Microscopic approach to large-amplitude deformation dynamics with local QRPA inertial functions

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  Microscopic determination of inertial functions in the quadrupole collective Hamiltonian

  Hinohara, KS, Nakatsukasa, Matsuo, Matsuyanagi, PRC **82**, 064313 (2010)

- **Application to oblate-prolate shape coexistence (72Kr)**

  KS & Hinohara, Nucl. Phys. A **849**, 53 (2011)

- **Application to shape transition in neutron-rich Cr isotopes around N=40**

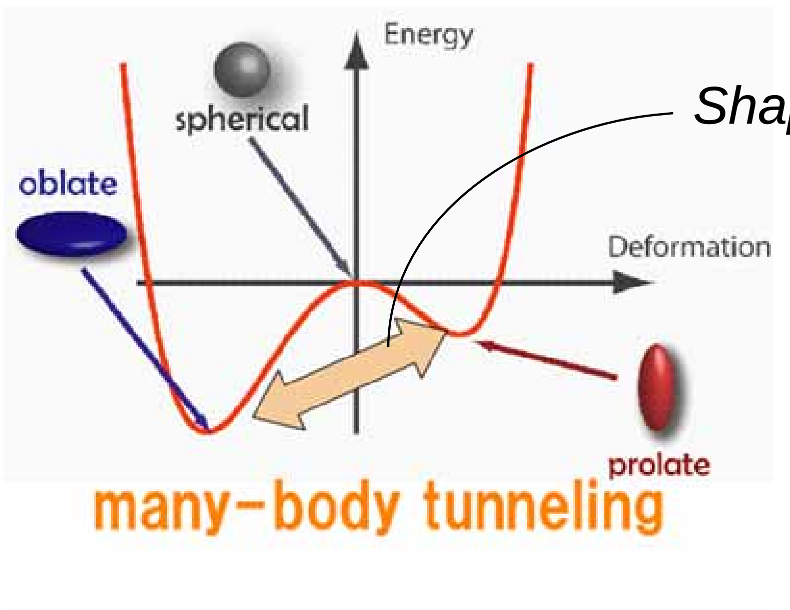
  KS et al., in preparation

- **Summary**

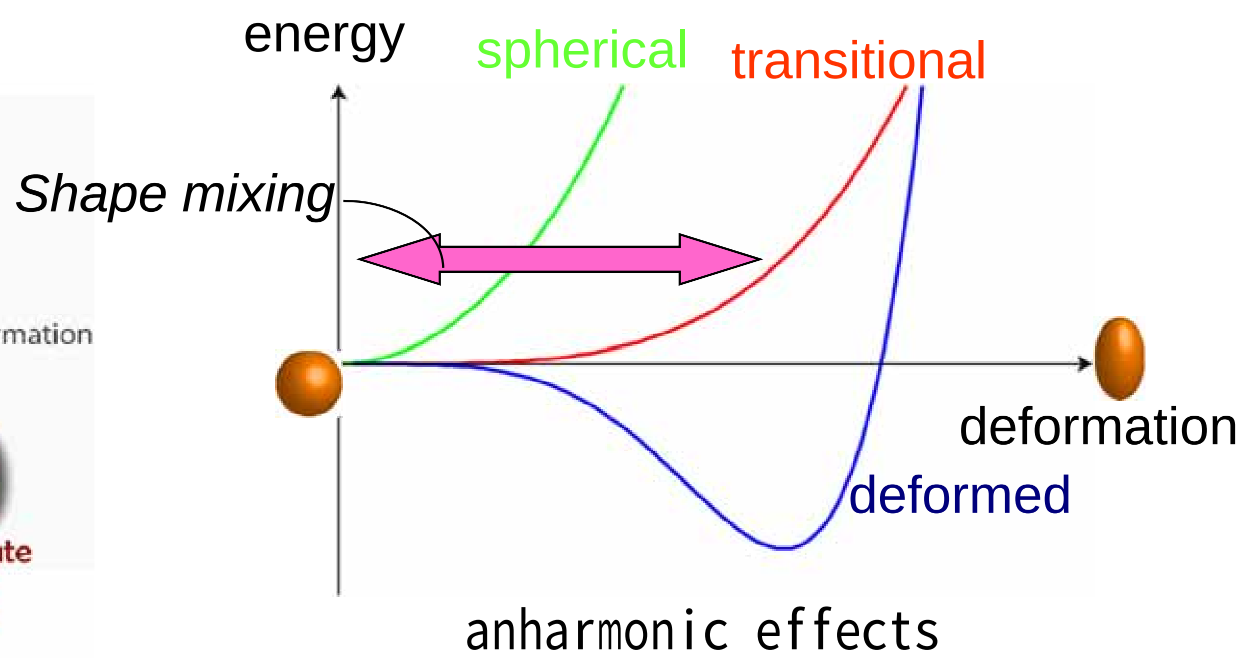
Microscopic approach to large-amplitude collective motion

A new method of determining the 5D quadrupole collective Hamiltonian:
Constrained HFB + Local QRPA method

Shape coexistence



transitional nuclei



Beyond small-amplitude approximation

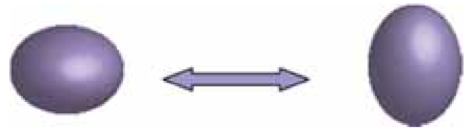
5D quadrupole collective Hamiltonian

(Generalized Bohr-Mottelson Hamiltonian) :

$$H = T_{\mathrm{vib}} + T_{\mathrm{rot}} + \boxed{V(\beta,\gamma)}$$

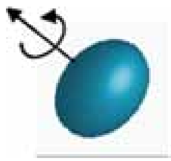
collective potential

$$T_{\mathrm{vib}} = \frac{1}{2}\boxed{D_{\beta\beta}(\beta,\gamma)}\dot{\beta}^2 + \boxed{D_{\beta\gamma}(\beta,\gamma)}\dot{\beta}\dot{\gamma} + \frac{1}{2}\boxed{D_{\gamma\gamma}(\beta,\gamma)}\dot{\gamma}^2$$



