Resonances and alpha-particle condensate in ^{16}O studied with Complex Scaling Method

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Contents

- Sketch and definition of alpha condensates in finite nuclei

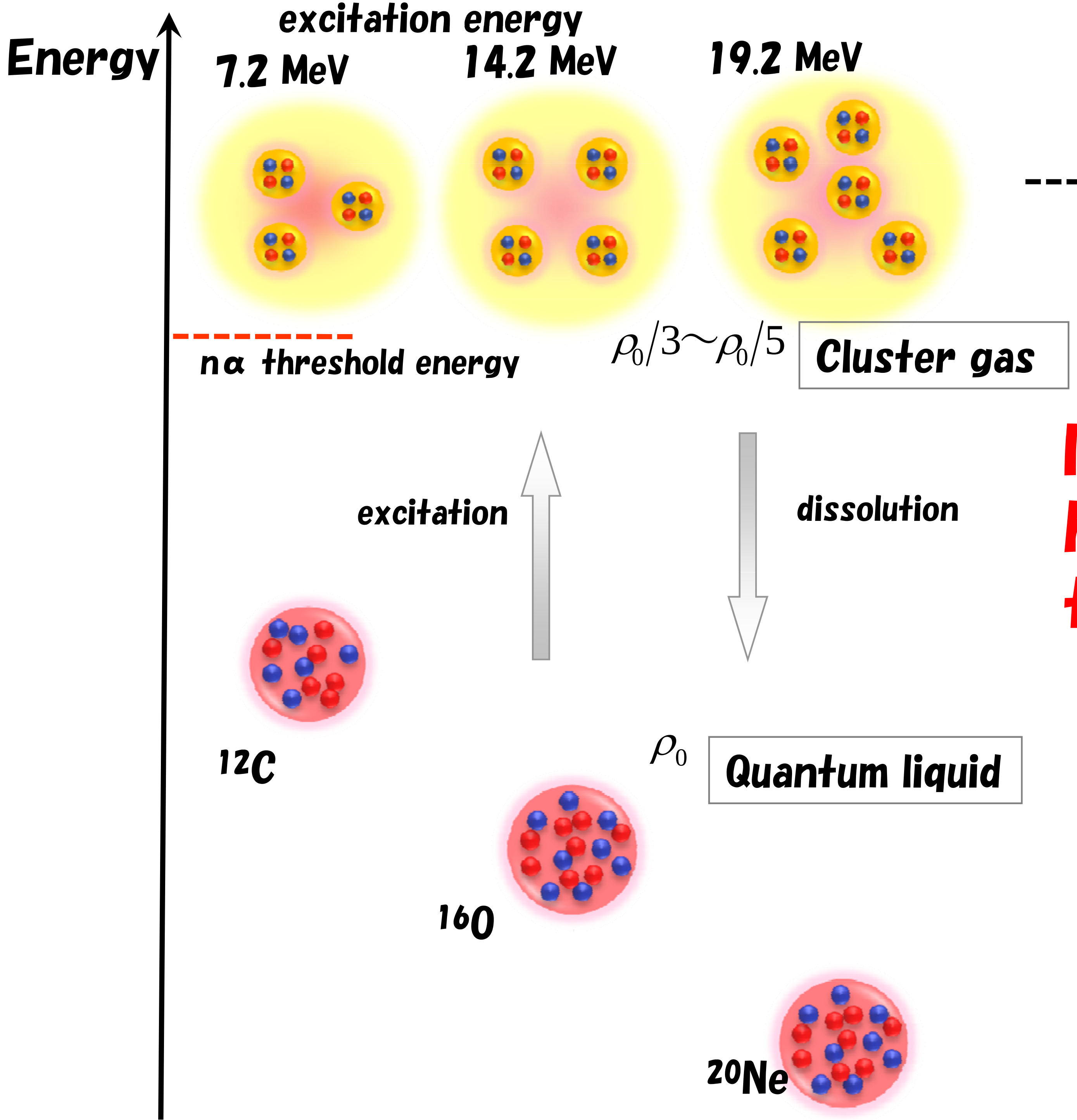
- 4α OCM with GEM (Gauss Expansion Method)

within bound state approximation
- Hoyle-analogue state and other cluster states ($J^{\pi}=0^{+}$)
- possible excitations of condensate ($J^{\pi}=1^{-}$, 2^{+}, 3^{-}, 4^{+})

Complex Scaling Method
- $J^{\pi}=0^{+}$ resonances (first naive results)

Summary

``gas phase'' in finite nuclei



Infinite nuclear matter

(low density)

$< \rho_0/5$

Crust of neutron star ?

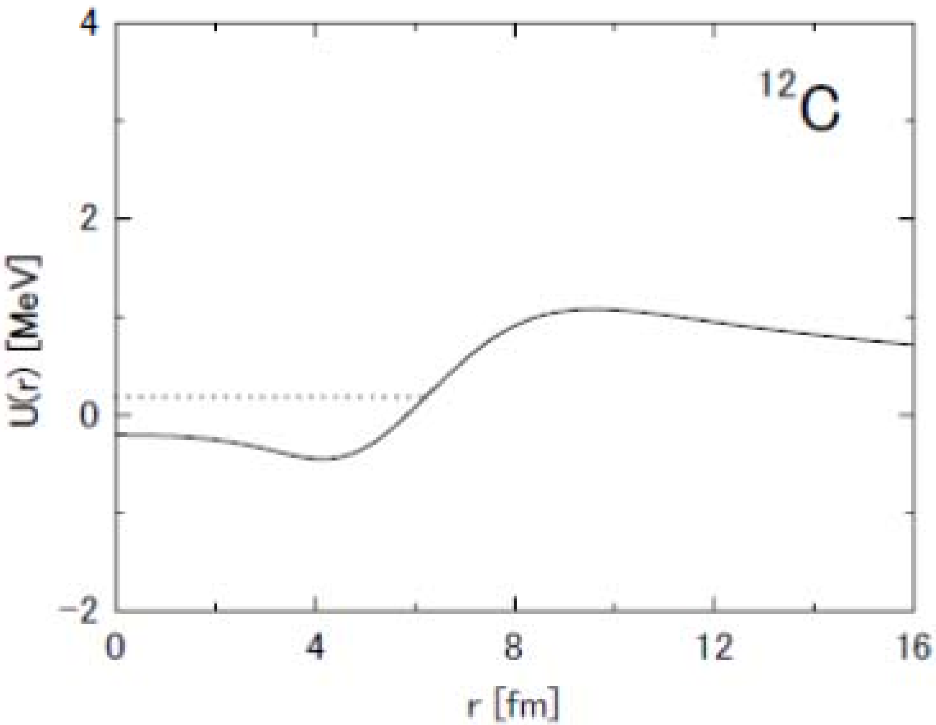
Investigation in heavier nuclei than ^{12}C

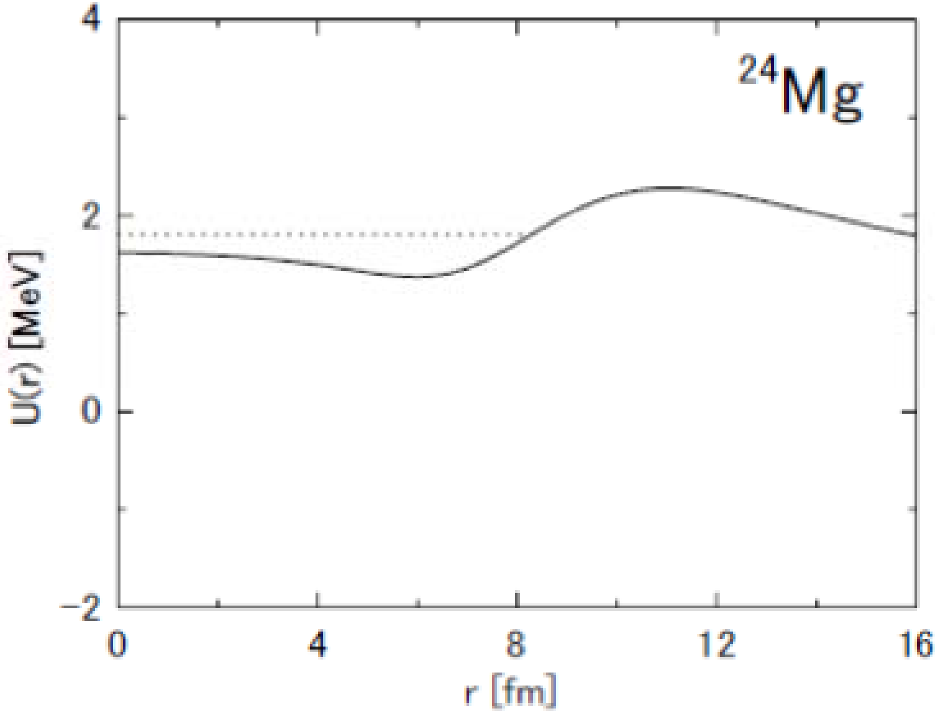
Single-α potential given via Gross-Pitaevskii approach

$$\Phi(n\alpha) = \prod_{i=1}^{n} \phi(\mathbf{r}_i), \quad \left[-\frac{\hbar^2}{2m_\alpha}\left(1-\frac{1}{n}\right)\nabla^2 + U(r) \right]\phi(\mathbf{r}) = \varepsilon\phi(\mathbf{r})$$

- Pure bosons
- $n\alpha$ structure are assumed

$$U(r) = (n-1)\int d\mathbf{r}' \left|\phi(\mathbf{r}')\right|^2 v_2(\mathbf{r}',\mathbf{r}) + \frac{(n-1)(n-2)}{2}\int d\mathbf{r}'' d\mathbf{r}' \left|\phi(\mathbf{r}'')\right|^2 \left|\phi(\mathbf{r}')\right|^2 v_3(\mathbf{r}'',\mathbf{r}',\mathbf{r})$$





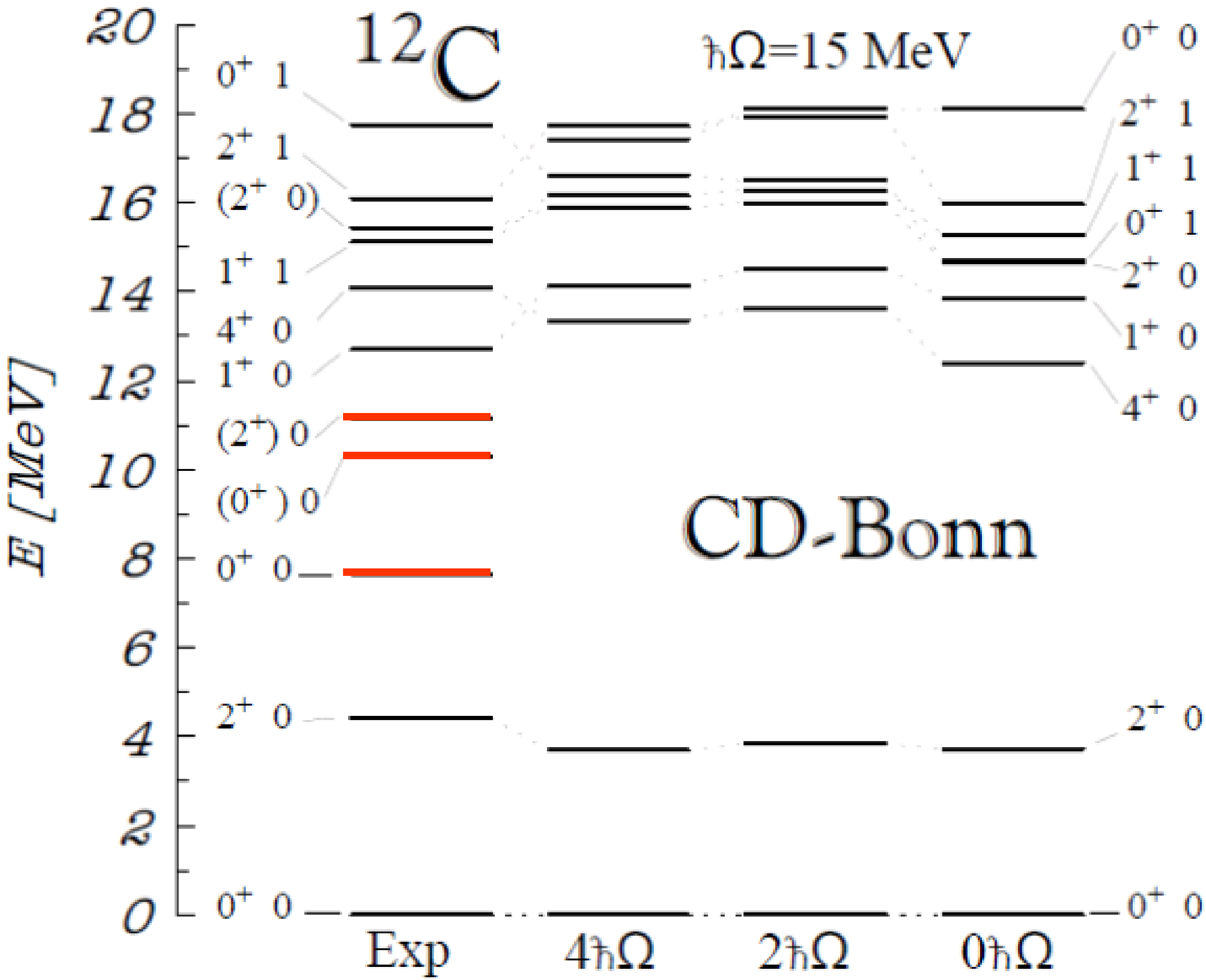
- Coulomb barrier → quasi-stable states
- The barrier position is more than 8 fm. Trapped into a loose potential (interaction range of Ali-Bodmer ~4 fm)
- Weakly interacting "gas-like" states, α condensates

T. Yamada, P. Schuck, PRC 69, 024309 (2004).

Typical mysterious 0^+ states in nuclear structure problem

0_2^+ state of ^{12}C (Hoyle state) indispensable to ^{12}C production in stars

Ab initio non-core shell model calculation



P. Navratil et al., PRL 84, 5728 (2001).

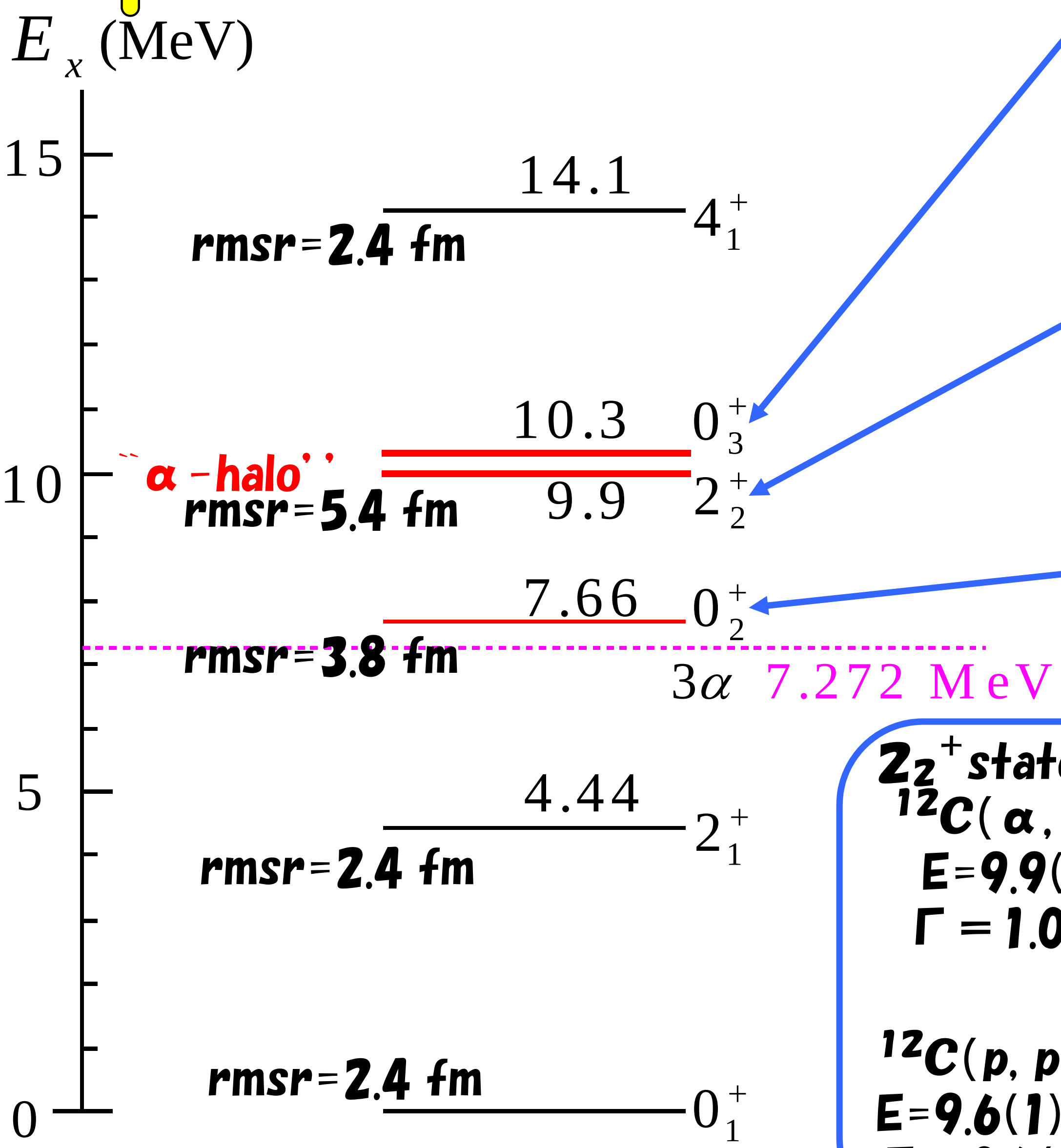
0_2^+ , 2_2^+ , 0_3^+ states : missing

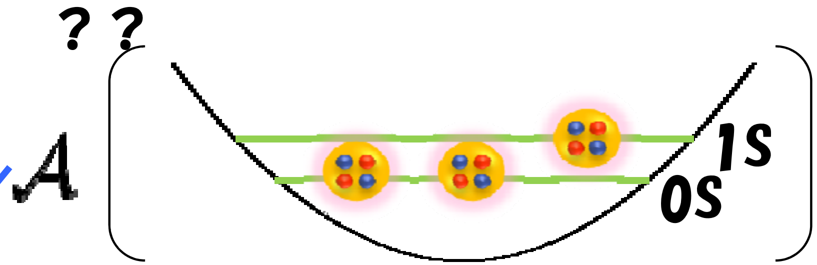
0_2^+ state: excitation energy is not lower than 20 MeV

