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Ab-initio reaction calculations for four-nucleon system

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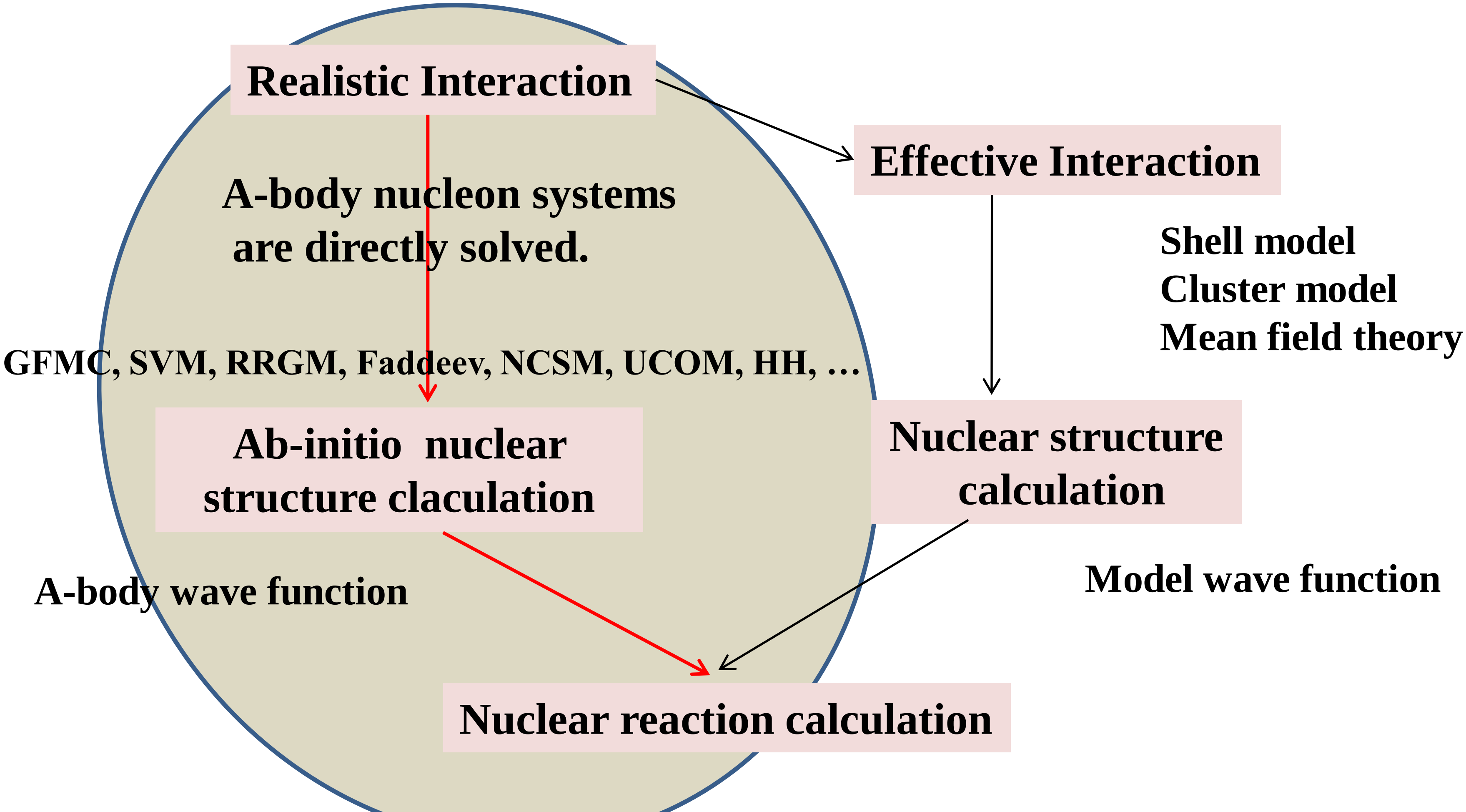
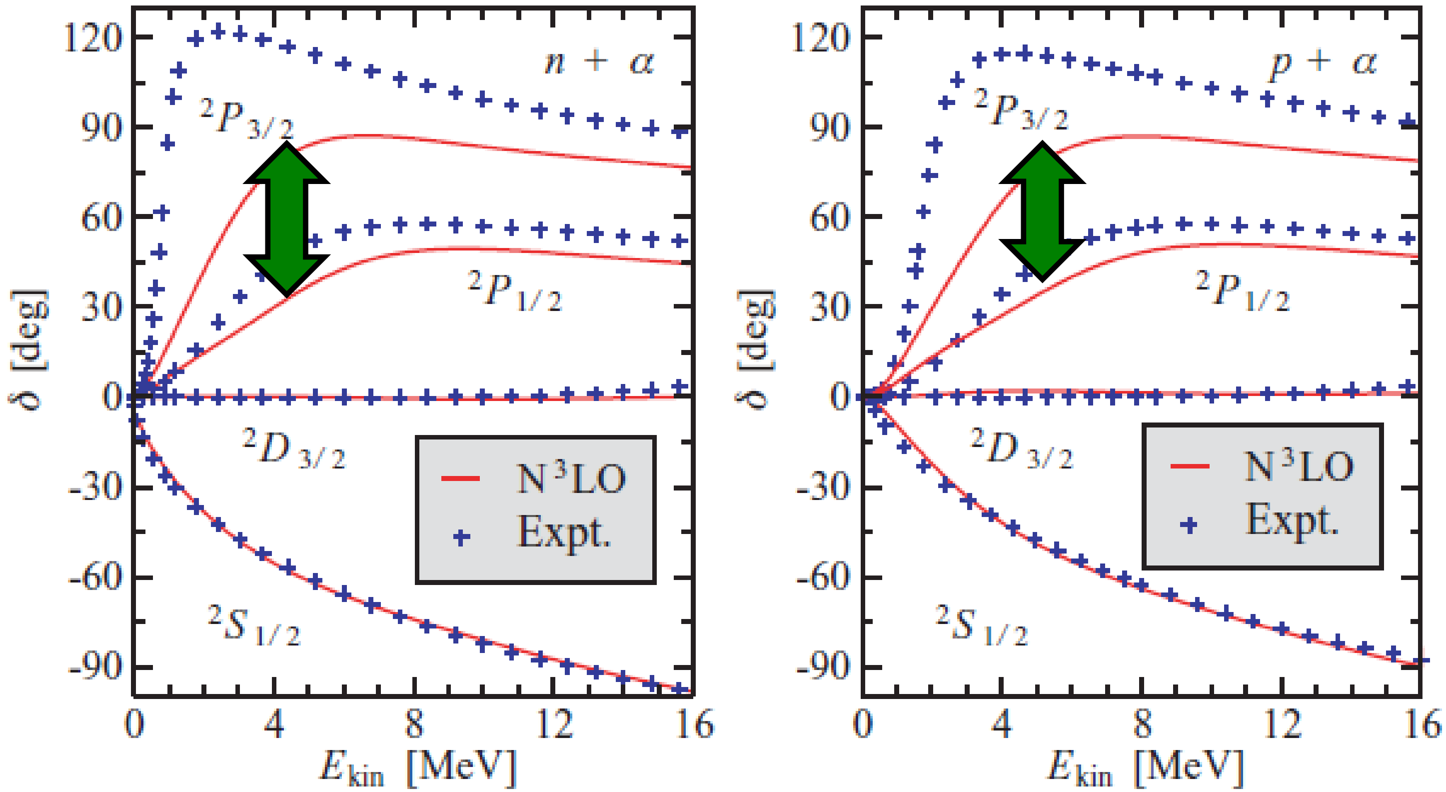
Few-body systems (in press)

Ab-intio nuclear reaction calculation



We want to know the relation between the conventional reaction calculation and the *ab initio* (type) calculation.

