Magnetic and Antimagnetic rotation in Covariant Density Functional Theory

Jie Meng 孟 杰

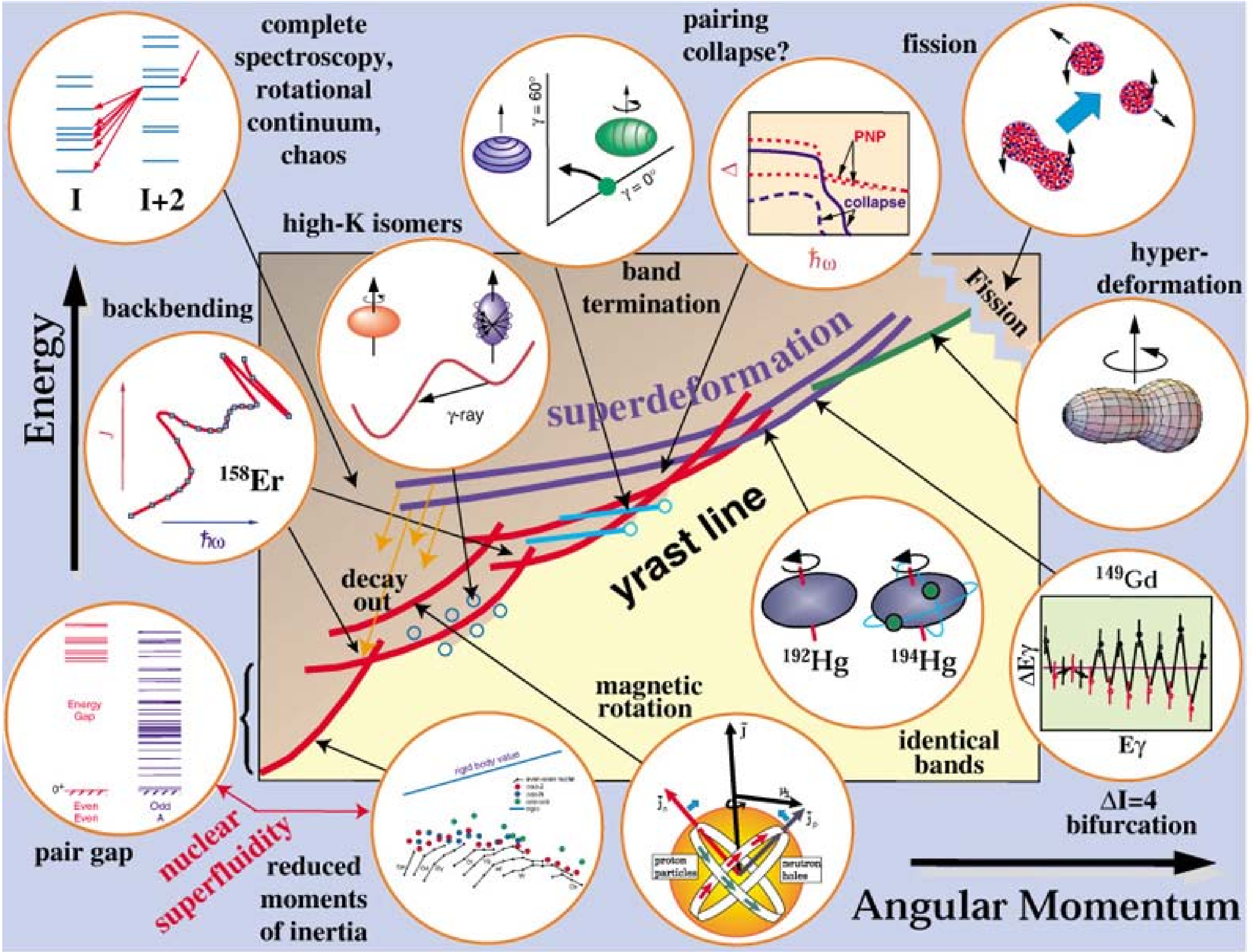
北京大学物理学院
School of Physics
Peking University (PKU)

北京航空航天大学物理与核能工程学院
School of Physics and Nuclear Energy Engineering
Beihang University (BUAA)

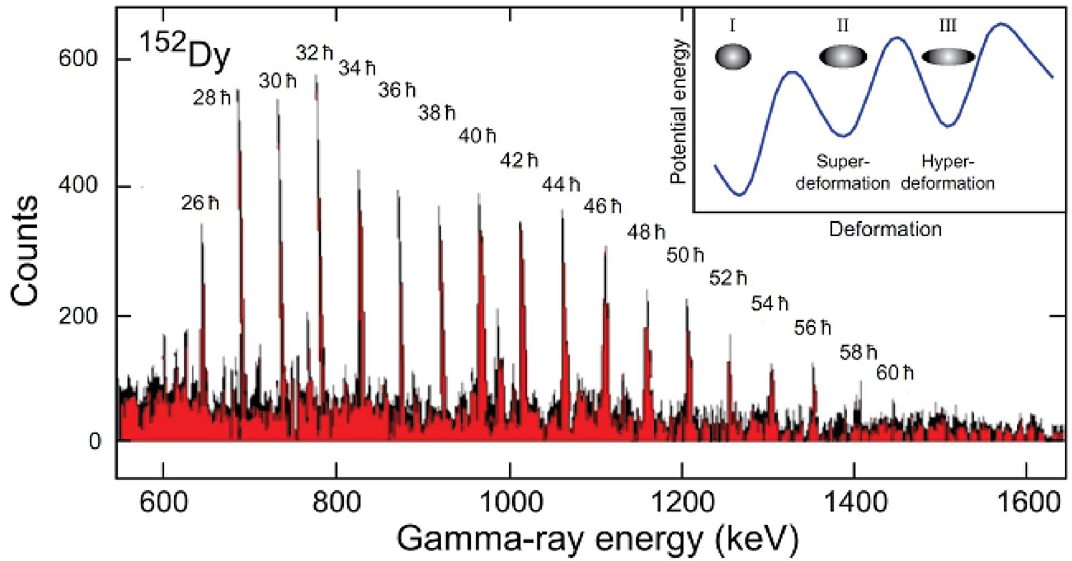
Outline

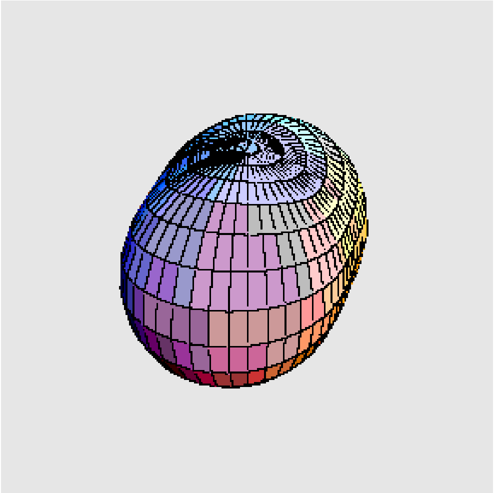
- Introduction
- Theoretical framework
- Magnetic rotation
- Antimagnetic rotation
- Summary & Perspectives

Angular Momentum World of Nucleus



Electric Rotation





Twin PRL1986

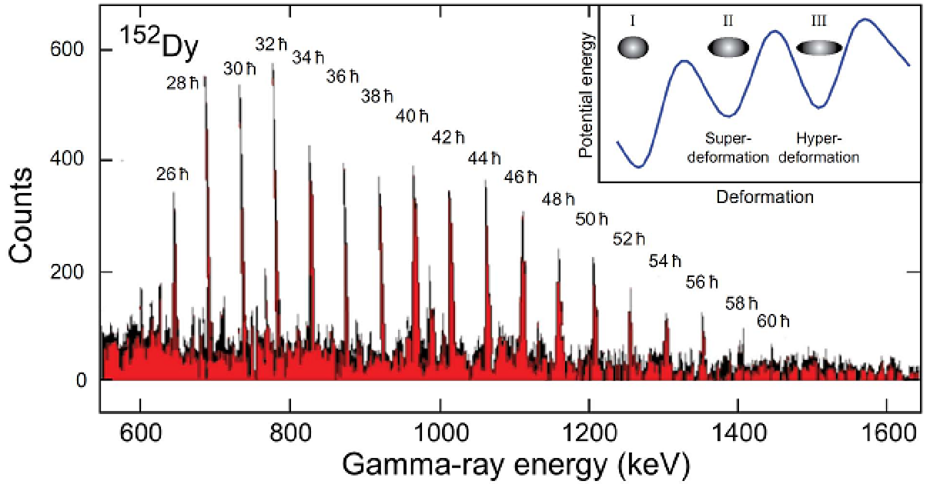
- ✓ **Substantial quadrupole deformation**
- ✓ **Strong electric quadrupole (E2) transitions**
- ✓ **Coherent collective rotation of many nucleons**

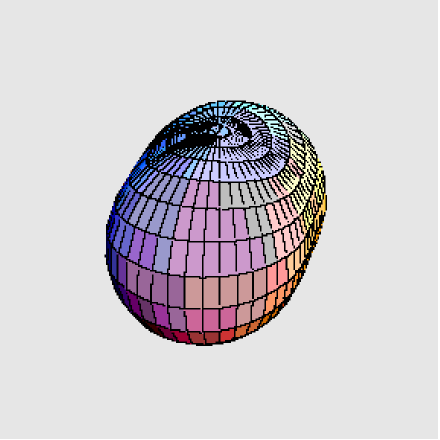
Bohr PR1951

Electric & Magnetic Rotation

ΔI = 2

E2 Transitions

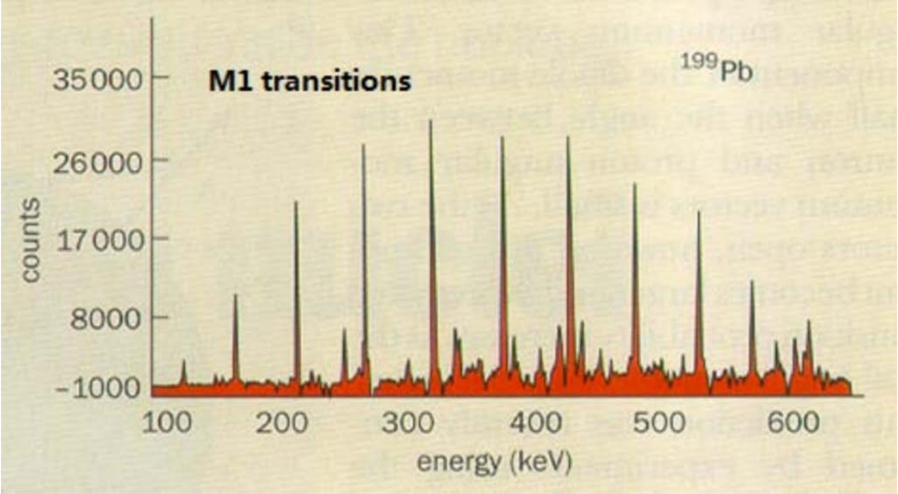


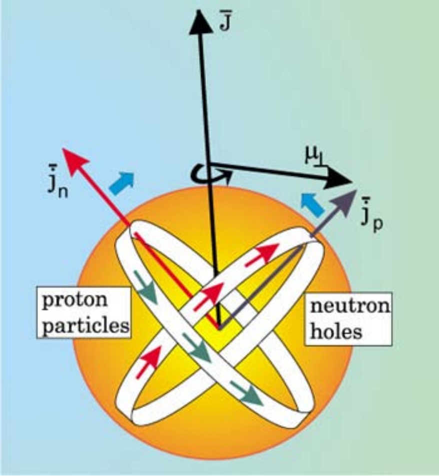


Twin PRL1986

ΔI = 1

M1 Transitions

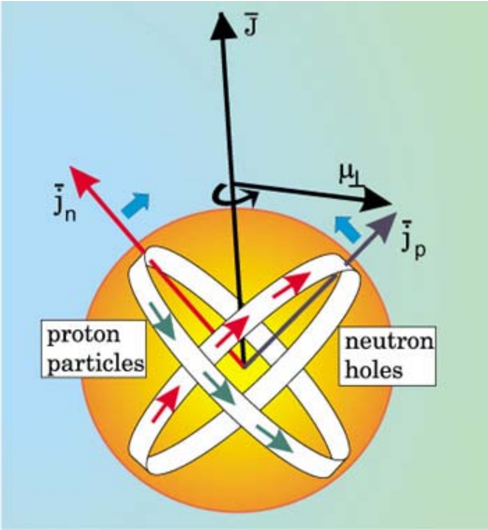


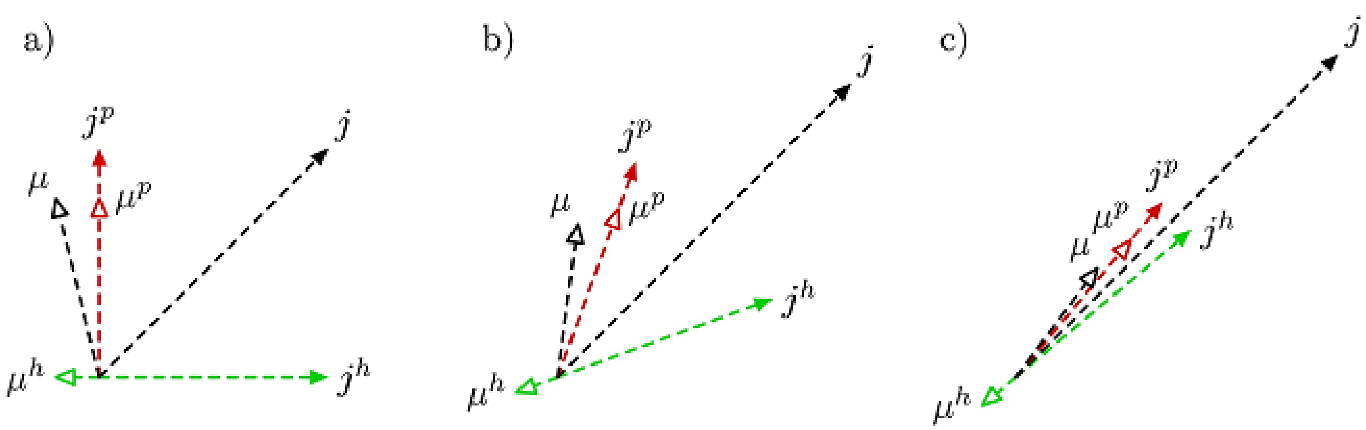


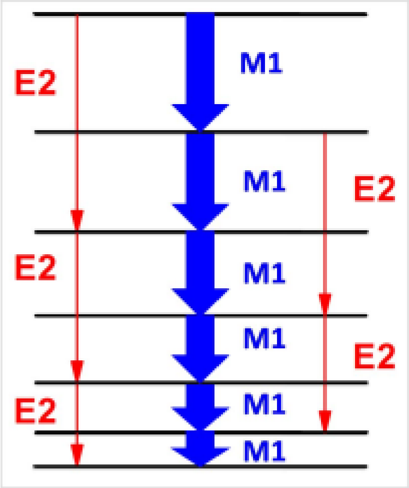
Hübel PPNP2005

Magnetic rotation

- ✓ near spherical or weakly deformed nuclei
- ✓ strong M1 and very weak E2 transitions
- ✓ rotational bands with $\Delta I = 1$
- ✓ shears mechanism