vibrational masses

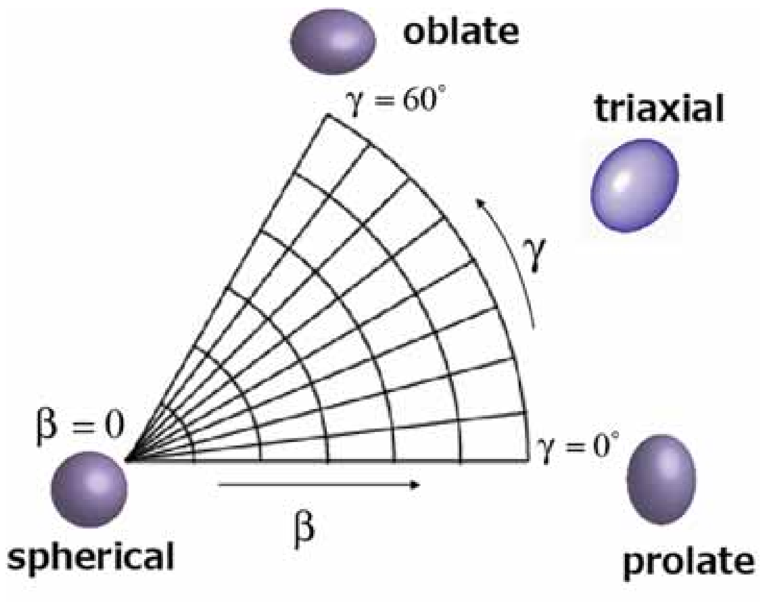
$$T_{\mathrm{rot}} = \sum_{k=1}^{3}\frac{1}{2}\boxed{J_k}\omega_k^2$$



rotational moments of inertia

2 deformation parameters (β, γ)

+ 3 Euler angles Ω



"Constrained HFB+ Local QRPA method" → Hinohara-san's talk

Hinohara, KS, Nakatsukasa, Matsuo, Matsuyanagi, PRC **82**, 064313 (2010)

- LQRPA inertial masses include the contribution from the time-odd components of the mean field unlike the cranking masses.

- "CHFB+ LQRPA" method is based on the Adiabatic SCC method

Matsuo, Nakatsukasa, and Matsuyanagi, Prog.Theor. Phys. 103(2000), 959.

N. Hinohara, et al, Prog. Theor. Phys. 117(2007) 451.

ASCC method

Moving-frame HFB eq. $|\varphi(q_1,q_2)\rangle$

Moving-frame QRPA eq. $\hat{Q}^{\alpha} \quad \hat{P}^{\alpha}$

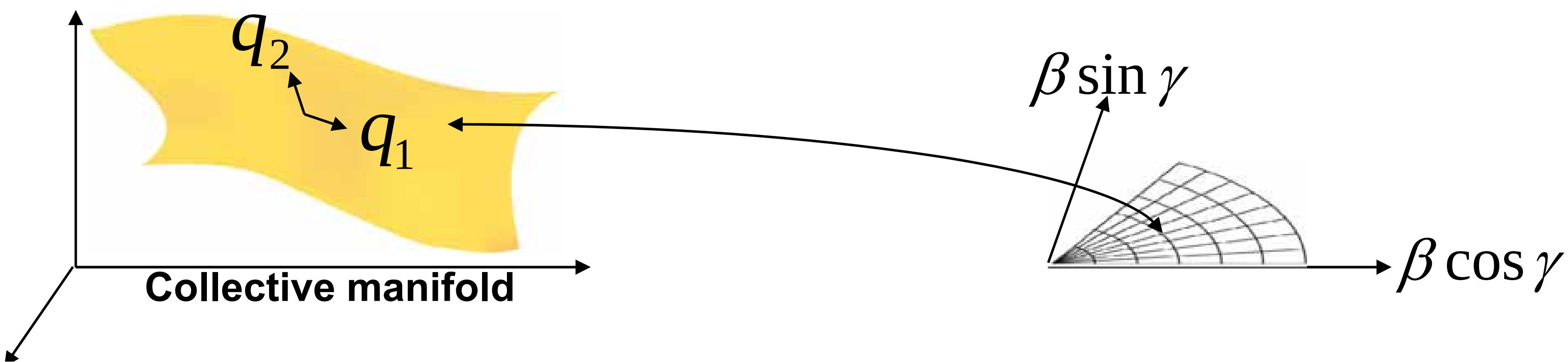
TDHFB phase space

→

CHFB+LQRPA method

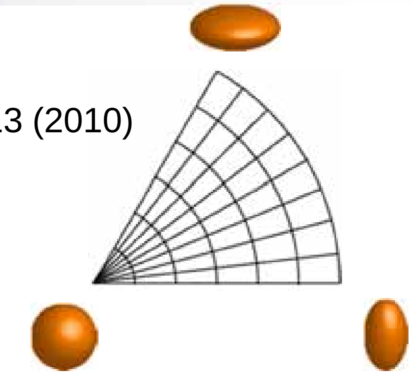
Constrained HFB eq. $|\varphi(\beta,\gamma)\rangle$

Local QRPA eq. $\hat{Q}^{\alpha} \quad \hat{P}^{\alpha}$



Constrained HFB + Local QRPA method

Hinohara, KS, Nakatsukasa, Matsuo, Matsuyanagi, Phys. Rev. C **82**, 064313 (2010)
KS & Hinohara, Nucl. Phys. A **849**, 53 (2011)



Constrained HFB (CHFB) equation:

$$\delta \langle \phi(\beta,\gamma)|\, \hat{H}_{\mathrm{CHFB}}(\beta,\gamma)\, |\phi(\beta,\gamma)\rangle = 0$$

$$\hat{H}_{\mathrm{CHFB}}(\beta,\gamma) = \hat{H} - \sum_{\tau=n,p} \lambda(\beta,\gamma)\tilde{N}^{(\tau)} - \sum_{m=0,2} \mu_{2m}(\beta,\gamma)\hat{D}^{(+)}_{2m}$$

→ $V(\beta,\gamma)$

Local QRPA (LQRPA) equations for vibration:

$$\delta \langle \phi(\beta,\gamma)|\, [\hat{H}_{\mathrm{CHFB}}(\beta,\gamma), \hat{Q}^i(\beta,\gamma)] - \frac{1}{i}\hat{P}_i(\beta,\gamma)\, |\phi(\beta,\gamma)\rangle = 0, \qquad (i = 1,2)$$

$$\delta \langle \phi(\beta,\gamma)|\, [\hat{H}_{\mathrm{CHFB}}(\beta,\gamma), \frac{1}{i}\hat{P}_i(\beta,\gamma)] - C_i(\beta,\gamma)\hat{Q}^i(\beta,\gamma)\, |\phi(\beta,\gamma)\rangle = 0.$$

→ $D_{\beta\beta}(\beta,\gamma)$ $D_{\beta\gamma}(\beta,\gamma)$ $D_{\gamma\gamma}(\beta,\gamma)$