Five nucleon scattering
(*ab-initio* type calculation)



S. Qaunglioni P. Navratil, Physical Review C 79, 044606 (2009).

NCSM/RGM framework

Purpose

We want to solve 4～6 nucleons reaction in *ab-initio* way by using a correlated Gaussian method.

The correlated Gaussian method with global vector representation has been applied more than A=4 system.

Method

(Single) global vector representation (GVR)

K. Varga, Y. Suzuki, and J. Usukura, FBS24(1998)81

Double global vector representation (DGVR)

Y. Suzuki, W. Horiuchi and W. Orabi, K. Arai, FBS42(2008)33

Triple global vector representation (TGVR)

S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont and D. Baye, FBS (in press).

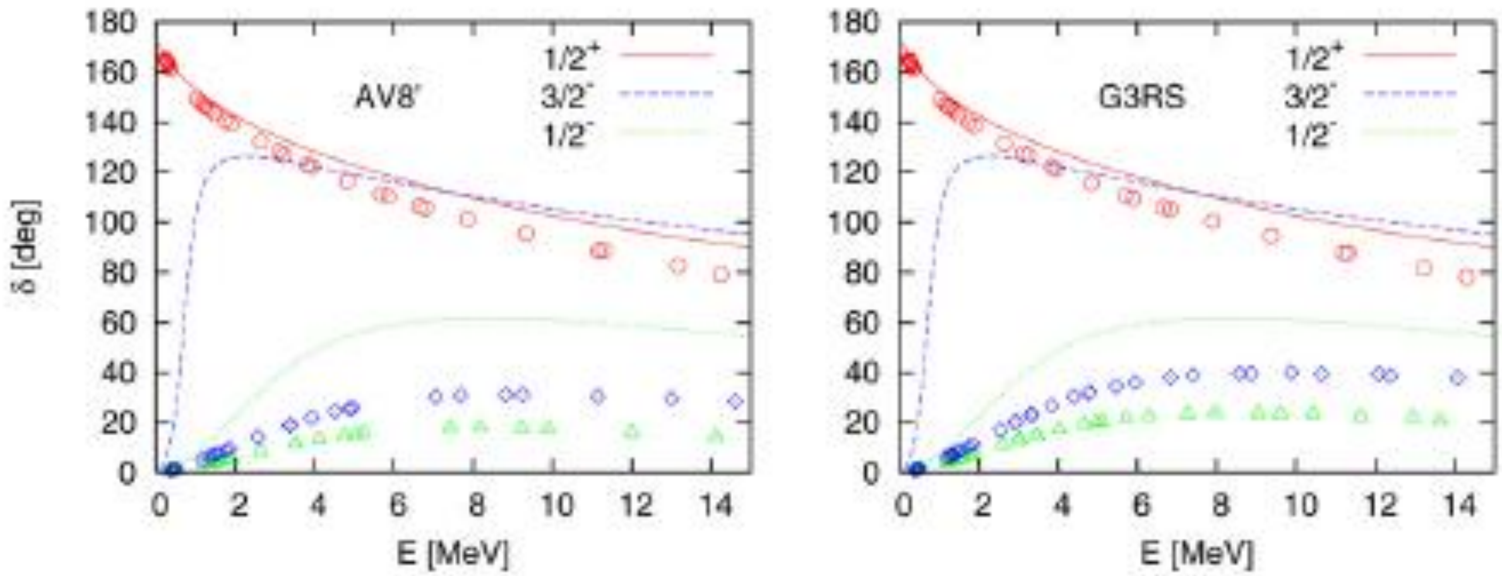
Microscopic R-Matrix Method (MRM)

D. Baye, P. -H.Heenen, M. Libert-Heinemann, NPA291(1977).

K. Kanada, K. Kaneko, S. Saito, Y.C. Tang, NPA444(1985).

α+n phase shift by DGVR

Y. Suzuki, W. Horiuchi, K. Arai, Nucl.Phys. A823(2009)1



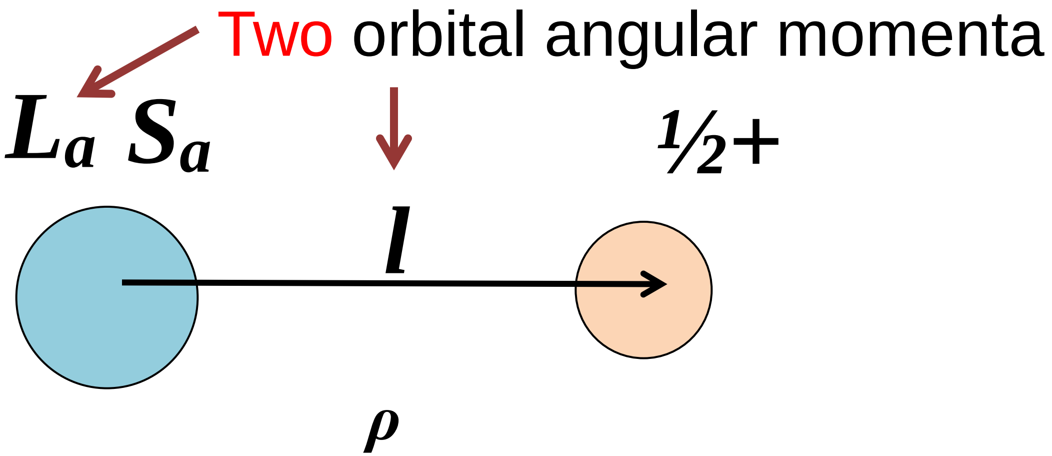
Spin-orbit splitting is small !

Three Nucleon Force and/or 3N+2N model space ?

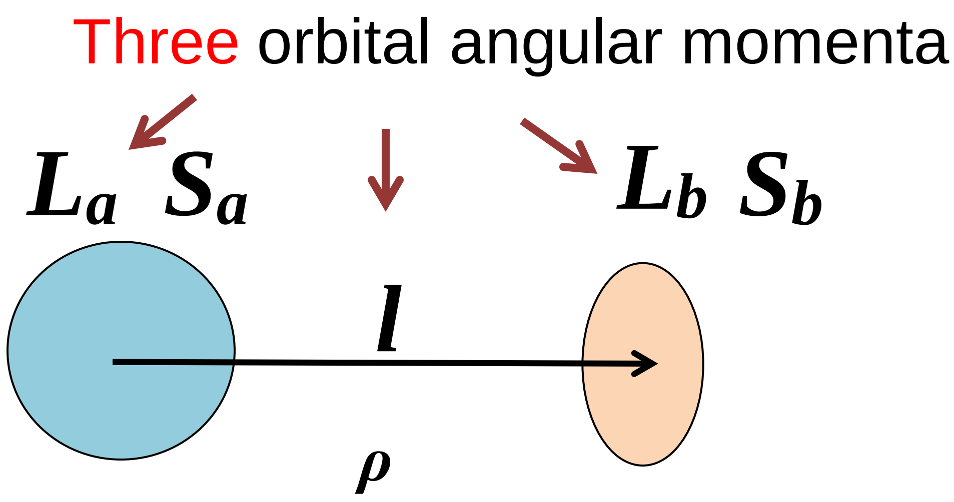
Merits of TGVR

1. We can describe the scattering states.

2. We can treat unnatural parity state, 0-.



Example
4He+n



Example
t+d

Hamiltonian(4-body case)

$$H = \sum_{i=1}^{4} T_i - T_{\rm cm} + \sum_{i<j}^{4} V_{ij} + \sum_{i<j<k}^{4} V_{ijk}\,,$$

Realistic Interaction: AV8' (+Coulomb+3NF)

V_{ij}: Central+LS+Tensor+Coulomb

Pudliner, Pandharipande, Carlson , Pieper, Wiringa: PRC56(1997)1720

V_{ijk}: Effective three nucleon force

Hiyama, Gibson, Kamimura, PRC 70(2003)031001

Effective Interaction: MN (+Coulomb)

V_{ij}: Central+Coulomb

Thompson, LeMere, Tang, NPA(1977)286

Correlated Gaussian function method with triple global vectors

First, we calculate matrix elements in LS coupled form.

