Pairing correlations in Exotic Nuclei

September 22, 2011, Kyoto, Japan

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1. Introduction
2. IS+IV pairing interactions
3. BEC-BCS crossover in Borromian Nuclei
4. Di-neutron correlations and Dipole response
5. Summary

Isospin (IS+IV) dependent Pairing Interaction, BCS-BEC

J. Margeuron, Orsay
K. Hagino, Tohoku University, Japan
P. Schuck, Orsay
J. Carbonell , Grenoble

Systematic Study

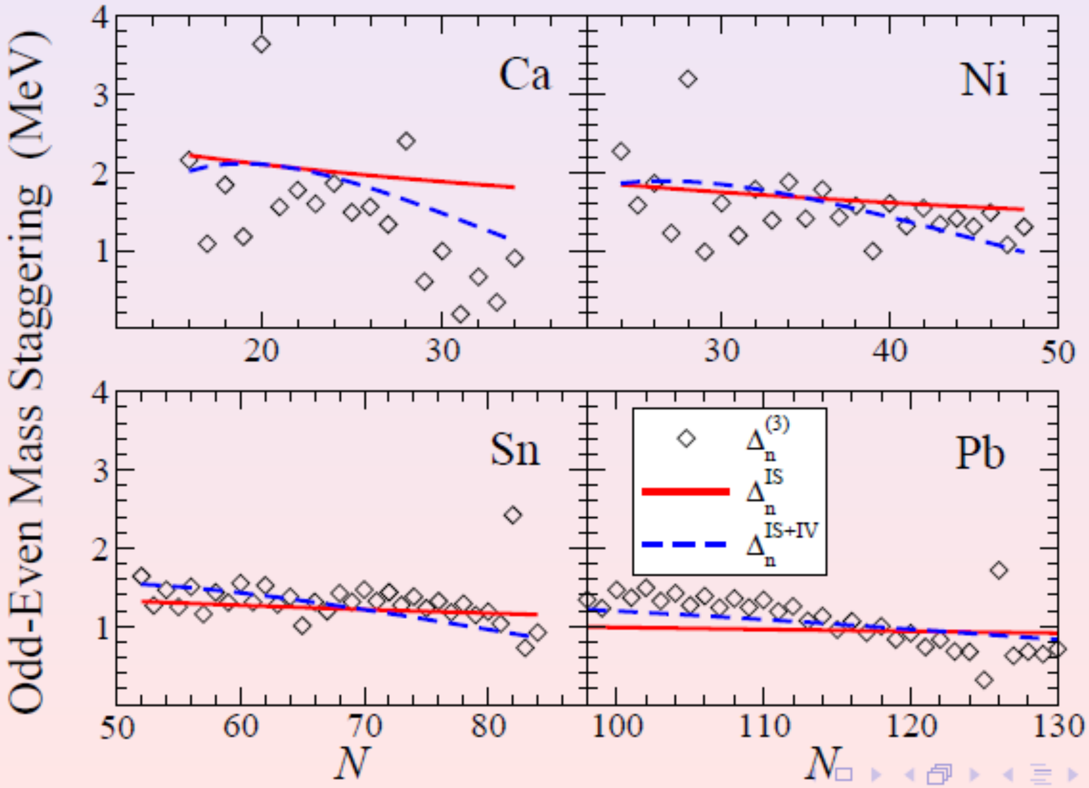
Carlos Bertulani, Texas A&M University at Commerce
Hong Feng Lu, China Agricultural University/UoA
Hongliang Lui, Peking University

Isospin effects in the pairing gaps

P. Vogel, B. Jonson, and P. G. Hansen, Phys. Lett. **139B**, 227 (1984).

$$\Delta_n^{\mathrm{IS}} = 13.3/A^{1/2} \tag{23}$$

$$\Delta_n^{\mathrm{IS+IV}} = [7.2 - 44(1-2Z/A)^2]/A^{1/3} \tag{24}$$



Odd-Even mass staggering $\Delta_3 = \frac{(-)^A}{2}\left(\mathrm{B(A)\text{-}2xB(A+1)+B(A+2)}\right)$

Isospin dependent Pairing Interaction

$$V_{pair}(1,2) = \frac{1-\mathrm{P}_o}{2} V_0 g_\tau \left[\rho, \beta\tau_z\right] \delta(\vec{r}_1 - \vec{r}_2)$$

$$g_\tau\left[\rho,\beta\tau_z\right] = g^1{}_\tau\left[\rho,\beta\tau_z\right] + g^2{}_\tau\left[\rho,\beta\tau_z\right]$$

$$g^1{}_\tau\left[\rho,\beta\tau_z\right] = 1 - \mathrm{f_s}(\beta\tau_z)\eta_s\left(\frac{\rho}{\rho_0}\right)^{\alpha_s} - \mathrm{f_n}(\beta\tau_z)\eta_n\left(\frac{\rho}{\rho_0}\right)^{\alpha_n}$$

$$\mathrm{f_s}(\beta\tau_z) = 1 - \mathrm{f_n}(\beta\tau_z),\ \mathrm{f_n}(\beta\tau_z) = \beta\tau_z = \frac{\rho_n(r) - \rho_p(r)}{\rho(r)}\tau_z$$

J.Margueron, HS and K. Hagino
PRC76,064316(2007) and PRC77,054309(2008)

Guideline for the parameters

$V_0 \rightarrow$ nn scattering length

$\eta_s, \alpha_s, \eta_n, \alpha_n \rightarrow$ pairing gaps in nuclear and neutron matter

No free parameters!

nn scattering length and pairing strength

G. F. Bertsch and H. Esbensen

T-matrix theory provides the formulas

$$k \cot \delta = -\frac{2}{\alpha\pi}\left[1 + \alpha k_c + \frac{\alpha k}{2} \ln \frac{k_c - k}{k_c + k}\right]$$

$$= -\frac{1}{a_{nn}} - \frac{k}{\pi} \ln \frac{k_c - k}{k_c + k}$$

$$V_0 = 2\pi^2 \alpha \hbar^2 / m$$

$$\alpha = 2a_{nn} / (\pi - 2k_c a_{nn})$$

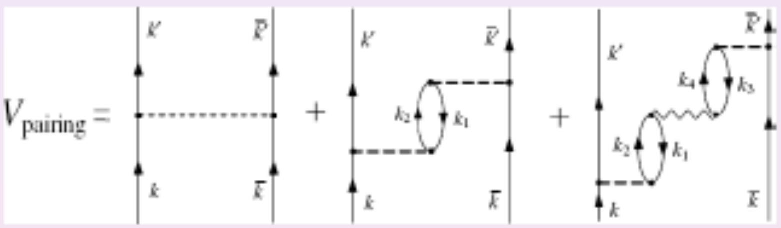
Pairing gap in uniform matter

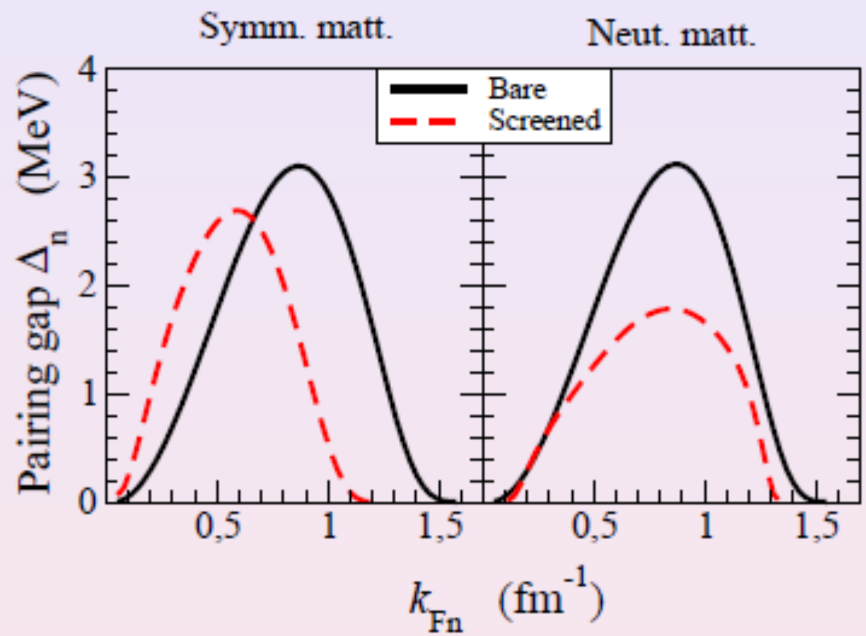
Microscopic treatment based on the realistic N-N interaction.

`Cao, Lombardo, Schuck, PRC 74, 064301 (2006)`

Bare:Argonne V_{18} +3body

Bare + medium polarization :





- reference calculation including only the bare NN interaction (bare),
- additional contribution from medium polarization effects (screened).

Medium polarization effects :

- In symmetric matter : shift the peak to lower densities.
- In neutron matter : reduction of the peak (/2).

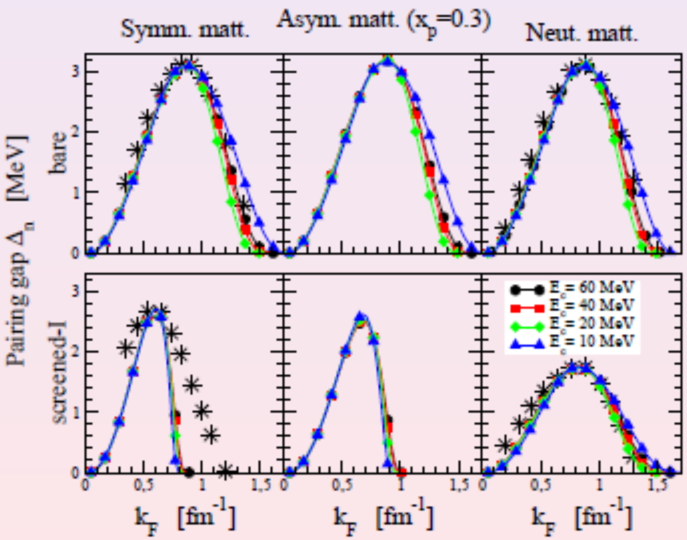
Adjustment of the density dependent term

Pairing gap in uniform matter obtained from microscopic treatment based on the realistic N-N interaction

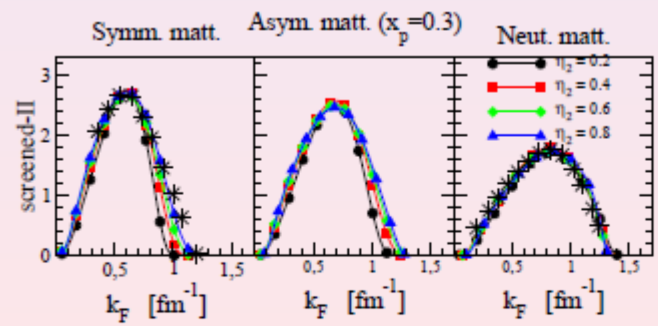
`Cao, Lombardo, Schuck, PRC 74, 064301 (2006)`

Result of the adjustment :

$g = g_1$



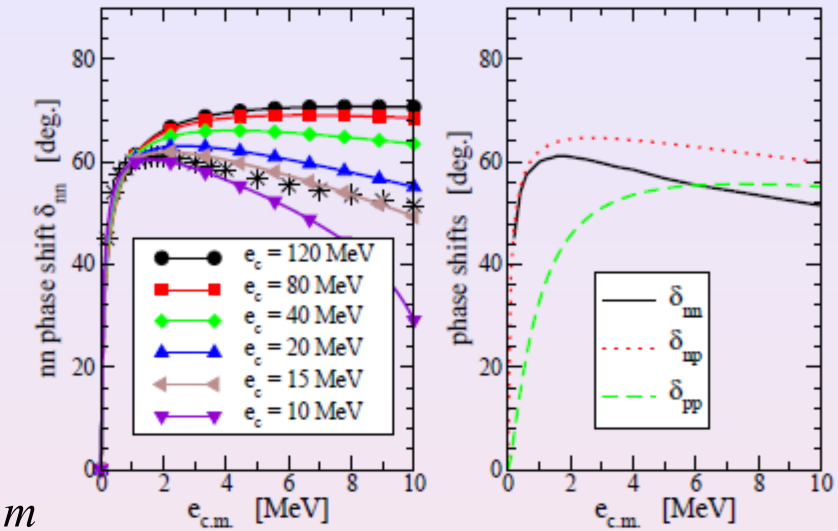
$g = g_1 + g_2$



Neutron Fermi momentum k_{Fn} : $\rho_n \equiv k_{Fn}^3/3\pi^2$.

Free interacion v_0

k_c and v_0 are adjusted so as to reproduce the low energy phase shift.



$$e_{C.M.} = \hbar^2 k^2 / m$$

e_c (MeV)	a_{nn} (fm)	r_{nn} (fm)	α (fm)	v_0 (MeV.fm^3)	v_0^* (MeV.fm^3)	v_0^∞ (MeV.fm^3)
120	−12.6	0.75	−0.55	−448	−458	−481
80	−13.0	0.92	−0.66	−542	−555	−589
40	−13.7	1.30	−0.91	−746	−767	−833
20	−15.0	1.83	−1.25	−1024	−1050	−1178

effective range $r_{nn} = 4/\pi k_c$ $\alpha = 2a_{nn}/(\pi - 2k_c a_{nn})$ $v_0 = 2\pi^2 \alpha \hbar^2 / m$ unitary limit $a_{nn} \to \infty$

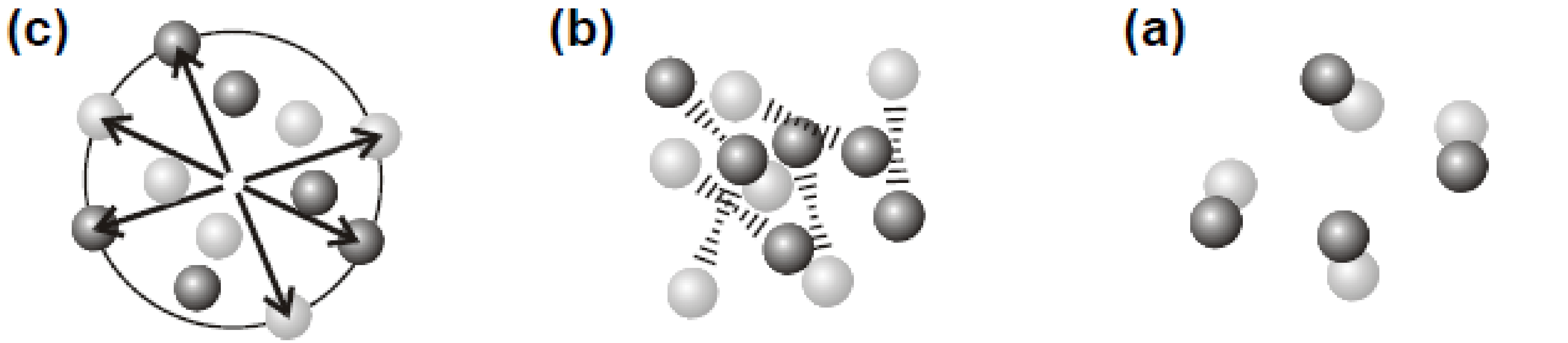
Parameters of the density dependent term

	$E_c = e_c/2$ (MeV)	η_s	α_s	η_n	α_n
bare	60	0.598	0.551	0.947	0.554
$g = g_1$	40	0.664	0.522	1.01	0.525
	20	0.755	0.480	1.10	0.485
	10	0.677	0.365	0.931	0.378
screened-I	60	7.84	1.75	0.89	0.380
$g = g_1$	40	8.09	1.69	0.94	0.350
	20	9.74	1.68	1.00	0.312
	10	14.6	1.80	0.92	0.230
screened-II	60	1.61	0.23	1.56	0.125
$g = g_1 + g_2$	40	1.80	0.27	1.61	0.122
$\eta_2 = 0.8$	20	2.06	0.31	1.70	0.122
	10	2.44	0.37	1.66	0.0939

$$g_n^1[\rho, I] = 1 - f_s(I)\eta_s \left(\frac{\rho}{\rho_0}\right)^{\alpha_s} - f_n(I)\eta_n \left(\frac{\rho}{\rho_0}\right)^{\alpha_n},$$

$f_s(I) = 1 - f_n(I)$ and $f_n(I) = I$.

