Description of many-body scattering states using complex scaling method

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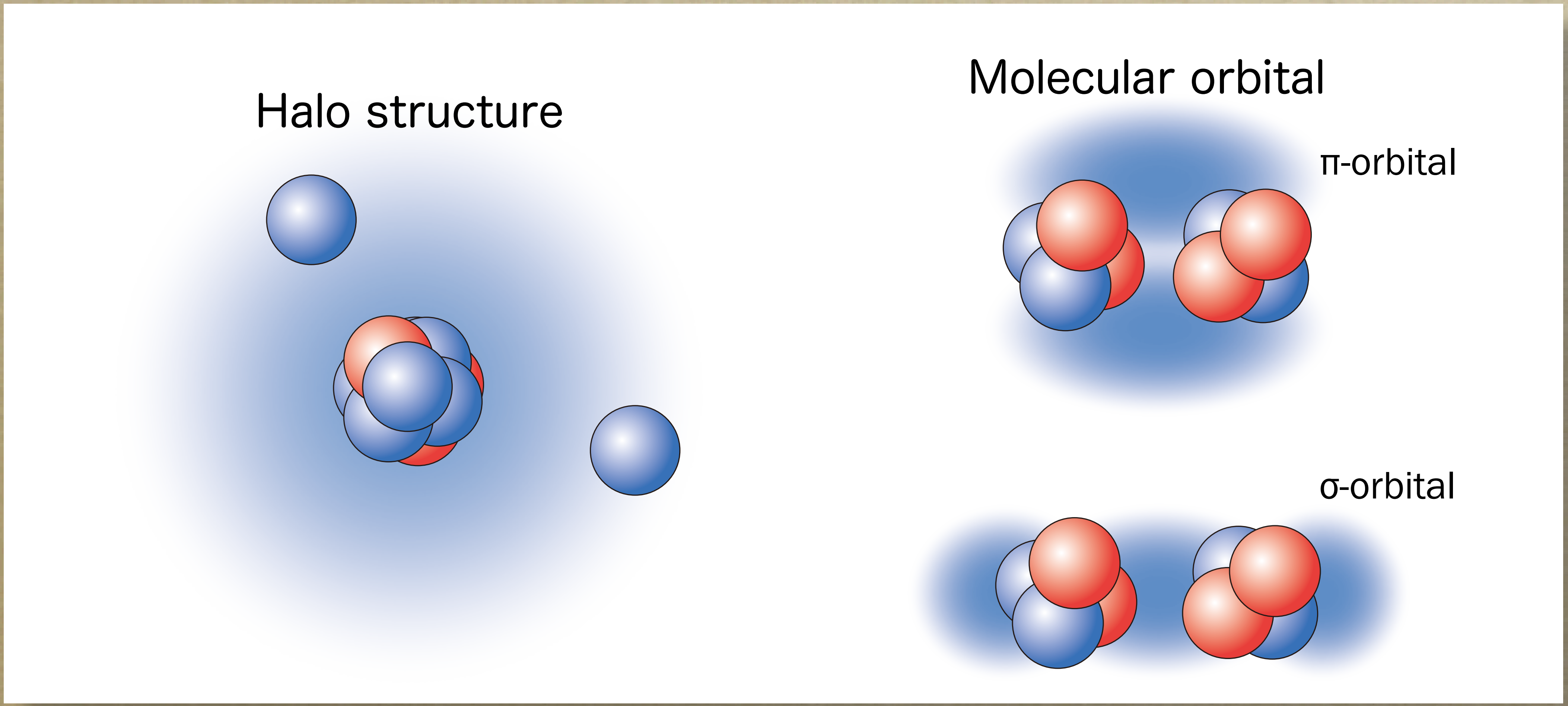
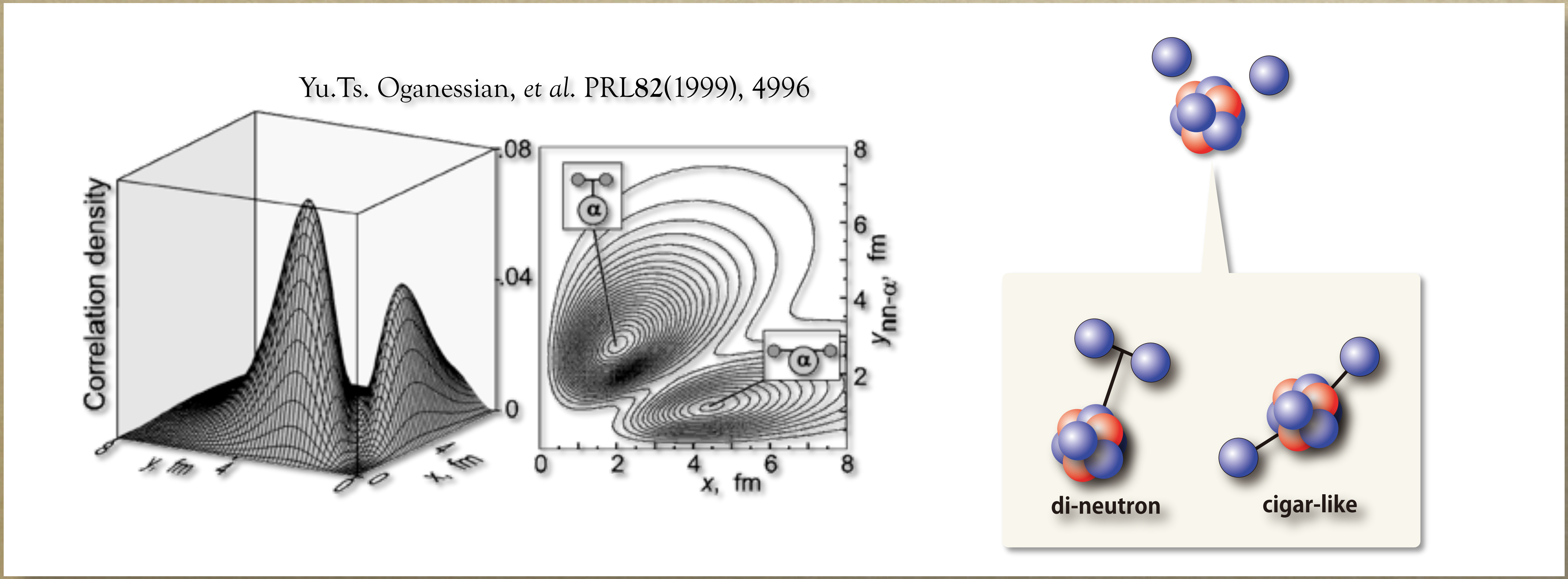
Recent developments in RI beam experiments