The typical excited states which resist a shell model description

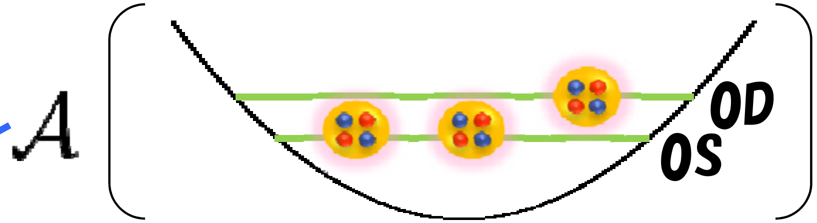
“BEC” in ^{12}C

Observed levels of ^{12}C

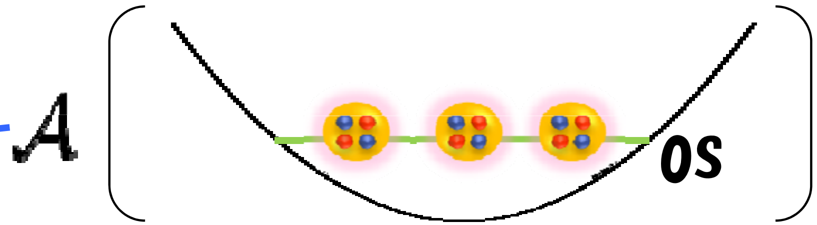




C. Kurokawa and K. Katō, PRC 71, 021301 (2005).



Y. Funaki et al., EPJA 24, 321 (2005).



2_2^+ state:

$^{12}C(\alpha, \alpha')$

E=9.9(3) MeV

Γ = 1.0(3) MeV

M. Itoh et al., NPA 738, 268 (2004).

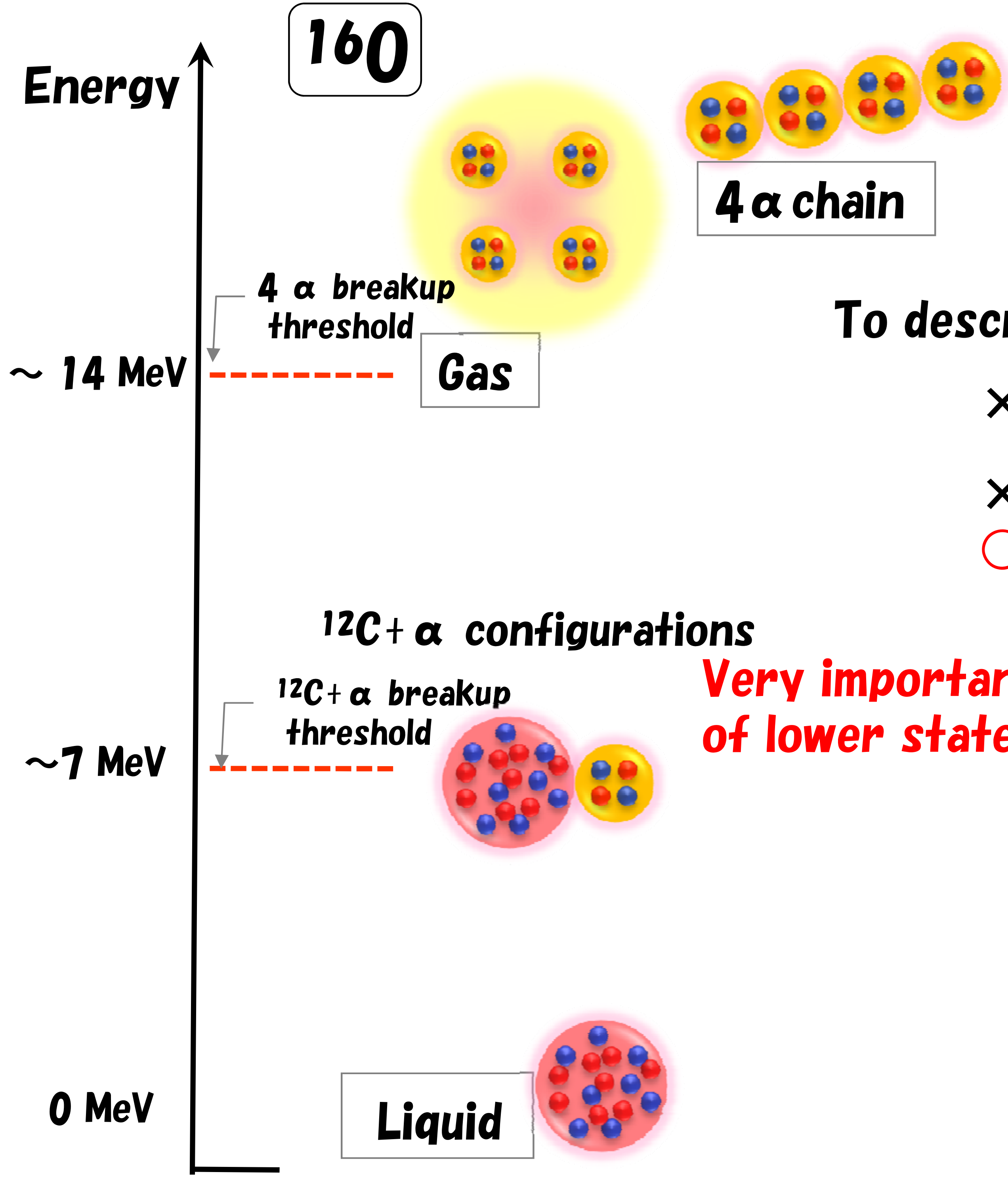
Alpha cond.model
E_{cal}=9.38 MeV
Γ_{cal}=0.64 MeV

$^{12}C(p, p')$

E=9.6(1) MeV

Γ =0.6(1) MeV *M. Freer et al., PRC80, 041303(R) (2009).*

Analogue to the Hoyle state in ^{16}O?



To describe 4α cluster states

× mean field model

× $^{12}C+\alpha$ cluster model

○ 4α cluster model

Very important : simultaneous description of lower states(g.s.、$^{12}C+\alpha$)

Fully solving 4 α -particles relative motions (4α OCM)

Present: Larger model space

$$\varphi_{\ell m}(\mathbf{r},\nu) = N_\ell(\nu) r^\ell \exp\left(-\nu r^2\right) Y_{\ell m}(\mathbf{r})$$

Gaussian basis (GEM)

E. Hiyama et al. Prog. Part. Phys. **51**, 223(2003).

$^{12}C+\alpha$: succeeded
dilute 4α: not reproduced

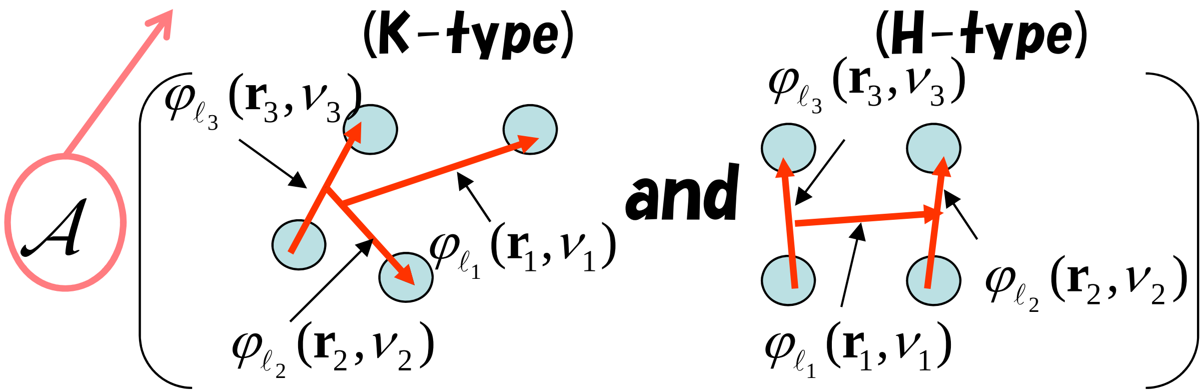
4α OCM (H. O. basis)

K. Fukatsu and K. Kato, PTP **87**, 151 (1992).

$^{12}C+\alpha$ coupled channel OCM (H. O. basis)

Y. Suzuki, PTP **55**, 1751 (1976);
56, 111 (1976).

Approximately taken into account



Adopted angular momentum channels: $[[l_3,l_2], l_1]$ ($l_3+l_2+l_1 \leqq 8$) (up to now, $\leqq 5$)
Including $l_3, l_2, l_1 = 4$

Total w.f.

$$\Psi_{\mathrm{OCM}}(0_k^+) = \sum_{\{l\}\{\nu\}} A^{(k)}_{l_1,l_2,l_{12},l_3}(\nu_1,\nu_2,\nu_3)\left[\left[\varphi_{\ell_1}(\boldsymbol{r}_1,\nu_1),\varphi_{\ell_2}(\boldsymbol{r}_2,\nu_2)\right]_{l_{12}},\varphi_{\ell_3}(\boldsymbol{r}_3,\nu_3)\right]_0$$

$A^{(k)}_{l_1,l_2,l_{12},l_3}(\nu_1,\nu_2,\nu_3)$: Determined by diagonalizing Hamiltonian

Hamiltonian of 4α OCM

$$H = T + \sum_{i<j}\left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{Coul}(r_{ij})\right] + V_{3\alpha} + V_{4\alpha} + V_{Pauli}$$

Pauli blocking operator on α - α motions

$$V_{Pauli} = \lim_{\lambda\to\infty} \lambda \sum_{2n+\ell<4} \sum_{ij} \left| u_{n\ell}(r_{ij}) \right\rangle \left\langle u_{n\ell}(r_{ij}) \right|$$

Pauli forbidden state: h.o.w.f.

2-body force (folding MHN force)

$$V_{2\alpha}(r) = \sum_n V_n^{(2)} \exp\left(-\beta_n^{(2)} r^2\right)$$

Coulomb force

$$V_{2\alpha}^{Coul}(r) = \frac{4e^2}{r} \operatorname{erf}(ar)$$

Phenomenological 3-body force (repulsive)

$$V_{3\alpha} = V^{(3)} \sum_{i<j<k} \exp\left[-\beta(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)\right]$$

$$V^{(3)} = 87.5 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

Phenomenological 4-body force (repulsive)

$$V_{4\alpha} = V^{(4)} \exp\left[-\beta(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)\right]$$

$$V^{(4)} = 12000 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

Energies from 4α threshold

	Cal. (MeV)	Exp. (MeV)
^{12}C(g.s.)	−7.32	−7.28
^{12}C(2_1^+)	−4.88	−2.84
^{12}C(4_1^+)	2.06	6.43
^{12}C(0_2^+)	0.70	0.38
^{16}O(g.s.)	−14.2	−14.44

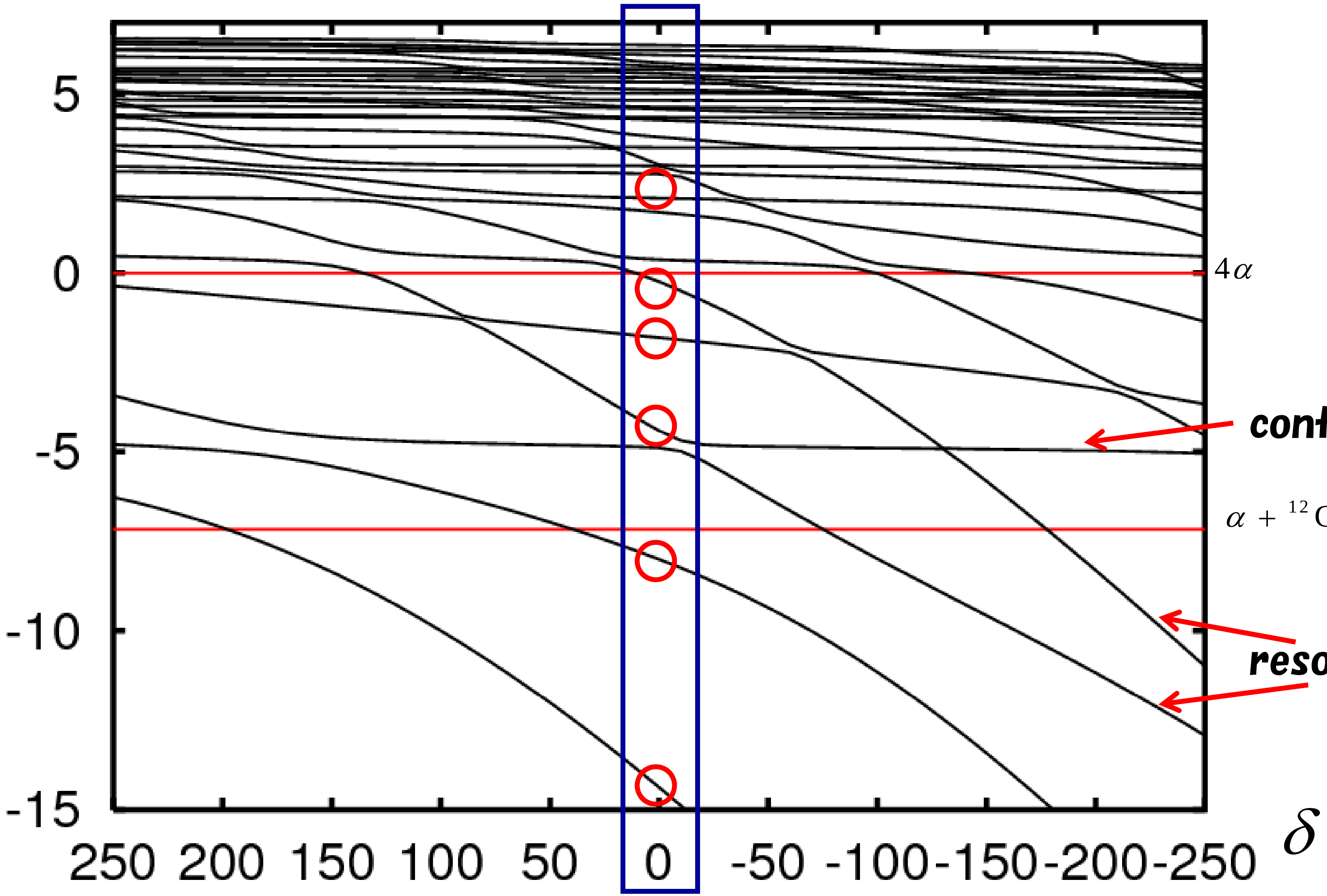
$$\left|\langle V_{3\alpha}\rangle\right|, \left|\langle V_{4\alpha}\rangle\right| < \frac{7}{100}\left|\langle V_{2\alpha}\rangle\right|$$