Nuclear Matter

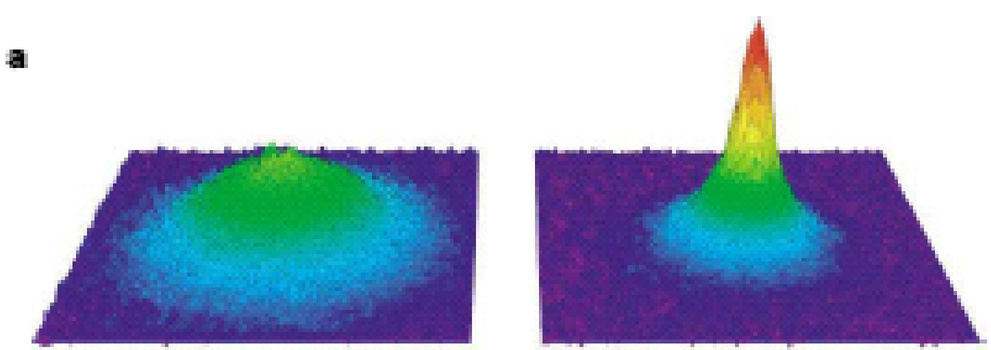


BCS superfluidity of Cooper pairs | **BCS - BEC crossover** | **BEC superfluidity of bound molecules**

$$|v_{\mathrm{pair}}| \to \infty$$

- Weakly interacting fermions
- Correlation in p space (large coherence length)

- Interacting "diatomic molecules"
- Correlation in r space (small coherence length)



cf. BEC of molecules in ^{40}K

M. Greiner et al., Nature 426('04)537

Description of the BEC phase

→ The BCS equations describe also the BEC phase.
Ph. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985)

Proof :
Eq. (17) and (18) go over Schrödinger-like Eq.

$$\frac{p^2}{m}\Psi_{pair} + [1 - 2n_n(k)]\frac{1}{V}\mathrm{Tr}v_{nn}\Psi_{pair} = 2\nu_n\Psi_{pair}. \qquad (19)$$

where $\Psi_{pair} = u_k v_k$ is the pair wave function.

At zero density, $2\nu_n$ is the binding energy of Ψ_{pair}.
→ strongly correlated (BEC state) if $\nu_n < 0$
The smooth change between BCS to BEC is also describe by the BCS equations.

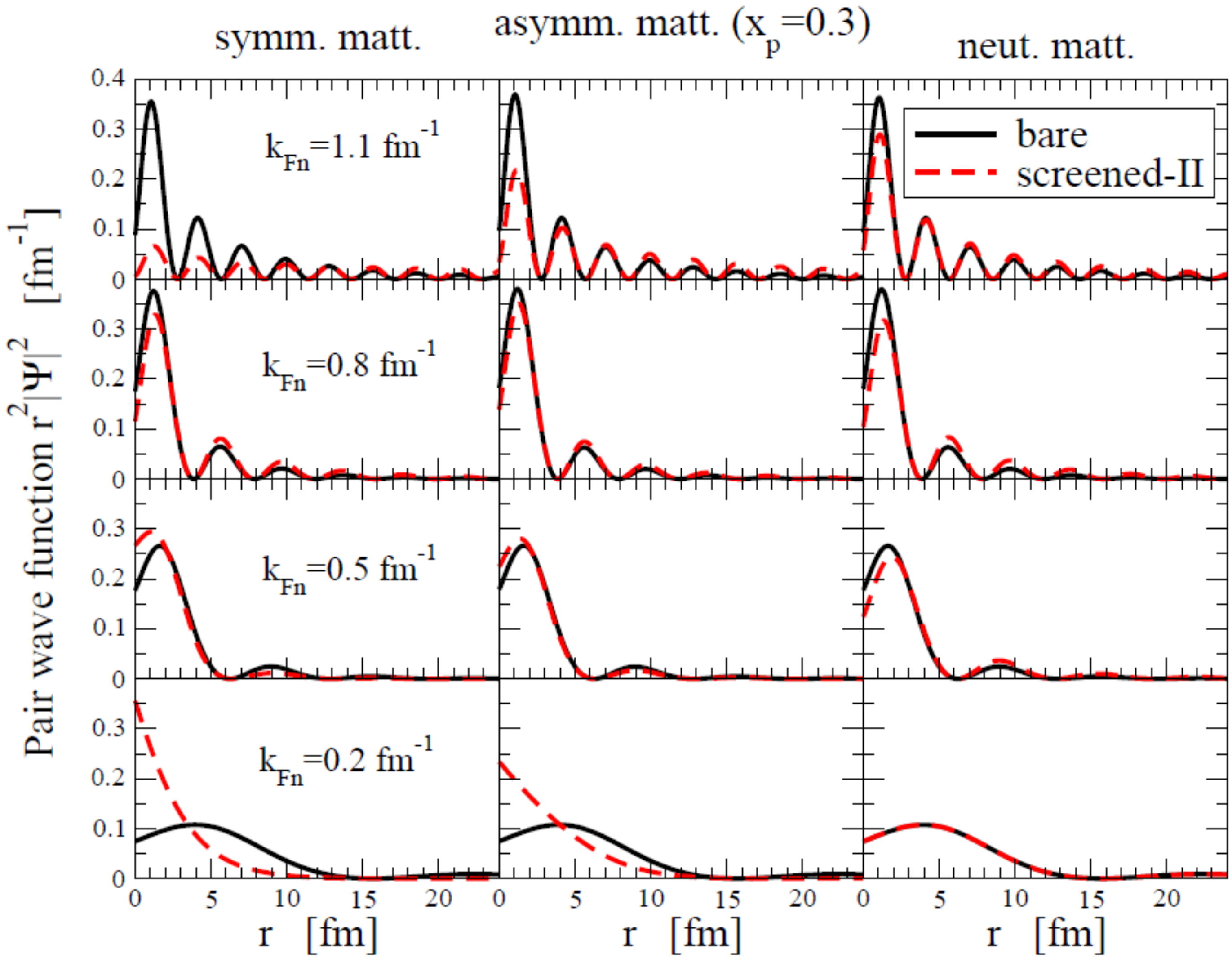


FIG. 5: (Color online) Neutron Cooper pair wave function $r^2|\Psi_{pair}(r)|^2$ as a function of the relative distance r between the pair partner at the Fermi momenta k_{Fn}=1.1, 0.8, 0.5 and 0.2 fm^{-1}.

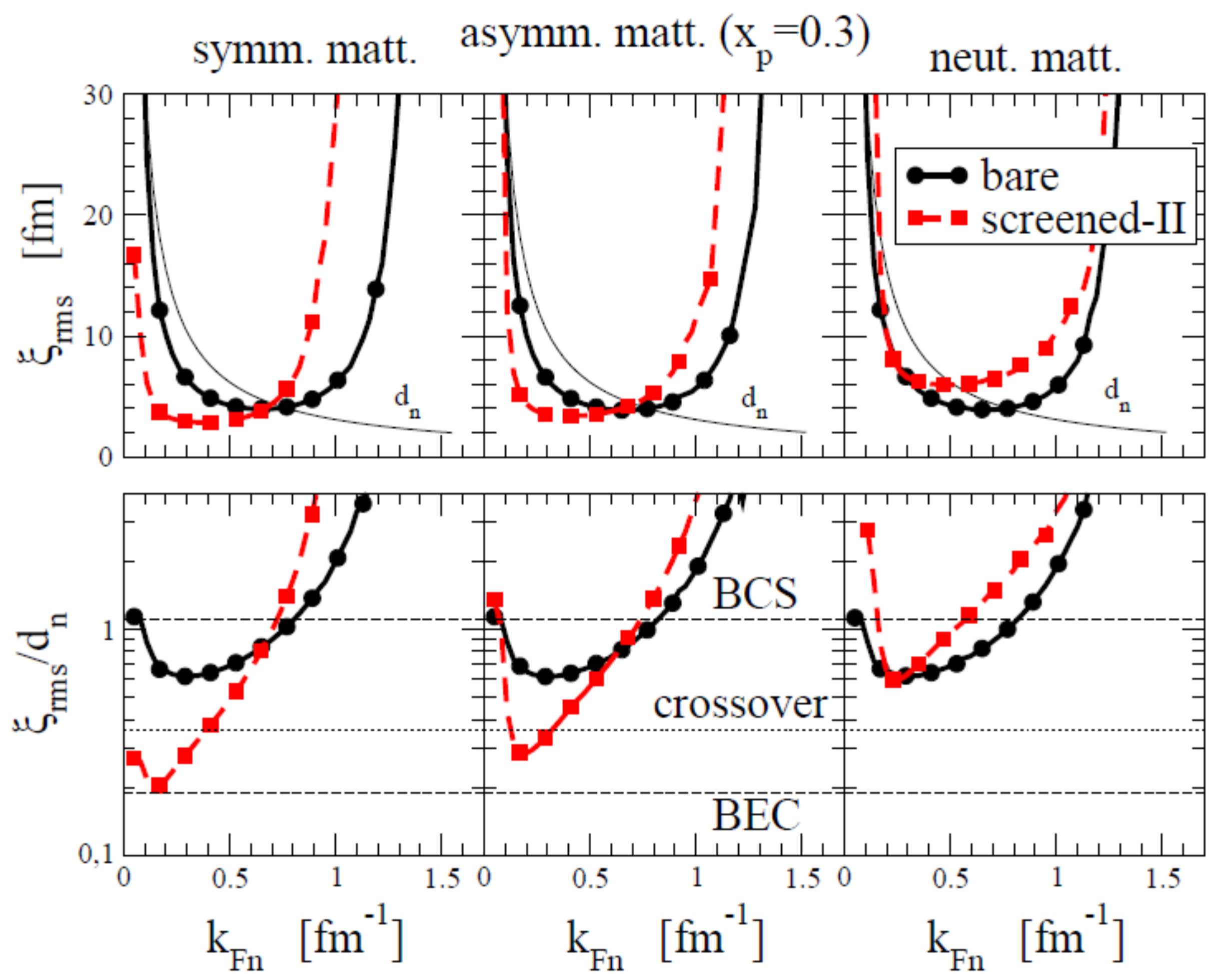


FIG. 8: (Color online) Top panels: Comparison between the rms radius ξ_{rms} of the neutron pair and the average inter-neutron distance $d_n = \rho_n^{-1/3}$ (thin line) as a function of the neutron Fermi momentum k_{Fn} in symmetric (left panel), asymmetric (central panel) and neutron matters (right panel). Bottom panels: The order parameter ξ_{rms}/d_n as a function of k_{Fn}. The boundaries of the BCS-BEC crossover are represented by the two dashed lines, while the unitary limit is shown by the dotted line. The two pairing interactions are used for the calculations.

Messages from Nuclear Matter Calculations

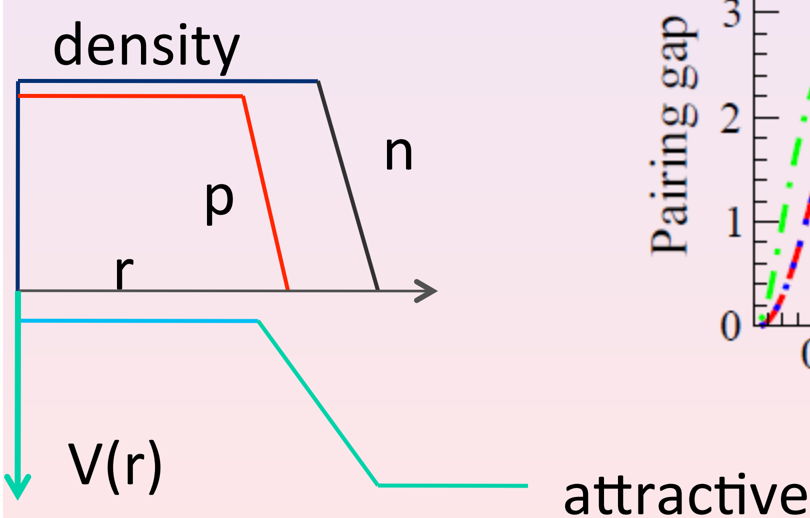
- New type of density dependent contact pairing interactions : reproduce microscopic pairing gaps in symmetric and neutron matter and depend on isospin-asymmetry.
- Medium polarization effects :
  $\longrightarrow$ reduction of the bare gap in neutron matter,
  $\longrightarrow$ strong attraction (quasi-BEC state) in low density symmetric matter.