- Recent developments in RI beam experiments provide us with the exciting opportunities to investigate the exotic structure in neutron-rich nuclei, such as:
  - Neutron halo structures,
  - Molecular and Atomic orbitals,
  - Violation of magic numbers, and etc.



Two-neutron halo structure

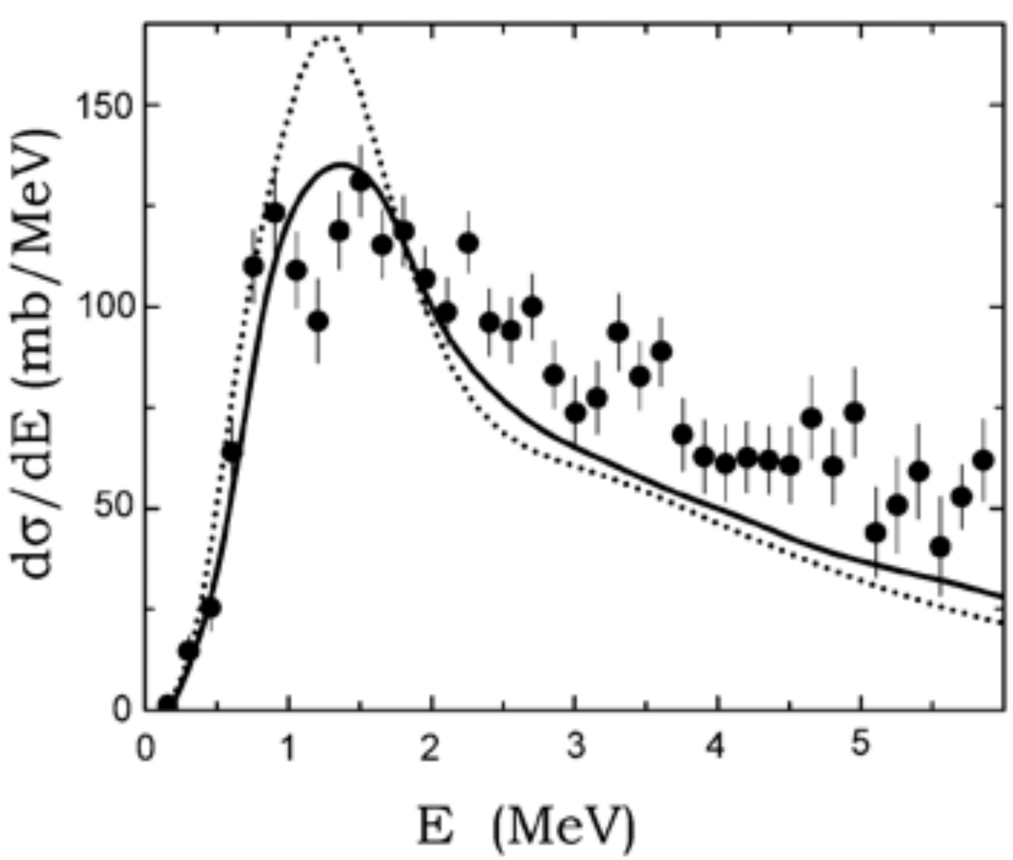
- In particular, the two-neutron halo structure is one of the interesting topics.
  - In the two-neutron halo nuclei, two halo neutrons are weakly bound by the core nucleus and spread out beyond the core.
  - In ^{6}He and ^{11}Li, the exotic n-n correlation, the so-called “dineutron”, has been suggested from the calculation using the core+n+n three-body model.