Assignment of resonances ($J^\pi = 0^+$)

$$(r_{ij} \equiv r_i - r_j)$$

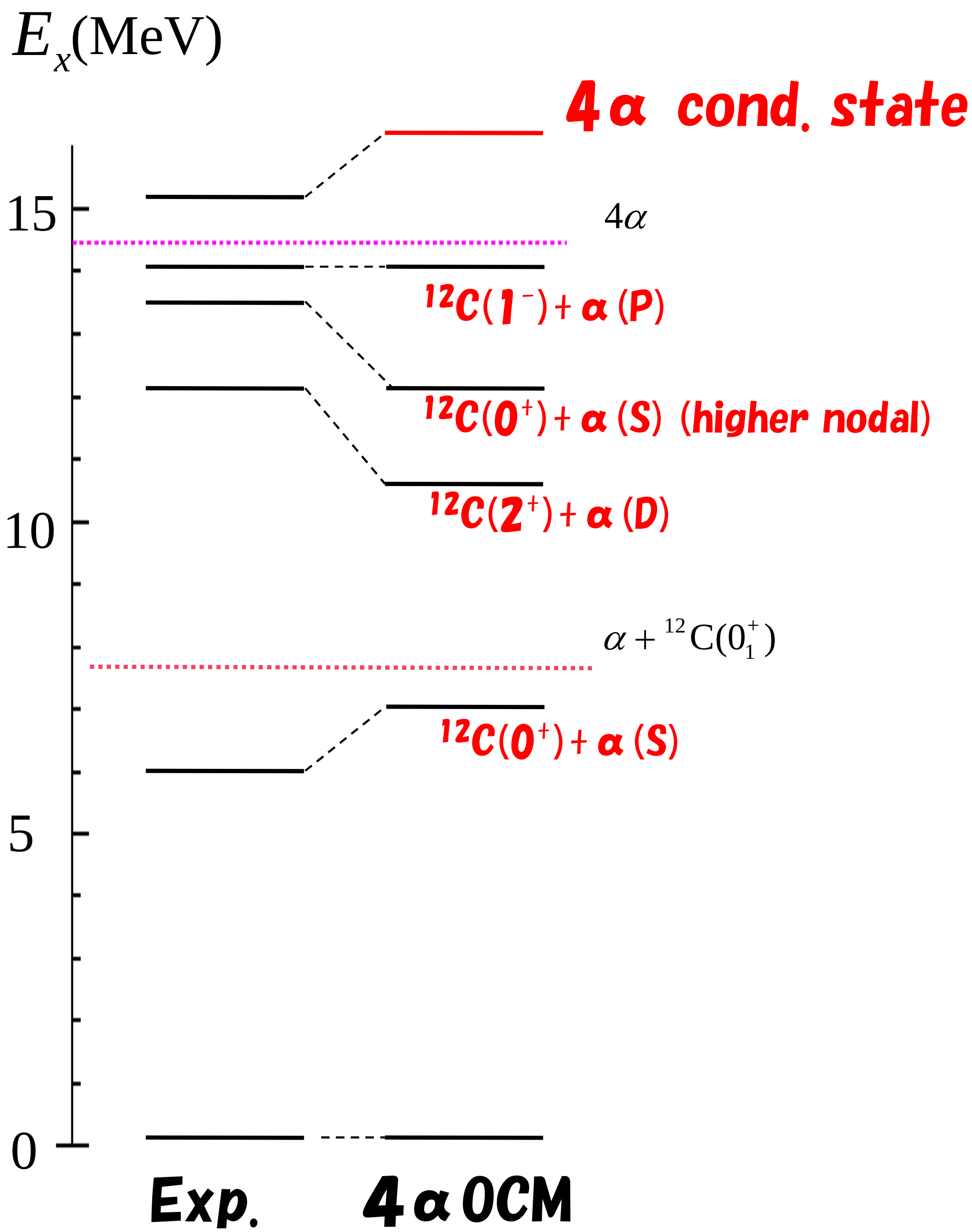
$$H' = H + \delta \cdot \exp\left[-0.05(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2) \right]$$

○ : adopted states



Physical situation

0^+ spectra, rms radii, monopole matrix elements



	R_{rms} (fm)	M(E0) (fm^2)	M(E0) (fm^2) Exp.
$(0_1^+)_{OCM}$	2.7		
$(0_2^+)_{OCM}$	3.0	3.9	3.55
$(0_3^+)_{OCM}$	3.1	2.4	4.03
$(0_4^+)_{OCM}$	4.0	2.4	no data
$(0_5^+)_{OCM}$	3.1	2.6	3.3
$(0_6^+)_{OCM}$	5.6	1.0	no data

Large monopole matrix element can be the evidence of cluster states.

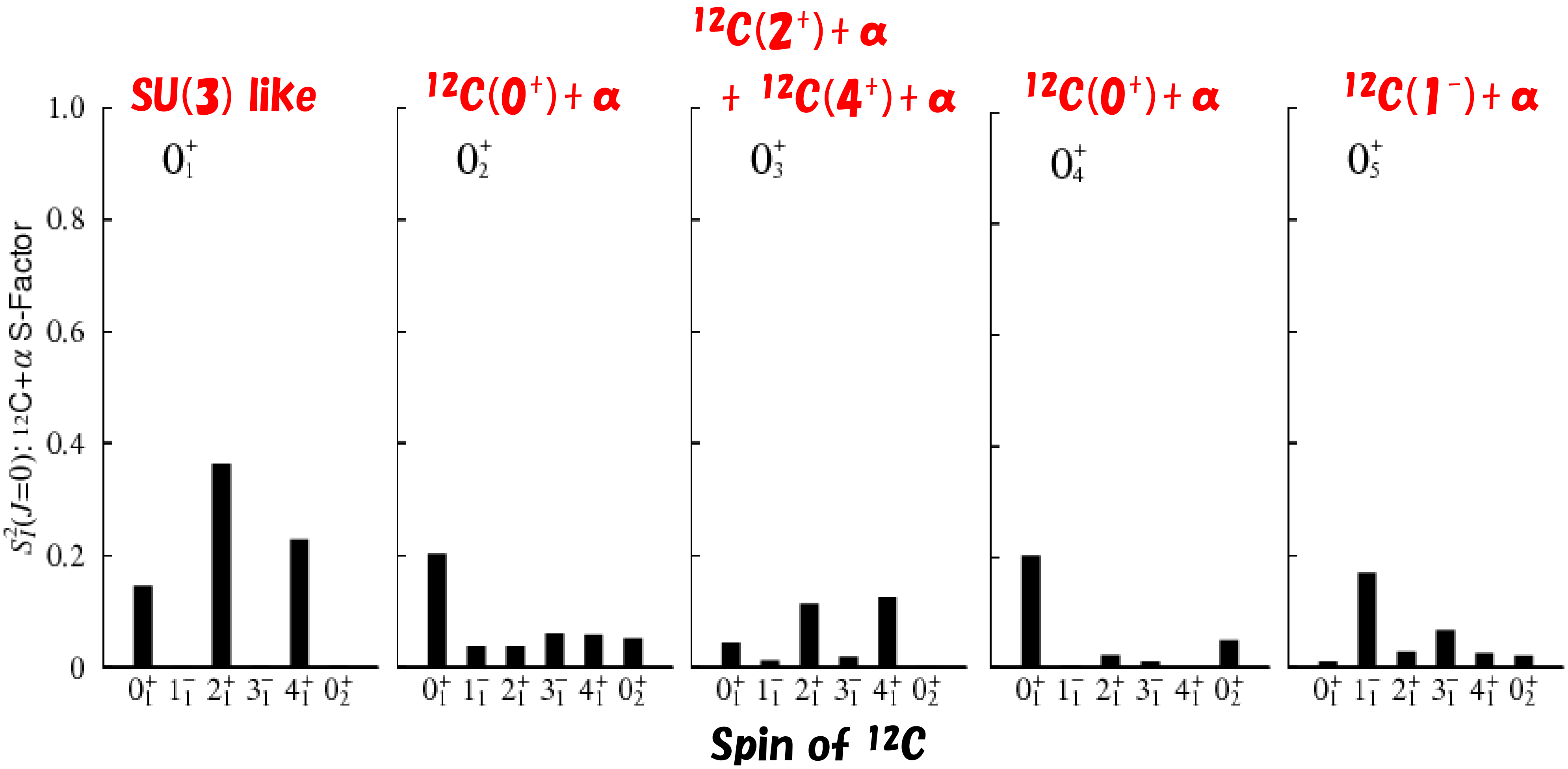
T. Yamada, Y. F. et al., PTP120, 1139 (2008).

0_4^+ state: T. Wakasa, Y. F. et al., PLB 653, 173 (2007).

S-factor for the lower lying states

$$r \times \mathcal{Y}_{IL,J=0}(r) = r \times \left\langle \left[\frac{\delta(r-r')}{rr'} Y_L(\hat{\boldsymbol{r}}') \Psi_{\mathrm{OCM}}(^{12}\mathrm{C}(I)) \right]_0 \middle| \Psi_{\mathrm{OCM}}(0_k^+) \right\rangle$$

$$\boxed{S_{IL}^2(J=0) = \int dr \left(r \times \mathcal{Y}_{IL,J=0}(r) \right)^2}$$



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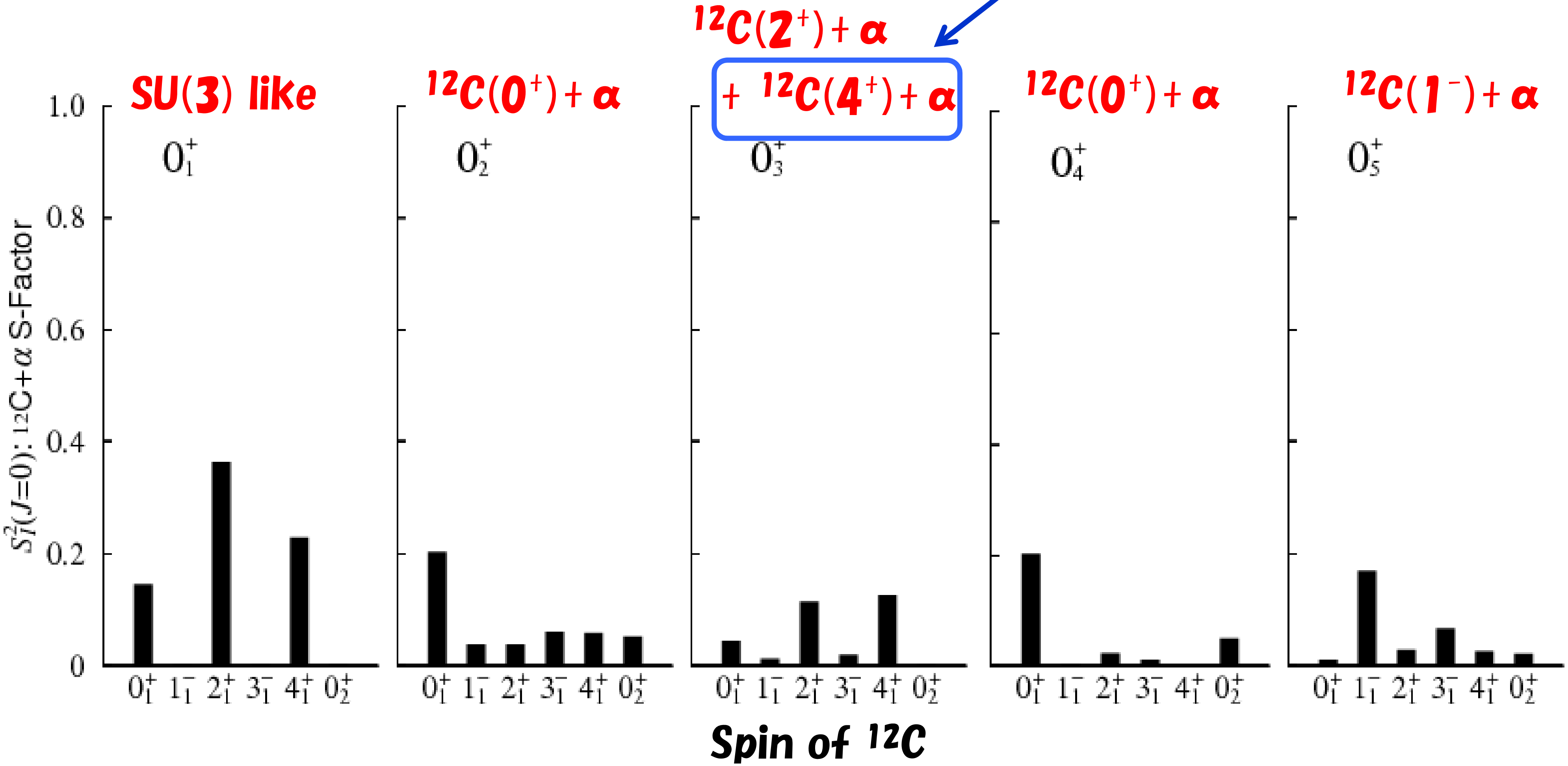
Might be occasionally mixed



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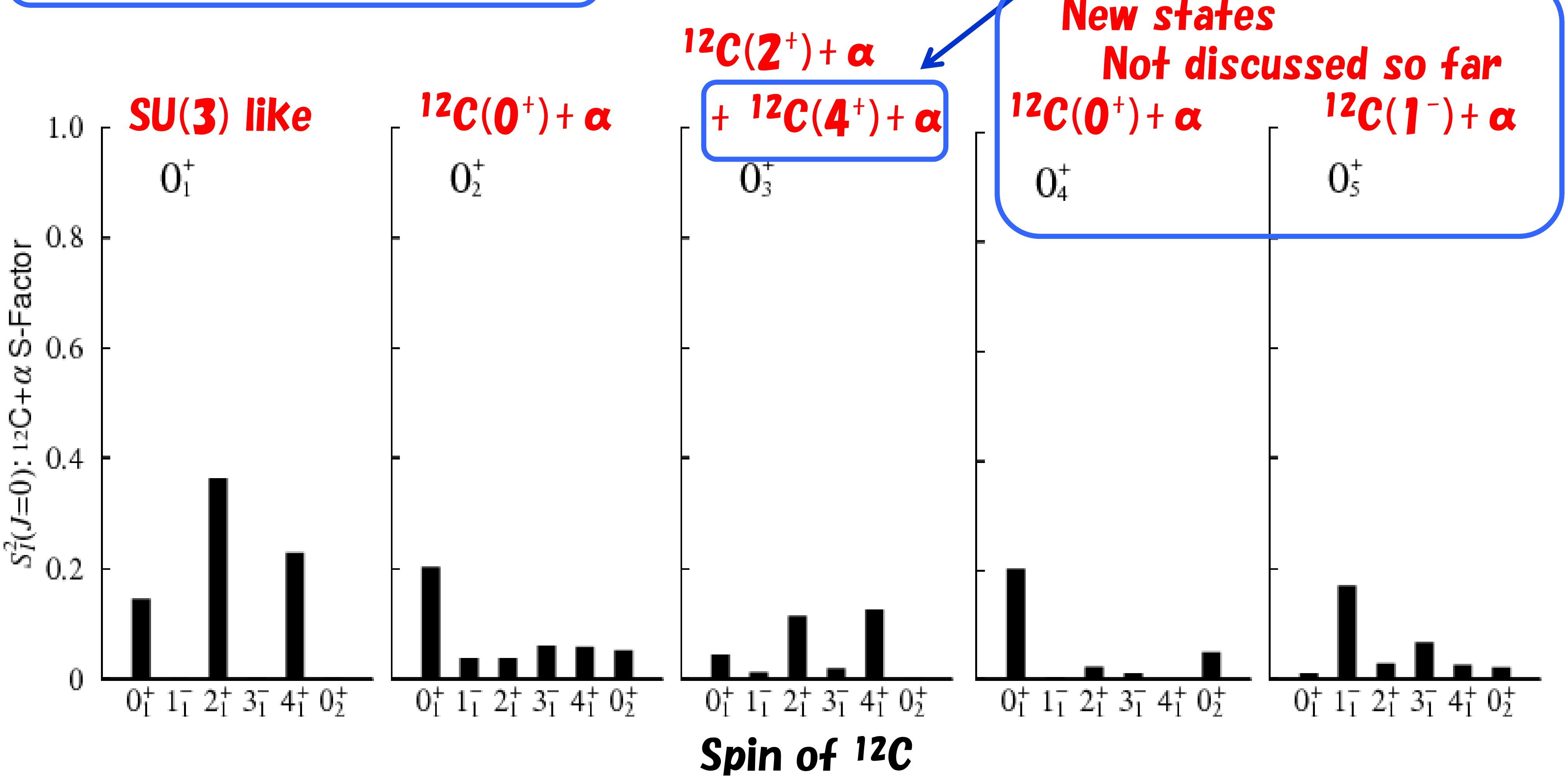
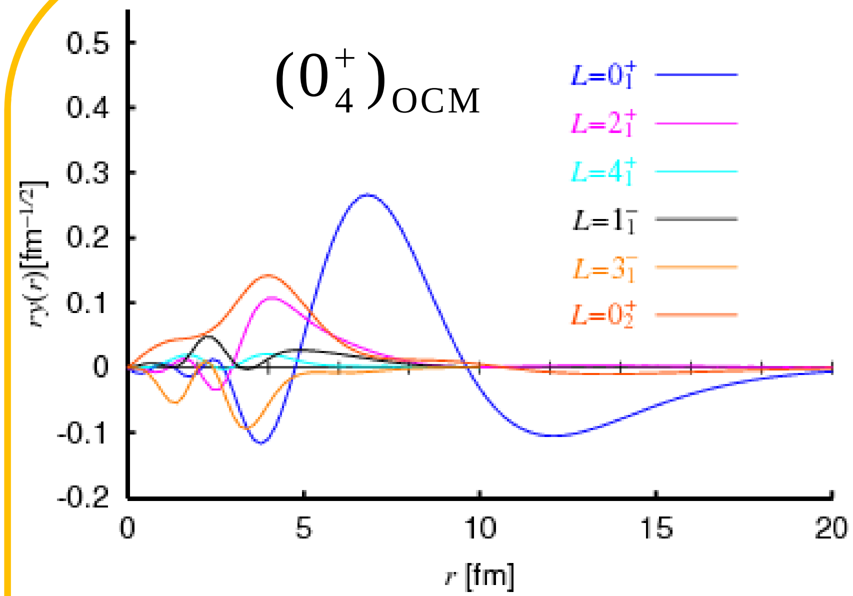


