

Alpha-Clusters in Nuclear Systems

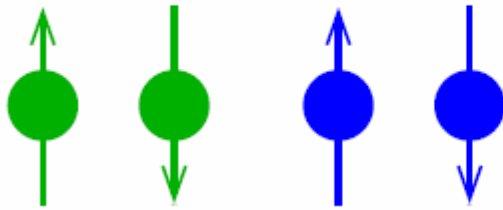
P. Schuck

**Y. Funaki, H. Horiuchi, G. Röpke,
A. Tohsaki , W. von Oertzen and T. Yamada**

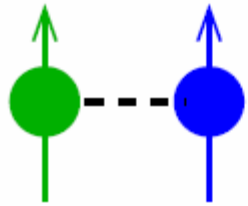
Contents:

- **General Aspects of Nuclear Clustering**
- **α – Particles and their Condensation**
- **${}^8\text{Be}$ and Hoyle State in ${}^{12}\text{C}^*$**
- **Extensions to Heavier Nuclei: ${}^{16}\text{O}$**
- **α – de – excitation of Compound States**
- **α 's in Compact Stars**
- **Conclusions, Outlook**

Clusters important aspect and richness of nuclear systems due to 4 Fermions :

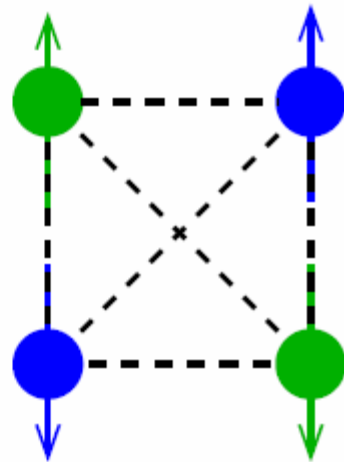


Dimer :

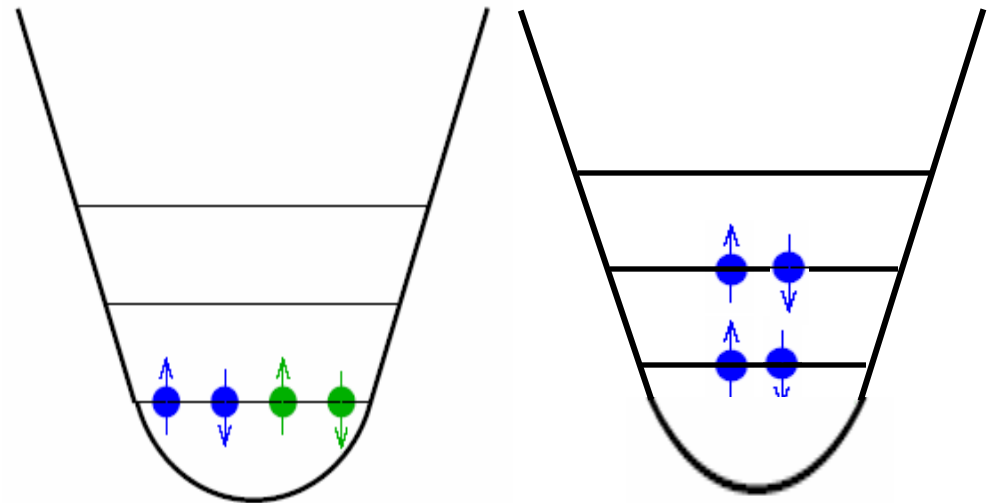


$$\frac{E}{A} = 1 \text{ MeV}$$

Quartet :



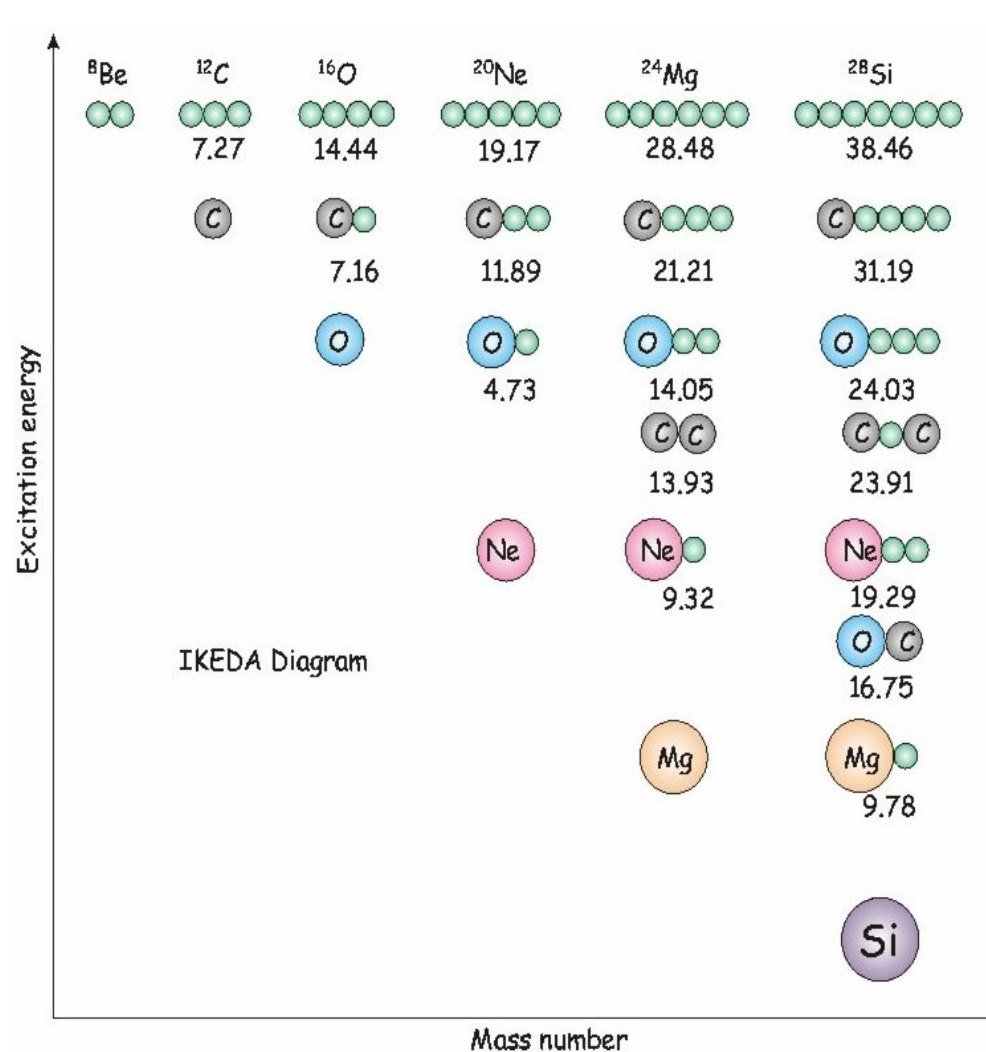
$$\frac{E}{A} = 7 \text{ MeV}, \quad E^* = 20 \text{ MeV}$$



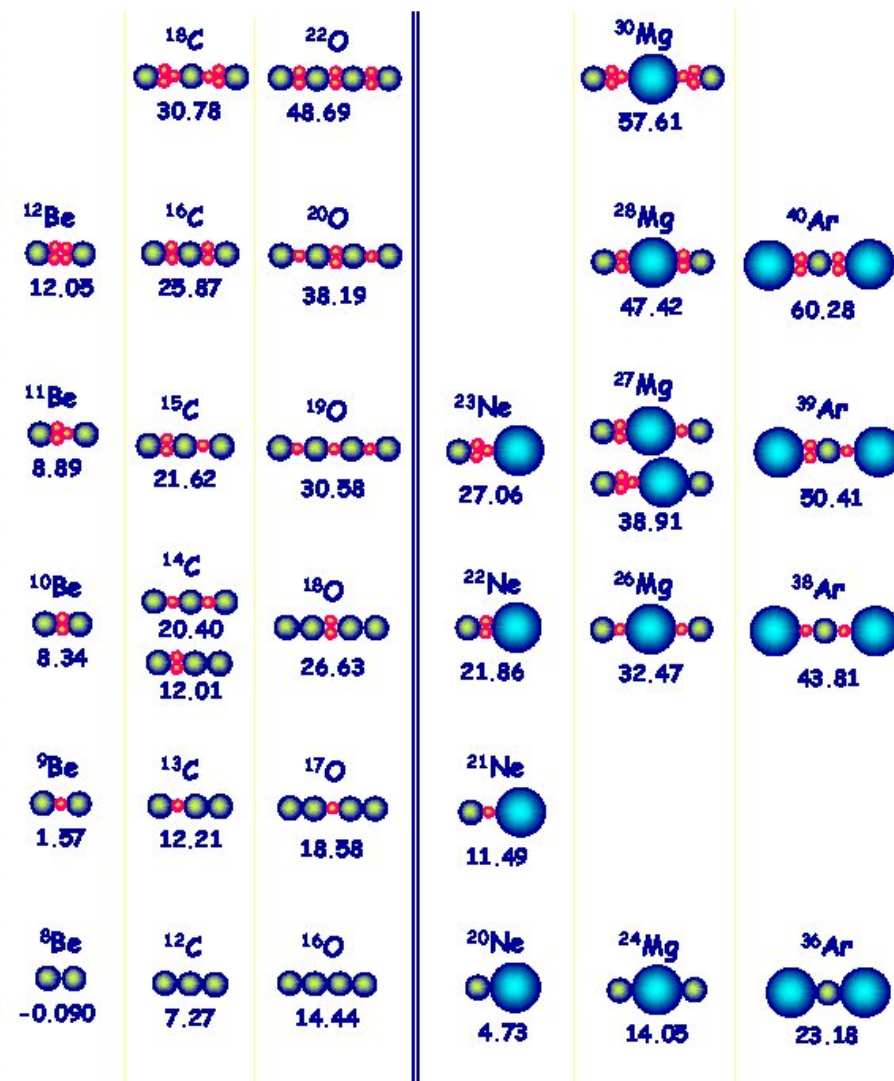
Proposal :

Trapping of 4 different species of Fermionic atoms.

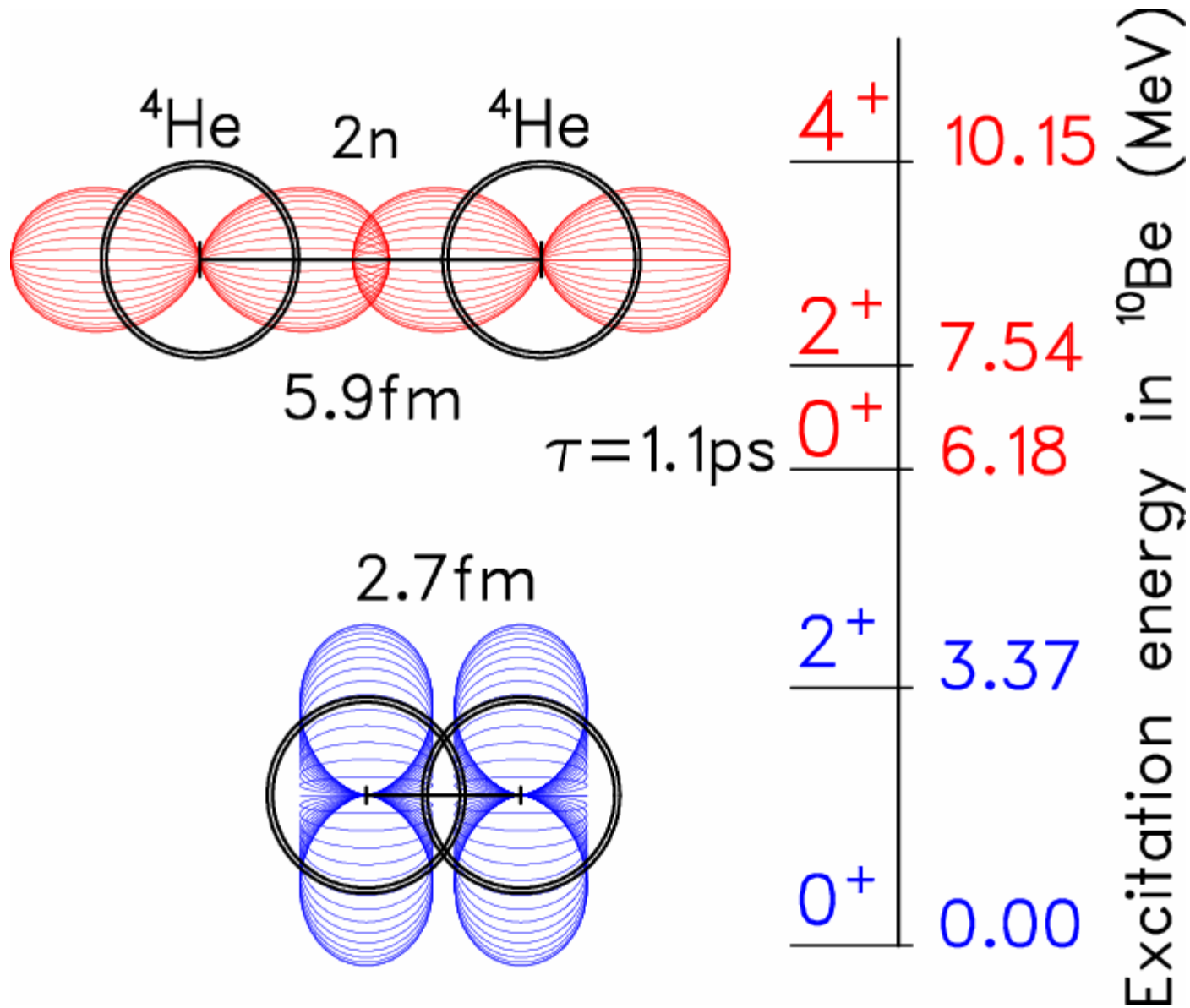
Ikeda



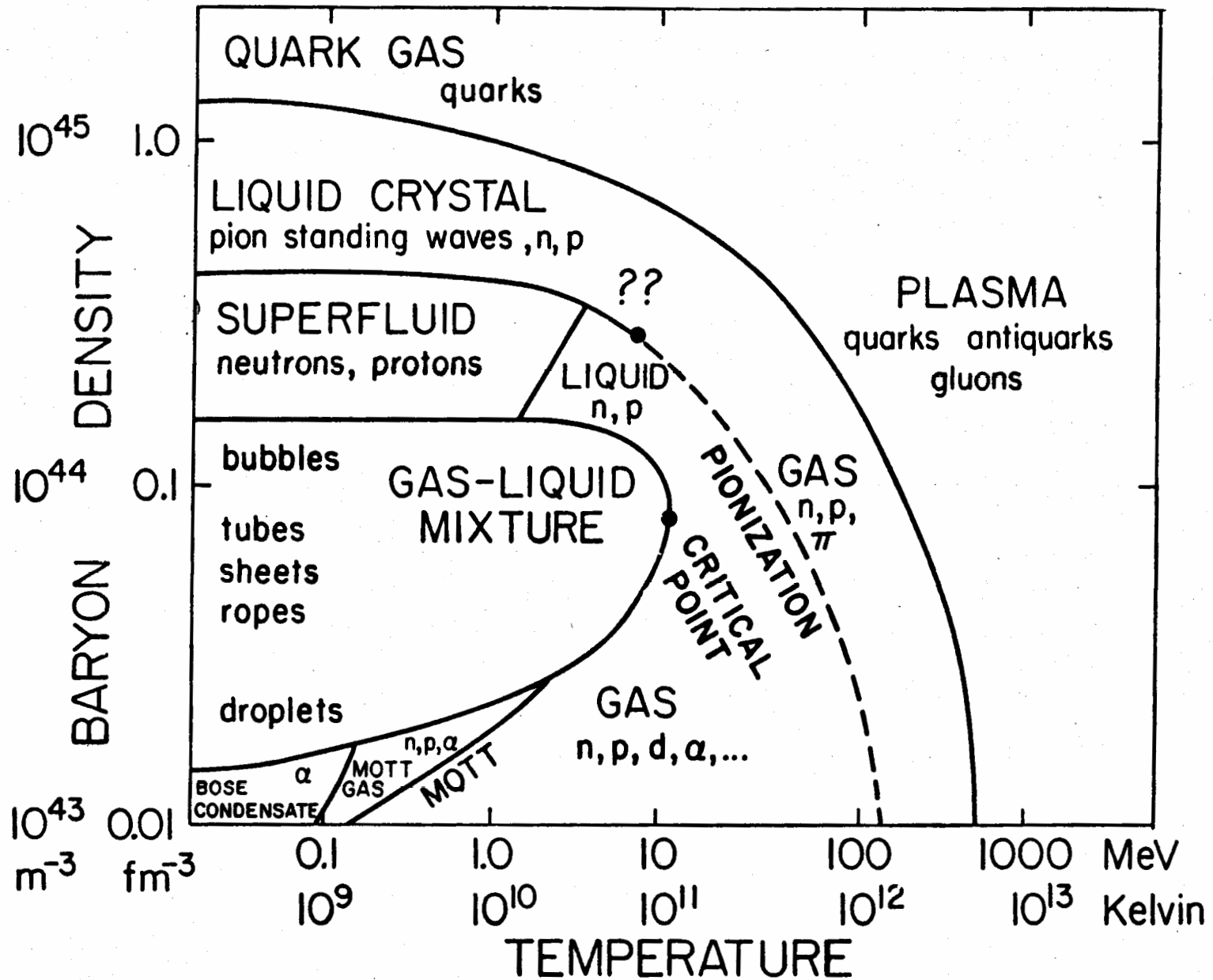
von Oertzen



H. G. Bohlen, M. Freer

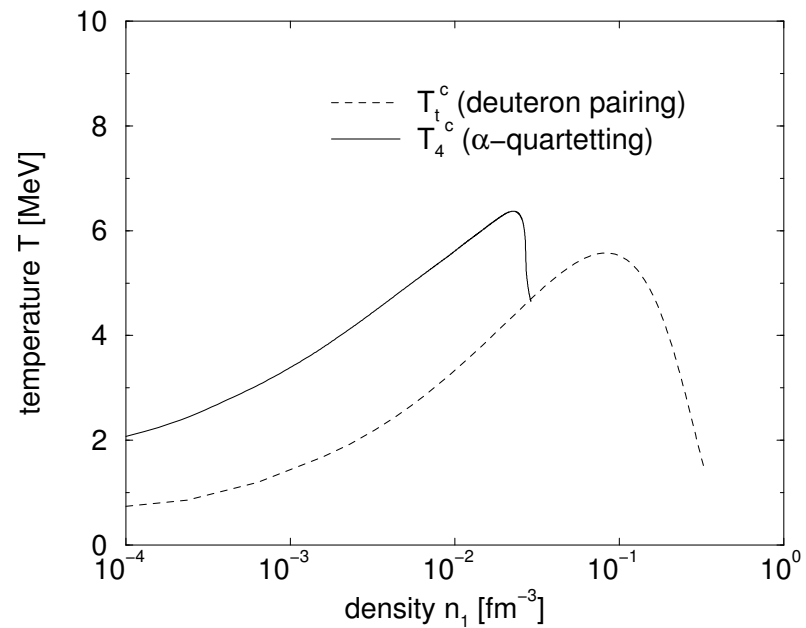
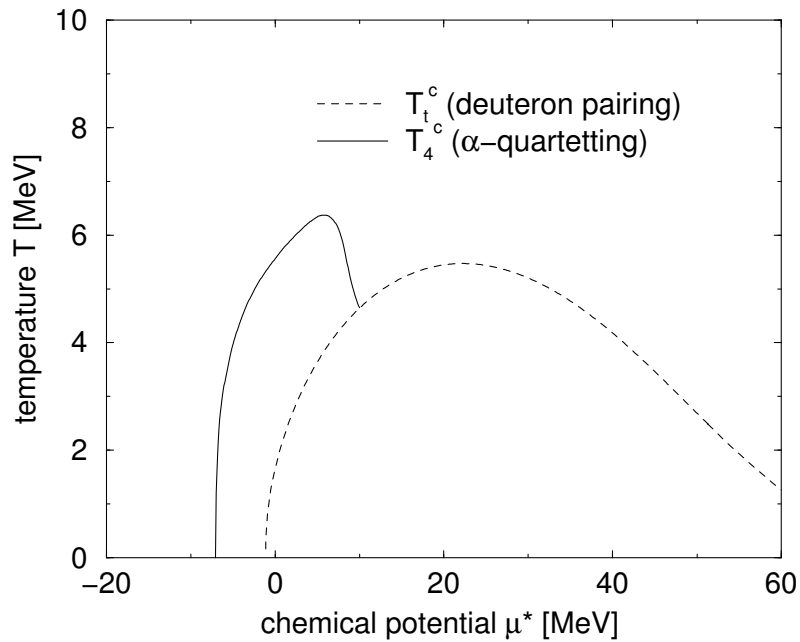
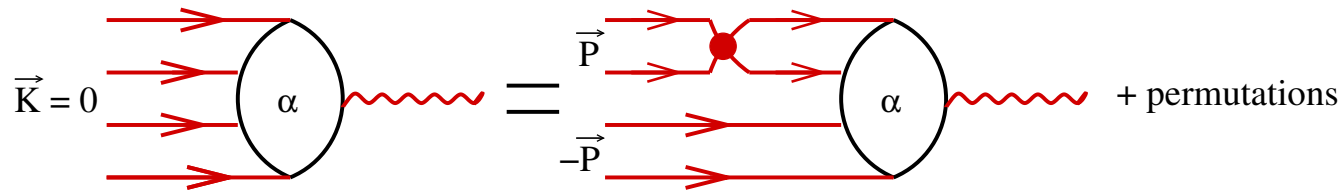


P. Siemens

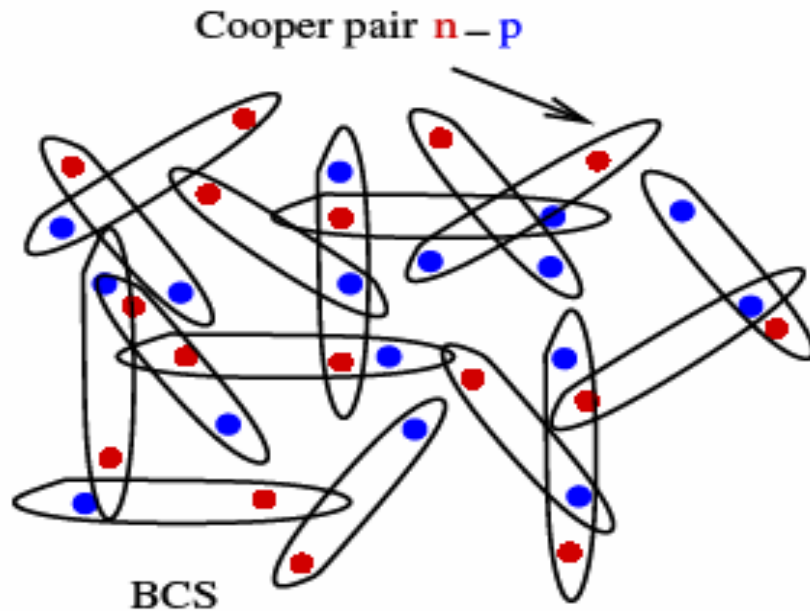


- α -Particle Condensation : G. Röpke, M. Beyer

$$(4\mu - \epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4) \psi_{1234} = (1 - f_1 - f_2) \sum_{1'2'} v_{121'2'} \psi_{1'2'34} + \text{permutations}$$



α -Condensation only at very low density !