Local QRPA equations for rotation:

$$\delta \langle \phi(\beta,\gamma)|\, [\hat{H}_{\mathrm{CHFB}}, \hat{\Psi}_k] - \frac{1}{i}(\mathcal{J}_k)^{-1}\hat{I}_k\, |\phi(\beta,\gamma)\rangle = 0$$

$$\langle \phi(\beta,\gamma)|\, [\hat{\Psi}_k(\beta,\gamma), \hat{I}_{k'}]\, |\phi(\beta,\gamma)\rangle = i\delta_{kk'}$$

→ $J_k(\beta,\gamma)$

Derivation of vibrational masses

$$\hat{P}^{\alpha} = \frac{1}{i}\frac{\partial}{\partial q_{\alpha}}$$

Vibrational part of collective Hamiltonian

$$\mathcal{H}_{\rm vib} = \frac{1}{2}\sum_{\alpha=1,2} \dot{q}_{\alpha}^{2}$$ ← Scale transformation B=1

q_2, q_1

collective coordinates

$$dq_{\alpha} = \sum_{m=0,2} \frac{\partial q_{\alpha}}{\partial D_{2m}^{(+)}} dD_{2m}^{(+)}$$

one-to-one correspondence

In terms of quadrupole deformation

$\sqrt{2}D_2^{(+)}$ **Quadrupole def.** $D_0^{(+)}$

$$\mathcal{H}_{\rm vib} = \frac{1}{2}M_{00}\dot{D}_{20}^{(+)2} + M_{02}\dot{D}_{20}^{(+)}\dot{D}_{22}^{(+)} + \frac{1}{2}M_{22}\dot{D}_{22}^{(+)2}$$

$$\frac{\partial \hat{D}_{2m}^{(+)}}{\partial q_{\alpha}} = \frac{\partial}{\partial q_{\alpha}} \langle \phi(\beta,\gamma) | \hat{D}_{2m}^{(+)} | \phi(\beta,\gamma) \rangle = \langle \phi(\beta,\gamma) | \left[\hat{D}_{2m}^{(+)}, \frac{1}{i}\hat{P}^{\alpha}(\beta,\gamma) \right] | \phi(\beta,\gamma) \rangle$$

LQRPA eq.

Criterion to choose 2 LQRPA modes:

Take a pair which gives the minimal $W = \beta^{-2}(D_{\beta\beta}D_{\gamma\gamma} - D_{\beta\gamma}^{2})$

Classical Quadrupole Collective Hamiltonian:

$$T = \frac{1}{2} D_{\beta\beta} \dot{\beta}^2 + D_{\beta\gamma} \dot{\beta}\dot{\gamma} + \frac{1}{2} D_{\gamma\gamma} \dot{\gamma}^2 + \sum_k \frac{1}{2} \mathcal{J}_k \omega_k^2$$

Pauli's prescription

Quantized quadrupole collective Hamiltonian):
(General Bohr-Mottelson Hamiltonian)

$$\hat{T} = \frac{-\hbar^2}{2\sqrt{WR}} \left\{ \frac{1}{\beta^3} \left[\partial_\beta \left(\beta^3 \sqrt{\frac{R}{W}} D_{\gamma\gamma} \partial_\beta \right) - \partial_\beta \left(\beta^3 \sqrt{\frac{R}{W}} D_{\beta\gamma} \partial_\gamma \right) \right] + \frac{1}{\sin 3\gamma} \left[-\partial_\gamma \left(\sqrt{\frac{R}{W}} \sin 3\gamma D_{\beta\gamma} \partial_\beta \right) + \partial_\gamma \left(\sqrt{\frac{R}{W}} \sin 3\gamma D_{\beta\beta} \partial_\gamma \right) \right] \right\} + \sum_k \frac{\hat{I}_k^2}{2\mathcal{J}_k}$$

$$W = \beta^{-2} (D_{\beta\beta} D_{\gamma\gamma} - D_{\beta\gamma}^2)$$
$$R = D_1 D_2 D_3$$

Collective Schrodinger equation:

$$\{\hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V\} \Psi_{\alpha IM}(\beta, \gamma, \Omega) = E_{\alpha I} \Psi_{\alpha IM}(\beta, \gamma, \Omega)$$

Collective wave function:

vib. **rot.**

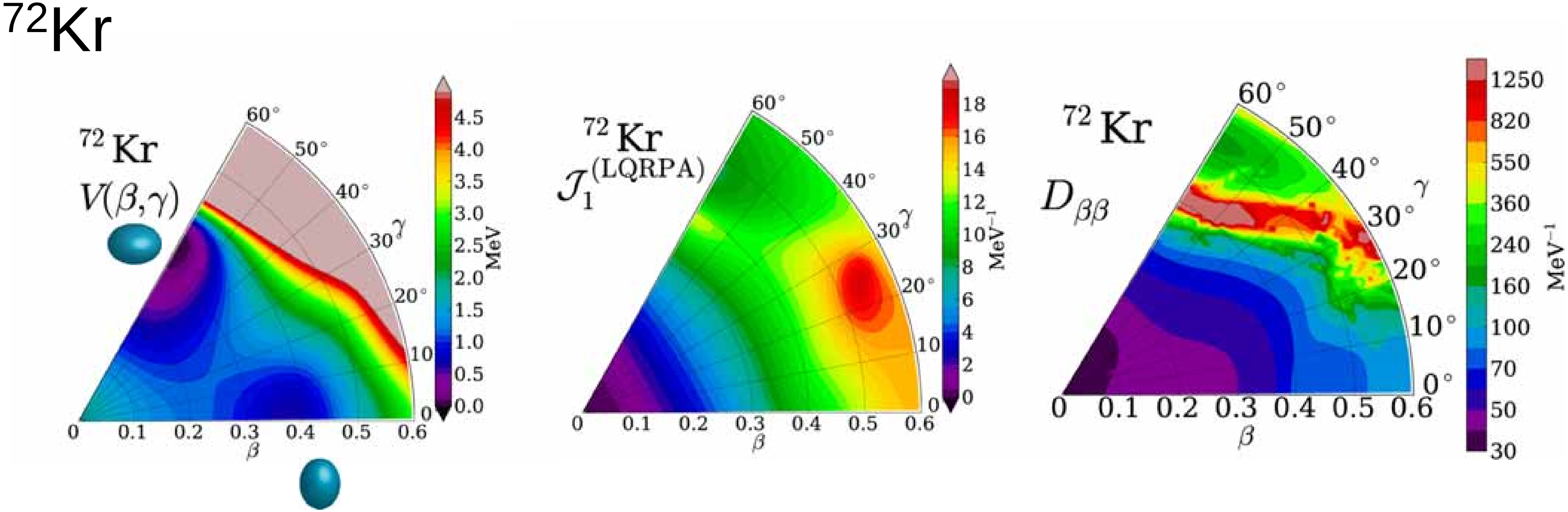
$$\Psi_{\alpha IM}(\beta, \gamma, \Omega) = \sum_{K=\text{even}} \Phi_{\alpha IK}(\beta, \gamma) \langle \Omega | IMK \rangle$$