LS-coupled basis function

$$\mathcal{A}\left[\left[\left[\psi^{(\mathrm{space})}_{L_a}\psi^{(\mathrm{space})}_{L_b}\right]_{L_{ab}}\chi_\alpha(\rho_\alpha)\right]_L\left[\psi^{(\mathrm{spin})}_{S_a}\psi^{(\mathrm{spin})}_{S_b}\right]_S\right]_{JM}\psi^{(\mathrm{isospin})}_{T_aM_{T_a}}\psi^{(\mathrm{isospin})}_{T_bM_{T_b}}$$

Next, we transform them to I(channel spin) coupled form.

Correlated Gaussian function with triple global vectors for four nucleon system

Unnatural parity 0^-

L1=L2=L12=L3=1

$$F_{L_1L_2(L_{12})L_3LM}(u_1,u_2,u_3,A,x)$$

$$= \underbrace{\exp\left(-\frac{1}{2}\tilde{x}Ax\right)}_{\text{Correlated Gaussian}} \left[\underbrace{\left[\mathcal{Y}_{L_1}(\widetilde{u_1}x)\mathcal{Y}_{L_2}(\widetilde{u_2}x)\right]}_{\text{Double global vector}}{}_{L_{12}}\mathcal{Y}_{L_3}(\widetilde{u_3}x)\right]_{LM}$$

Correlated Gaussian   Double global vector   New extension

$$\mathcal{Y}_{L_iM_i}(\widetilde{u_i}x) = |\widetilde{u_i}x|^{L_i}Y_{L_iM_i}(\widehat{\widetilde{u_i}x}) \qquad \widetilde{u_i}x = \sum_{j=1}^{N-1}(u_i)_j x_j$$

For H-type, we can choose, $\widetilde{u_1}=(1,0,0)$, $\widetilde{u_2}=(0,1,0)$ and $\widetilde{u_3}=(0,0,1)$

We also write the K-type basis function in the same form.

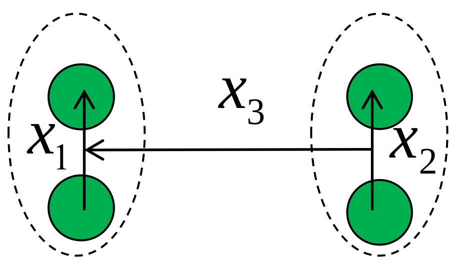
$$\exp\left(-\frac{1}{2}\widetilde{x'}A_Kx'\right)\left[\left[\mathcal{Y}_{L_1}(x'_1)\mathcal{Y}_{L_2}(x'_2)\right]_{L_{12}}\mathcal{Y}_{L_3}(x'_3)\right]_{LM}$$

$$x' = U_{KH}x \qquad \widetilde{u_1}=(1,0,0),\ \widetilde{u_2}=(0,-\tfrac{1}{2},1) \text{ and } \widetilde{u_3}=(0,\tfrac{2}{3},\tfrac{2}{3})$$

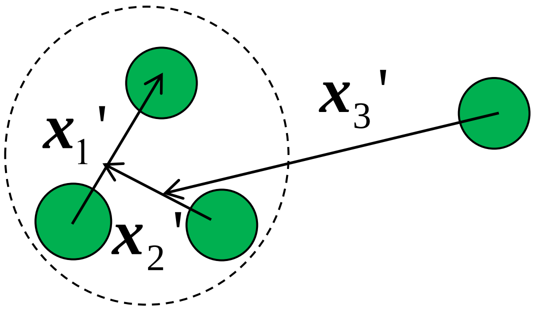
$$A = (u_1u_2u_3)A_K\begin{pmatrix}\widetilde{u_1}\\ \widetilde{u_2}\\ \widetilde{u_3}\end{pmatrix} = \widetilde{U_{KH}}A_KU_{KH}$$

Transformation of Jacobi coordinate

H-type



K-type



$$\boldsymbol{x}_{\mathrm{i}} = \sum_{j=1}^{4} \boldsymbol{U}_{ij}\, \boldsymbol{r}_j \qquad (\mathrm{i}=1,2,3,4) \qquad \bar{\boldsymbol{x}} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)$$

$$\boldsymbol{U}_H = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \qquad \boldsymbol{U}_K = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1/2 & 1/2 & -1 & 0 \\ 1/3 & 1/3 & 1/3 & -1 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \qquad \boldsymbol{U}_K \boldsymbol{U}_H^{-1} = \begin{pmatrix} \boldsymbol{U}_{KH} & 0 \\ 0 & 1 \end{pmatrix}$$

H-type $\Rightarrow$ K-type

$$\boldsymbol{x}' = \boldsymbol{U}_{KH}\ \boldsymbol{x}$$

Permutation symmetry on F-function

(N-1)×(N-1) matrix

$$
\begin{aligned}
&PF_{L_1L_2(L_{12})L_3LM}(u_1,u_2,u_3,A,x)\\
&=F_{L_1L_2(L_{12})L_3LM}(u_1,u_2,u_3,A,x_P) \quad \longleftarrow \quad x_P=\mathcal{P}x\\
&=F_{L_1L_2(L_{12})L_3LM}(\widetilde{\mathcal{P}}u_1,\widetilde{\mathcal{P}}u_2,\widetilde{\mathcal{P}}u_3,\widetilde{\mathcal{P}}A\mathcal{P},x)
\end{aligned}
$$

We only replace a set of 4 variables, (u1,u2,u3, A) as the transformation of the Jacobi coordinate.

$$
< F_{L_4L_5(L_{45})L_6L'}(u_4,u_5,u_6,A',x)\,|\,O\,|\,F_{L_1L_2(L_{12})L_3L}(u_1,u_2,u_3,A,x) >
$$

Few body system (in press)

We can describe the matrix elements for A≧4 systems in a unified way by using TGVR.

Microscopic R-matrix Method

All of the pseudo excited states of clusters are taken in account

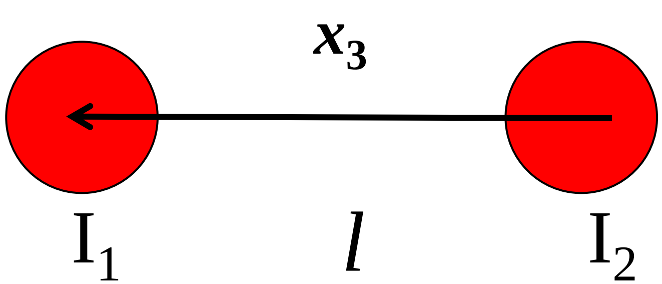
$$\Psi^{(H-type)} = \mathrm{A}\{\Phi_{I_1}^{2N}(\boldsymbol{x}_1, \boldsymbol{x}_2)\,\Phi_{I_2}^{2N}\chi_{\boldsymbol{l}}(\boldsymbol{x}_3)\}$$

$$\Psi^{(K-type)} = \mathrm{A}\{\Phi_{I'_1}^{3N}(\boldsymbol{x}'_1, \boldsymbol{x}'_2)\,\Phi_{I'_2}^{N}\chi_{\boldsymbol{l}'}(\boldsymbol{x}'_3)\}$$

$\chi_l(\boldsymbol{x}_3)$: Cluster relative wave function

(expanded by Gaussian basis functions)

Microscopic R-matrix method



$[[I_1\ I_2]_I\ l\ ]_{JM}$

$$\begin{cases} \boldsymbol{a} : \ \text{channel raidus } (13 \sim 15fm) \\ \boldsymbol{x}_3 < \boldsymbol{a} \ \text{---} \ \text{Gaussian expansion} \\ \boldsymbol{x}_3 > \boldsymbol{a} \ \text{---} \ I_l(k\boldsymbol{a})\,\delta_{\alpha\alpha'} - S_{\alpha\alpha'}\,O_l(k\boldsymbol{a}) \quad \textbf{or} \quad W_{\boldsymbol{l}+1/2,\eta}(2k\boldsymbol{a}) \end{cases}$$

e.g. D. Baye, P. -H.Heenen, M. Libert-Heinemann, NPA291(1977).