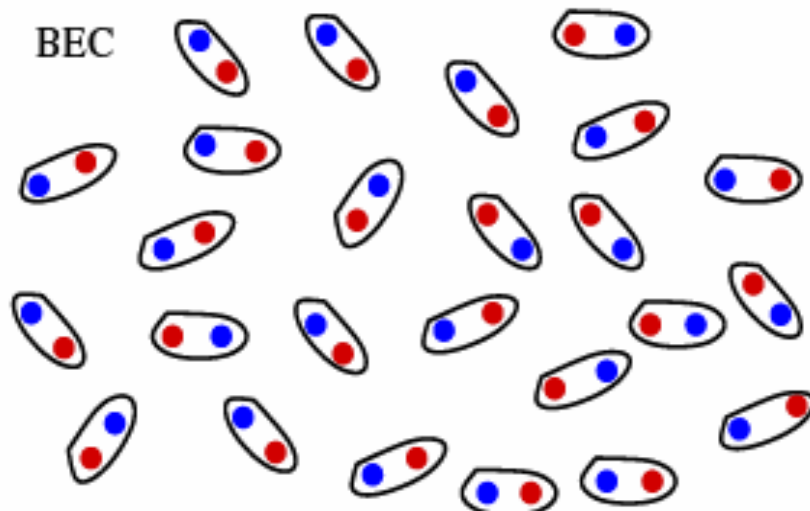
High Density

$n-p$ Cooper pairs

Strongly overlapping

not Bosons

Low density : \rightleftharpoons smooth transition



α - Particles
Only Exist
in Low Density
BEC Phase

gas of Deuterons

\sim Bosons

Finite nuclei ?

Exact ${}^8\text{Be}$:

Density : $\frac{\rho_0}{3}$

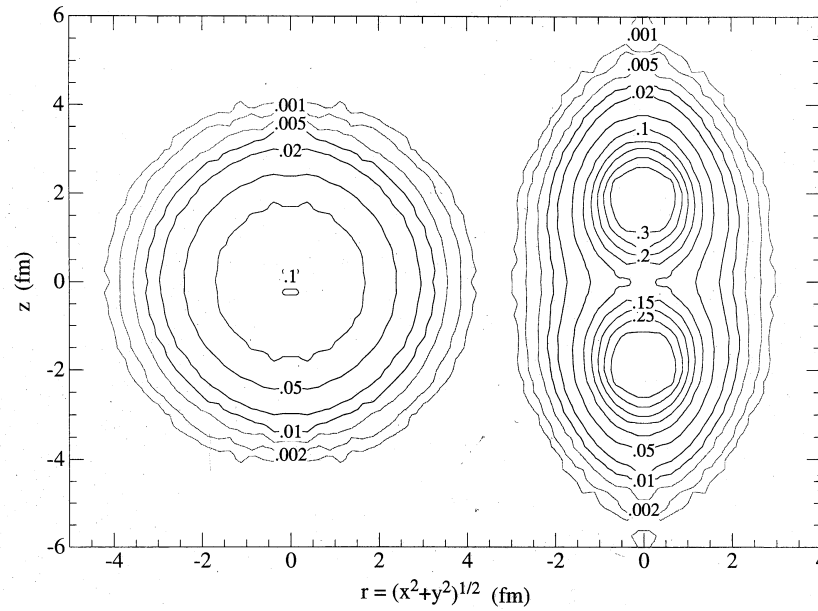
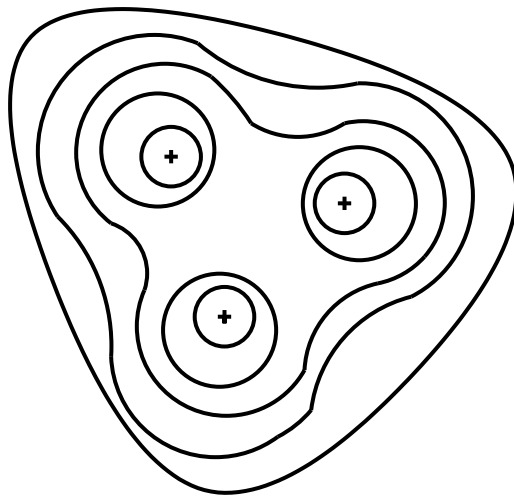


Fig. 15 (Wiringa, et al.)

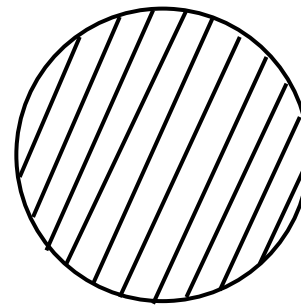
3 rd α -particle



V

collapse
⇒

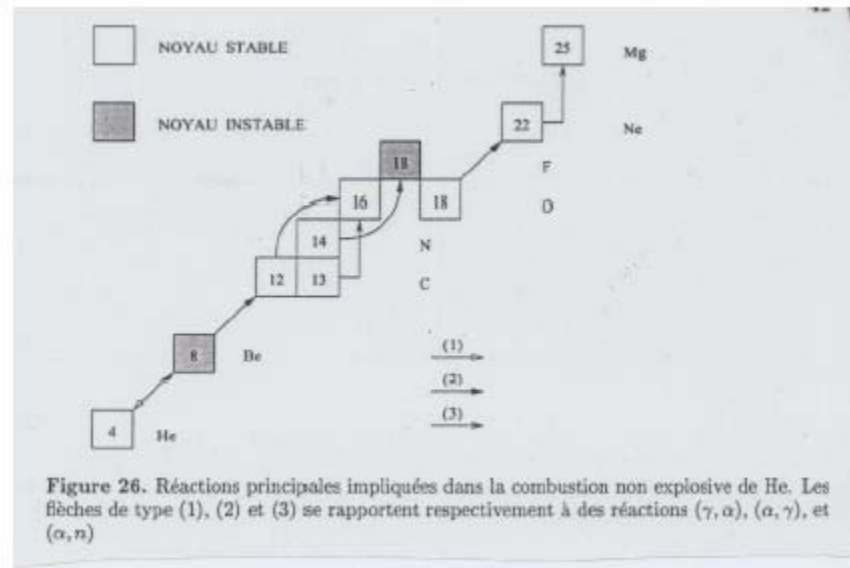
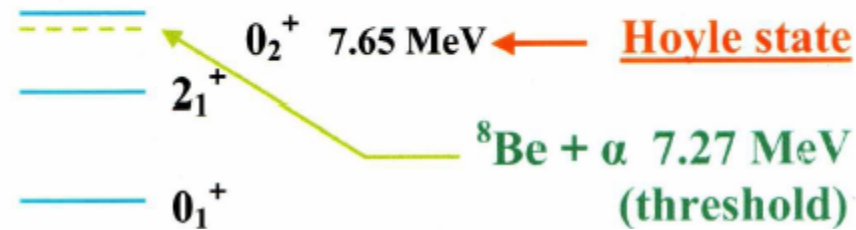
Fermi gas



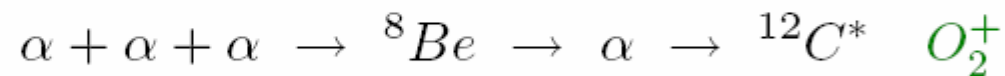
compact ground state $V/3$

${}^{12}\text{C}$

Does a dilute 3α $^{12}\text{C}^*$ state exist ?
 Similar to $^8\text{Be} + \alpha$?



At $T = 10^8\text{K}$ helium burning
 thermal equilibrium



O_2^+ : dilute 3α state hypothesis !

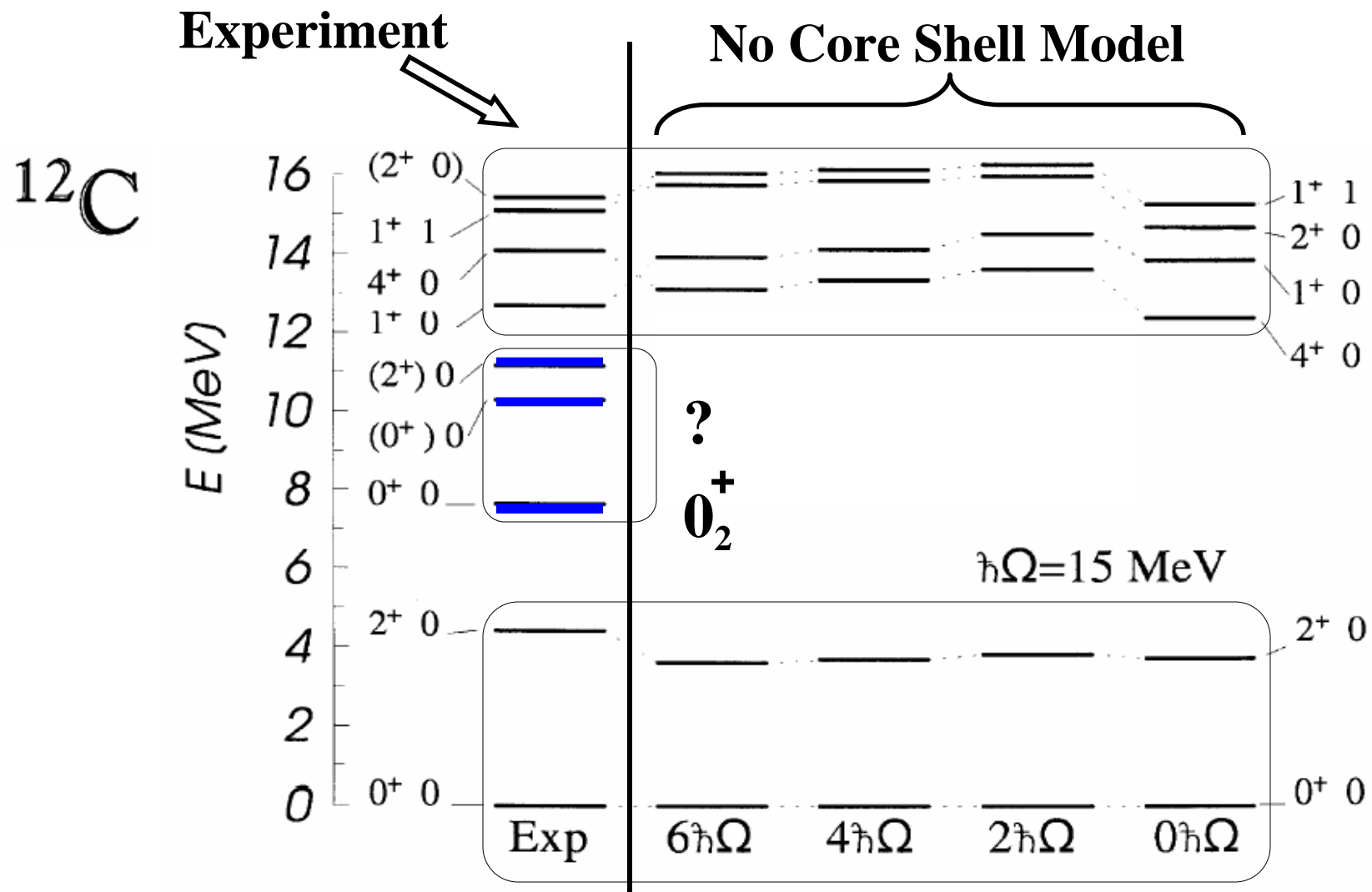
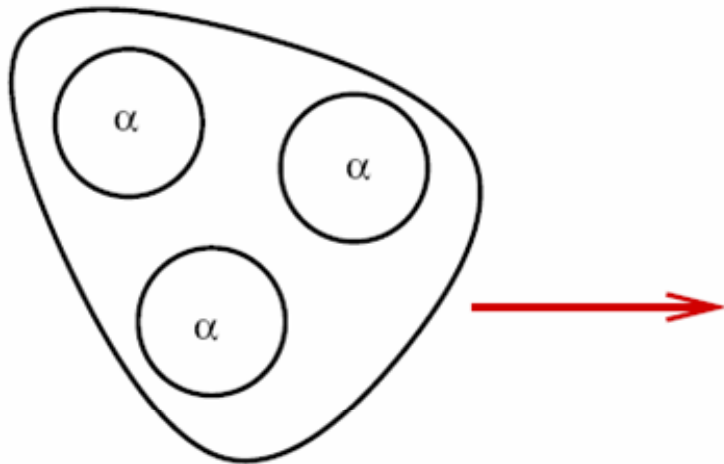


Figure 4. Experimental and NCSM excitation spectra for ^{12}C for different model space sizes.

it seems impossible to get Hoyle state from shell model calculation !

45 MeV B. Barret

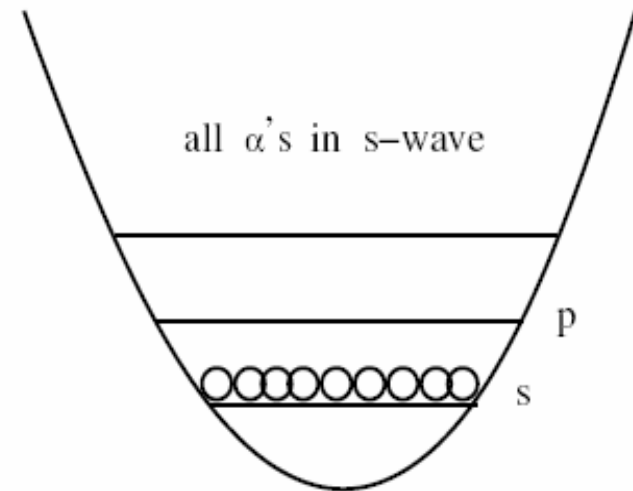
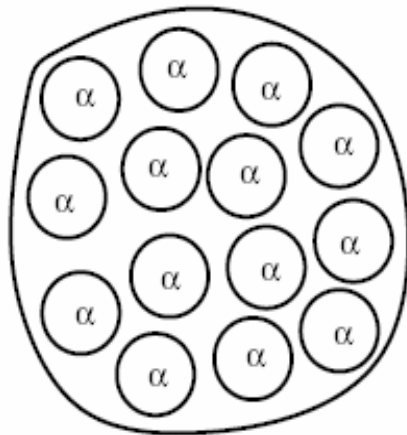
If O_2^+ in ^{12}C dilute α - state



then α -condensate

infinite matter $\rho_{crit} \sim \frac{\rho_0}{3}$

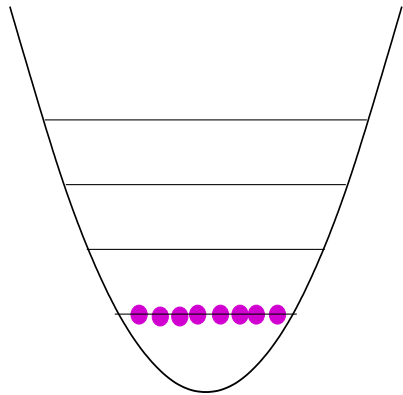
Conjecture: all $n.\alpha$ nuclei possess excited $n\alpha$ condensed state



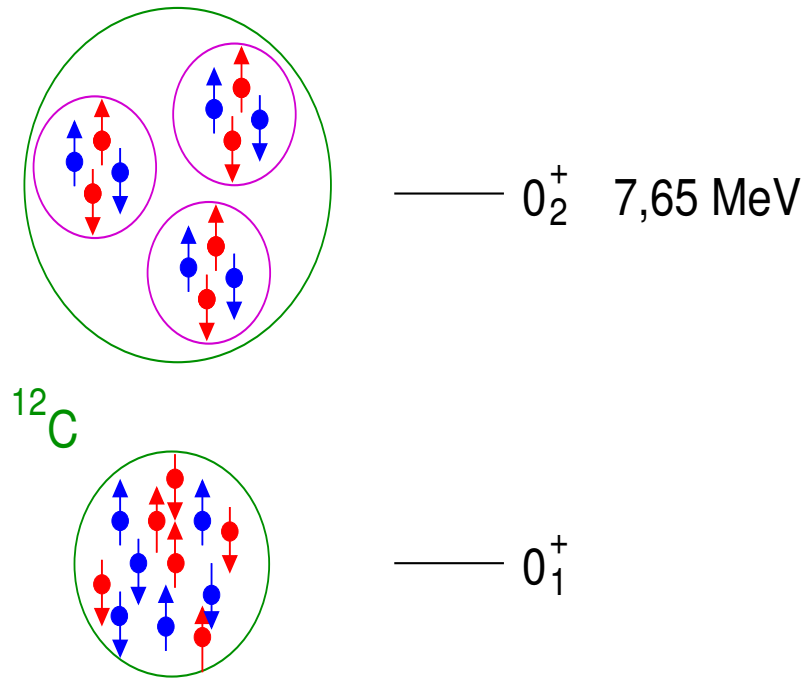
Analogy with atoms in traps ! $\rho(r) = N|\phi_0(r)|^2$

$N = 10^6$

Bosons

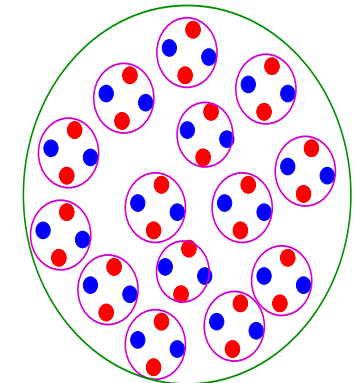


Back to nuclei

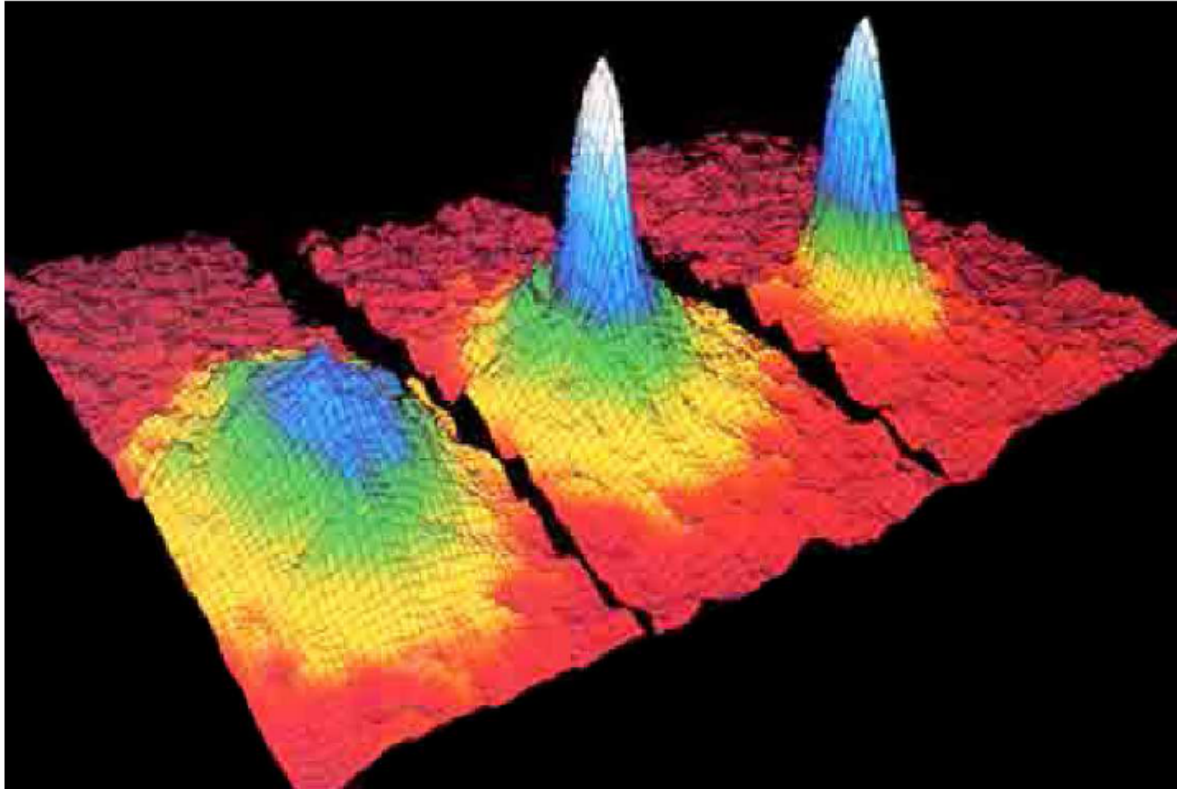


● proton
● neutron
○ alpha

many α 's
→ condensate



strong cluster phenomena in lighter nuclei



Theoretical Description

Ideal Bose condensate : $|0\rangle = b_0^\dagger b_0^\dagger \cdots b_0^\dagger |vac\rangle$

