The coexistence of cluster and shell model structures with AMD+GCM

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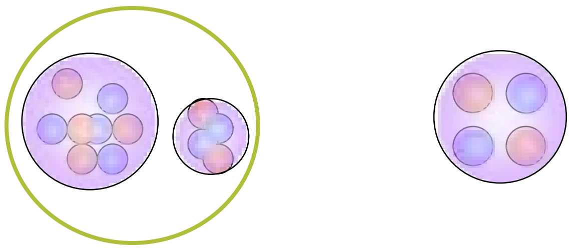
Introduction

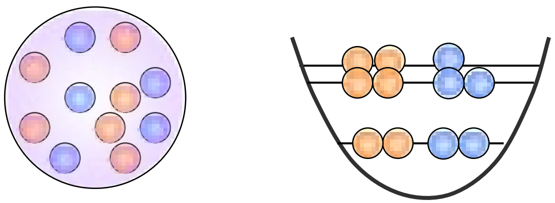
Cluster and shell-model structures

Cluster structure



The cluster means a spatially localized subsystem composed of strongly correlated nucleons.

Shell-model structure

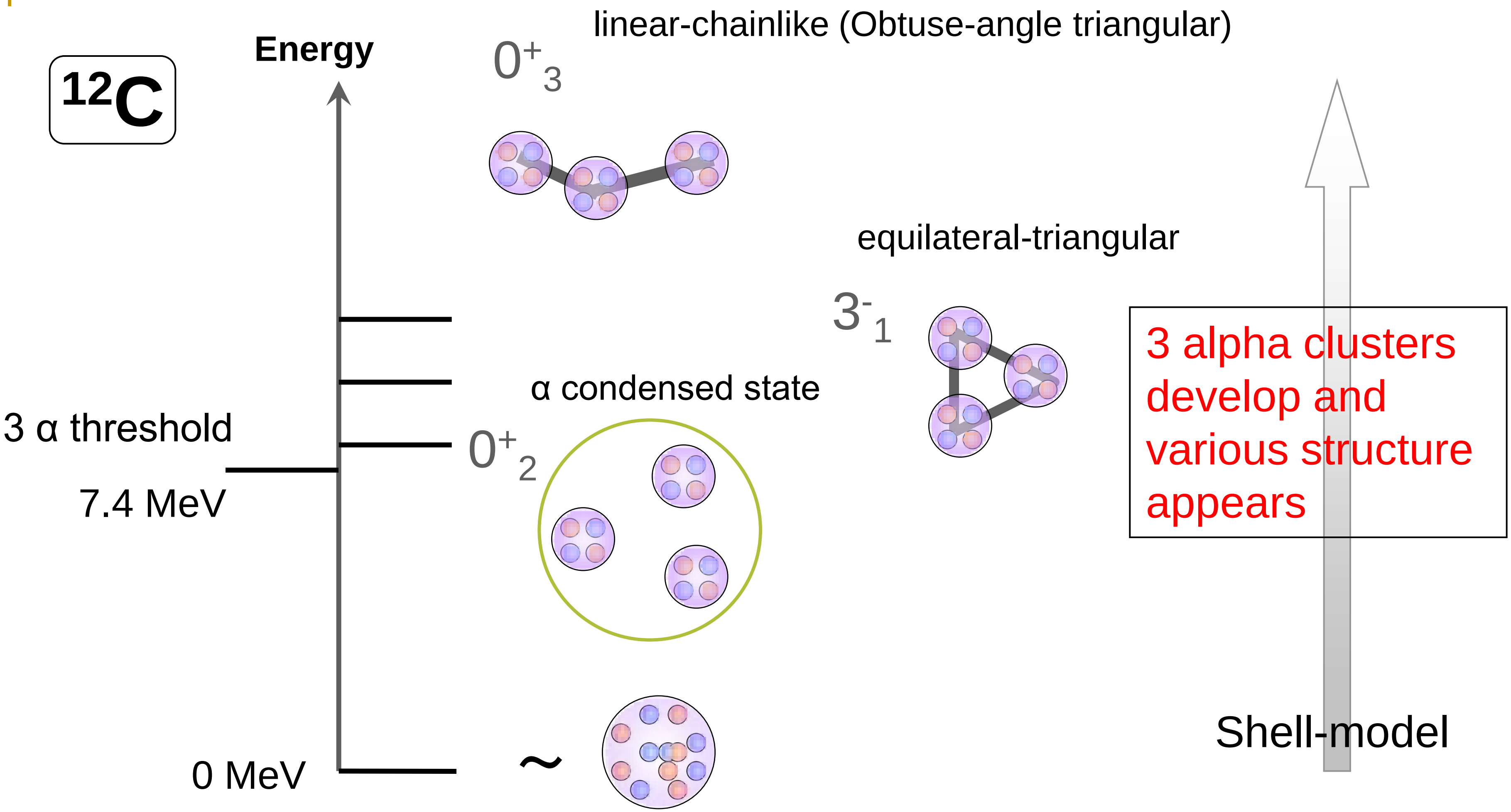


Nucleons make an one-center mean field and move in this field independently.

Very contrasting at correlation.

In light nuclei, cluster and shell model structures often coexist.

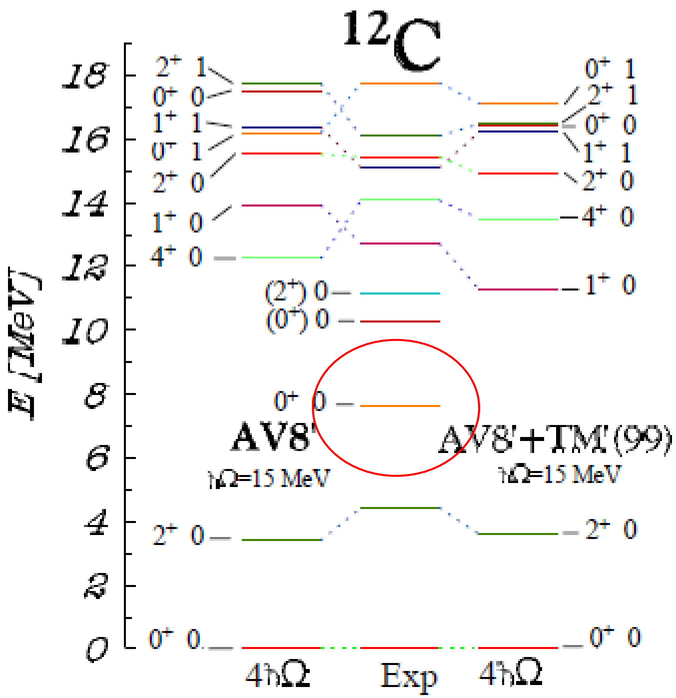
The coexistence of cluster and shell-model structures



E. Uegaki, et al. Prog. Theor. Phys. **57**, 1262 (1977)
M. Kamimura, et al. J. Phys. Soc. Jpn. **44** (1978), 225.
A. Tohsaki, et al. Phys. Rev. Lett. **87**, 192501 (2001)
T. Neff, et al. Nuc. Phys. **A738**, 357 (2004)
Y. Kanada-En'yo, Prog. Theor. Phys. **117**, 655 (2007) etc