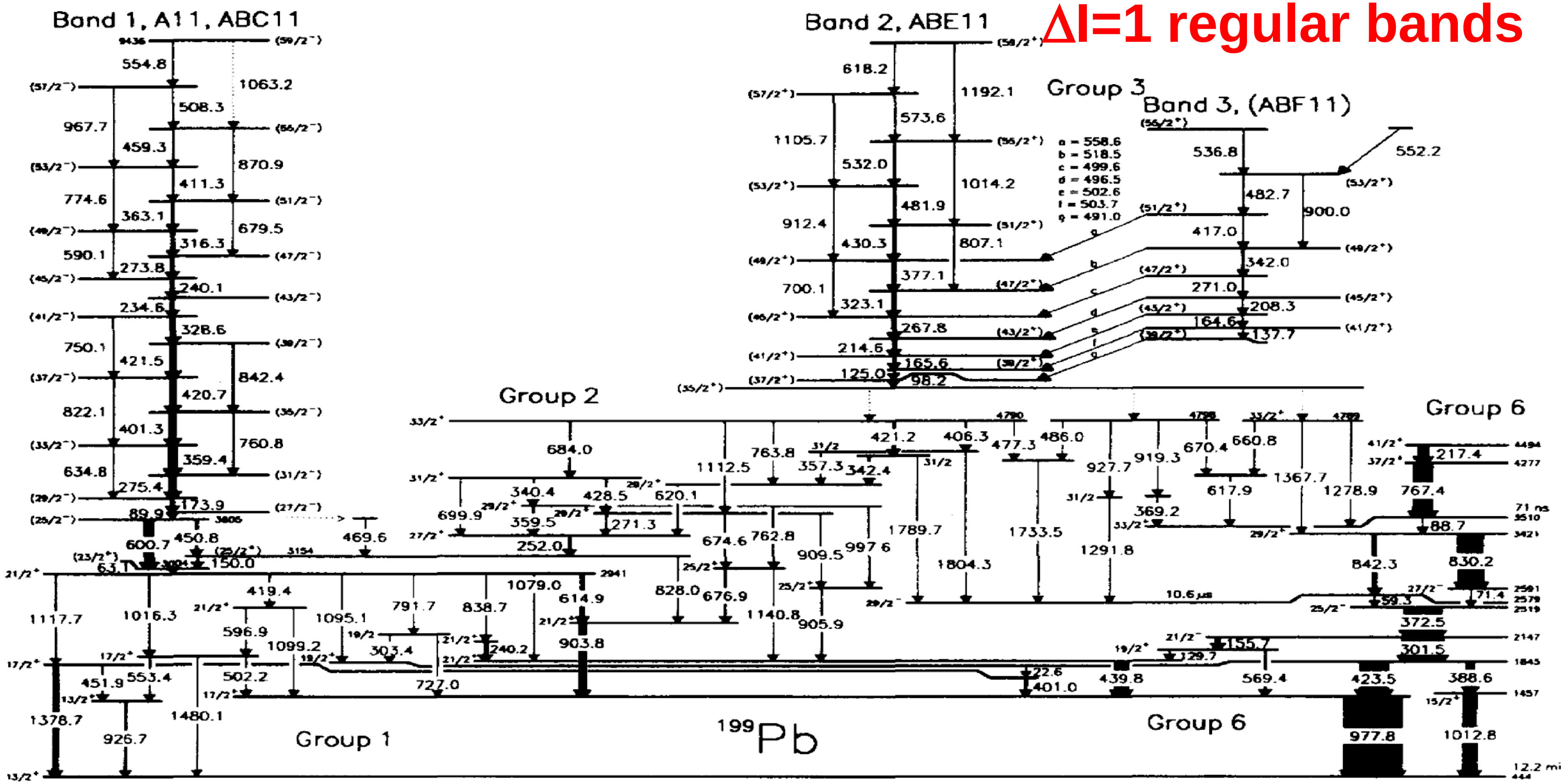


First attempt in verify MR

Nuclear Physics A 595 (1995) 499–512

Lifetimes of shears bands in ^{199}Pb

ΔI=1 regular bands



ΔI = 1 Enhanced magnetic dipole transition

508 *M. Neffgen et al. / Nuclear Physics A 595 (1995) 499–512*

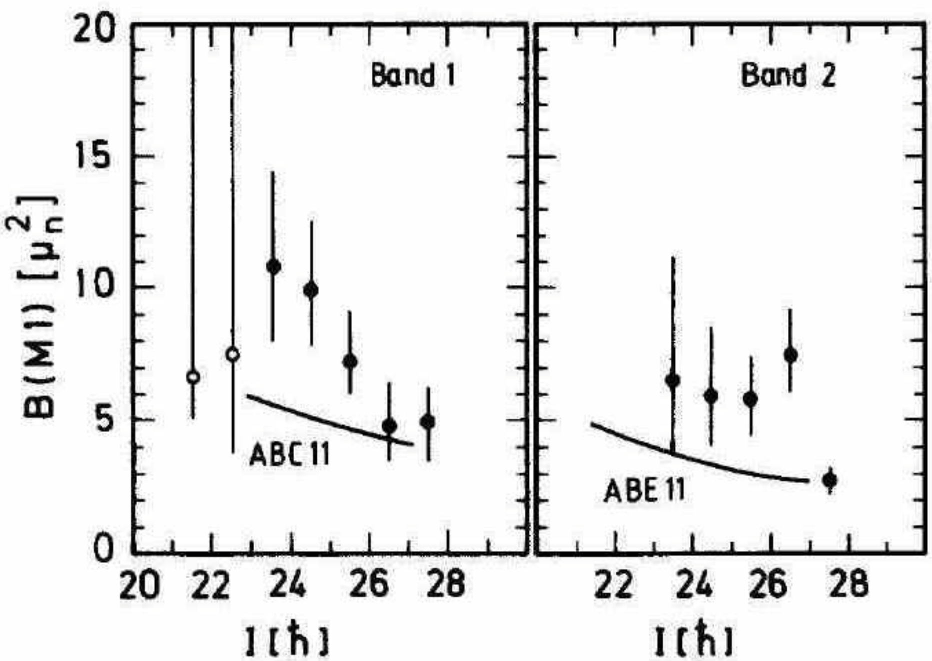


Fig. 5. Experimental (points) and calculated (lines) reduced magnetic dipole transition probabilities for bands 1 and 2 in ^{199}Pb as a function of spin. Open circles: transitions in the band-crossing region.

How does B(M1) change with spin I ?

Good agreement between TAC and PRM

Z. Phys. A 356, 263–279 (1996)

ZEITSCHRIFT

Interpretation and quality of the tilted axis cranking approximation

Stefan Frauendorf[1], Jie Meng[1,2,*]

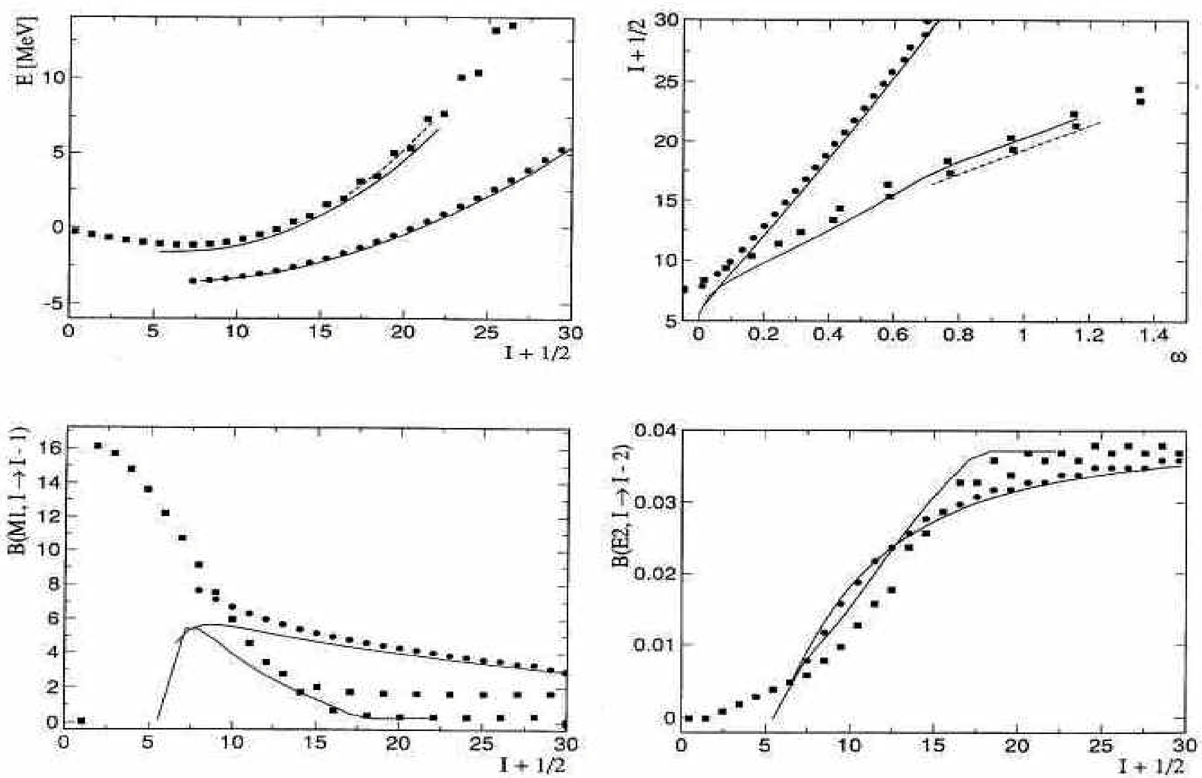
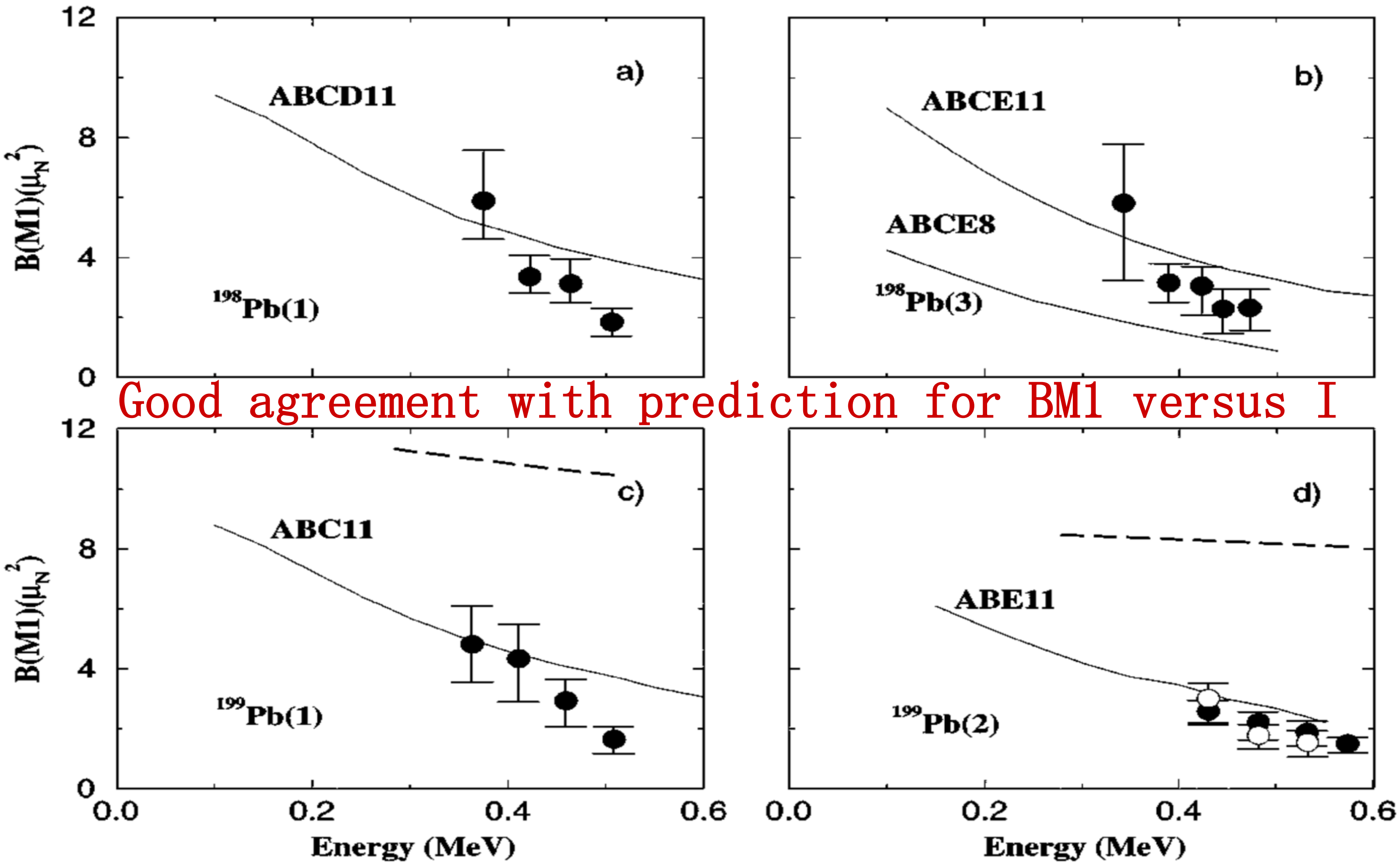


Fig. 8. Energy, angular momentum, $B(M1)$ and $B(E2)$ values for lowest band of the combination of a proton RAL hole with a neutron DAL hole. Circles: PRM $C = 0.25$ MeV, squares: PRM $C = 0.10$ MeV, full lines: TAC, dashed lines: PAC signature

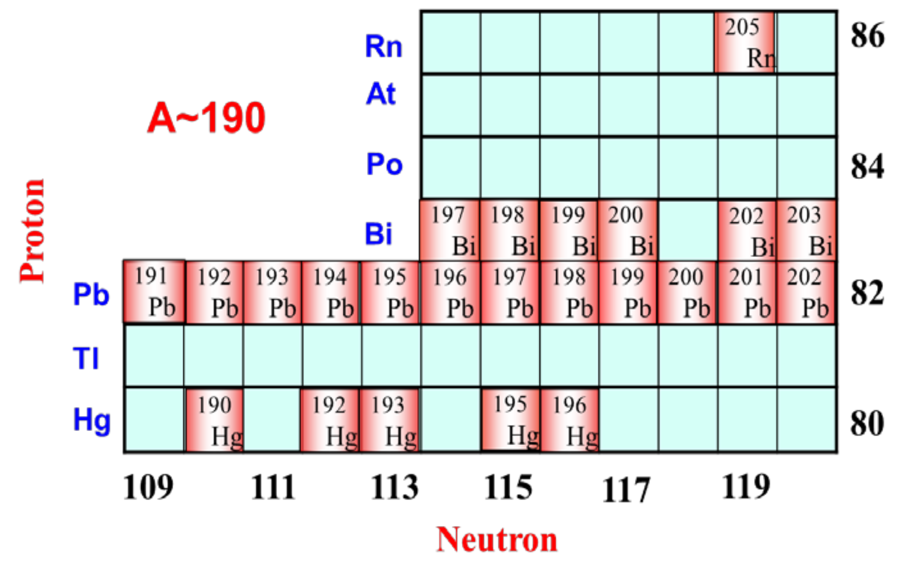
VOLUME 78, NUMBER 10 PHYSICAL REVIEW LETTERS 10 MARCH 1997

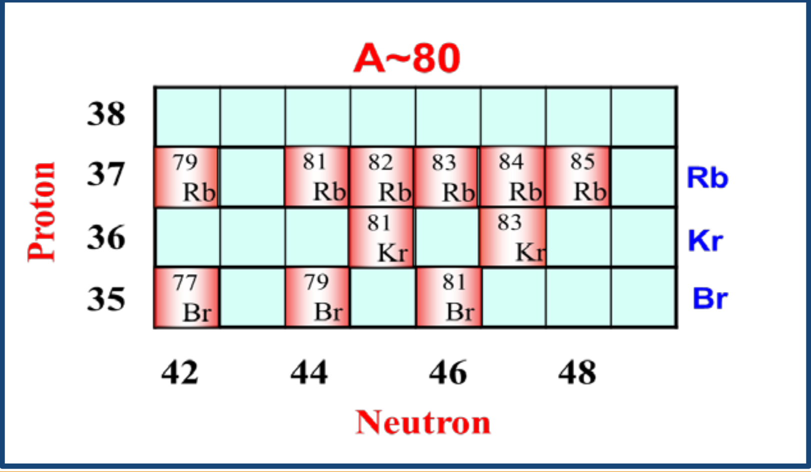
Evidence for "Magnetic Rotation" in Nuclei: Lifetimes of States in the M1 bands of ^{198,199}Pb

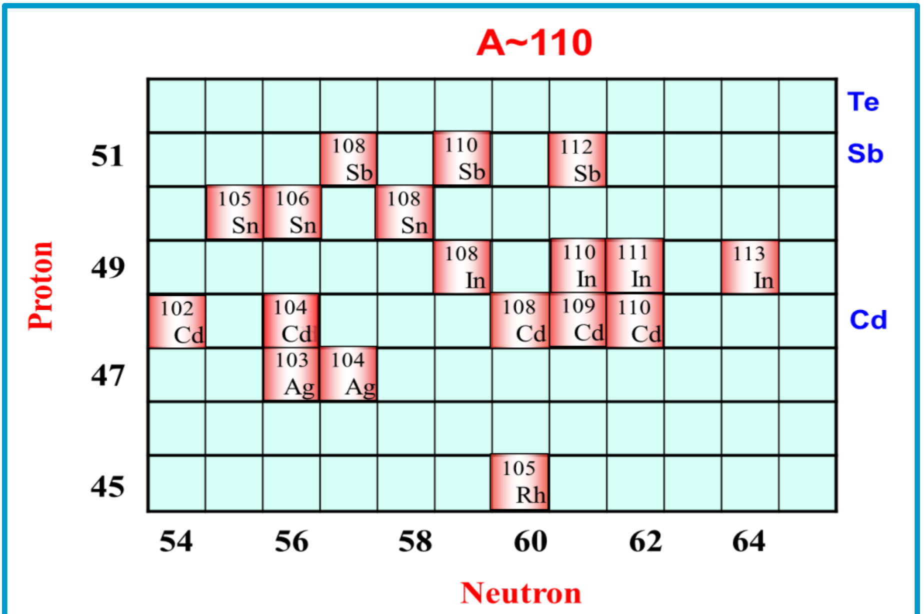


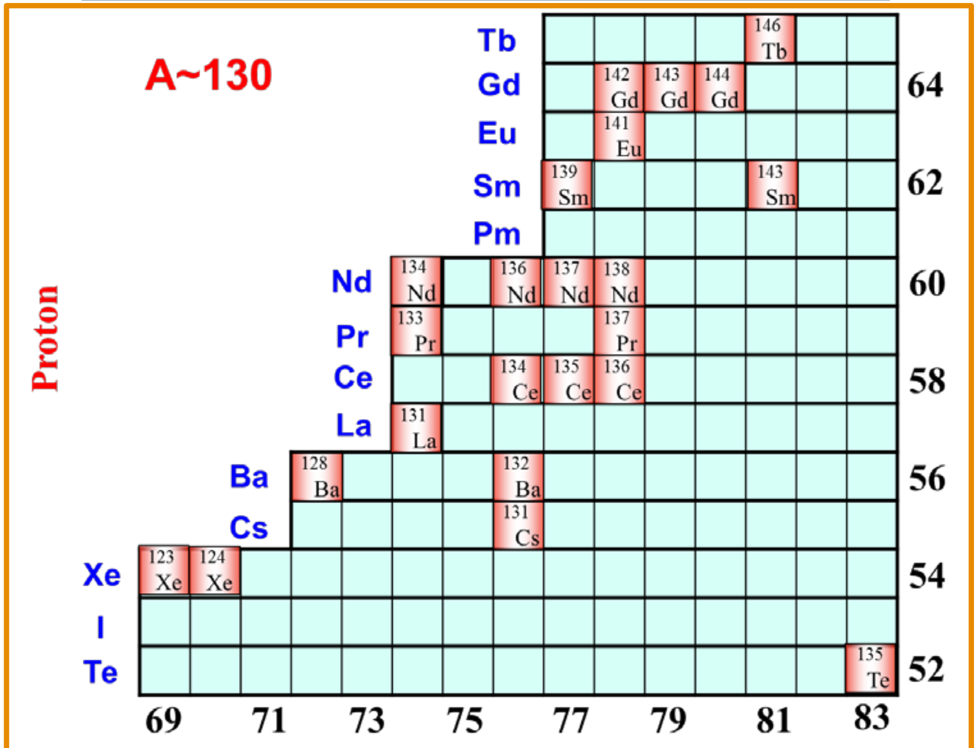
Good agreement with prediction for BM1 versus I