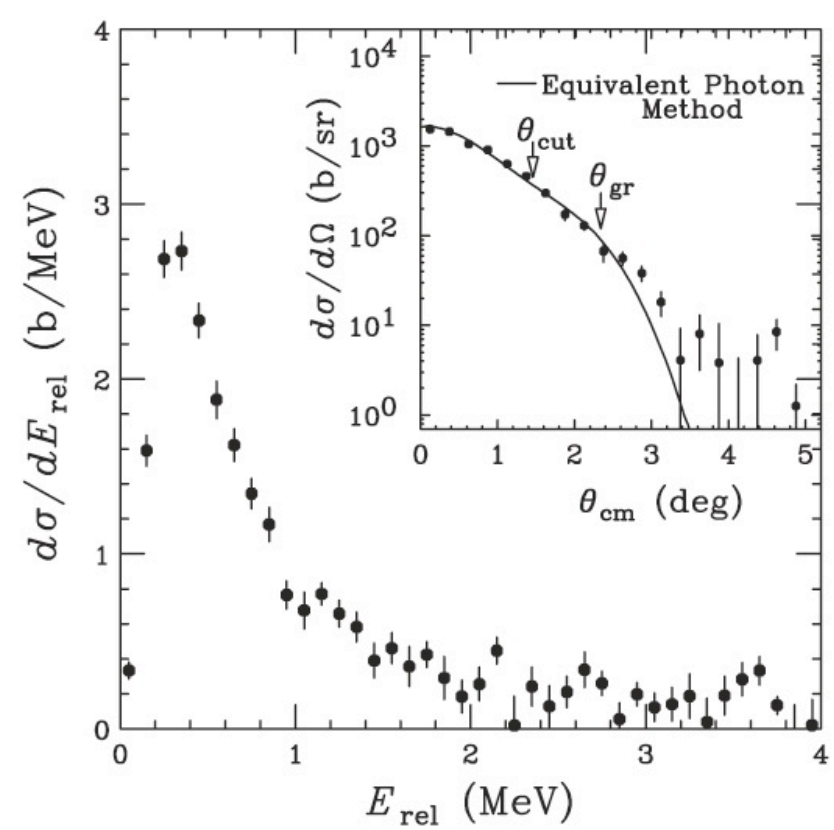
Two-neutron halo structure

- Experimentally, two-neutron halo structure has been investigated by using the Coulomb breakup reactions.
  - The characteristic low-lying enhancement has been observed in the Coulomb breakup cross sections.
  - This enhancement is responsible to the weakly-bound halo neutrons?

^{6}He breakup: T. Aumann et al., PRC 59, 1252 (1999).

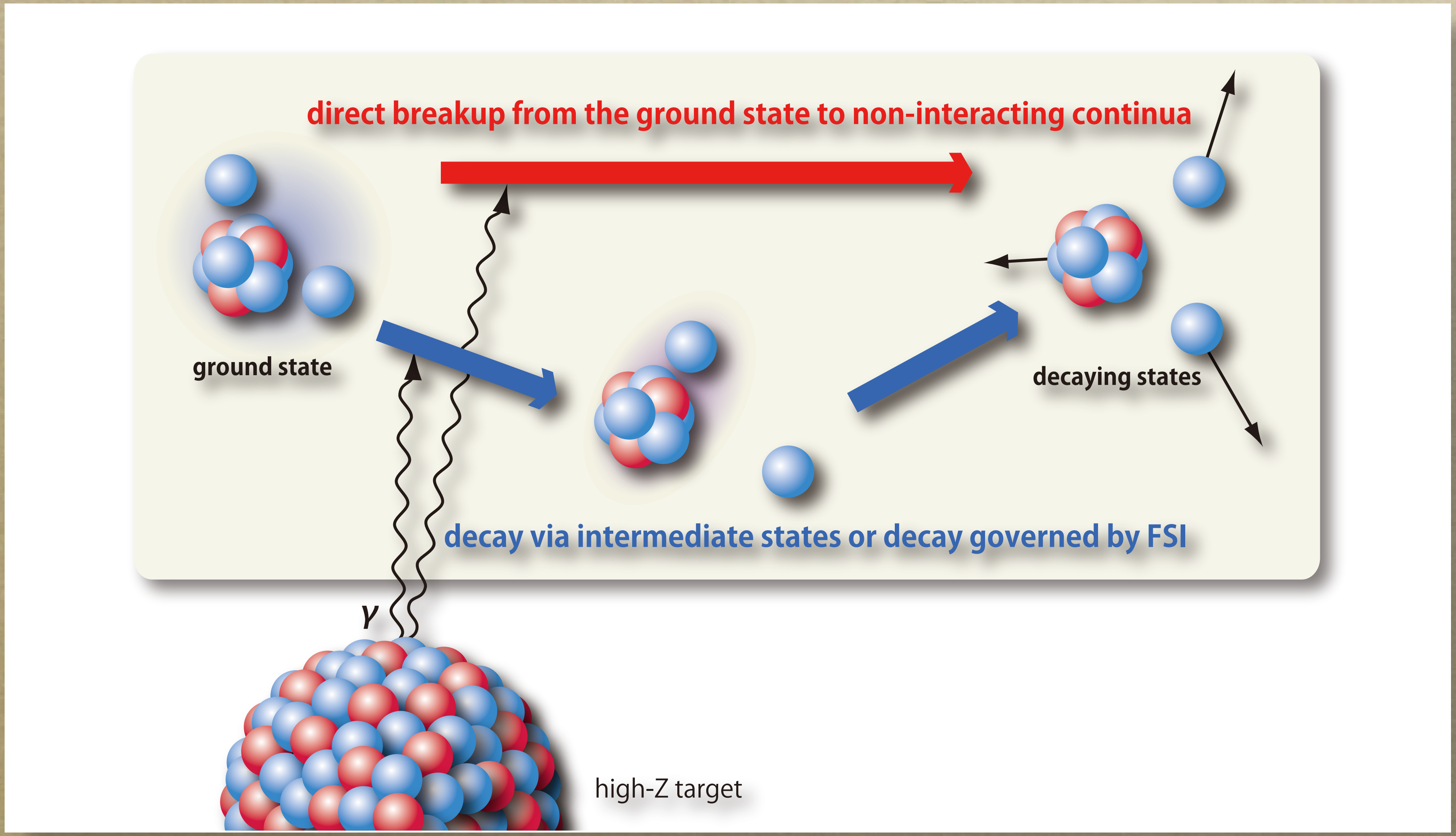


^{11}Li breakup: T. Nakamura et al., PRL 96, 252502 (2006).



To investigate the two-neutron halo nuclei

- Two-neutron halo nuclei are weakly-bound systems, and hence, they are broken up to the core+n+n three-body scattering states.
  - To investigate the structure of the two-neutron halo nuclei, it is necessary to describe the scattering states of the core+n+n system and to understand the mechanism of the three-body breakups.