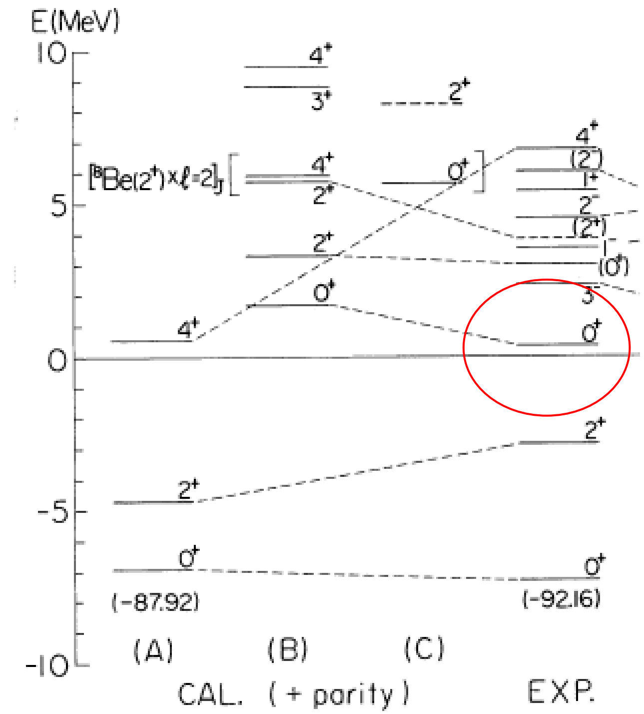
Ab initio calculation

No Core Shell Model



P. Navratil, et al. Phys. Rev. C **68**, 034305 (2003).
J. Phys. G: Nucl. Part. Phys. **36** 083101 (2009).

3α Cluster Model

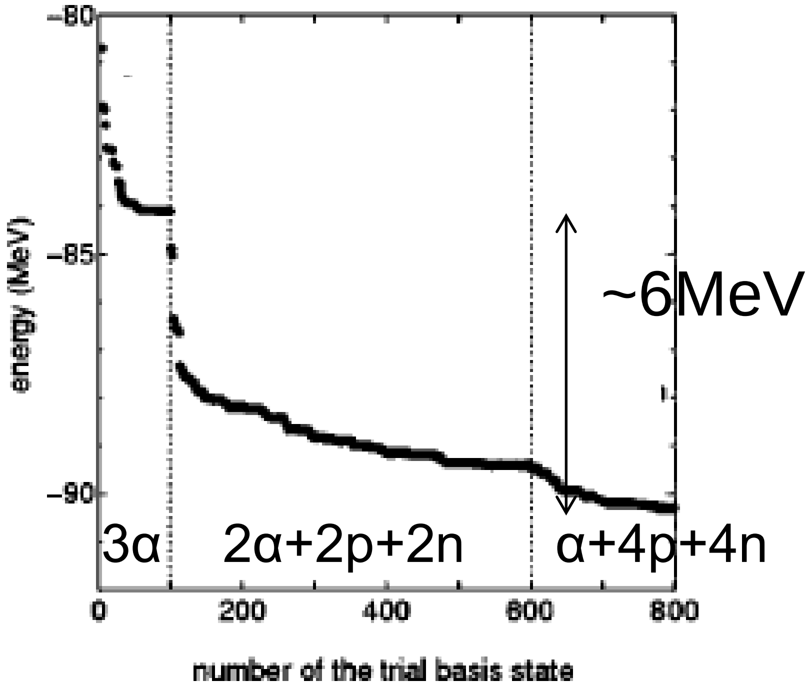


E. Uegaki, et al. Prog. Theor. Phys. **57**, 1262 (1977).

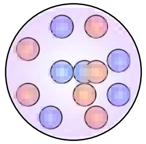
No Core Shell Model still have not been able to reproduce cluster developed states.

Breaking of cluster (Shell model structure)

^{12}C



0^+_1



Shell model structure

It is important to include the α breaking (Shell-model structure).

N.itagaki, et al. Phys. Rev. C **70**, 054307 (2004).

Purpose of our study

・Propose a method which can describe various cluster and shell-model structures.

β-γ constraint AMD + GCM

・To check the applicability, we applied this method to ^{12}C and ^{10}Be.

・Importance of the triaxiality γ.

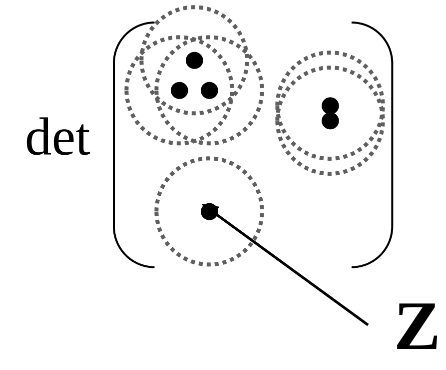
T.S. and Y. kanada-En'yo, Prog. Theor. Phys. **123**, 303 (2010).

Framework

β-γ constraint AMD + GCM

AMD（Antisymmetrized Molecular Dynamics）

a wave function of A-body system



$$\Phi_{\mathrm{AMD}} = \det[\varphi_1, \varphi_2, \cdots, \varphi_{\mathrm{A}}]$$

$$\varphi_i = \phi(\mathbf{Z}_i)\chi(\boldsymbol{\xi}_i)$$

spatial

$$\phi(\mathbf{Z}_i) \propto \exp[-\nu(\mathbf{r} - \frac{\mathbf{Z}_i}{\sqrt{\nu}})^2]$$

spin and isospin

$$\chi(\boldsymbol{\xi}_i) = \begin{pmatrix} \xi_{i\uparrow} \\ \xi_{i\downarrow} \end{pmatrix} \times (\mathrm{p}\ {}_{\mathrm{or}}\ \mathrm{n})$$

Set of variational parameters

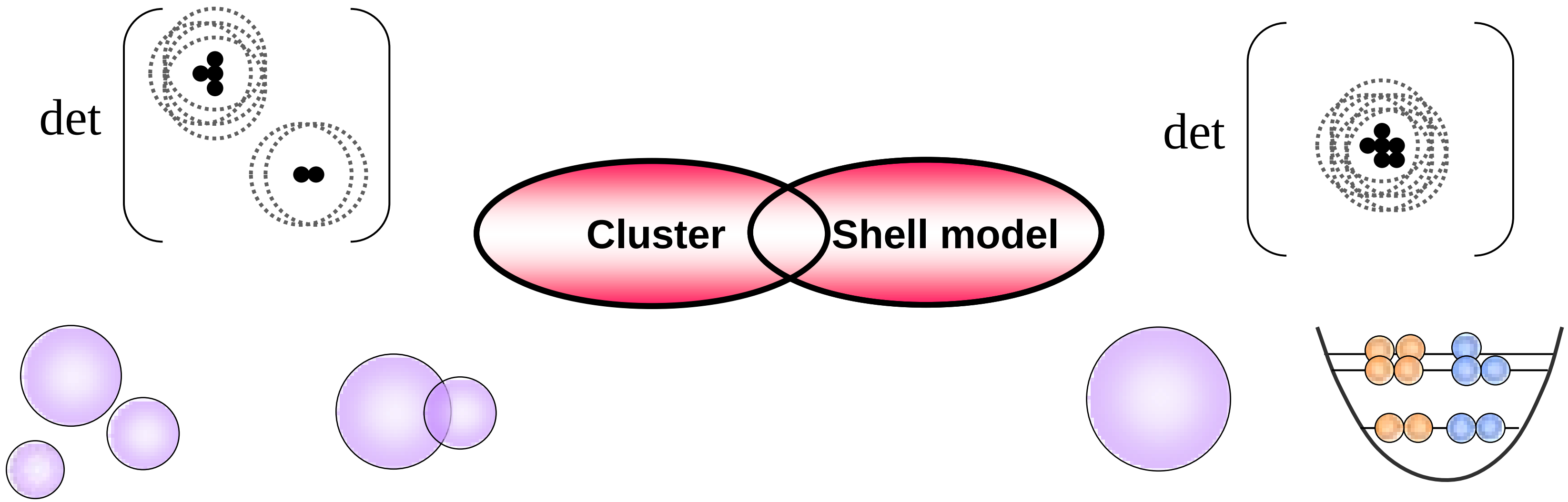
$$Z = \{\mathbf{Z}_i, \boldsymbol{\xi}_i\}$$

$\mathbf{Z}_i$: center of Gaussian wave packets

$\boldsymbol{\xi}_i$: spin direction

β-γ constraint AMD + GCM

Model space of AMD



Cluster structure

Shell-model structure

AMD can describe both of cluster structures
and shell-model structures.