Application to oblate-prolate shape coexistence

68,70,72Se: Hinohara, KS, Nakatsukasa, Matsuo, Matsuyanagi, Phys. Rev. C **82**, 064313 (2010)

72,74,76Kr: KS & Hinohara, Nucl. Phys. A **849**, 53 (2011)

^{72}Kr



Collective potential | Moment of inertia | Vib. inertial mass

Model details:

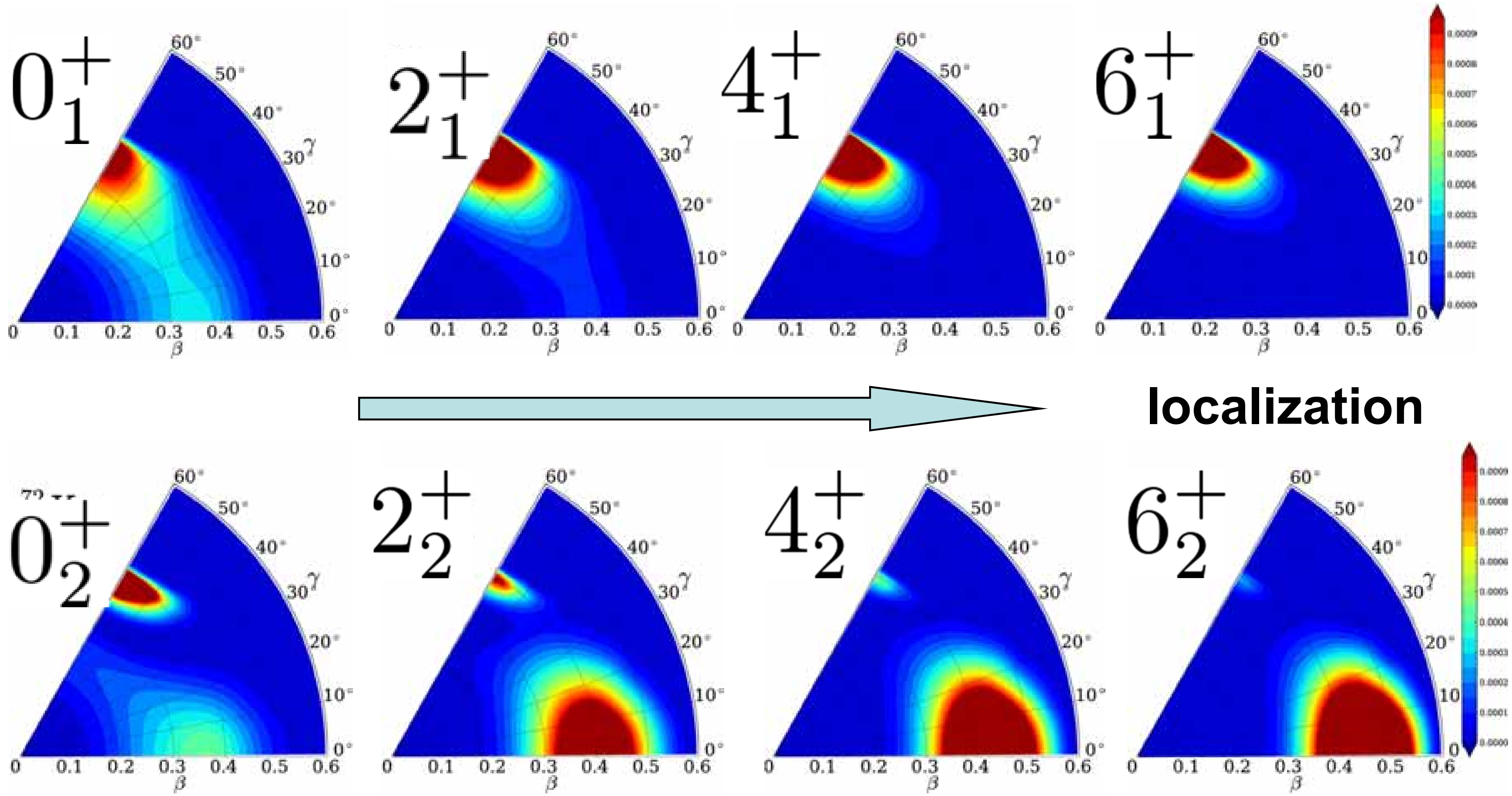
- Effective interaction: P+Q model including quadrupole-pairing int.
- Model space: two major shells (Nsh=3,4)
- Single-particle energy: modified oscillator
- Interaction strength: adjusted to reproduce Skyrme-HFB gaps and deformation property

Collective wave functions squared x β^4 $\quad \beta^4 \sum_K |\Phi_{IKk}(\beta,\gamma)|^2$ $\quad ^{72}$Kr