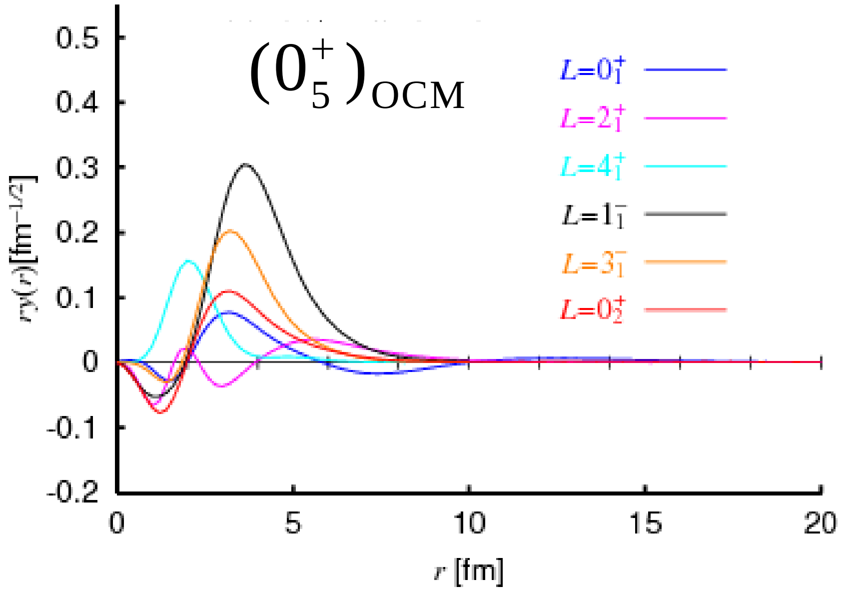
Reduced width amplitudes of 0_4^+ and 0_5^+ states obtained with 4α OCM



- Very well developed α cluster structure
- $^{12}C(g.s.)+\alpha$ component is dominant.

  higher nodal structure

Assigned to the new state observed by Wakasa et al.

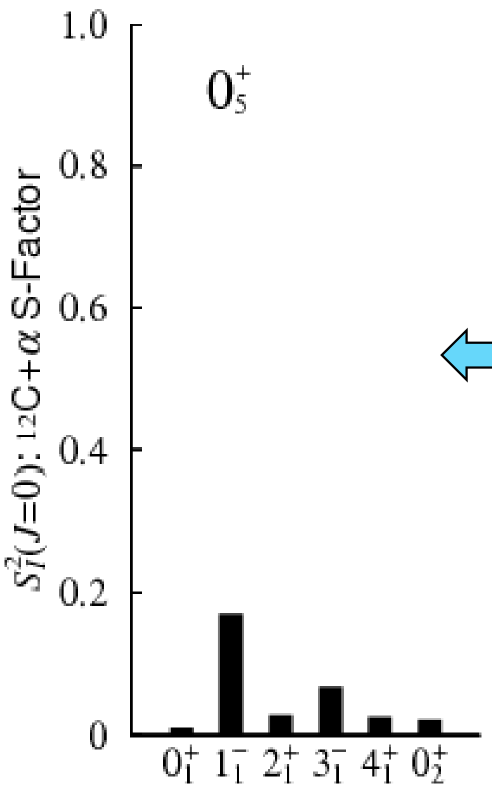


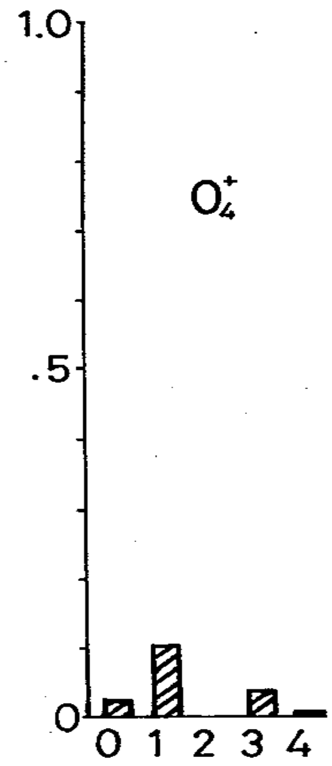
- Large amplitude in surface region(~5 fm)
- developed α cluster structure
- mixing of $^{12}C(1^-)+\alpha$, $^{12}C(3^-)+\alpha$ and $^{12}C(0_2^+)+\alpha$ structures

- New (not discussed so far) $\alpha+^{12}C$ cluster states.

- $\alpha+^{12}C$ dynamics survives up to around the 4α threshold.

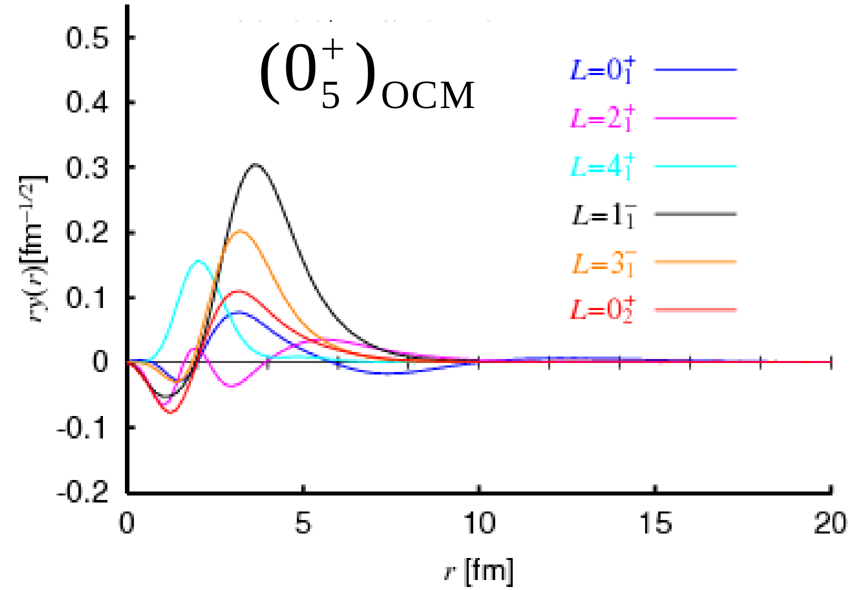
Reduced width amplitudes of 0_4^+ and 0_5^+ states obtained with 4α OCM





K. Fukatsu and K. Kato, PTP **87**, 151 (1992).

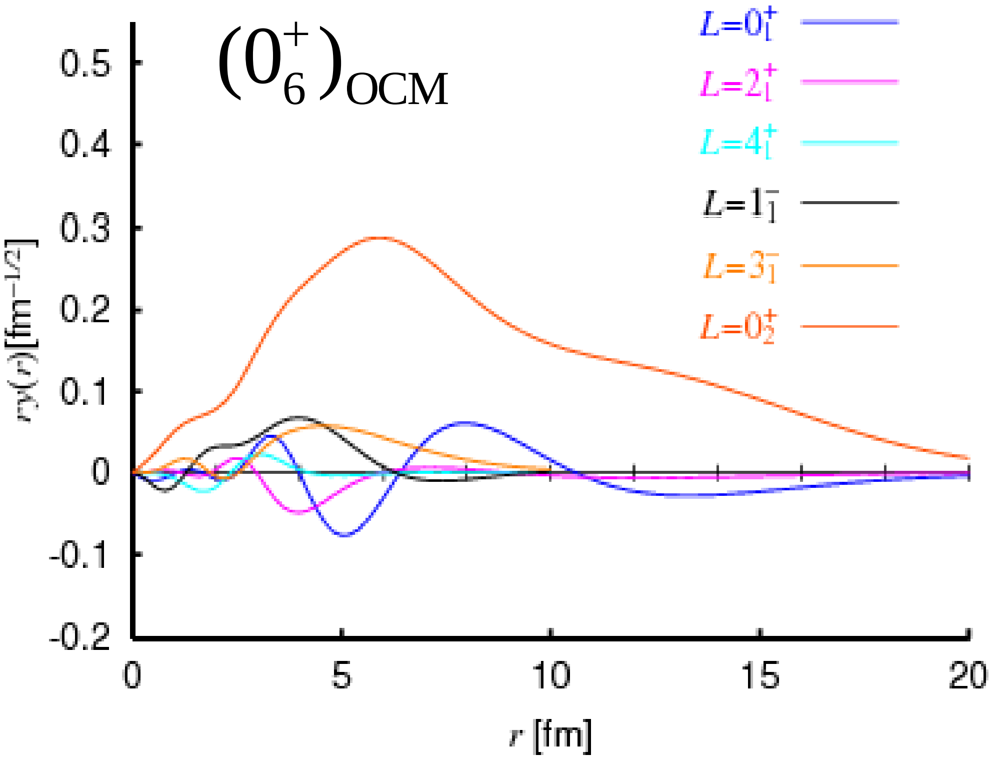
Not discussed here but result of calculation is only shown.



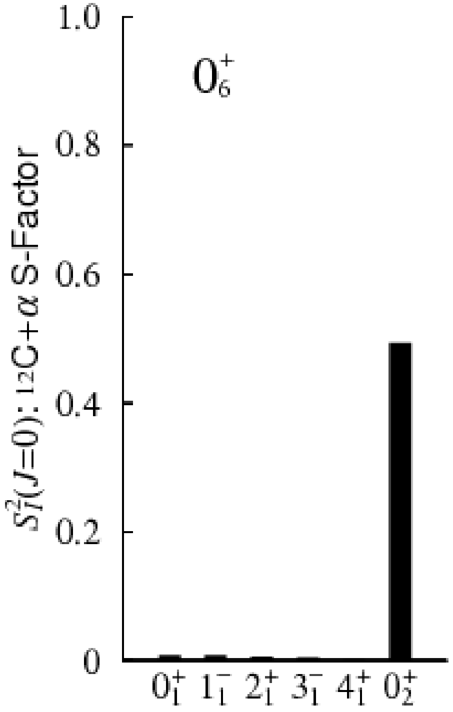
- **Large amplitude in surface region(~5 fm)**
- **developed α cluster structure**
- **mixing of $^{12}C(1^-)+\alpha$, $^{12}C(3^-)+\alpha$ and $^{12}C(0_2^+)+\alpha$ structures**

Reduced width amplitudes of 0_6^+ state obtained with 4α OCM

Defined as $r\times\mathcal{Y}_{IL,J=0}(r)=r\times\left\langle\left[\frac{\delta(r-r')}{rr'}Y_L(\hat{\boldsymbol{r}}')\Psi_{\mathrm{OCM}}(^{12}\mathrm{C}(I))\right]_0\middle|\Psi_{\mathrm{OCM}}(0_k^+)\right\rangle$



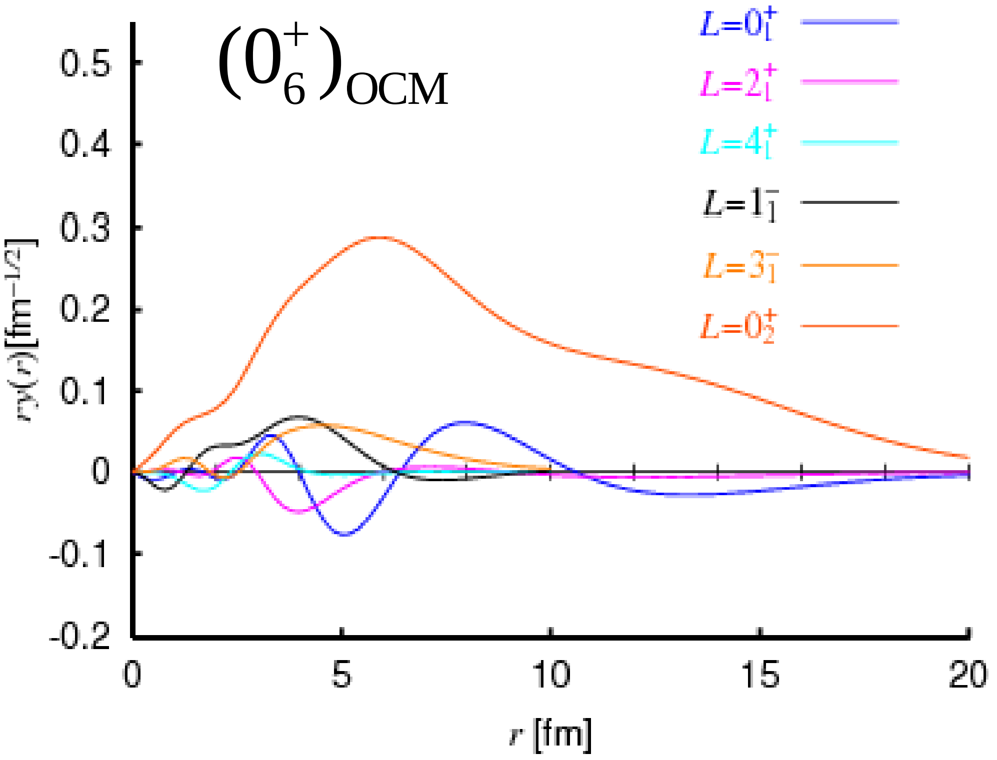
$S_{IL}^2(J=0)=\int dr\left(r\times\mathcal{Y}_{IL,J=0}(r)\right)^2$

