Clustering and Correlations in ^{12}C and Neutron Matter

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Clustering in 12C 0+ states: ground- and Hoyle states

Homogeneous Neutron Matter:
- Cold Atoms and Low-Density Neutron Matter
- Higher-Density Matter and Neutron Stars

Response:
- Spin Response and Neutrino Emissivity

Inhomogeneous Matter (drops):
- Neutrons Confined in External Fields

VMC: Variational Monte Carlo
  assumed form for wave function
  Monte Carlo for integration

GFMC: Green's function Monte Carlo
  sample imaginary-time path integral
  explicit spin-isospin sums

AFDMC: Auxiliary-Field Diffusion (Green's fn)
  Monte Carlo
  sample space and spins

AFMC: lattice calculations using auxiliary fields

^{12}C ground state

$$\Psi_0 = \exp[-H\tau]\ \Psi_T$$

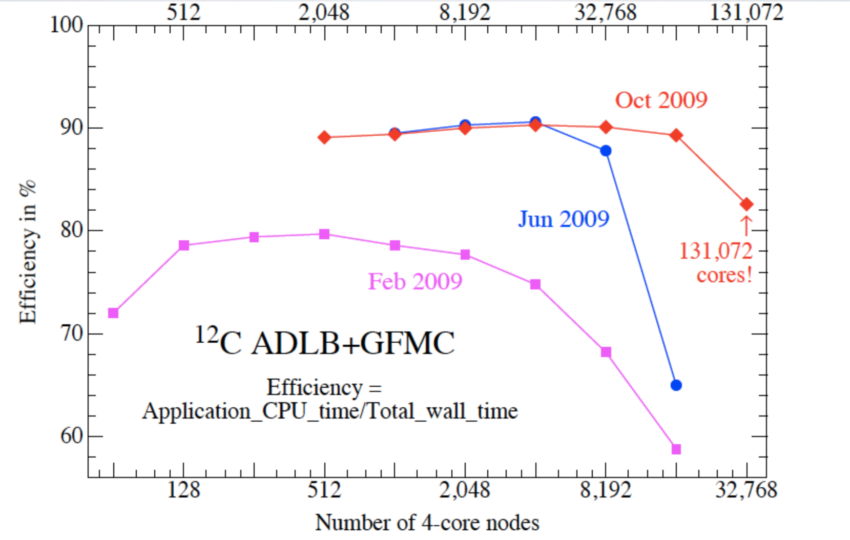
The `Jastrow' part of the trial wave function is a
major part of the entire calculation

There are 5 LS basis J=0+ basis states in the 0p shell
can be constructed by operators on (p3/2)^8 state.

Can also make an explicitly triple-alpha state
(1 in 0s and 2 in 0p shells) with Jastrow correlations

This basis of 6 sates works well for the ground state

Asynchronous Dynamic Load Balancing
Pieper and Lusk, SCIDAC Review



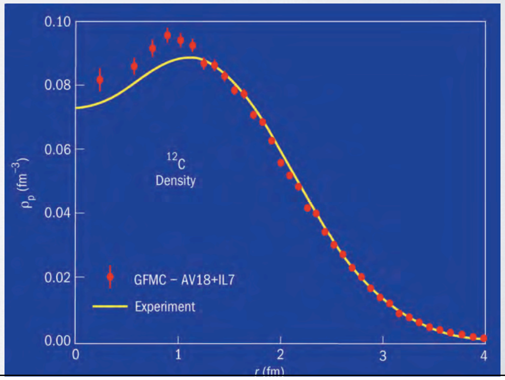
Carbon 12 ground state

computed from: $\psi_0 = \exp[-H\tau]\,\psi_T$

Trial state should incorporate flexible long-distance physics

ψ_T has 5 simplest shell-model states
+ alpha-particle `cluster' state

Good description of ground state energy/density



Trial state for 2nd 0+ (Hoyle) state

Calculation by S. Pieper (ANL)

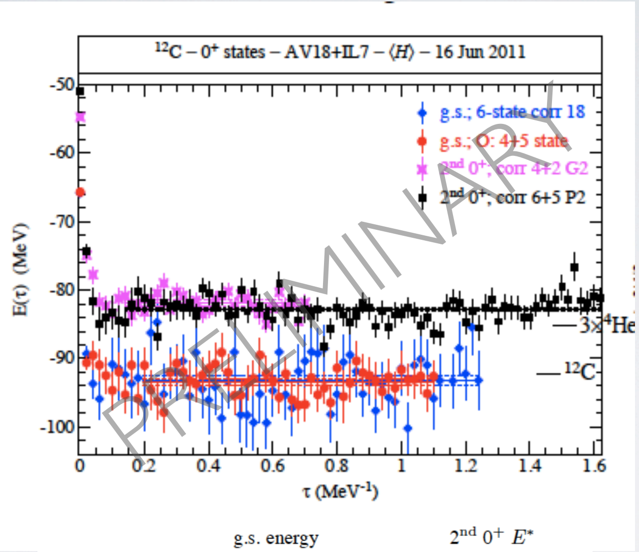
states in G.S. trial state +
- alphas in 0s, 0p, 1s-0d shell
- also try a pair in 1s-0d shell

total of 11 states to be diagonalized

Initial diagonalization for ground-state ψ_T (same results)
Compute GFMC g.s. overlaps with these states,
diagonalize overlaps (10 states) to get 2nd 0+