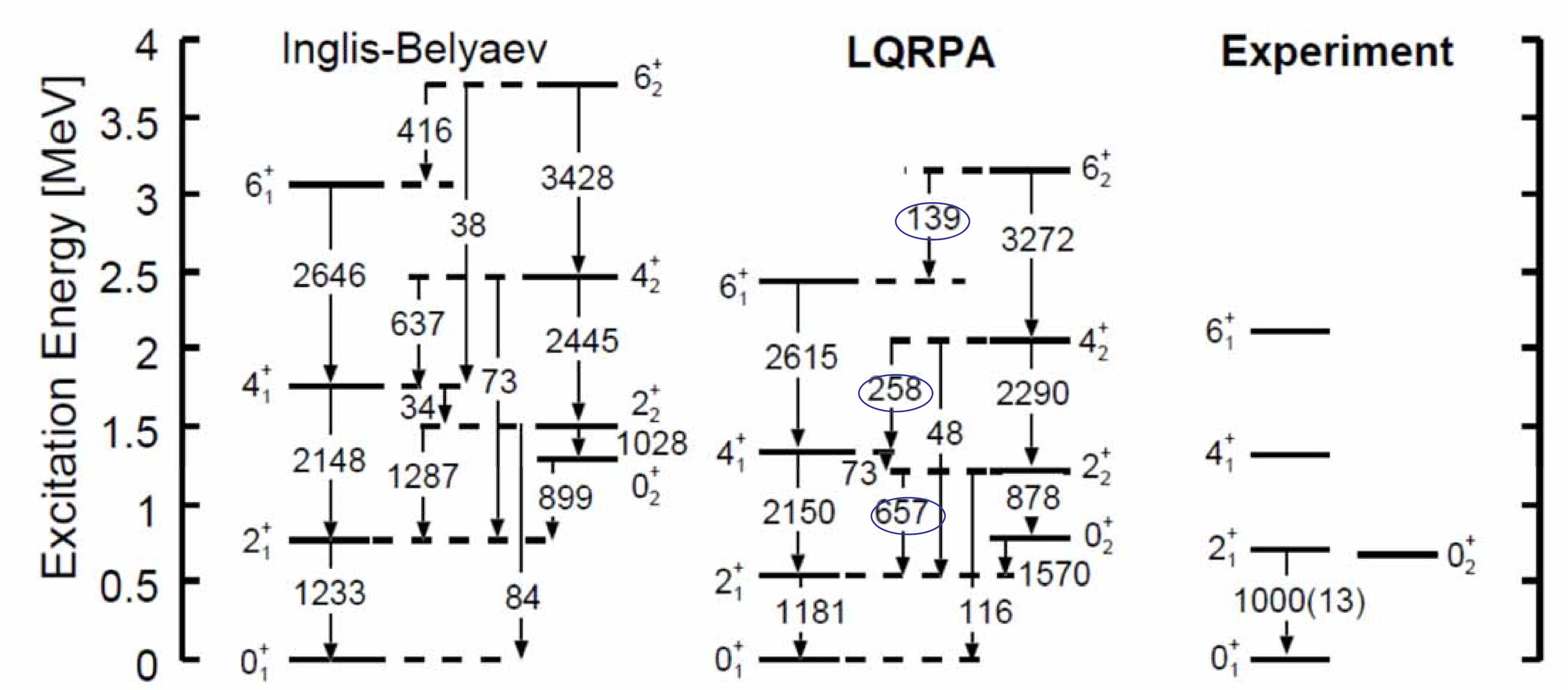
Normalization of w. f.:

$$\int d\beta d\gamma |\Phi_{\alpha I}(\beta,\gamma)|^2 |G(\beta,\gamma)|^{\frac{1}{2}} = 1$$

$$|G(\beta,\gamma)|^{\frac{1}{2}} d\beta d\gamma = 2\beta^4 \sqrt{W(\beta,\gamma)R(\beta,\gamma)} \sin 3\gamma d\beta d\gamma$$



Excitation energies and B(E2) values (72Kr)



( ) …B(E2) e^2 fm^4

effective charge: e_{pol} =0.658

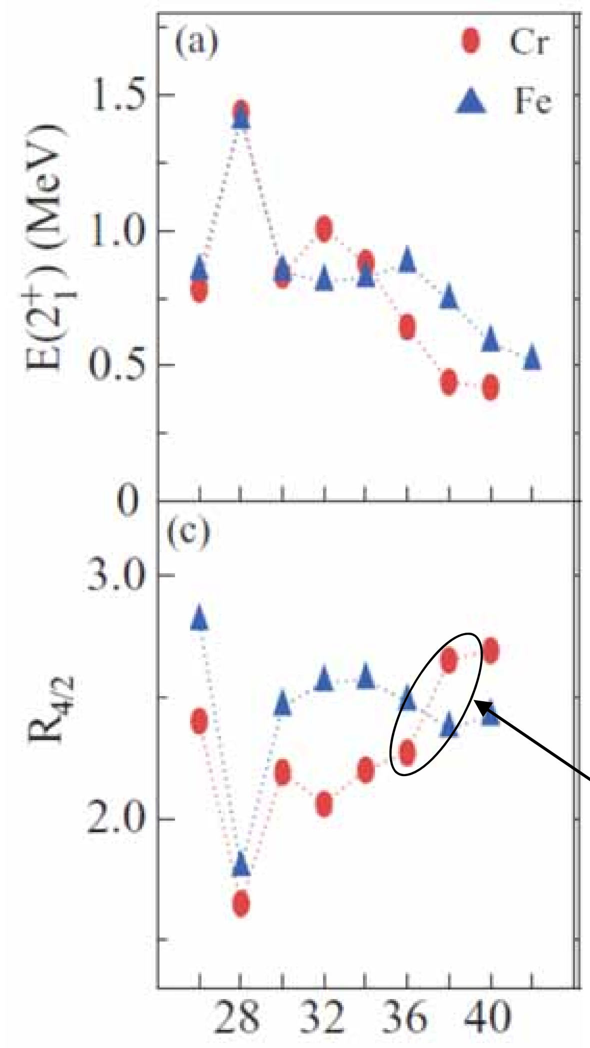
EXP：Fischer et al., Phys.Rev.**C67** (2003) 064318,
Bouchez, et al., Phys.Rev.Lett.90 (2003) 082502.
Gade, et al., Phys.Rev.Lett.**95** (2005) 022502, **96** (2006) 189901

Time-odd mean field contribution lowers excitation energies

Interband transitions become weaker as angular momentum increases.

Development of deformation in Cr isotopes around N∼40

Experimental 2^+ excitation energies & $E(4_1^+)/E(2_1^+)$ ratios



$E(2_1^+)$ decreases with increasing N toward N=40

$R_{4/2}=E(4_1^+)/E(2_1^+)$ increases with increasing N

Sudden rise in $R_{4/2}$ from $^{60}Cr_{36}$ to $^{62}Cr_{38}$

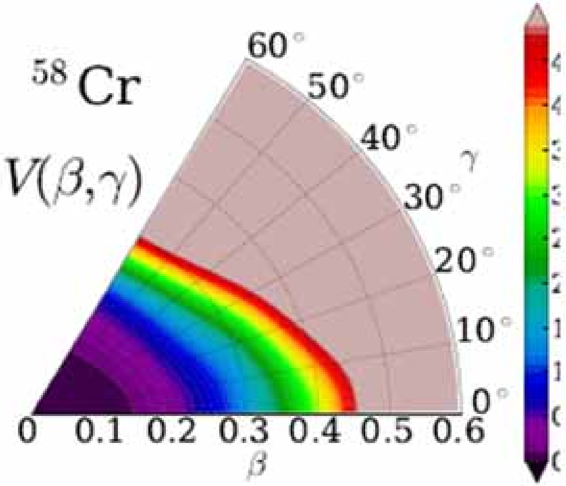
N∼40

onset of deformation ?