α -particle condensate : $|\Phi_{\alpha C}\rangle = C_\alpha^\dagger C_\alpha^\dagger \cdots C_\alpha^\dagger |vac\rangle$

In r -space :

$$\langle \vec{r}_1, \vec{r}_2, \cdots, \vec{r}_{4n} | \Phi_{\alpha C} \rangle = \mathcal{A} \left\{ \Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) \Phi(\vec{r}_5, \vec{r}_6, \vec{r}_7, \vec{r}_8) \cdots \Phi(\vec{r}_{4n-3}, \vec{r}_{4n-2}, \vec{r}_{4n-1}, \vec{r}_{4n}) \right\}$$

In comparison with pairing :

$$\langle \vec{r}_1, \vec{r}_2, \cdots | \text{BCS} \rangle = \mathcal{A} \left\{ \Phi(\vec{r}_1, \vec{r}_2) \Phi(\vec{r}_3, \vec{r}_4) \cdots \right\}$$

Variational ansatz for $\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4)$: $\Phi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) = e^{-\frac{2}{B^2} \vec{R}^2} \phi_\alpha(\vec{r}_i - \vec{r}_j)$

Center of mass : $\vec{R} = \frac{1}{4}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4)$

Intrinsic α -wave function :

$$\phi_\alpha(\vec{r}_i - \vec{r}_j) = e^{-\frac{1}{8b^2} \{(\vec{r}_4 - \vec{r}_1)^2 + (\vec{r}_4 - \vec{r}_2)^2 + (\vec{r}_4 - \vec{r}_3)^2 + \dots\}}$$

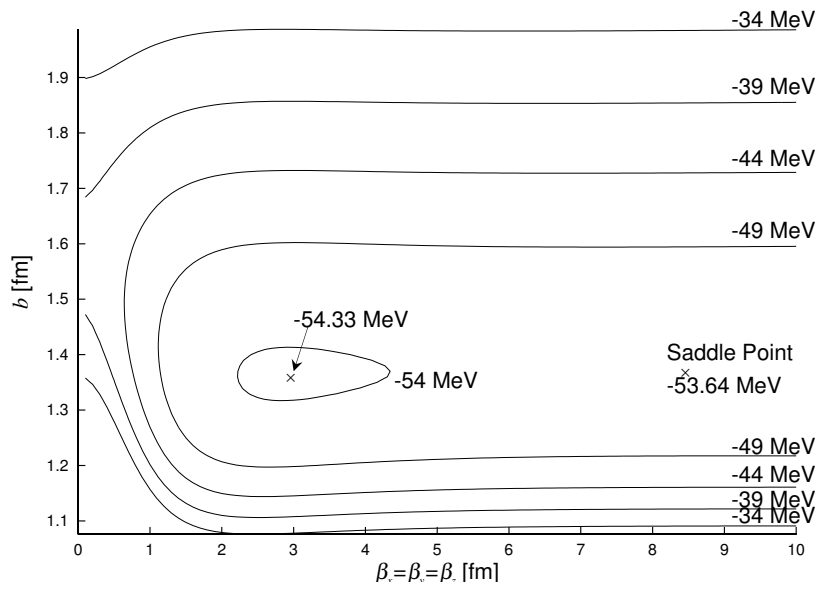
Two variational parameters : B, b

Two limits : $B = b$ $|\Phi_{\alpha C}\rangle =$ Slater determinant

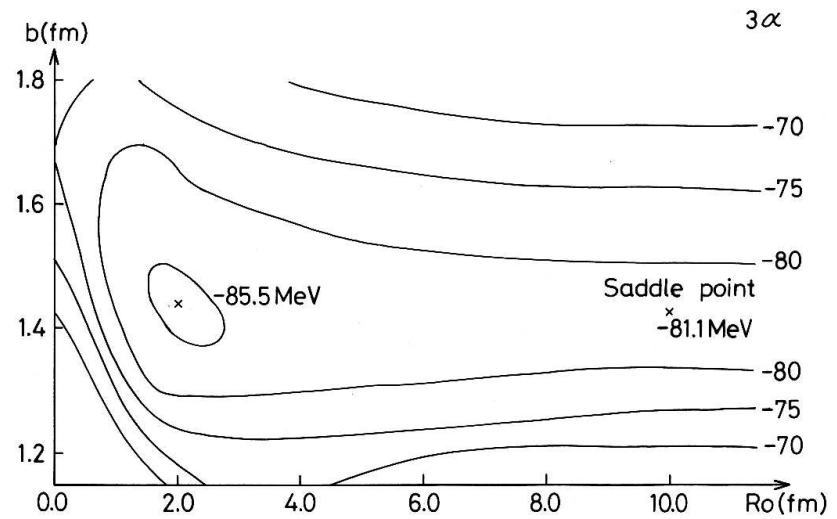
$B \gg b$ $|\Phi_{\alpha C}\rangle =$ gas of independent α -particles

Two dimensional surface : $E(B, b) = \frac{\langle \Phi_{\alpha C} | H | \Phi_{\alpha C} \rangle}{\langle \Phi_{\alpha C} | \Phi_{\alpha C} \rangle}$

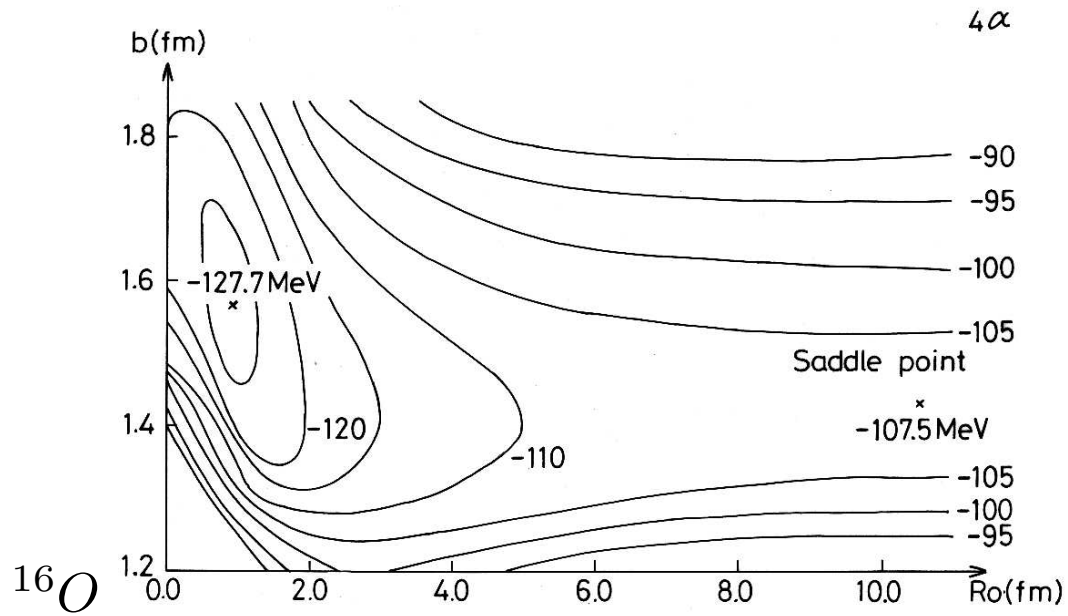
8B



${}^{12}C$



${}^{16}O$



Hamiltonian :

$$H = \begin{array}{ccccccc} T & + & V_{N-N} & + & V_C & + & V_{N-N-N} \\ \text{Kin. energy} & & \text{Gaussien} & & \text{Coulomb} & & \text{Gaussian} \end{array}$$

Quantization of energy surface $E(B, b)$:

Force : A. Tohsaki \sim 1990 no adjustable parameters !

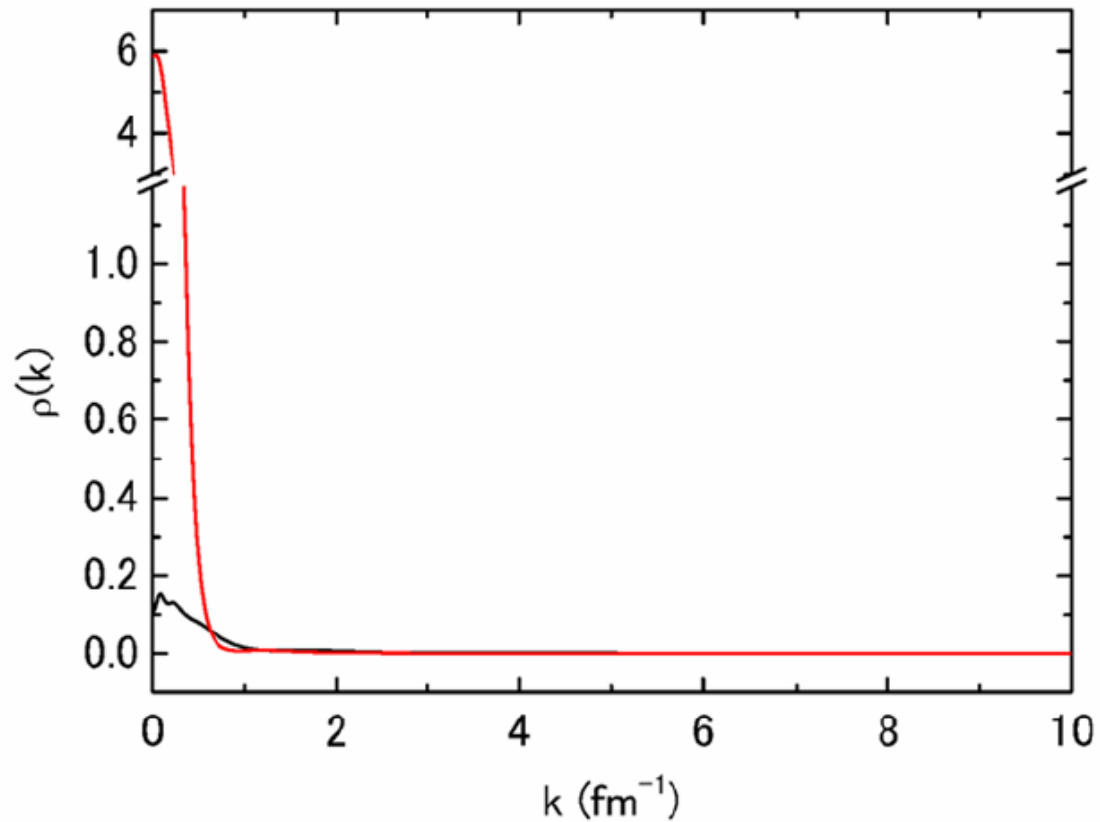
Hill-Wheeler : $|\psi\rangle = \sum_B f_B |\Phi_{\alpha C}(B)\rangle$

Without adjustable parameters :

$$\begin{array}{l} {}^{12}\text{C} : (E_{O_2^+} - E_{3\alpha}) = \text{Theory} + 0.50 \text{ MeV} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{Exp.} + 0.38 \text{ MeV} \\ {}^{16}\text{O} : (E_{O_5^+} - E_{4\alpha}) = \text{Theory} - 0.70 \text{ MeV} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{Exp.} - 0.44 \text{ MeV} \end{array}$$

r.m.s. of O_2^+ in ^{12}C 3.83 fm
 ground state 2.40 fm

Momentum Distribution



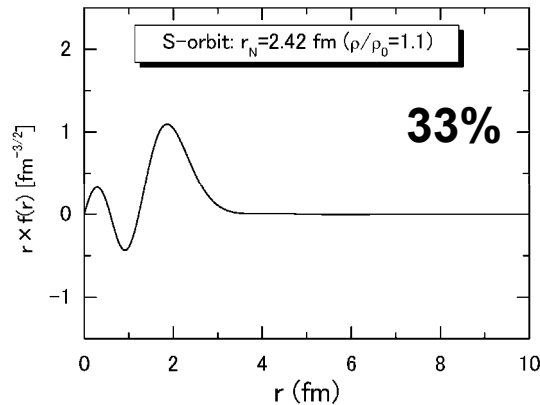
$$\frac{V_{O_2^+}}{V_{O_1^+}} \sim 3 - 4$$

	<i>Exp.</i>	<i>Cal.</i>
$M(O_2^+ \rightarrow O_1^+)$	$5.4 \pm 0.2 fm^2$	6.7
$R_{rms}(O_1^+)$	$2.43 fm$	2.40
$R_{rms}(O_2^+)$		3.47

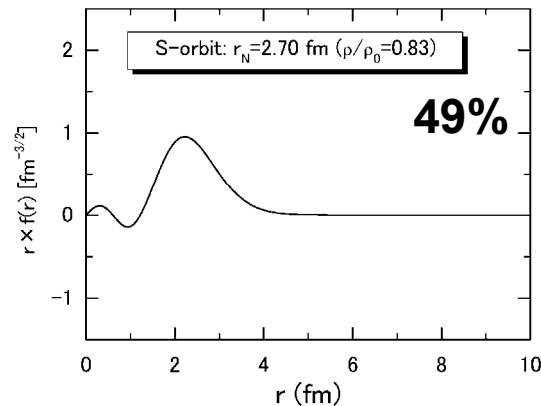
Radial behavior of S-wave α orbit vs. R_{rms}

$$R_{\text{rms}} = 2.43 \text{ fm} \rightarrow 4.84 \text{ fm} \quad (\rho/\rho_0 = 1.1 \rightarrow 0.14)$$

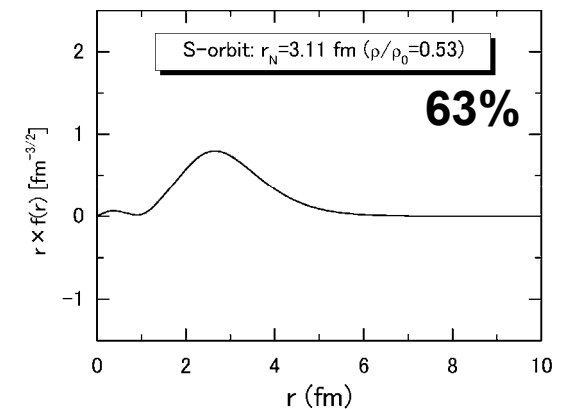
$R_{\text{rms}} = 2.43 \text{ fm} \quad (\rho/\rho_0 = 1.1)$



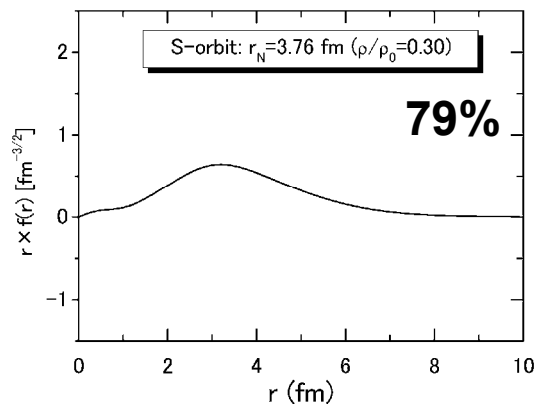
$R_{\text{rms}} = 2.70 \text{ fm} \quad (\rho/\rho_0 = 0.83)$



$R_{\text{rms}} = 3.11 \text{ fm} \quad (\rho/\rho_0 = 0.53)$



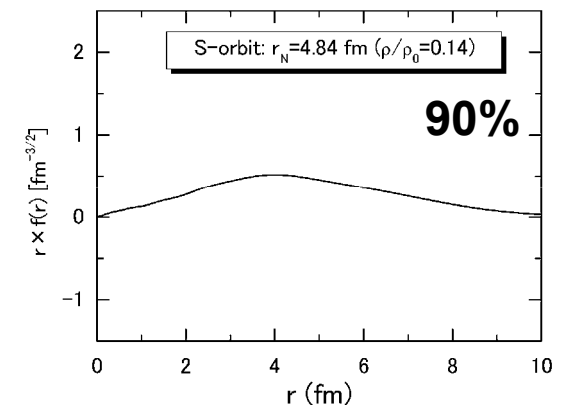
$R_{\text{rms}} = 3.76 \text{ fm} \quad (\rho/\rho_0 = 0.30)$

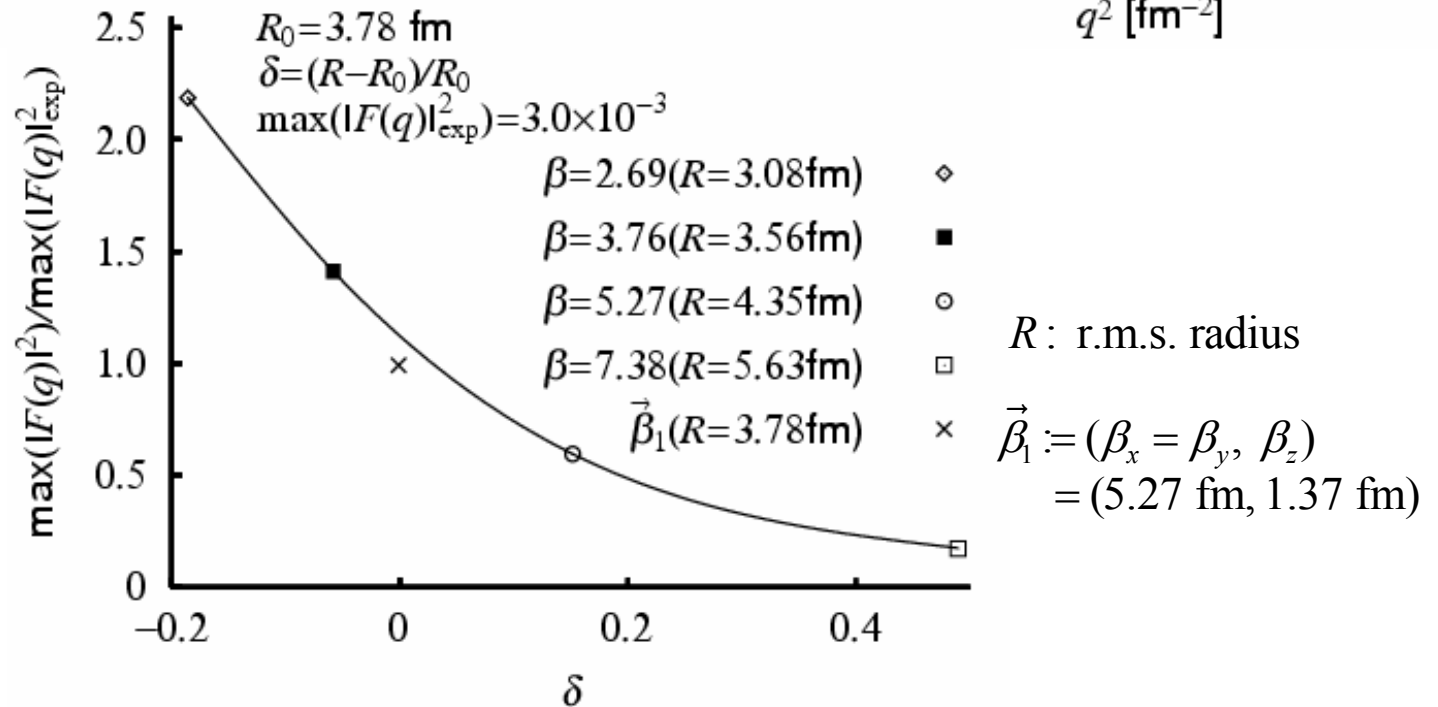
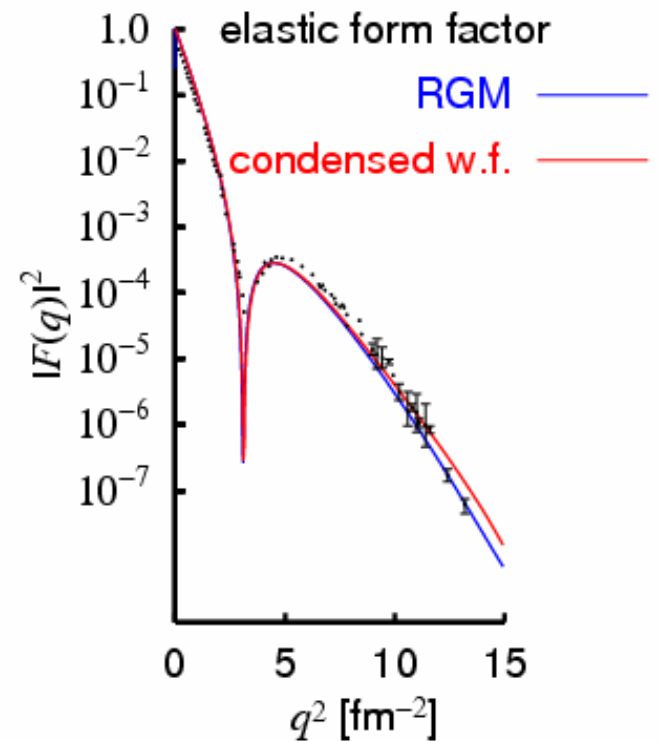
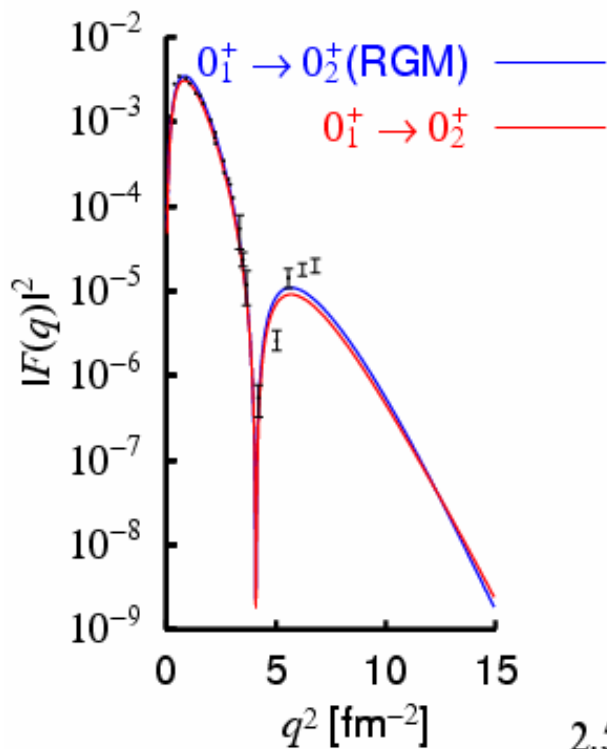


Increasing R_{rms} ,
we see smooth change
from **2S** to **0S** orbit.