Experiment: MR



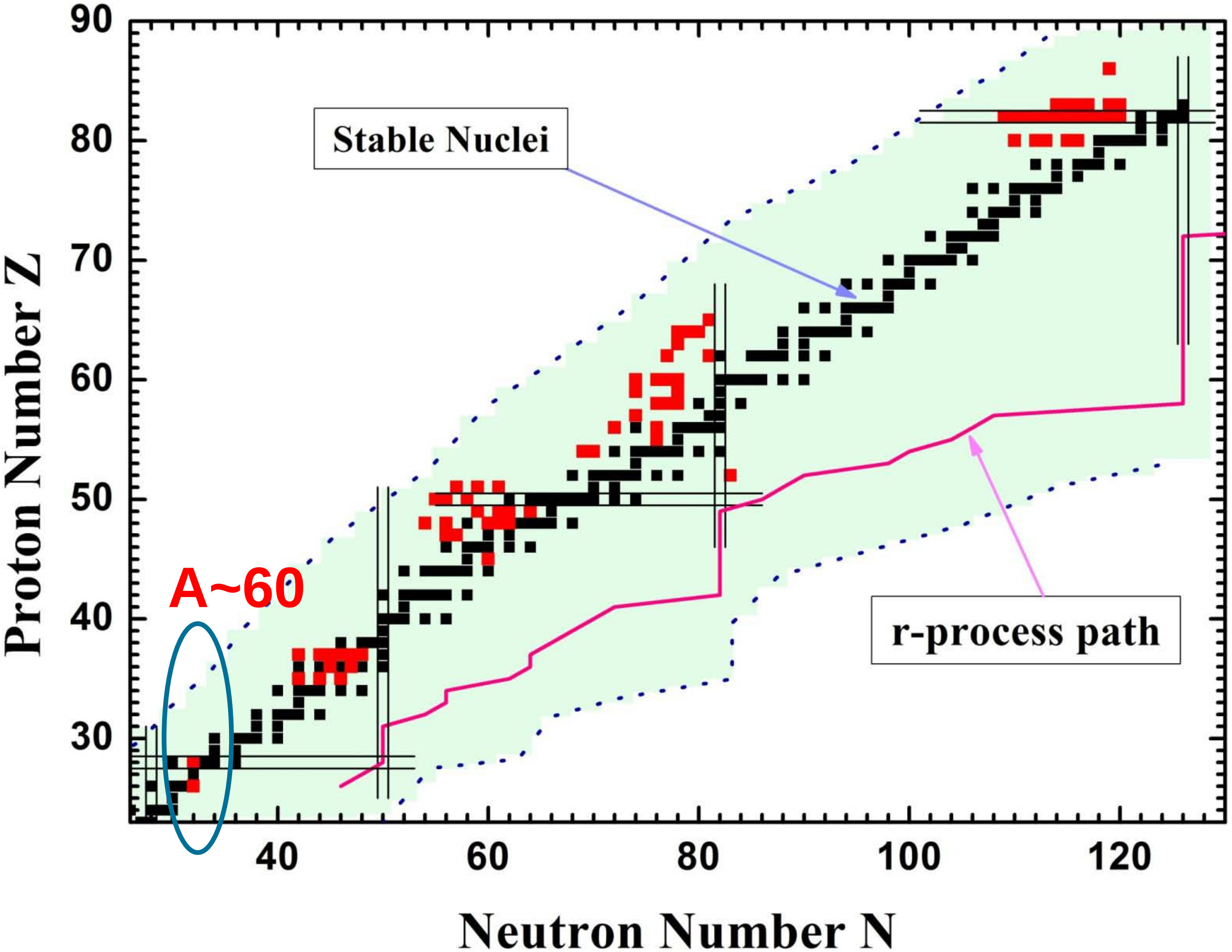






Experiment: MR

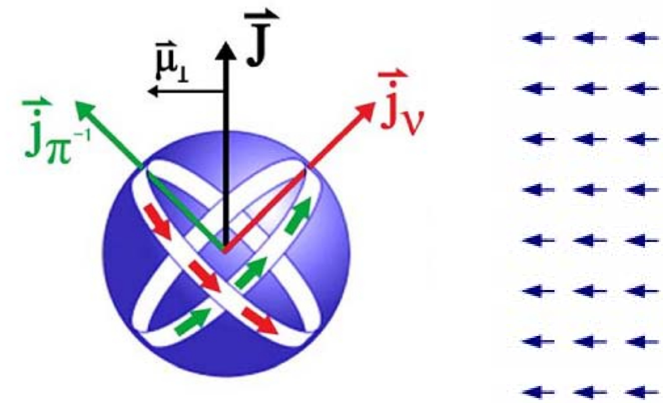
Magnetic rotation: 78 nuclei



Antimagnetic Rotation (AMR)

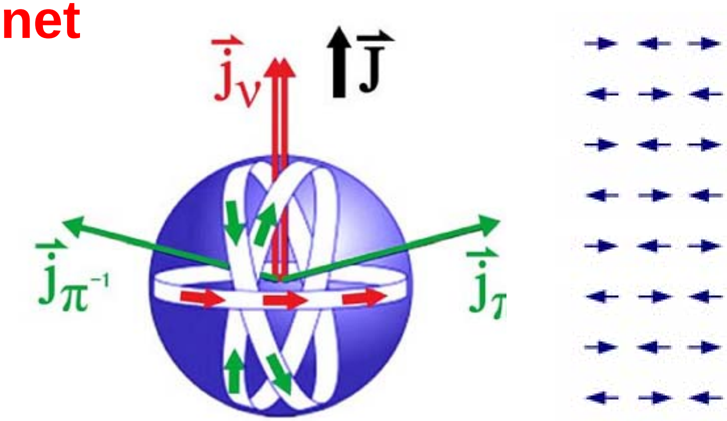
Magnetic rotation ⟺ Ferromagnet

- ✓ rotational bands with $\Delta I = 1$
- ✓ near spherical nuclei; weak E2 transitions
- ✓ strong M1 transitions
- ✓ B(M1) decrease with spin
- ✓ shears mechanism



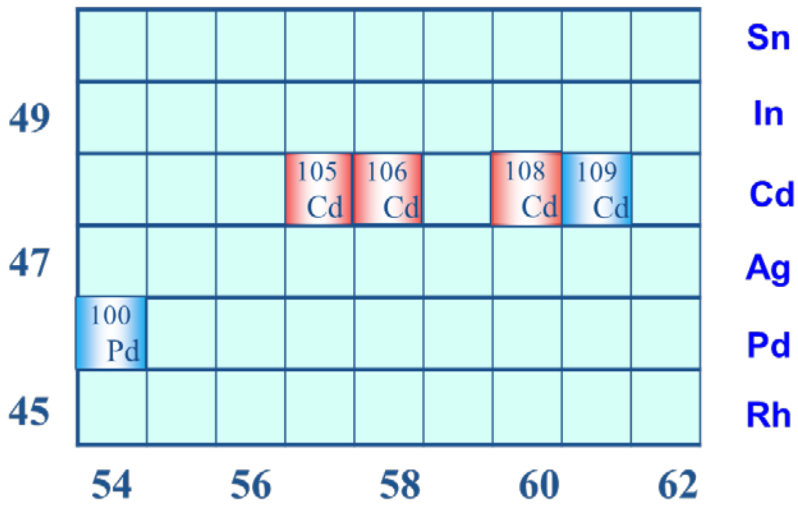
Antimagnetic rotation ⟺ Antiferromagnet

- ✓ rotational bands with $\Delta I = 2$
- ✓ near spherical nuclei; weak E2 transitions
- ✓ no M1 transitions
- ✓ B(E2) decrease with spin
- ✓ two “shears-like” mechanism



Experiment: AMR

Antimagnetic rotation: 3 nuclei

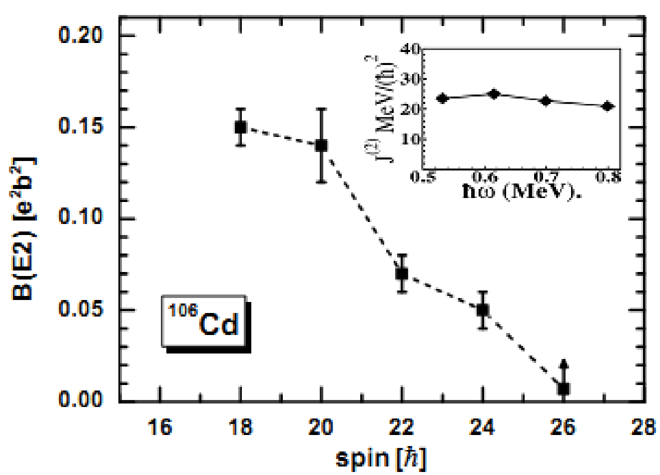


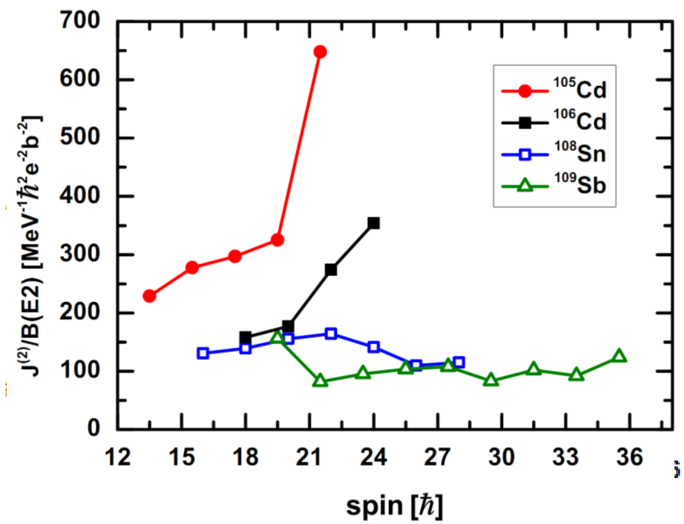
?

Other mass regions

Small B(E2)
Decrease tendency

Large $J^{(2)}/B(E2)$
Increase tendency





Simons PRL2003; Simons PRC2005

Theory

✓ **Semiclassical particle plus rotor model**

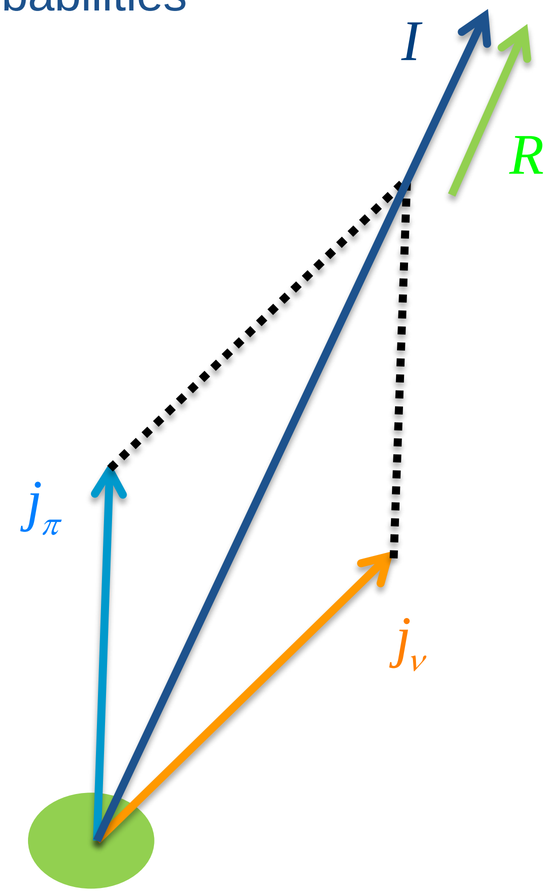
Clark ARNPS2000

simple geometry for the energies and transition probabilities

$$E(I)=\frac{(\vec{I}-\vec{j}_{\pi}-\vec{j}_{\nu})^{2}}{2\Im}+V_{2}P_{2}(\theta)$$

Core Rotor

Particle shears mechanism



Theory

✓ **Semiclassical particle plus rotor model**

Clark ARNPS2000

simple geometry for the energies and transition probabilities

✓ **Pairing-plus-quadrupole tilted axis cranking (TAC) model**

Frauendorf NPA1993; Frauendorf NPA2000

semi-phenomenological Hamiltonian

$$H' = H - \vec{\omega} \cdot \vec{J}$$

$$H = H_{sph} - \frac{\chi}{2} \sum_{\mu=-2}^{2} Q_\mu^+ Q_\mu - G P^+ P - \lambda N$$

Theory

- ✓ **Semiclassical particle plus rotor model**

  Clark ARNPS2000

  simple geometry for the energies and transition probabilities

- ✓ **Pairing-plus-quadrupole tilted axis cranking (TAC) model**

  Frauendorf NPA1993; Frauendorf NPA2000

  semi-phenomenological Hamiltonian

Phenomenological investigations

- ➢ polarization effects are neglected or only partially considered
- ➢ nuclear currents are treated without self-consistency
- ➢ adjusted to show MR/AMR in some way or another

A fully self-consistent microscopic investigation?

DFT: Cranking version

TAC based on Covariant Density Functional Theory

✓ **Meson exchange version:**

3-D Cranking: ***Madokoro, Meng, Matsuzaki, Yamaji, PRC 62, 061301 (2000)***

2-D Cranking: ***Peng, Meng, Ring, Zhang, PRC 78, 024313 (2008)***

✓ **Point coupling version:** Simple and more suitable for systematic investigations

2-D Cranking: ***Zhao, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)***

TAC based on Skyrme Density Functional Theory

3-D Cranking: ***Olbratowski, Dobaczewski, Dudek, Płóciennik, PRL 93, 052501(2004)***

2-D Cranking: ***Olbratowski, Dobaczewski, Dudek, Rzaca-Urban, Marcinkowska, Lieder, APPB 33, 389(2002)***

Fully self-consistent microscopic investigations

- fully taken into account polarization effects
- self-consistently treated the nuclear currents
- without any adjustable parameters for rotational excitations

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Density functional theory in nuclear physics

$$E[\hat{\rho}] = \langle \Psi | H | \Psi \rangle \approx \langle \Phi | H_{eff}(\rho) | \Phi \rangle$$

Skyrme
Gogny

$|\Phi\rangle$ Slater determinant $\Longleftrightarrow$ $\hat{\rho}$ density matrix

$$|\Phi\rangle = \mathcal{A}\{\varphi_1(\mathbf{r_1}) \dots \varphi_(\mathbf{r_A})\} \Longleftrightarrow \hat{\rho}(\mathbf{r}, \mathbf{r}') = \sum_{i=1}^{A} |\varphi_i(\mathbf{r})\rangle\langle\varphi_i(\mathbf{r}')|$$

Mean field:

$$\hat{h} = \frac{\delta E}{\delta \hat{\rho}}$$

Eigenfunctions:

$$\hat{h}|\varphi_i\rangle = \varepsilon_i |\varphi_i\rangle$$

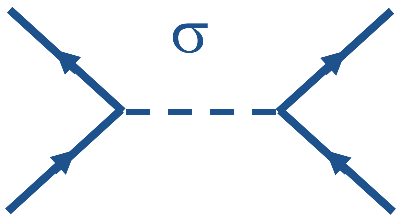
Interaction:

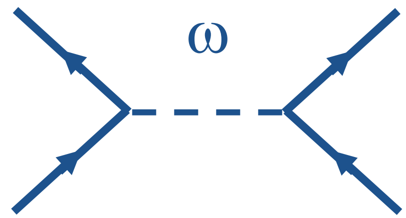
$$\hat{V} = \frac{\delta^2 E}{\delta \hat{\rho} \delta \hat{\rho}}$$