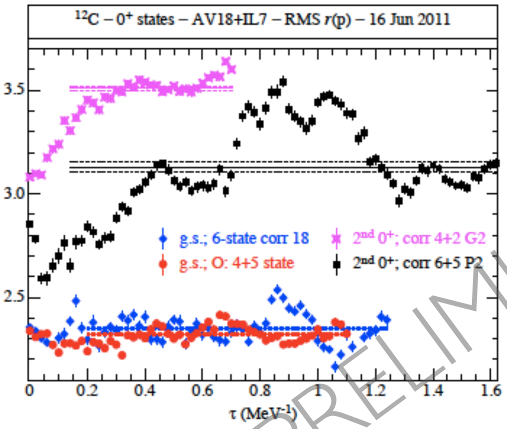
PRELIMINARY

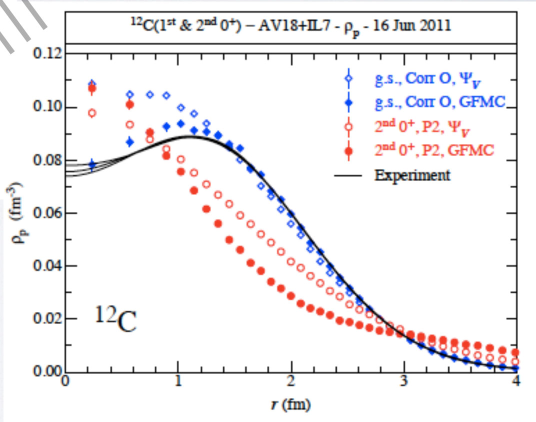
Ground and Excited State Energies



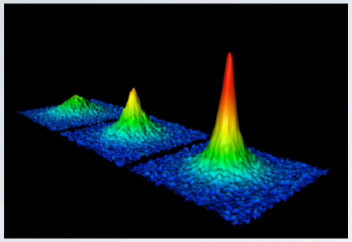
	g.s. energy			2nd 0$^+$ E^*		
	VMC	GFMC	Expt.	VMC	GFMC	Expt.
AV18	–44.9(2)	–73.2(5)		10.0(3)	7.9(6)	
AV18+IL7	–65.7(2)	–93.3(4)	–92.16	14.7(2)	10.4(5)	7.65

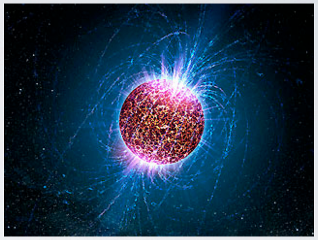
RMS radii / Charge Density





Correlations in Homogeneous Neutron Matter

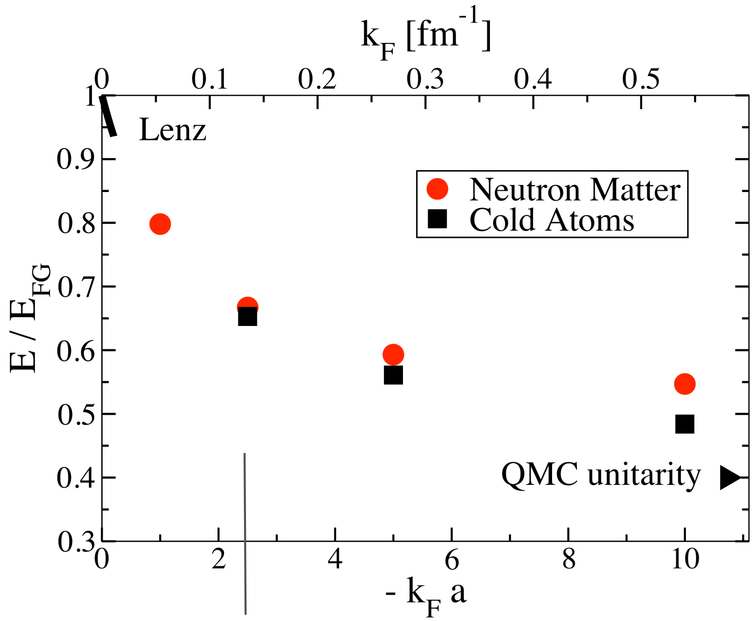




Low-Density (dilute) near free Fermions to near Unitarity
range of the interaction < interparticle spacing

Analytically known at extremely low density
E / E_{FG} rapidly decreases to ~ 1/2 with increasing k_F a

Higher density EOS important for neutron star mass/radius



$k_F r_e \sim 0.3$

A. Gezerlis, J. C., 2008,2010

Improved Lattice Methods and Unitary Gas

E/E(FG) = 0.372(5)
no fixed-node error

At finite (small) effective range:

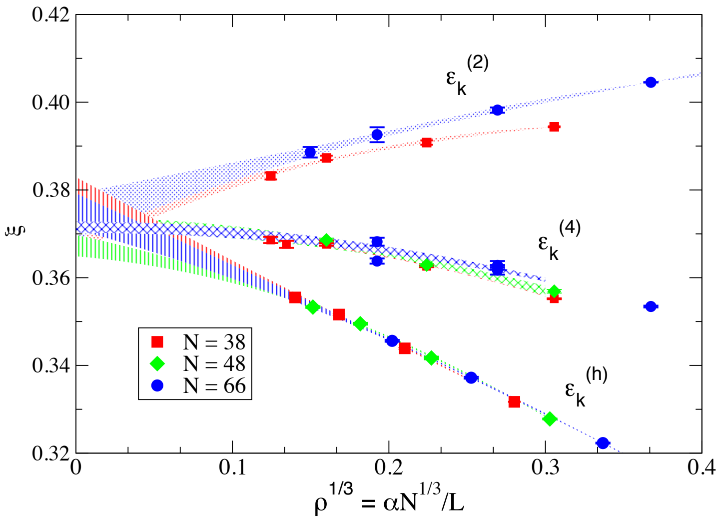
$$E \;/\; E_{FG} \;=\; \xi \;+\; \mathcal{S}\; k_F\; r_e$$

ξ and $\mathcal{S}$ are universal parameters

Can measure neutron matter EOS (including effective range corrections) in cold atoms

Unitary Fermi Gas (lattice)