β-γ constraint AMD + GCM

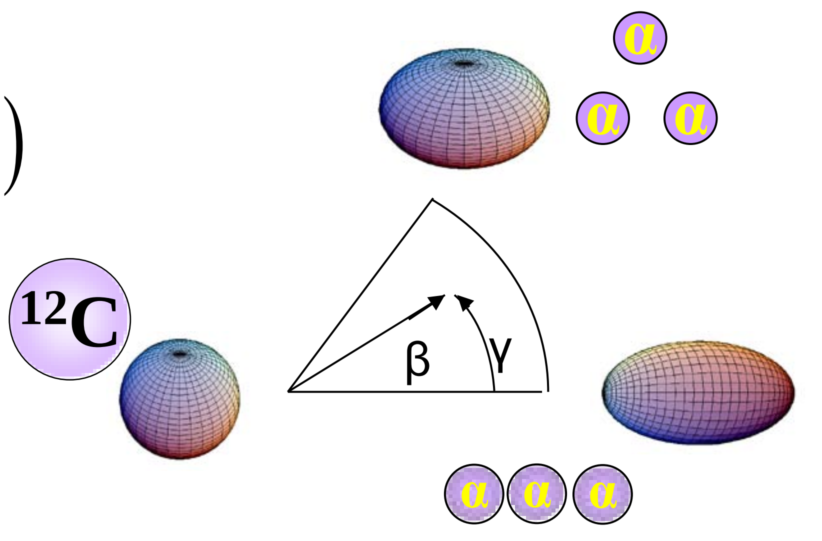
Constraints

The quadrupole deformation (β, γ)

$$\beta\cos\gamma = \frac{\sqrt{5\pi}}{3}\frac{2<z^2>-<x^2>-<y^2>}{R^2}$$

$$\beta\sin\gamma = \sqrt{\frac{5\pi}{3}}\frac{<x^2>-<y^2>}{R^2}$$

$$R^2 = \frac{5}{3}\left(<x^2>+<y^2>+<z^2>\right)$$



β-γ constraint AMD + GCM

Parity and angular momentum projections

$$P^{\pm} = \frac{1 \pm P}{2}$$

$$P^{J}_{MK} = \frac{2J+1}{8\pi^2}\int d\Omega D^{J*}_{MK}(\Omega)R(\Omega)$$

In this study, we performed the variation after the parity projection (VAP).
After the variation, we project the obtained wave function onto the total-angular-momentum eigenstates (PAV).

GCM (Generator Coordinate Method)

$J^{\pm}$ state

$$\left|\Phi^{J\pm}_{n}\right\rangle = \sum_{K}\sum_{i} f_n\left(\beta_i, \gamma_i, K\right) P^{J}_{MK}\left|\Phi^{\pm}\left(\beta_i, \gamma_i\right)\right\rangle$$

Effective interaction

Hamiltonian

$$H^{\mathrm{eff}} = \sum_i t_i - T_{\mathrm{CM}} + \sum_{i<j} v_{ij}^{\mathrm{central}} + \sum_{i<j} v_{ij}^{\mathrm{LS}} + \sum_{i<j} v_{ij}^{\mathrm{Coulomb}}$$

The central force : The Volkov No.2

$$v_{ij}^{\mathrm{central}} = (v_1 \exp[-(\frac{r_{ij}}{a_1})^2] + v_2 \exp[-(\frac{r_{ij}}{a_2})^2]) X_{ij}$$

$$X_{ij} = W + BP_\sigma - HP_\tau - MP_\sigma P_\tau \quad (W = 0.4, M = 0.6, B = H = 0.125)$$

$$v_1 = -60.65[\mathrm{MeV}], a_1 = 1.80[\mathrm{fm}], v_2 = 61.14[\mathrm{MeV}], a_2 = 1.01[\mathrm{fm}]$$

The LS force : The LS part of the G3RS

$$v_{ij}^{\mathrm{LS}} = (u_1 \exp[-(\frac{r_{ij}}{a_1})^2] + u_2 \exp[-(\frac{r_{ij}}{a_2})^2]) P(S=1) P(T=1) \mathbf{L} \cdot \mathbf{S}$$

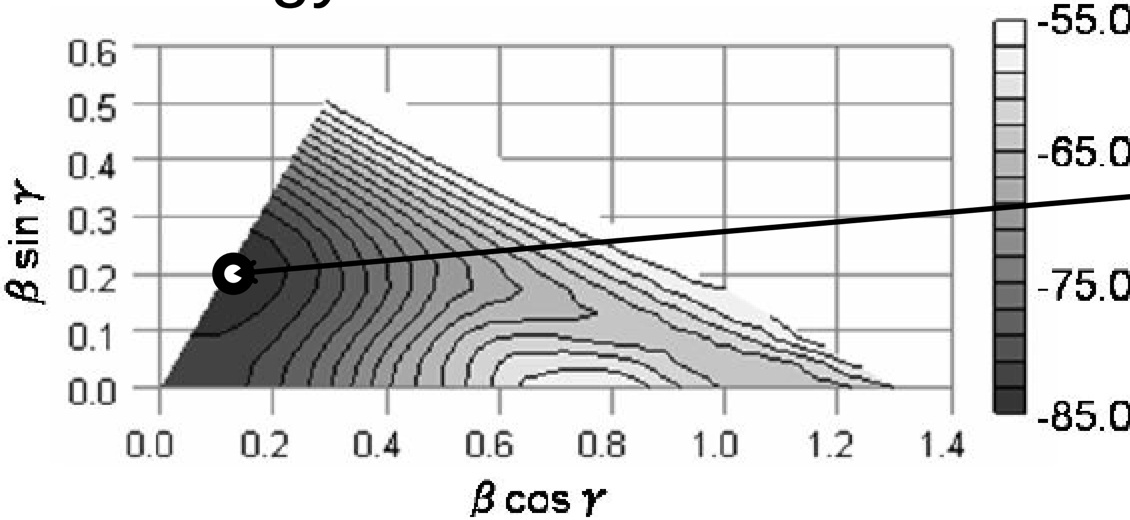
$$P(S=1) = \frac{1+P_\sigma}{2}, P(T=1) = \frac{1+P_\tau}{2}$$

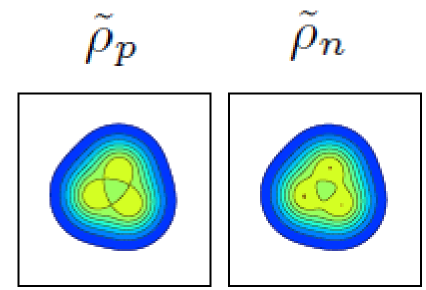
$$u_1 = 1600[\mathrm{MeV}], a_1 = 0.447[\mathrm{fm}], u_2 = -1600[\mathrm{MeV}], a_2 = 0.600[\mathrm{fm}]$$