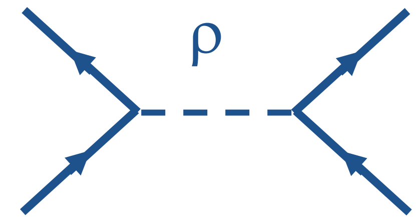
Extensions: Pairing correlations, Covariance
Relativistic Hartree Bogoliubov (RHB) theory

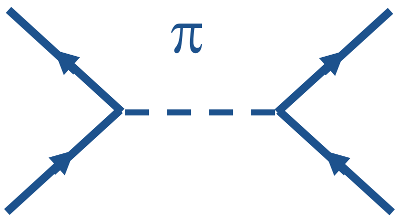
Starting point of CDFT

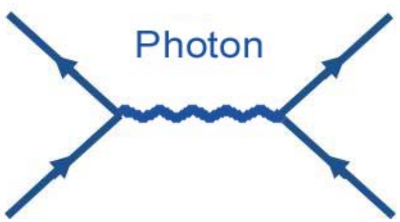
Nucleons are coupled by exchange of mesons via an effective Lagrangian with all relativistic symmetries, used in a mean field concept **and** no-sea approximation











meson	J^{π}	T
π	0^-	1
σ	0^+	0
ω	1^-	0
ρ	1^-	1

Brief introduction of CDFT

Lagrangian:

$$L=\bar{\psi}[i\gamma^{\mu}\partial_{\mu}-M-g_{\sigma}\sigma-\gamma^{\mu}(g_{\omega}\omega_{\mu}+g_{\rho}\vec{\tau}\bullet\vec{\rho}_{\mu}+e\tfrac{1-\tau_3}{2}A_{\mu})-\tfrac{f_{\pi}}{m_{\pi}}\gamma_5\gamma^{\mu}\partial_{\mu}\vec{\pi}\bullet\vec{\tau}]\psi$$

$$+\frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma-\frac{1}{2}m_{\sigma}^{2}\sigma^{2}-\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu}+\frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu}-\frac{1}{4}\vec{R}_{\mu\nu}\bullet\vec{R}^{\mu\nu}$$

$$+\frac{1}{2}m_{\rho}^{2}\vec{\rho}^{\mu}\cdot\vec{\rho}_{\mu}+\frac{1}{2}\partial_{\mu}\vec{\pi}\bullet\partial^{\mu}\vec{\pi}-\frac{1}{2}m_{\pi}^{2}\vec{\pi}\bullet\vec{\pi}-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$\Omega^{\mu\nu}=\partial^{\mu}\omega^{\nu}-\partial^{\nu}\omega^{\mu}$$

$$\vec{R}^{\mu\nu}=\partial^{\mu}\vec{\rho}^{\nu}-\partial^{\nu}\vec{\rho}^{\mu}$$

$$F^{\mu\nu}=\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu}$$

Hamiltonian:

$$H=\bar{\psi}(-i\boldsymbol{\gamma}\bullet\nabla+M)\psi+\frac{1}{2}\int d^{4}y\sum_{i=\sigma,\omega,\rho,\pi,A}\bar{\psi}(x)\bar{\psi}(y)\Gamma_{i}D_{i}(x,y)\psi(y)\psi(x)$$

$$=T+V$$

$$\Gamma_{\sigma}(1,2)\equiv-g_{\sigma}(1)g_{\sigma}(2),\qquad \Gamma_{\rho}(1,2)\equiv+(g_{\rho}\gamma_{\mu}\vec{\tau})_{1}\cdot(g_{\rho}\gamma^{\mu}\vec{\tau})_{2},$$

$$\Gamma_{\omega}(1,2)\equiv+(g_{\omega}\gamma_{\mu})_{1}(g_{\omega}\gamma_{\mu})_{2},\ \Gamma_{\pi}(1,2)\equiv-(\tfrac{f_{\pi}}{m_{\pi}}\vec{\tau}\gamma_{5}\gamma_{\mu}\partial^{\mu})_{1}\cdot(\tfrac{f_{\pi}}{m_{\pi}}\vec{\tau}\gamma_{5}\gamma_{\nu}\partial^{\nu})_{2}$$

$$\Gamma_{\text{em}}(1,2)\equiv+\frac{e^{2}}{4}(\gamma_{\mu}(1-\tau_{3}))_{1}(\gamma^{\mu}(1-\tau_{3}))_{2}$$

Brief introduction of CDFT

$$H = T + \sum_{i=\sigma,\omega,\rho,\pi,A} V_i$$

$$\psi(x) = \sum_i [f_i(\mathbf{x})e^{-i\varepsilon_i t}c_i + g_i(\mathbf{x})e^{i\varepsilon_i' t}d_i^\dagger]$$

$$\psi^\dagger(x) = \sum_i [f_i^\dagger(\mathbf{x})e^{i\varepsilon_i t}c_i^\dagger + g_i^\dagger(\mathbf{x})e^{-i\varepsilon_i' t}d_i\,]$$

$$T = \int d\mathbf{x} \sum_{\alpha\beta} \bar{f}_\alpha(-i\gamma\cdot\nabla + M)f_\beta c_\alpha^\dagger c_\beta,$$

Hartree

$$V_i = \frac{1}{2}\int d\mathbf{x}_1 d\mathbf{x}_2 \sum_{\alpha\beta;\alpha'\beta'} c_\alpha^\dagger c_\beta^\dagger c_{\beta'} c_{\alpha'} \bar{f}_\alpha(1)\bar{f}_\beta(2)\Gamma_i(1,2)D_i(1,2)f_{\beta'}(2)f_{\alpha'}(1)$$

Fock

Energy density functional:

$$|\Phi_0\rangle = \prod_\alpha c_\alpha^\dagger |0\rangle$$

$$E = \langle\Phi_0|H|\Phi_0\rangle = \langle\Phi_0|T|\Phi_0\rangle + \sum_{i=\sigma,\omega,\rho,\pi,A}\langle\Phi_0|V_i|\Phi_0\rangle$$

$$= E_k + E_\sigma^D + E_\sigma^E + E_\omega^D + E_\omega^E + E_\rho^D + E_\rho^E + E_\pi + E_{\rm em}^D + E_{\rm em}^E$$

Equations of motion in RMF theory

For system with time invariance:

$$\left[\alpha\cdot\boldsymbol{p}+V(\boldsymbol{r})+\beta\left(M+S(\boldsymbol{r})\right)\right]\psi_i=\varepsilon_i\psi_i$$

$$\begin{cases} V(\boldsymbol{r})=g_\omega\omega(\boldsymbol{r})+g_\rho\tau_3\rho(\boldsymbol{r})+e\dfrac{1-\tau_3}{2}A(\boldsymbol{r}) \\ S(r)=g_\sigma\sigma(\boldsymbol{r}) \end{cases}$$

Same footing for

- Deformation
- Rotation
- Pairing (RHB,BCS,SLAP)
- …

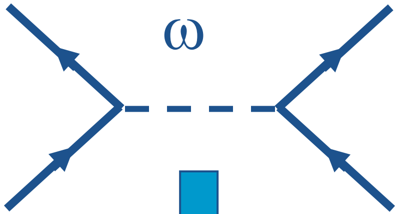
$$\left[-\Delta+m_\sigma^2\right]\sigma=-g_\sigma\rho_s-g_2\sigma^2-g_3\sigma^3$$

$$\left[-\Delta+m_\omega^2\right]\omega=g_\omega\rho_b-c_3\omega^3$$

$$\left[-\Delta+m_\rho^2\right]\rho=g_\rho\left[\rho_b^{(n)}-\rho_b^{(p)}\right]-d_3\rho^3$$

$$\begin{cases} \rho_s(\boldsymbol{r})=\sum_{i=1}^A\bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_v(\boldsymbol{r})=\sum_{i=1}^A\psi_i^+(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_3(\boldsymbol{r})=\sum_{i=1}^A\psi_i^+(\boldsymbol{r})\tau_3\psi_i(\boldsymbol{r}) \\ \rho_c(\boldsymbol{r})=\sum_{i=1}^A\psi_i^+(\boldsymbol{r})\dfrac{1-\tau_3}{2}\psi_i(\boldsymbol{r}) \end{cases}$$

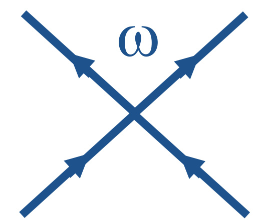
RMF theory with Point-Coupling interaction



$$H=\bar{\psi}_i\left(-i\gamma\bullet\nabla+M\right)\psi_i+\frac{1}{4}F^{i\nu}F_{i\nu}$$
$$+\frac{1}{2}((\nabla\sigma)^2+m_\sigma^2\sigma^2)+g_\sigma\sigma\rho_s+\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4$$
$$+\frac{1}{2}g_\omega\omega_0\rho_v+\frac{1}{2}g_\rho\vec{\rho}_0\rho_3$$

$$g_\omega\omega=\frac{1}{1-\Delta/m_\omega^2}\frac{g_\omega^2}{m_\omega^2}\rho_v=\frac{g_\omega^2}{m_\omega^2}\rho_v+\frac{g_\omega^2}{m_\omega^4}\Delta\rho_v+\cdots\approx\alpha_v\rho_v+\delta_v\Delta\rho_v$$

$$H=\bar{\psi}_i\left(-i\gamma\Box\nabla+M\right)\psi_i+\frac{1}{4}F^{i\nu}F_{i\nu}$$
$$+\frac{1}{2}\alpha_S\rho_S^2+\frac{1}{2}\delta_S\rho_S\Delta\rho_S+\frac{1}{3}\beta_S\rho_S^3+\frac{1}{4}\gamma_S\rho_S^4$$
$$+\frac{1}{2}\alpha_V\rho_V^2+\frac{1}{2}\delta_V\rho_V\Delta\rho_V+\frac{1}{2}\alpha_{TV}\rho_{V}^2+\frac{1}{2}\delta_{TV}\rho_{TV}\Delta\rho_{TV}$$



Equations of motion in RMF-PC theory

For system with time invariance:

$$\left[\alpha\cdot\boldsymbol{p}+V(\boldsymbol{r})+\beta\left(M+S(\boldsymbol{r})\right)\right]\psi_i=\varepsilon_i\psi_i$$

$$\begin{cases} V(\boldsymbol{r})=\alpha_V\rho_V(\boldsymbol{r})+\gamma_V\rho_V^3(\boldsymbol{r})+\delta_V\Delta\rho_V(\boldsymbol{r})+\tau_3\alpha_{TV}\rho_{TV}(\boldsymbol{r})+\tau_3\delta_{TV}\Delta\rho_{TV}(\boldsymbol{r})+e\dfrac{1-\tau_3}{2}A(\boldsymbol{r}) \\ S(r)=\alpha_S\rho_S+\beta_S\rho_S^2+\gamma_S\rho_S^3+\delta_S\Delta\rho_S \end{cases}$$

$$\begin{cases} \rho_s(\boldsymbol{r})=\sum_{i=1}^{A}\bar{\psi}_i(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_v(\boldsymbol{r})=\sum_{i=1}^{A}\psi_i^{+}(\boldsymbol{r})\psi_i(\boldsymbol{r}) \\ \rho_3(\boldsymbol{r})=\sum_{i=1}^{A}\psi_i^{+}(\boldsymbol{r})\tau_3\psi_i(\boldsymbol{r}) \\ \rho_c(\boldsymbol{r})=\sum_{i=1}^{A}\psi_i^{+}(\boldsymbol{r})\dfrac{1-\tau_3}{2}\psi_i(\boldsymbol{r}) \end{cases}$$

Without Klein-Gordon equation

Tilted axis cranking CDFT

General Lagrangian density

$$
\begin{aligned}
L =& \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - m)\psi \\
&-\frac{1}{2}\alpha_{S}(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_{V}(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi) - \frac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)(\bar{\psi}\vec{\tau}\gamma^{\mu}\psi) \\
&-\frac{1}{3}\beta_{S}(\bar{\psi}\psi)^{3} - \frac{1}{4}\gamma_{S}(\bar{\psi}\psi)^{4} - \frac{1}{4}\gamma_{V}[(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)]^{2} \\
&-\frac{1}{2}\delta_{S}\partial_{\nu}(\bar{\psi}\psi)\partial^{\nu}(\bar{\psi}\psi) - \frac{1}{2}\delta_{V}\partial_{\nu}(\bar{\psi}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\gamma^{\mu}\psi) - \frac{1}{2}\delta_{TV}\partial_{\nu}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi)\partial^{\nu}(\bar{\psi}\vec{\tau}\gamma_{\mu}\psi) \\
&-e\frac{1-\tau_{3}}{2}\bar{\psi}\gamma^{\mu}\psi A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}
\end{aligned}
$$

Transformed to the frame rotating with the uniform velocity

$$
\Omega = (\Omega_x, 0, \Omega_z) = (\Omega\cos\theta_{\Omega}, 0, \Omega\sin\theta_{\Omega})
$$

$$
x^{\alpha} = \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} \rightarrow \tilde{x}^{\alpha} = \begin{pmatrix} \tilde{t} \\ \tilde{\mathbf{x}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{R} \end{pmatrix} \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix}
$$

Koepf NPA1989; Kaneko PLB1993; Madokoro PRC1997

TAC RMF:equations of motion

Dirac Equation

$$\left[\alpha\cdot(-i\nabla-\vec{V}(\boldsymbol{r}))+\beta\left(M+S(\boldsymbol{r})\right)+V(\boldsymbol{r})-\Omega\cdot J\right]\psi_i=\varepsilon_i\psi_i$$

Potential

$$\begin{cases} S(r)=\alpha_S\rho_S+\beta_S\rho_S^2+\gamma_S\rho_S^3+\delta_S\Delta\rho_S \\ V^\mu(\boldsymbol{r})=\alpha_V j_V^\mu(\boldsymbol{r})+\gamma_V(j_V^\mu)^3(\boldsymbol{r})+\delta_V\Delta j_V^\mu(\boldsymbol{r})+\tau_3\alpha_{TV}j_{TV}^\mu(\boldsymbol{r})+\tau_3\delta_{TV}\Delta j_{TV}^\mu(\boldsymbol{r})+e\dfrac{1-\tau_3}{2}A^\mu(\boldsymbol{r}) \end{cases}$$

Spatial components of vector field are involved due to the time-reversal invariance broken

Observables

Binding energy

$$E_{\text{tot}} = \sum_{k=1}^{A} \epsilon_k - \int d^3 r \left\{ \frac{1}{2}\alpha_S \rho_S^2 + \frac{1}{2}\alpha_V j_V^\mu (j_V)_\mu + \frac{1}{2}\alpha_{TV} j_{TV}^\mu (j_{TV})_\mu \right.$$
$$+ \frac{2}{3}\beta_S \rho_S^3 + \frac{3}{4}\gamma_S \rho_S^4 + \frac{3}{4}\gamma_V (j_V^\mu (j_V)_\mu)^2 + \frac{1}{2}\delta_S \rho_S \Delta \rho_S + \frac{1}{2}\delta_V (j_V)_\mu \Delta j_V^\mu$$
$$\left. + \frac{1}{2}\delta_{TV} j_{TV}^\mu \Delta (j_{TV})_\mu + \frac{1}{2} e j_p^0 A_0 \right\} + \sum_{k=1}^{A} \langle k | \Omega J | k \rangle$$

Angular momentum

$$\langle \hat{J} \rangle^2 = I(I+1)$$

Quadrupole moments and magnetic moments

$$Q_{20} = \sqrt{\frac{5}{16\pi}} \langle 3z^2 - r^2 \rangle, \quad Q_{22} = \sqrt{\frac{15}{32\pi}} \langle x^2 - y^2 \rangle$$

$$\mu = \sum_i^A \int d^3 r [\frac{mc^2}{\hbar c} Q_i \psi_i^{?}(r) r \times \alpha \psi_i(r) + \kappa_i \psi_i^\dagger(r) \beta \Sigma \psi_i(r)]$$

Where $Q_p = 1, \quad Q_n = 0, \quad \kappa_p = 1.793, \quad \kappa_n = -1.913$

Observables

Binding energy

$$E_{\text{tot}} = \sum_{k=1}^{A} \epsilon_k - \int d^3 r \left\{ \frac{1}{2}\alpha_S \rho_S^2 + \frac{1}{2}\alpha_V j_V^\mu (j_V)_\mu + \frac{1}{2}\alpha_{TV} j_{TV}^\mu (j_{TV})_\mu \right.$$
$$+ \frac{2}{3}\beta_S \rho_S^3 + \frac{3}{4}\gamma_S \rho_S^4 + \frac{3}{4}\gamma_V (j_V^\mu (j_V)_\mu)^2 + \frac{1}{2}\delta_S \rho_S \Delta \rho_S + \frac{1}{2}\delta_V (j_V)_\mu \Delta j_V^\mu$$
$$\left. + \frac{1}{2}\delta_{TV} j_{TV}^\mu \Delta (j_{TV})_\mu + \frac{1}{2} e j_p^0 A_0 \right\} + \sum_{k=1}^{A} \langle k | \Omega J | k \rangle$$

