Clustering and Correlations in $^{12}$C and Neutron Matter

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Clustering in $^{12}$C 0+ states: ground- and Hoyle states

Homogeneous Neutron Matter:
- Cold Atoms and Low-Density Neutron Matter
- Higher-Density Matter and Neutron Stars

Response:
- Spin Response and Neutrino Emissivity

Inhomogeneous Matter (drops):
- Neutrons Confined in External Fields
VMC: Variational Monte Carlo
   assumed form for wave function
   Monte Carlo for integration

GFMC: Green's function Monte Carlo
   sample imaginary-time path integral
   explicit spin-isospin sums

AFDMC: Auxiliary-Field Diffusion (Green’s fn)
   Monte Carlo
   sample space and spins

AFMC: lattice calculations using auxiliary fields
$^{12}\text{C}$ ground state

$$\Psi_0 = \exp[-H\tau] \Psi_T$$

The 'Jastrow' part of the trial wave function is a major part of the entire calculation.

There are 5 LS basis $J=0+$ basis states in the 0p shell can be constructed by operators on $(p^{3/2})^8$ state.

Can also make an explicitly triple-alpha state (1 in 0s and 2 in 0p shells) with Jastrow correlations

This basis of 6 sates works well for the ground state.
Asynchronous Dynamic Load Balancing
Pieper and Lusk, SCIDAC Review

Efficiency in %

Number of 4-core nodes

12C ADLB+GFMC

Efficiency = Application_CPU_time/Total_wall_time

Oct 2009
Jun 2009
Feb 2009
131,072 cores!
Carbon 12 ground state
computed from: \( \psi_0 = \exp \left[ -H \tau \right] \psi_T \)

Trial state should incorporate flexible long-distance physics

\( \psi_T \) has 5 simplest shell-model states
+ alpha-particle `cluster' state

Good description of ground state energy/density
Trial state for 2nd 0+ (Hoyle) state
Calculation by S. Pieper (ANL)

states in G.S. trial state +
  alphas in 0s, 0p, 1s-0d shell
  also try a pair in 1s-0d shell
  total of 11 states to be diagonalized

Initial diagonalization for ground-state $\Psi_T$ (same results)
Compute GFMC g.s. overlaps with these states,
diagonalize overlaps (10 states) to get 2nd 0+

PRELIMINARY
RMS radii / Charge Density

![Graphs showing RMS radii and charge density for different states of 12C.](image-url)
Low-Density (dilute) near free Fermions to near Unitarity range of the interaction $< \text{interparticle spacing}$

Analytically known at extremely low density $E / E_F$ rapidly decreases to $\sim 1/2$ with increasing $k_F a$

Higher density EOS important for neutron star mass/radius
$k_F \approx 0.3$

A. Gezerlis, J. C., 2008, 2010

[Graph showing $E/E_{eq}$ vs $k_F a$ with data points for Neutron Matter and Cold Atoms, and a note on QMC unitarity]
Improved Lattice Methods
and Unitary Gas

$E/E(\text{FG}) = 0.372(5)$
no fixed-node error

At finite (small) effective range:

$$\frac{E}{E_{\text{FG}}} = \xi + S k_F r_e$$

$\xi$ and $S$ are universal parameters

Can measure neutron matter EOS (including effective range corrections) in cold atoms
Unitary Fermi Gas (lattice)

Up to $27^3$ lattice, 66 particles

Universality of effective range correction

Lattice method (points) compared to continuum (DMC)

\[ S = 0.11(0.03) \]
Pairing Gap at low density
Summary of Gap calculations
Low-moderate density EOS
s-wave pairing gap closes
Neutrino Emissivity

Response of neutron matter to spin excitations determines emissivity:

\[ S(k, \omega) = \langle 0 | \sum_i \exp[i k \cdot r_i] \sigma_i \cdot \sum_j \exp[i k \cdot r_j] \sigma_j | 0 \rangle \]