Yamada & Schuck, EPJA 2005

$R_{\text{rms}} = 4.84 \text{ fm} \quad (\rho/\rho_0 = 0.14)$



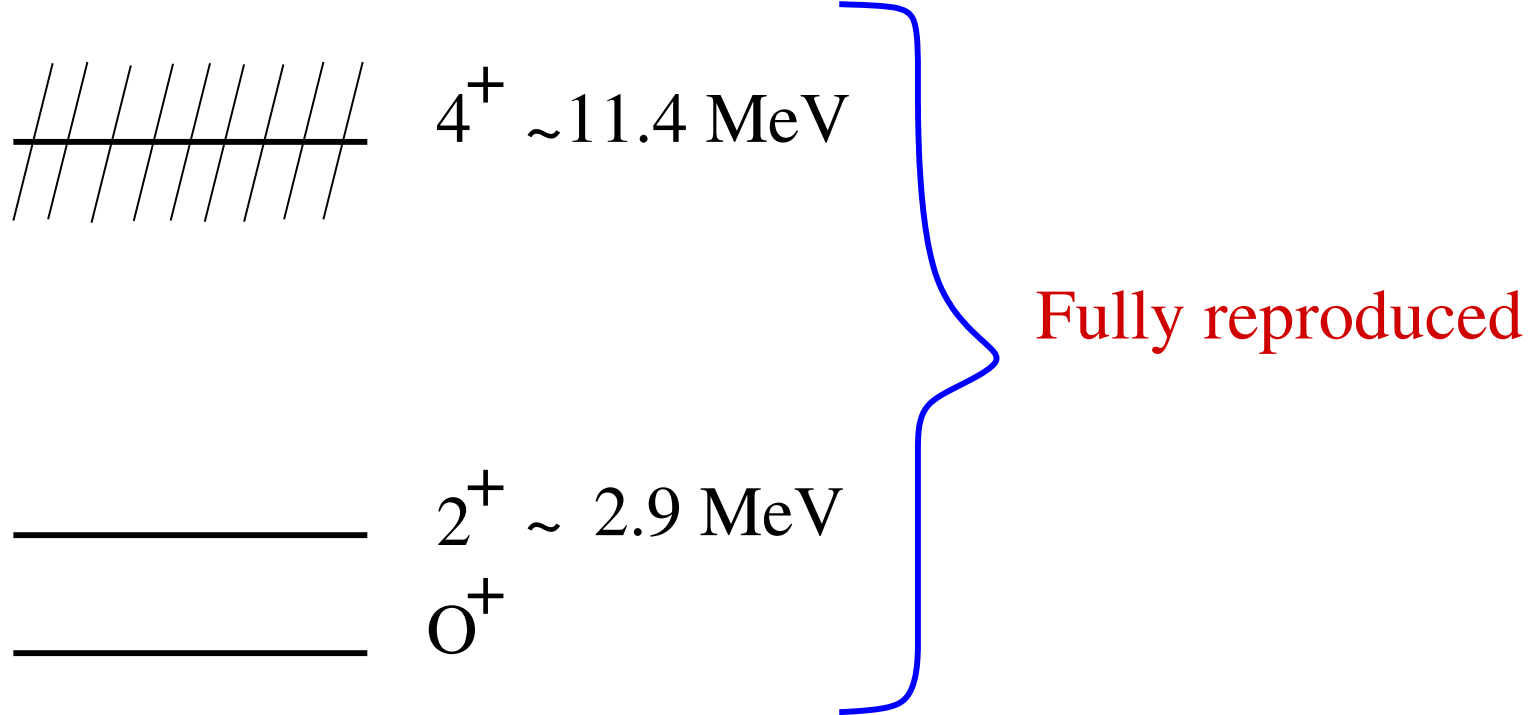


Some more numbers :

		Theory	Exp.
^{12}C :	O_1^+	-89.52	-92.16
	O_2^+	-81.79	-84.51
		7.73	7.65

	^8Be	^{12}C	^{16}O	^{20}Ne	
Threshold states	2α	3α	4α	5α	
$E - E_{\text{thresh.}}$	O_1^+	O_2^+	O_3^+	O_4^+] theory
	-0.17	0.50	-0.7	1.8	
	O_1^+	O_2^+	O_5^+	?] experimental
	0.09	0.38	-0.44	?	

Spectrum of ${}^8\text{Be}$:



${}^{12}\text{C}$: Second excited 2^+ : 2_2^+

It has been discovered recently by Itoh *et al.*

2.6 MeV above 3 α threshold

Width $\sim 1 \text{ MeV}$: resonance in continuum

Theory : We start with deformed α condensate state :

$$\Phi_{n\alpha} \propto \mathcal{A} \prod_{i=1}^n \exp \left\{ -\frac{2 X_{ix}^2}{B_x^2} - \frac{2 X_{iy}^2}{B_y^2} - \frac{2 X_{iz}^2}{B_z^2} \right\} \Phi_{\alpha_i}$$

Then projection on good angular momentum

Then Hill Wheeler or GCM

For width : ACCC method

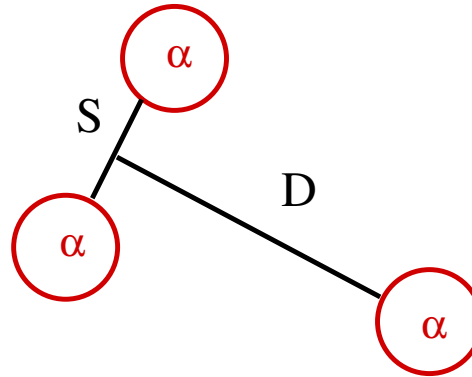
	Position	Width
<u>Experiment :</u>	$2.6 \pm 0.3 \text{ MeV}$	$1.0 \pm 0.3 \text{ MeV}$
<u>Theory :</u>	2.1 MeV	0.64 MeV

With in error bars !

$$\text{RMS : } \quad 4.43 \text{ fm}$$
$$\frac{V_{2_2^+}}{V_{0_1^+}} \sim 8 \quad !!$$

α - halo !

Internal structure :



Extremely dilute 3α state

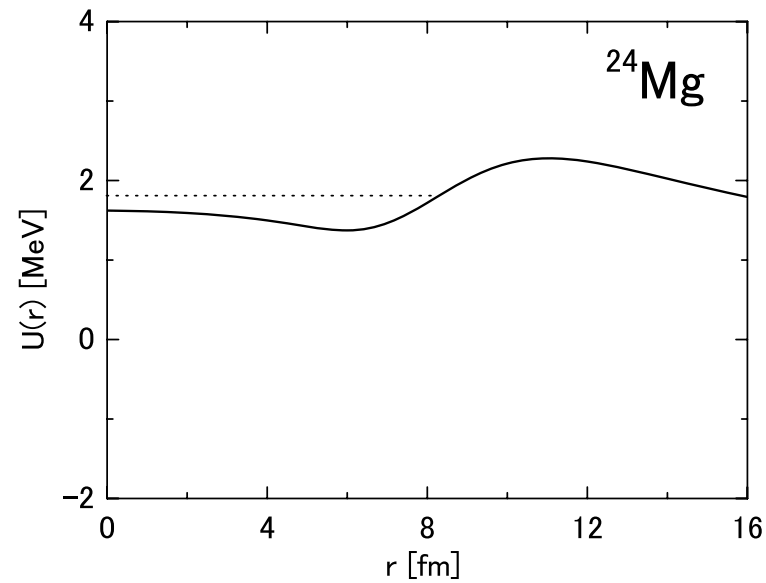
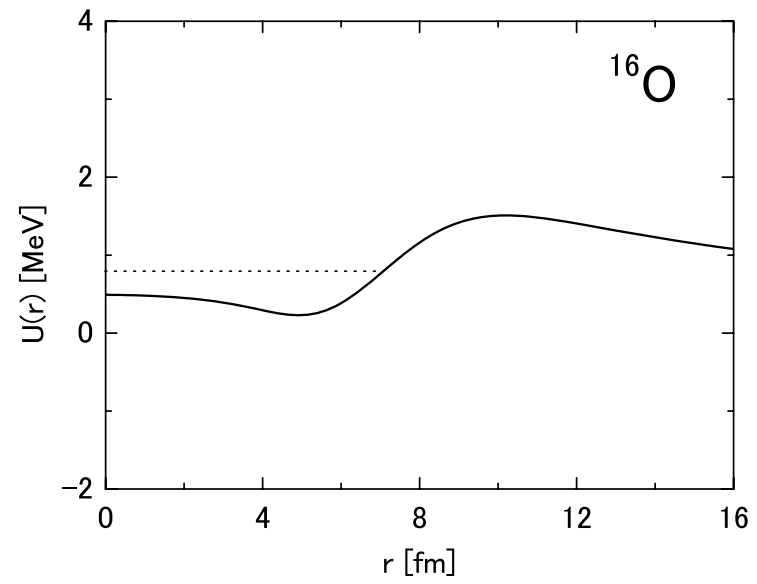
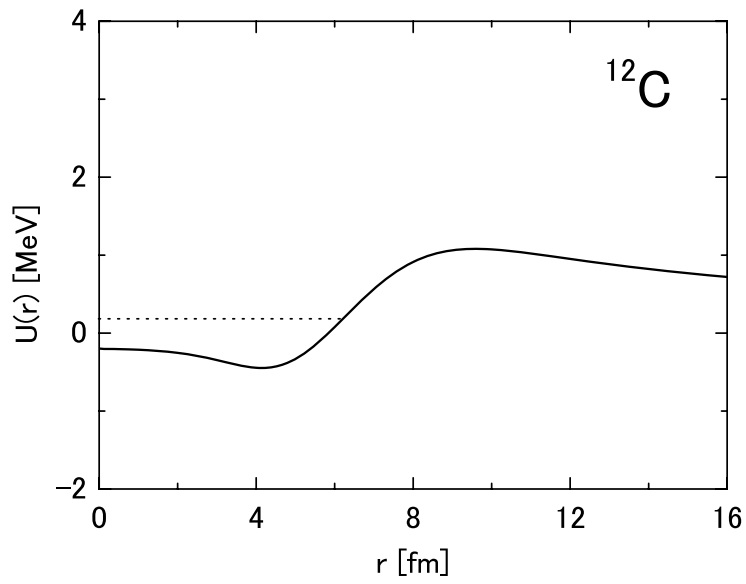
Suggests a pure Boson picture

$$|\phi_0\rangle = b_0^+ b_0^+ b_0^+ \dots |vac\rangle$$

Hartree – Fock (Gross Pitaevsky eq)
for ideal bosons (α ' s) :

$$\left[-\frac{\hbar^2}{2m_\alpha} \Delta + N \int d^3 r' v(\vec{r} - \vec{r}') |\phi_0(\vec{r}')|^2 \right] \phi_0(\vec{r}) = \epsilon_0 \phi_0(\vec{r})$$

effective $\alpha - \alpha$ + Coulomb
T. Yamada



Estimate for maximum number

$$N_{limit}^{\alpha} \simeq 10 \quad \Rightarrow \quad {}^{40}\text{Ca}^{**}$$

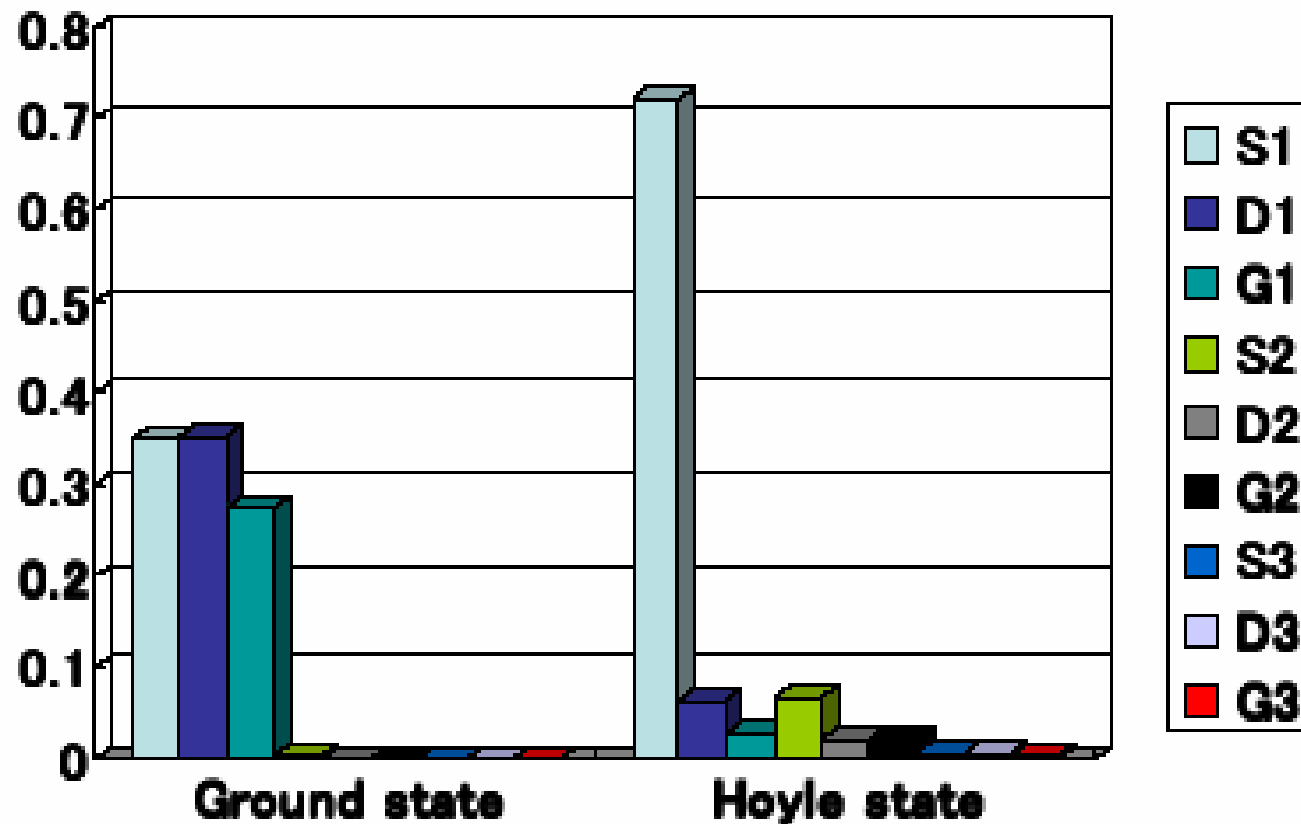
Boson occupancy :

α -particle density matrix :

$$\rho_{\alpha}(\vec{R}, \vec{R}'), \quad \vec{R} : \text{c.m. of } \alpha$$

Diagonalization :

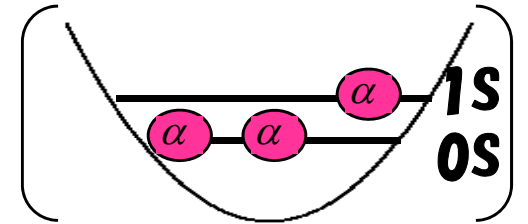
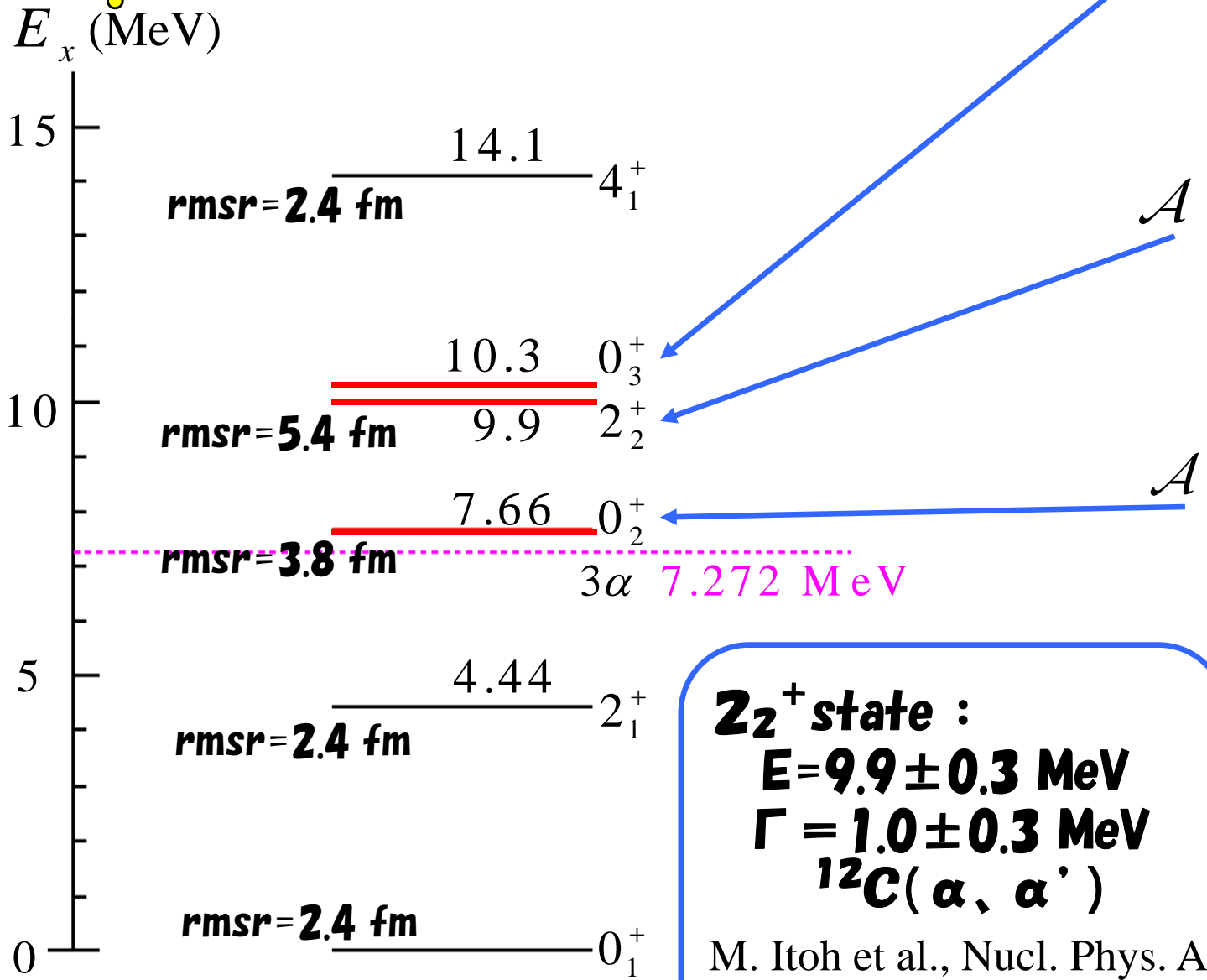
$^{12}\text{C} : O_2^+ \quad 70\% \quad \text{S-wave occupancy}$



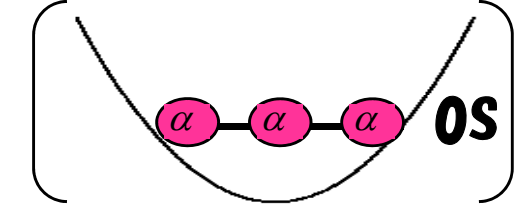
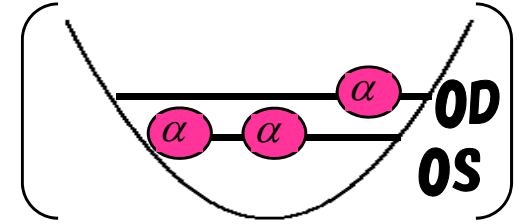
"BEC" in ^{12}C

??
A

Observed levels of ^{12}C



C. Kurokawa and K. Katō,
PRC 71, 021301 (2005).



2_2^+ state :
 $E = 9.9 \pm 0.3$ MeV
 $\Gamma = 1.0 \pm 0.3$ MeV
 $^{12}\text{C}(\alpha, \alpha')$
 M. Itoh et al., Nucl. Phys. A
 738 (2004) 268-272

α cond. + ACCC
 $E = 9.38$ MeV
 $\Gamma = 0.64$ MeV
Volkov No. 1 force
is adopted
 Y. F. et al., EPJA
 24, 321 (2005).