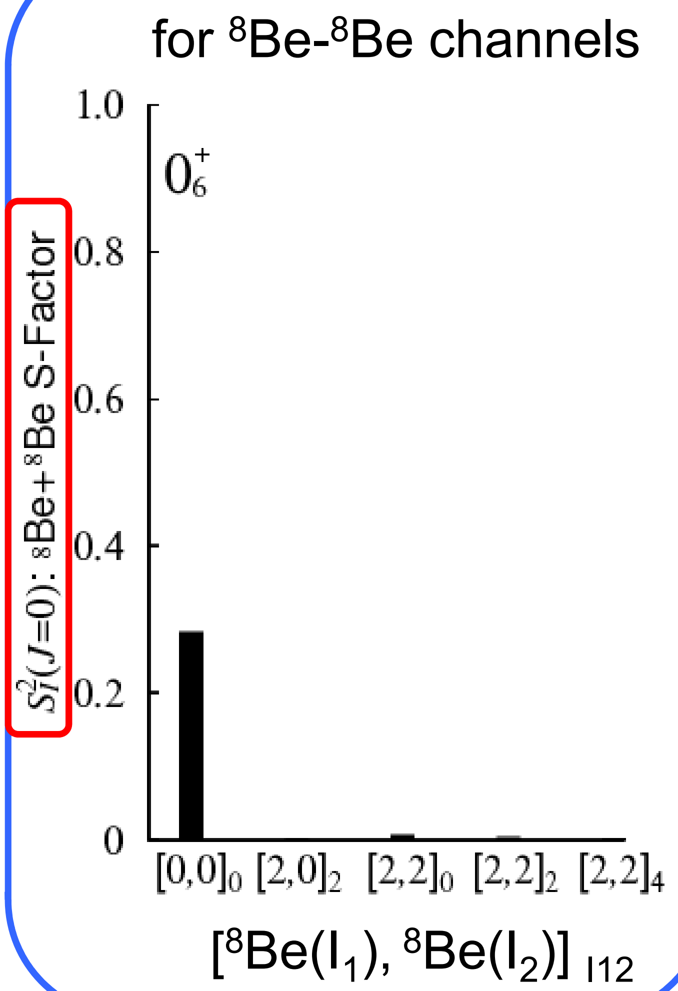


I: Spin of ^{12}C

α + ^{12}C(Hoyle) configuration is dominant.
^{12}C(Hoyle): 3α condensate → **4α condensate**

Reduced width amplitudes of 0_6^+ state obtained with 4α OCM



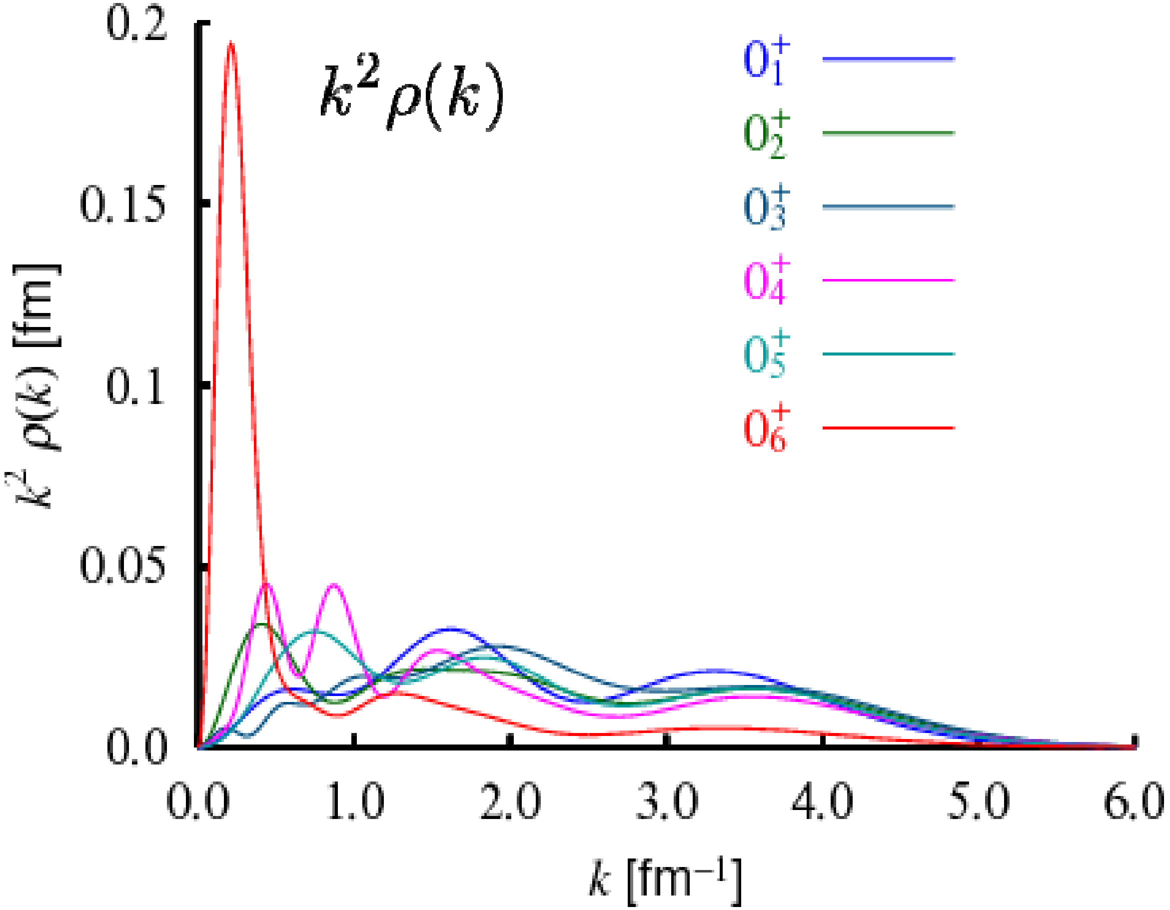


$\alpha + {}^{12}C$(Hoyle) configuration is dominant.
^{12}C(Hoyle): 3α condensate → 4α condensate

Momentum distributions of the α particles

$$\rho(k) = \int d\boldsymbol{r} d\boldsymbol{r}' \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}} \rho(\boldsymbol{r},\boldsymbol{r}') \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}'}}{(2\pi)^{3/2}}$$

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{4}\sum_{i=1}^{4} \langle \Psi_{\mathrm{OCM}}(0_k^+)|\delta(\boldsymbol{r}_i - \boldsymbol{X}_G - \boldsymbol{r}')\rangle\langle\delta(\boldsymbol{r}_i - \boldsymbol{X}_G - \boldsymbol{r})|\Psi_{\mathrm{OCM}}(0_k^+)\rangle$$



$\boldsymbol{r}_i$: coordinate of the i-th α particle

$\boldsymbol{X}_G$: coordinate of total center-of-mass

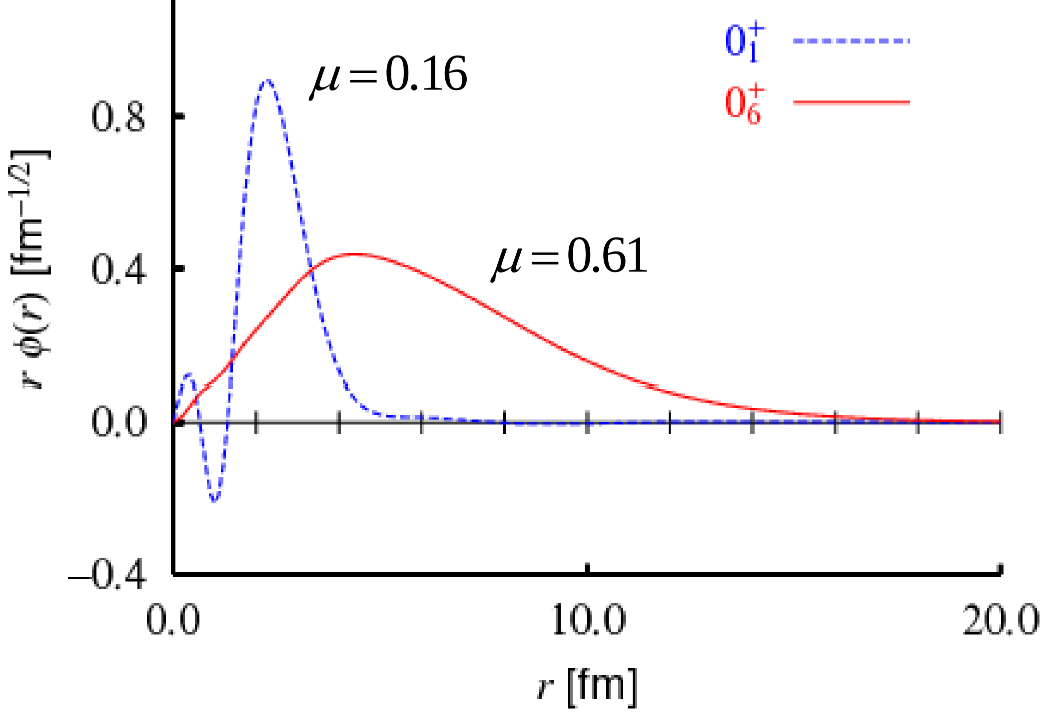
0_6^+ : delta-function-like peak at zero momentum

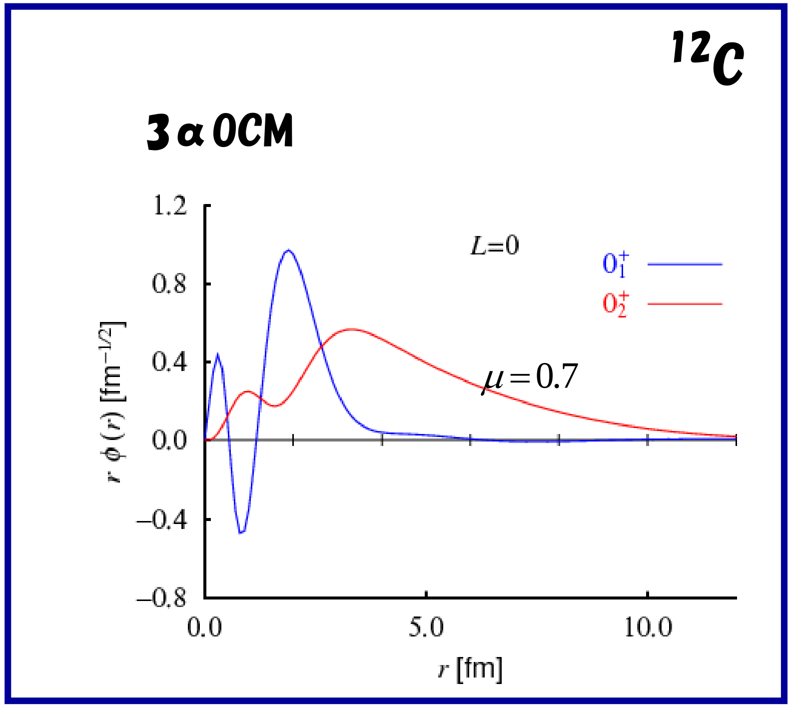
4α condensate state character.

de Broglie w.l. $\lambda = \frac{2\pi}{\sqrt{\langle k^2\rangle}} \geq 20$ fm

Single-α occupancy and single-α orbit for the 0_1^+ and 0_6^+ states
(Only the S orbit (L=0) with the largest occupancy)

$$\int dr' \rho(r, r') \phi(r') = \mu\ \phi(r)$$





Similar to ^{12}C case!

0_6^+ : Large 0S occupancy ! (61%)
Largely extended 0S orbital, large occupancy.
Mean-field-like structure of α particles.

Typical nature of the α condensate!

0_1^+ : α particles are dissolved . Reflecting shell structure of nucleons.
SU(3) configuration : 2S nodal behaviour

Y. F. et al., PRL 101, 081502 (2008).

Alpha decay widths

$\Gamma(0_4^+)_{OCM} \sim 0.8$ MeV
$\Gamma(0_5^+)_{OCM} < 0.2$ MeV
$\Gamma(0_6^+)_{OCM} \fallingdotseq 0.2$ MeV
(calculated based on R-matrix theory)

$$\Gamma = \sum \Gamma_L = \sum P_L \cdot \gamma_L^2(a)$$

$$\gamma_L^2(a) \propto (a\mathcal{Y}_L(a))^2$$

P_L : penetration factor

$\gamma_L^2(a)$: reduced width

a: channel radius

$\Gamma(0_4^+$ at 13.6 MeV$) = 0.6$ MeV
$\Gamma(0_5^+$ at 14.0 MeV$) = 0.19$ MeV
$\Gamma(0_6^+$ at 15.2 MeV$) = 0.17$ MeV

$0_4^+ - 0_6^+$: Consistent with experiment

Y. F. et al., PRC80, 0864326 (2009).

The reason why 0_6^+ is narrow in spite of the high excitation

P_L : depends on decay energy

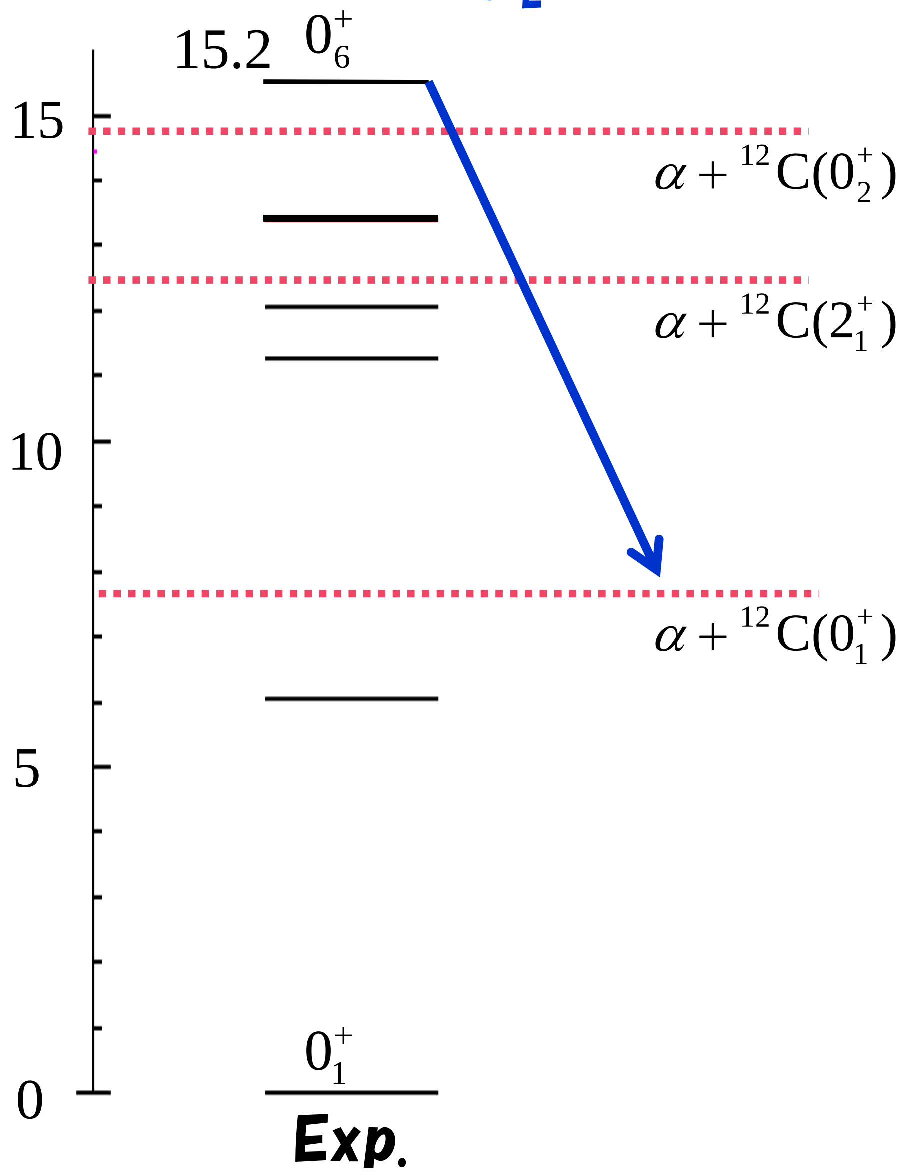
$(\gamma_L)^2$: $\alpha + {}^{12}C(L)$ components

$\alpha + {}^{12}C(L = 0_1^+)$

P_L : Large

$(\gamma_L)^2$: Small

$\Gamma_L = P_L (\gamma_L)^2$: Suppressed



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0_6^+ : Small width, quasi stable

$\Rightarrow$ M. Itoh (CYRIC)