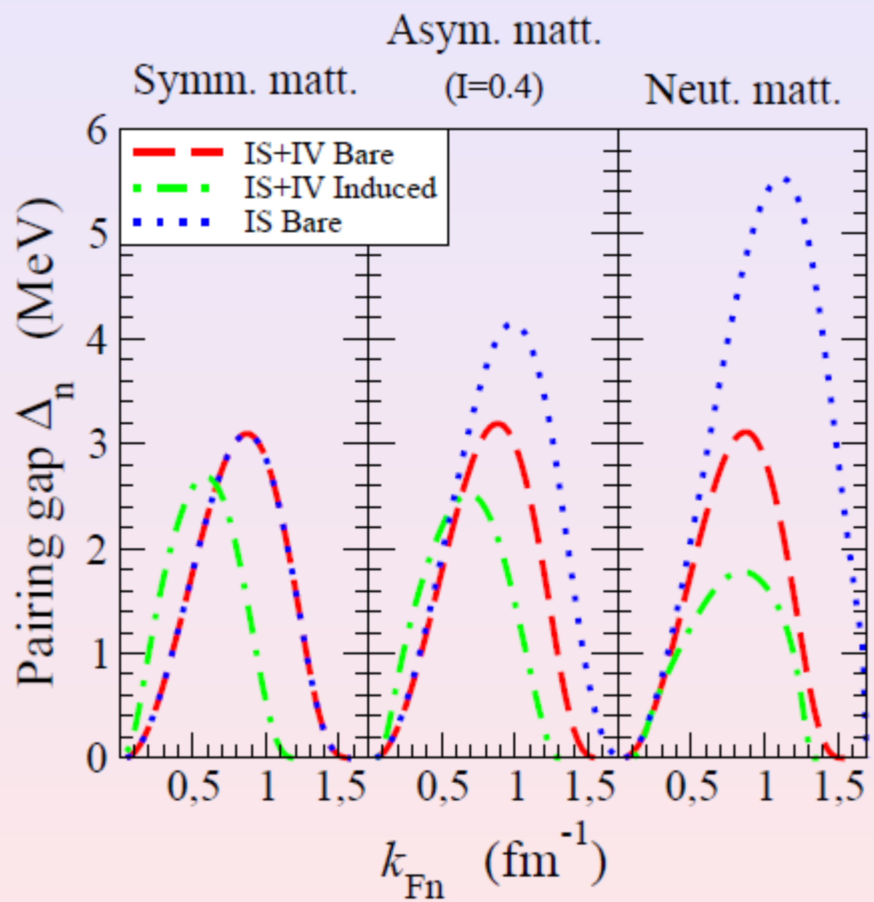
BCS-BEC crossover phenomena

J. Margueron, H. Sagawa, K. Hagino,
Phys. Rev. C 76, 064316(2007)

IS+IV Pairing in Finite Nuclei

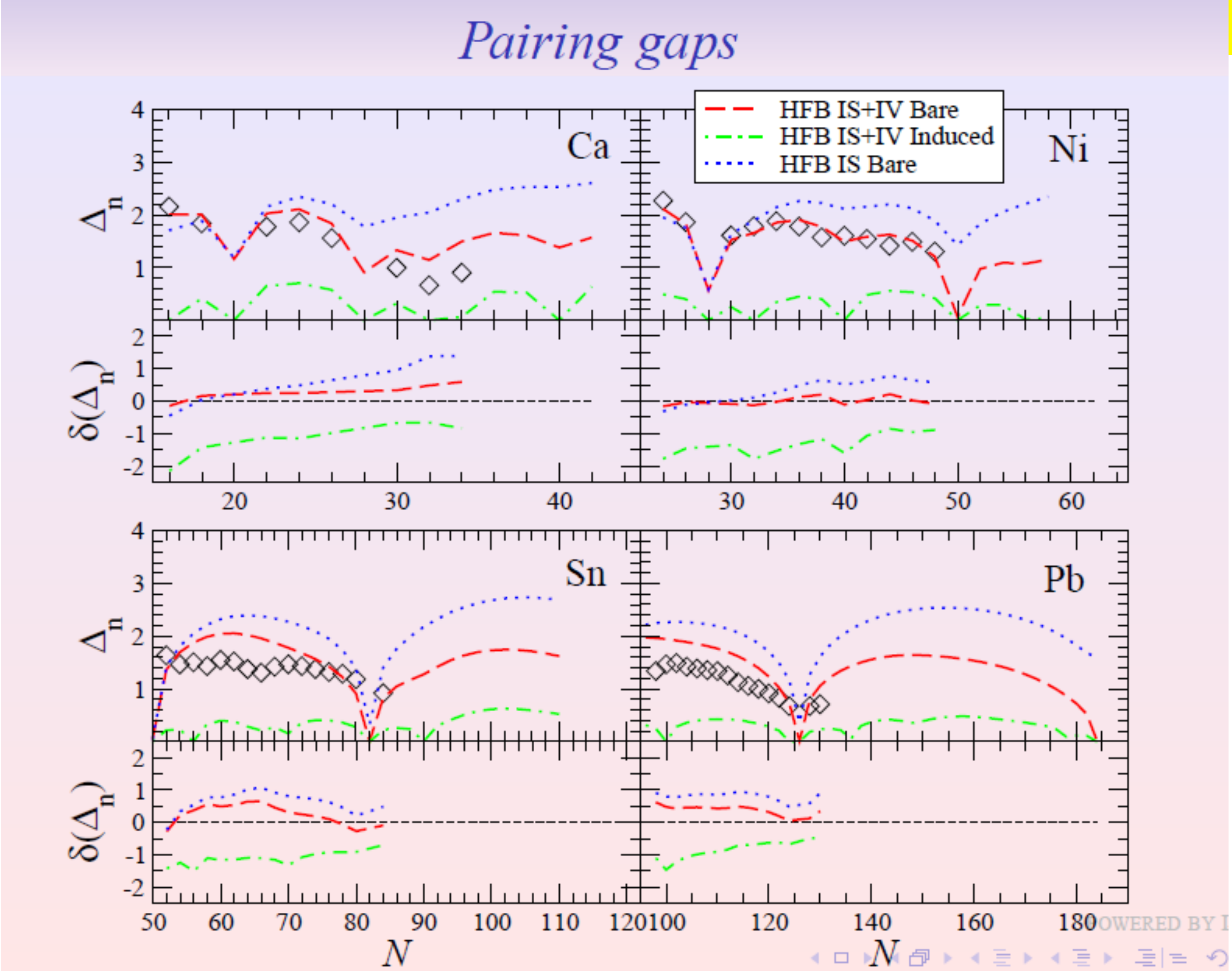
Question 2 :
→ Role of the isospin-dependent term ?





HFB calculations with canonical basis

SLy4+isospin dependent pairing interactionsCa, Ni, Sn, Pb isotopes



J.Margueron, HS and K. Hagino、 PRC77,054309(2008)

Deformed HF+BCS with odd particle blocking (EV8-odd)

SkP +isospin dependent pairing
SLy4+isospin dependent pairing

Odd-even mass difference and isospin dependent pairing interaction

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[2] *College of Science, China Agricultural University, Beijing, P.R.China*
[3] *Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, 965-8580 Fukushima, Japan*
(Dated: August 12, 2009)

PRC80, 027303(2009)

Deformed HF+BCS with blocking

EV8-ODD

EV8 (**P. Bonche, H. Flocard and P.H. Heenen**, *CPC* 171 (2005) 49)

1 – code has only 6,500 lines

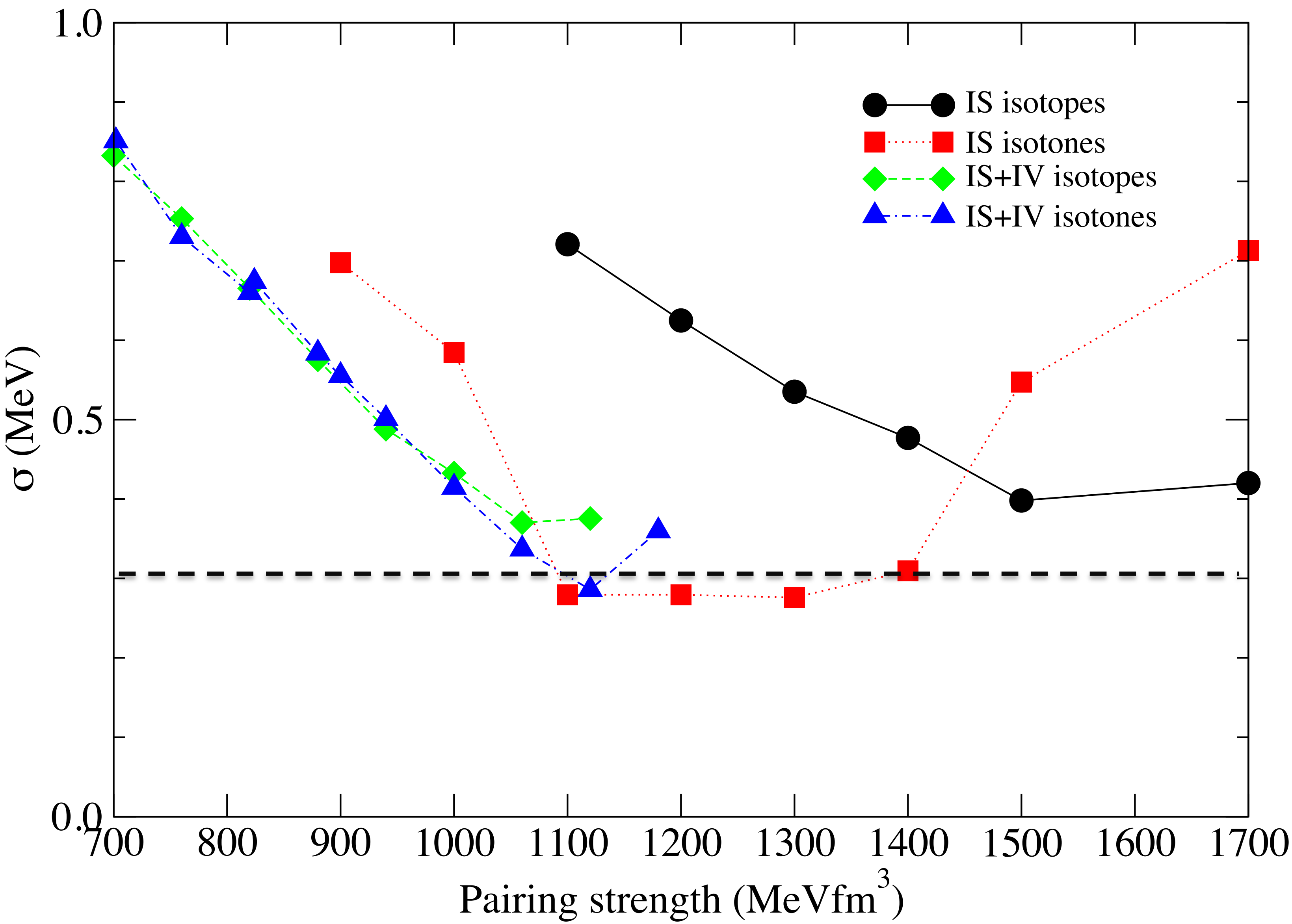
2 - HF+BCS, 3-d coordinate mesh, axial symmetry, small continuum space

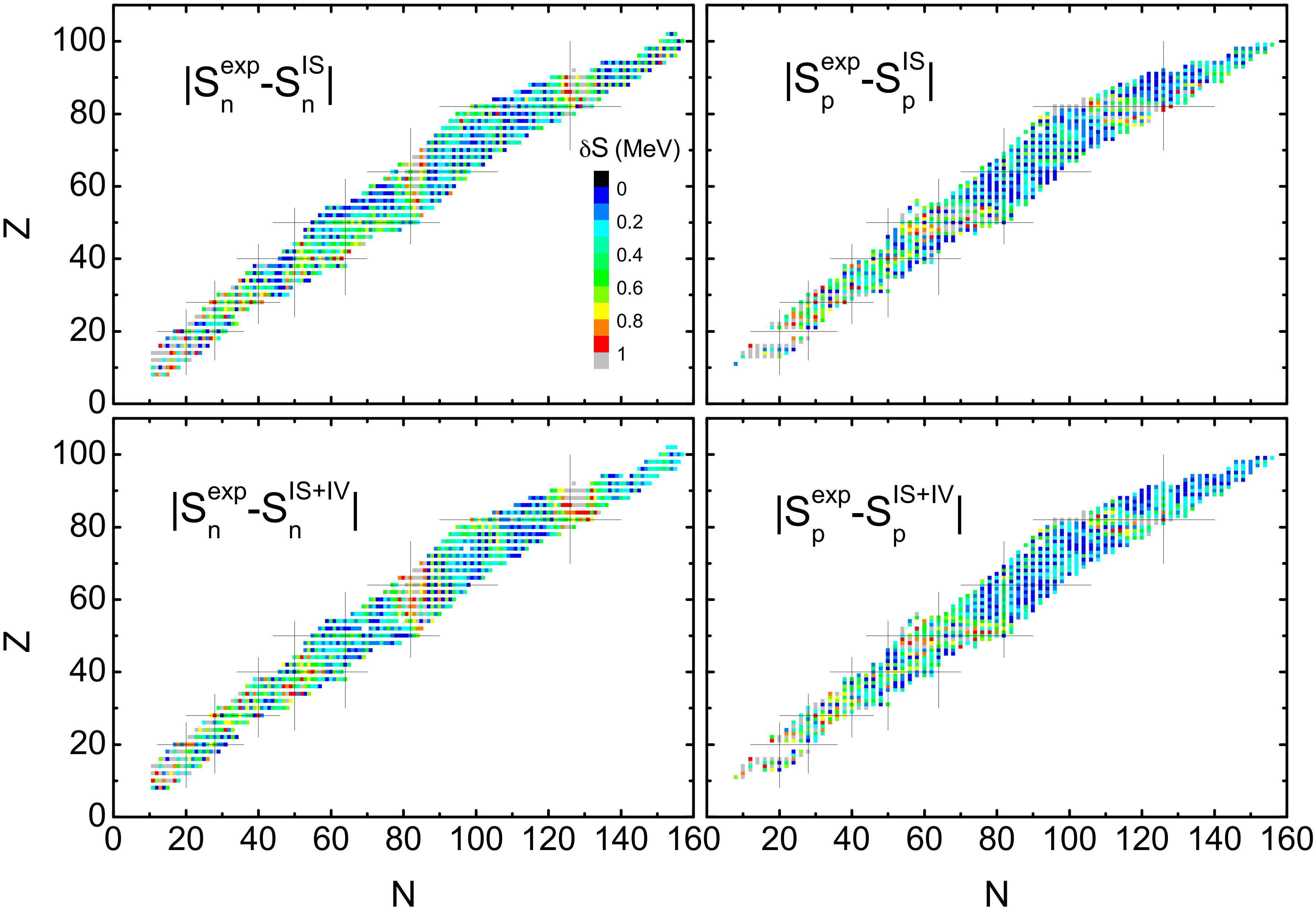
EV8 ODD:

3 – Blocking implemented by Bertsch (odd N) and Bertulani (odd Z).

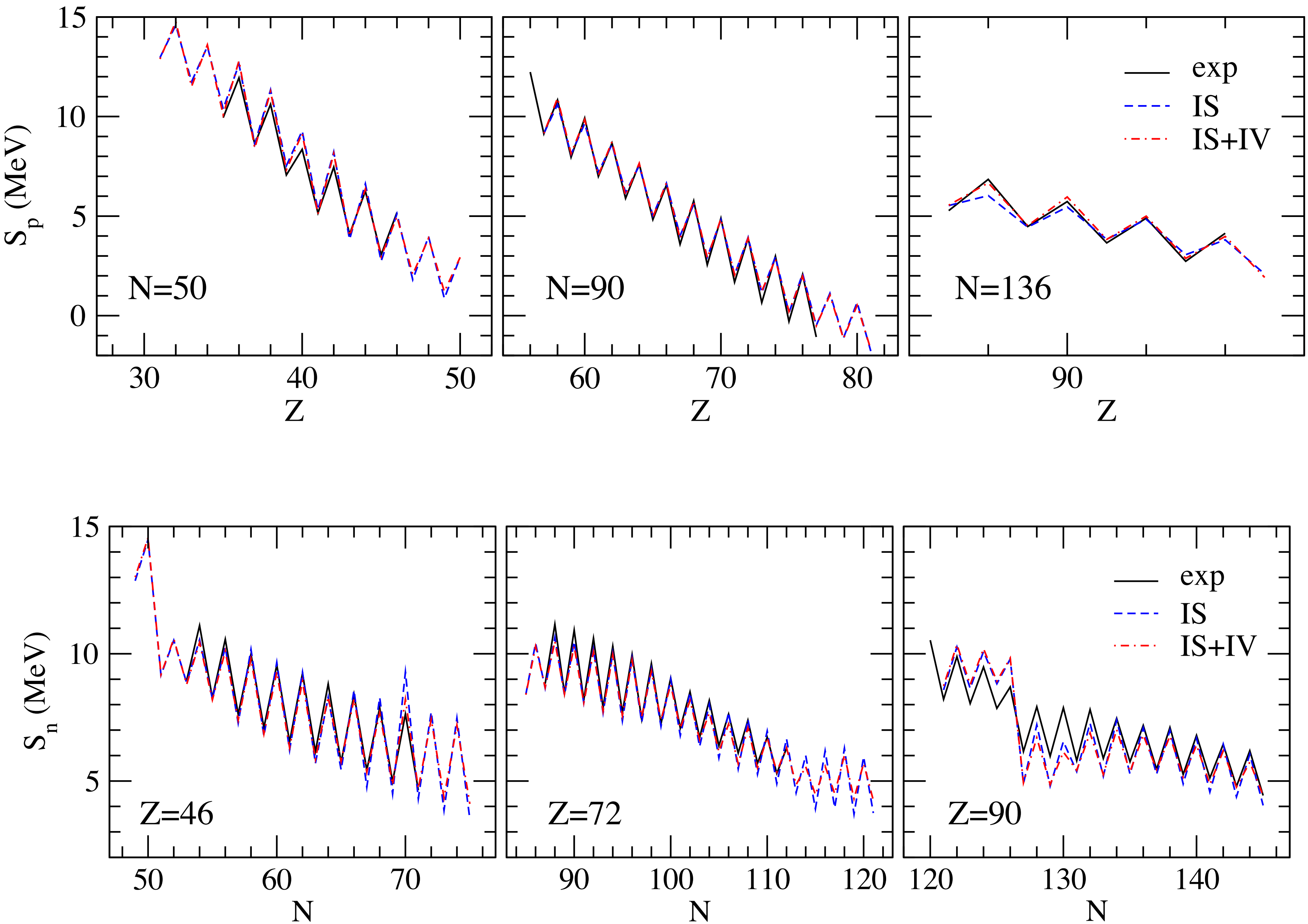
A systematic study of pairing gaps varying the coupling strength