Hamiltonian of 4 α OCM

$$H = T + \sum_{i < j} \left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{Coul}(r_{ij}) \right] + V_{3\alpha} + V_{4\alpha} + V_{Pauli}$$

Equation of motion

$$\delta \left[\langle \Phi_L(^{16}\text{O}) | H - E | \Phi_L(^{16}\text{O}) \rangle \right] = 0$$

Pauli blocking operator on $\alpha - \alpha$ motions

$$V_{Pauli} = \lim_{\lambda \rightarrow \infty} \lambda \sum_{2n+l < 4} \sum_{ij} |u_{nl}(r_{ij})\rangle \langle u_{nl}(r_{ij})|$$

Pauli forbidden state: h.o.w.f.

2-body force (folding MHN force)

$$V_{2\alpha}(r) = \sum_n V_n^{(2)} \exp(-\beta_n^{(2)} r^2)$$

Coulomb force

$$V_{2\alpha}^{Coul}(r) = \frac{4e^2}{r} \text{erf}(ar)$$

Phenomenological 3-body force (repulsive)

$$V_{3\alpha} = V^{(3)} \sum_{i < j < k} \exp[-\beta(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)]$$

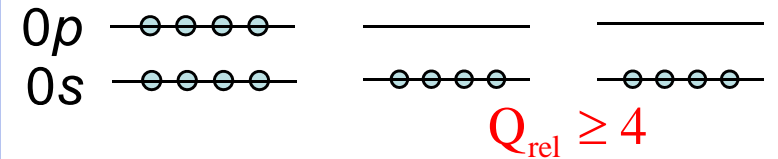
$$V^{(3)} = 87.5 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

Phenomenological 4-body force (repulsive)

$$V_{4\alpha} = V^{(4)} \exp[-\beta(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)]$$

$$V^{(4)} = 12000 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

$${}^8\text{Be}(Q \geq 4) \rightarrow \alpha(Q=0) + \alpha(Q=0)$$



$\alpha - \alpha$ motion with $Q_{rel} < 4$ should be eliminated

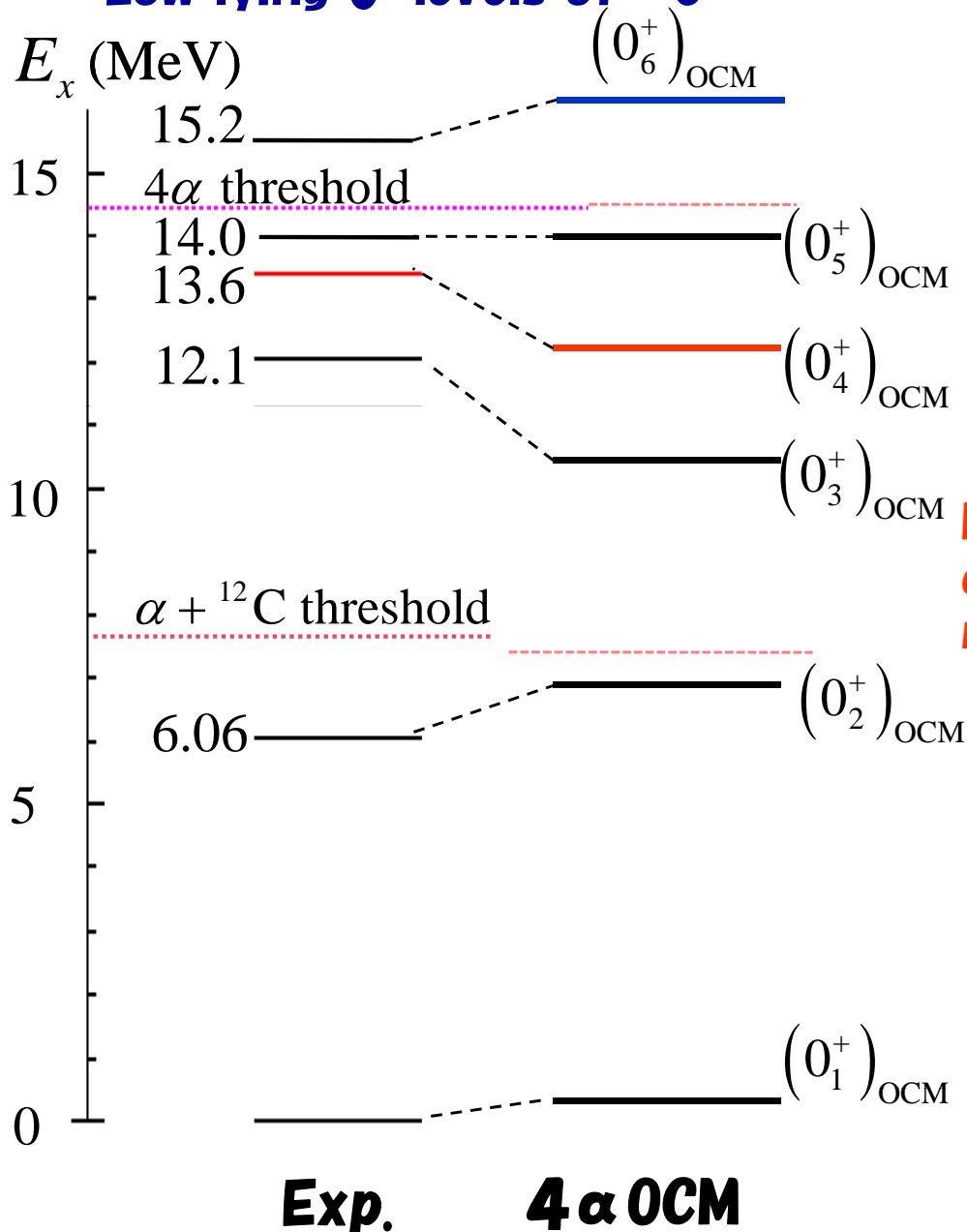
Q_{rel} : H.O. quantum number regarding $\alpha - \alpha$ motion

Energies from 4 α threshold

	Cal. (MeV)	Exp. (MeV)
${}^{12}\text{C}(\text{g.s.})$	<u>-7.32</u>	<u>-7.28</u>
${}^{12}\text{C}(2_1^+)$	-4.88	-2.84
${}^{12}\text{C}(4_1^+)$	2.06	6.43
${}^{12}\text{C}(0_2^+)$	0.70	0.38
${}^{16}\text{O}(\text{g.s.})$	<u>-14.2</u>	<u>-14.44</u>

Energy levels, rms radii, monopole matrix elements and density distribution.

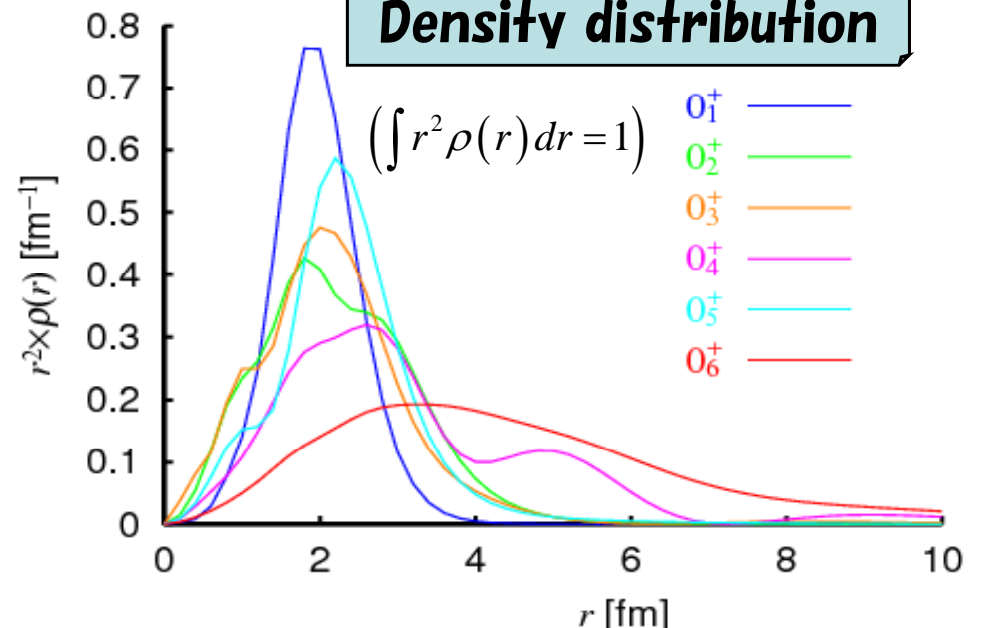
Low lying 0^+ levels of ^{16}O



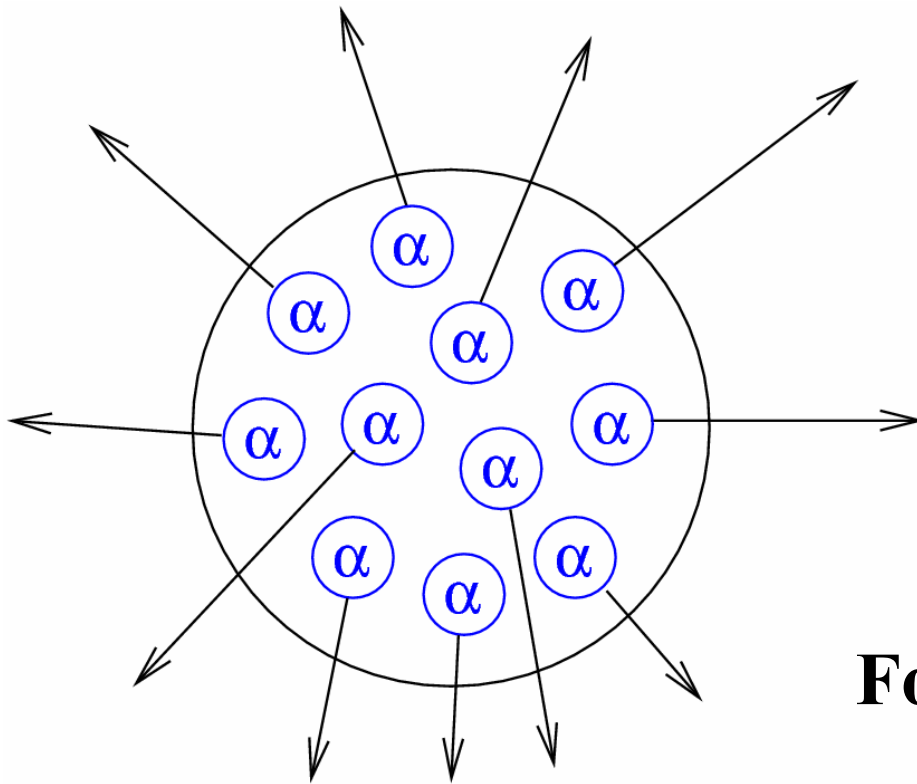
	R_{rms} (fm)	$M(E0)$ (fm ²)	$M(E0)$ (fm ²) Exp.
$(0_1^+)_{\text{OCM}}$	2.7		
$(0_2^+)_{\text{OCM}}$	3.1	4.2	0_2^+: 3.55
$(0_3^+)_{\text{OCM}}$	2.9	4.1	0_3^+: 4.03
$(0_4^+)_{\text{OCM}}$	3.9	2.4	0_4^+: no data
$(0_5^+)_{\text{OCM}}$	3.1	2.0	0_5^+: 3.3
$(0_6^+)_{\text{OCM}}$	5.4	1.4	0_6^+: no data

Large monopole matrix element can be the evidence of cluster states (Yamada, Y.F. et al., nucl-th/0703045)

Density distribution



Expanding Many α – Particle Coherent State:



**All α 's can be
detected in coincidence**

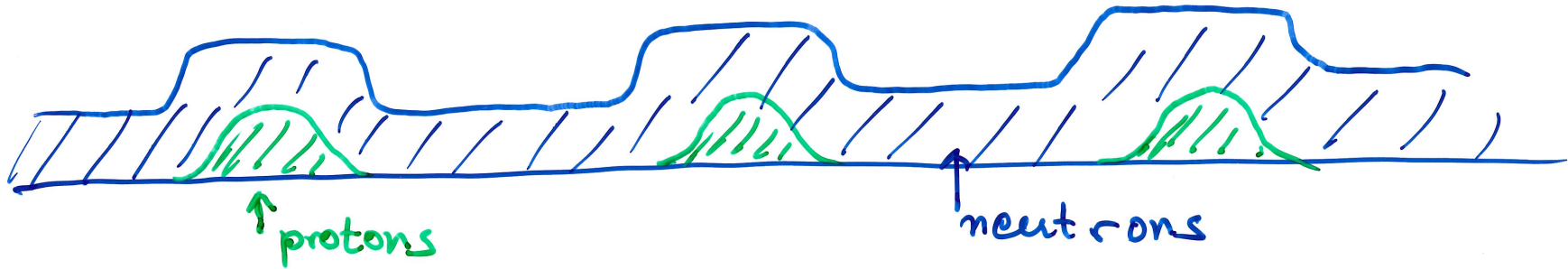
→ Coherent State Established

For example, $^{40}\text{Ca}^* \rightarrow 10 \alpha$'s

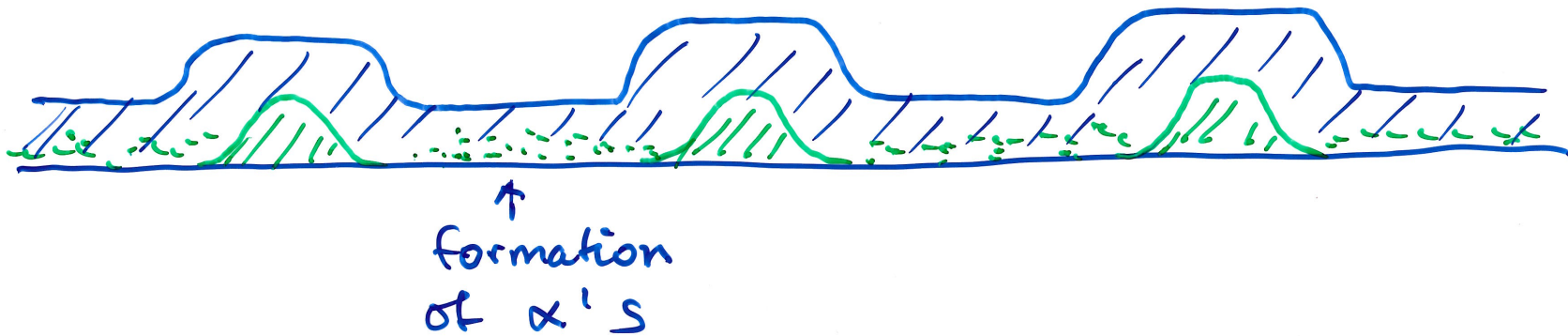
Search at Orsay

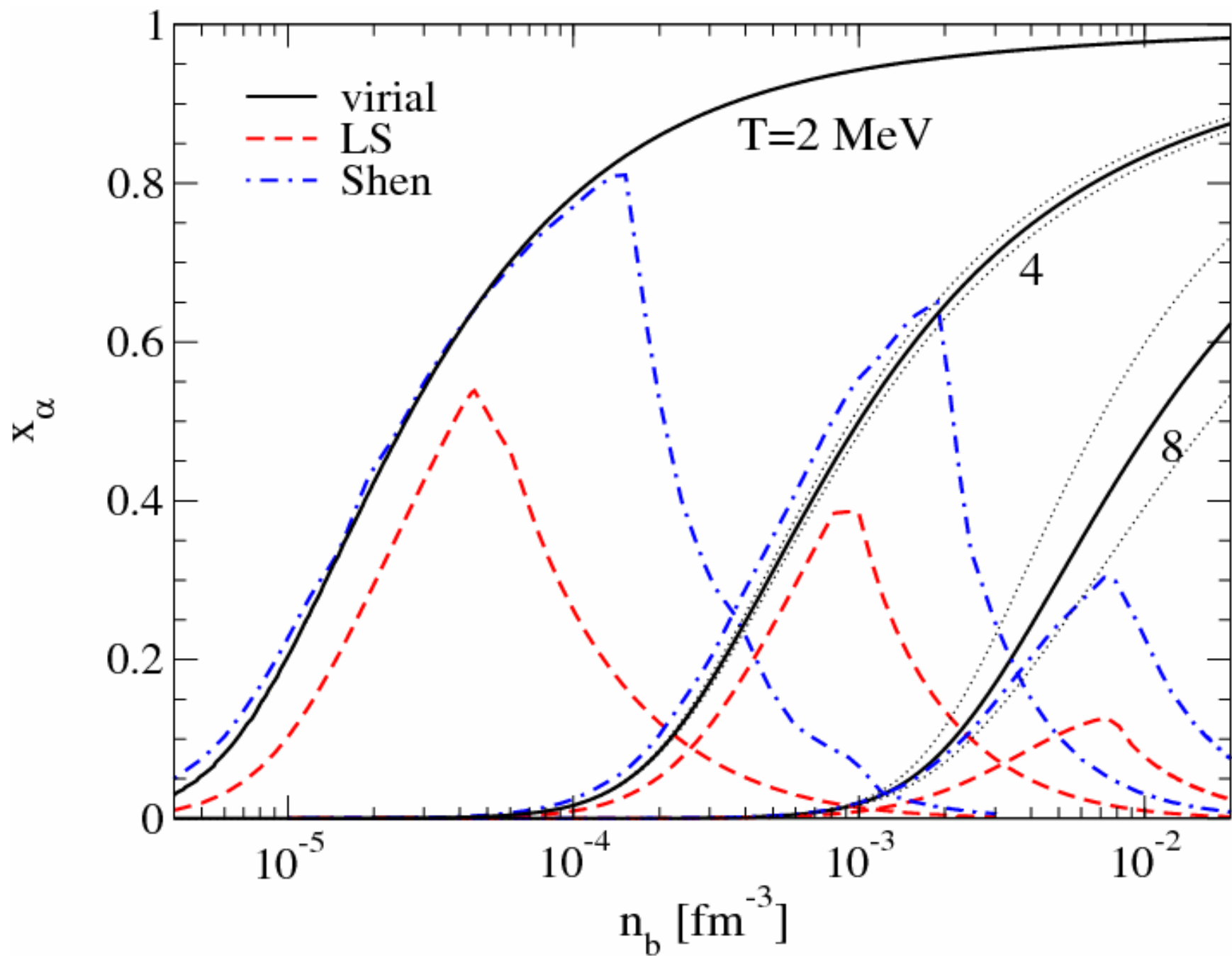
Alpha particle condensation in crust of neutron stars?