P_L : depends on decay energy

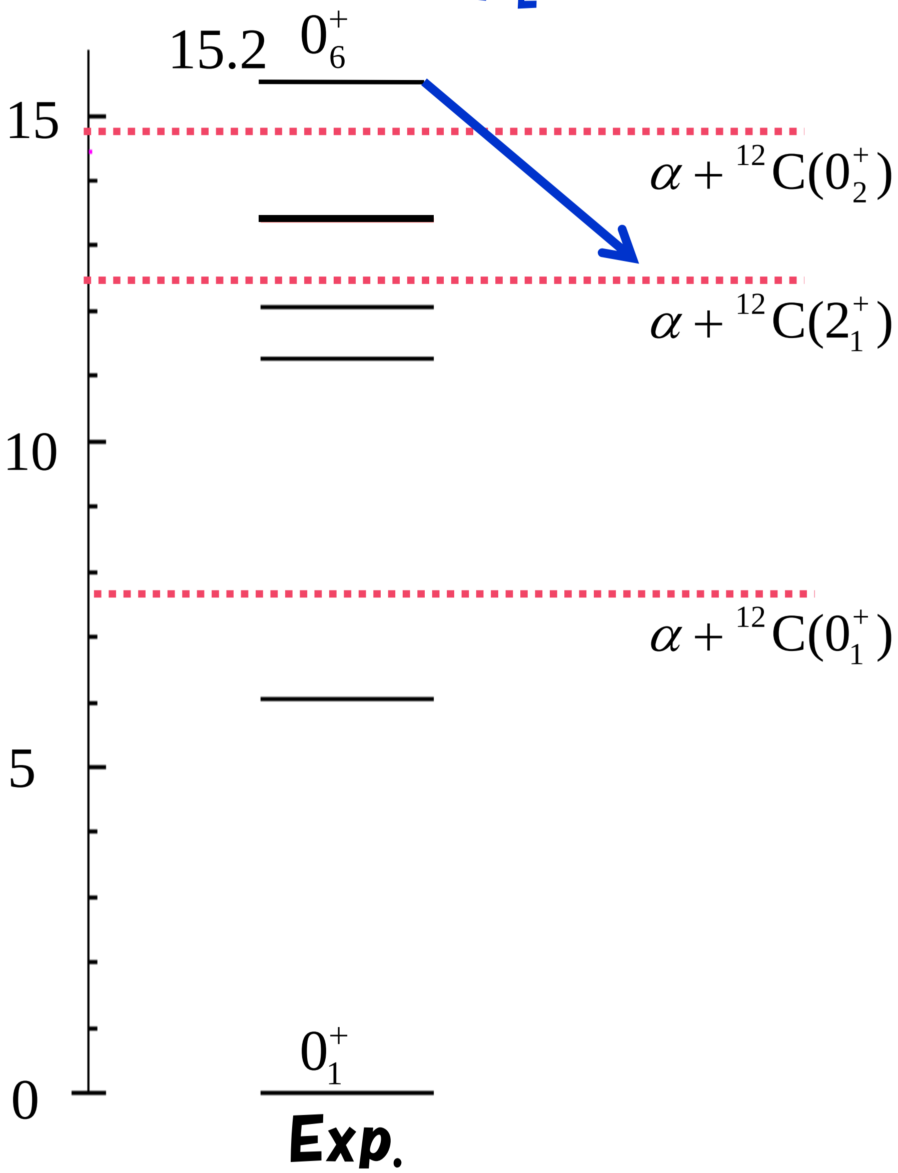
$(\gamma_L)^2$: $\alpha + {}^{12}C(L)$ components

$\alpha + {}^{12}C(L=2_1^+)$

P_L : Medium

$(\gamma_L)^2$: Small

$\Gamma_L = P_L\,(\gamma_L)^2$: Suppressed



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0_6^+ : Small width, quasi stable

⇒ M. Itoh (CYRIC)

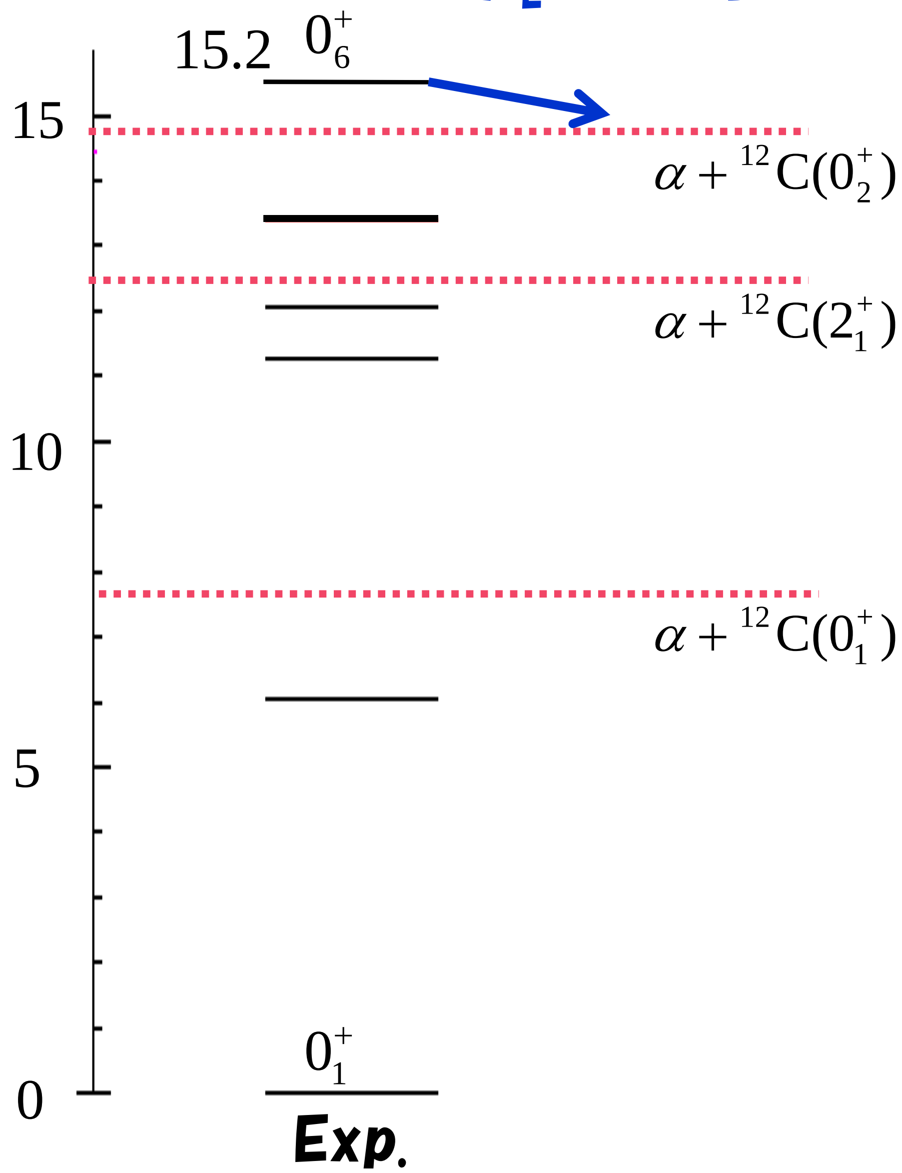
0_6^+ : Decays into all channels are suppressed!

P_L : depends on decay energy

$(\gamma_L)^2$: $\alpha + {}^{12}C(L)$ components

$\alpha + {}^{12}C(L=0_2^+)$
P_L : Small
$(\gamma_L)^2$: Large

$\Gamma_L = P_L\,(\gamma_L)^2$: Suppressed



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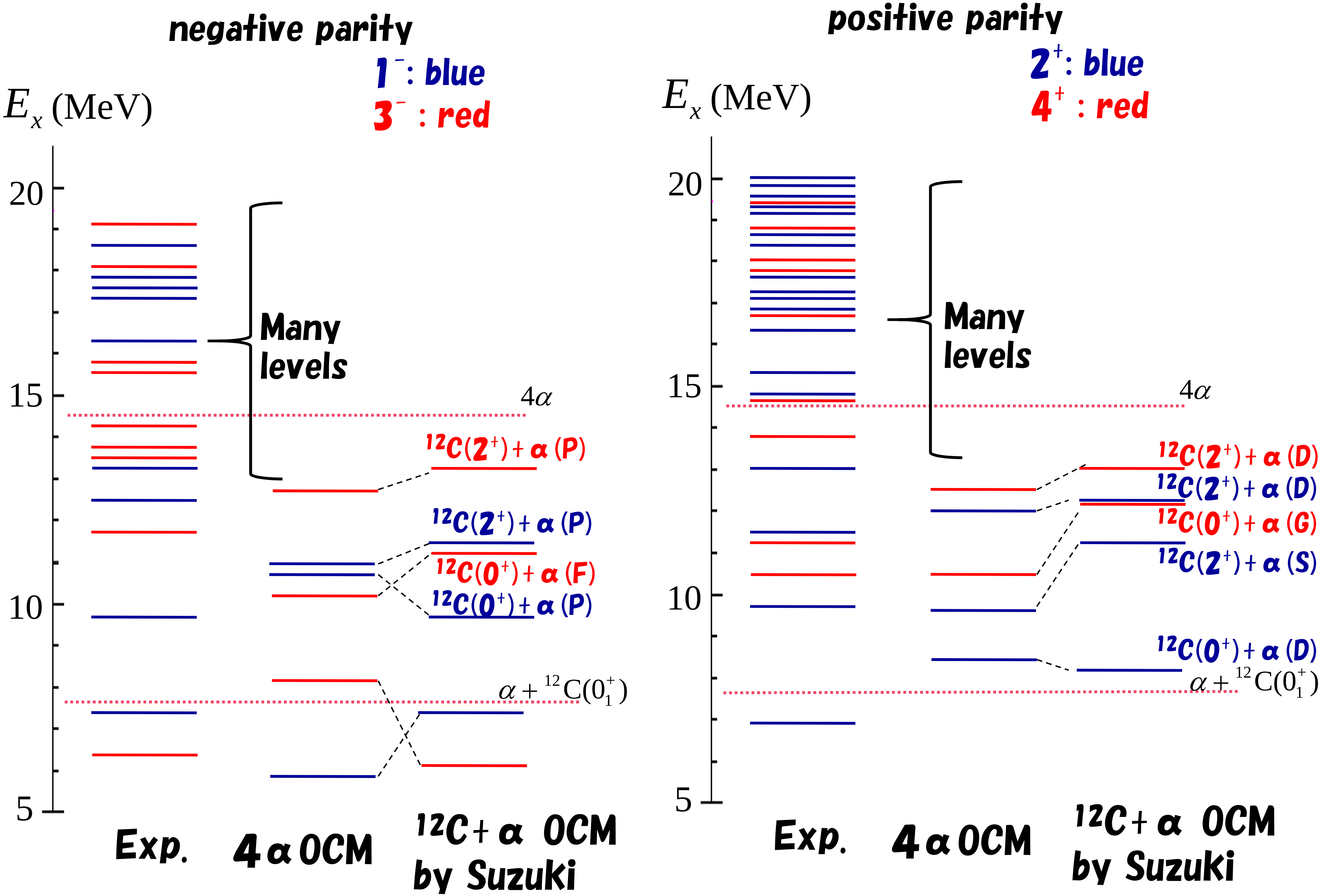
a: channel radius

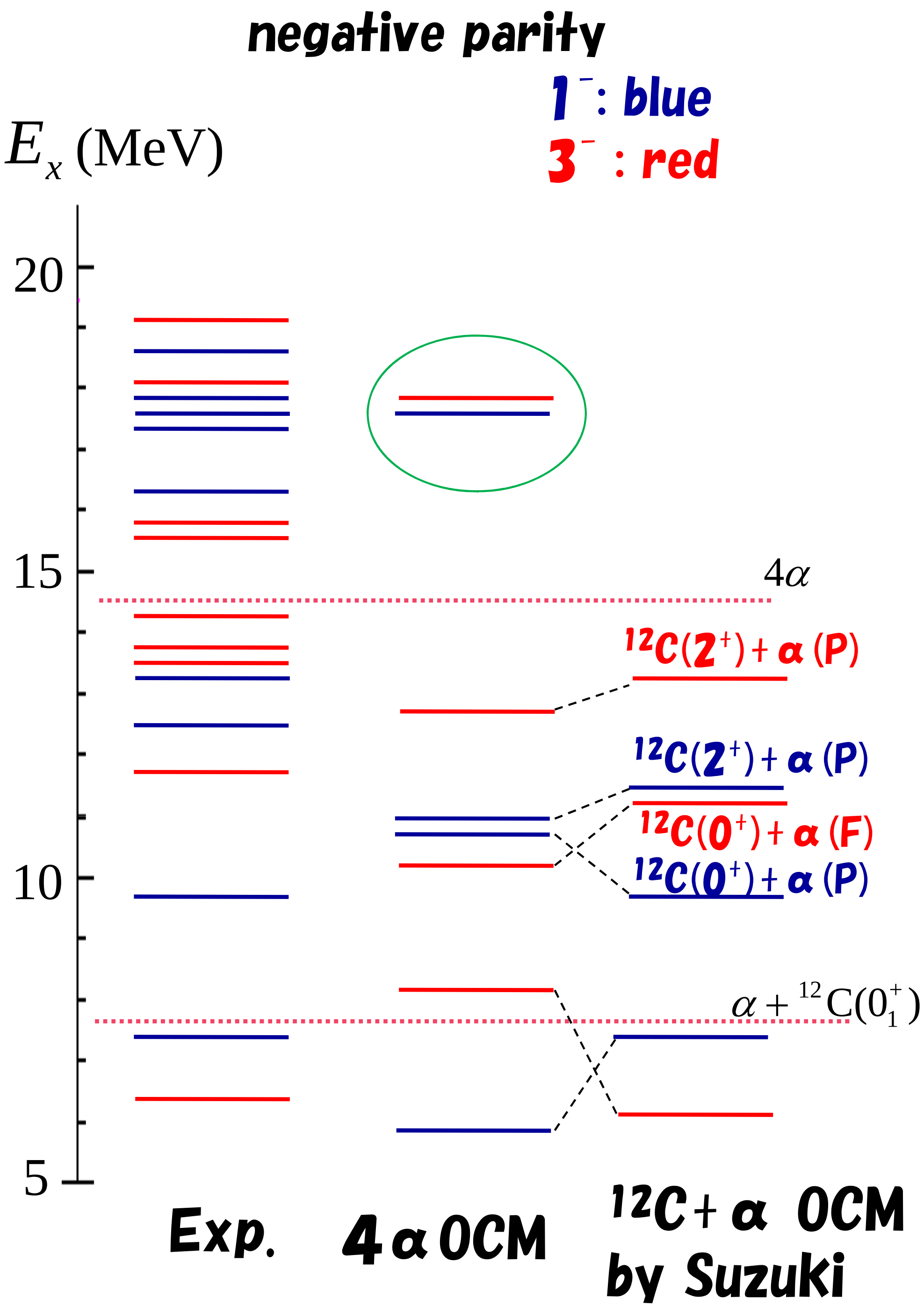
$\Gamma(0_4^+$ at 13.6 MeV$) = 0.6$ MeV
$\Gamma(0_5^+$ at 14.0 MeV$) = 0.19$ MeV
$\Gamma(0_6^+$ at 15.2 MeV$) = 0.17$ MeV

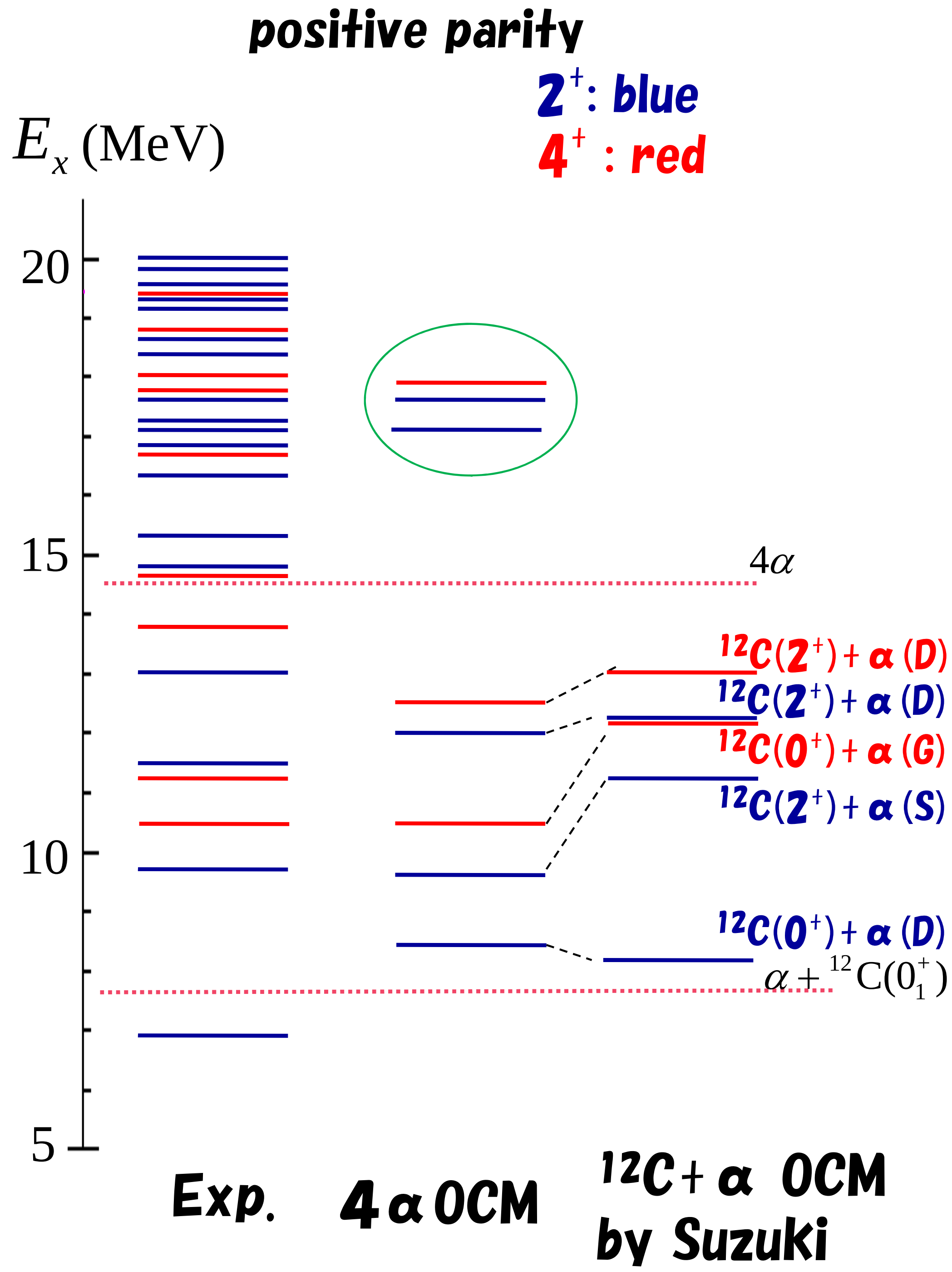
In heavier systems, the analogue states may survive with long-lives(narrow width)!