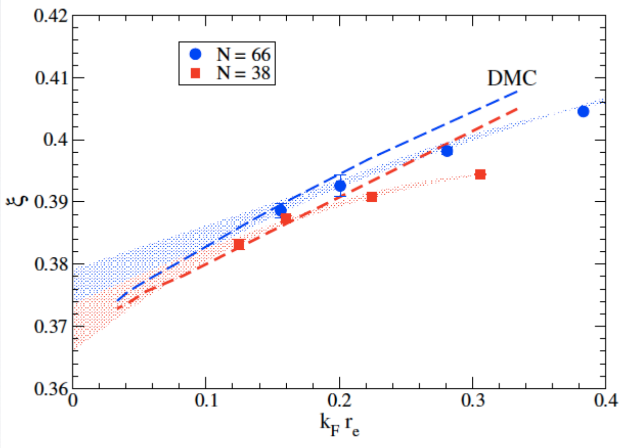
E/E_{FG}



up to 27^3 lattice, 66 particles

K.E. Schmidt, S. Zhang, S. Gandolfi, J.C, 2011

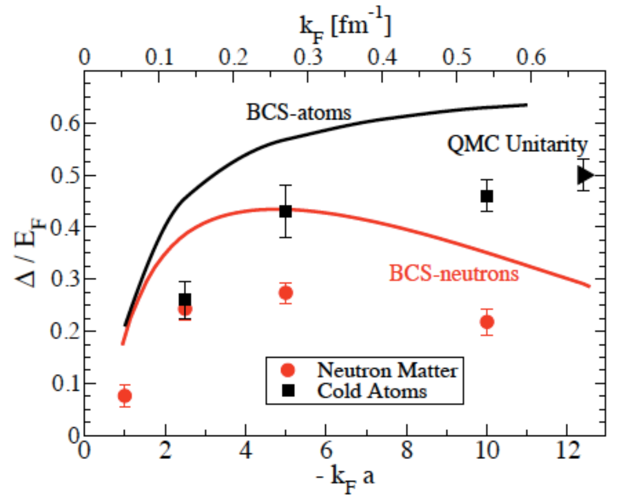
Universality of effective range correction



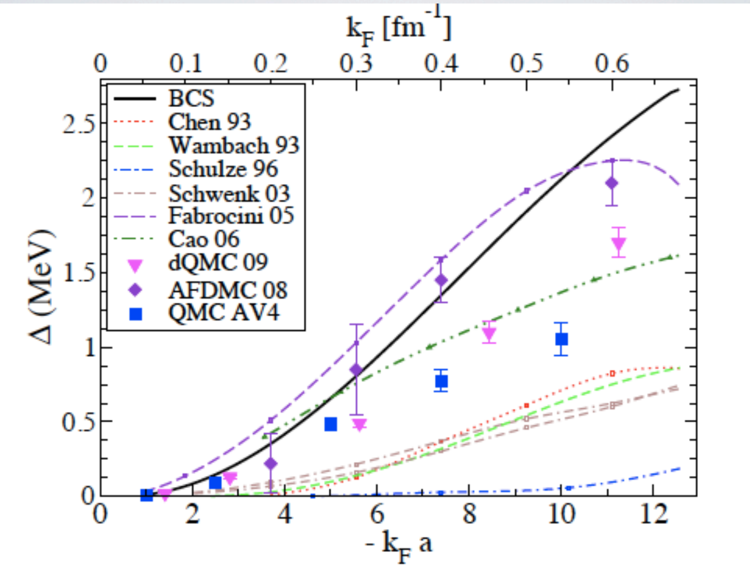
Lattice method (points) compared to continuum (DMC)

$$\mathcal{S} = 0.11(0.03)$$

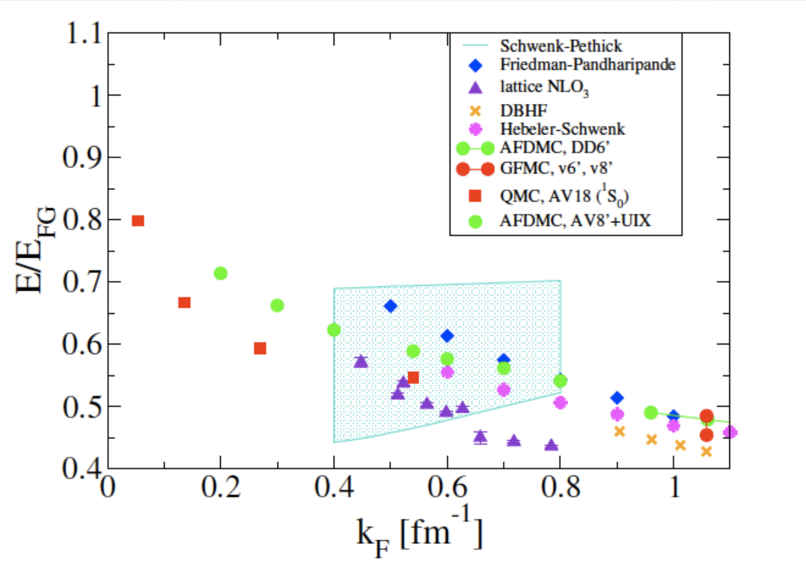
Pairing Gap at low density



Summary of Gap calculations



Low-moderate density EOS
s-wave pairing gap closes



Neutrino Emissivity

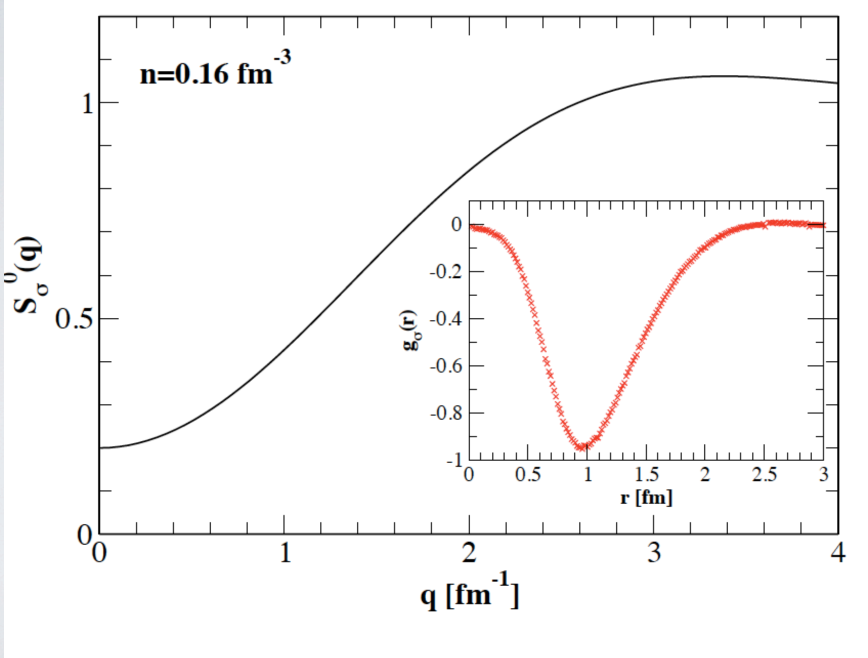
Response of neutron matter to spin excitations determines emissivity:

$$S(k,\omega) = \langle 0 \mid \sum_i \exp[ik\cdot r_i]\sigma_i \cdot \sum_j \exp[ik\cdot r_j]\sigma_j \mid 0\rangle$$