$T=0$

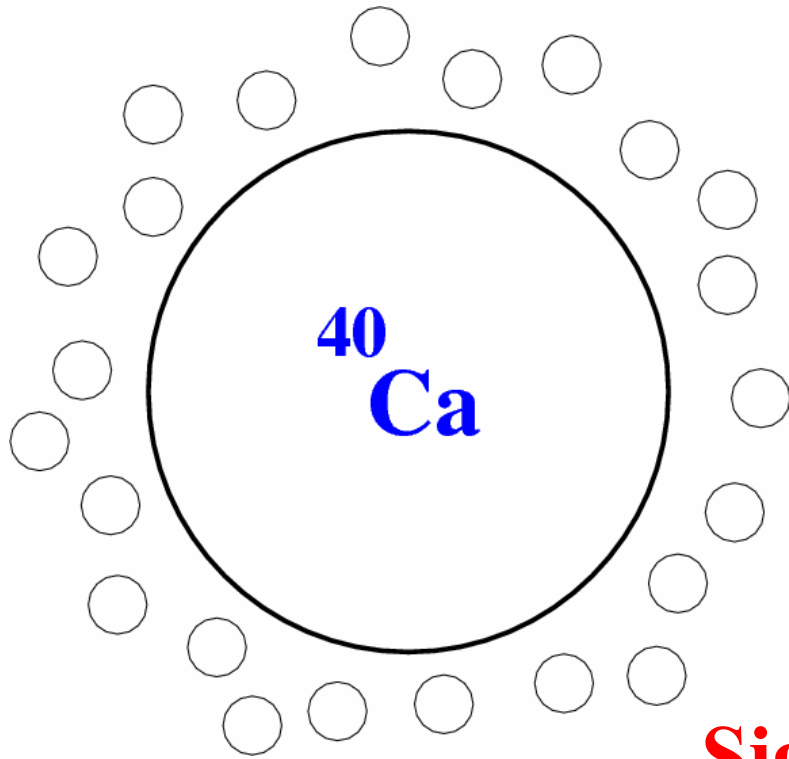


$T>0$





Compound Nuclei with α – Gas:

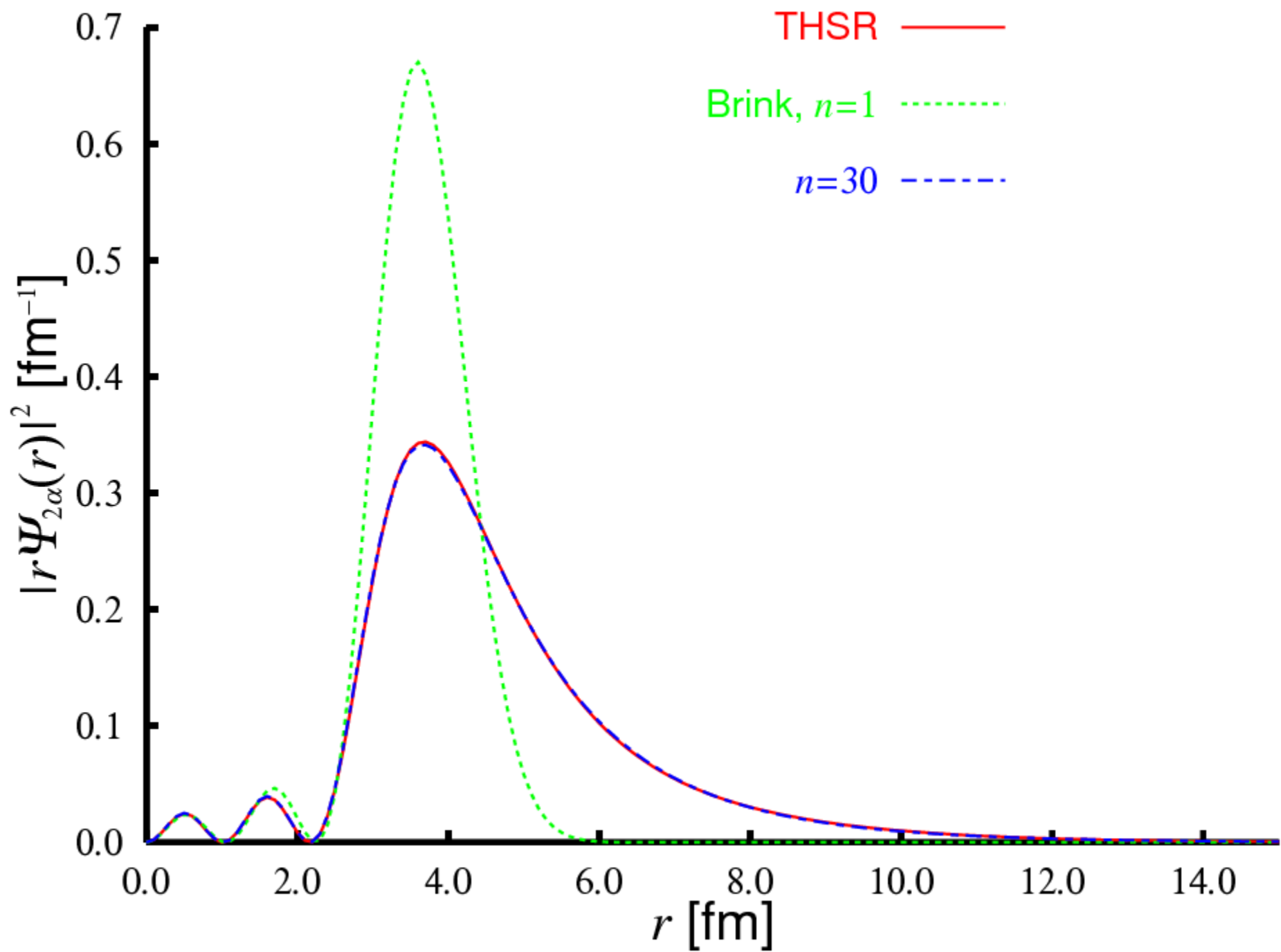


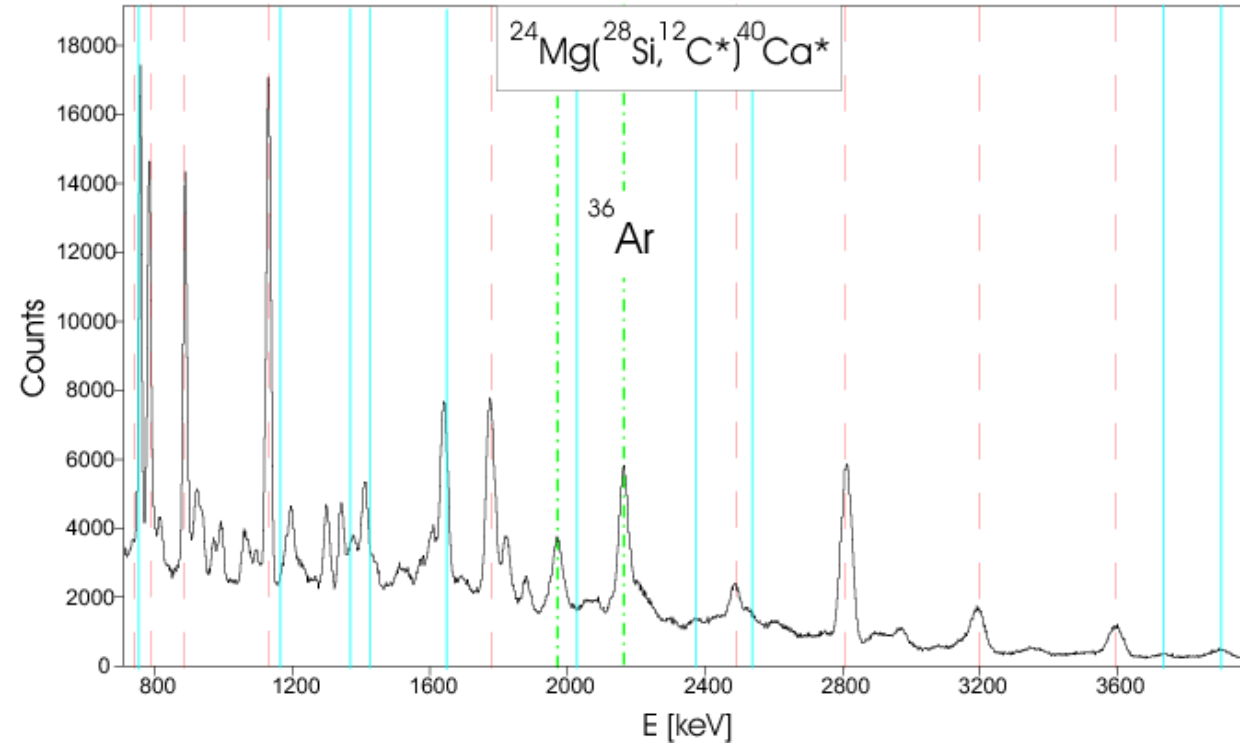
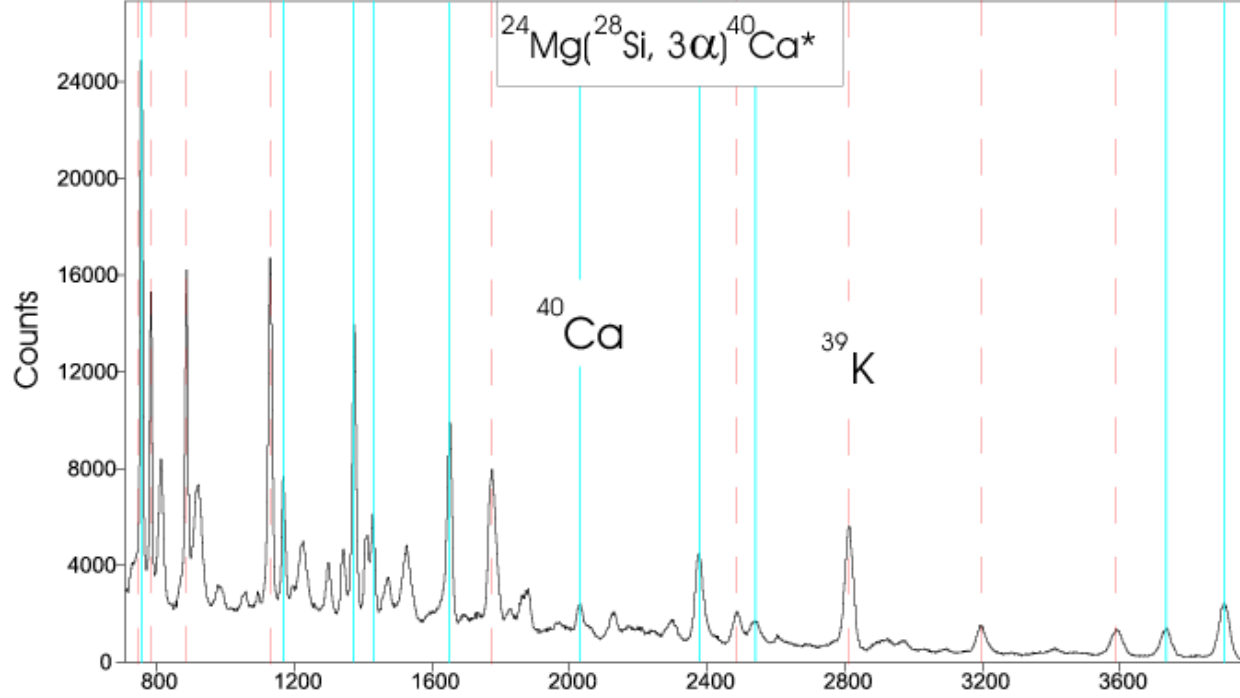
Disintegration into 2 α 's

Enhanced \rightarrow

Sign of α – Particle Condensate

(W. V. O.)





Critical Temperature for α -particle Condensation in symmetric and asymmetric nuclear matter

T. Sogo, P. Schuck, G. Röpke

Well known: T_c for pairing

Gap eq.
$$\Delta_p = \sum_{p'} V_{pp'} \frac{\Delta_{p'}}{2E_{p'}} (1 - 2f(E_{p'}))$$

$$E_p = \sqrt{(\varepsilon_p - \mu)^2 + \Delta_p^2} \quad f(\omega) = \frac{1}{e^{\omega/T} + 1}$$

Critical temperature $\Delta_p \rightarrow 0$

$$2\varepsilon_p \kappa_p - (1 - 2f(\varepsilon_p - \mu)) \sum_{p'} V_{pp'} \kappa_{p'} = 2\mu \kappa_p$$

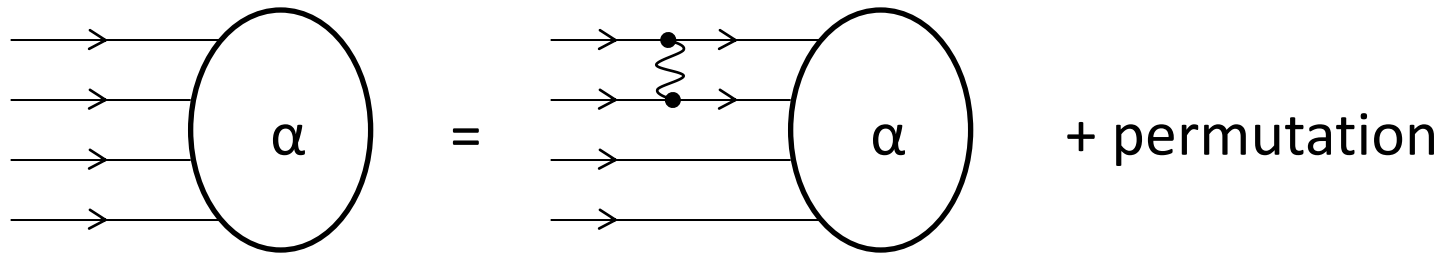
Thouless criterion!

two body equation

Generalization to four body (α -particle) condensation

eq. for T_c

$$\begin{aligned}
 & (4\mu - \varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4)\psi_{1234} \\
 &= (1 - f_1 - f_2)v_{121'2'}\psi_{1'2'34} + (1 - f_1 - f_3)v_{131'3'}\psi_{1'23'4} \\
 &+ (1 - f_1 - f_4)v_{141'4'}\psi_{1'234'} + (1 - f_2 - f_3)v_{232'3'}\psi_{12'3'4} \\
 &+ (1 - f_2 - f_4)v_{242'4'}\psi_{12'34'} + (1 - f_3 - f_4)v_{343'4'}\psi_{123'4'}
 \end{aligned}$$

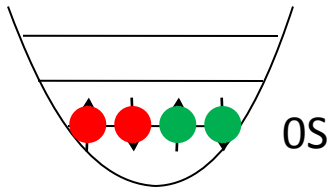


Solution with realistic NN force: R. Lazauskas, PRC**79**,1051801

Heavy numerical calculation! Approximation demanded!

Trial: momentum projected mean field

$$\psi_{1234} = \delta(\vec{K} - \vec{k}_1 - \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \varphi(k_1) \varphi(k_2) \varphi(k_3) \varphi(k_4)$$



For condensation $\vec{K} = 0$!

Only one unknown single particle wave function $\varphi(k)$

+ Effective interaction:

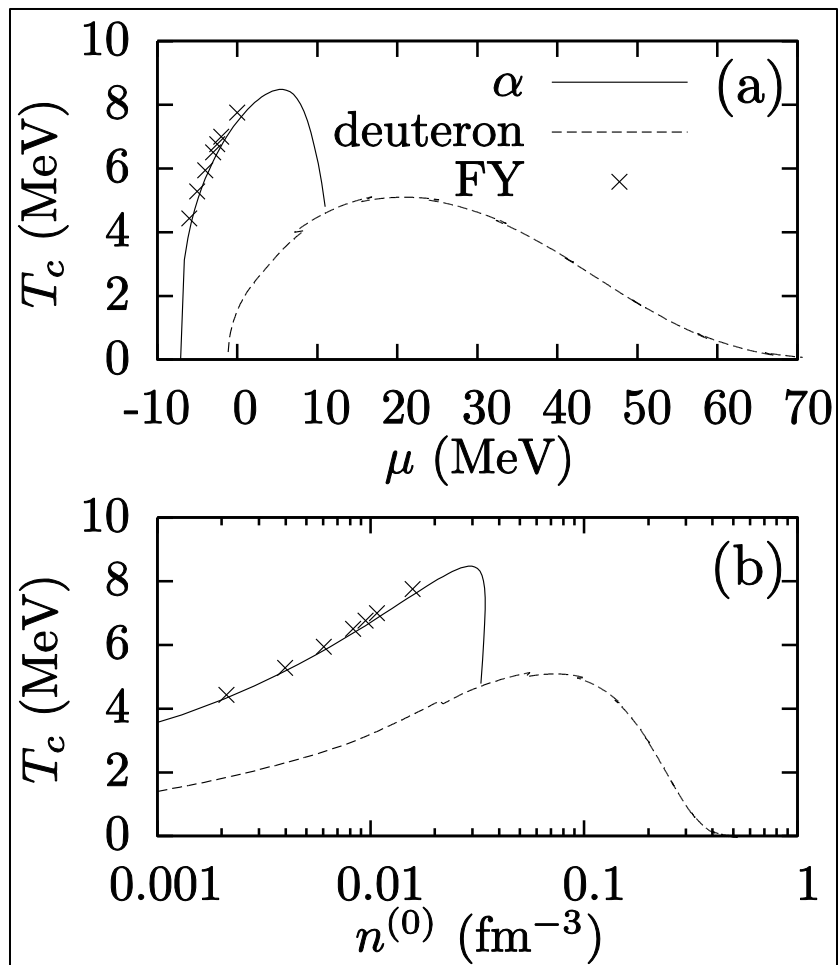
$$v_{1234} = \lambda w(\vec{k}_1 - \vec{k}_2) w(\vec{k}_3 - \vec{k}_4) \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \quad w(\vec{k}) = e^{-k^2 / (4k_0^2)}$$

λ, k_0 : such that binding and radius of α -particle correct!

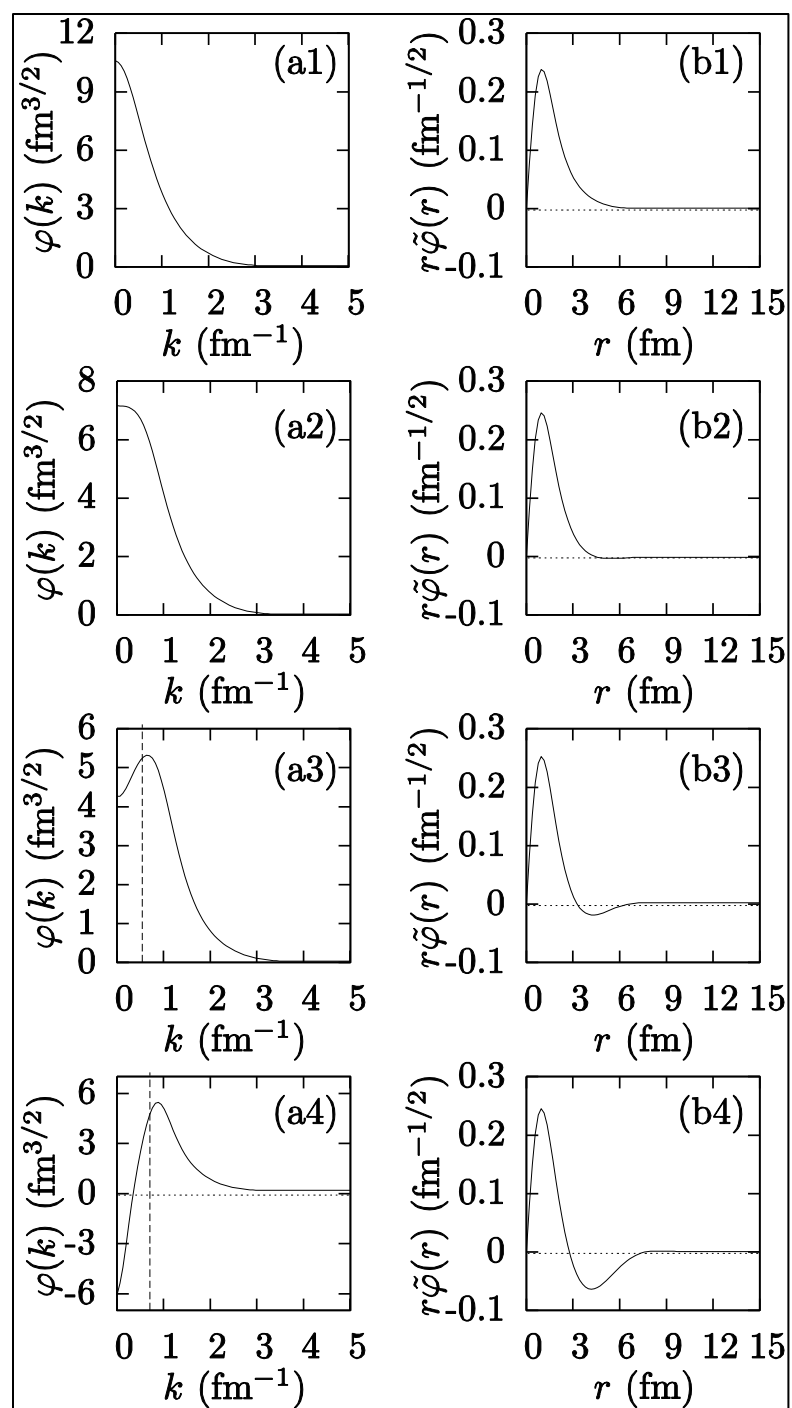
↳ HF type of eq. for $\varphi(k)$

$$\varphi(k) = - \frac{3B[\varphi, T, \mu]}{A[\varphi, T, \mu] + 3C[\varphi, T, \mu]}$$