Energy spectra for non-zero spins(preliminary)

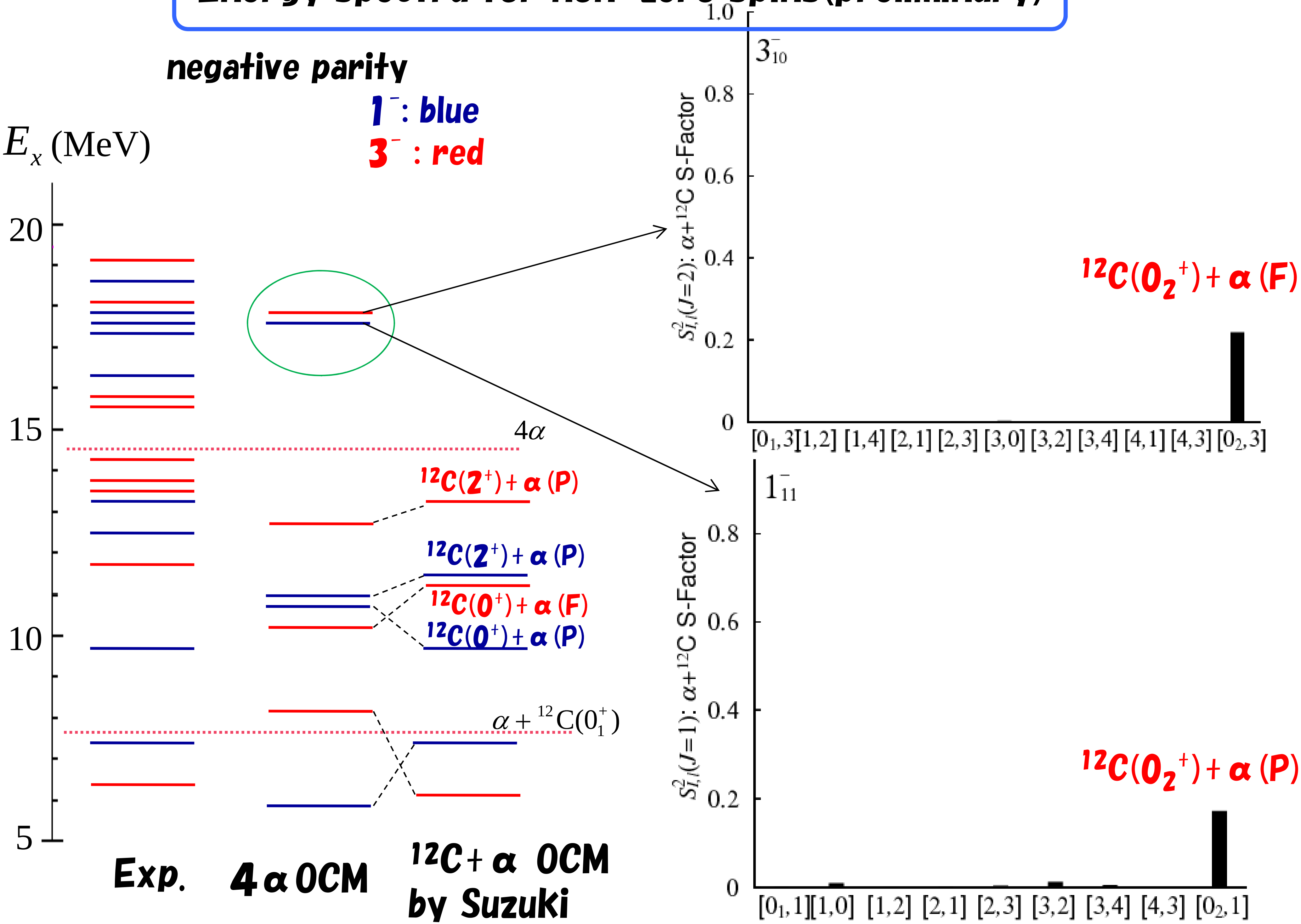


Energy spectra for non-zero spins(preliminary)

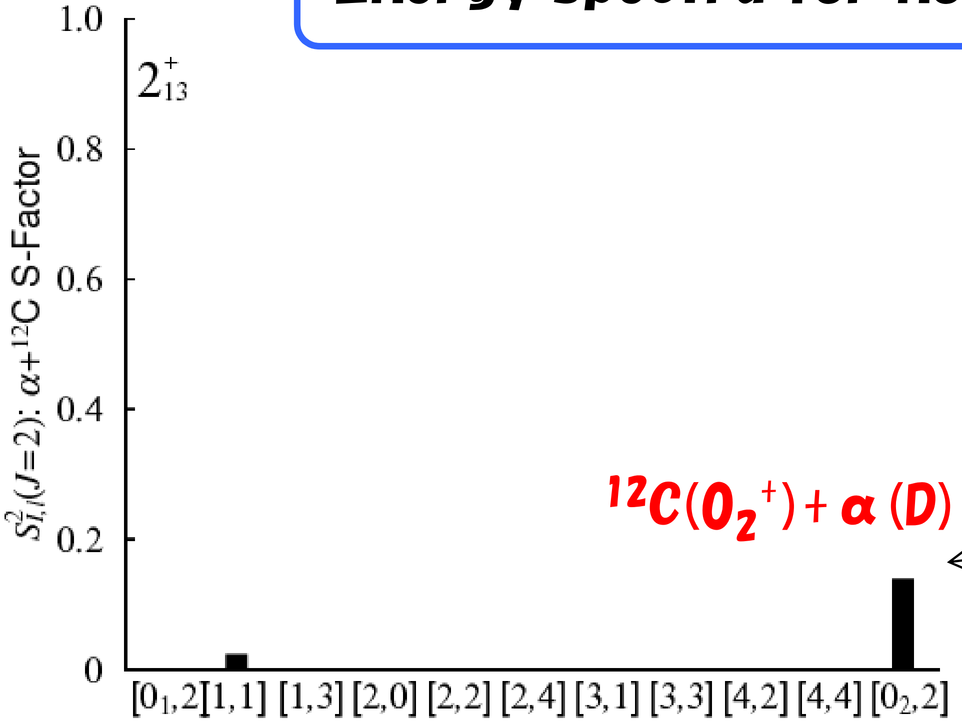


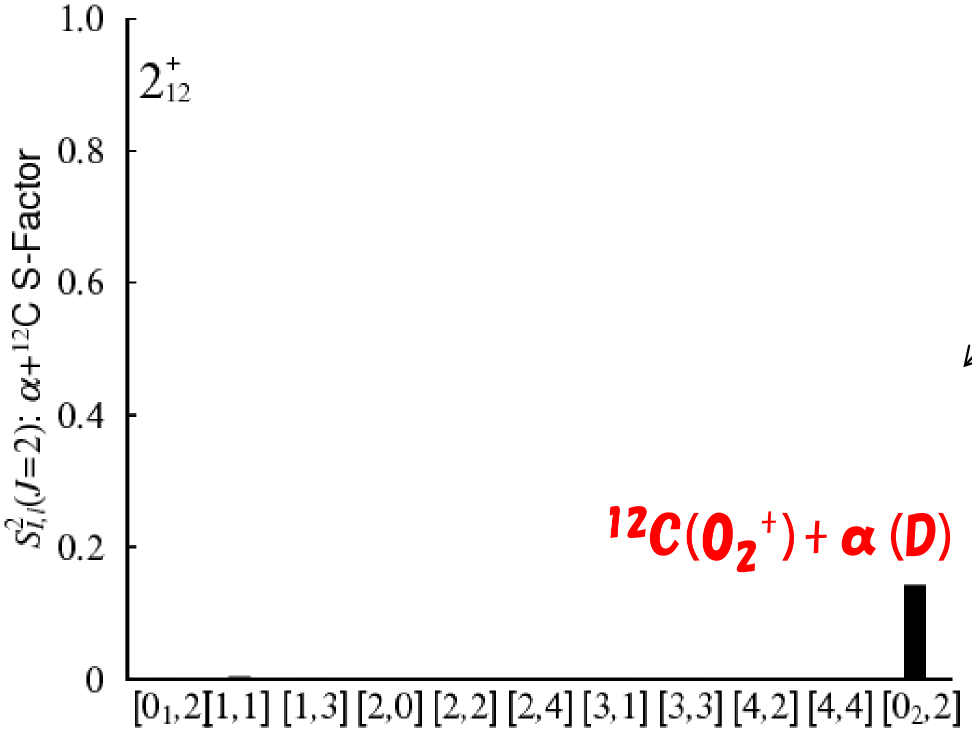


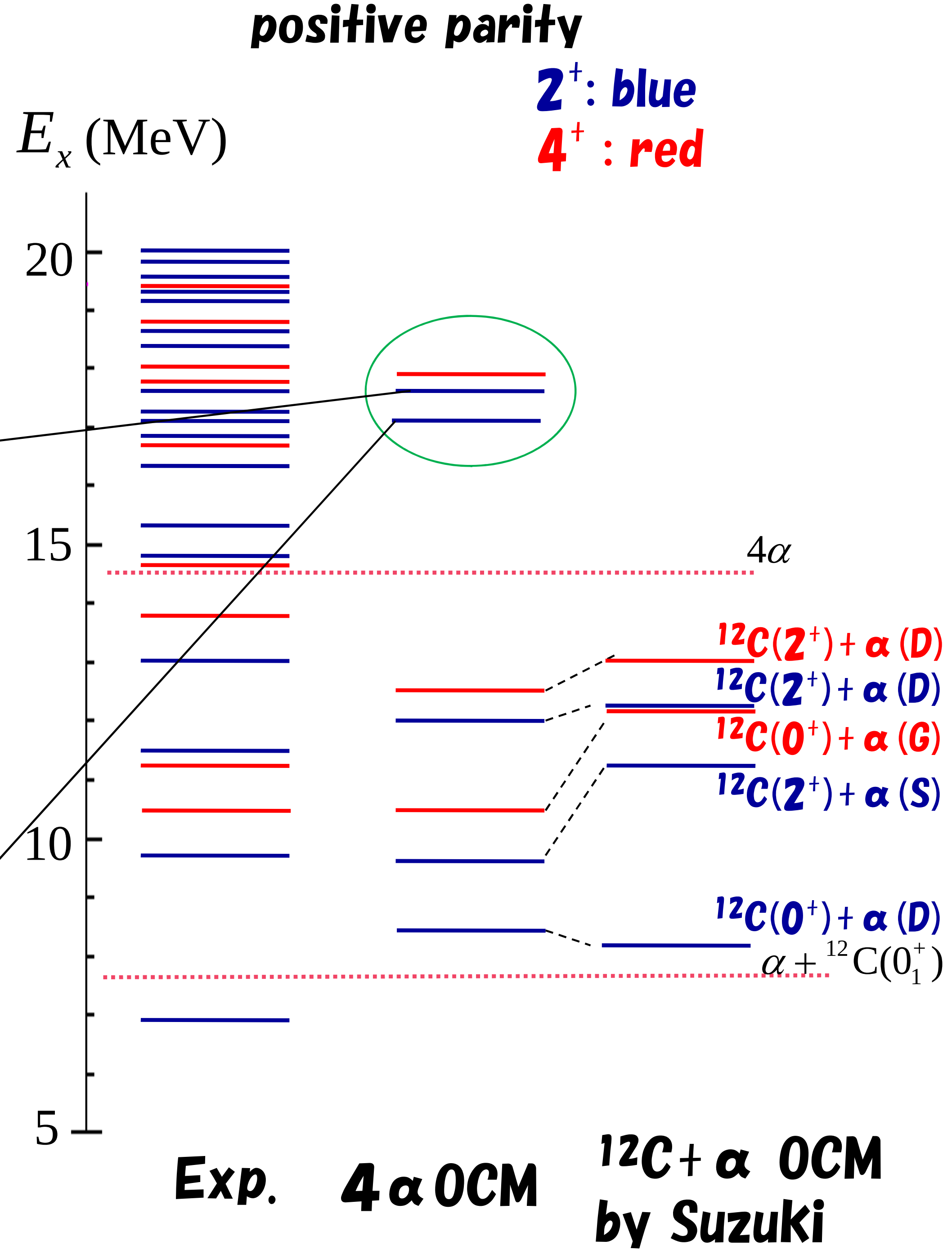
Energy spectra for non-zero spins(preliminary)



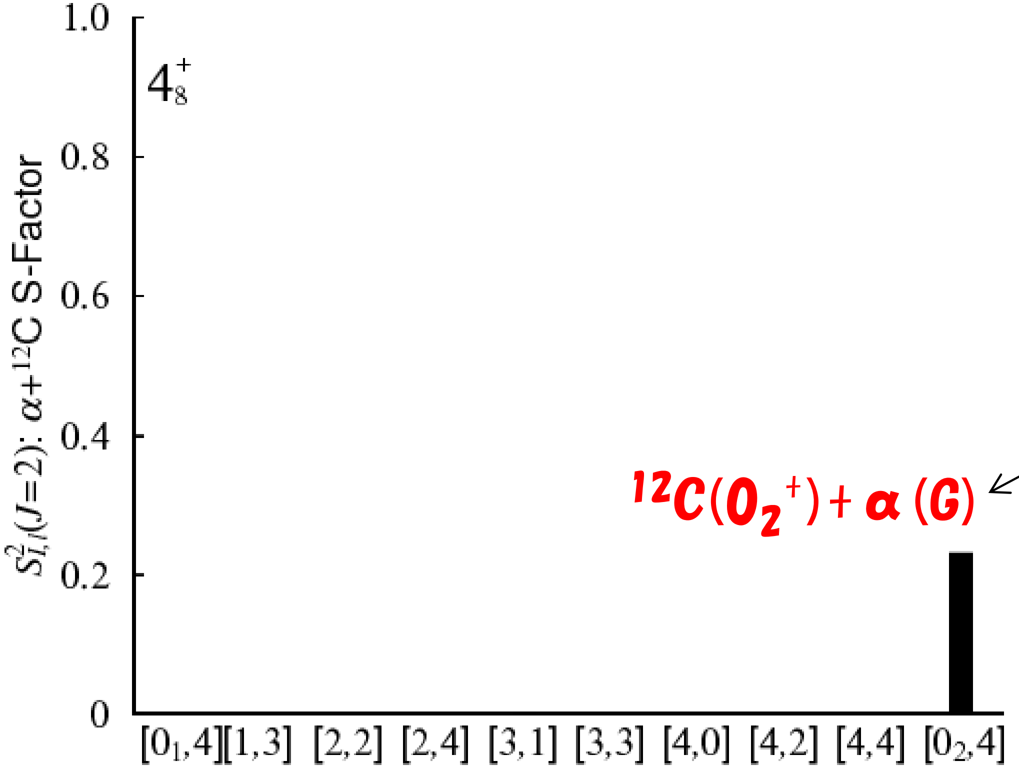
Energy spectra for non-zero spins(preliminary)

