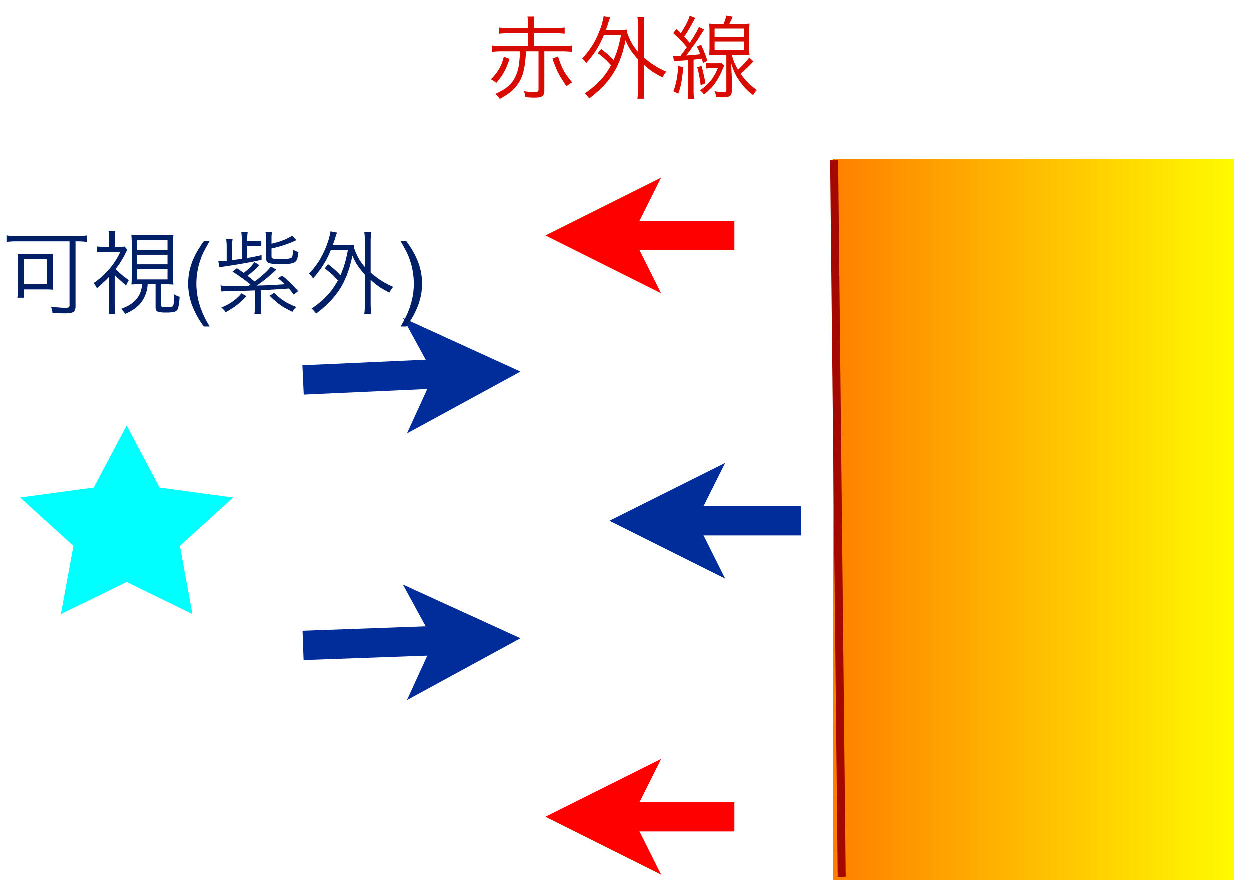






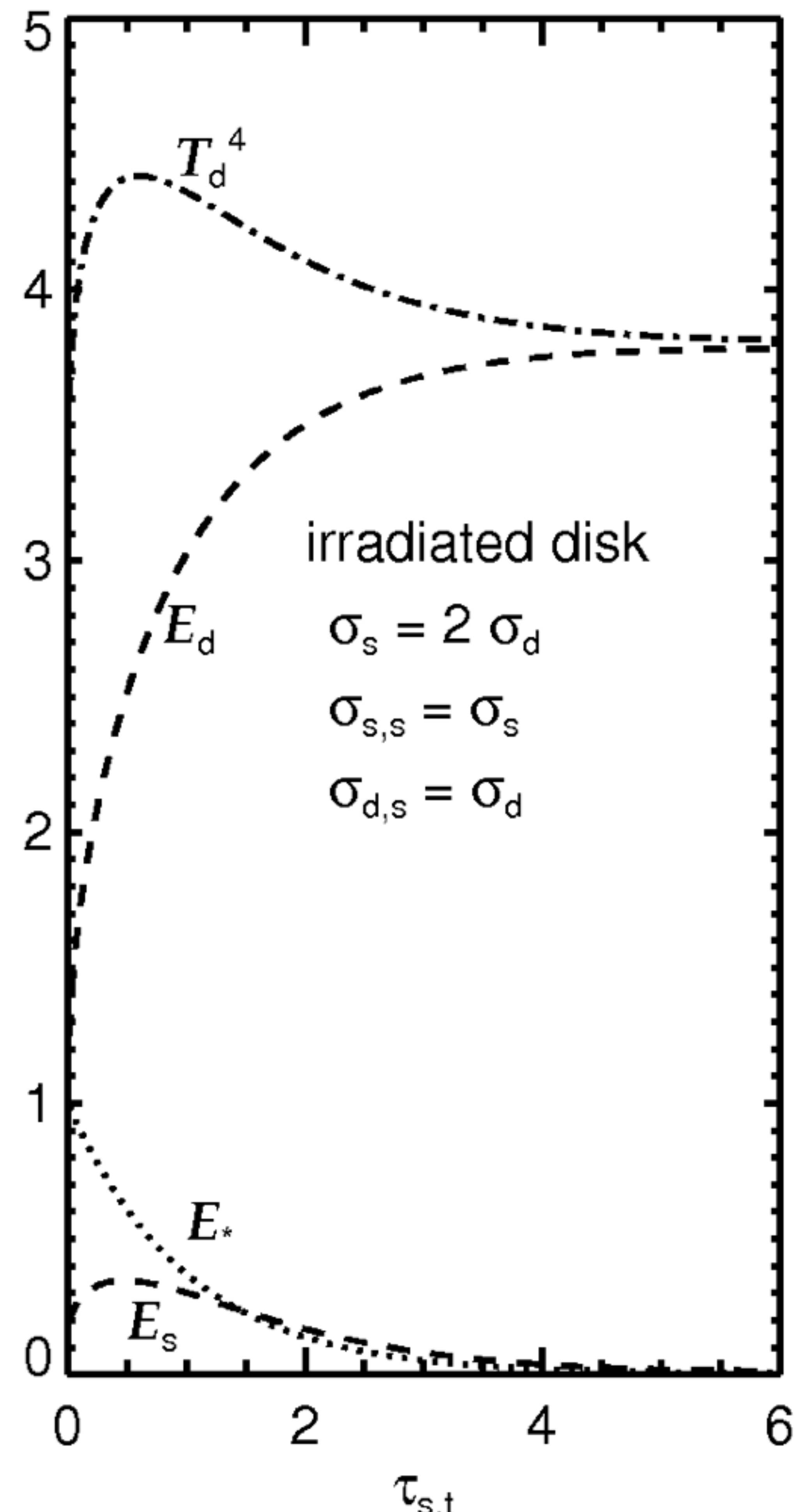
SEDも計算できる

ex. 照射された原始惑星円盤からの赤外線