C. Bertulani, Honglinag Lui, HS et al. (2011).

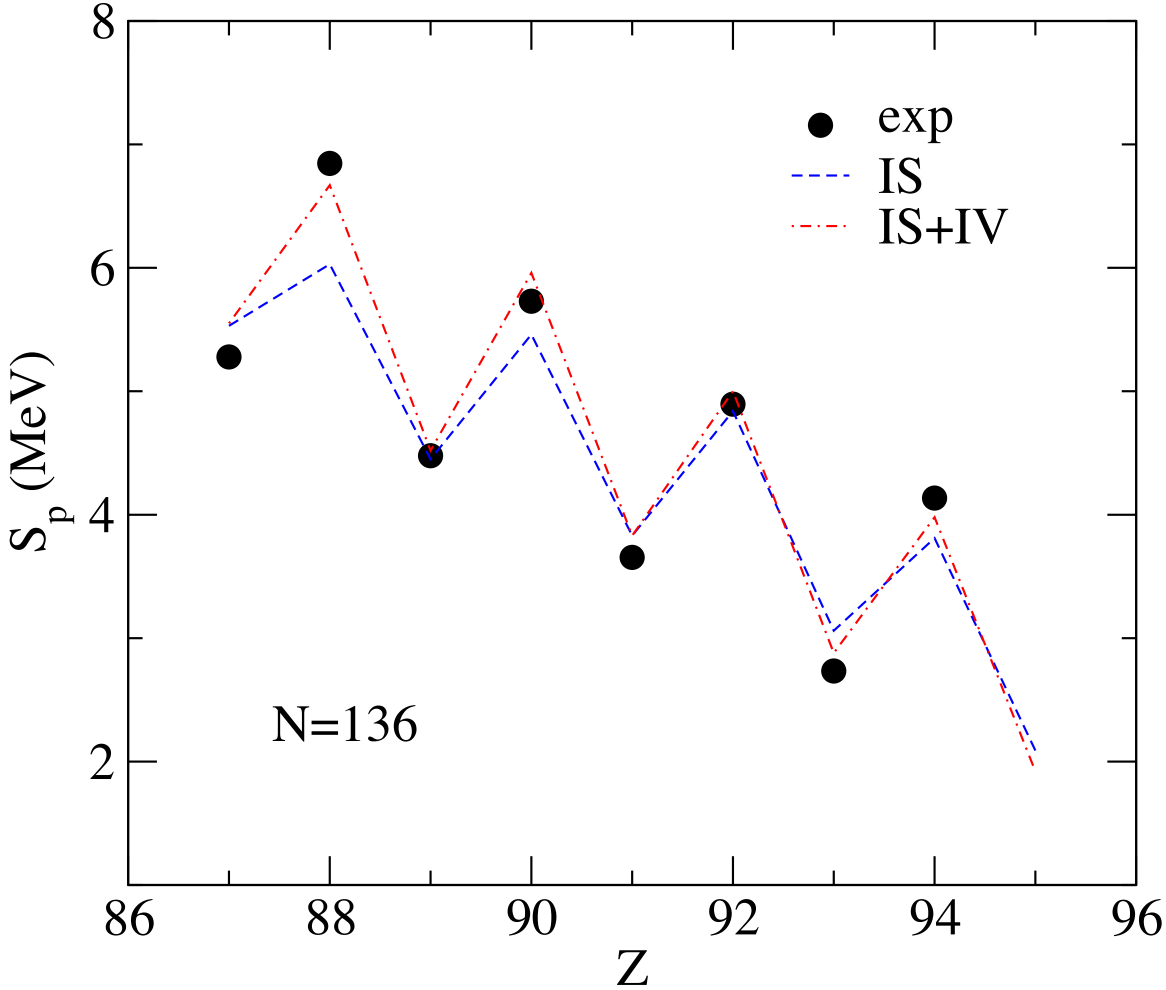




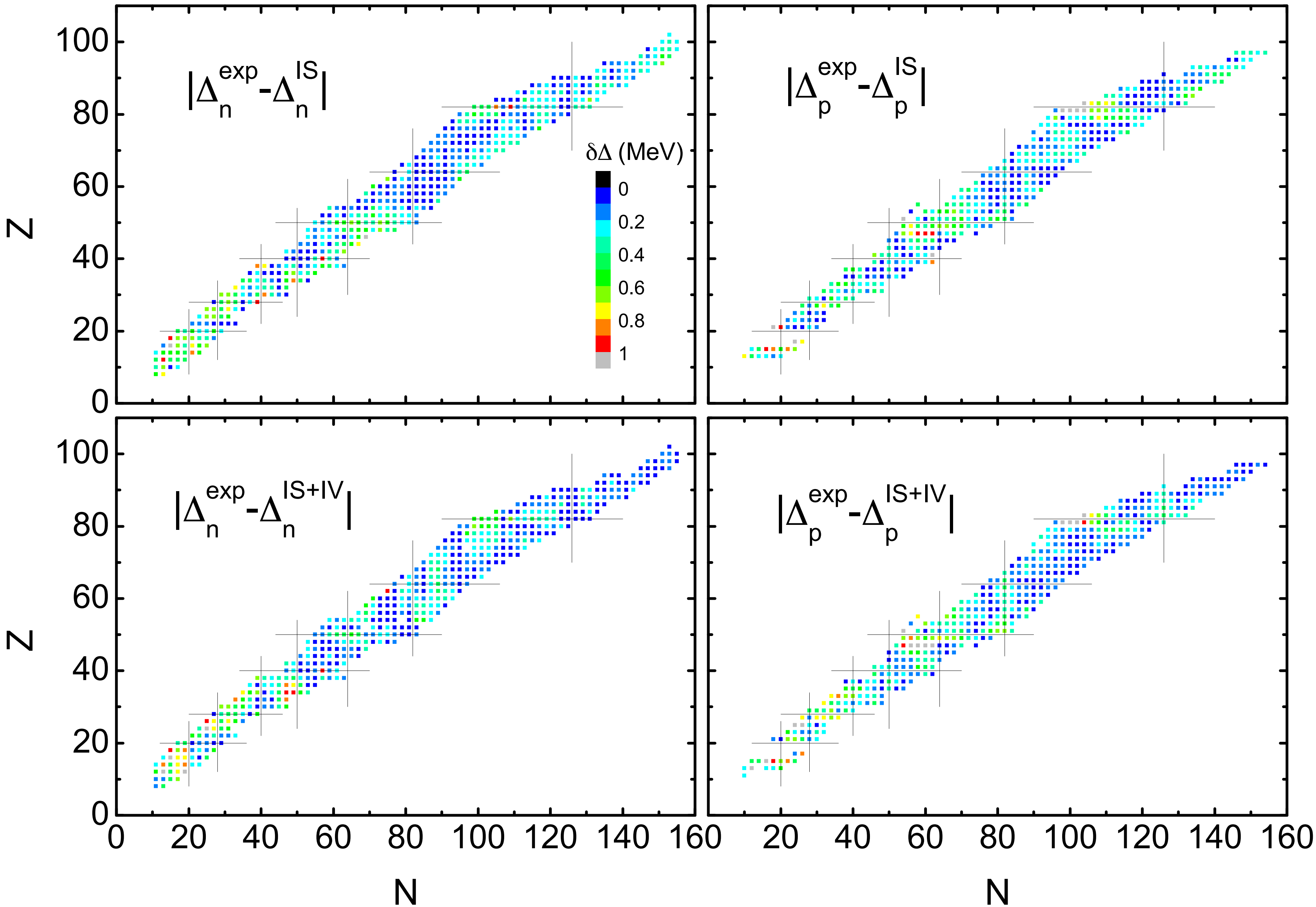
$|S_n^{exp}-S_n^{IS}|$
$|S_p^{exp}-S_p^{IS}|$
$|S_n^{exp}-S_n^{IS+IV}|$
$|S_p^{exp}-S_p^{IS+IV}|$
δS (MeV)
0
0.2
0.4
0.6
0.8
1
Z
N
0
20
40
60
80
100
120
140
160



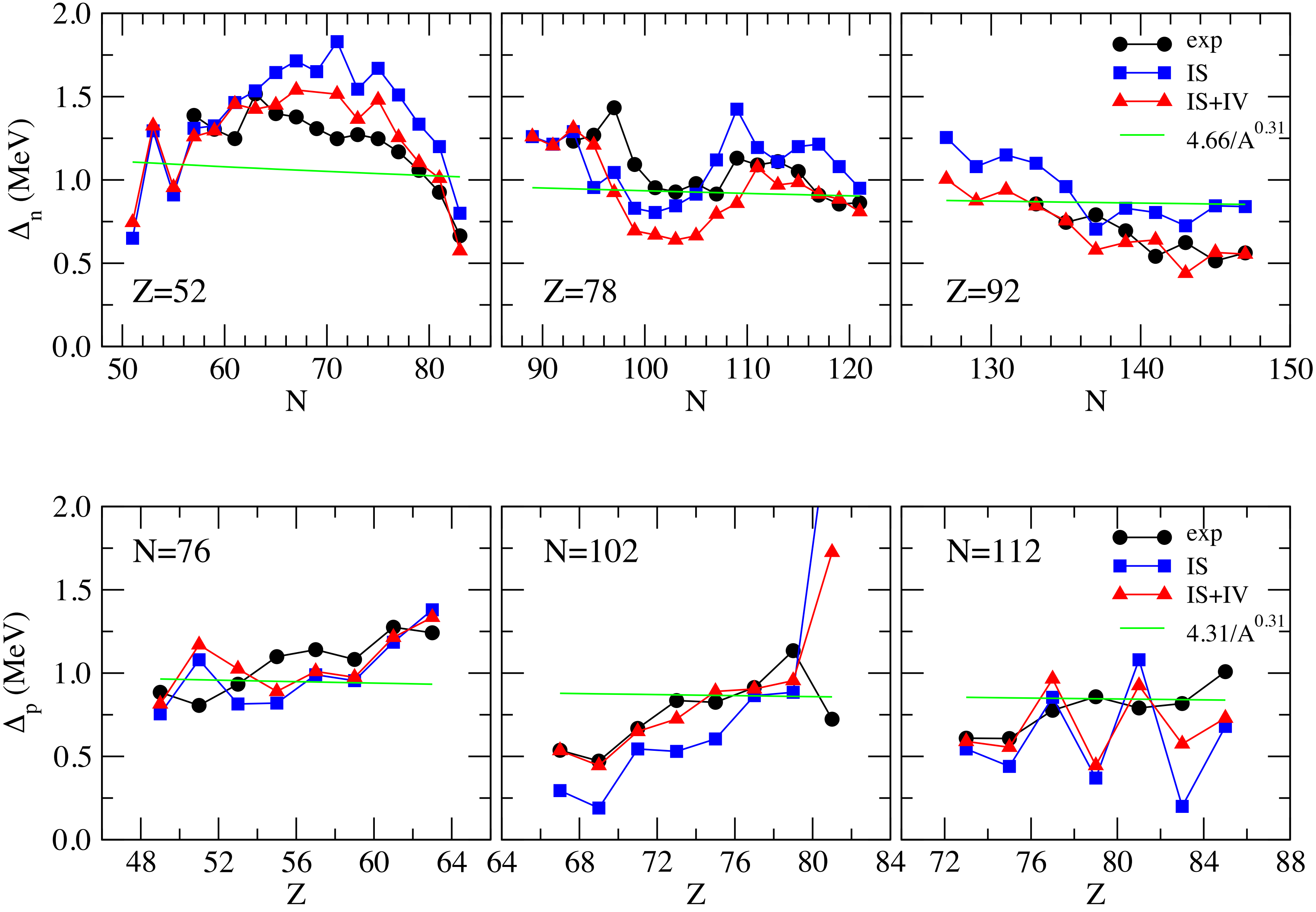
S_p (MeV)
N=50
N=90
N=136
Z
exp
IS
IS+IV
S_n (MeV)
Z=46
Z=72
Z=90
N



exp
IS
IS+IV
N=136
S_p (MeV)
Z
8
6
4
2
86
88
90
92
94
96



$|\Delta_n^{exp}-\Delta_n^{IS}|$
$|\Delta_p^{exp}-\Delta_p^{IS}|$
$|\Delta_n^{exp}-\Delta_n^{IS+IV}|$
$|\Delta_p^{exp}-\Delta_p^{IS+IV}|$
δΔ (MeV)
0
0.2
0.4
0.6
0.8
1
Z
N
0
20
40
60
80
100
120
140
160



Δn (MeV)
N
Z=52
Z=78
Z=92
exp
IS
IS+IV
4.66/A^0.31
Δp (MeV)
Z
N=76
N=102
N=112
4.31/A^0.31

Average Gaps for low and high isospin

TABLE I:

Data set			Low isospin	High isospin	Difference
Neutrons	$Z = 52$	Exp	1.36	1.08	-0.28
		IS	1.52	1.41	-0.11
		IS+IV	1.40	1.19	-0.21
	$Z = 78$	Exp	1.13	0.99	-0.14
		IS	0.96	1.16	0.20
		IS+IV	0.87	0.91	0.04
	$Z = 92$	Exp	0.77	0.56	-0.21
		IS	0.90	0.80	-0.10
		IS+IV	0.70	0.55	-0.15
Protons	$N = 76$	Exp	1.19	0.93	-0.26
		IS	1.13	0.87	-0.26
		IS+IV	1.13	0.98	-0.15
	$N = 102$	Exp	0.96	0.63	-0.33
		IS	0.79	0.39	-0.40
		IS+IV	0.92	0.59	-0.33
	$Z = 112$	Exp	0.87	0.66	-0.21
		IS	0.58	0.61	0.03
		IS+IV	0.67	0.70	0.03

Perspectives

- IS+IV pairing interaction looks promising for the fitting purpose.

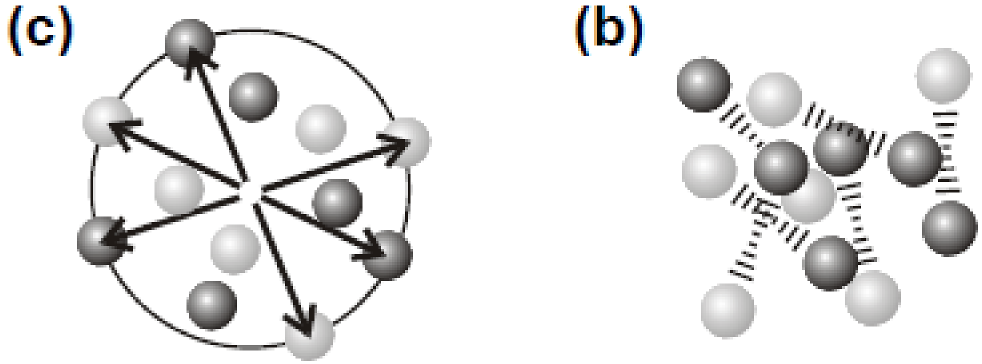
- Extension to include $((N\text{-}Z)/A)^2$ term + Two-body Coulomb.

  (M. Yamagami) (M. Yamagami and H. Nakada)

- Better EDF with pairing correlations: Pairing correlation properties should be isolated from the full functional.

- Are we much better than LDM for pairing gap residuals as a systematics?

  yes !