Angular momentum

$$\langle \hat{J} \rangle^2 = I(I+1)$$

Quadrupole moments and magnetic moments

B(M1) and B(E2) transition probabilites

$$B(M1) = \frac{3}{8\pi}\mu_\perp^2 = \frac{3}{8\pi}(\mu_x \sin\theta_J - \mu_z \cos\theta_J)^2 \qquad B(E2) = \frac{3}{8}\left[Q_{20}\cos^2\theta_J + \sqrt{\frac{2}{3}} Q_{22}(1+\sin^2\theta_J) \right]^2$$

RMF parameterizations

Meson Exchange

Nonlinear parameterizations:

$$M, m_\sigma, m_\omega, m_\rho, g_\sigma, g_\omega, g_\rho, g_2, g_3, c_3, d_3$$

NL3, NLSH, TM1, TM2, PK1, …

Density dependent parameterizations:

$$M, m_\sigma, m_\omega, m_\rho, g_\sigma(\rho), g_\omega(\rho), g_\rho(\rho)$$

TW99, DD-ME1, DD-ME2, PKDD, …

Point Coupling

Nonlinear parameterizations:

$$M, \alpha_S, \alpha_V, \alpha_{TV}, \delta_S, \delta_V, \delta_{TV}, \beta_S, \gamma_S, \gamma_V$$

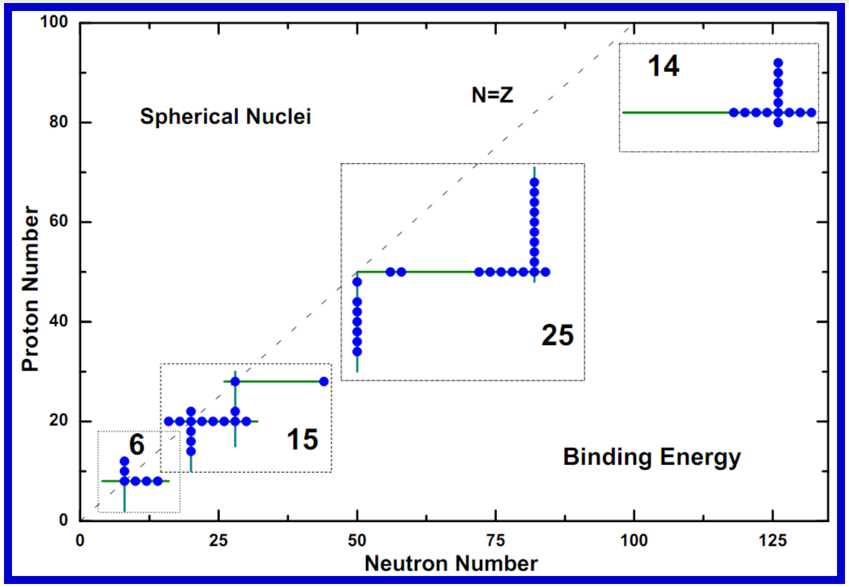
PC-LA, PC-F1, PC-PK1 …

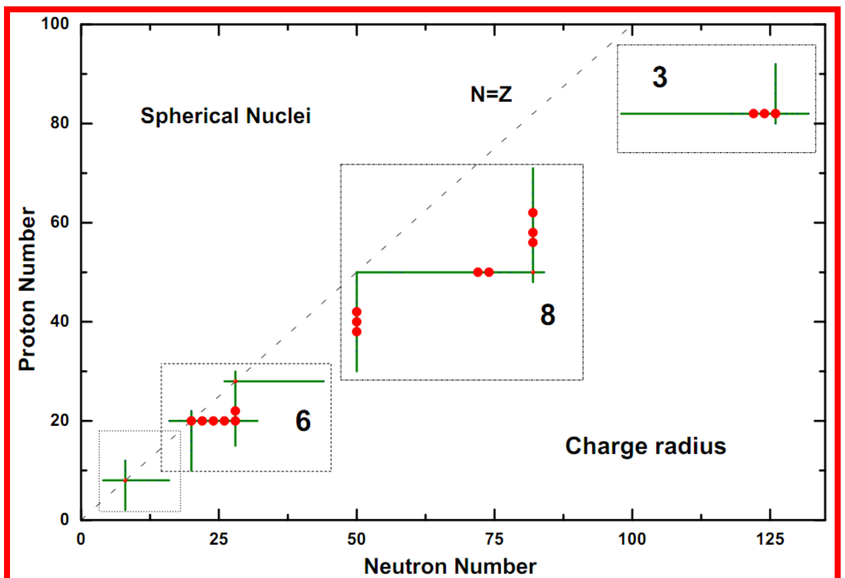
Density dependent parameterizations:

$$M, \delta_S, \alpha_S(\rho), \alpha_V(\rho), \alpha_{TV}(\rho)$$

DD-PC1, …

Parameterizations: PC-PK1



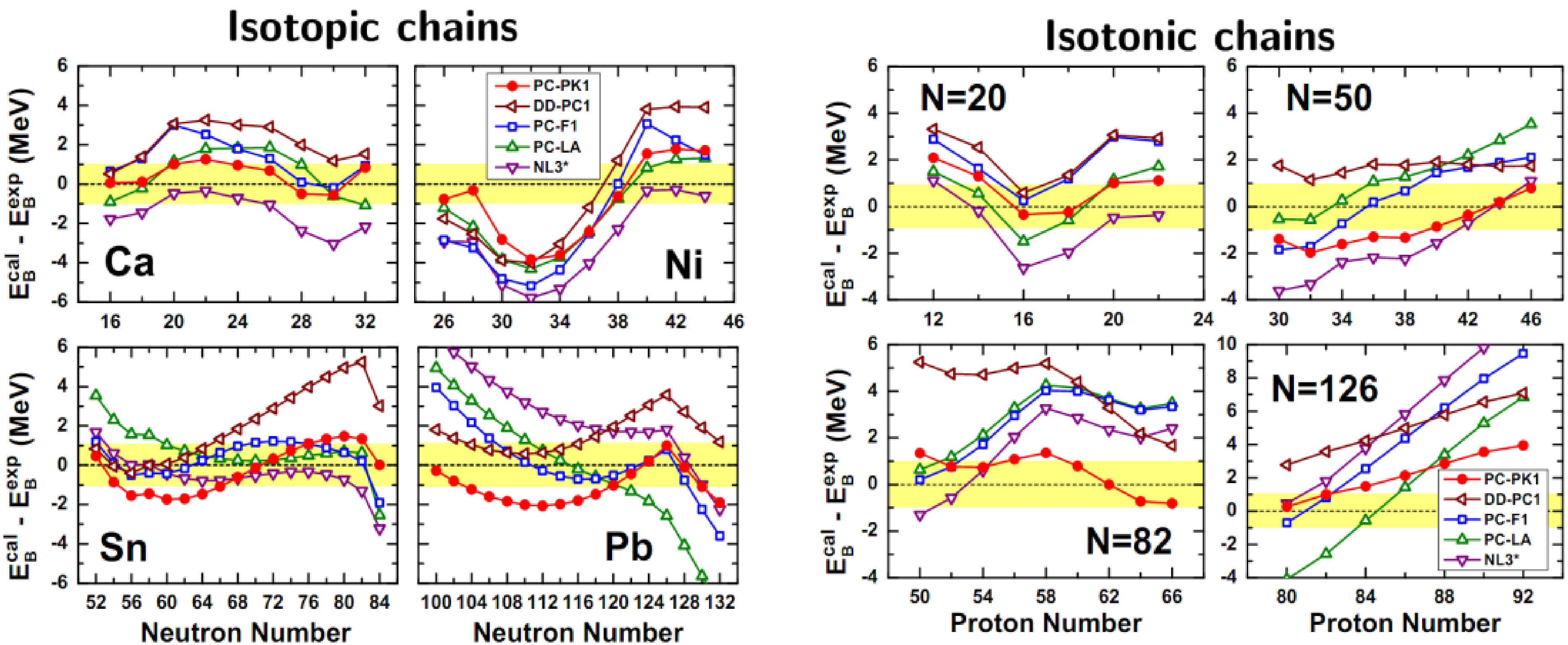


Coupl.	Cons.	PC-PK1	Dimension
α_S	$[10^{-4}]$	-3.96291	MeV^{-2}
β_S	$[10^{-11}]$	8.66530	MeV^{-5}
γ_S	$[10^{-17}]$	-3.80724	MeV^{-8}
δ_S	$[10^{-10}]$	-1.09108	MeV^{-4}
α_V	$[10^{-4}]$	2.69040	MeV^{-2}
γ_V	$[10^{-18}]$	-3.64219	MeV^{-8}
δ_V	$[10^{-10}]$	-4.32619	MeV^{-4}
α_{TV}	$[10^{-5}]$	2.95018	MeV^{-2}
δ_{TV}	$[10^{-10}]$	-4.11112	MeV^{-4}
V_n	$[10^{0}]$	-349.5	$\mathrm{MeV\ fm}^{3}$
V_p	$[10^{0}]$	-330	$\mathrm{MeV\ fm}^{3}$

rms error	PC-PK1	PC-F1	PC-LA	Remarks
σ_{BE}	1.25	2.60	3.46	60 Nuclei
σ_{rc}	0.016	0.017	0.023	17 Nuclei
σ_{BE}	1.39	2.36	4.51	16 Nuclei
σ_{rc}	0.017	0.012	0.02	11 Nuclei

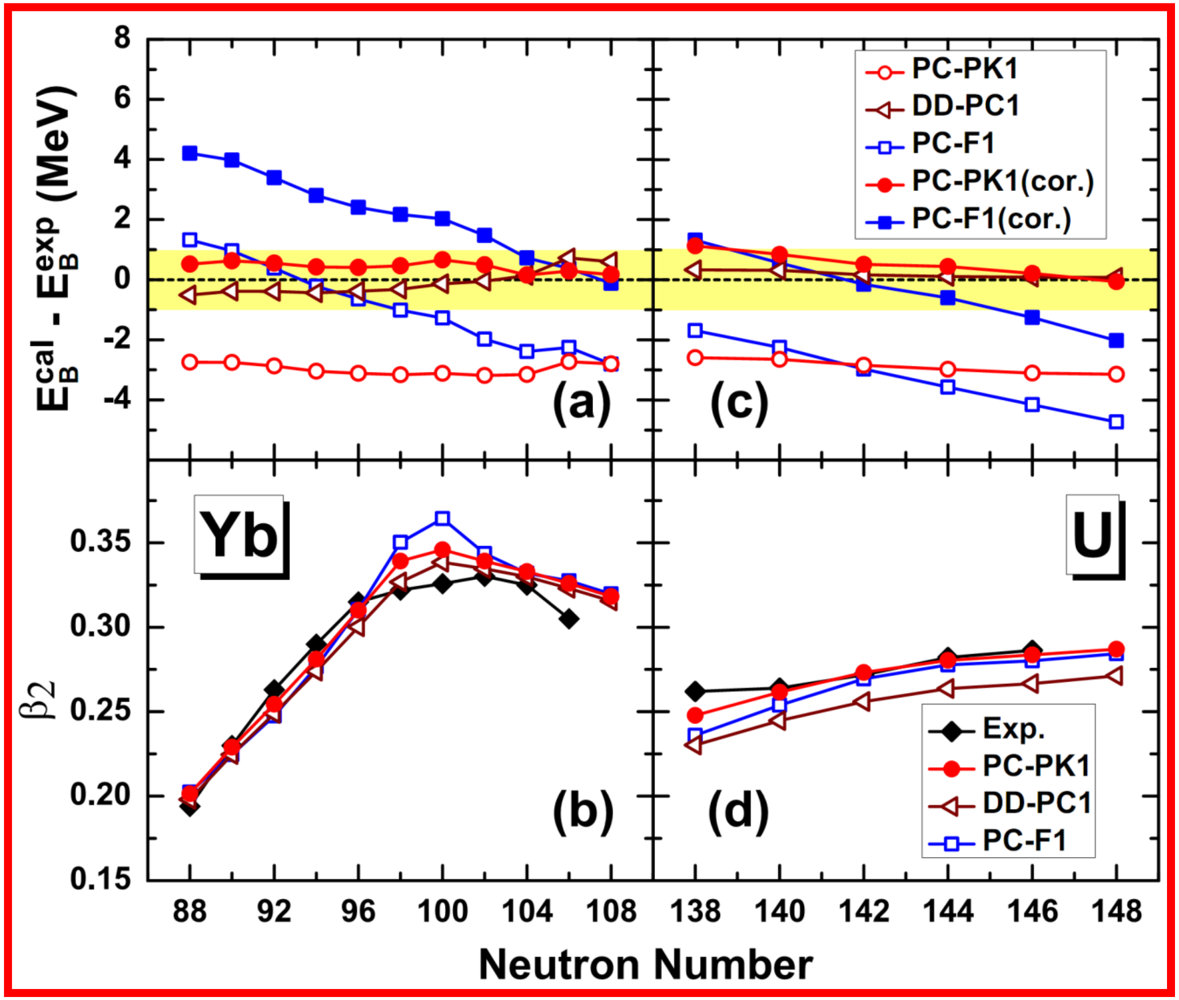
Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)

Spherical nuclei



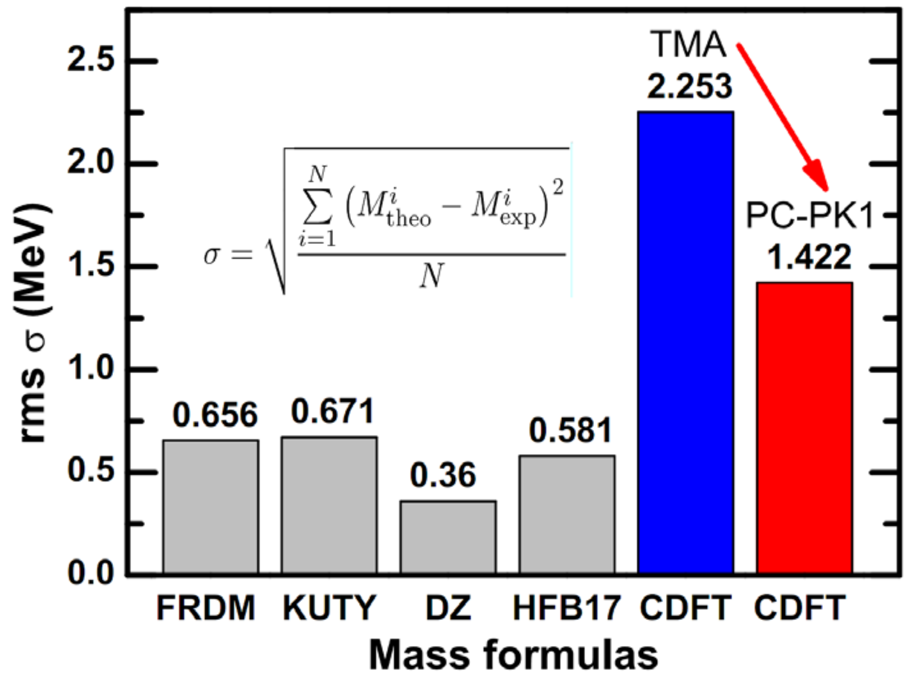
Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)

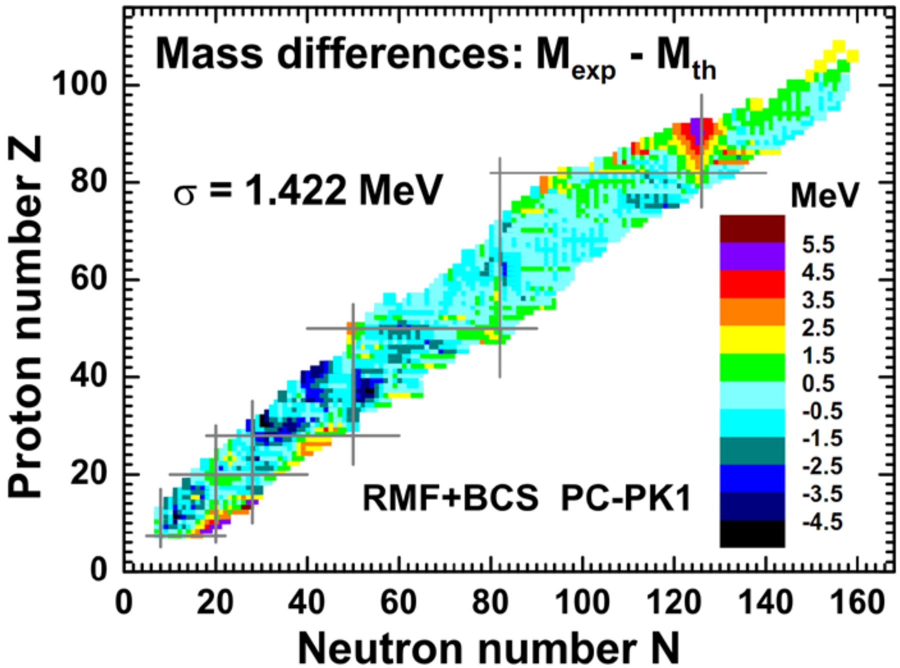
Deformed nuclei



Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)