Structures in ^{12}C

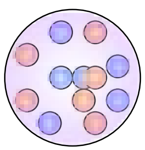
Calculated results of ^{12}C

+ energy surface

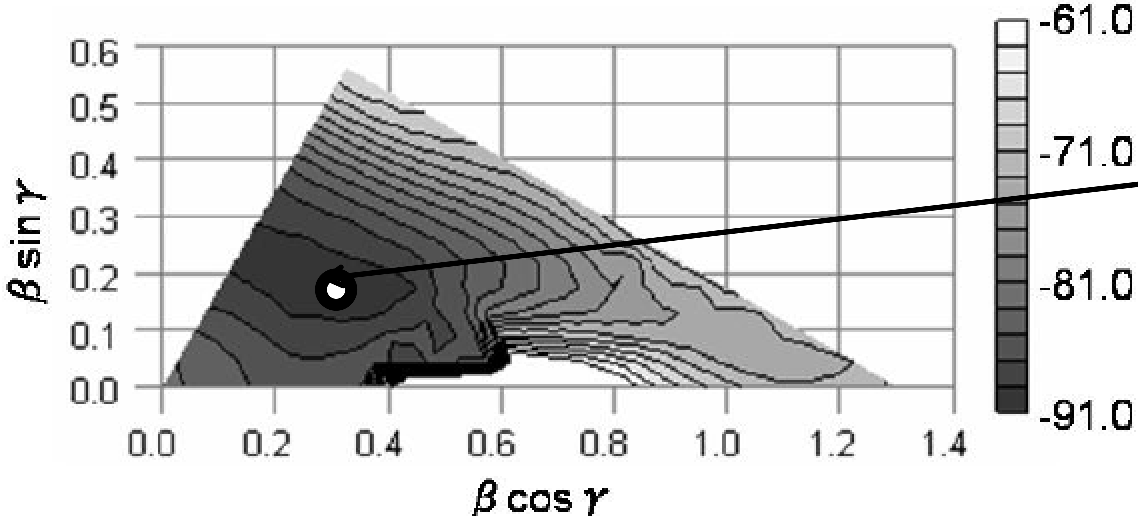


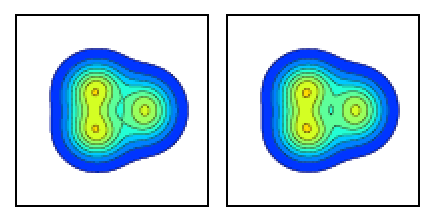


Shell-model



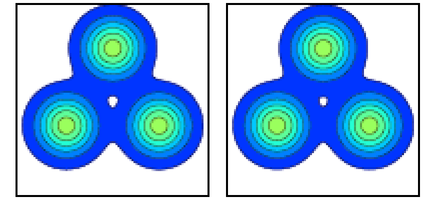
0$^+$ energy surface

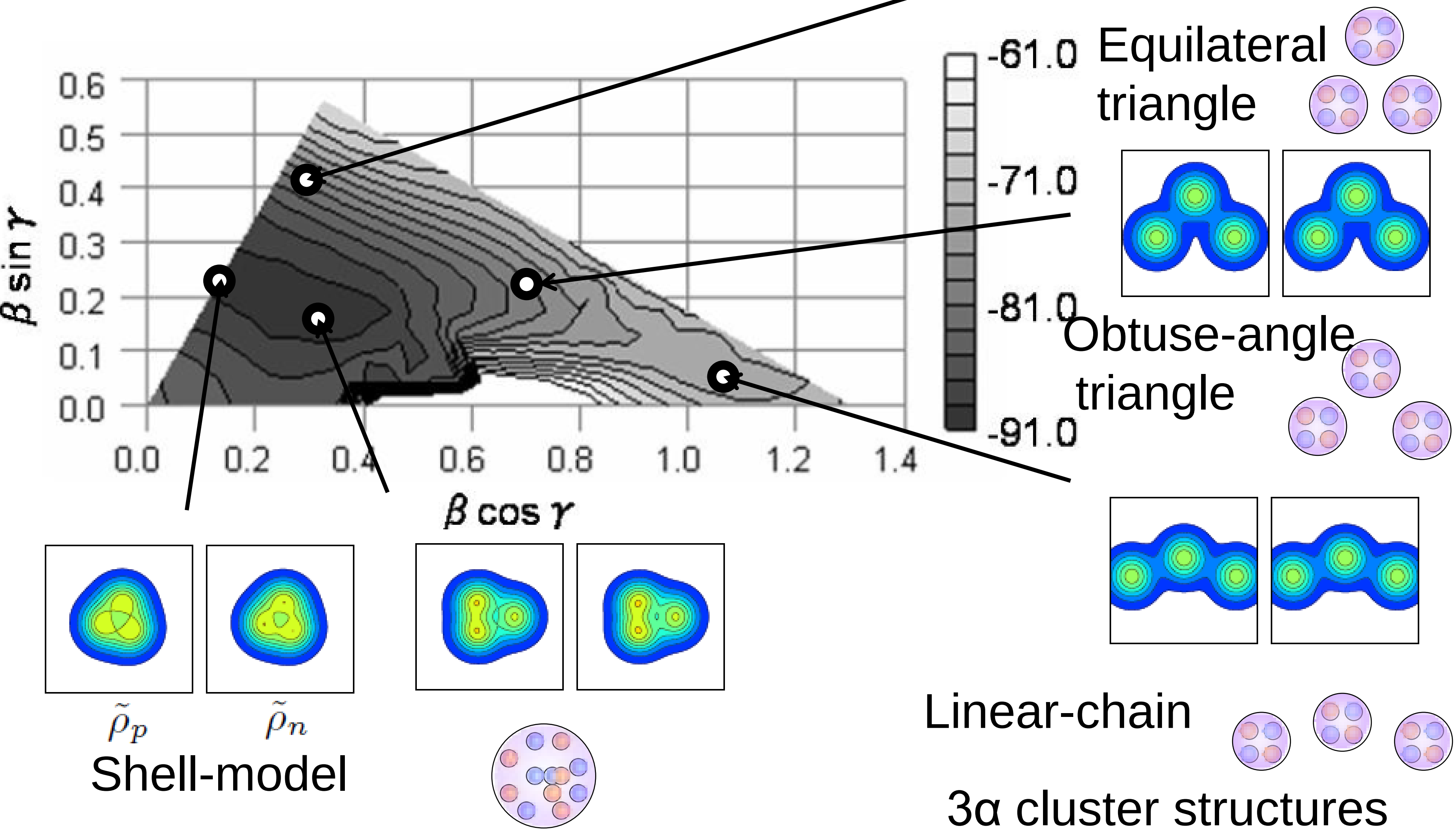




Calculated results of ^{12}C

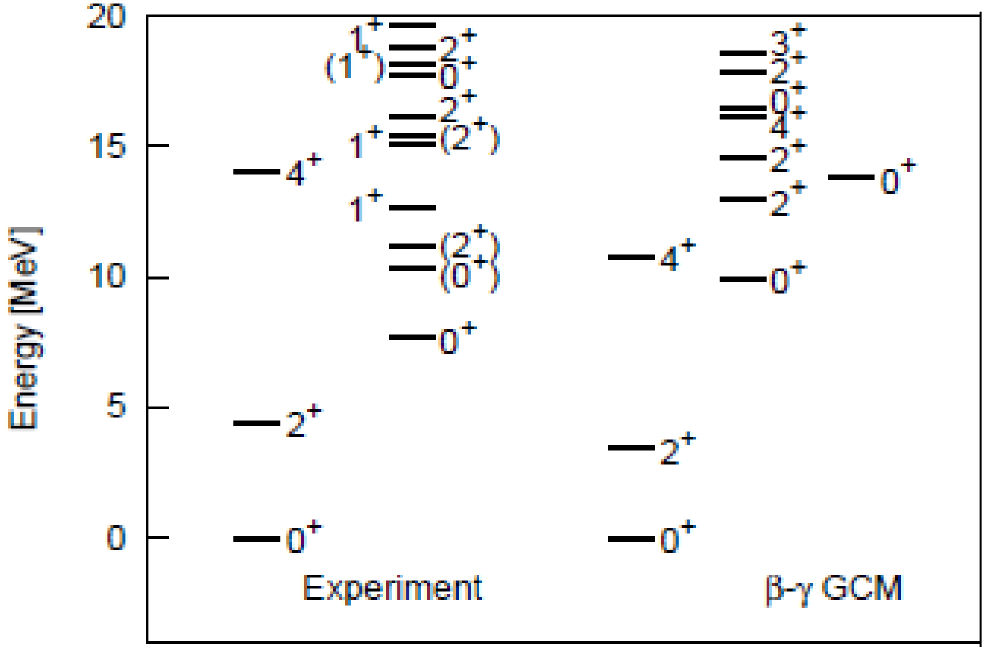
Structures on the β-γ plane





Calculated results of ^{12}C