At k=0 no response (or emissivity) without tensor and spin-orbit correlations

$$Q \;=\; \frac{C_A^2 G_F^2 n}{20\pi^3} \int_0^\infty d\omega\ \omega^6\ e^{-\omega/T} S_\sigma(\omega)$$

`Short-range' correlations



Use Sum Rules to constrain the response:

$$S_\sigma^{-1} = \frac{\chi_\sigma}{2n} = \frac{\chi_\sigma^F}{2n(1+G_0)}$$

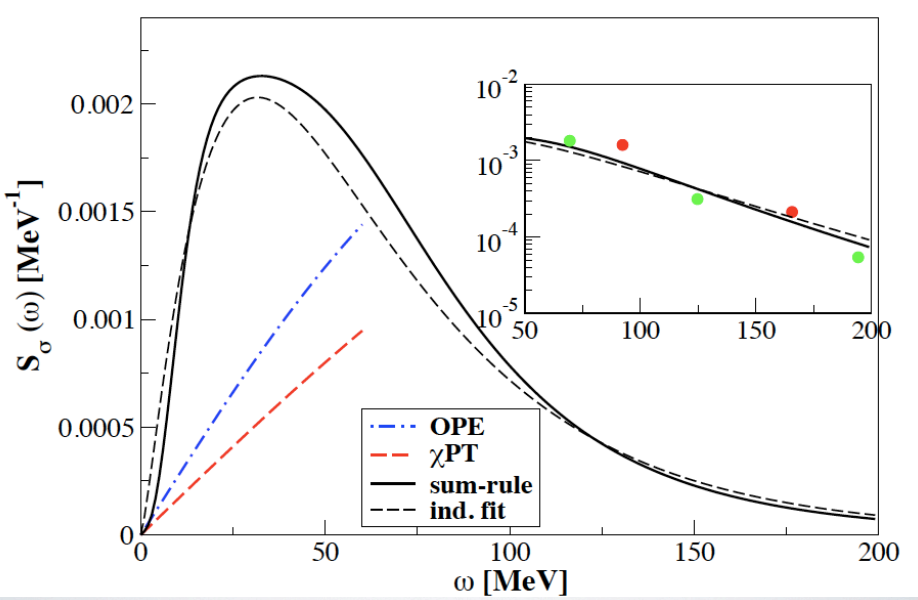
$$S_\sigma^{0} = 1 + \lim_{q\to 0}\frac{4}{3N}\sum_{i\neq j}^{N}\langle 0|e^{-i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_j})}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j|0\rangle$$

$$S_\sigma^{+1} = -\frac{4}{3N}\lim_{q\to 0}\langle 0|[H_N, \boldsymbol{s}(\mathbf{q})]\cdot\boldsymbol{s}(-\mathbf{q})|0\rangle$$

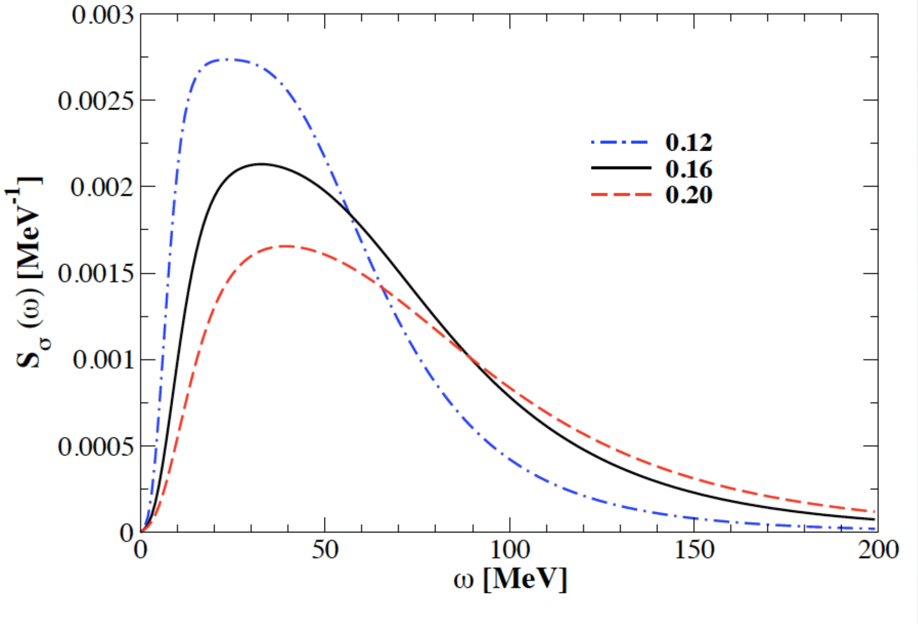
Static and energy-weighted sum rules from ground-state expectations
Inverse energy-weighted from spin susceptibility

Low-Energy shape constrained by Q.P. model
High-Energy tail constrained by 2-body physics