Nuclear Mass





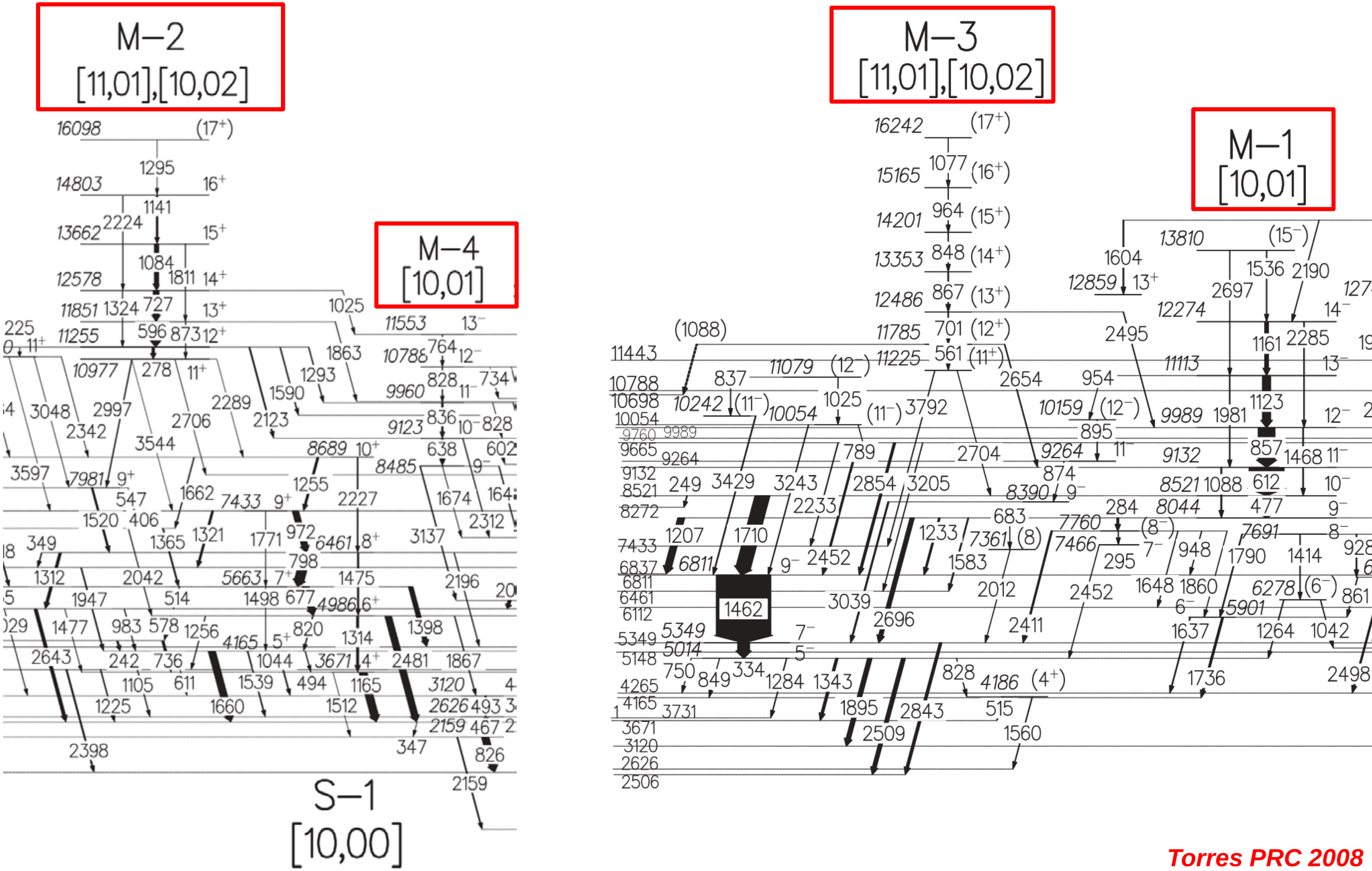
Exp value for 2149 nuclei from Audi et al. NPA2003

Outline

- Introduction
- Theoretical framework
- Magnetic rotation
- Antimagnetic rotation
- Summary & Perspectives

MR: ^{60}Ni

lightest nucleus with magnetic rotation



Torres PRC 2008

MR: ^{60}Ni

- **Harmonic oscillator shells: Nf = 10**
- **Parameter set: PC-PK1**
- **Configurations:**

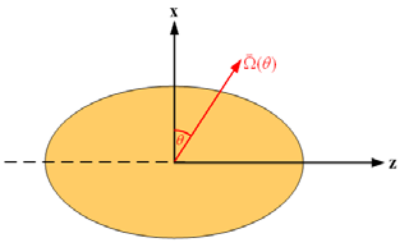
M-1	Config1	$\pi[(1f_{7/2})^{-1}(fp)^1]$	$\nu[(1g_{9/2})^1(fp)^3]$
	Config1*	$\pi[(1f_{7/2})^{-1}(fp)^1]$	$\nu[(1g_{9/2})^1(fp)^4(1f_{7/2})^{-1}]$
M-2	Config2	$\pi[(1f_{7/2})^{-1}(1g_{9/2})^1]$	$\nu[(1g_{9/2})^1(fp)^3]$
M-3	Config3	$\pi[(1f_{7/2})^{-1}(fp)^1]$	$\nu[(1g_{9/2})^2(fp)^2]$
	Config3*	$\pi[(1f_{7/2})^{-2}(fp)^2]$	$\nu[(1g_{9/2})^2(fp)^3(1f_{7/2})^{-1}]$

Zhao, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

Numerical Details

- **Symmetry**

	$\mathcal{P}$	$\mathcal{P}_x\mathcal{T}$	$\mathcal{P}_x$	$\mathcal{P}_z\mathcal{T}$	$\mathcal{P}_z$	$\mathcal{P}_y\mathcal{T}$	$\mathcal{P}_y$
J_x	√	×	√	√	×	√	×
J_z	√	√	×	×	√	√	×



$(\mathcal{P}_y = \mathcal{P}\mathcal{R}_y(\pi))$

- **Identification of the energy levels**

$$|n_x\ n_y\ n_z\ n_s\rangle \rightarrow |nljm_z\rangle \rightarrow |nljm_x\rangle$$

Cartesian basis **Spherical basis**

- **Constrained intrinsic framework**

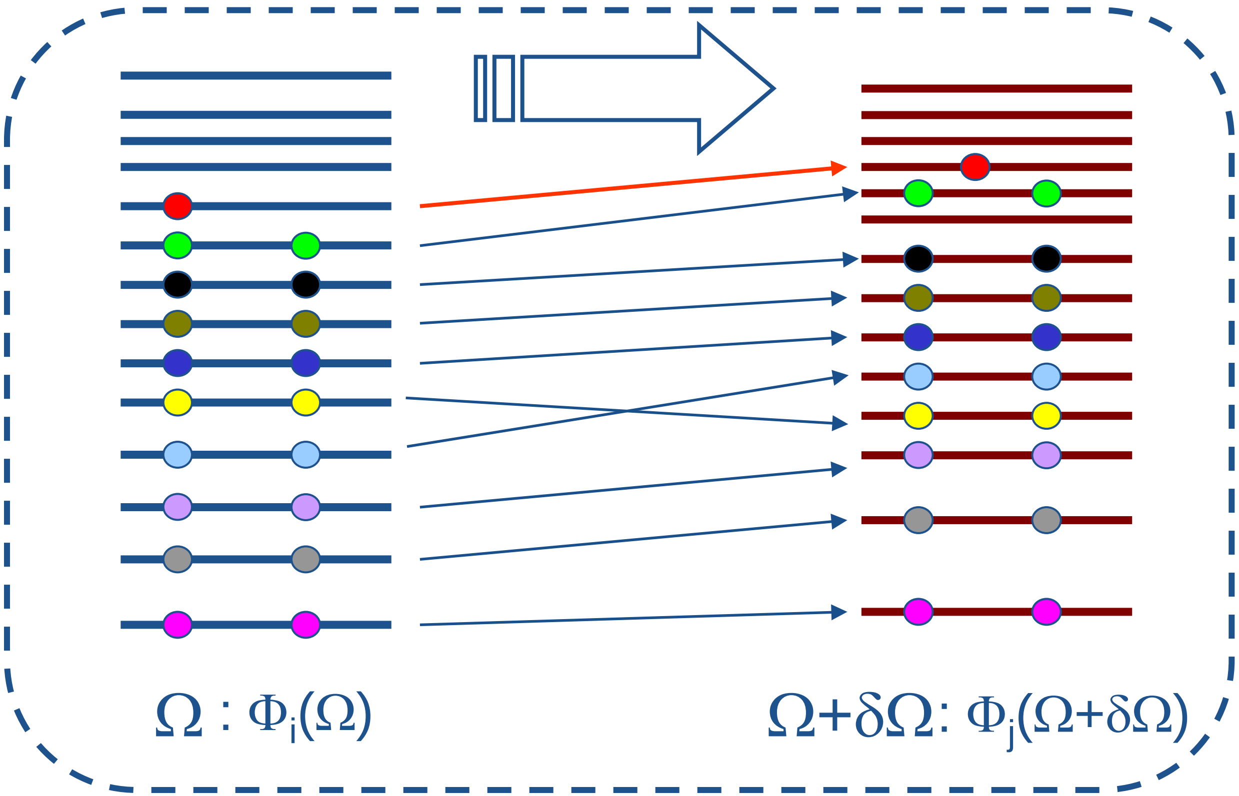
$$\langle H'\rangle = \langle H\rangle + \frac{1}{2}C\left(\langle Q_{2-1}\rangle - a_{2-1}\right)^2 \qquad a_{2-1} = 0$$

Enforce principal axes to be identical with the x, y, and z axis.

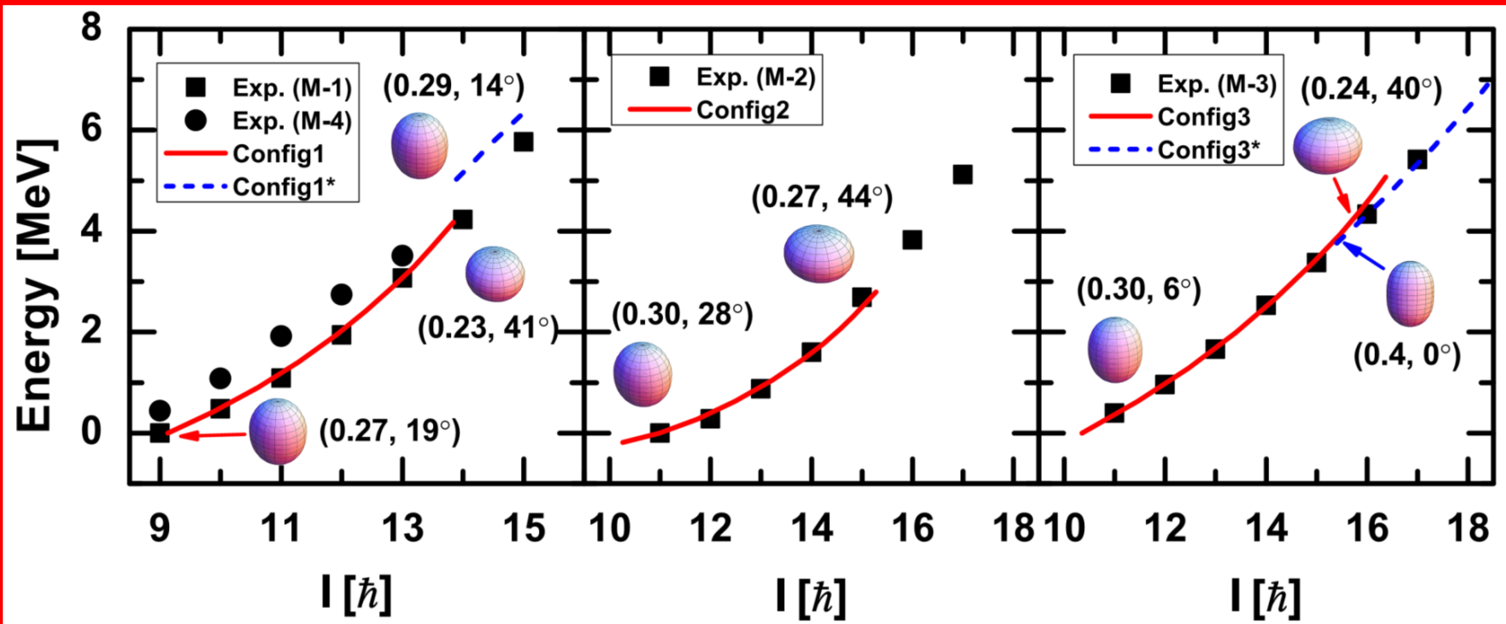
Numerical Details

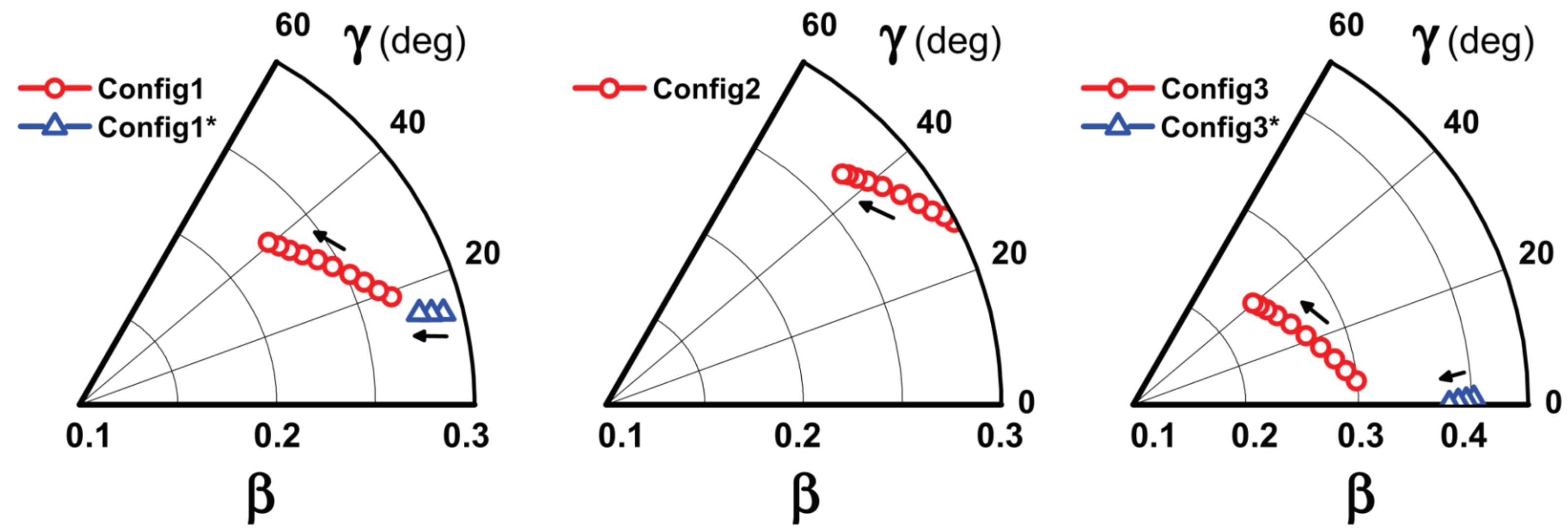
- **Parallel transport principle**

$$\left\langle \phi_j(\Omega+\delta\Omega) \middle| \phi_i(\Omega) \right\rangle \approx$$