Energy levels in ^{12}C



(A) $B(E2)$

Transitions	β-γ GCM	Experiment
$2_1^+ \to 0_1^+$	6.0	7.59 ± 0.42
$2_1^+ \to 0_2^+$	1.9	2.6 ± 0.4
$2_2^+ \to 0_1^+$	1.1	
$2_2^+ \to 0_2^+$	58	

unit: e^2fm^4

(B) Root-mean-square radii

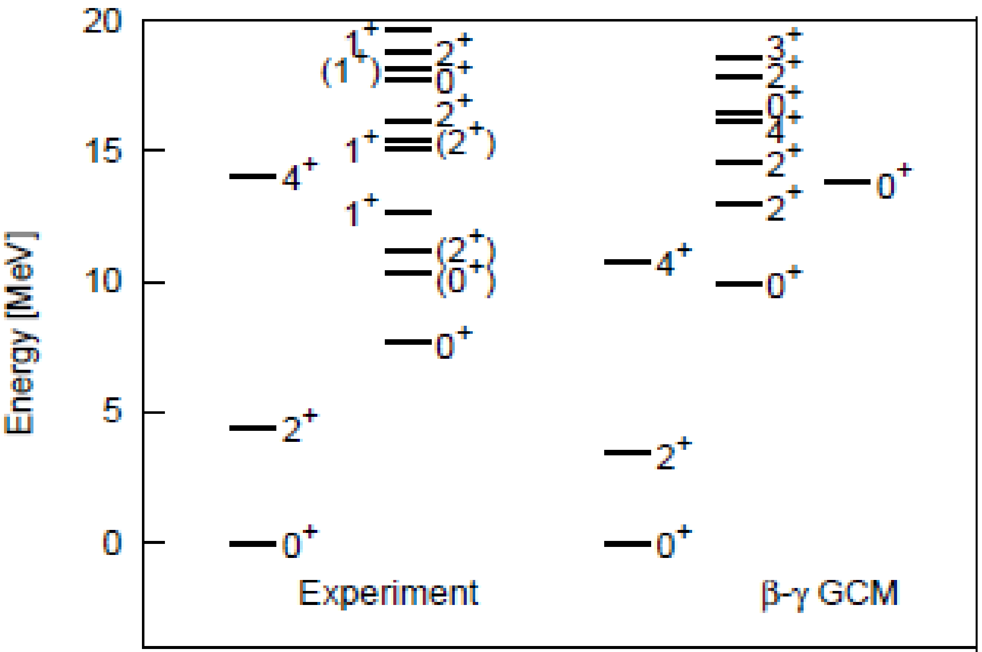
States	β-γ GCM	Experiment
0_1^+	2.31	2.35 ± 0.02
		2.31 ± 0.02
0_2^+	2.90	
0_3^+	3.26	

unit: fm

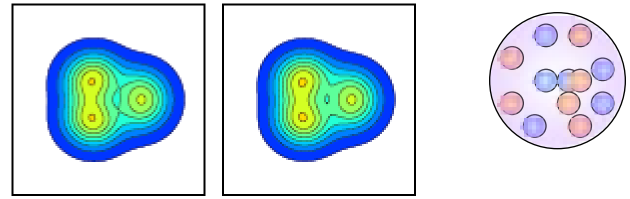
Our results reproduce the experimental values (B(E2) and rms radii) well.