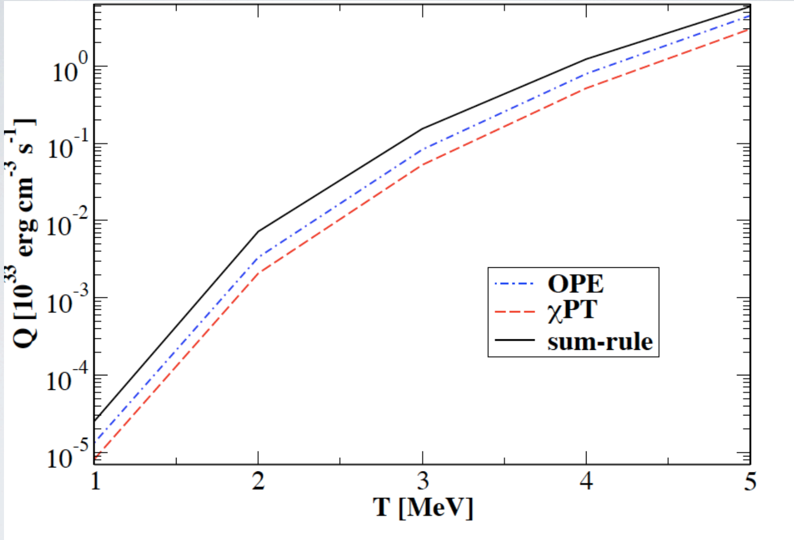
Response compared to previous calculations saturation density



Density Dependence



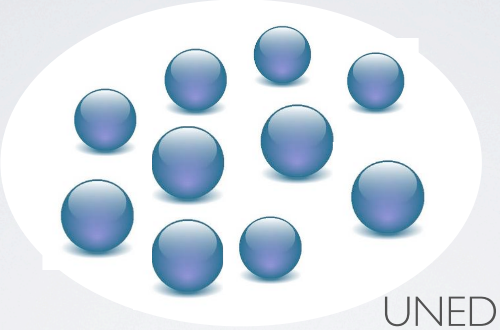
Emissivity vs. Temperature:



Emissivity actually increased compared to previous results

Inhomogeneous Matter
`Neutron Drops’

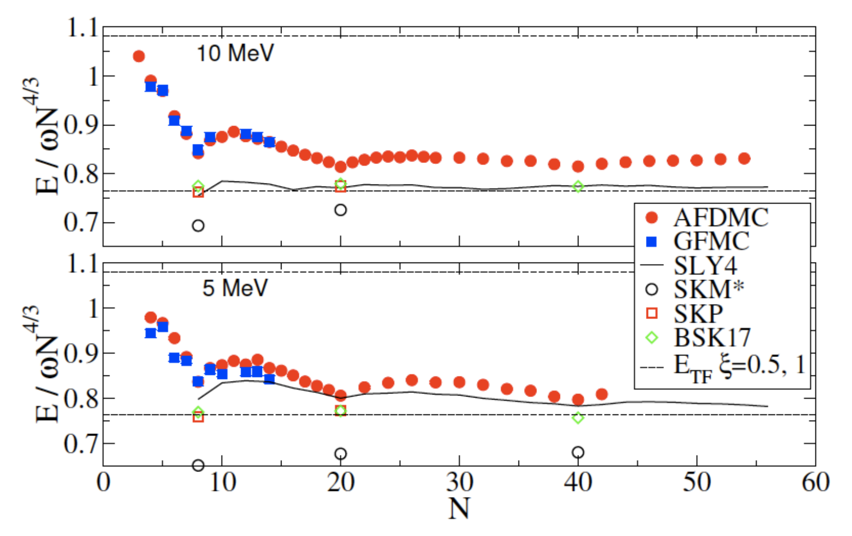
N = 6 to 50 Neutrons
Harmonic Oscillator and Wood-Saxon external wells



UNEDF SCIDAC project

Explore very large isospin limit of the density functional.
Examine gradient, spin-orbit, and pairing terms at
Low to Moderate densities