Physical channels for scattering states in ^{4}He

Table 1. Channel spins ($^{2I+1}\ell_J$) of physical d+d, t+p, and h+n channels for $J \leq 2$ and $\ell \leq 2$.

channel \ J^π	0^+	1^+	2^+	0^-	1^-	2^-
$d(1^+)+d(1^+)$	$^1S_0$ $^5D_0$	$^5D_1$	$^5S_2$ $^1D_2$ $^5D_2$	$^3P_0$	$^3P_1$	$^3P_2$
$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$, $h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$	$^1S_0$	$^3S_1$ $^3D_1$	$^1D_2$ $^3D_2$	$^3P_0$	$^1P_1$ $^3P_1$	$^3P_2$

Important channels in the scattering states

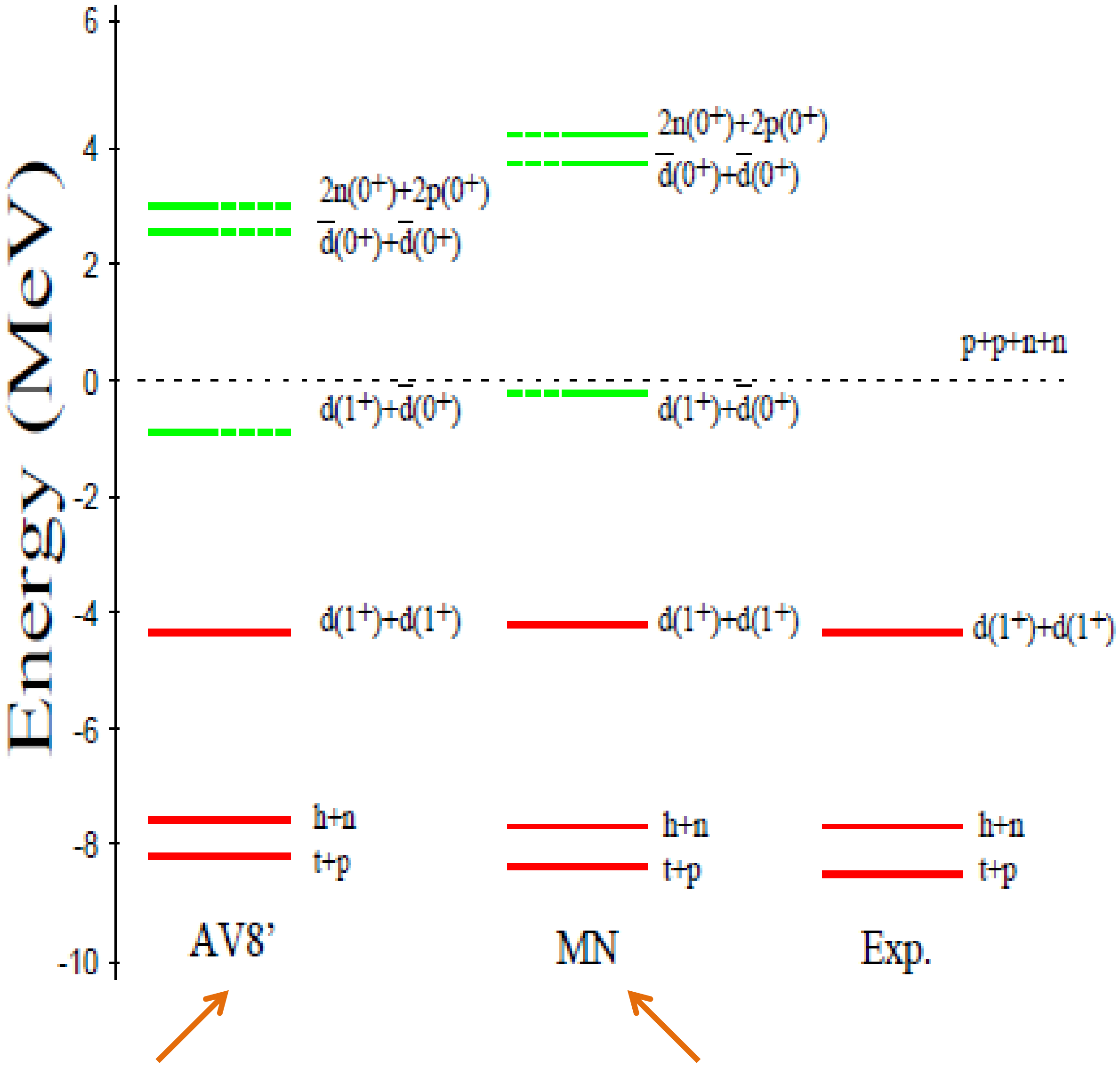
But they are not cpmplete because of distortion of clusters in the interaction region.

The basis function for the sub-system is determined by SVM

potential	cluster	present				literature			
		N_k	E (MeV)	$R^{\rm rms}$ (fm)	P_D (%)	E (MeV)	$R^{\rm rms}$ (fm)	P_D (%)	
AV8′ (with TNF)	$d(1^+)$	8	−2.18	1.79	5.9	−2.24	1.96	5.8	
	$t(\frac{1}{2}^+)$	30	−8.22	1.69	8.4	−8.41	-	-	Hiyama
	$h(\frac{1}{2}^+)$	30	−7.55	1.71	8.3	−7.74	-	-	
	^{4}He(0^+)	(2370)	−27.99	1.46	13.8	−28.44	-	14.1	
MN	$d(1^+)$	4	−2.10	1.63	0	−2.20	1.95	0	Horiuchi
	$t(\frac{1}{2}^+)$	15	−8.38	1.70	0	−8.38	1.71	0	
	$h(\frac{1}{2}^+)$	15	−7.70	1.72	0	−7.71	1.74	0	
	^{4}He(0^+)	(1140)	−29.94	1.41	0	−29.94	1.41	0	

present

Threshold positions in the present calculation



with three-nucleon force

without three-nucleon force

Included channels in the present calculation

model			channel
FULL	$2N+2N$	I	$d(1^+)+d(1^+)$
			$d(1^+)+d^*(1^+)$
			$d^*(1^+)+d^*(1^+)$
		II	$\bar{d}(0^+)+\bar{d}(0^+)$
			$\bar{d}(0^+)+d^*(0^+)$
			$d^*(0^+)+d^*(0^+)$
		III	$d^*(2^+)+d^*(1^+)$
			$d^*(2^+)+d^*(2^+)$
		IV	$d^*(3^+)+d^*(1^+)$
			$d^*(3^+)+d^*(2^+)$
			$d^*(3^+)+d^*(3^+)$
		V	$2n(0^+)+2p(0^+)$
			$2n(0^+)+2p^*(0^+)$
			$2n^*(0^+)+2p(0^+)$
			$2n^*(0^+)+2p^*(0^+)$
	$3N+N$	1	$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$
			$t^*(\frac{1}{2}^+)+p(\frac{1}{2}^+)$
		2	$h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$
			$h^*(\frac{1}{2}^+)+n(\frac{1}{2}^+)$

Thanks to the reduction of basis function by SVM for the sub-system. We can reduce the dimension of matrix elements very much!