Theoretical attempt to describe the breakup of 2n halo

- There are several theoretical works to describe the three-body scattering states and to investigate the breakup mechanism of two-neutron halos.
  - For examples, Hyperspherical Harmonics, Faddeev method, and etc.

In this talk, I would like

- to introduce the method to describe the many-body scattering state using the complex scaling method, and then,
- to show its applications to the scattering problems for A=6 systems.

Complex scaling method
and
Complex-scaled solutions
of the Lippmann-Schwinger equation

Complex scaling method (CSM)

- CSM is a powerful tool to investigate the many-body resonances on the same footing as the bound-state case.
  - In CSM, the relative coordinates and momenta are transformed as follows.

$$U(\theta) : \mathbf{r} \to \mathbf{r}e^{i\theta}, \qquad \mathbf{k} \to \mathbf{k}e^{-i\theta}$$

  - Then, we obtain the complex-scaled Schroedinger equation as

$$\hat{H}\chi(\mathbf{r}) = E\chi(\mathbf{r}) \to \hat{H}^\theta \chi^\theta(\mathbf{r}) = E^\theta \chi^\theta(\mathbf{r})$$

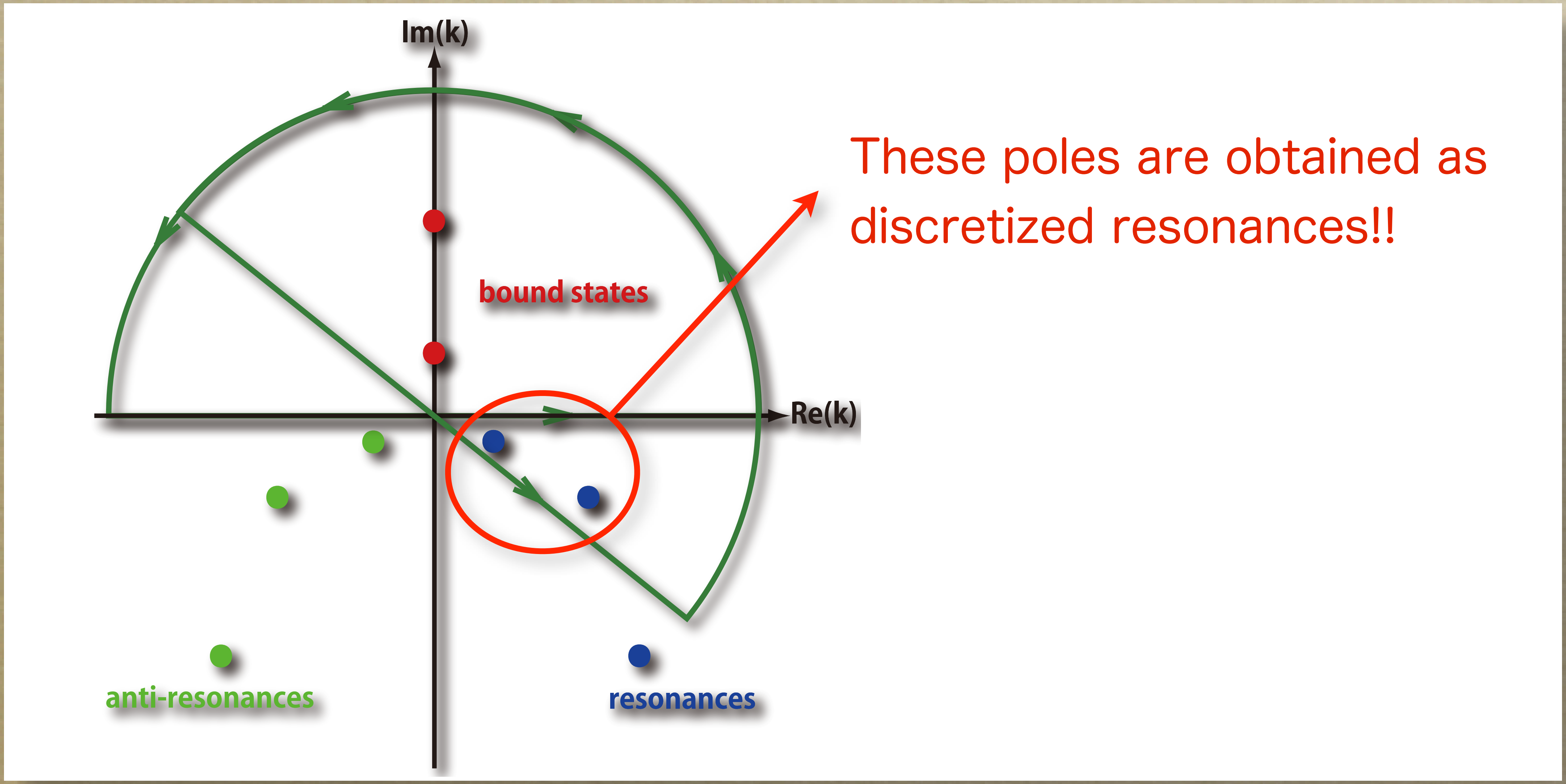
  - Here, the complex-scaled wave function and Hamiltonian are given as

$$\chi^\theta(\mathbf{r}) = U(\theta)\chi(\mathbf{r}) = e^{\frac{3}{2}i\theta}\chi(\mathbf{r}e^{i\theta})$$