Results: Symmetric nuclear matter



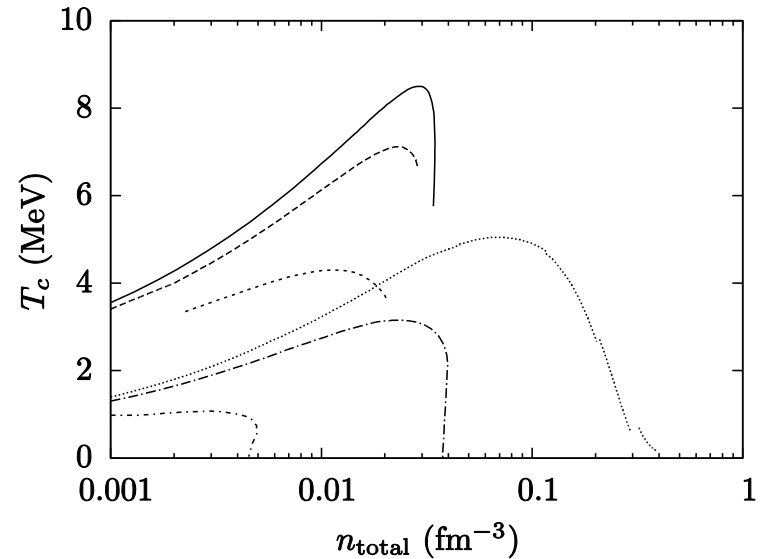
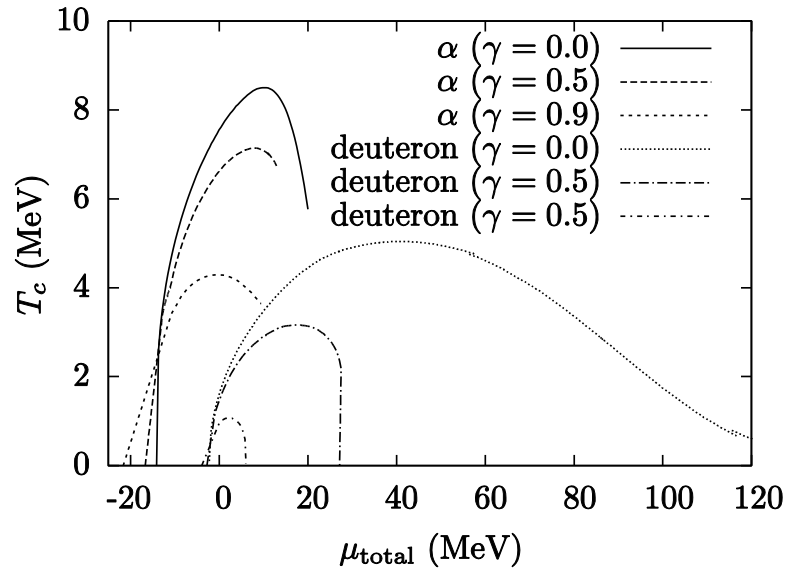
[T. Sogo, et. al., PRC **79**, 051301 (2009)]



We generalize to asymmetric matter, like in compact stars.

Same formulation with projected mean field

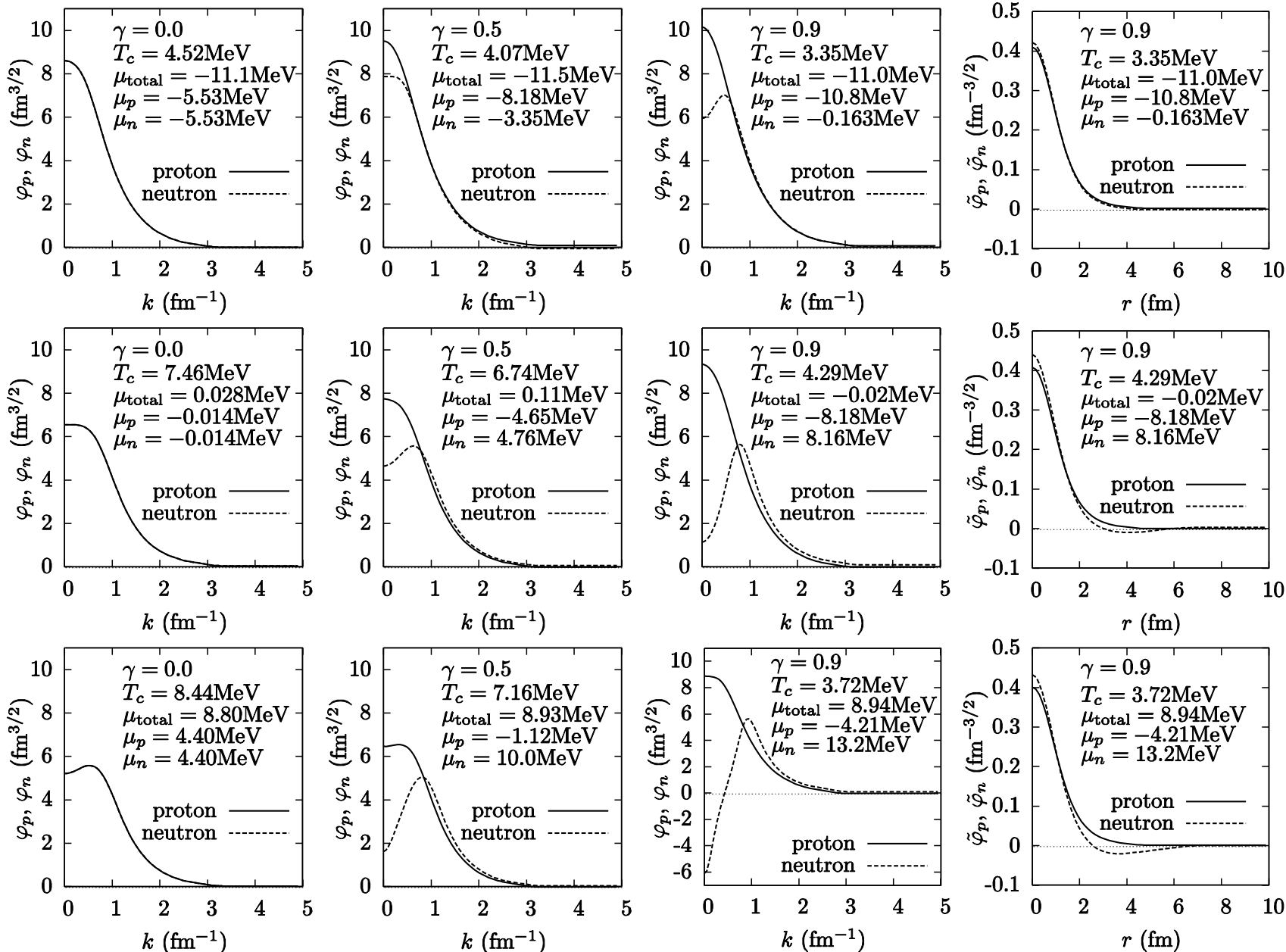
Now $\varphi_p \neq \varphi_n \rightarrow$ two coupled eqs.



$$\gamma = \frac{n_n - n_p}{n_n + n_p}$$

Critical temperature quite high
even for very strong asymmetry ($\gamma=0.9$)!

Wave functions (Asymmetric nuclear matter)



In BCS we have the following two coupled eqs.:

$$hU + \Delta V = EU$$

$$\Delta U - hV = EV$$

In first eq we can eliminate V component using second eq.:

$$V = \frac{1}{E + h} \Delta U$$

insert into first eq.:

$$(E - h)U + \frac{\Delta\Delta}{E + h}U = 0$$

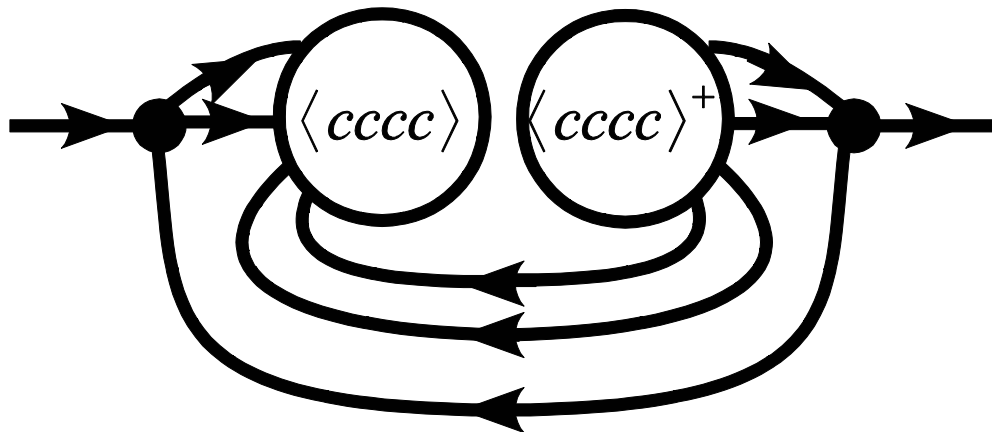
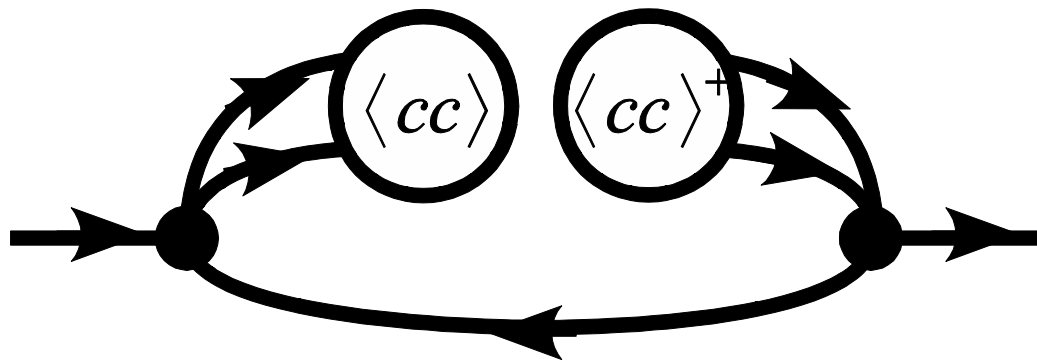
The effective single particle potential

$$M^{BCS} = \frac{|\Delta|^2}{E + h}$$

is called the **single particle mass operator in BCS approximation**

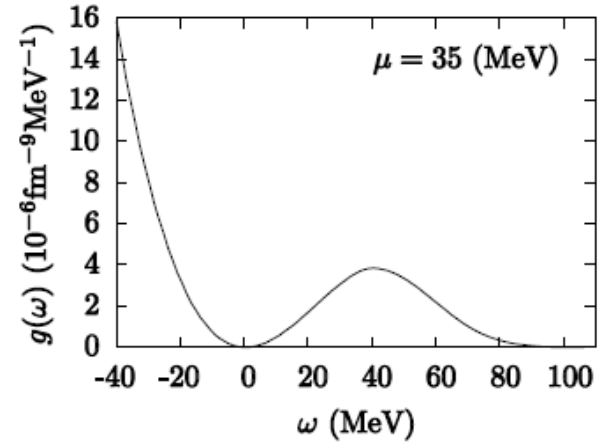
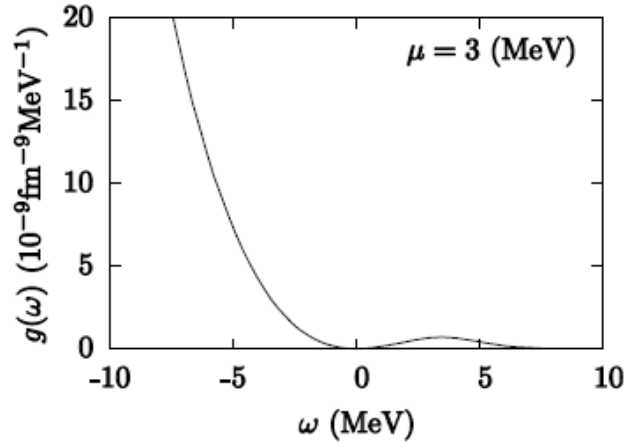
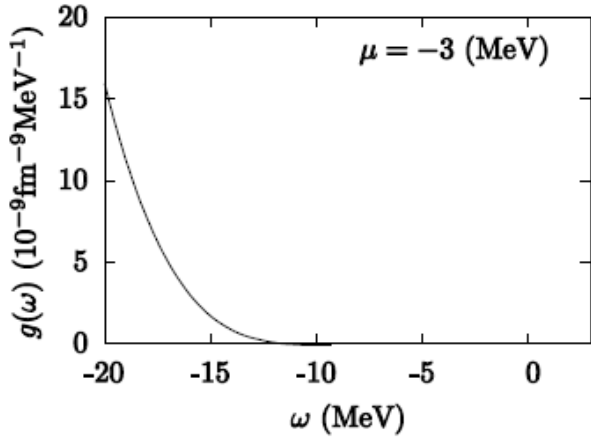
Full nonlinear quartet order parameter equation + solution

Pairing and quartet mass operators:



Three hole level density $\bar{f} = 1 - f$

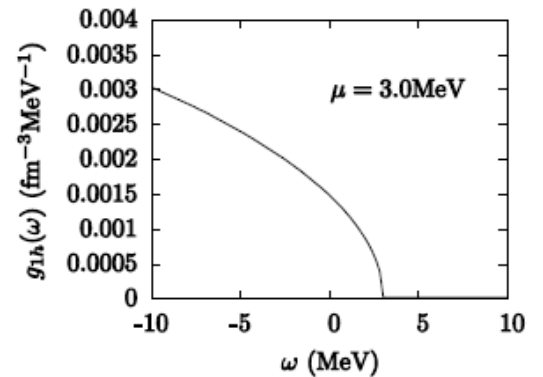
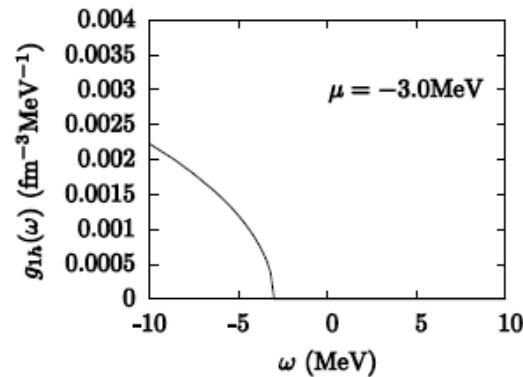
$$g(\omega) = \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3k_3}{(2\pi)^3} (\bar{f}_1 \bar{f}_2 \bar{f}_3 + f_1 f_2 f_3) \delta(\omega + \varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$



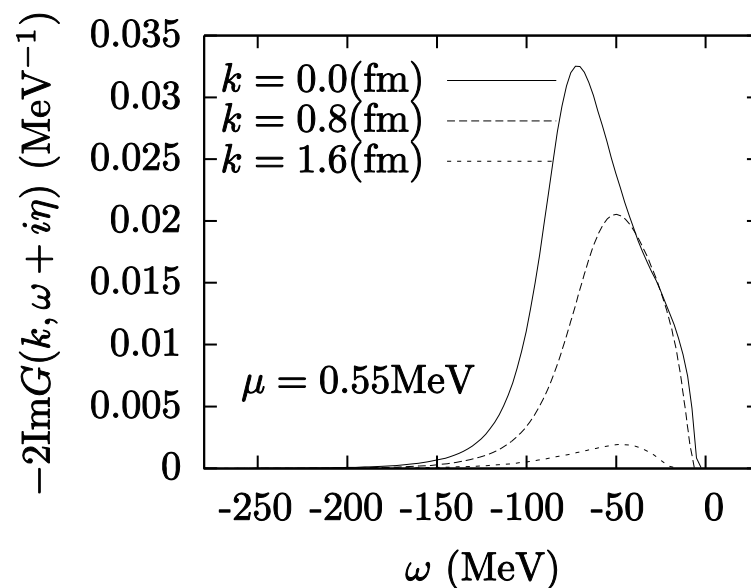
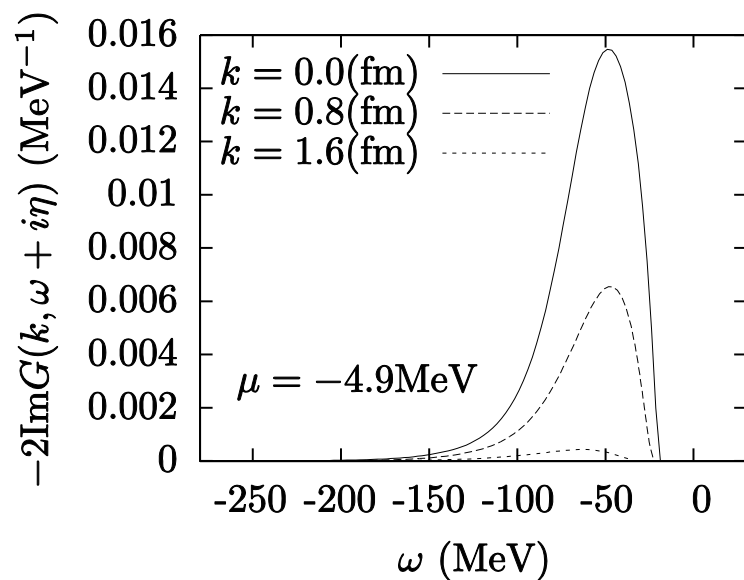
c.f. one hole level density

$$g_{1h}(\omega) = \int \frac{d^3k}{(2\pi)^3} \pi [f_k + \bar{f}_k] \delta(\omega + \varepsilon)$$

$$= \begin{cases} \frac{m}{2\pi} \sqrt{2m(\mu - \omega)} & (\mu > \omega) \\ 0 & (\mu < \omega) \end{cases}$$



Spectral function of single-particle GF



No sharp quasiparticle pole!

Conclusion

Projected product ansatz allows to tackle with the difficult quartet condensation process.

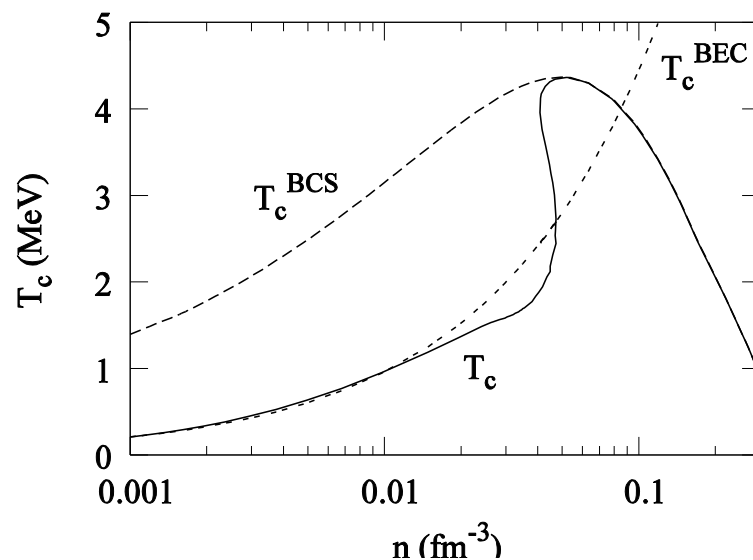
Still missing :

Theoretical description of the transition weakly bound quartets (present) to strongly bound ones (BEC)

For pairing, this has been achieved by Nozières and Schmitt-Rink.

Transition between weakly bound pairs and strongly bound molecules seems to be smooth.

However, BEC transition temperature much lower than BCS one.



What will it be for α -particle? Future task to be solved!