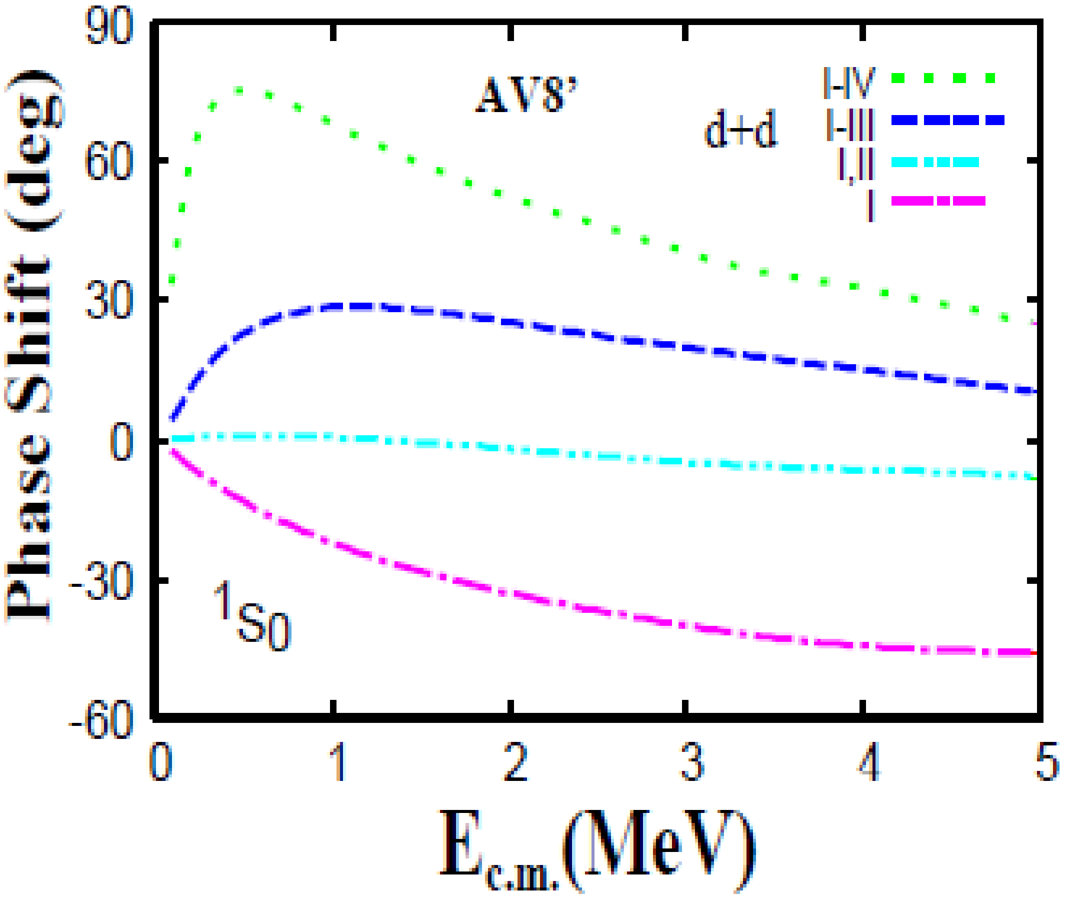
Dimensions of matrix elements for FULL in the LS-coupled case

0+	6660
1+	16680
2+	22230
0-	4200
1-	11670
2-	12480

For 2+, it takes about 200 days with 1CPU(1Core). And we need about 20Gbyte memory for the MRM calculation.

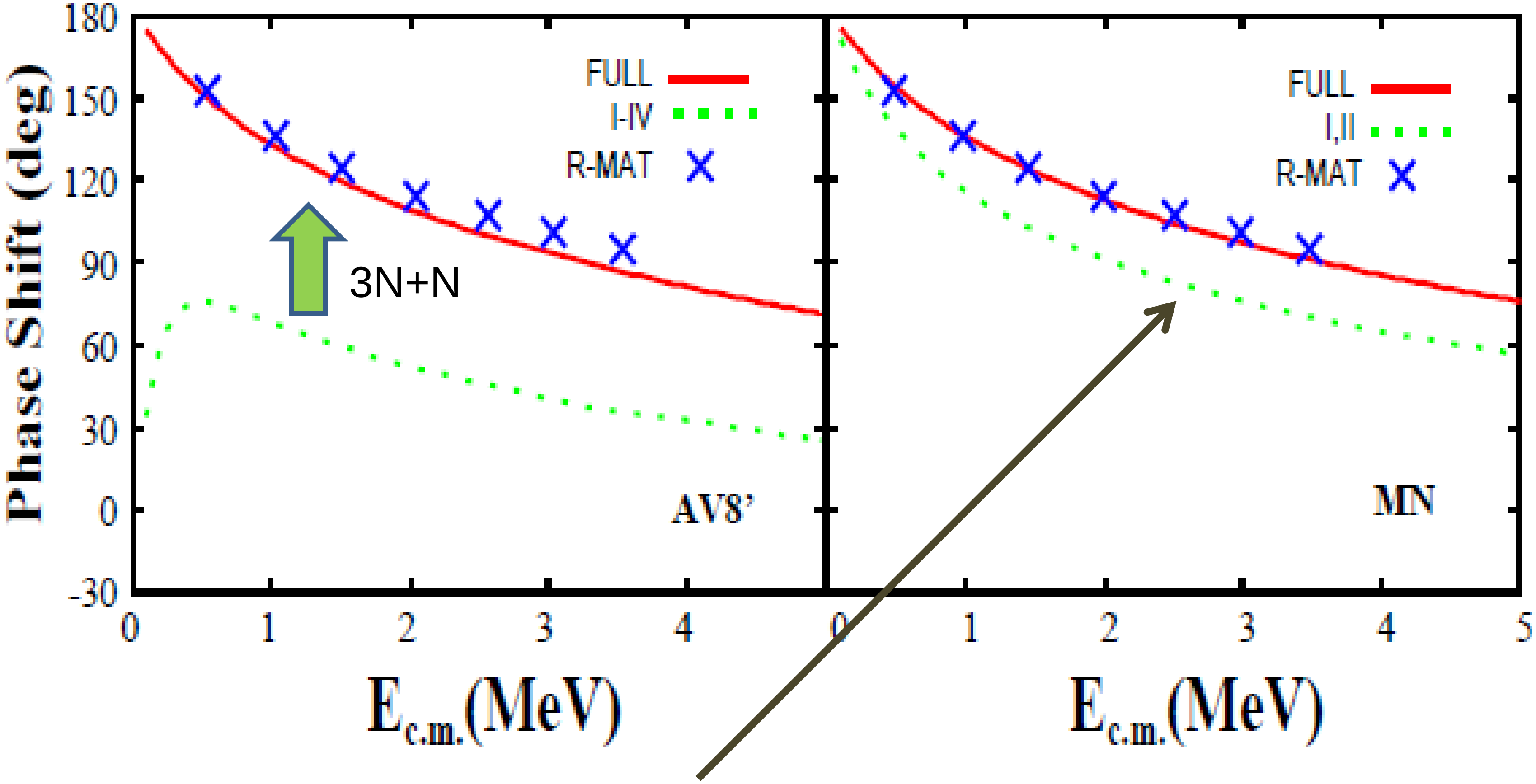
All pseudo states (discretized continuum state) are employed in the MRM calculation.