Calculated results of ^{12}C

Structures of 0^+ states

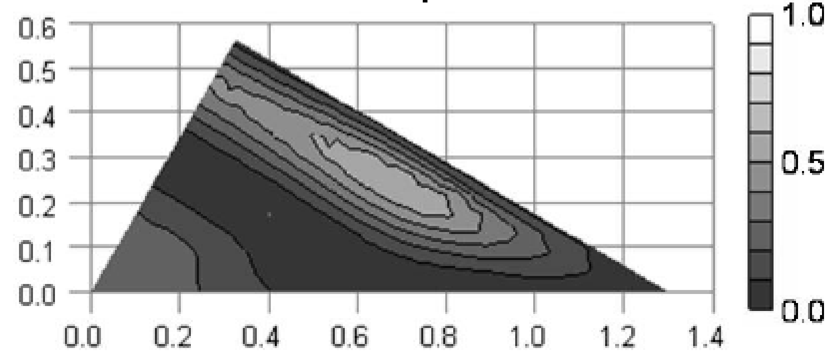


0^+_1: Shell-model-like

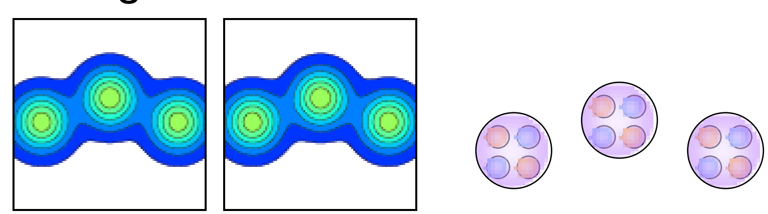


0^+_2: Various 3α cluster configurations

GCM amplitude



0^+_3: Linear-chainlike

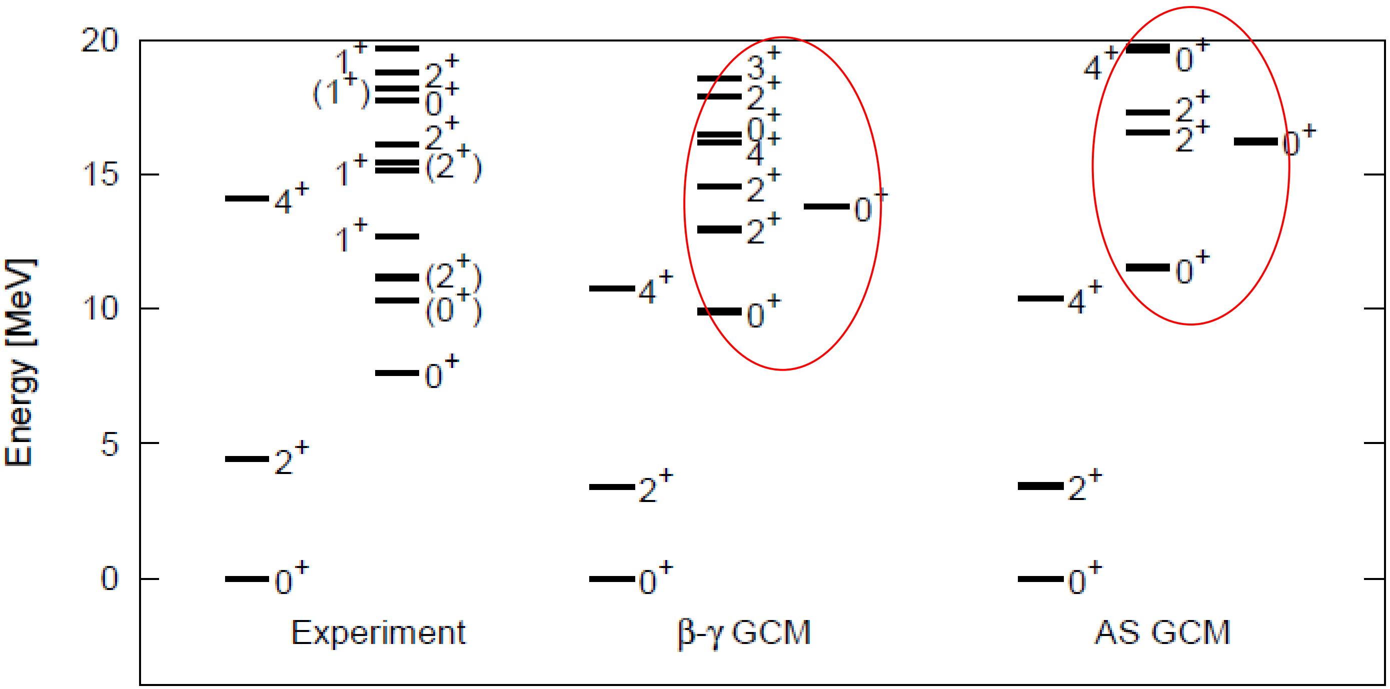


The present results is consistent with the earlier works.

⇒β-γ constraint AMD+GCM is effective for describing various cluster and shell structures.

Importance of triaxiality

Comparison with the axial symmetric GCM



(A) $B(E2)$

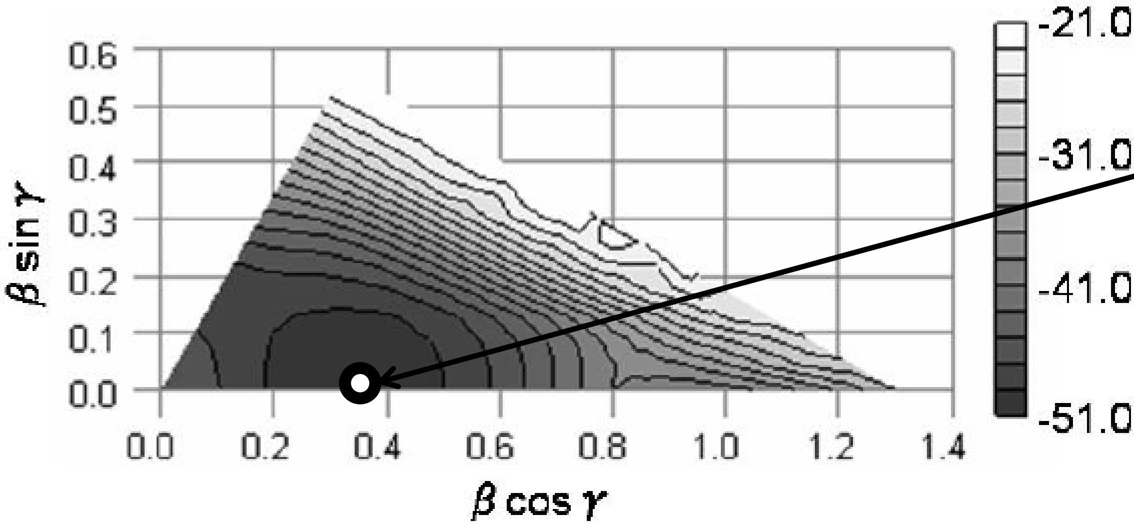
Transitions	β-γ GCM	AS GCM	Experiment
$2_1^+ \to 0_1^+$	6.0	5.3	7.59 ± 0.42
$2_1^+ \to 0_2^+$	1.9	1.5	2.6 ± 0.4
$2_2^+ \to 0_1^+$	1.1	0.4	
$2_2^+ \to 0_2^+$	58	11	

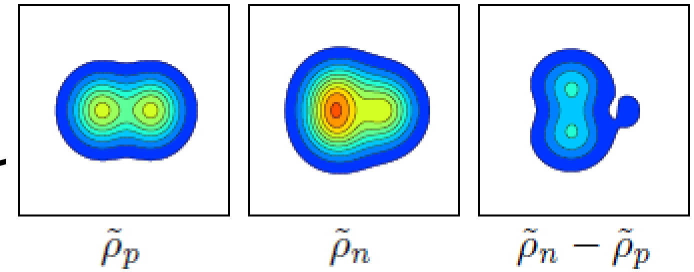
The triaxial basis has an essential role in the description of excited states.