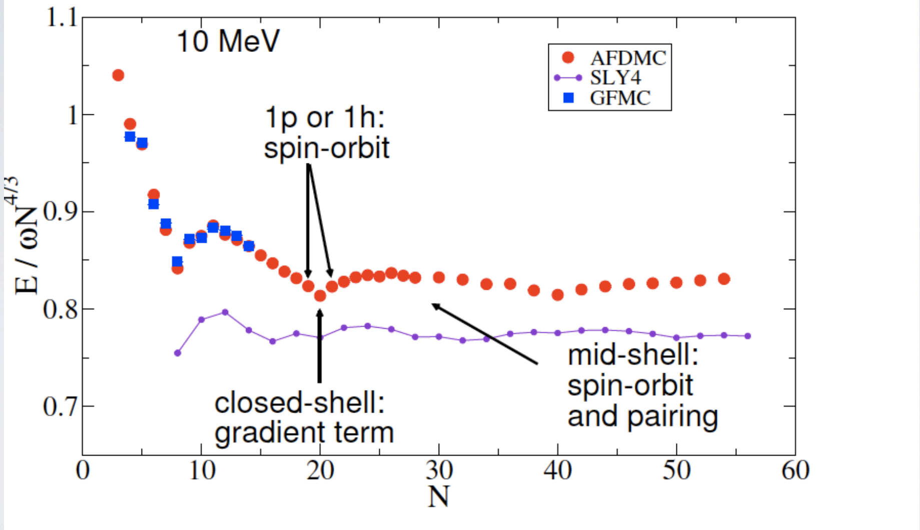
Harmonic Oscillator External Potential

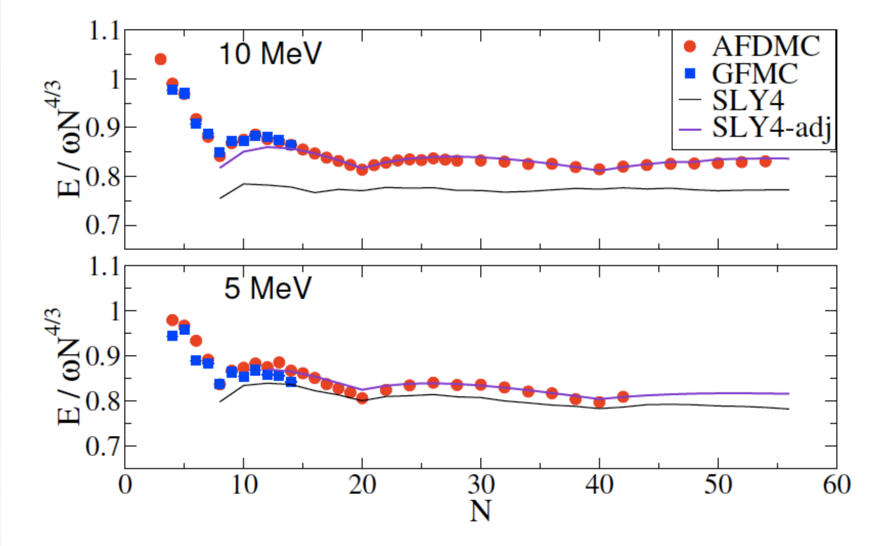


Gandolfi, Pieper, JC, PRL 2011

\`Traditional' Skyrme models overbind neutron drops

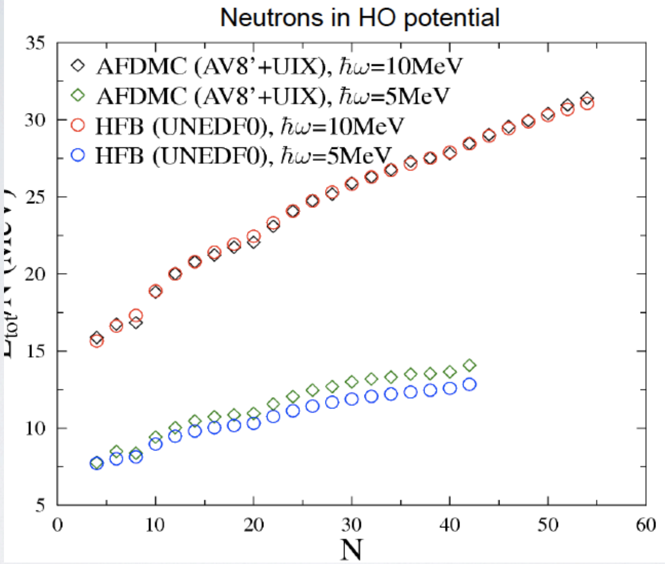
Closed shells determined by EOS + gradient terms





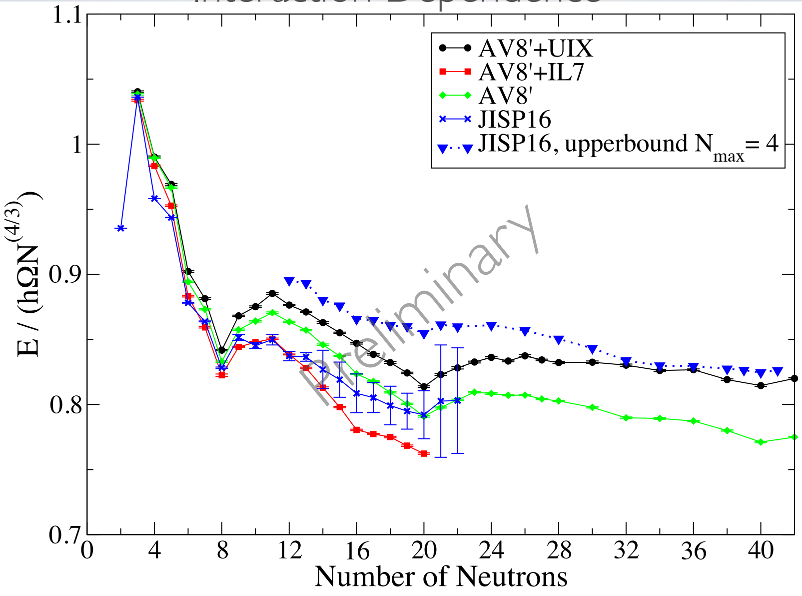
Repulsive gradient terms required to fit neutron drops
also smaller spin-orbit, pairing interactions

UNEDF0 functional



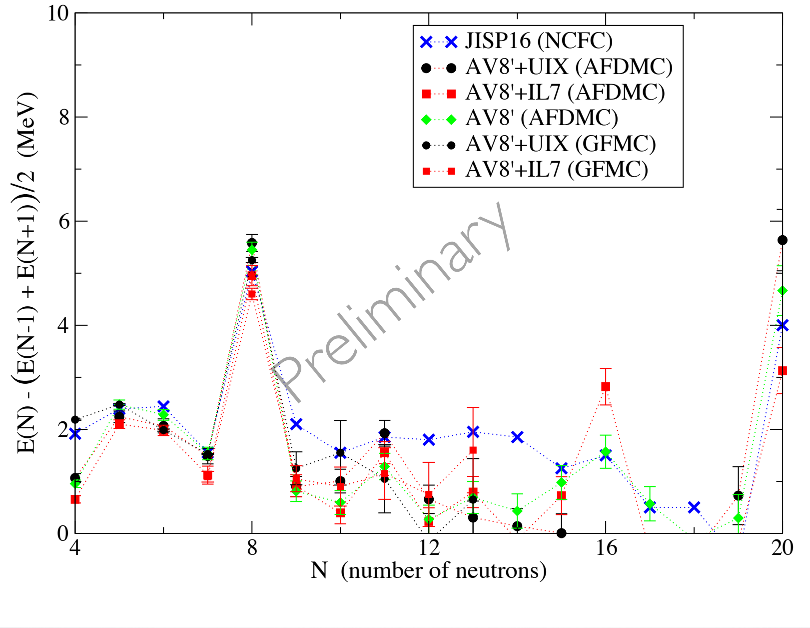
Marcus Kortelainen, Aizu 2010 workshp

Interaction Dependence



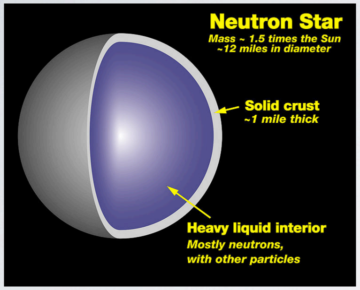
J. Vary, P. Maris, S. Pieper, S. Gandolfi, J.C., in preparation