$^1S_0$ d+d elastic phase shift within d+d channel



	channel
I	$d(1^+)+d(1^+)$
	$d(1^+)+d^*(1^+)$
	$d^*(1^+)+d^*(1^+)$
II	$\bar{d}(0^+)+\bar{d}(0^+)$
	$\bar{d}(0^+)+d^*(0^+)$
	$d^*(0^+)+d^*(0^+)$
III	$d^*(2^+)+d^*(1^+)$
	$d^*(2^+)+d^*(2^+)$
IV	$d^*(3^+)+d^*(1^+)$
	$d^*(3^+)+d^*(2^+)$
	$d^*(3^+)+d^*(3^+)$

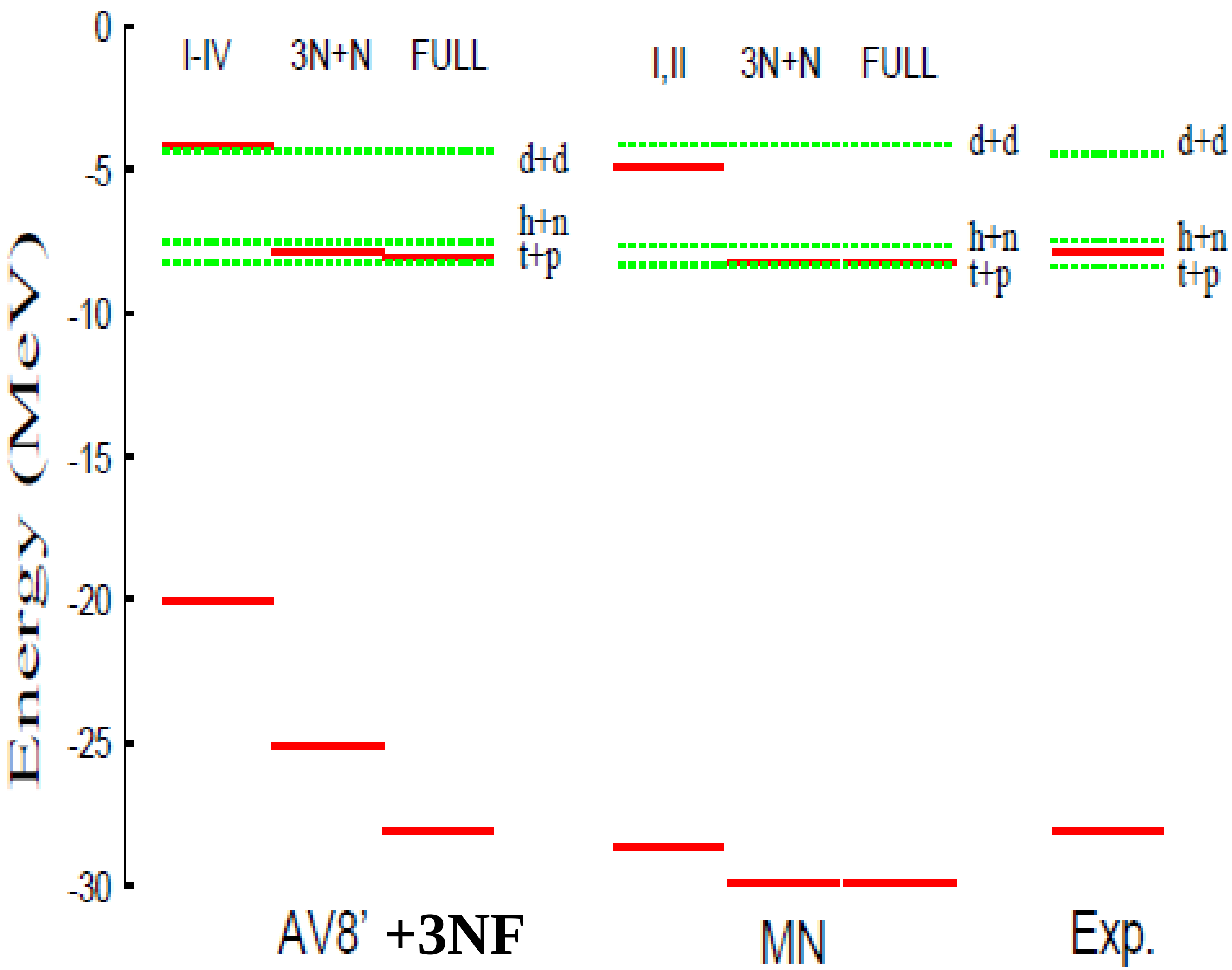
$^{1}S_0$ d+d elastic phase shift (0+)



For effective interaction, d+d scattering picture is good!

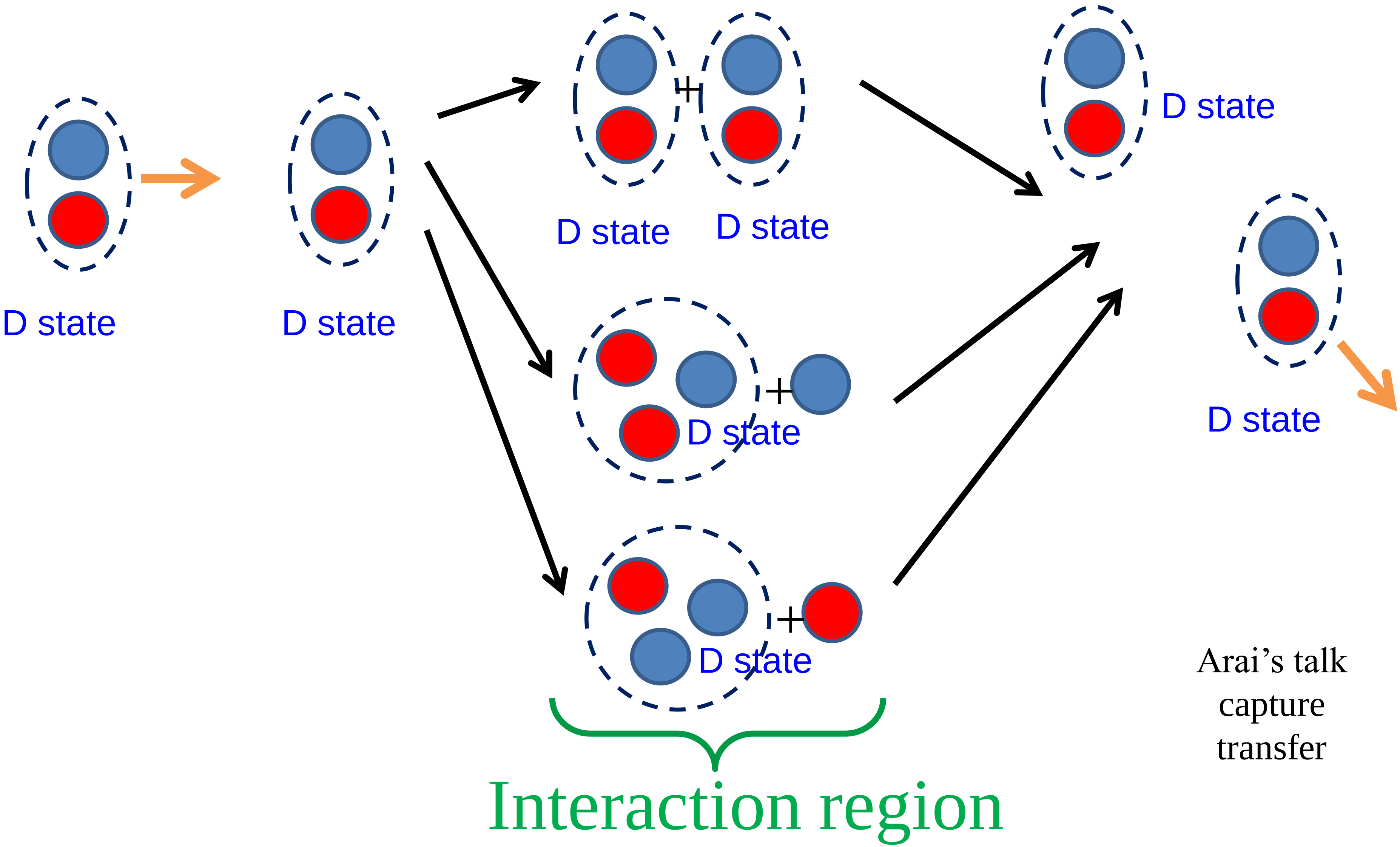
R.-Matrix analyses : Hofmann, Hale, PRC77(2008)044002