BCS superfluidity of Cooper pairs

BCS - BEC crossover

BEC superfluidity of bound molecules

$$|v_{\mathsf{pair}}| \to \infty$$

- Weakly interacting fermions
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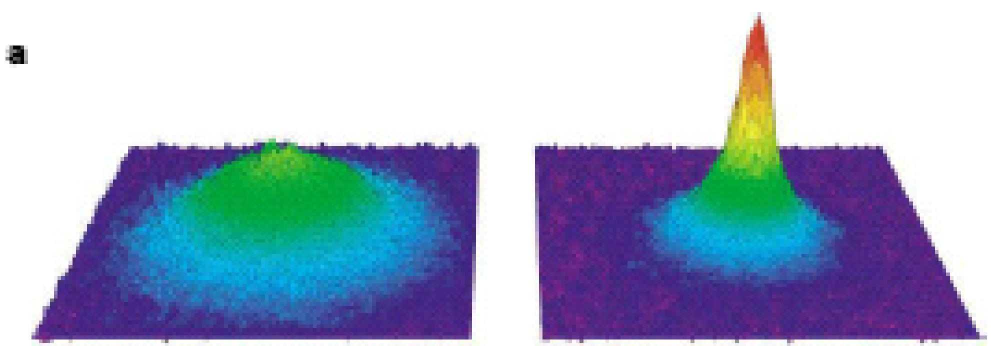
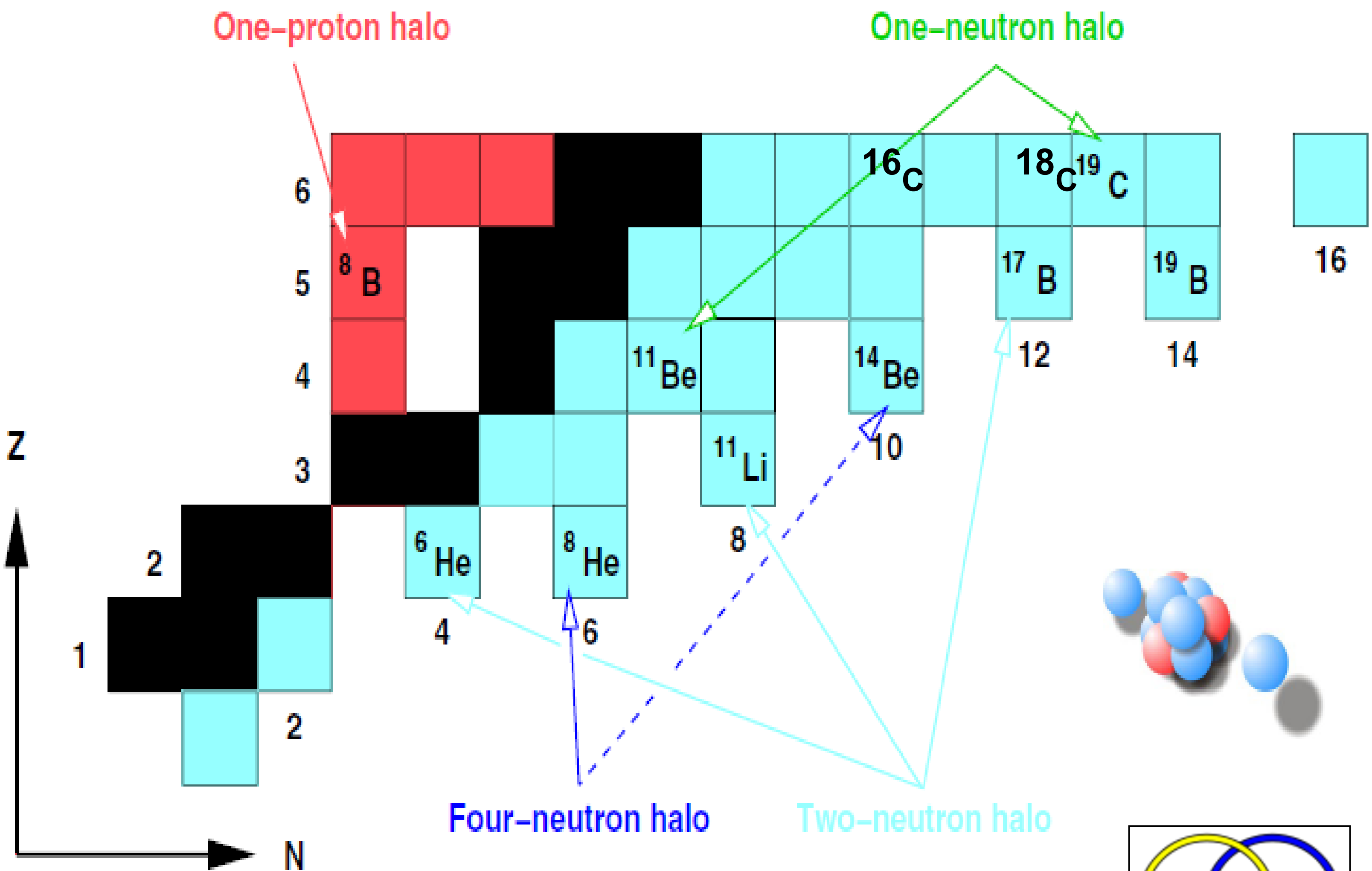
- Interacting "diatomic molecules"
- Correlation in r space (small coherence length)



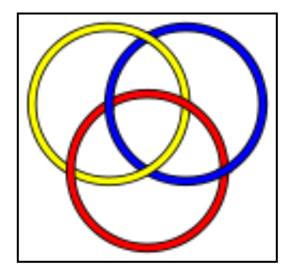
cf. BEC of molecules in ^{40}K

M. Greiner et al., Nature 426('04)537



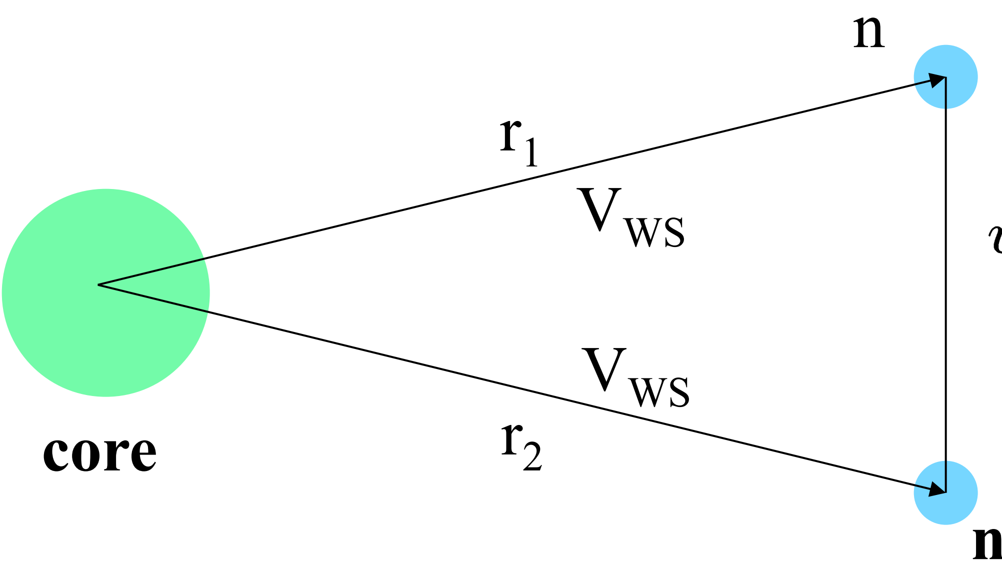
Borromian Nuclei

(any two body systems are not bound, but three body system is bound)



Three-body model

H. Esbensen, G.F. Bertsch, K. Hencken, *Phys. Rev. C56('99)3054*



Density-dependent delta-force

$$v(\boldsymbol{r}_1, \boldsymbol{r}_2) = v_0(1 + \alpha\rho(r)) \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$v_0 \longleftarrow a_{nn}$

$\alpha \longleftarrow S_{2n}$

$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}$$

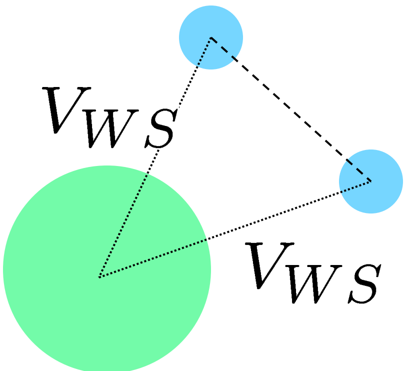
(note) recoil kinetic energy of the core nucleus

Important for dipole excitation

- Hamitonian diagonalization with WS basis
- Continuum can be included by solving Green's functions
- Pauli blocking is properly taken into account.

→ Application to ^{11}Li, ^{6}He

Two-particle wave functions (J=0 pairs)



$$\hat{h}\,\psi_{nljm}(\boldsymbol{r}) = \epsilon_{nlj}\,\psi_{nljm}(\boldsymbol{r})$$

$$\Psi^{(2)}_{nn'lj}(\boldsymbol{r},\boldsymbol{r}') = \sum_m \langle jmj-m|00\rangle \psi_{nljm}(\boldsymbol{r})\psi_{n'lj-m}(\boldsymbol{r}')$$

Hamiltonian diagonalization

$$\Psi_{gs}(\boldsymbol{r},\boldsymbol{r}') = \mathcal{A}\sum_{nn'lj} \alpha_{nn'lj}\,\Psi^{(2)}_{nn'lj}(\boldsymbol{r},\boldsymbol{r}')$$

- **Continuum: box discretization**
- **Energy cut-off:** $\epsilon_{nlj} + \epsilon_{n'lj} \leq \frac{A_c+1}{A_c} E_{\rm cut}$

Application to ^{11}Li, and ^{6}He

^{11}Li, ^{6}He: Typical Borromean nuclei

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^{11}Li: $a_{nn} = -15$ fm, $E_{\text{cut}} = 30$ MeV, $R_{box} = 40$ fm

$$a_{n-{}^9Li}(s) = -30 + (+12/-31)\, fm$$

(Efimov states ?)

WS: adjusted to $p_{3/2}$ energy in ^{8}Li & n-^{9}Li elastic scattering

Parity-dependence ⟵ to increase the s-wave component

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^{6}He: $a_{nn} = -15$ fm, $E_{\text{cut}} = 40$ MeV, $R_{box} = 30$ fm

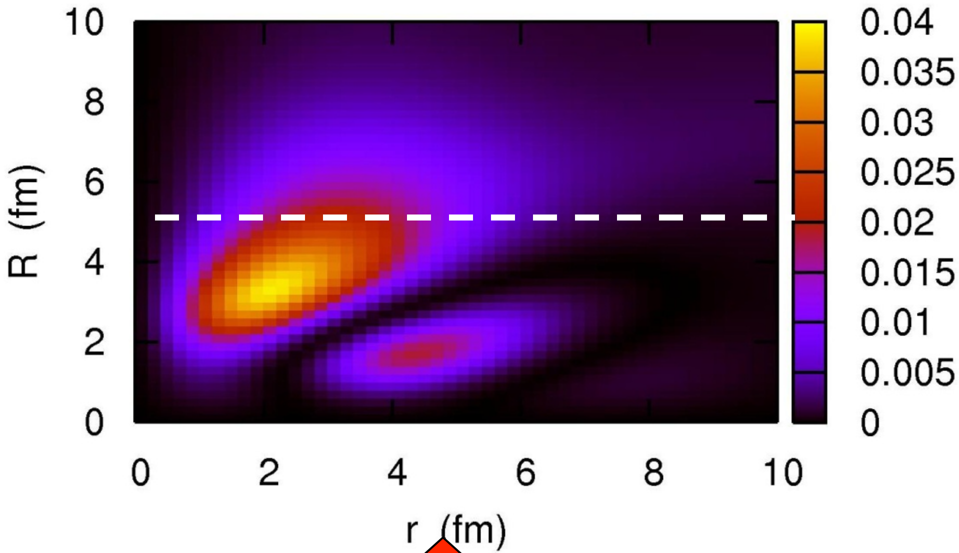
$$a_{n-{}^4He}(s) = 4.97 \pm 0.12\, fm$$

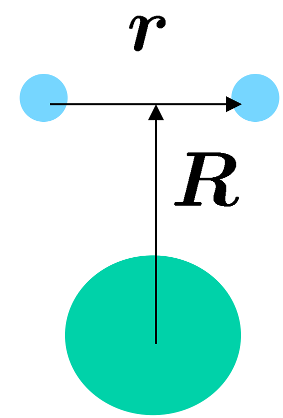
WS: adjusted to n-α elastic scattering

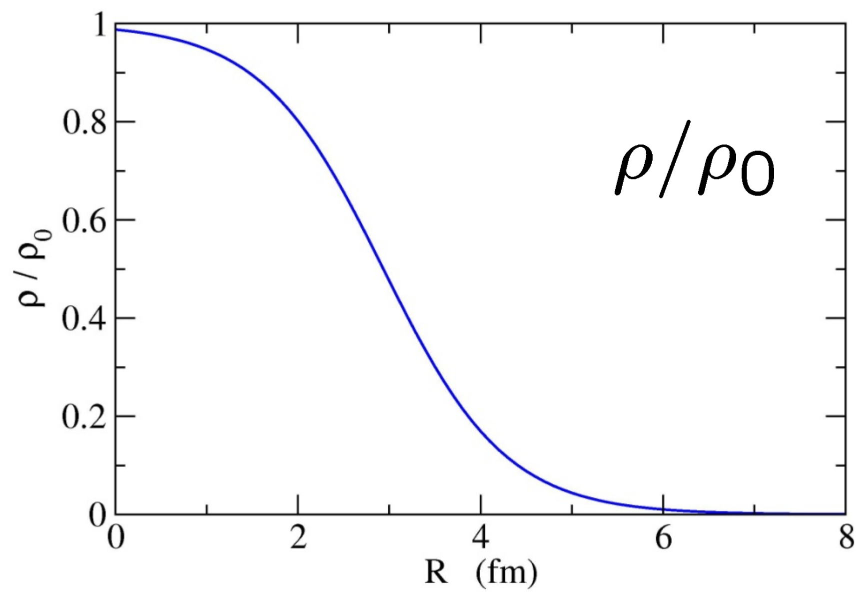
Coexistence of BCS-BEC like behaviour of Cooper Pair in ^{11}Li

$$\Psi^{(S=0)}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\sum_L f_L(r,R)[Y_L(\hat{\boldsymbol{r}})Y_L(\hat{\boldsymbol{R}})]^{(00)}$$

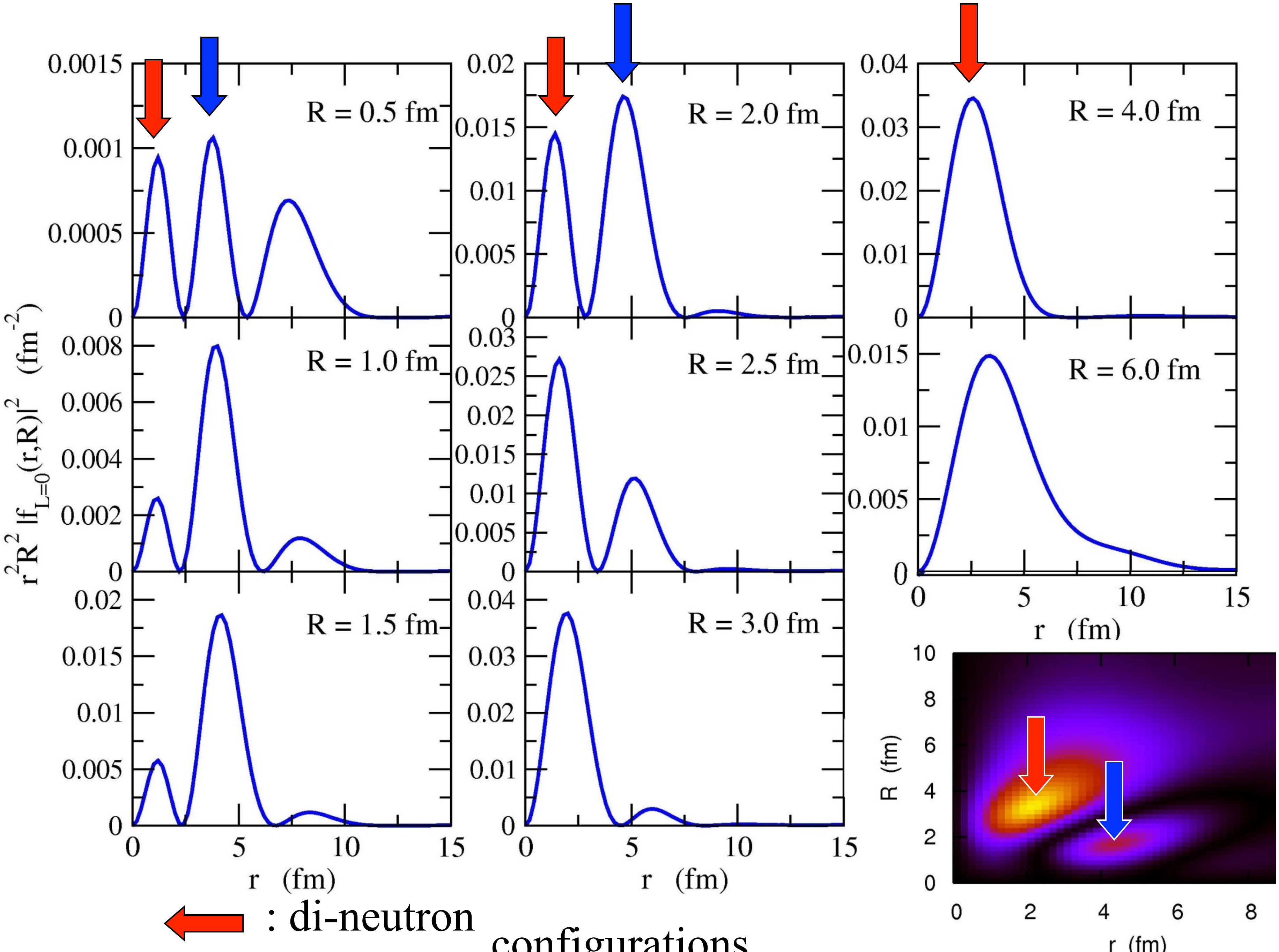
$r^2R^2|f_{L=0}(r,R)|^2$







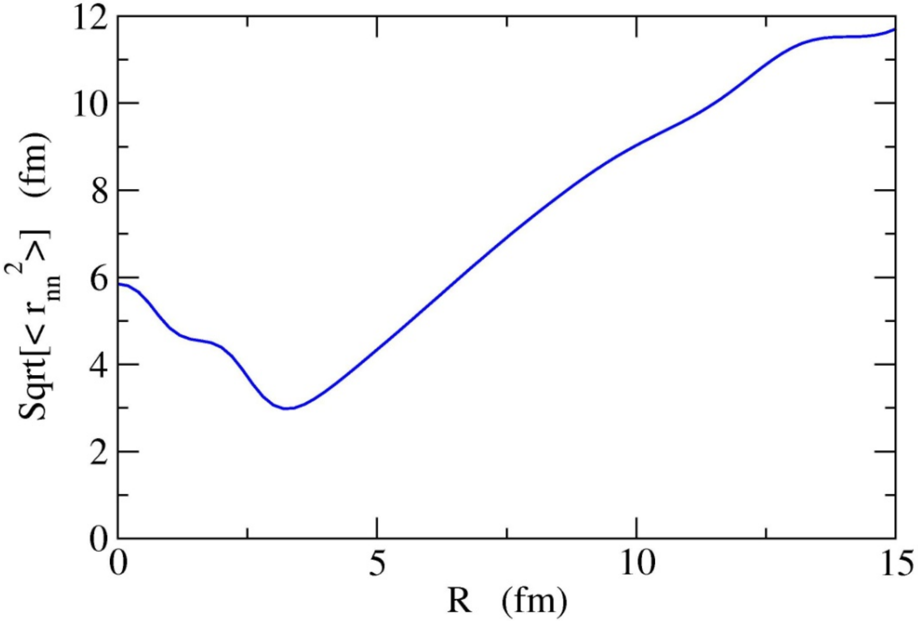
Probing the behavior
at several densities