Conclusions, Outlook:

- **Loosely Bound 3 – α State in $^{12}\text{C}^*$ Finally Established**
- **More α – Particle Condensates Very Likely to Exist**
- **Nuclei Unique Fermi Systems for Cluster Effects. Eventually also in Future Cold Atom Experiments**
- **May be α – Condensation Important in Proto – Neutron stars**

Collaborators:

Y. Funaki, H. Horiuchi, G. Röpke, A. Tohsaki,

T. Yamada, W. von Oertzen

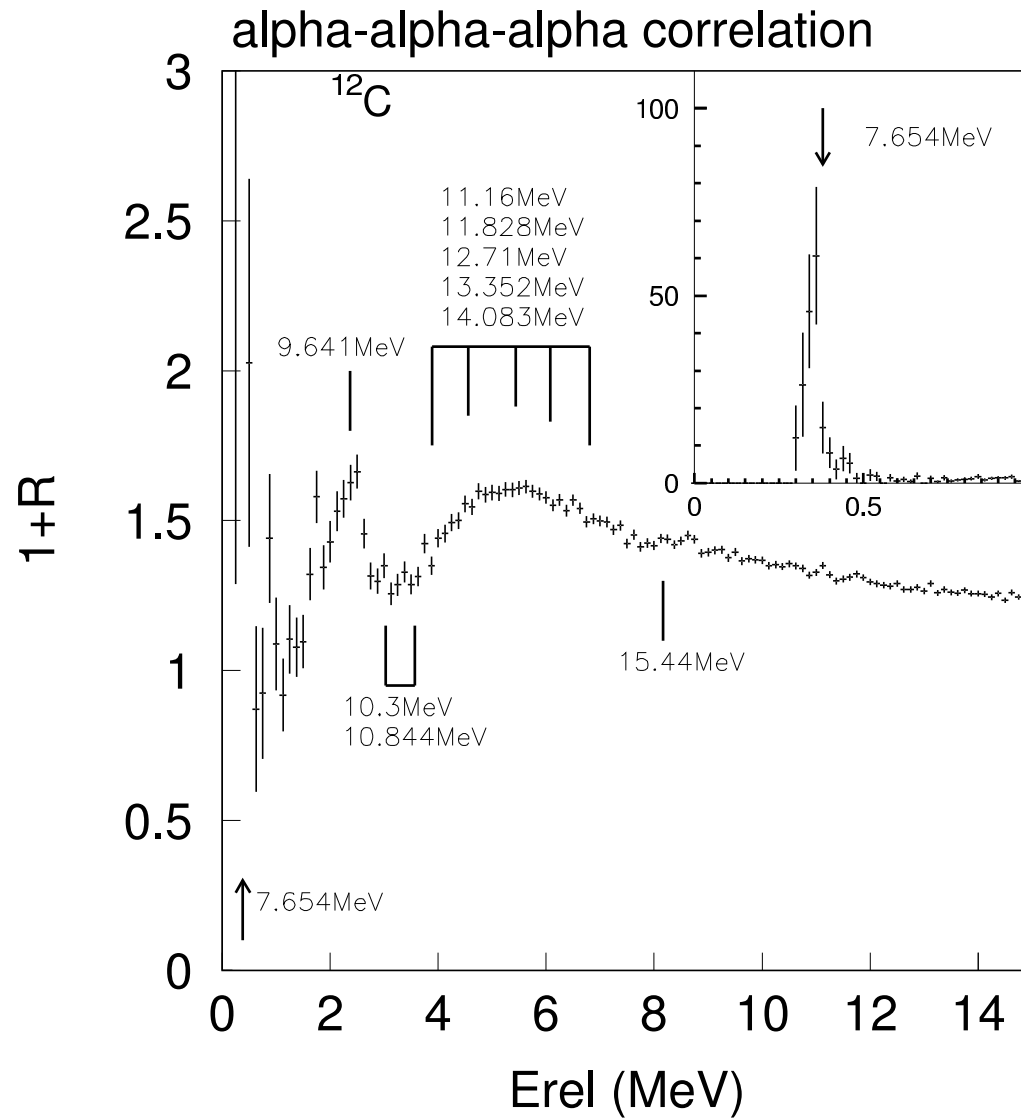
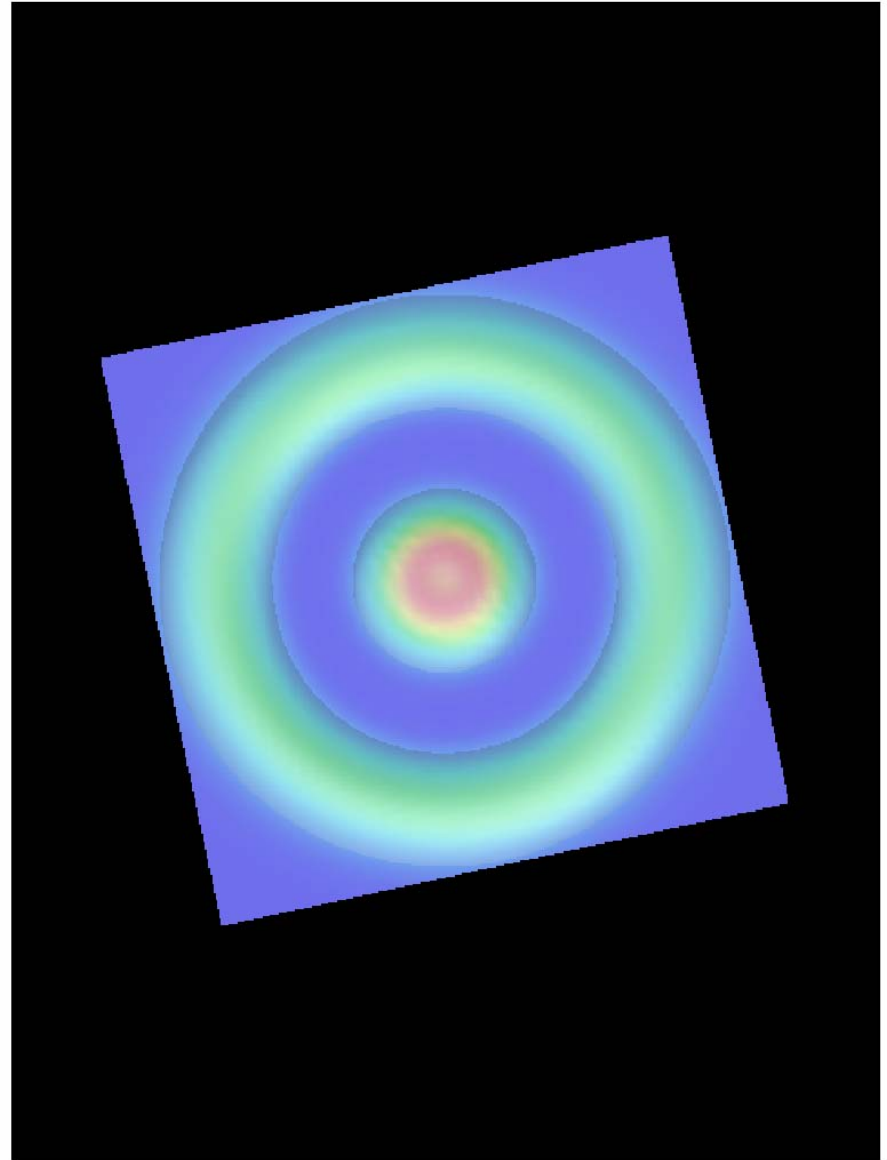
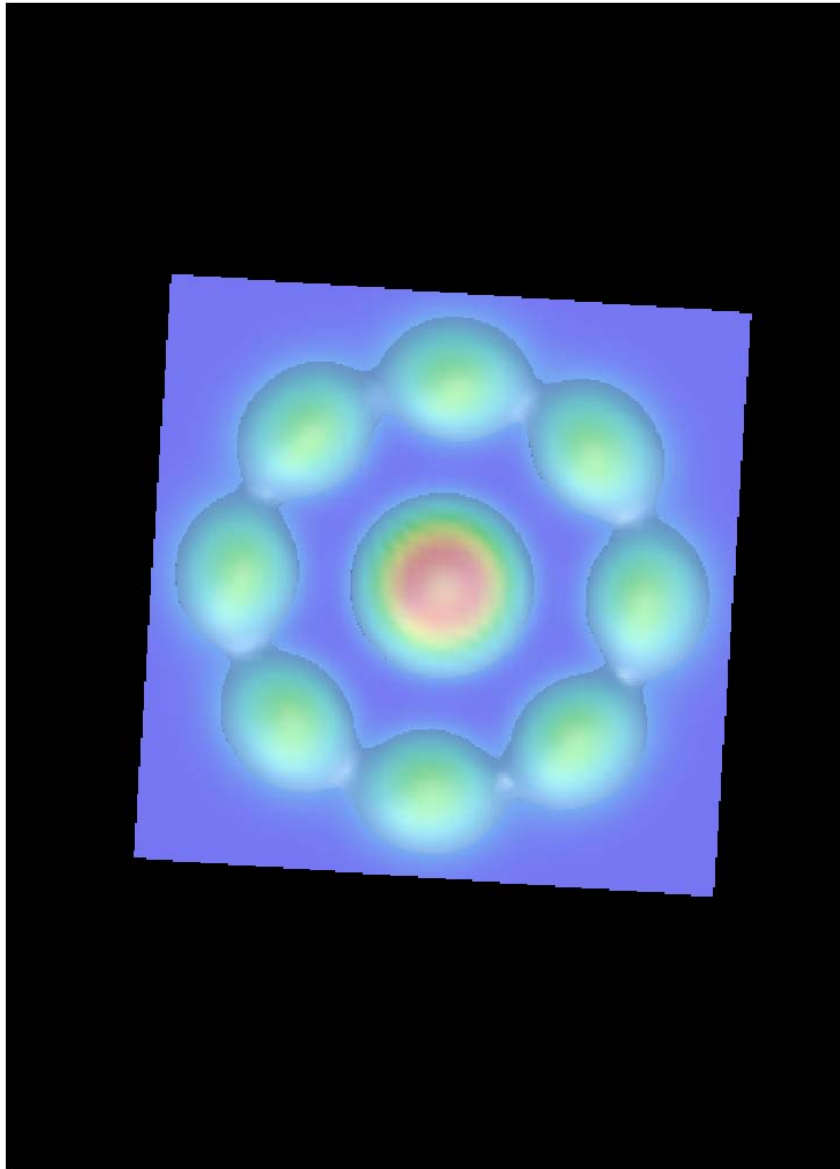
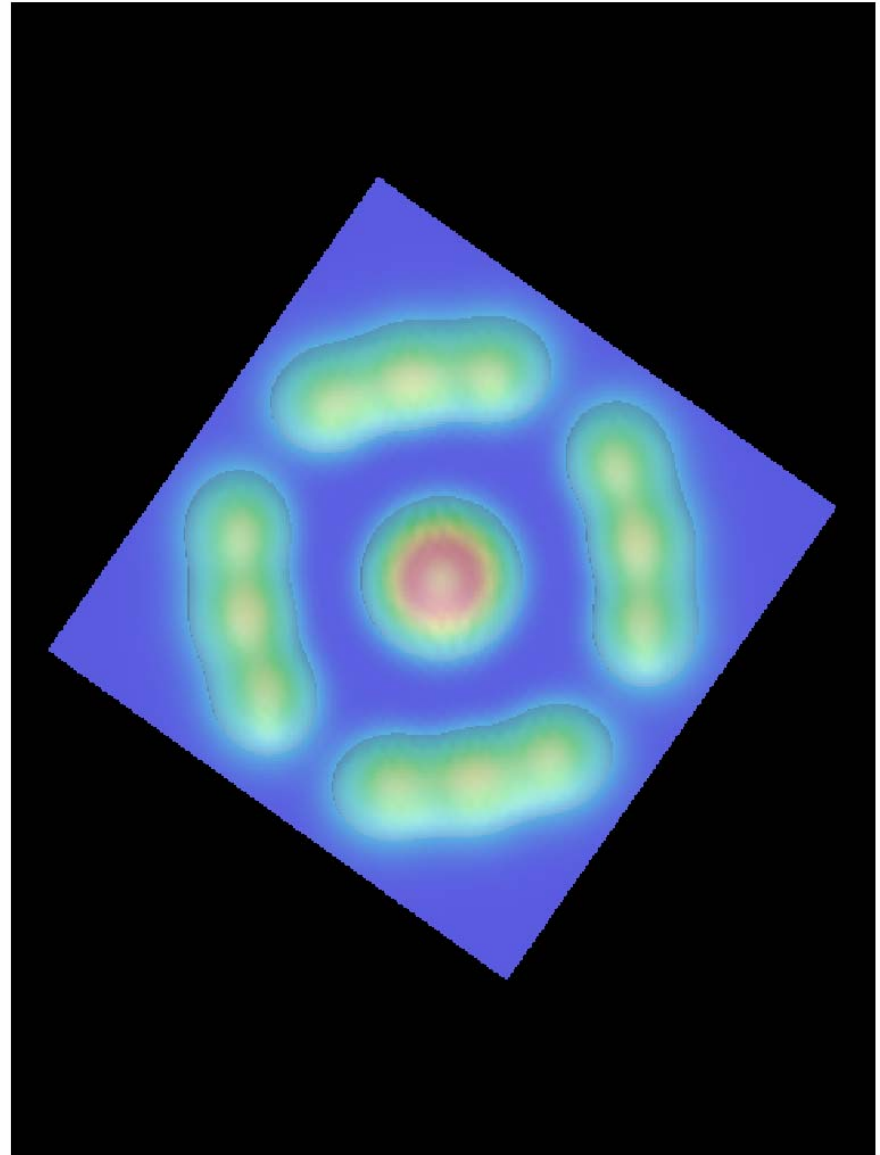
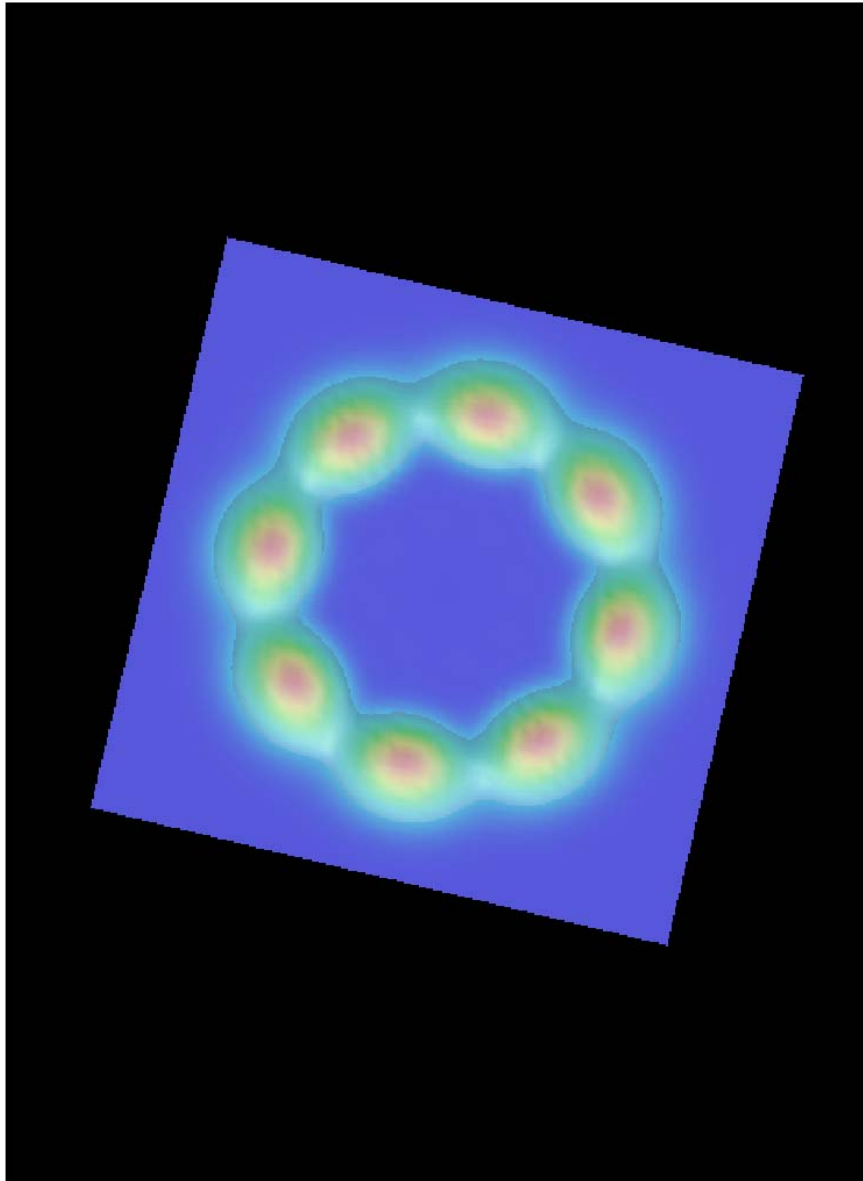
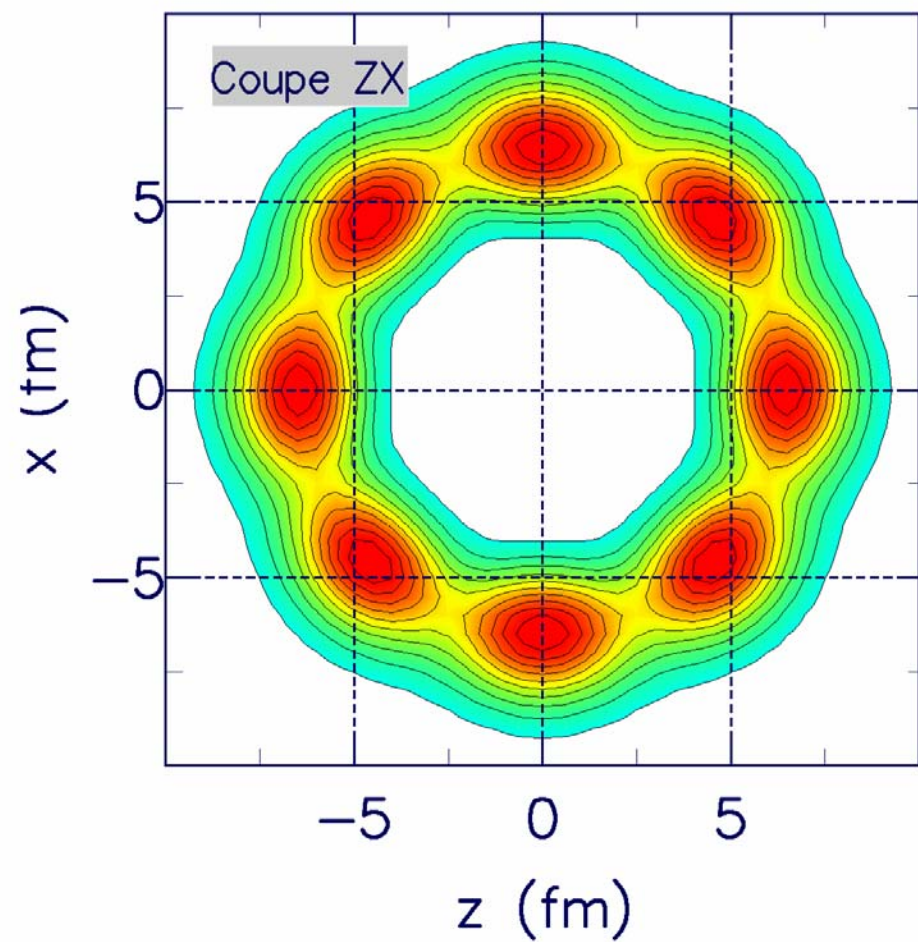


Figure 5.27: The α - α - α correlation function is shown. Resonances from the excited states of ^{12}C are labelled with the first peak seen more clearly in the inner upright panel.

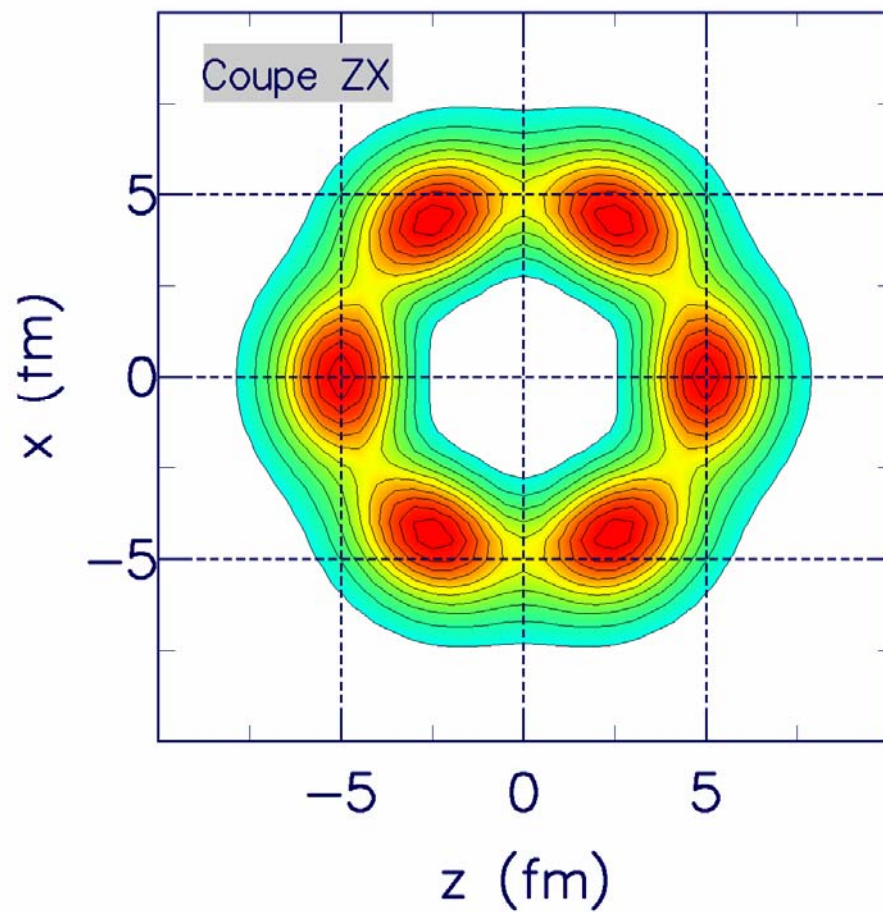




^{32}S



^{24}Mg



^{20}Ne