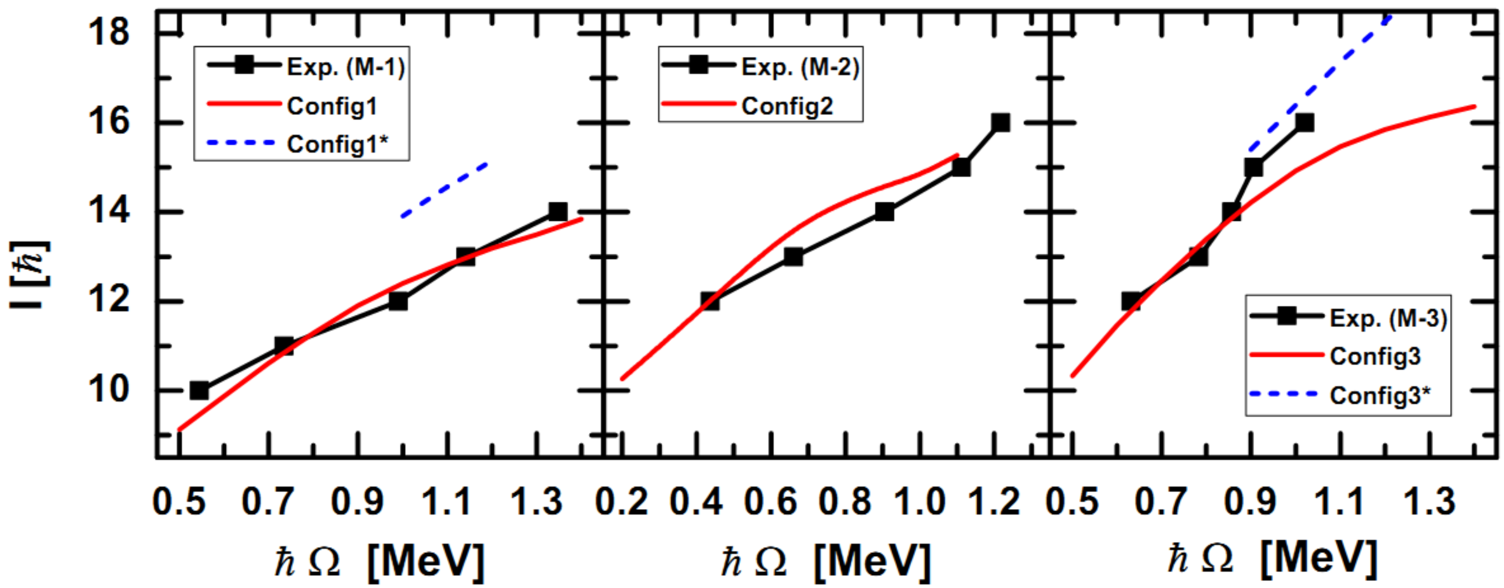
MR: ^{60}Ni





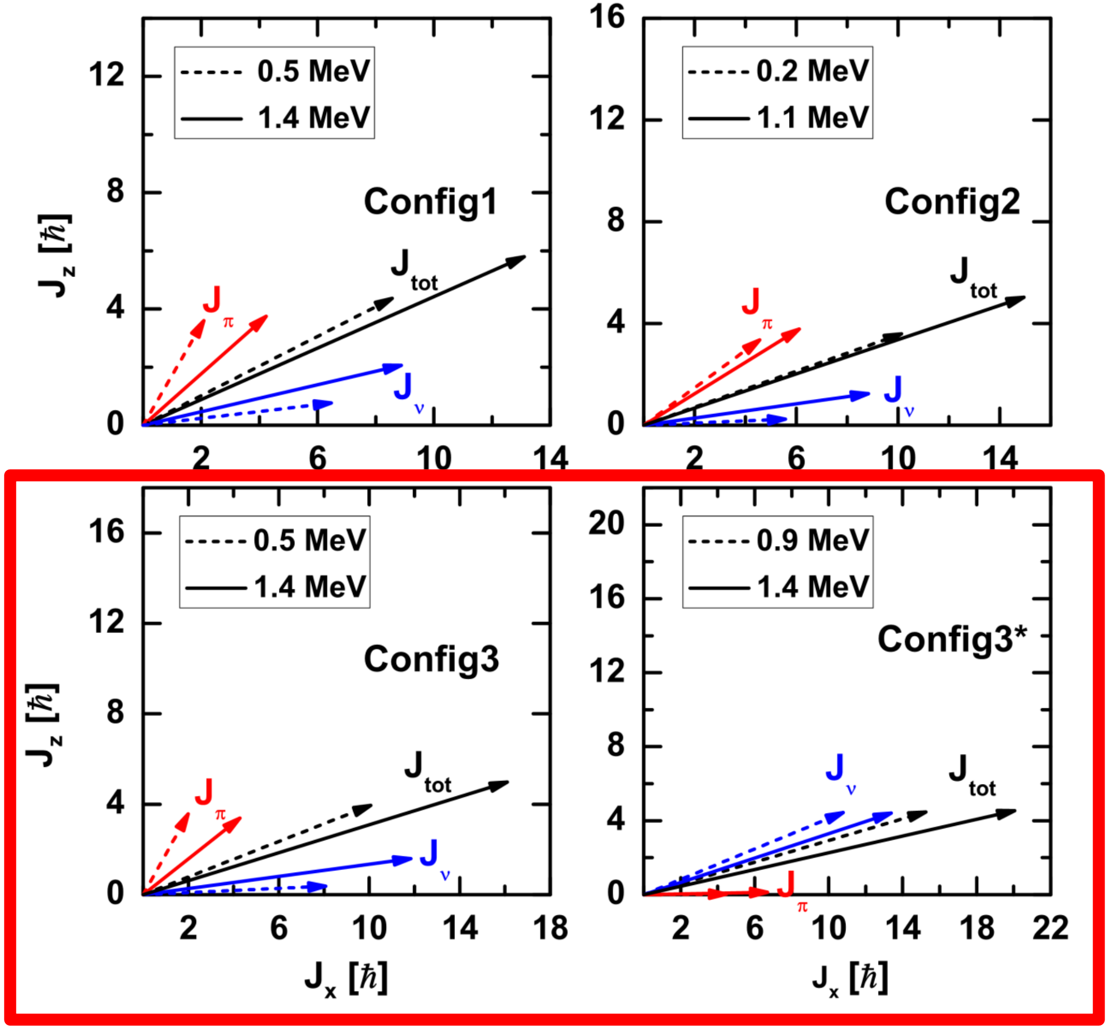
Zhao, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

MR: ^{60}Ni



Zhao, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

MR: ^{60}Ni



Magnetic Rotation

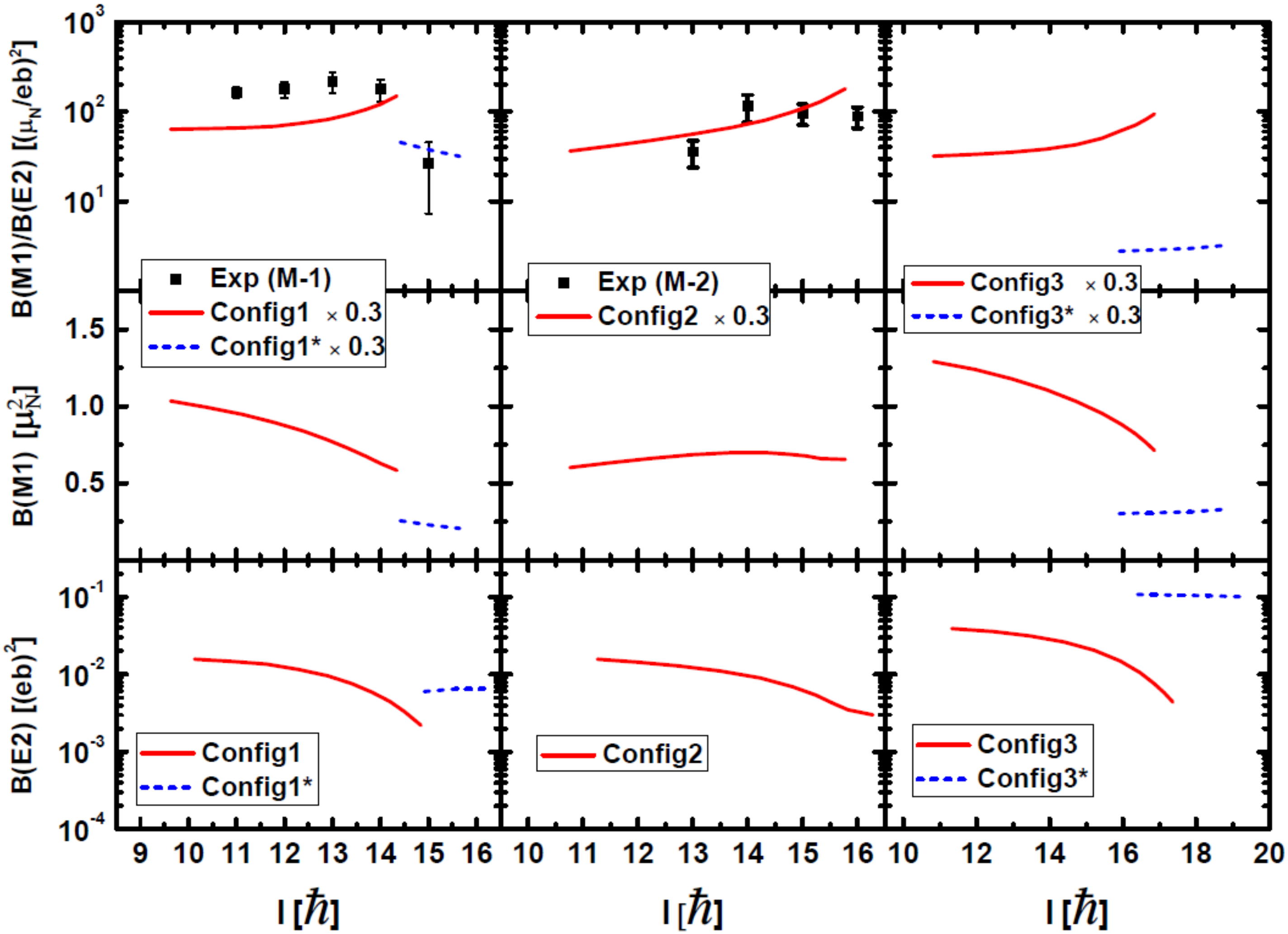
↓

Electric Rotation

Shears mechanism

Zhao, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

MR: ^{60}Ni



Electromagnetic transition properties

Zhao, Zhang, Peng, Liang, Ring, Meng, PLB 699, 181 (2011)

Outline

- Introduction
- Theoretical framework
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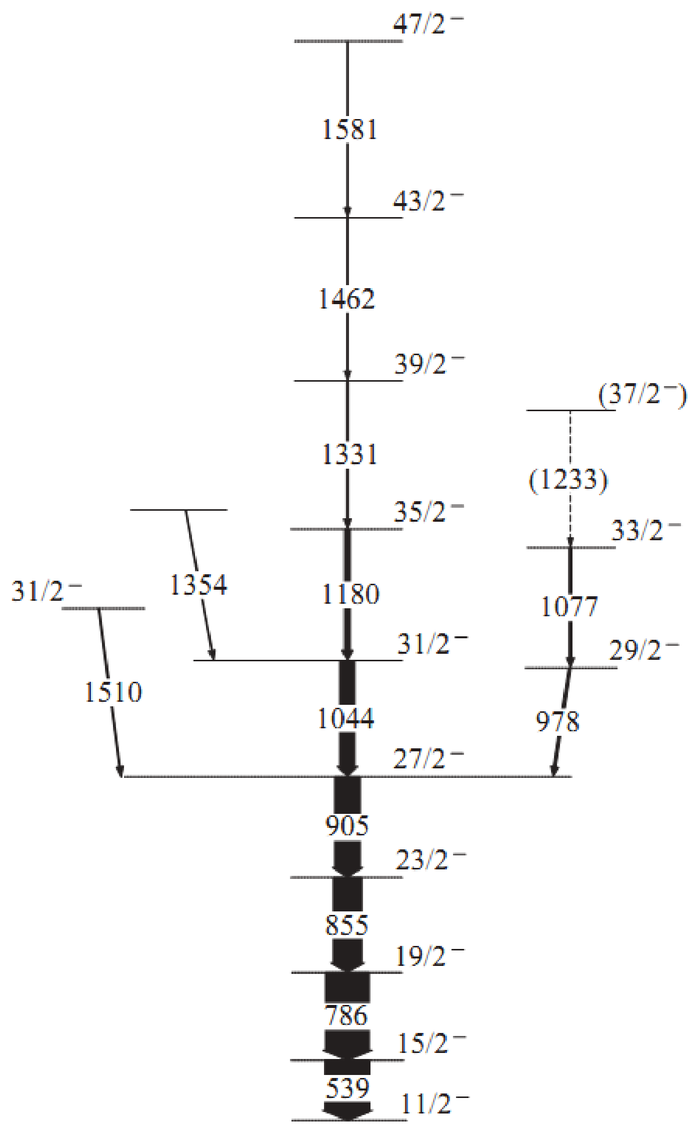
AMR: ^{105}Cd

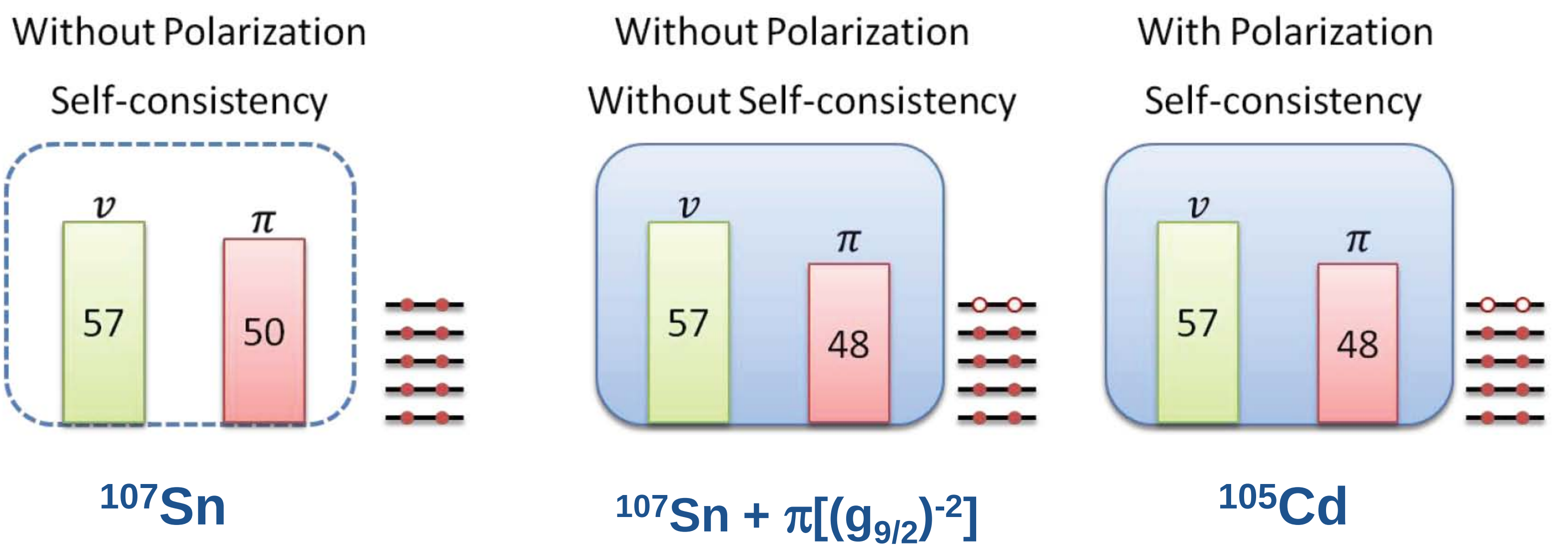
first odd-A nucleus with antimagnetic rotation

Choudhury PRC2010



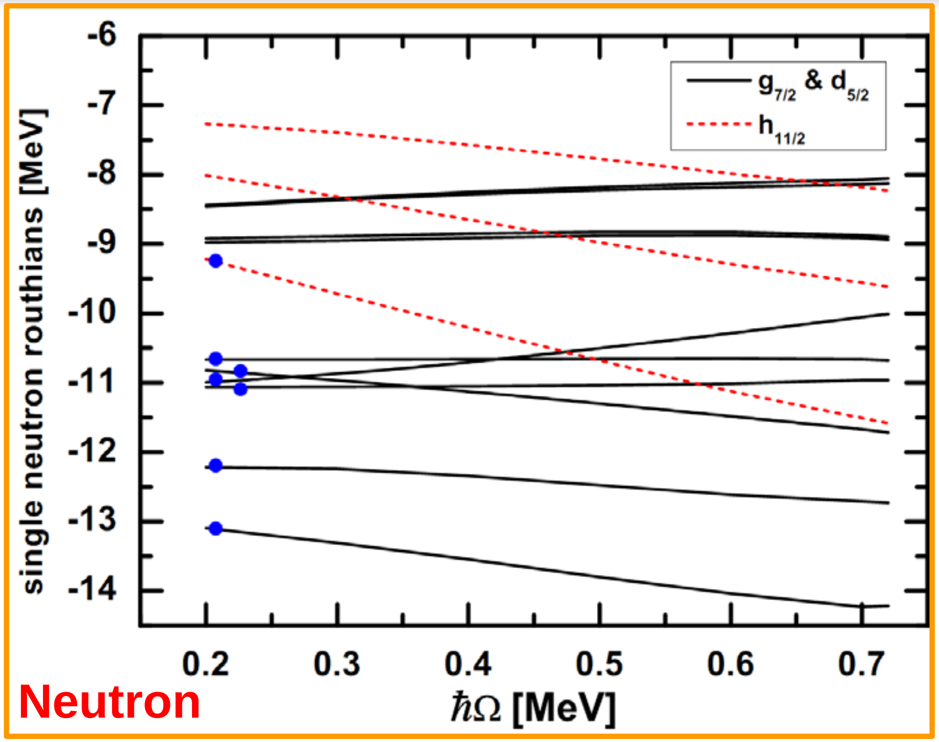
AMR: ^{105}Cd

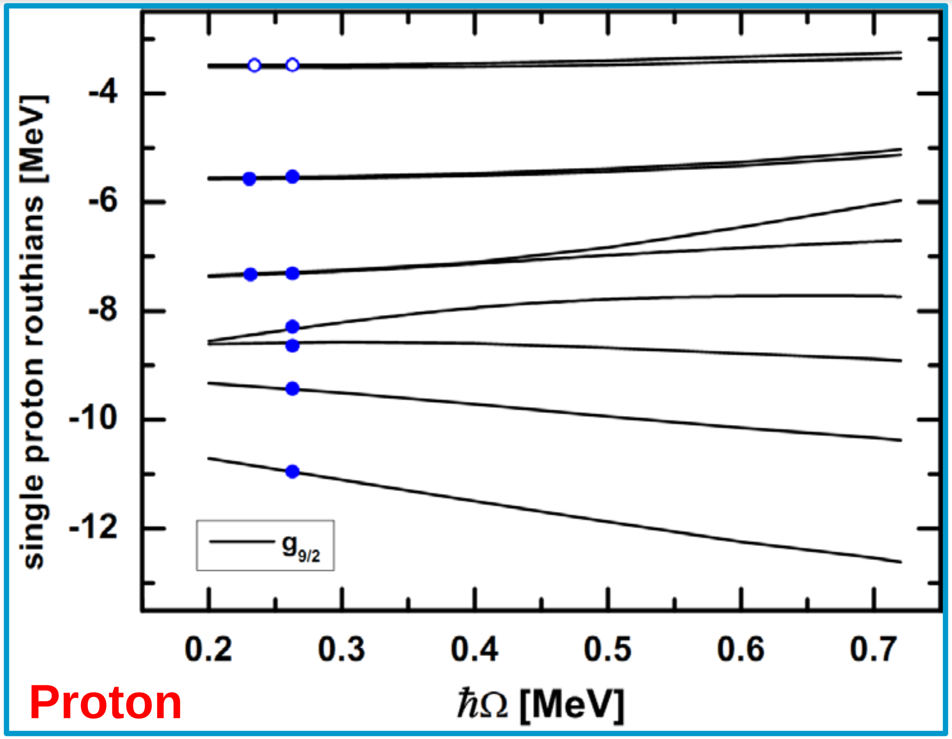
- **Harmonic oscillator shells: Nf = 10**
- **Parameter set: PC-PK1**
- **Configurations:** $\nu[h_{11/2}(g_{7/2})^{2}] \otimes \pi[(g_{9/2})^{-2}]$ *Choudhury PRC2010*
- **Polarizations:**