Structures in ^{10}Be

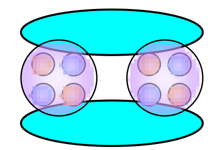
Calculated results of ^{10}Be

+ energy surface

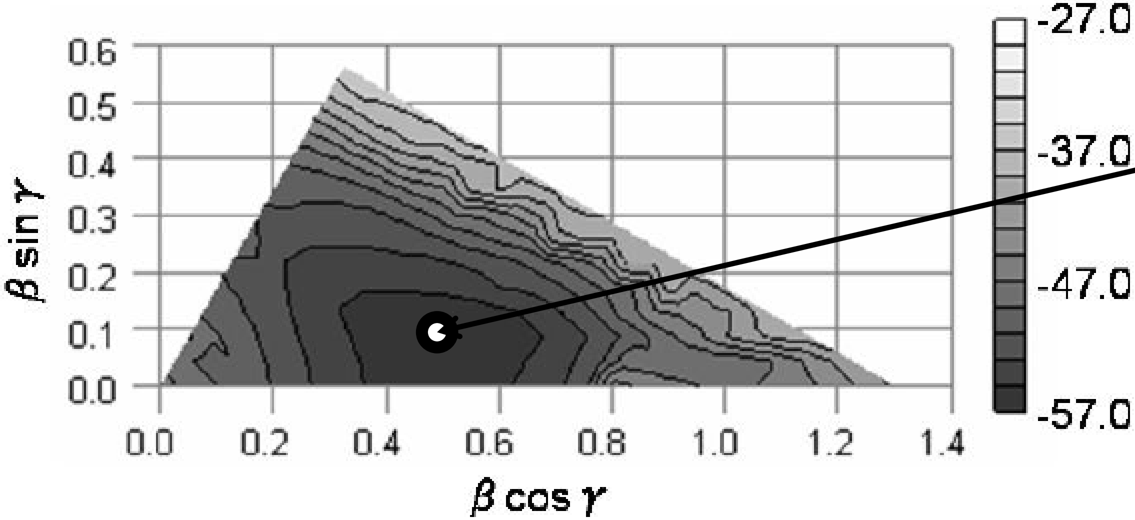


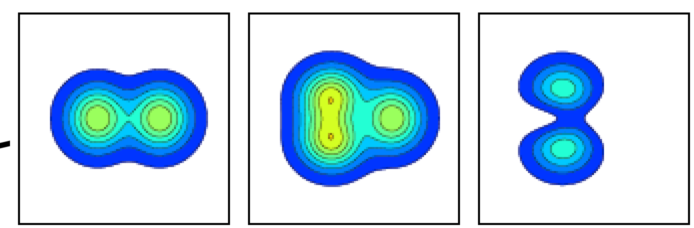


$2\alpha+2n(\pi_{3/2})^2$



0^+ energy surface

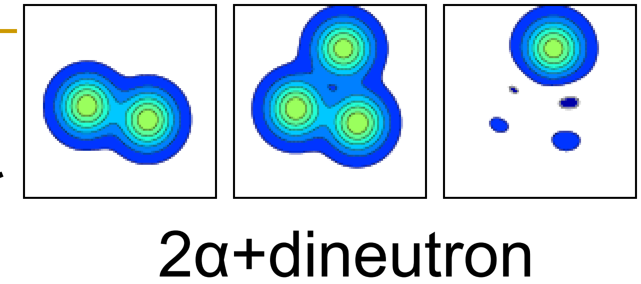


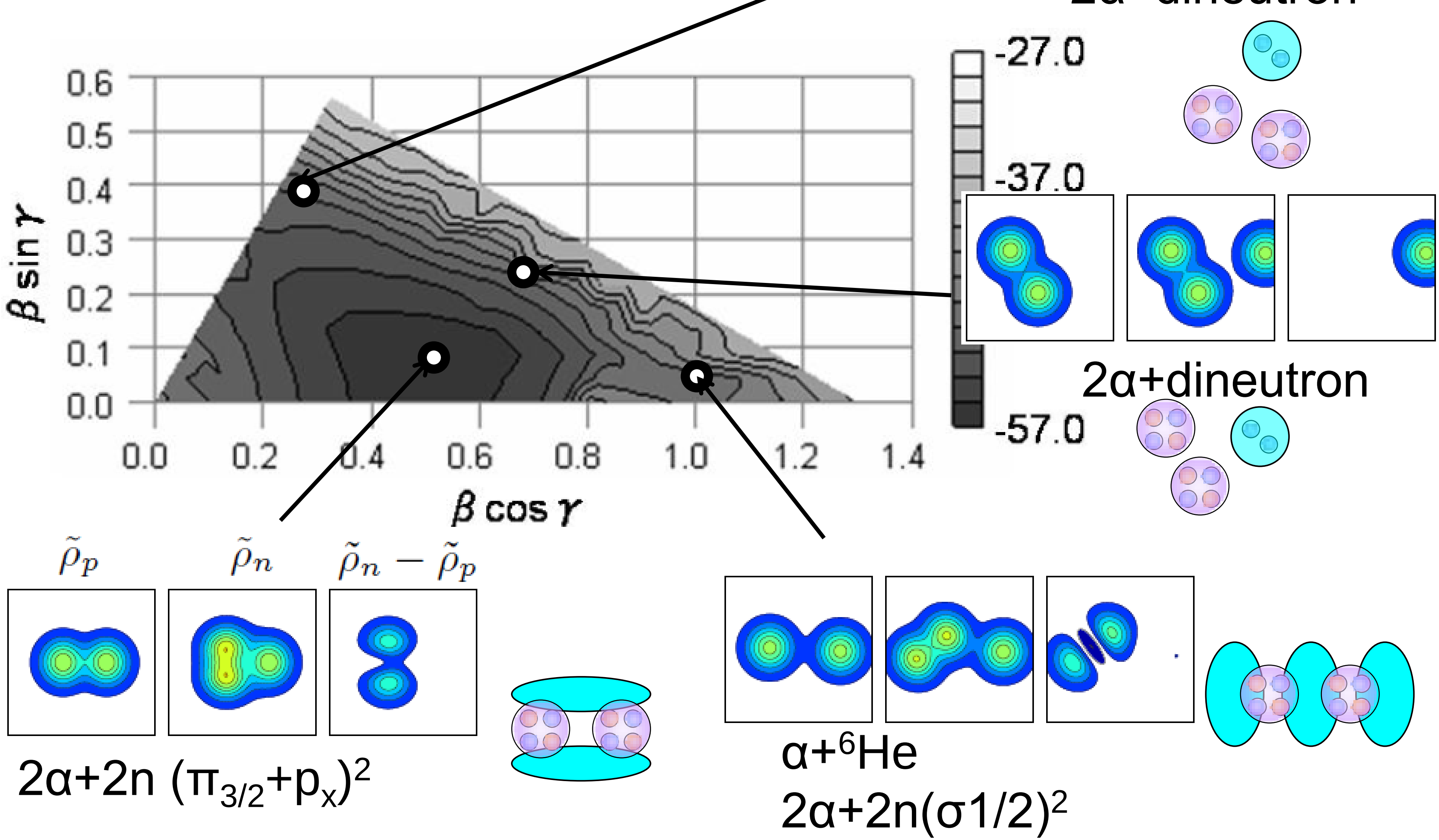


$2\alpha+2n(\pi_{3/2}+p_x)^2$

Calculated results of ^{10}Be

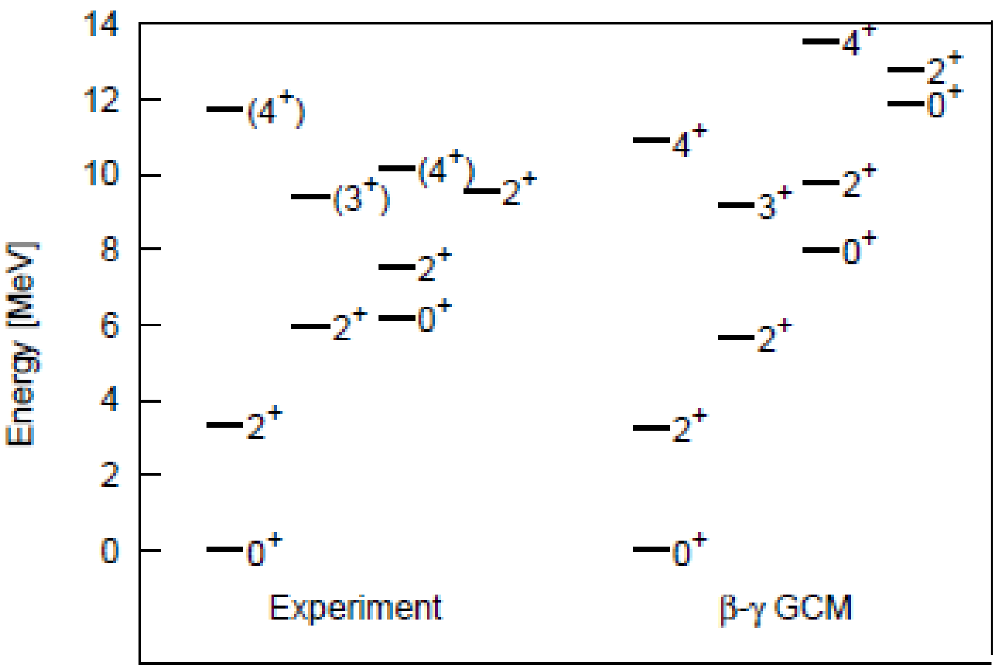
Structures on the β-γ plane





Calculated results of ^{10}Be

Energy levels in ^{10}Be



(A) $B(E2)$

Transitions	β-γ GCM	Experiment
$2_1^+ \to 0_1^+$	9.4	10.24 ± 0.97
$2_1^+ \to 0_2^+$	1.2	0.64 ± 0.23
$2_2^+ \to 0_1^+$	0.7	
$2_2^+ \to 2_1^+$	4.2	