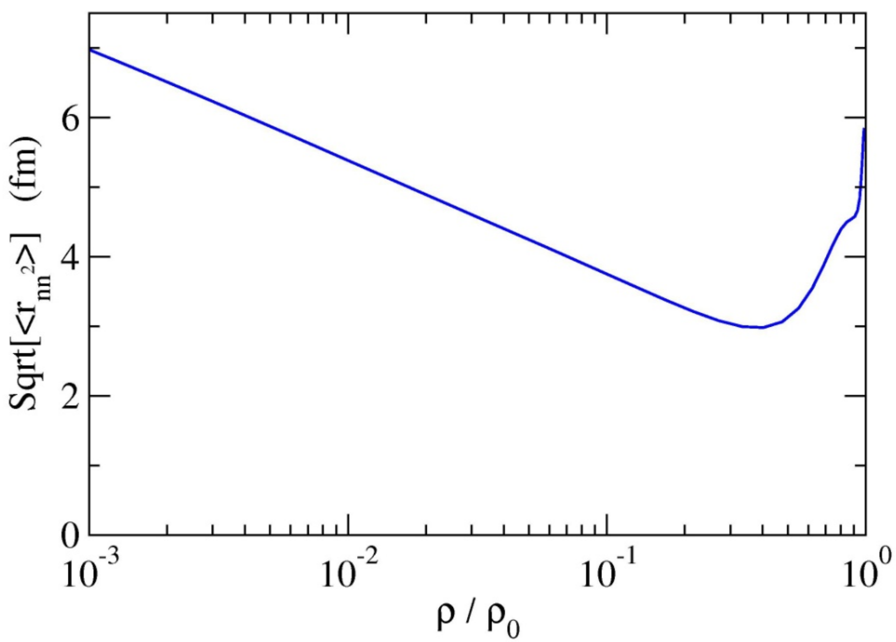


: di-neutron configurations

: cigar-like

$$\sqrt{\langle r_{nn}^2 \rangle}(R) = \sqrt{\frac{\int r^4 dr\, |f_{L=0}(r,R)|^2}{\int r^2 dr\, |f_{L=0}(r,R)|^2}}$$





Matter Calc.

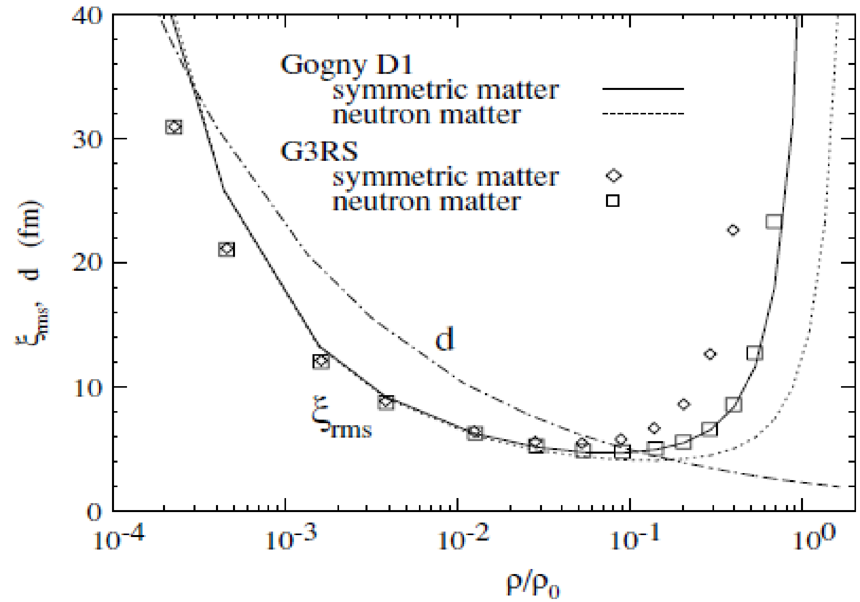
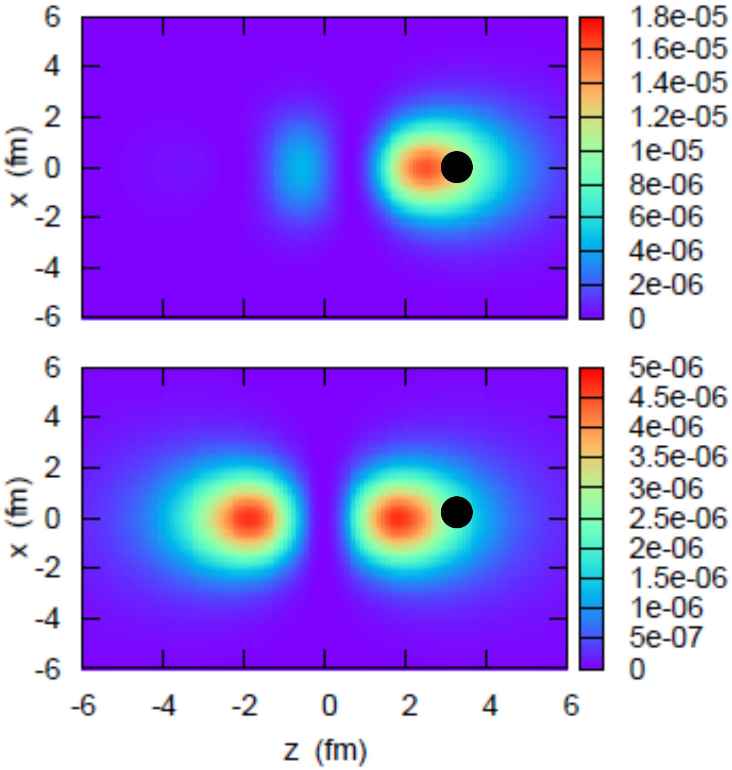


FIG. 3. The rms radius $\xi_{\rm rms}$ of the neutron Cooper pair in uniform matter, plotted as a function of the neutron density ρ/ρ_0. The results for symmetric nuclear and neutron matter obtained with the Gogny D1 force are shown by the solid and dotted curves, respectively, while the results for symmetric nuclear and neutron matter with the G3RS force are shown by the diamond and square symbols, respectively. The average interneutron distance $d = \rho^{-1/3}$ is plotted with the dotted-dashed curve.

M. Matsuo, PRC73('06)044309

Two-particle density



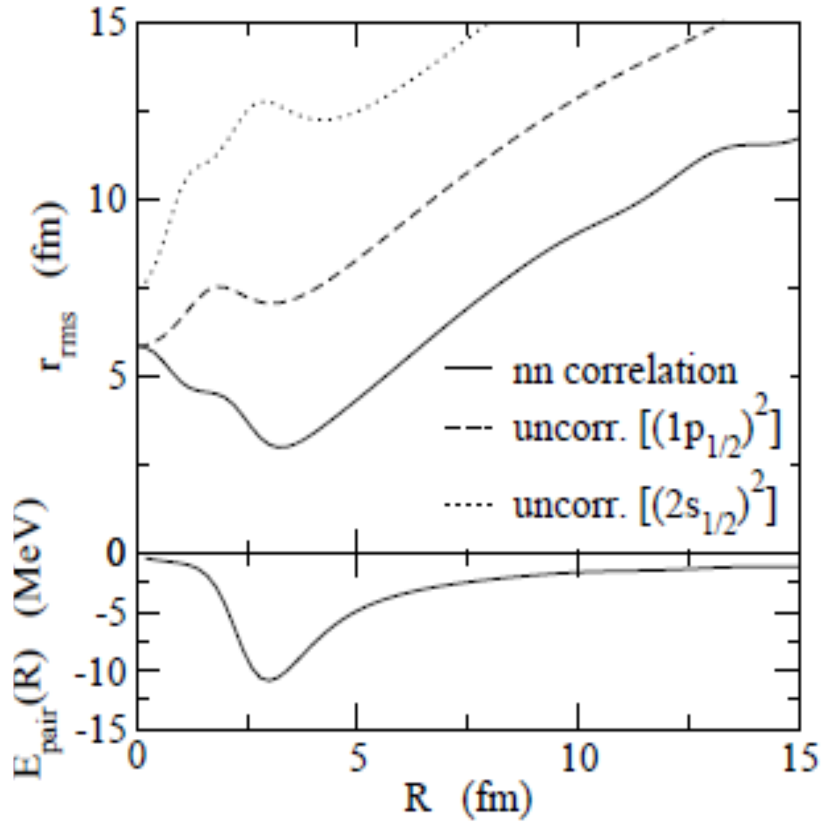


FIG. 2: (Color online) A two-dimensional (2D) plot for the two-particle density for the correlated pair (the upper panel) and for the uncorrelated $[(1p_{1/2})^2]$ configuration (the lower panel). It represents the probability distribution for the spin-up neutron when the spin-down neutron is at $(z, x) = (3.4, 0)$ fm.

Experimental proof of di-neutron

Dipole Excitations

Response to the dipole field:

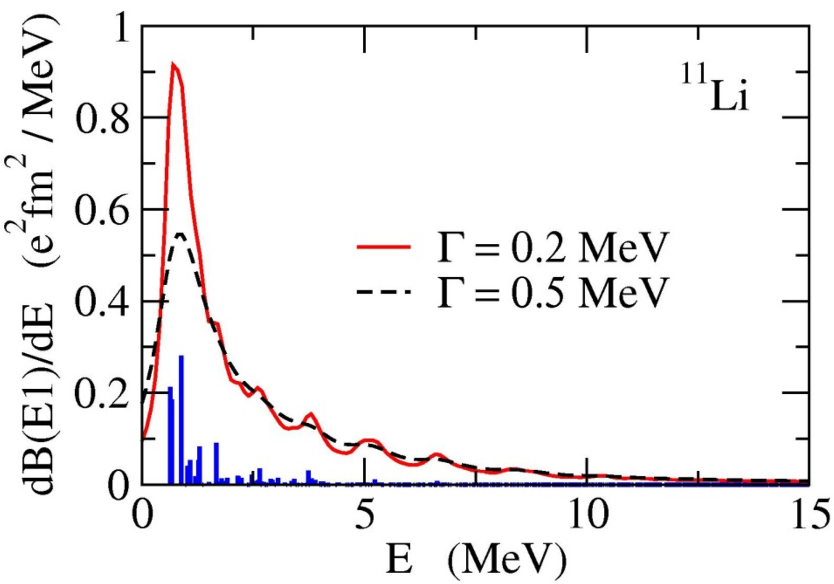
$$B_k(E1) = 3\,|\langle \Psi^k_{1^-} | \hat{D}_0 | \Psi_{gs} \rangle|^2$$

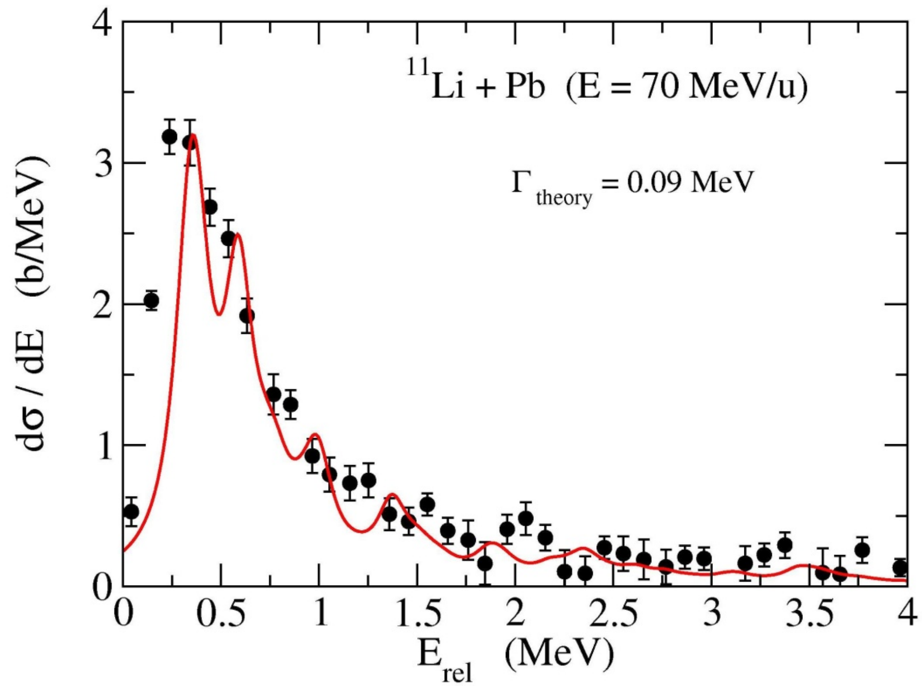
$$\hat{D}_M = -\frac{Ze}{A} \sum_{i=1,2} r_i Y_{1M}(\hat{\boldsymbol{r}}_i)$$

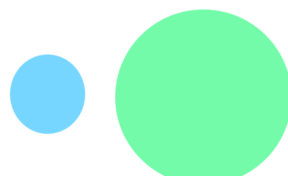
Smearing:

$$B(E1) = \sum_k \frac{\Gamma}{\pi} \frac{B_k(E1)}{(E - E_k)^2 + \Gamma^2}$$

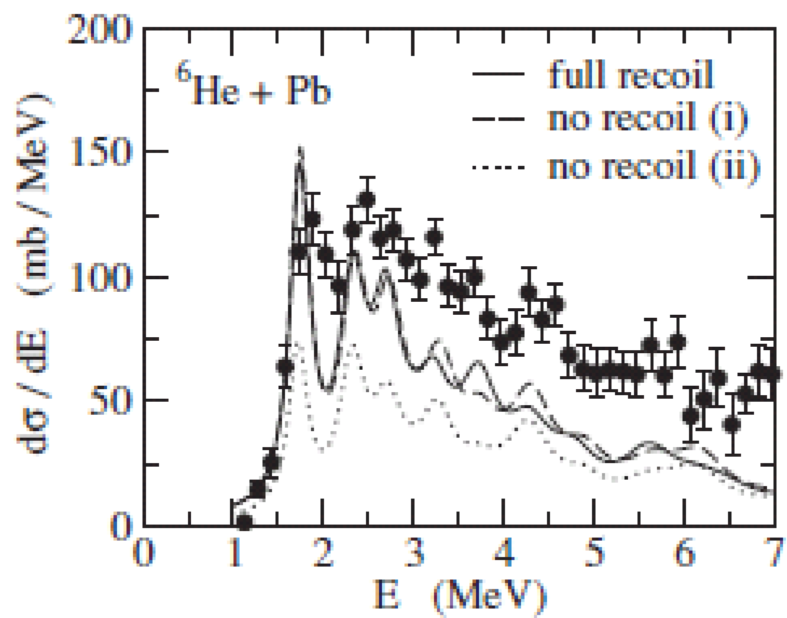
Comparison with expt. data (^{11}Li)