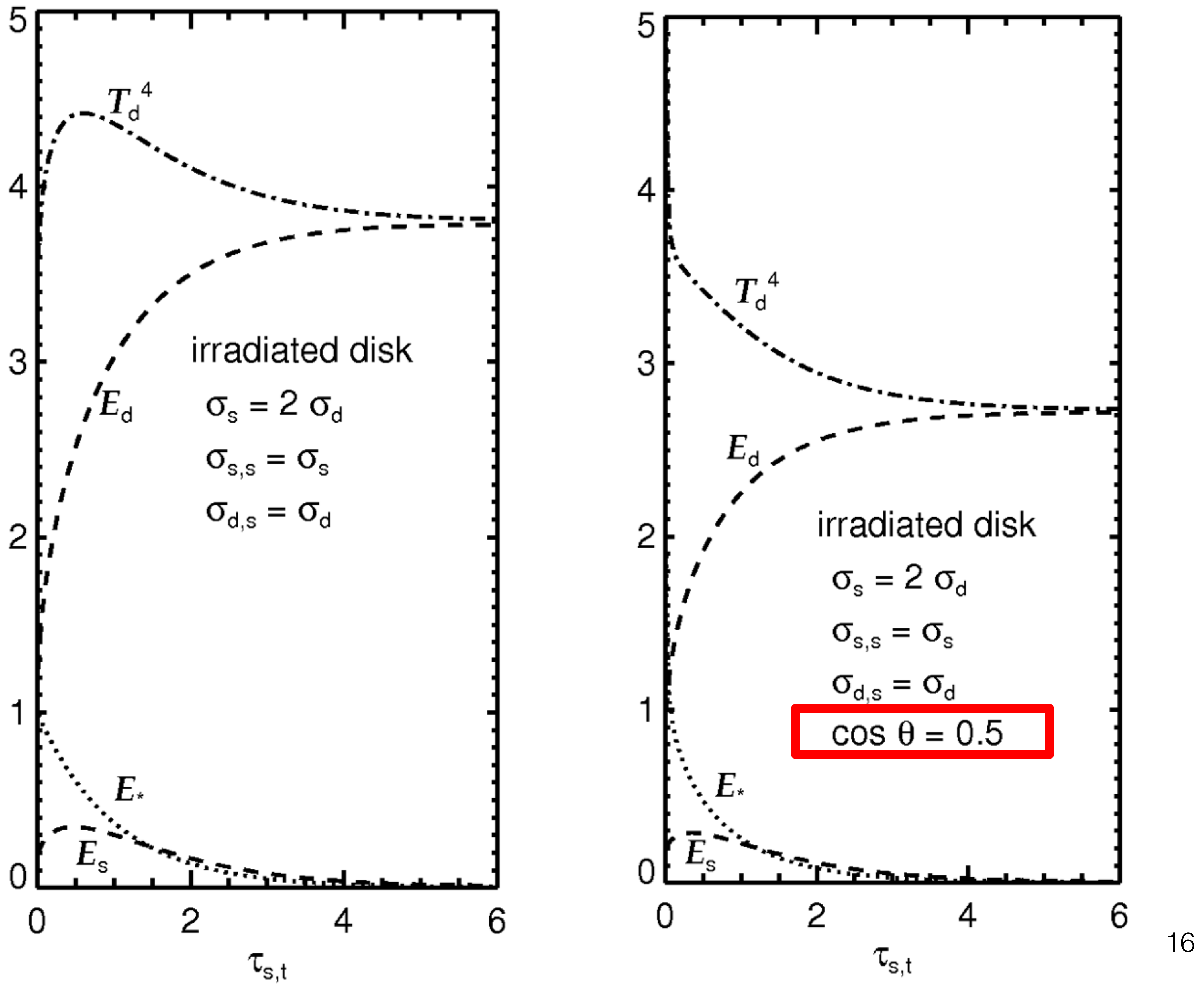


cf. Calvet et al. ’91
but

$$\chi = \frac{3 + 4f^2}{5 + 2\sqrt{4 - 3f^2}}$$



高温層の場所は入射角度に依る



まとめ

- M1 model は 影, 散乱, 伝播が扱える
  - 相対論的な天体では陽解法で計算可能
- 再構成法は「良い」数値流束を与える
  - 安定: E > |F| > 0 が保たれる (Δt<Δx/c)
  - 光学的に厚いところで拡散近似に収束
  - 「影」がシャープ