$$\hat{H}^\theta = U(\theta)\hat{H}U^{-1}(\theta)$$

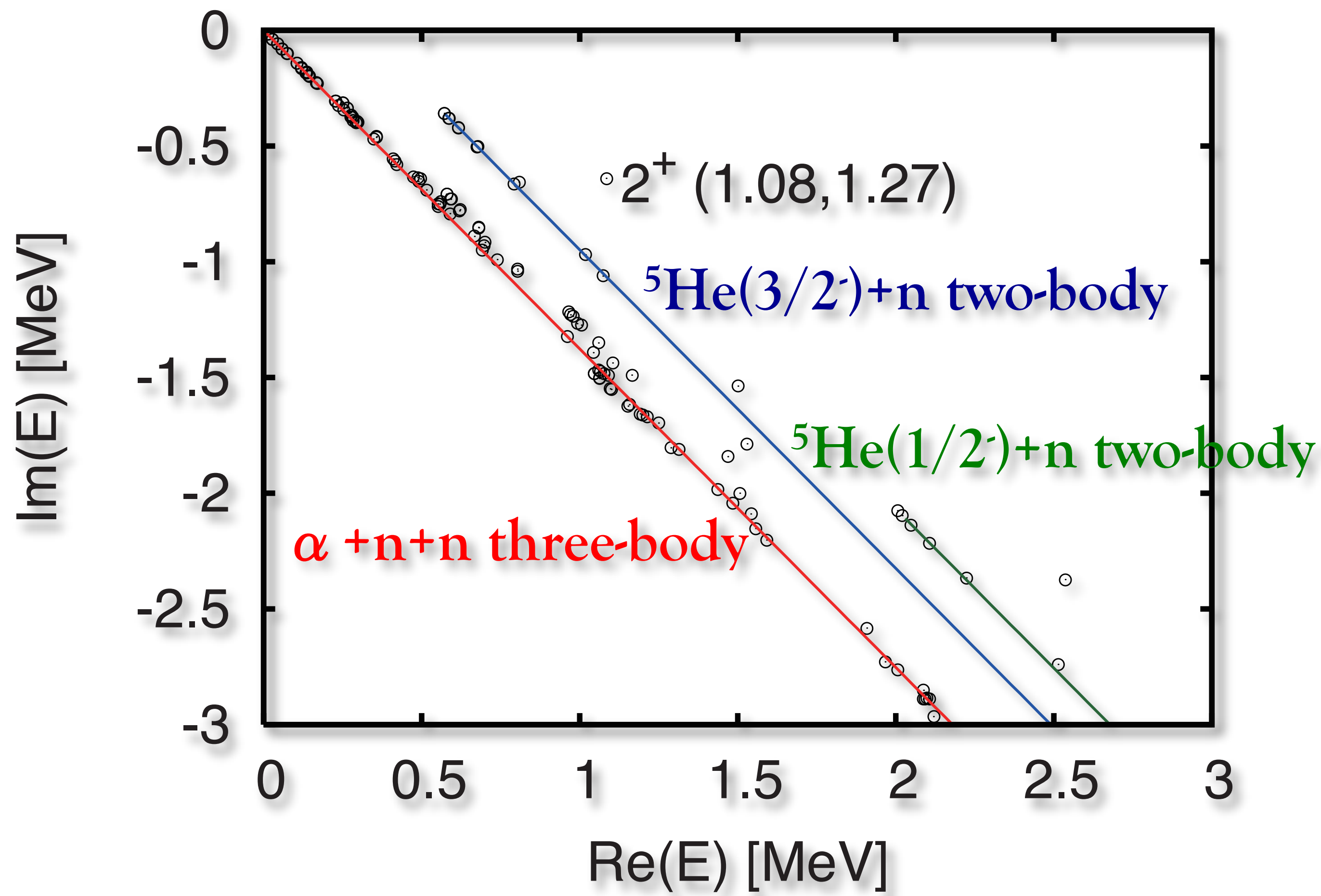
Complex scaling method (CSM)

- Under the transformation in CSM, the resonance poles are obtained as the discretized states as well as the bound states.
  - By rotating the contour of the integral pass in the momentum plane, we resonance poles in the S-matrix are found as the residues.



The obtained spectra in CSM

- In CSM, the energy eigenvalues are obtained as complex numbers, and their imaginary parts impose the outgoing boundary conditions.
  - The resonance has the energy of E_r - Γ/2.
  - The continuum states are classified into several families corresponding to the decaying channels.



ex) obtained spectra of 2^+ states of ^6He

Description of scattering states with CSM

- The behavior of the energy eigenvalues in CSM indicates that CSM is useful to describe the many-body scattering states.
- We develop the method to describe the many-body scattering states by combining CSM with the Lippmann-Schwinger equation.
  - We start with the formal solutions of the Lippmann-Schwinger equation.

$$\Psi^{(\pm)} = \Phi_0 + \lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} \pm i\varepsilon} \hat{V} \Phi_0$$

  - To take into account the outgoing boundary conditions in the Green's function, we employ the complex-scaled Green's function, which is given as

$$\lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} + i\varepsilon} = U^{-1}(\theta) \frac{1}{E - \hat{H}^{\theta}} U(\theta)$$