T. Aumann et al., PRC59



E_{peak}=0.66 MeV

B(E1) = 1.31 e^{2}fm^2 (E < 3.3 MeV)

T. Nakamura et al., PRL96,252502(2006)

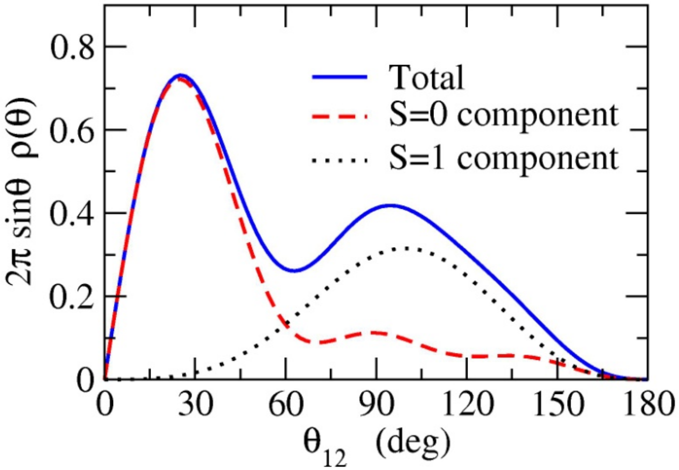
$E_{\text{peak}} \sim$ 0.6 MeV

B(E1) = (1.42 +/- 0.18) e^{2}fm^2

(E < 3.3 MeV)

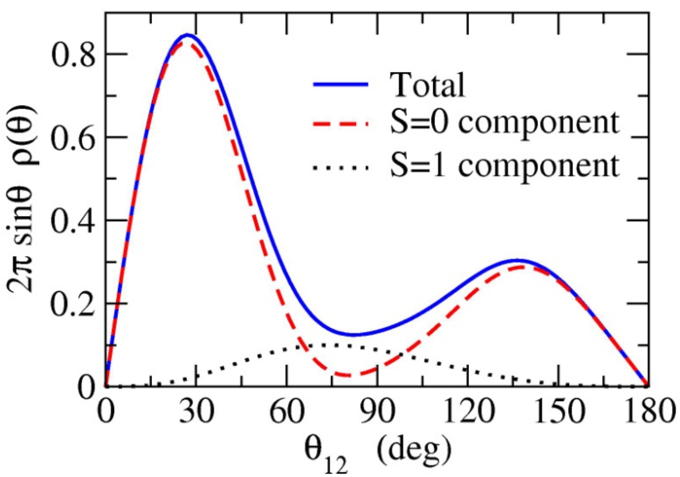
Geometry of Borromean nuclei

^{11}Li

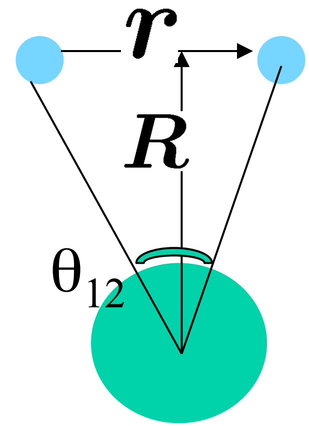


$\rightarrow \langle\theta_{12}\rangle = 65.29$ deg.

^{6}He



$\rightarrow \langle\theta_{12}\rangle = 66.33$ deg.



"experimental" mean opening angle

$\sqrt{\langle R^2\rangle} \leftarrow$ **B(E1)**

$\sqrt{\langle r^2\rangle} \leftarrow$ **matter radius**

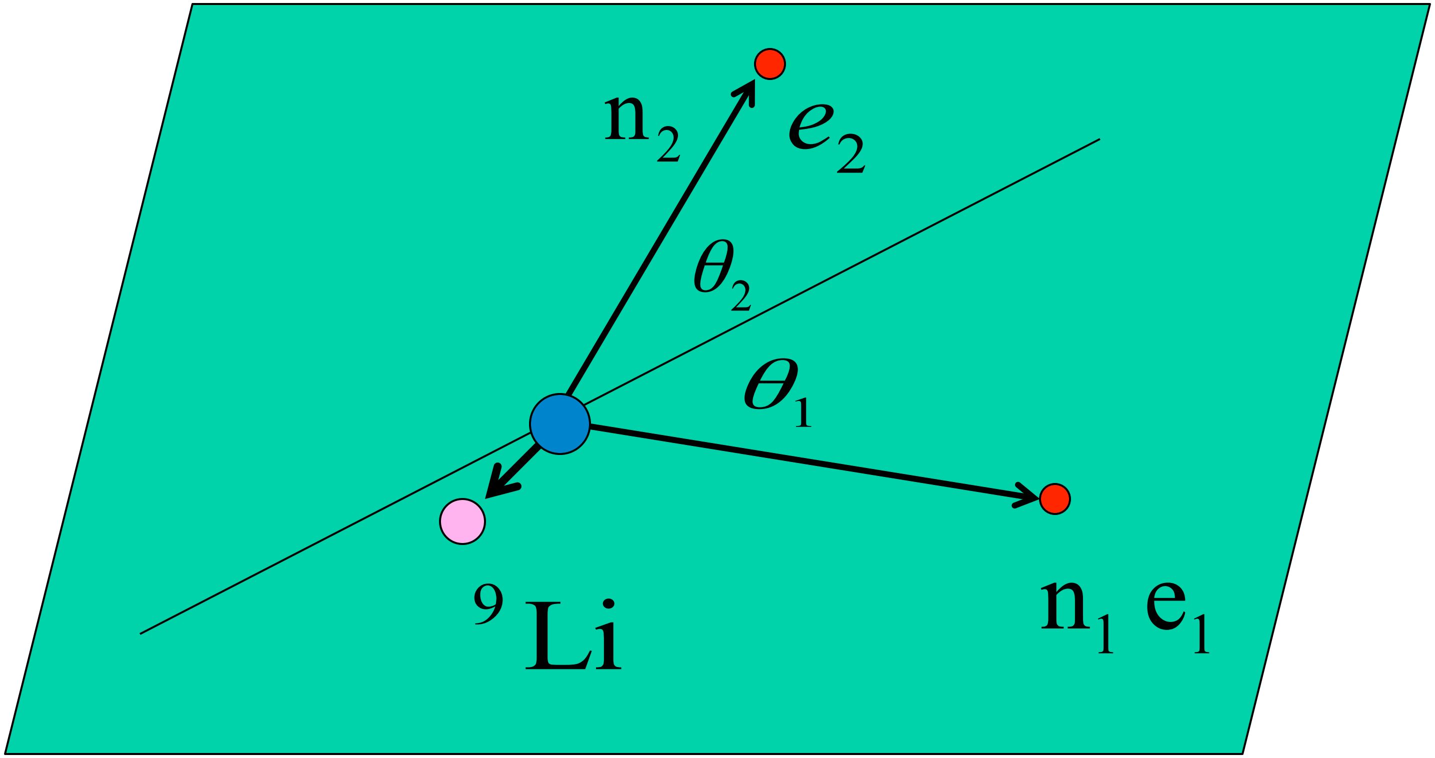
$$\langle\theta_{12}\rangle = 65.2^{+11.4}_{-13.0} \quad (^{11}\text{Li})$$

$$= 74.5^{+11.2}_{-13.1} \quad (^{6}\text{He})$$

K.Hagino and H. S.,PRC76('07)047302

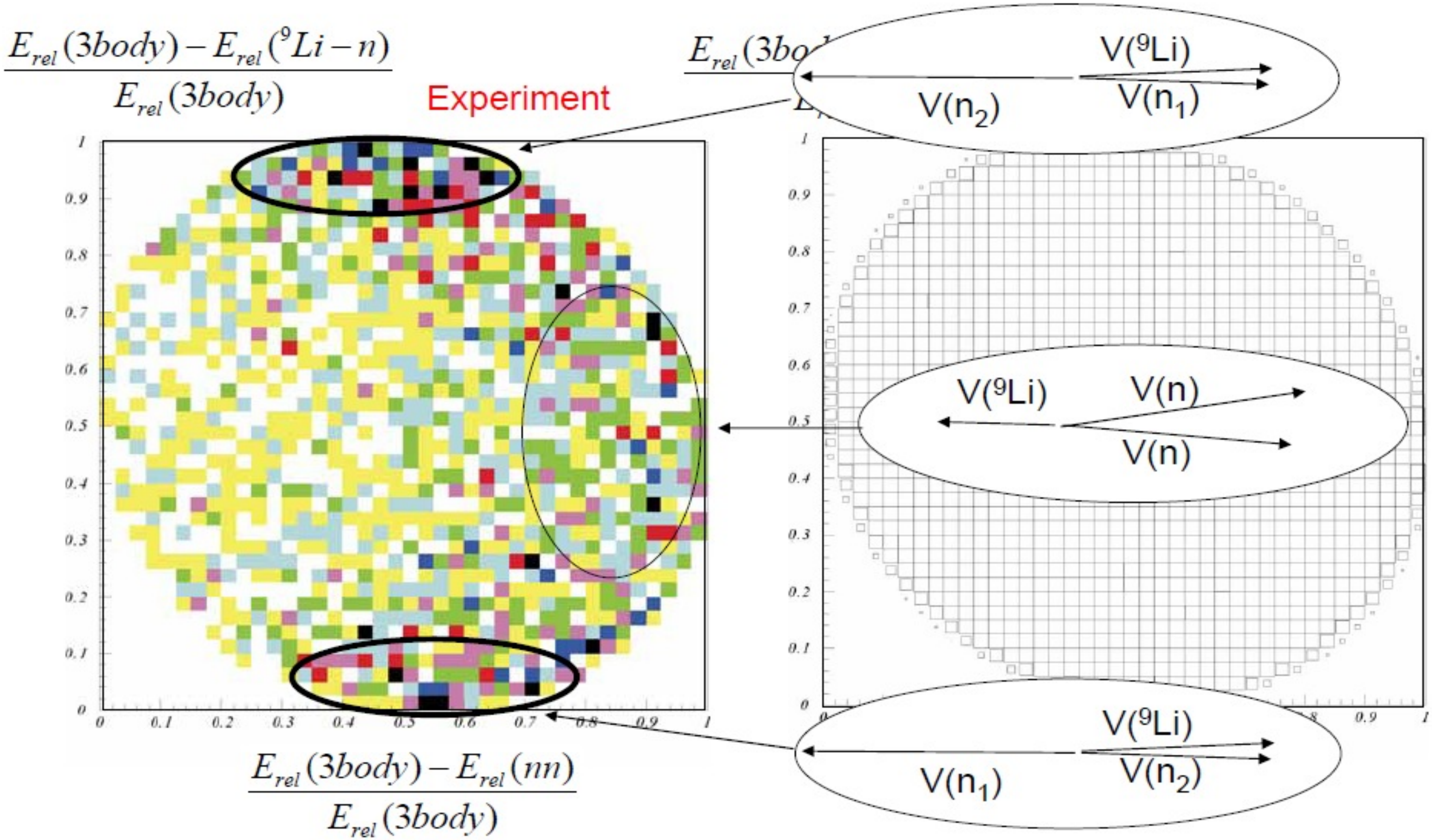
C.A. Bertulani and M.S. Hussein,
PRC76('07)051602

^{11}Li three-body break-up cross sections



K. Hagino, H.S.,T.Nakamura and S.Shimoura, PRC80,031301(R)(2009)

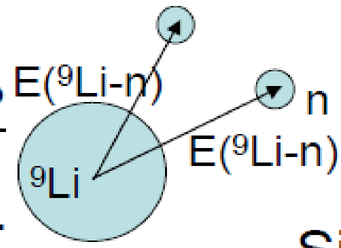
Dalitz Plot of Triple coincidence experiments



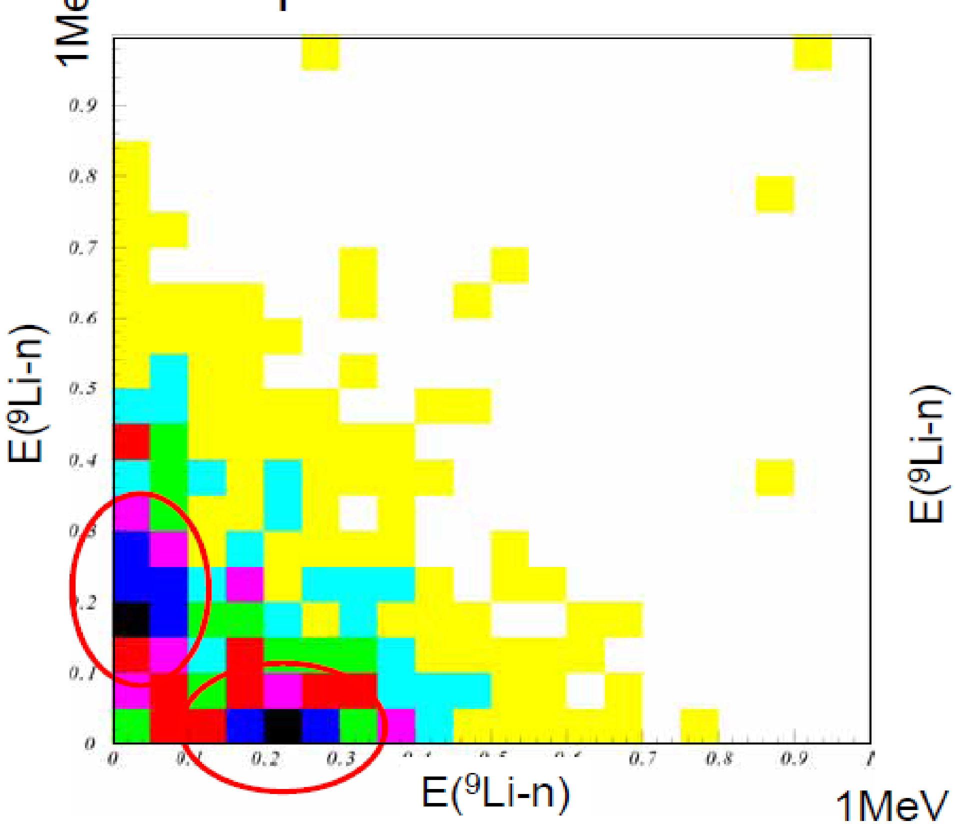
Preliminary **Exp. T. Nakamura et al., to be published**

Nakamura-san's slides

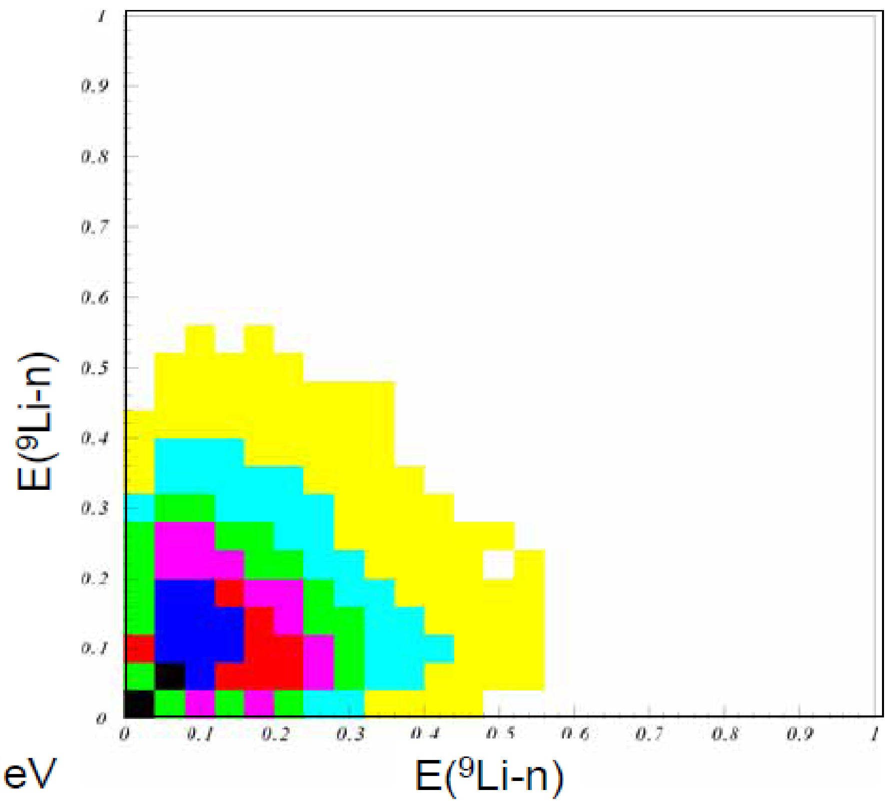
<u>Further Correlation?</u>



Experimental Result



Simulation (Phase Space)