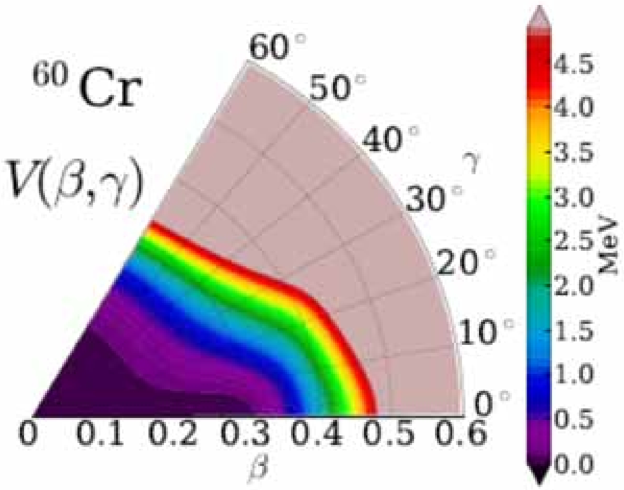
Gade et al., Phys.Rev.**C81** (2010) 051304(R),

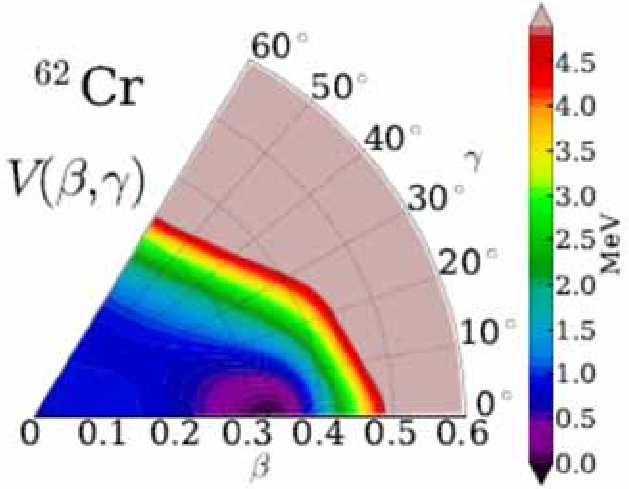
Collective potential $V(\beta,\gamma)$

$^{58}Cr_{34}$

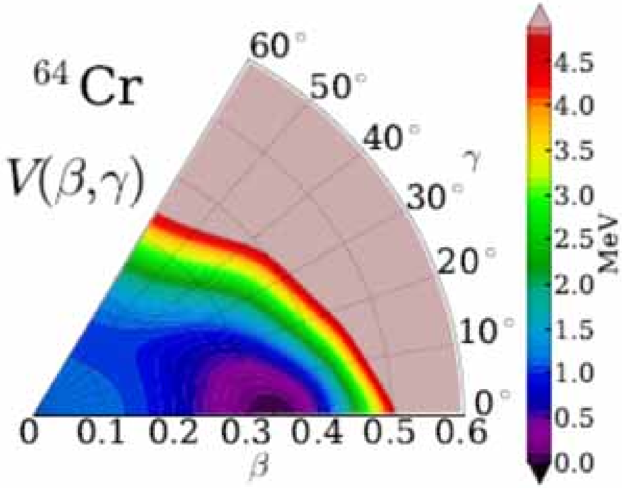


$^{60}Cr_{36}$

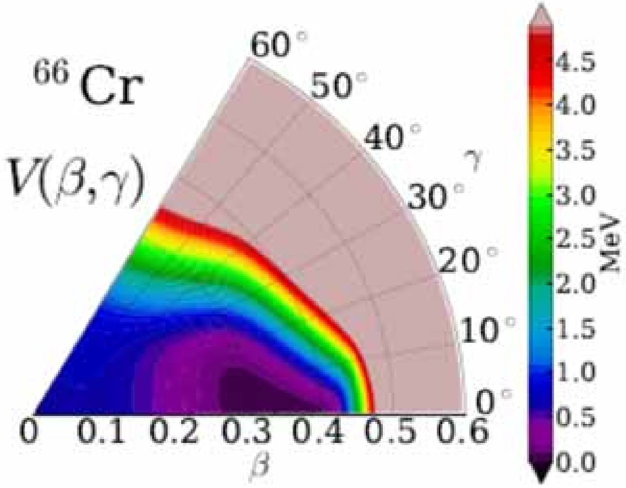




$^{62}Cr_{38}$



$^{64}Cr_{40}$



$^{66}Cr_{42}$

Model details:

Eff. Int. : P+Q model including the quadrupole pairing

Model space: 2 major shells

Int. parameters:
62Cr adjusted to reproduce the Skyrme-HFB gaps and deformation

Others : simple A -dep. assumed

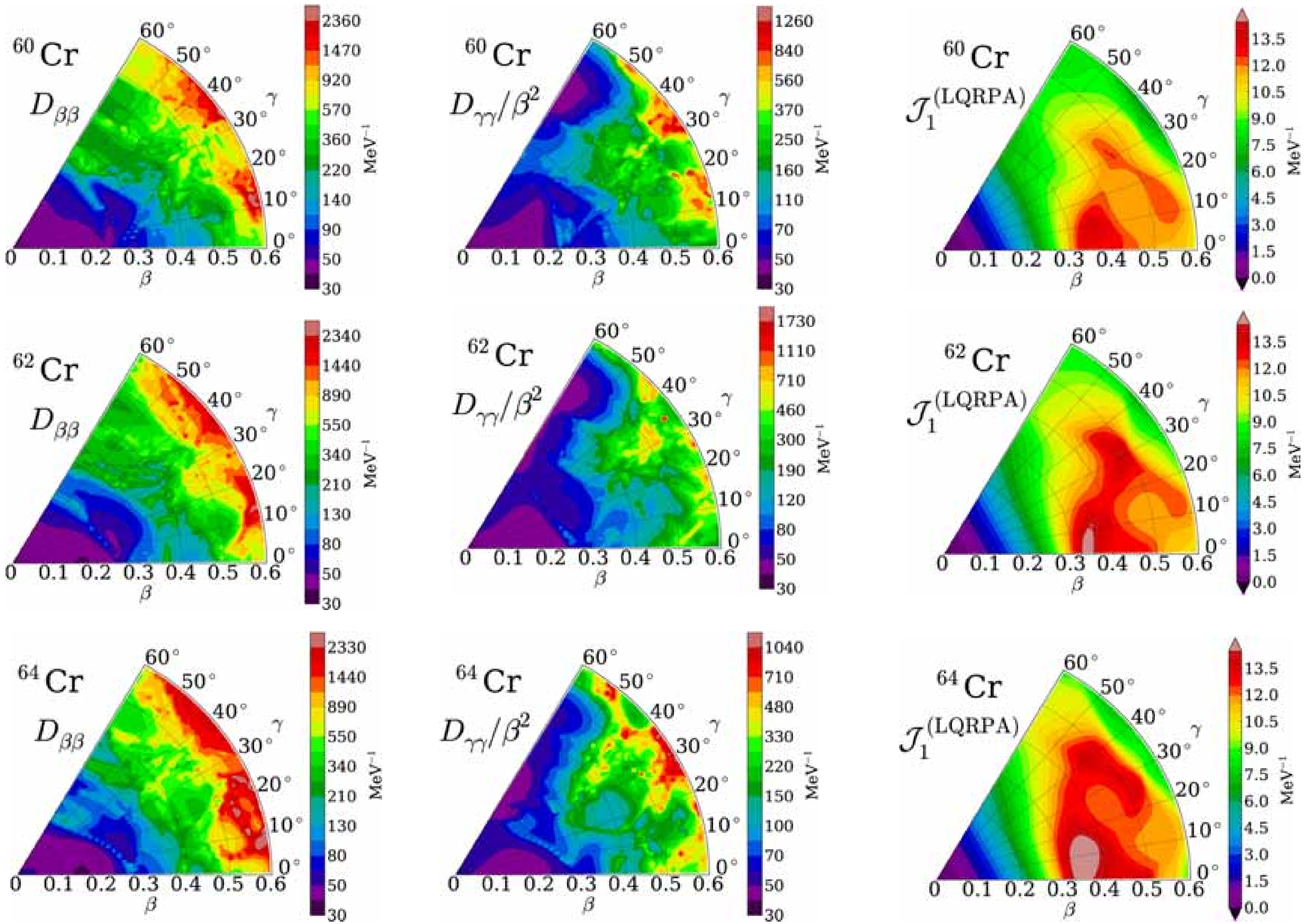
LQRPA vibrational masses: **LQRPA rotational MoI**

$D_{\beta\beta}(\beta,\gamma)$ $D_{\gamma\gamma}(\beta,\gamma)/\beta^2$ $J_1(\beta,\gamma)$