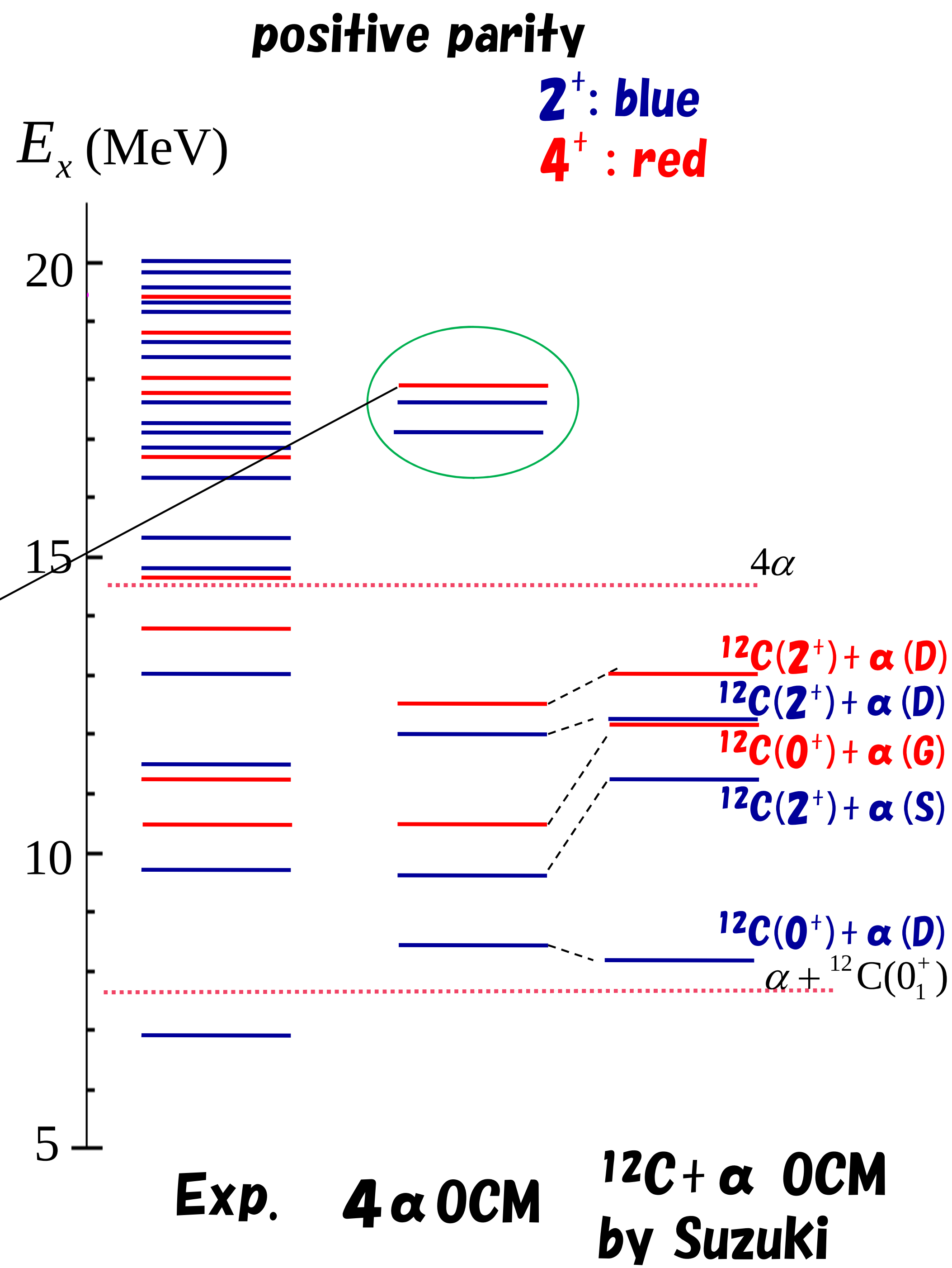






Energy spectra for non-zero spins(preliminary)

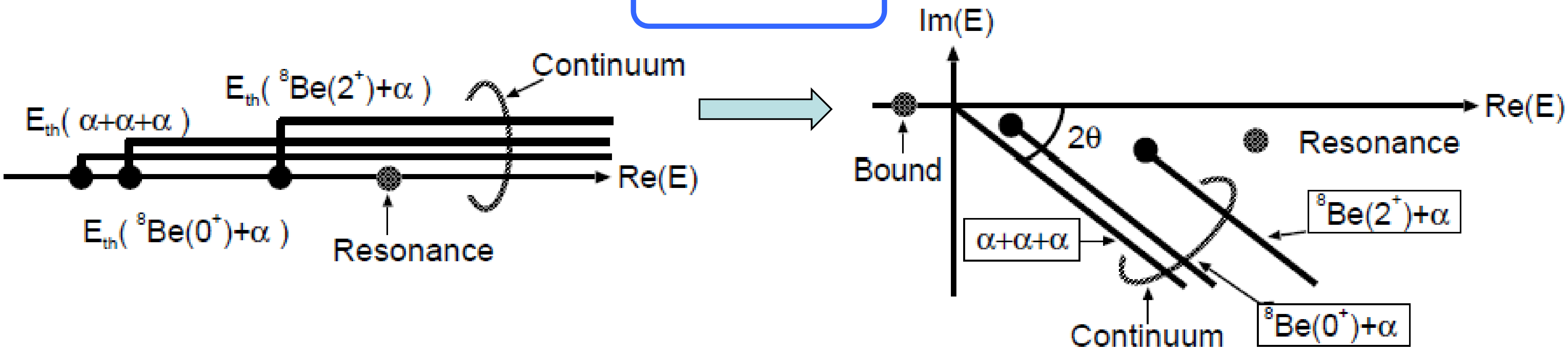




Complex Scaling Method(CSM)

Unique way to handle many-body resonances

$$\boldsymbol{r} \rightarrow e^{i\theta}\boldsymbol{r}$$



•**Resonances appear as poles in complex energy plane.**

$$E \rightarrow E - \frac{i}{2}\Gamma$$

•**Calculations can be done such as in the bound state approximation.**
•**Boundary conditions of the bound, resonance and continuum states are replaced with those in the bound states.**

Complex Scaling Method(CSM)

To be done・・・

・Threshold states (continuums) should be described well.

・Diagonalization of complex non-hermitian (dense) matrix

Appearing thresholds

(below the 4alpha threshold)

$^{12}C(0_1^+)+\alpha$, $^{12}C(2_1^+)+\alpha$,

(around the 4alpha threshold)

4α, $^8Be(0_1^+)+2\alpha$, $^8Be(0_1^+)+{}^8Be(0_1^+)$, $^{12}C(0_2^+)+\alpha$,

$^8Be(2_1^+)+2\ \alpha$, $^{12}C(2_2^+)+\alpha$, $^{12}C(0_3^+)+\alpha$, $^{12}C(3_1^-)+\alpha$, ・・・

All these continuums should sufficiently be included in the model space.

$(\nu^{-2}=0.5\sim 100\ [\mathrm{fm}])$

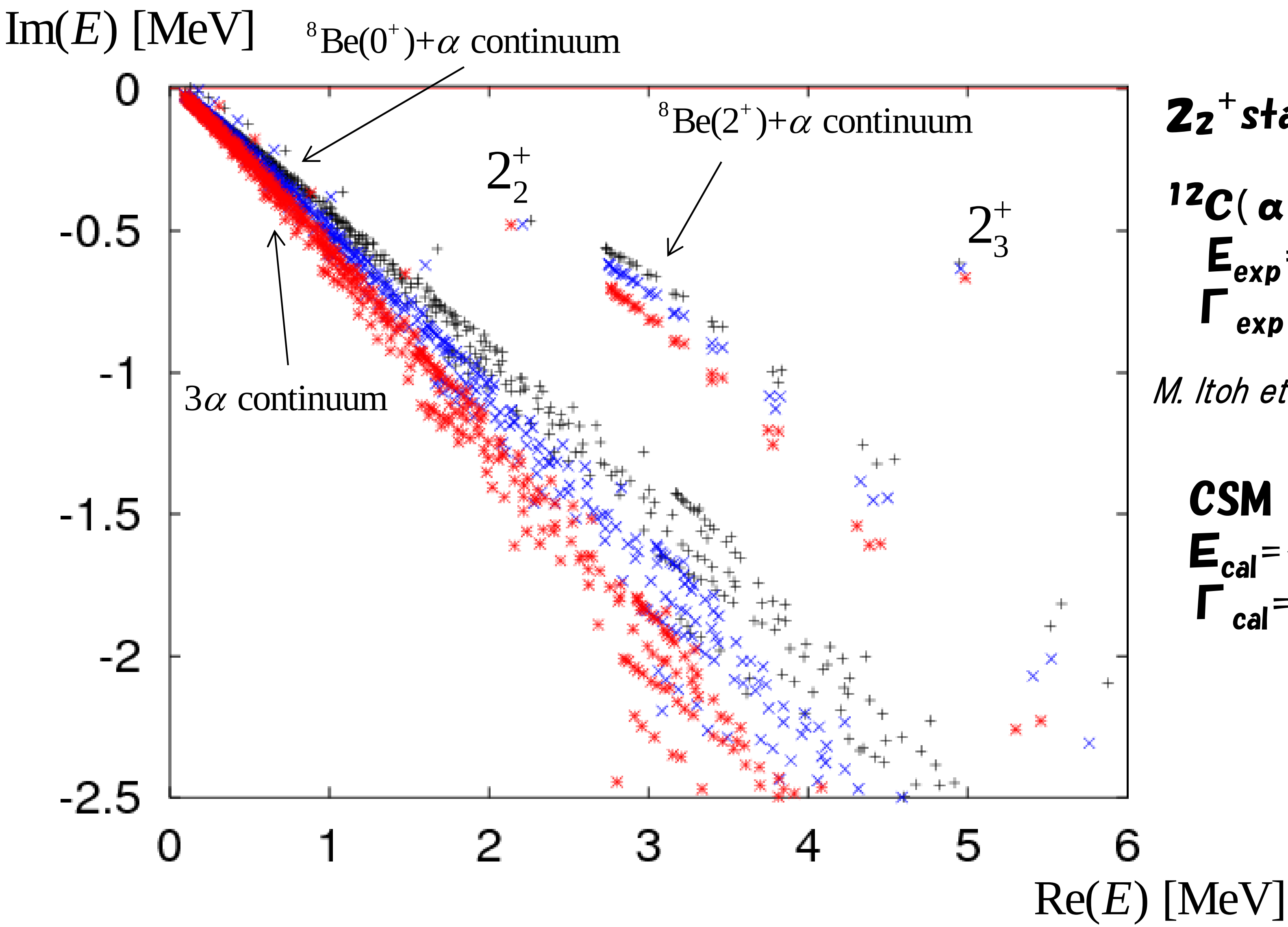
13, 4 gaussian bases for one Jacobi coordinate $\rightarrow 14^3=2{,}744$

$[[l_3,l_2]_{l_{32}},l_1]_J : l=0,1,2,3,4 \rightarrow\sim 20$ channels ($J^\pi=0^+$ case)

totally about 50000 dimensions

Example of CSM(^{12}C、 2^+ resonances)

$$\boldsymbol{r} \to e^{i\theta}\boldsymbol{r}\ (\theta = 14.4^\circ\,(\text{black}), 16.2^\circ\,(\text{blue}), 18.0^\circ\,(\text{red}))$$



2_2^+ state:

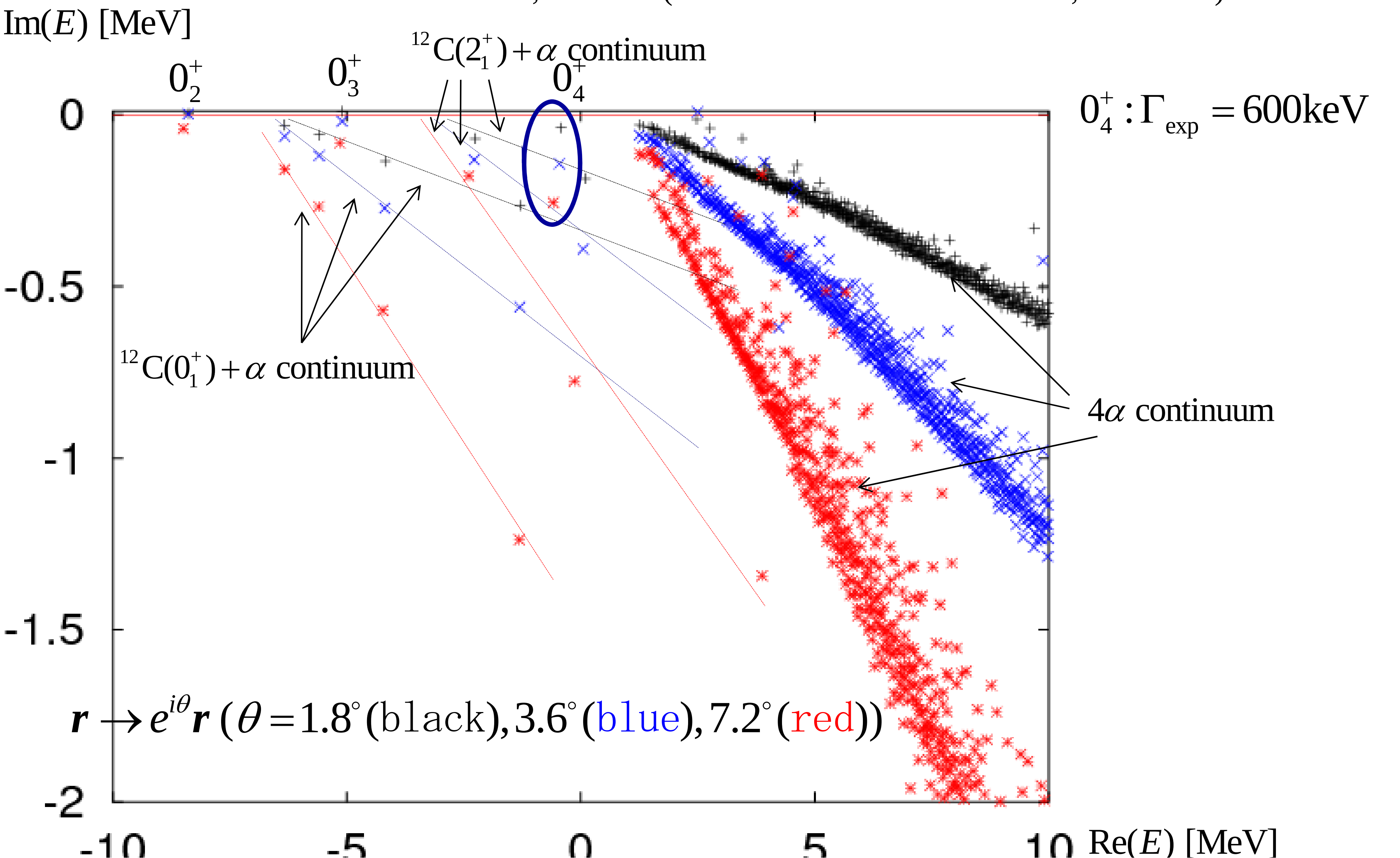
^{12}C(α, α')
E_{exp}=2.6(3) MeV
Γ_{exp}= 1.0(3) MeV

M. Itoh et al., NPA 738, 268 (2004).

CSM
E_{cal}=～2.1 MeV
Γ_{cal}=～1.0 MeV

CSM(^{16}O、0^+ resonances)(preliminary)

($\nu^{-2} = 0.5 \sim 15$ [fm]) 10 gaussians for one Jacobi $\to 10^3$
relative angular momenta :16 channels
16,000dim.(matrix elements :208CPU's, 10 hours)



Summary

Investigation of loosely bound alpha gas states in finite nuclei

- It is well established that the Hoyle state is the 3α condensate state.

- More α-particle condensate states very likely to exist.

  Analogue state in ^{16}O to the Hoyle state (found with 4α OCM calc.)
  as the sixth 0^+ state
  Assigned to 15.2 MeV state?
  More experimental information is needed.

- Hoyle analogs for non-zero spin states are promising.
  (spin excitations of 4α condensates)

  Problem is continuum mixing

On going issue: beyond bound state approximation
4-alpha CSM (Complex Scaling Method) with T2K-Tsukuba (up to 512cpu's)

Larger model space should be taken
(in particular much more Gaussian bases: ν^{-2}(fm) should be enlarged
up to 100 fm)
Then, total dim. are about 50,000 dim. (eigenvalue problem of non-hermitian matrices)
very technically, looking for a subroutine of diagonalization (not prepared in ScaLAPACK!)

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