Description of scattering states with CSM

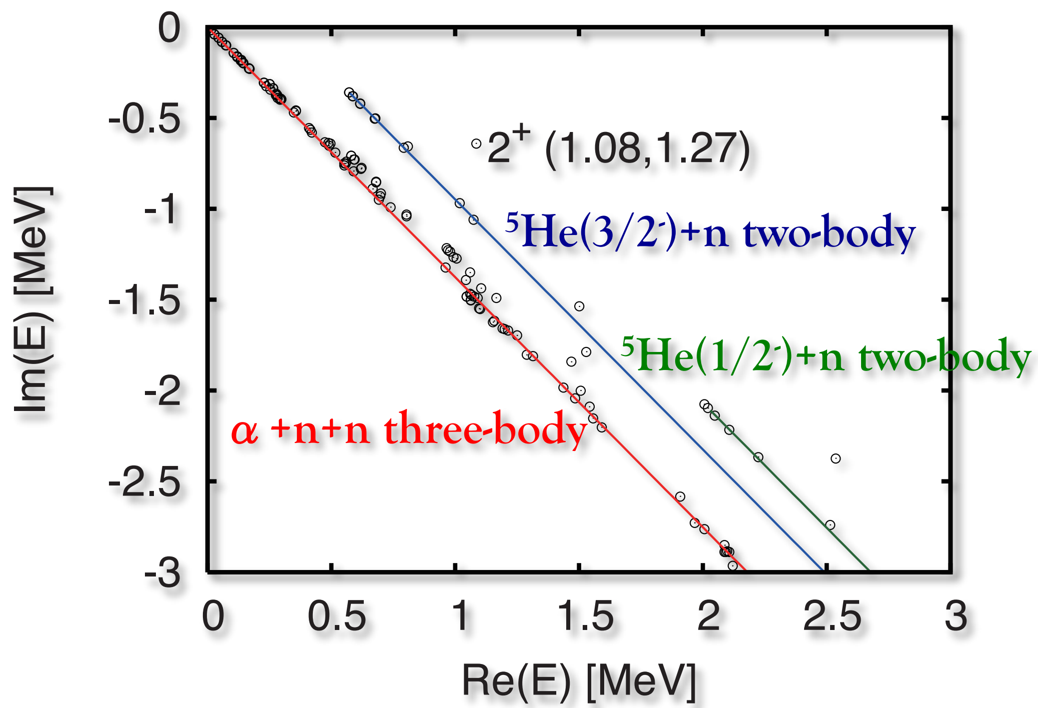
- Here, we expand the complex-scaled Green's function with the complete set constructed with the solved eigenstates and eigenvalues of H^θ.

$$U^{-1}(\theta)\frac{1}{E-\hat{H}^\theta}U(\theta)=\sum_n U^{-1}(\theta)|\chi_n^\theta\rangle\frac{1}{E-E_n^\theta}\langle\tilde{\chi}_n^\theta|U(\theta)$$

We solve the Schroedinger equation of H^θ by using the few-body technique in similar manner to the bound-state case.

$$\hat{H}^\theta\chi_n^\theta=E_n^\theta\chi_n^\theta$$

Here, we employ the orthogonality condition model and use the Gaussian basis functions.



Outgoing boundary conditions are taken into account in the imaginary parts of energy eigenvalues.

Description of scattering states with CSM

- Combining the Lippmann-Schwinger equation with the complex-scaled Green's function, we can describe the scattering states as follows.

$$|\Psi^{(+)}\rangle = |\Phi_0\rangle + \sum_n U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E - E_n^\theta} \langle\tilde{\chi}_n^\theta|U(\theta)\hat{V}|\Phi_0\rangle$$

$$\langle\Psi^{(-)}| = \langle\Phi_0| + \sum_n \langle\Phi_0|\hat{V}U^{-1}(\theta)|\chi_n^\theta\rangle \frac{1}{E - E_n^\theta} \langle\tilde{\chi}_n^\theta|U(\theta)$$

- We refer this solution to the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).
  - The advantage in CSLS is that we can solve many-body scattering problems
    - in similar manner to the bound-state cases
    - without explicit enforcement of the boundary conditions

Applications of CSLS
to the scattering problems for A=6 systems

1. *α+d elastic scattering*
2. *Coulomb breakup reaction of ^{6}He*

1. α+d elastic scattering

- Setup
  - Hamiltonian

$$\hat{H} = \sum_{i=1}^{3} t_i - T_{\mathrm{c.m.}} + \sum_{i=1}^{2} V_{\alpha N}(\mathbf{r}_i) + V_{NN} + V_{\alpha NN}$$

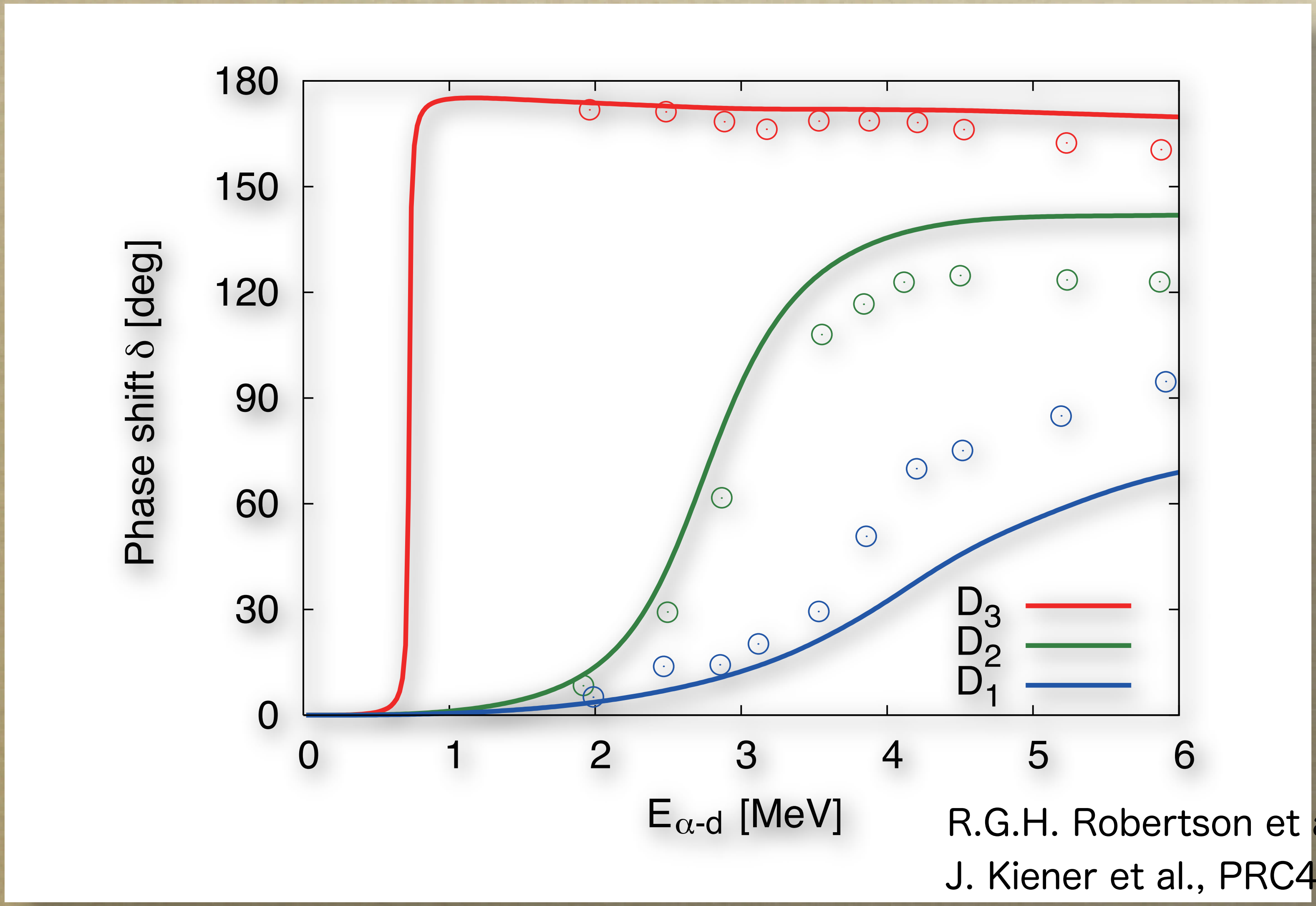
$$\text{where} \quad V_{\alpha N}\text{: KKNN potential,} \quad V_{NN}\text{: AV8'}$$