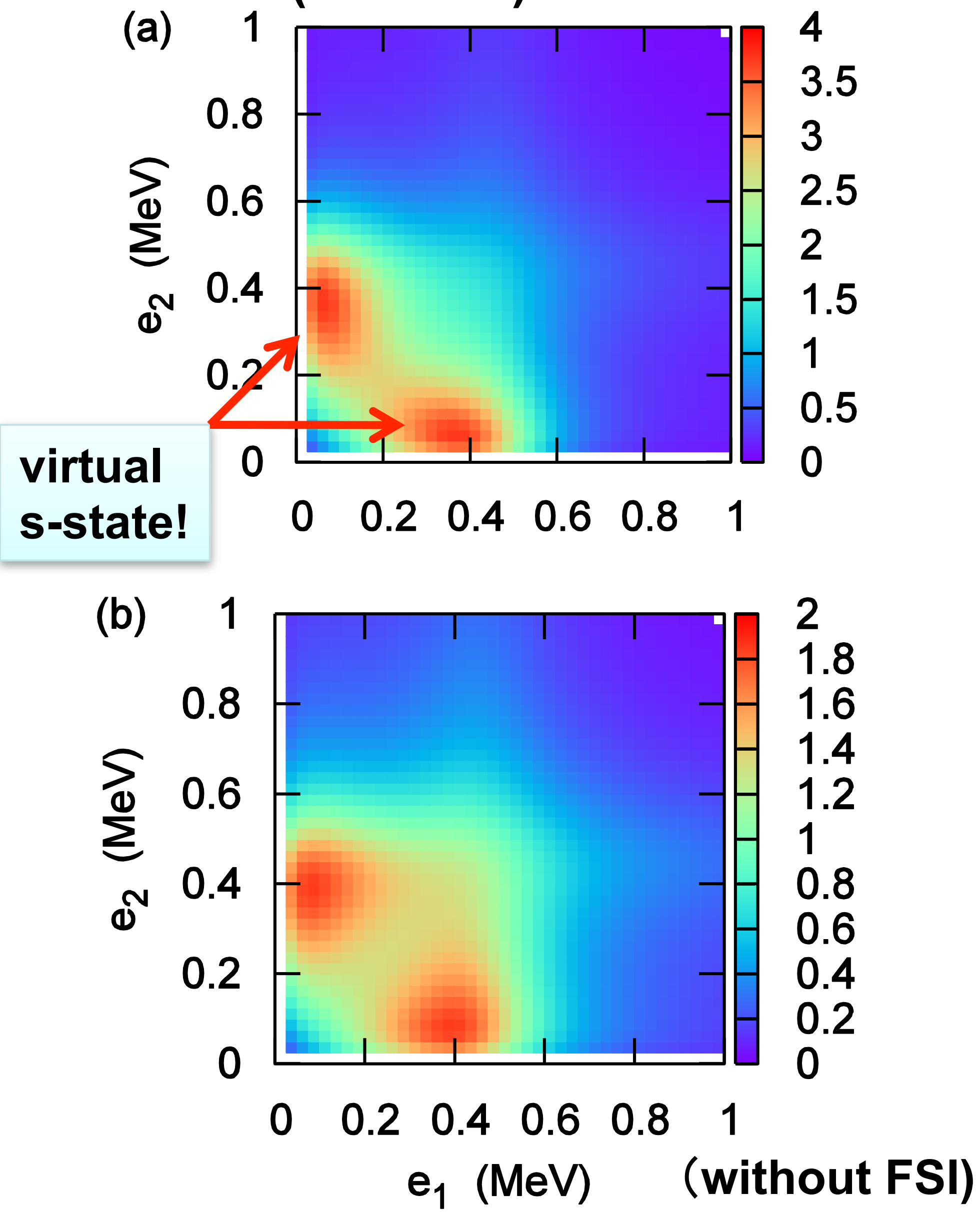


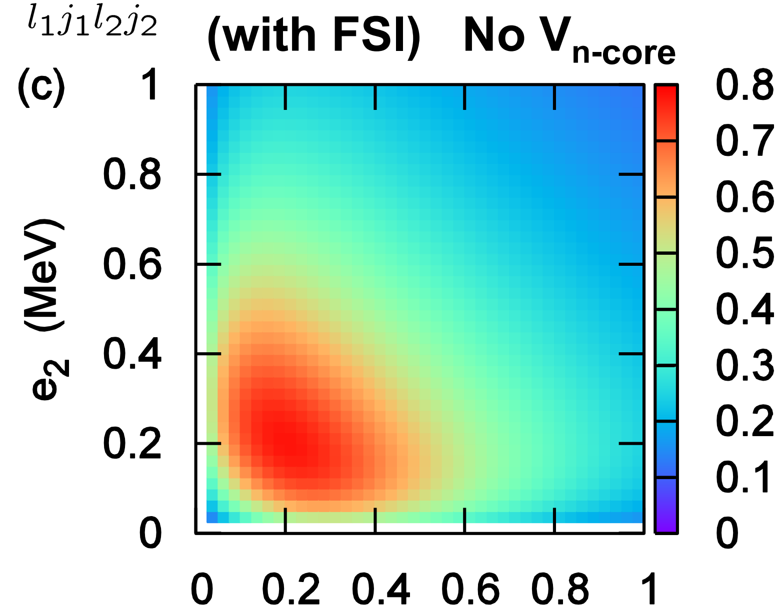
preliminary

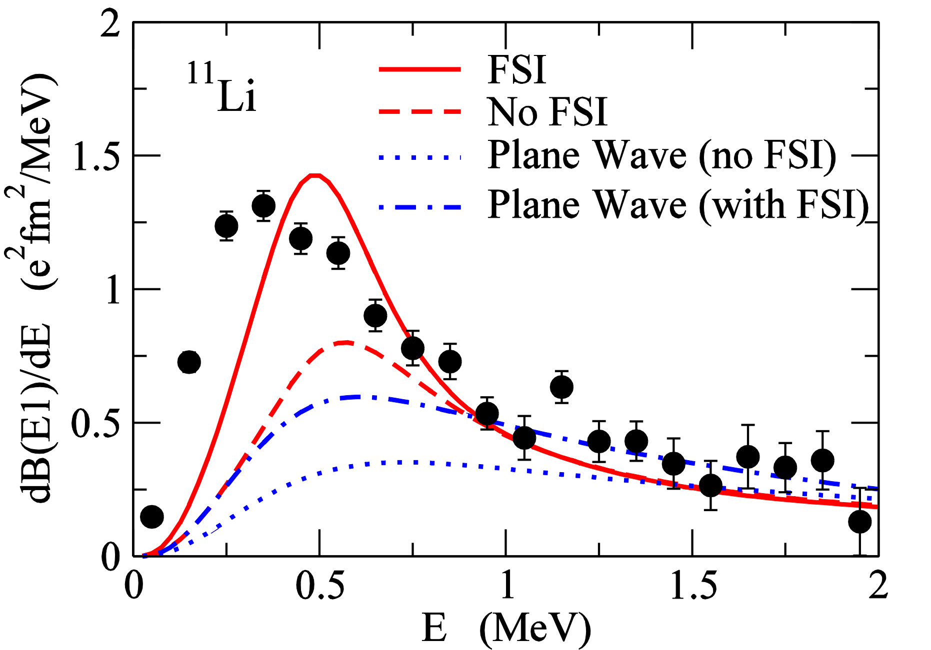
Double differential strength function for ^{11}Li

full calculation (with FSI)

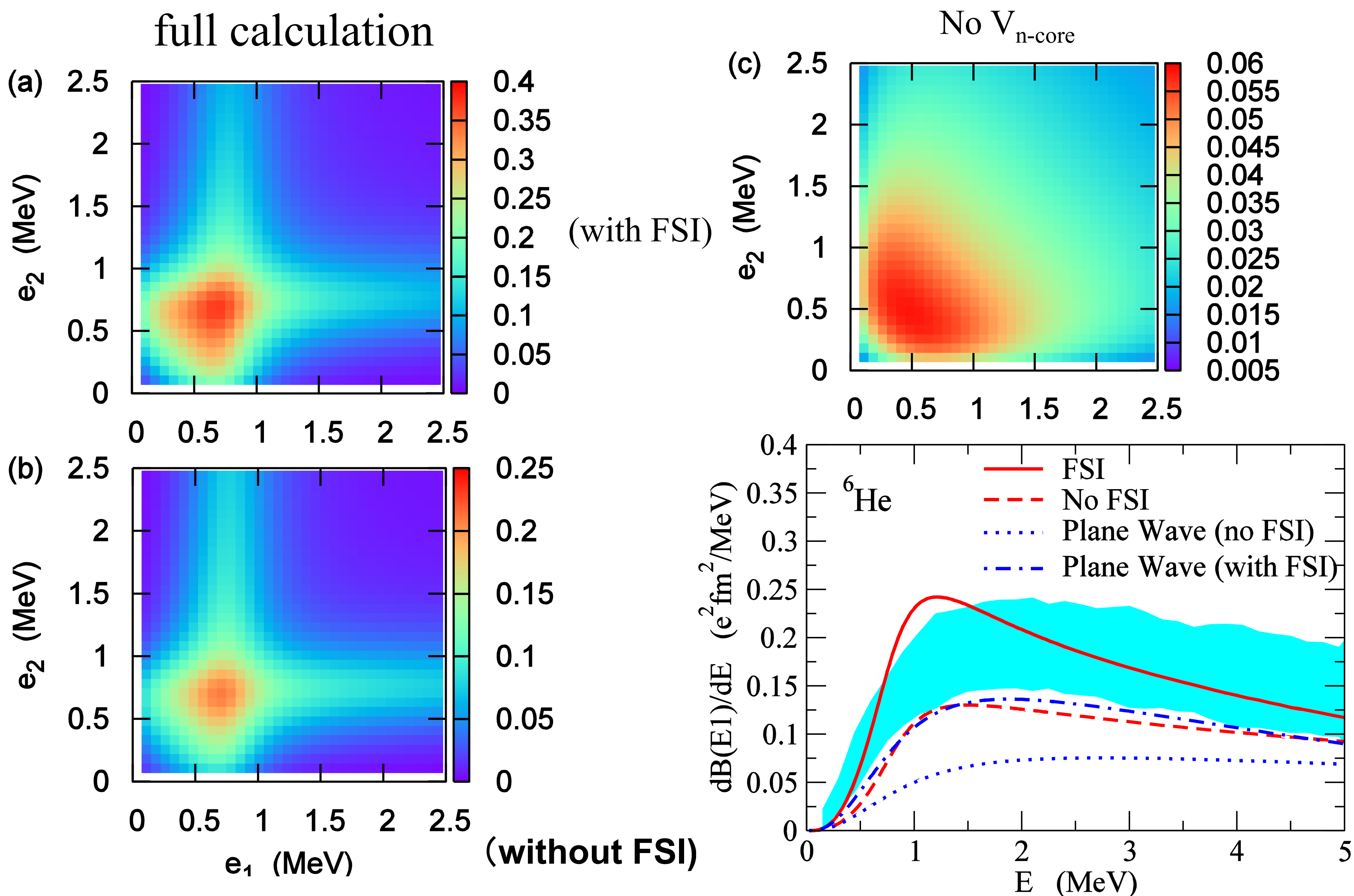
$$\frac{d^2B(E1)}{de_1\,de_2} = \sum_{l_1j_1l_2j_2} |\langle[(e_1j_1l_1)(e_2j_2l_2)]^{(J=1)}|\hat{O}_{E1}|\Psi_{gs}\rangle|^2$$







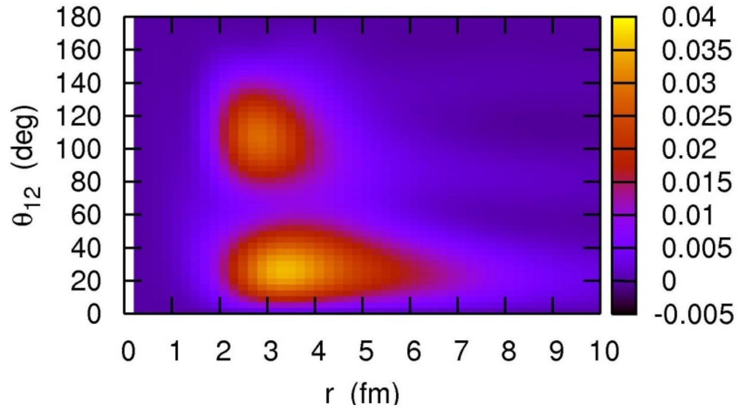
$$\frac{d^2B(E1)}{de_1\,de_2} = \sum_{l_1 j_1 l_2 j_2} |\langle[(e_1 j_1 l_1)(e_2 j_2 l_2)]^{(J=1)}|\hat{O}_{E1}|\Psi_{gs}\rangle|^2$$

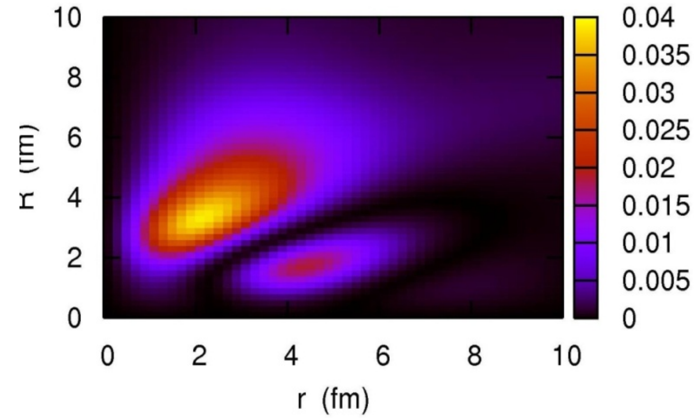


Summary I

➢ Application of three-body model to Borromean nuclei

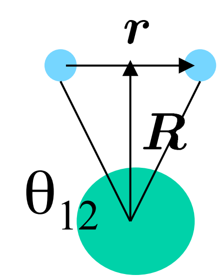
^{11}Li





➢ E1 response and geometry of Borromean nuclei

- strong pair correlations in di-neutrons



➢ Di-neutron wave function for each R

- **Concentration of a Cooper pair on the nuclear surface**
- **Relation to BCS-BEC crossover phenomenon**

➢ n-n coincidence cross sections from 11Li* and 6He*

- **importance of n-core interaction**
- **clear evidence of virtual s-state in n-9Li system**
- **correlation angle is determined experimentally.**

Summary II

- ➢Strong di-proton correlations is found in 17Ne
- ➢Two body Coulomb interaction decreases 13% of di-proton correlations.

Recent publications:

- **Di-neutron correlations in ^{11}Li and ^{6}He**

➢K.Hagino and H. S., PRC72('05) 044321.
➢K.H.agino and H. S., PRC75('07)021301(R).
➢K.Hagino, H. S., J. Carbonell, and P. Schuck, PRL99('07)022506.
➢H. Esbensen, K.Hagino, P. Mueller, and H. S., PRC76('07)024302.
➢K.Hagino and H. S., PRC76('07) 047302.

- **energy and angular n-n coincidence cross sections**

➢K.Hagino, H.S. ,T. Nakamura and S. Shimoura, PRC80,031301 (R)(2009).

- **Di-proton correlations in 17Ne**

➢T.Oishi, K. Hagino and HS, PRC82,024315 (2010).