Energy Levels of 0^+ state in ^{4}He



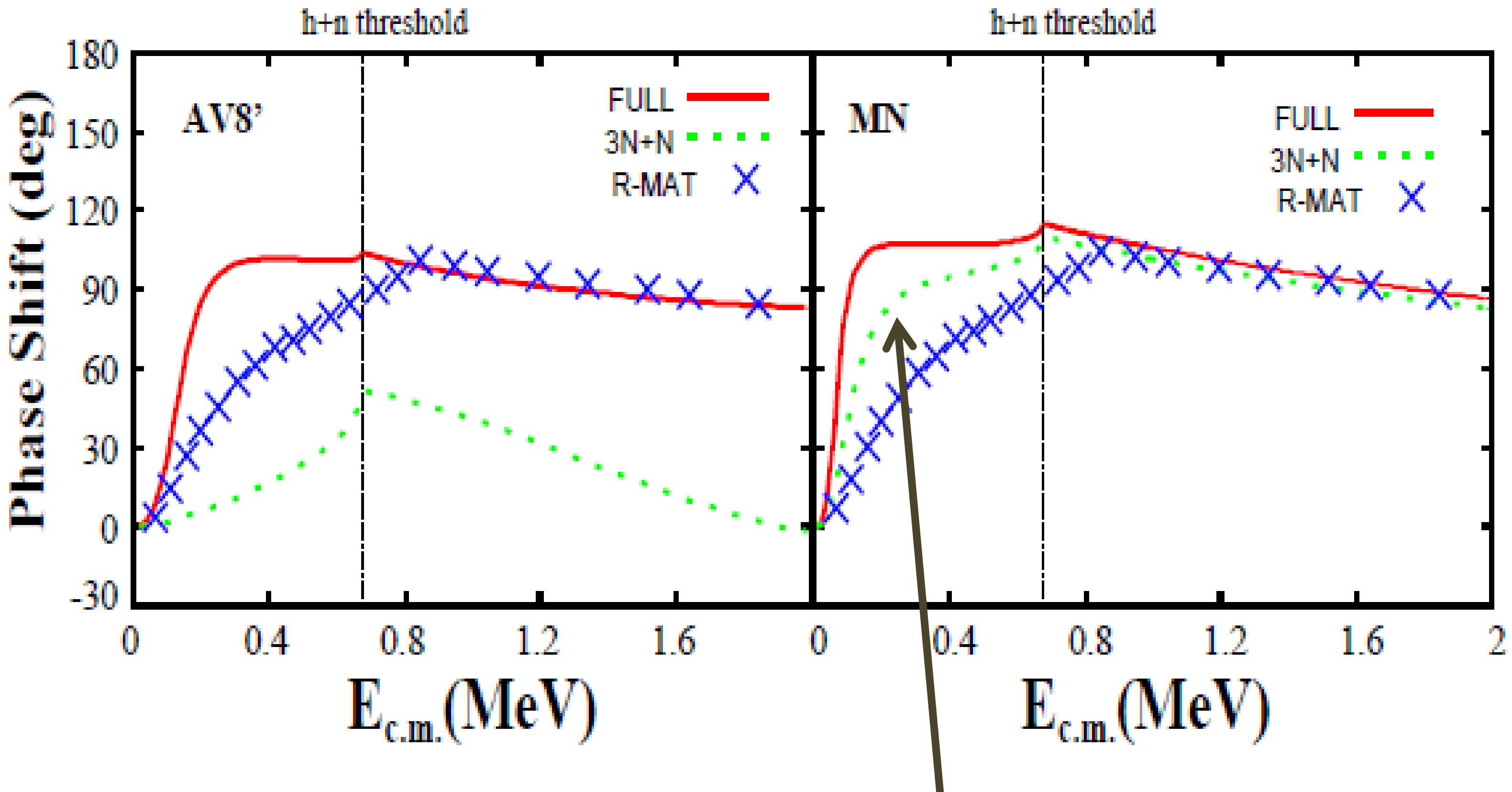
Coupling between d+d channel and 3N+N channels

Tensor force makes the coupling in the scattering strong



Arai's talk
capture
transfer

$^1S_0$ t+p elastic phase shift (0+)



For effective interaction, t+p scattering picture is good!