N=36

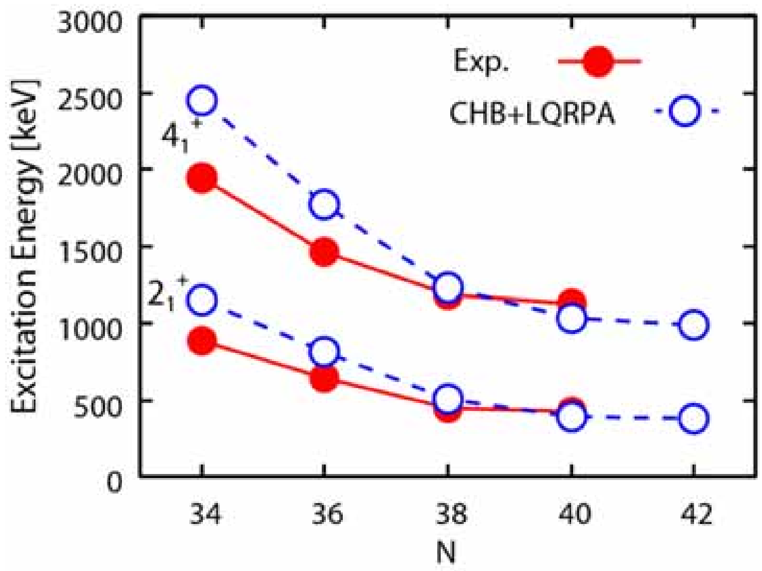
N=38

N=40

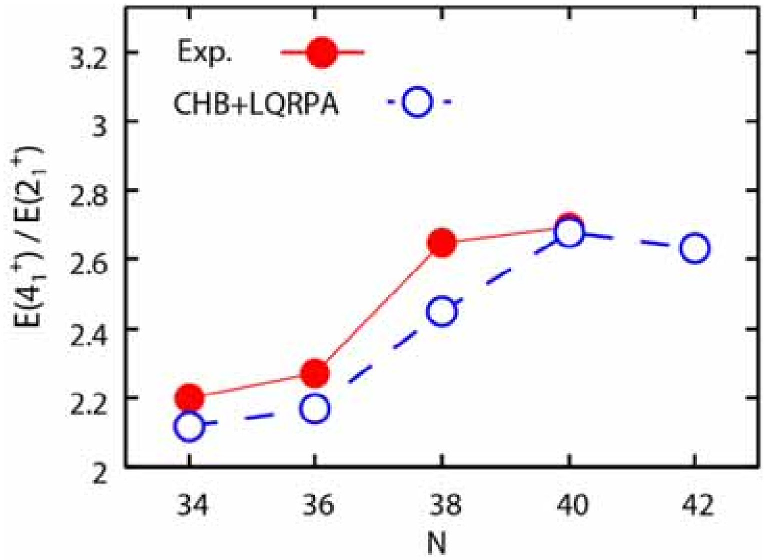


Application to Cr isotopes

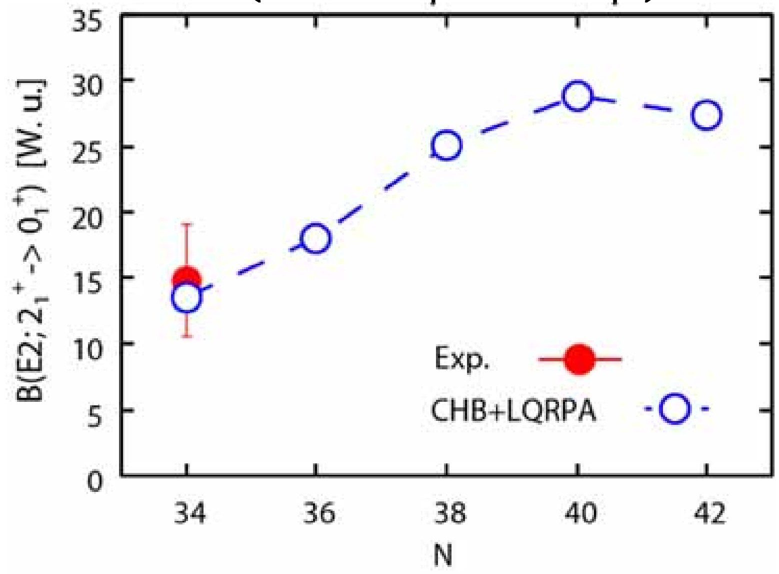
$E(2_1^+)$ & $E(4_1^+)$



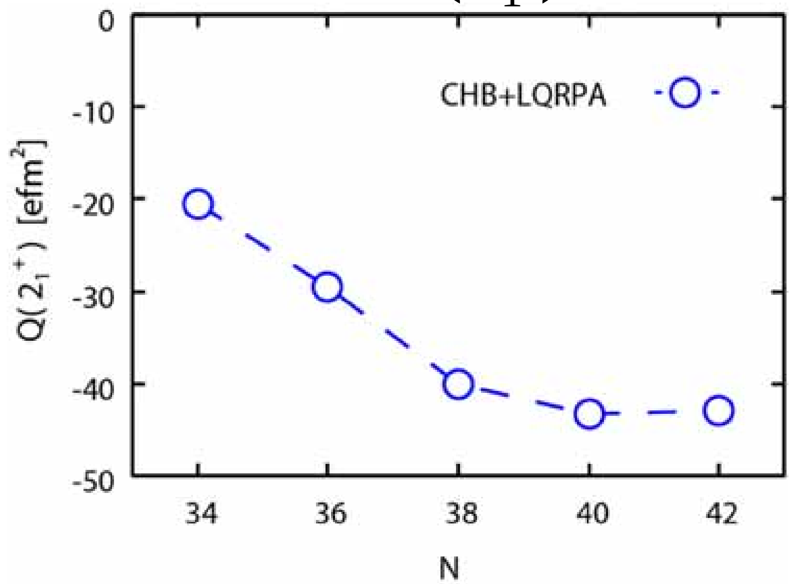
$E(4_1^+)/E(2_1^+)$



$B(E2; 2_1^+ \to 0_1^+)$



$Q(2_1^+)$



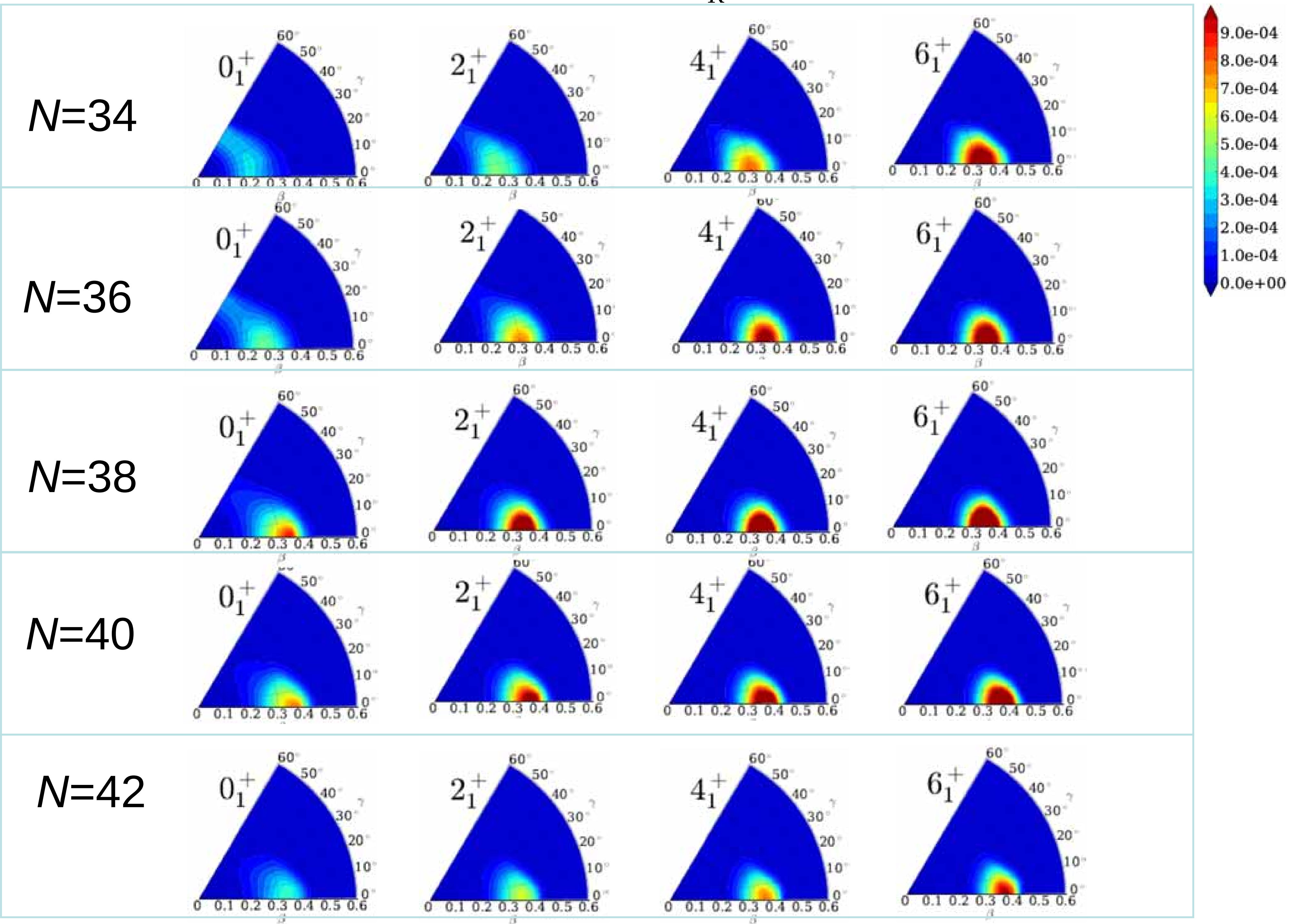
(e_n, e_p) =(0.5, 1.5)

EXP：Gade et al., Phys.Rev.**C81** (2010) 051304(R), S. Zhu et al., Phys. Rev. C74 (2006) 064315.

N. Aoi et al., PRL 102, 012502 (2009)

A. Bürger et al., PLB 622 (2005) 29

Collective wave functions squared $\beta^4 \sum_K \left|\Phi_{\alpha IK}(\beta,\gamma)\right|^2$ (ground band)



Summary

- We have proposed a method (CHFB+LQRPA method) for determining the inertial functions in the 5D quadrupole collective Hamiltonian.

- LQRPA inertial masses contain the contribution from the time-odd component of the mean field, which increases the inertial masses.

- Application to shape mixing dynamics

  oblate-prolate shape coexistence in 72Kr

  shape transition in neutron-rich Cr isotopes around N=40.

  role of rotational motion

Future work

2D CHFB +LQPRA method with Skyrme EDF