  - Basis functions to construct the complete set
    - Gaussian basis functions, whose ranges are taken up to 20 fm
  - The obtained properties of the ground state
    - Matter radius: 2.33 fm exp.) 2.44 ± 0.07 fm
    - Charge radius: 2.47 fm exp.) 2.56 ± 0.05 fm

A. Dobrovolsky et al., NPA766 (2006), 1.
G.C. Li et al., NPA81 (1971), 583.

Elastic phase shifts of α+d scattering

- The obtained elastic phase shifts for D-wave scattering of the α+d system in comparison with the observed data.
  - Our calculated results reproduce the observed trend in the phase shifts.



R.G.H. Robertson et al., PRL47(1981), 1867.
J. Kiener et al., PRC44(1991), 2195.

2. Coulomb breakup reaction of ^{6}He

- Setup
  - Hamiltonian

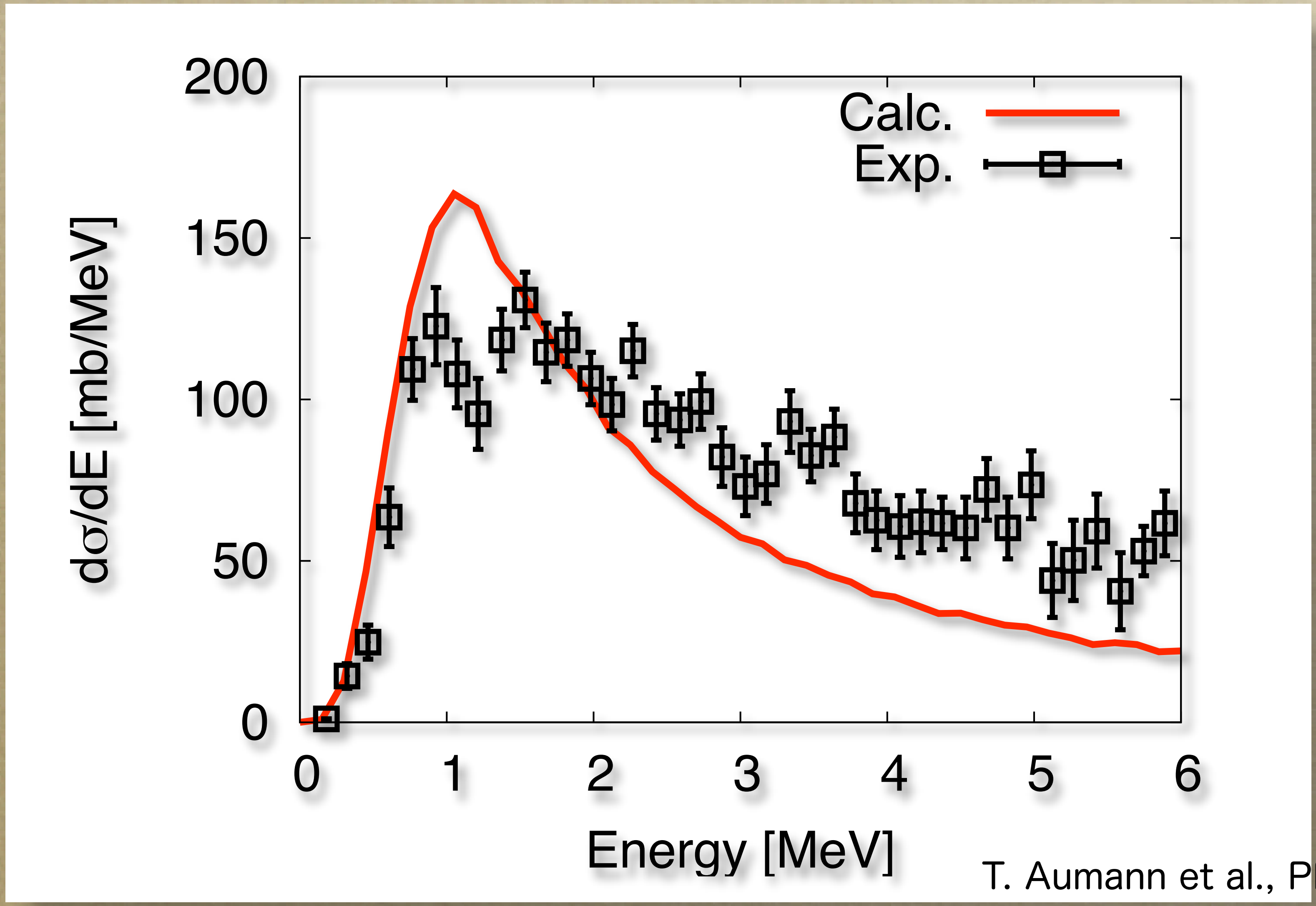
$$\hat{H} = \sum_{i=1}^{3} t_i - T_{\mathrm{c.m.}} + \sum_{i=1}^{2} V_{\alpha N}(\mathbf{r}_i) + V_{NN} + V_{\alpha NN}$$