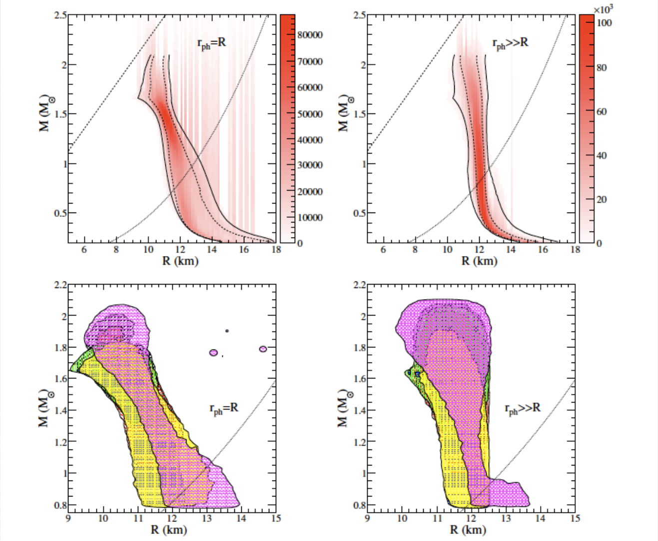
Pairing Gaps in Neutron Drops



J. Vary, P. Maris, S. Pieper, S. Gandolfi, J.C.

Neutron Matter at Higher Density Determines Neutron Star Mass/Radius





$k_F \sim 1.7\ \mathrm{fm}^{-1}$ at nuclear matter density

Steiner, Lattimer and Brown, 2010

Approach

Nucleons-only model w/ reasonable resolution to treat systems up to 2-3 x saturation density

AFDMC can treat neutrons w/ 2 and 3-body forces
  - includes superfluid pairing, short-range correlations,...
  - more accurate and more flexible than FHNC

Try to characterize uncertainty due to unknown interaction terms: vary strength and range of short-range TNI

Interaction Model (TNI)

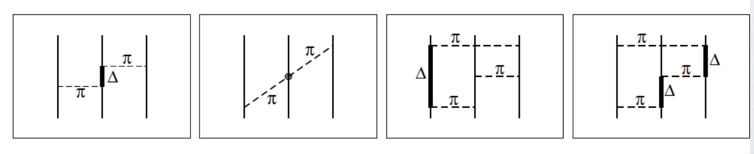
Longest Range Part: 2π TNI

$$V_{2\pi} \;=\; \sum_{cyc} T_\pi(r_{12})T_\pi(r_{23})\,\{S_{12},S_{23}\} \;+\; Y_\pi(r_{12})Y_\pi(r_{23})\;\;\{\sigma_1\cdot\sigma_2,\sigma_2\cdot\sigma_3\}$$

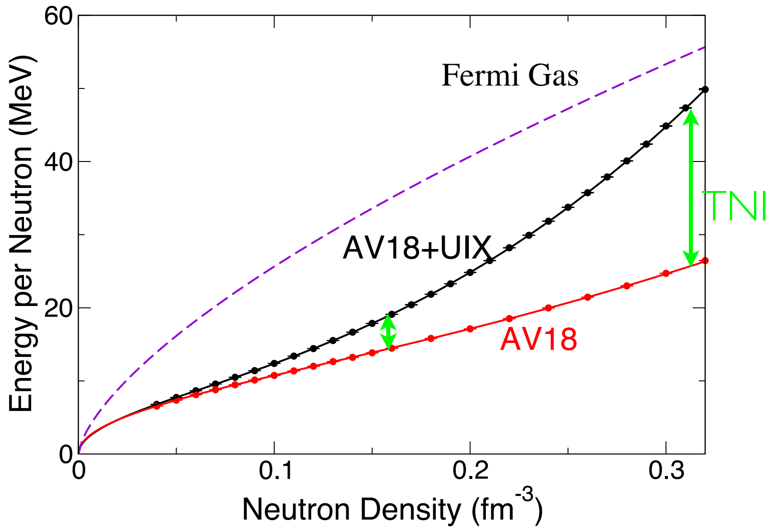
(in neutron matter)