At k=0 no response (or emissivity) without tensor and spin-orbit correlations

\[ Q = \frac{C_A^2 G_F^2 n}{20 \pi^3} \int_0^{\infty} d\omega \, \omega \, e^{-\omega/T} S_\sigma(\omega) \]
`Short-range' correlations

\[ S_0(q) \]

\[ n=0.16 \text{ fm}^{-3} \]

\[ q [\text{fm}^{-1}] \]

\[ r [\text{fm}] \]
Use Sum Rules to constrain the response:

\[
S_{\sigma}^{-1} = \frac{\chi_\sigma}{2n} = \frac{\chi_\sigma^F}{2n(1 + G_0)}
\]

\[
S_{\sigma}^{0} = 1 + \lim_{q \to 0} \frac{4}{3N} \sum_{i \neq j}^{N} \langle 0 | e^{-i \mathbf{q} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \sigma_i \cdot \sigma_j | 0 \rangle
\]

\[
S_{\sigma}^{+1} = -\frac{4}{3N} \lim_{q \to 0} \langle 0 | [H_N, s(\mathbf{q}) \cdot s(-\mathbf{q})] | 0 \rangle
\]

Static and energy-weighted sum rules from ground-state expectations
Inverse energy-weighted from spin susceptibility

Low-Energy shape constrained by Q.P. model
High-Energy tail constrained by 2-body physics
Response compared to previous calculations
saturation density
Density Dependence

![Graph showing density dependence with curves labeled 0.12, 0.16, and 0.20.](image)
Emissivity vs. Temperature:

Emissivity actually increased compared to previous results.
Inhomogeneous Matter
`Neutron Drops'

N = 6 to 50 Neutrons
Harmonic Oscillator and Wood-Saxon external wells

Explore very large isospin limit of the density functional.
Examine gradient, spin-orbit, and pairing terms at
Low to Moderate densities

UNEDF SCIDAC project
Harmonic Oscillator External Potential

`Traditional` Skyrme models overbind neutron drops

Gandolfi, Pieper, JC, PRL 2011
Closed shells determined by EOS + gradient terms

10 MeV

$E / \omega N^{4/3}$

1p or 1h: spin-orbit

closed-shell: gradient term

mid-shell: spin-orbit and pairing
Repulsive gradient terms required to fit neutron drops also smaller spin-orbit, pairing interactions
UNEDF0 functional

Neutrons in HO potential

Marcus Kortelainen, Aizu 2010 workshop
Interaction Dependence

\[ \frac{E}{\hbar\Omega N^{4/3}} \]

- AV8' + UIX
- AV8' + IL7
- AV8'
- JISP16
- JISP16, upperbound \( N_{\text{max}} = 4 \)

Pairing Gaps in Neutron Drops

J. Vary, P. Maris, S. Pieper, S. Gandolfi, J.C.
Neutron Matter at Higher Density Determines Neutron Star Mass/Radius

$k_F \sim 1.7 \text{ fm}^{-1}$ at nuclear matter density

Steiner, Lattimer and Brown, 2010
Approach

Nucleons-only model w/ reasonable resolution to treat systems up to 2-3 x saturation density

AFDMC can treat neutrons w/ 2 and 3-body forces includes superfluid pairing, short-range correlations,... more accurate and more flexible than FHNC

Try to characterize uncertainty due to unknown interaction terms: vary strength and range of short-range TNI
Interaction Model (TNI)

Longest Range Part: \(2\pi \text{TNI}\)

\[
V_{2\pi} = \sum_{\text{cyc}} T_{\pi}(r_{12})T_{\pi}(r_{23}) \{ S_{12}, S_{23} \} + Y_{\pi}(r_{12})Y_{\pi}(r_{23}) \{ \sigma_1 \cdot \sigma_2, \sigma_2 \cdot \sigma_3 \}
\]

(in neutron matter)

+ s-wave \(2\pi \text{TNI}\)

Add \(3\pi \text{TNI}\) terms from Illinois models

+ shorter range terms
TNI quite small (~ 4 MeV) at saturation density
moderate at 2x saturation density (< 1/2 $E_{FG}$)
Very small contribution from $2\pi$ TNI
Fix (a)symmetry energy or neutron matter energy at saturation density
Causality: $R > 2.9 (\text{GM}/c^2)$

$\rho_{\text{central}} = 2 \rho_0$

$\rho_{\text{central}} = 3 \rho_0$

$\rho_{\text{central}} = 4 \rho_0$

$\rho_{\text{central}} = 5 \rho_0$

$E_{\text{sym}} = 30.5 \text{ MeV (NN)}$
Strong Correlation between Symmetry Energy and its Density Dependence

Tsang, et al 2009
Conclusions:

Realistic description of homogeneous and inhomogeneous neutron matter achievable

Being built into theories of atomic nuclei, neutron star crust, etc.

Future:

Superfluid protons in neutron-star matter

Nuclear EW response (matter, nuclei)