Zhao, Peng, Liang, Ring, Meng PRL 177, 122501(2011)

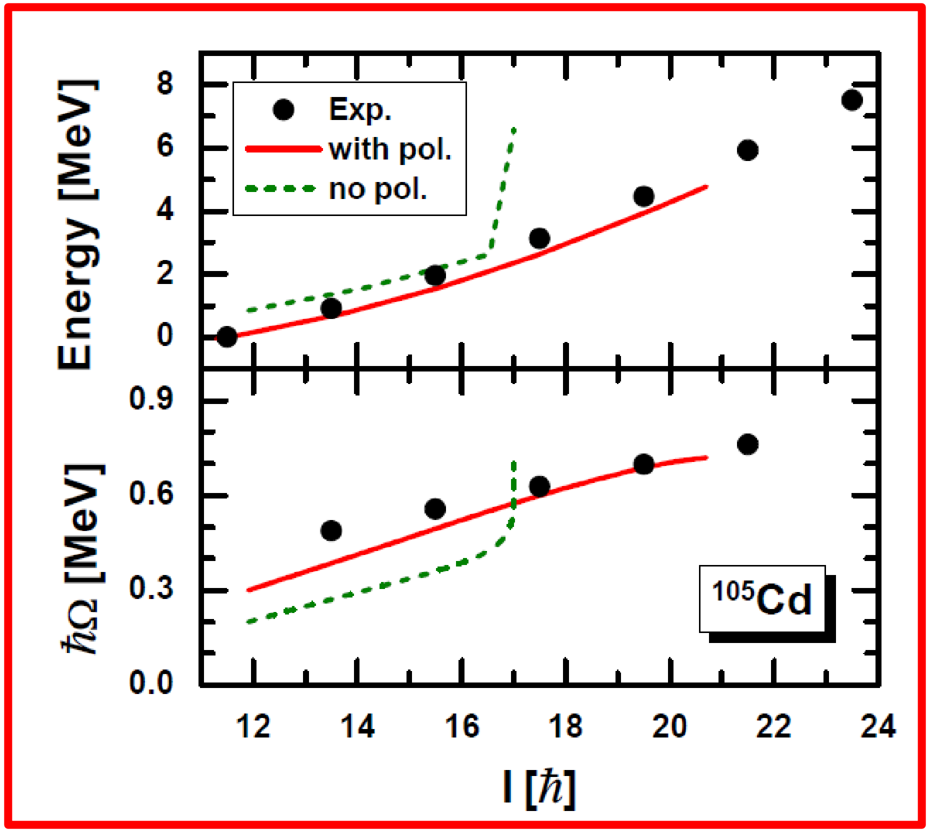
AMR: Single particle routhians

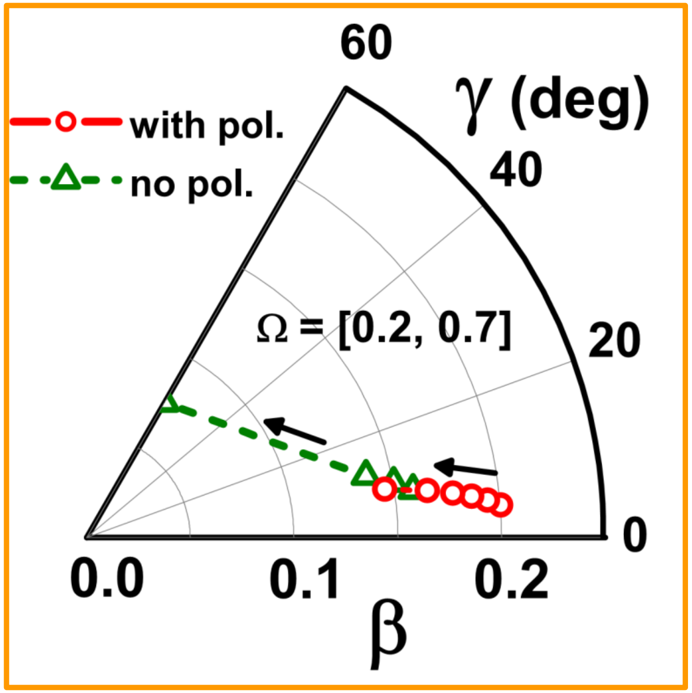




- Time reversal symmetry broken ⟹ energy splitting
- For proton, two holes in the top of $g_{9/2}$ shell
- For neutron, one particle in the bottom of $h_{11/2}$ shell, the other six are distributed over the (gd) shell with strong mixing
- This configuration is similar to $\nu[h_{11/2}(g_{7/2})^2] \otimes \pi[(g_{9/2})^{-2}]$, but not exactly

AMR: ^{105}Cd





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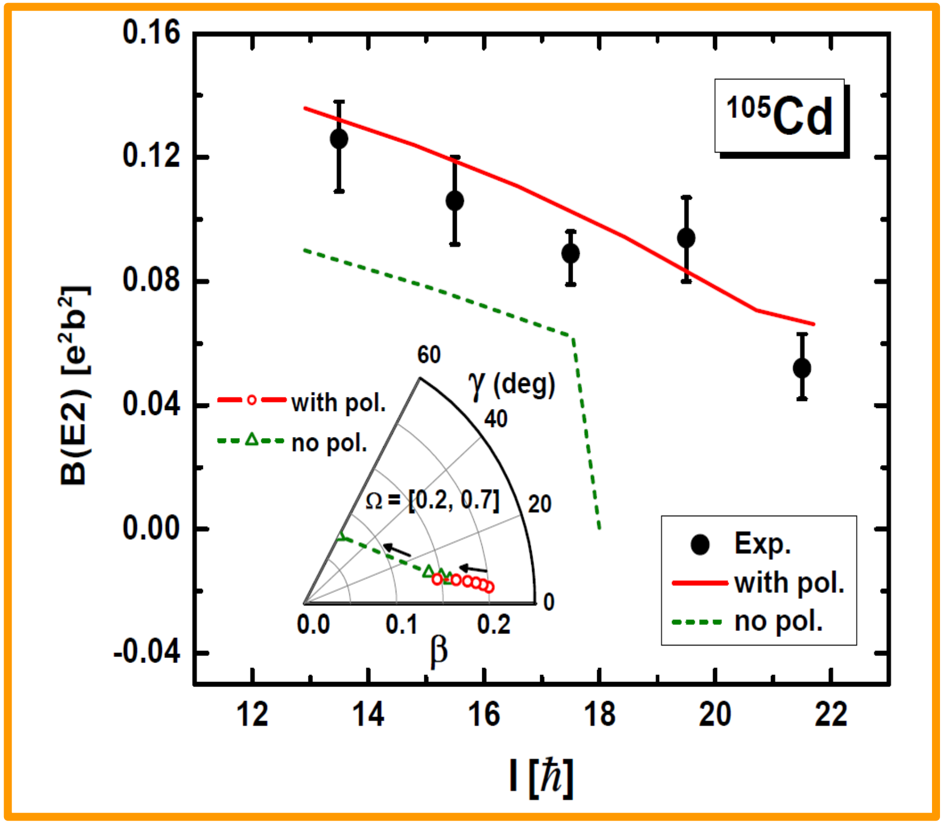
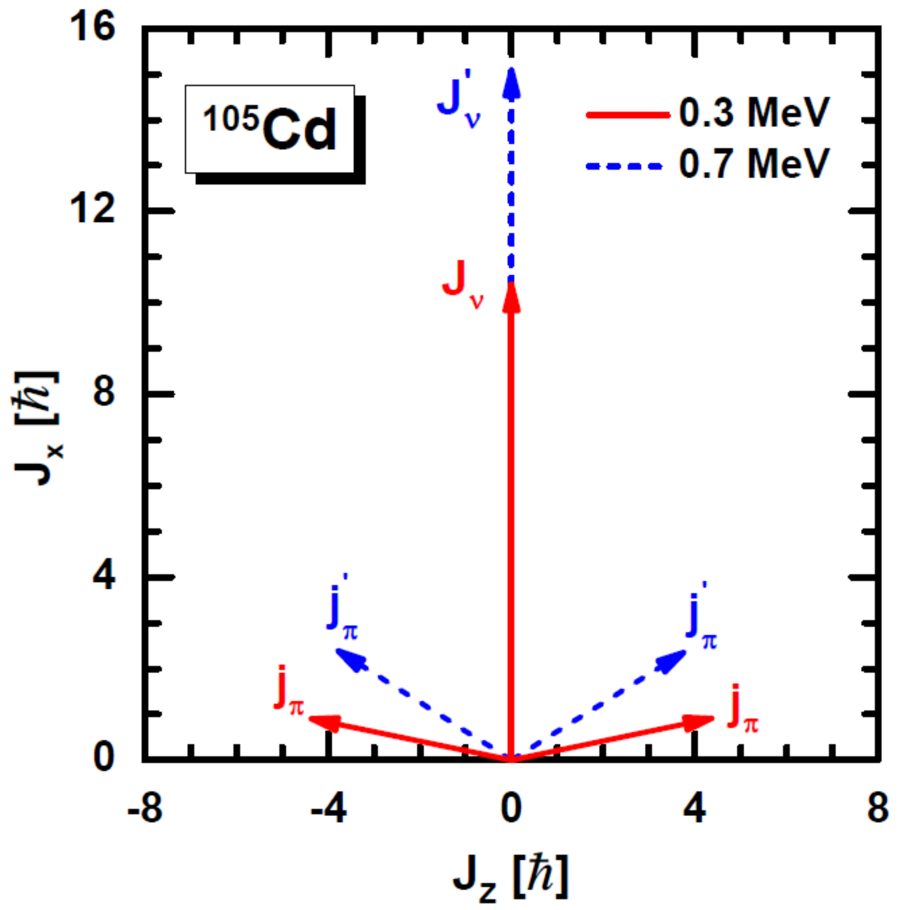
AMR: ^{105}Cd

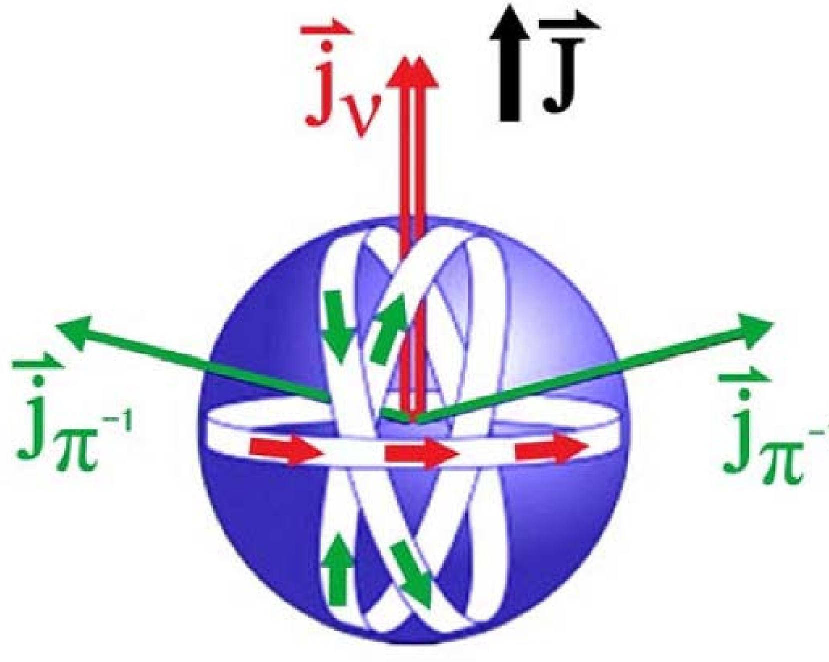


Polarization effects play important roles in the description of AMR, especially for E2 transitions.

Zhao, Peng, Liang, Ring, Meng PRL 177, 122501(2011)

AMR: ^{105}Cd

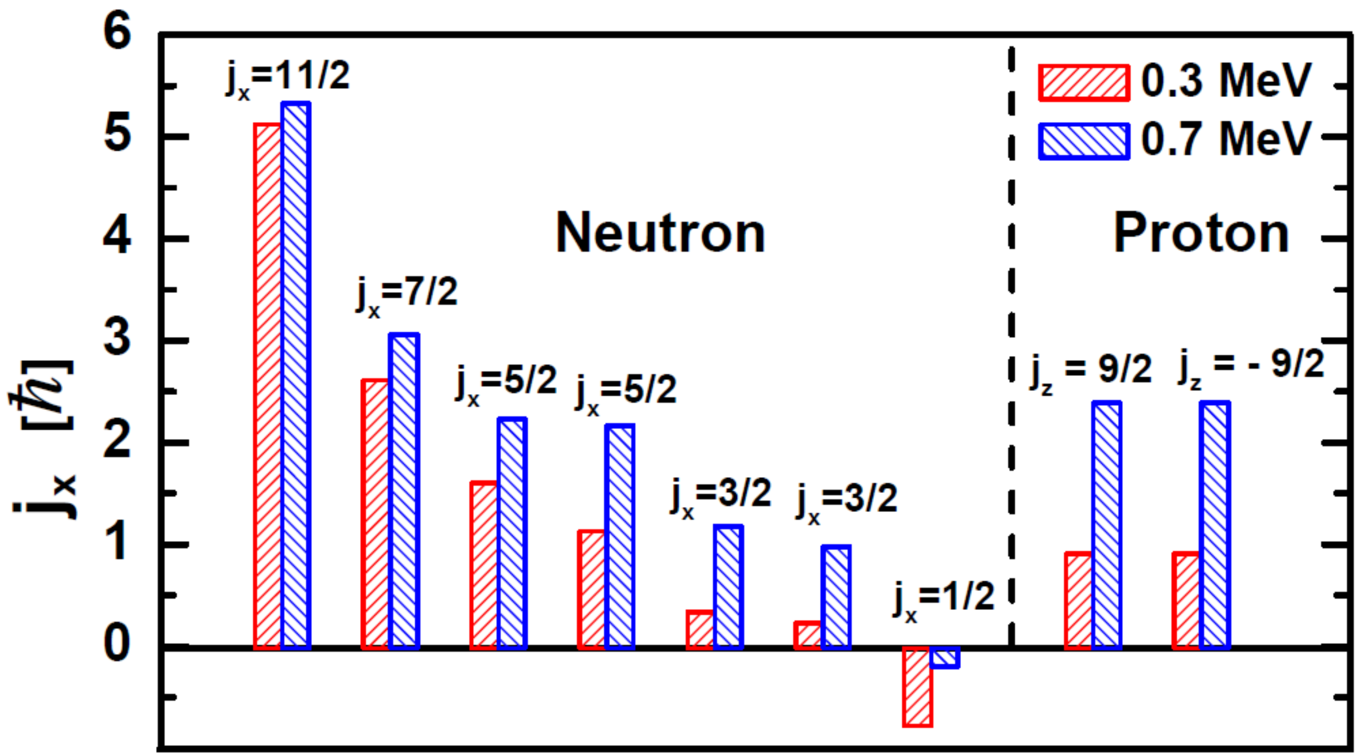




Two "shears-like" mechanism

Zhao, Peng, Liang, Ring, Meng   PRL 177, 122501(2011)

AMR: ^{105}Cd



In the microscopic point of view, increasing angular momentum results from the alignment of the proton holes and the mixing within the neutron orbitals.

Zhao, Peng, Liang, Ring, Meng PRL 177, 122501(2011)

Outline

- Introduction
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Summary and Perspectives

- Covariant density functional theory has been extended to describe rotational excitations including MR and AMR.

- PC-PK1:

  fitted with BE & Rc for 60 nuclei

  provide better description for the isospin dependence of BE

- ^{60}Ni: MR

  reproduce well the E, I, B(M1), and B(E2) values, inclduing the transition from electric rotation to magnetic rotation

- ^{105}Cd: AMR

  reproduce well the AMR pictures, E, I, and B(E2) values in a fully self-consistent microscopic way for the first time and find that polarization effects play important roles

Summary and Perspectives

? Pairing effects

? Transition from the electric to magnetic rotation

? Anti-magnetic rotation in other mass regions

? High-K isomer

Thank You!