+ s-wave 2π TNI

Add 3π TNI terms from Illinois models
+ shorter range terms



AV18 and AV18 + UIX

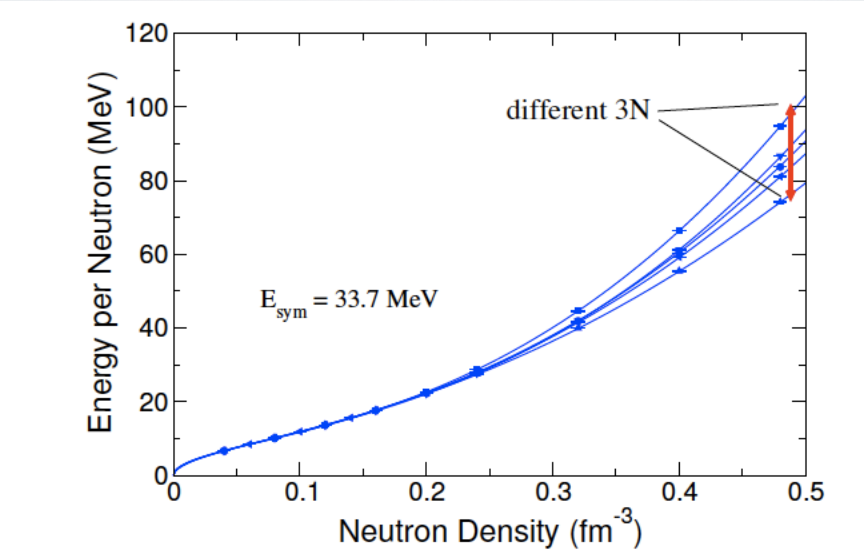


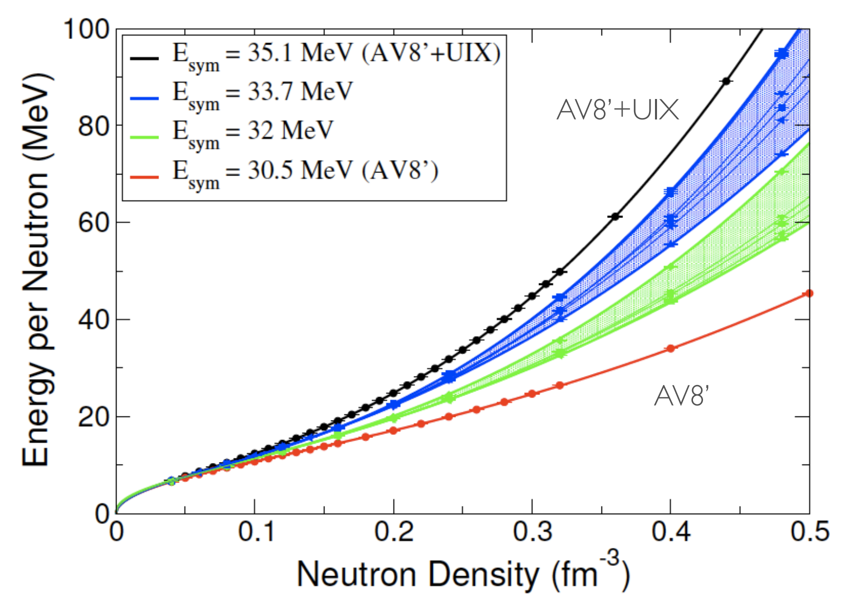
TNI quite small (~ 4 MeV) at saturation density

moderate at 2x saturation density (< 1/2 E_{FG})

Very small contribution from 2π TNI

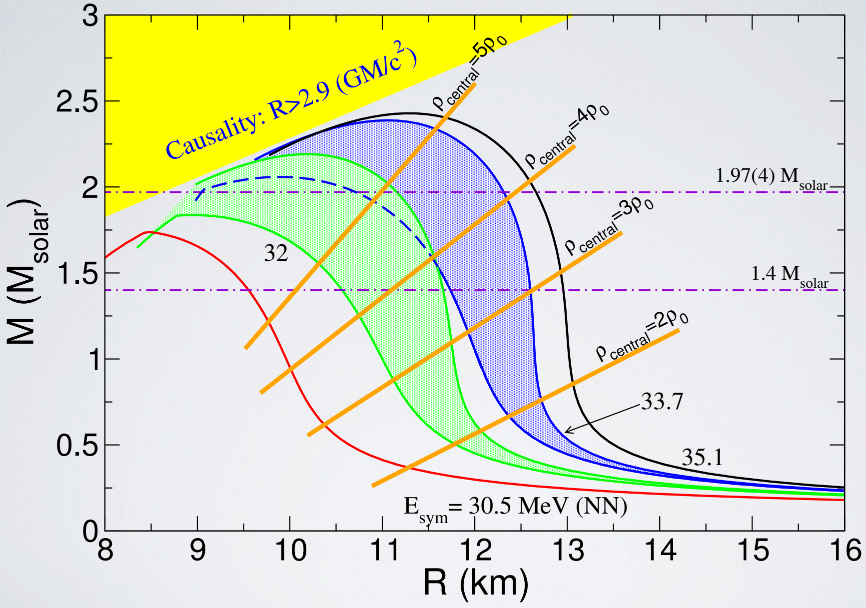
Fix (a)symmetry energy
or neutron matter energy at saturation density



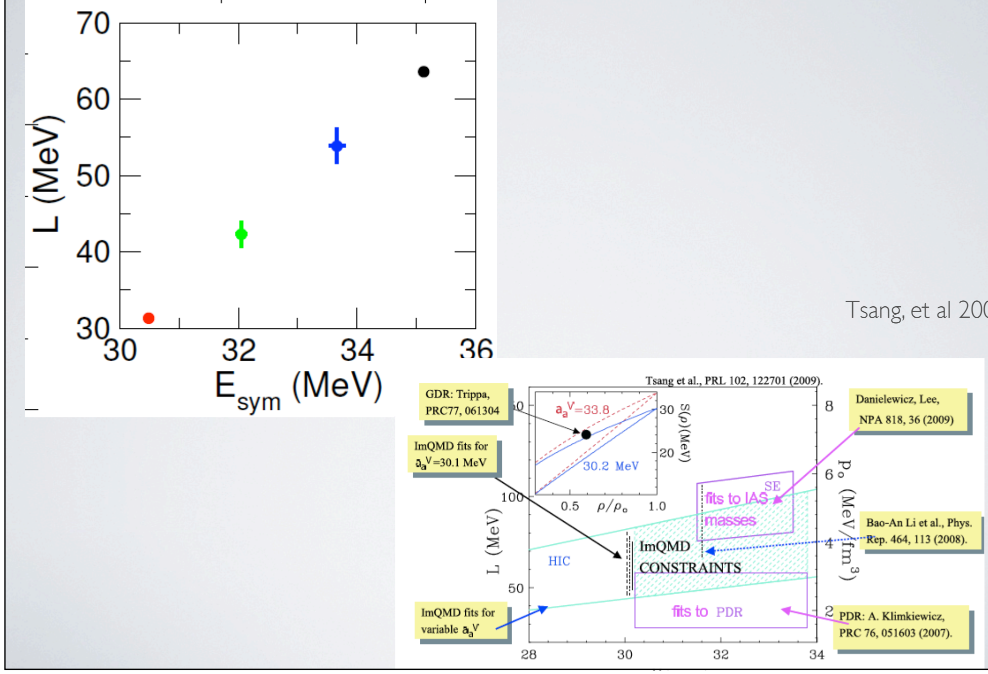


Gandolfi, Carlson, Reddy 2010

Mass/Radius Bands for Different Symmetry Energies



Strong Correlation between Symmetry Energy and its Density Dependence



Tsang, et al 2009

Tsang et al., PRL 102, 122701 (2009).

GDR: Trippa, PRC77, 061304

ImQMD fits for a_a^V=30.1 MeV

ImQMD fits for variable a_a^V

Danielewicz, Lee, NPA 818, 36 (2009)

Bao-An Li et al., Phys. Rep. 464, 113 (2008).

PDR: A. Klimkiewicz, PRC 76, 051603 (2007).

Conclusions:

Realistic description of homogeneous and inhomogeneous neutron matter achievable

Being built into theories of atomic nuclei, neutron star crust, etc.

Future:

Superfluid protons in neutron-star matter

Nuclear EW response (matter, nuclei)