$$\text{where} \quad V_{\alpha N}\text{: KKNN potential,} \quad V_{NN}\text{: Minnesota force}$$

  - Basis functions to construct the complete set
    - Gaussian basis functions, whose ranges are taken up to 20 fm
  - The obtained properties of the ground state
    - Matter radius: 2.46 fm   exp.) 2.48 ± 0.03 fm
    - Charge radius: 2.04 fm   exp.) 2.068(11) fm

Coulomb breakup cross section of ^{6}He

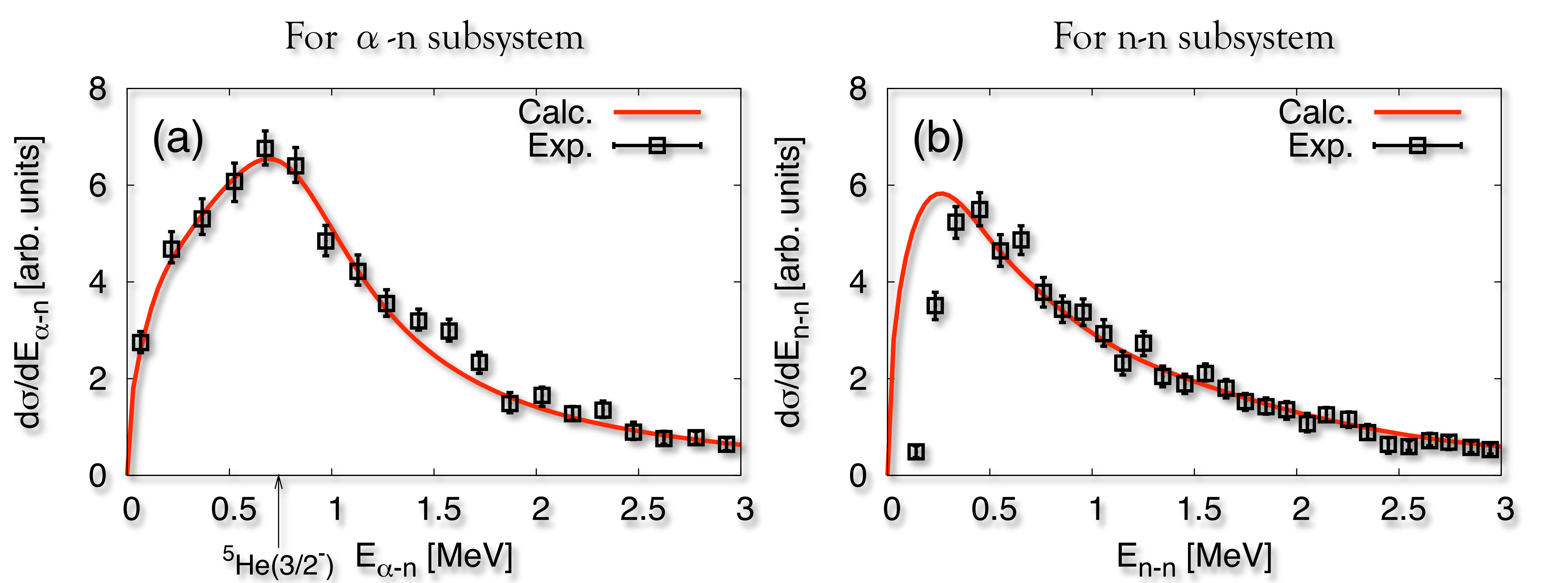
- The obtained Coulomb breakup cross section of ^{6}He in comparison with the experimental data.
  - The low-lying enhancement in the cross section is well reproduced.
  - CSLS is also capable of investigating the three-body scattering states of halo nuclei.



T. Aumann et al., PRC 59, 1252 (1999).

Invariant mass spectra for binary subsystems

- The invariant mass spectra for α-n and n-n subsystems.
  - The observed trend in the invariant mass spectra are well reproduced.
  - CSLS enables us to investigate the structures not only of the total system but also of the binary subsystems.



T. Aumann et al., PRC 59, 1252 (1999).

Summary

- Complex scaling method is a powerful tool to investigate not only the resonances but also the scattering states of nuclear many-body systems.
  - We develop the method to describe the many-body scattering states, which is referred to the complex-scaled solutions of the Lippmann-Schwinger equation (CSLS).
  - CSLS enables us to solve the three-body scattering problems
    - in similar manner to the bound-state cases
    - without explicit enforcement of boundary conditions
  - CSLS reasonably reproduces the scattering properties of A=6 systems, such as:
    - elastic phase shifts of α+d scattering
    - Coulomb breakup cross section of ^{6}He
    - Invariant mass spectra for binary subsystems in ^{6}He