unit: e^2fm^4

(B) Root-mean-square radii

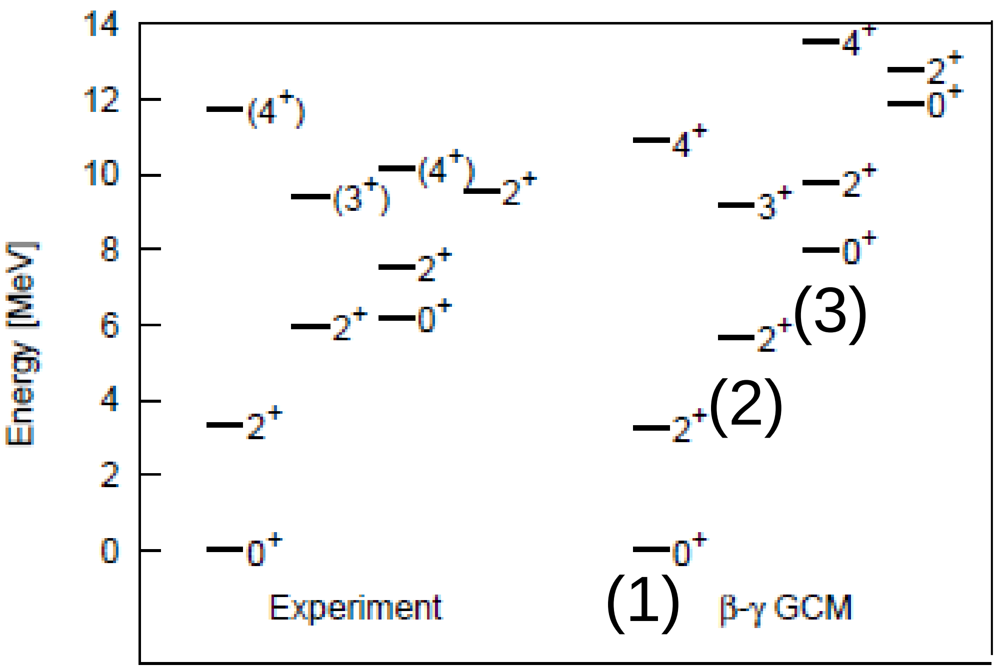
States	β-γ GCM	Experiment
0_1^+	2.39	2.30 ± 0.02
0_2^+	2.98	
0_3^+	2.96	

unit: fm

Our results reproduce the experimental results well.

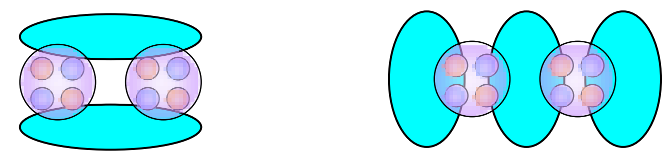
Calculated results of ^{10}Be

Structures of ^{10}Be



(1) $2\alpha+2n(\pi_{3/2}+p_x)^2$
(2) $K=2^+$ side band
(3) $\alpha+^6He$ or $2\alpha+2n(\sigma_{1/2})^2$

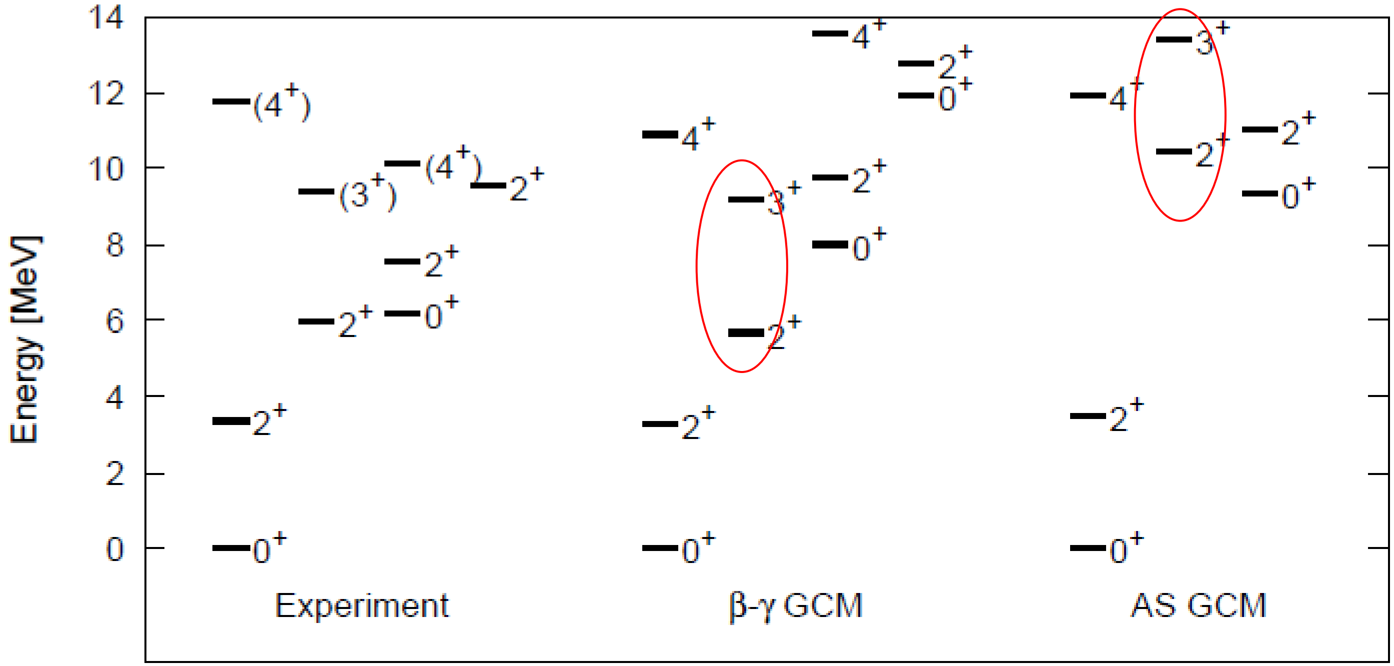
(1,2) (3)



The present results is consistent with the earlier works.

Importance of triaxiality

Comparison with the axial symmetric GCM



(A) $B(E2)$

Transitions	β-γ GCM	AS GCM	Experiment
$2_1^+ \to 0_1^+$	9.4	7.8	10.24 ± 0.97
$2_1^+ \to 0_2^+$	1.2	0.1	0.64 ± 0.23
$2_2^+ \to 0_1^+$	0.7	0.1	
$2_2^+ \to 2_1^+$	4.2	0.8	

The triaxial basis has an essential role in the description of excited states.

Summary

- β-γ constraint AMD+GCM

  This method is effective for describing various cluster and shell model structures.

- Structures in ^{12}C and ^{10}Be

  To describe excited states, the triaxial basis play an important role.

- Future work

  Structures of other nuclei

  ^{14}C, T.S. and Y. kanada-En'yo, PRC **82**, 044301 (2010).

  ^{11}B, in preparation