More elaborate interaction (AV18) case by Hofmann

Hofmann, Hale, PRC77(2008)044002

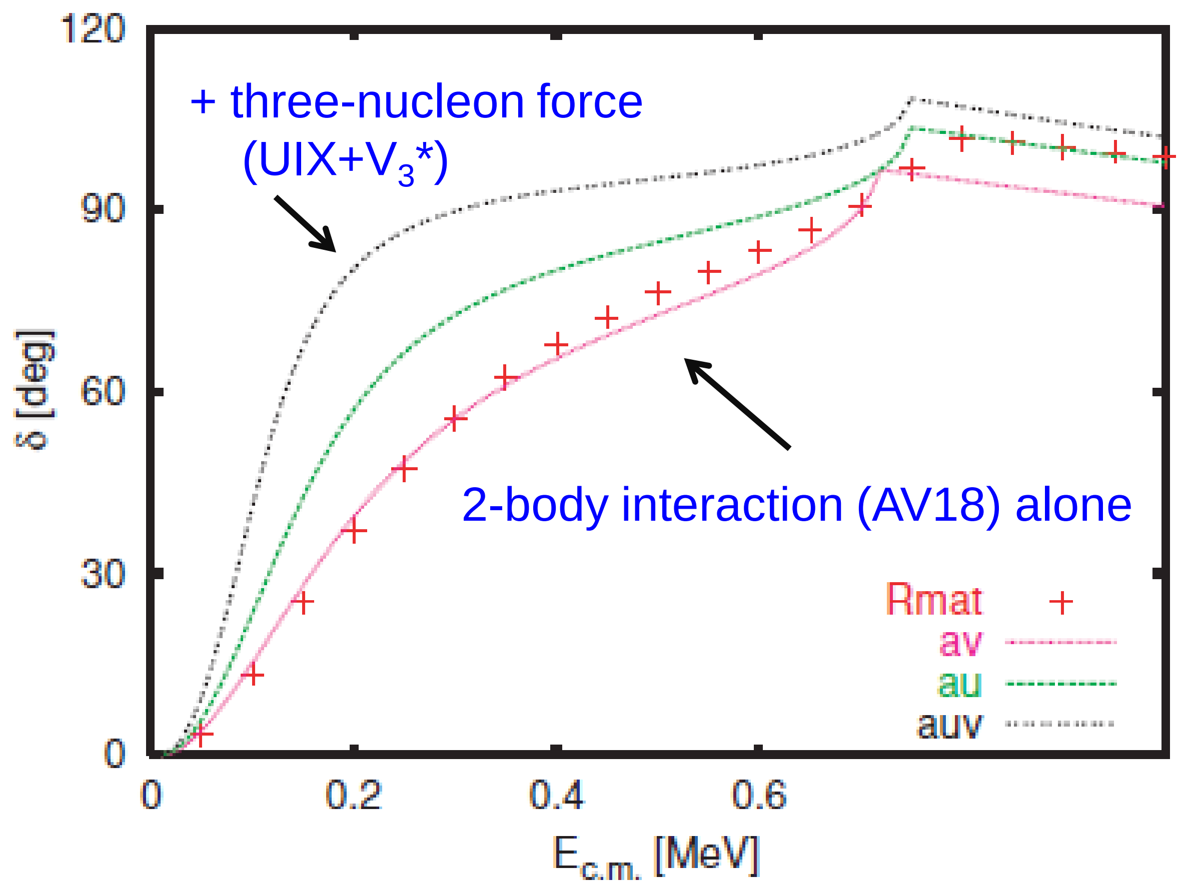
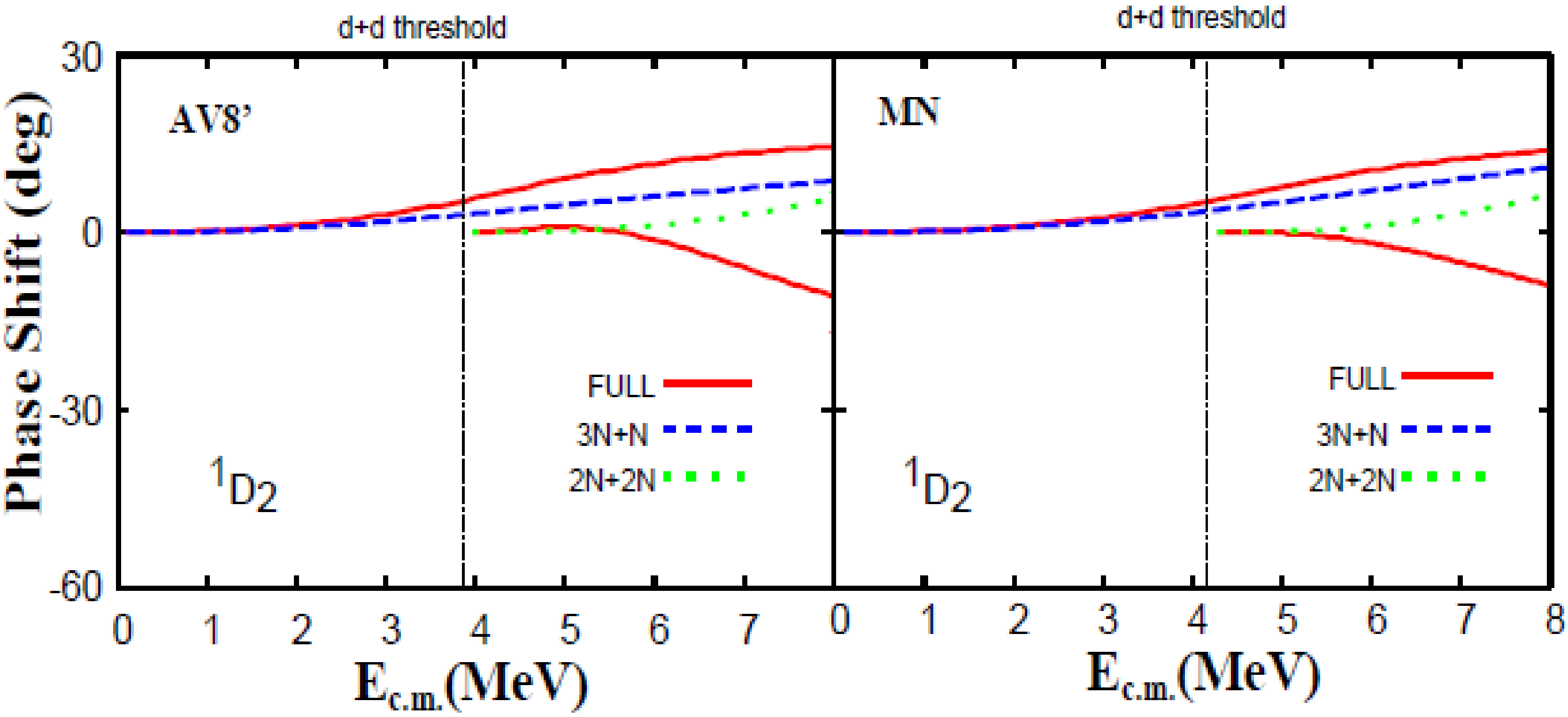


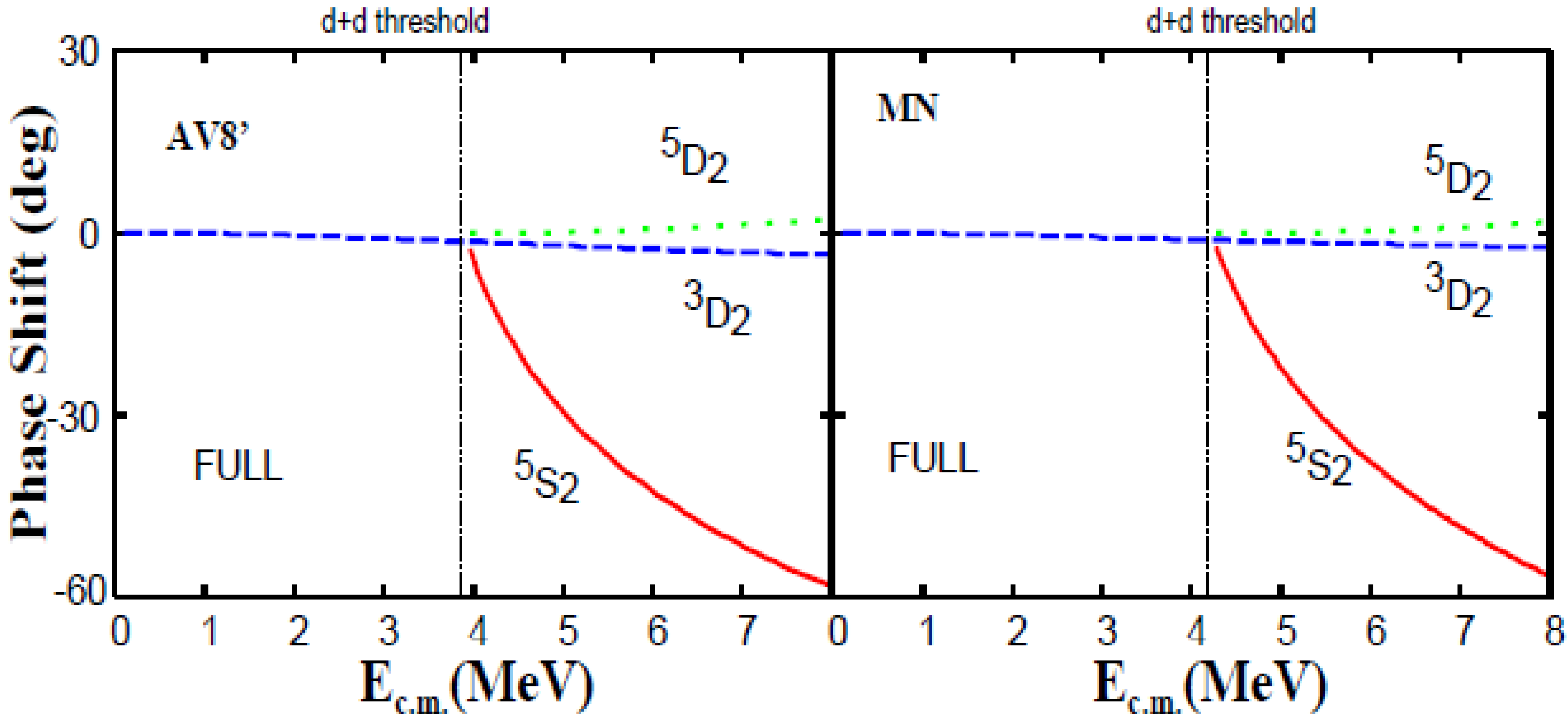
FIG. 1. (Color online) Low-energy triton-proton 0^+ phase shifts calculated using AV18 (av), AV18 and UIX (au), and additionally V_3^* (auv) compared with R-matrix (Rmat) results.

$^1D_2$ elastic phase shift (2+)



Phase shift with Realistic interaction is not different so much from effective interaction for $^1D_2$

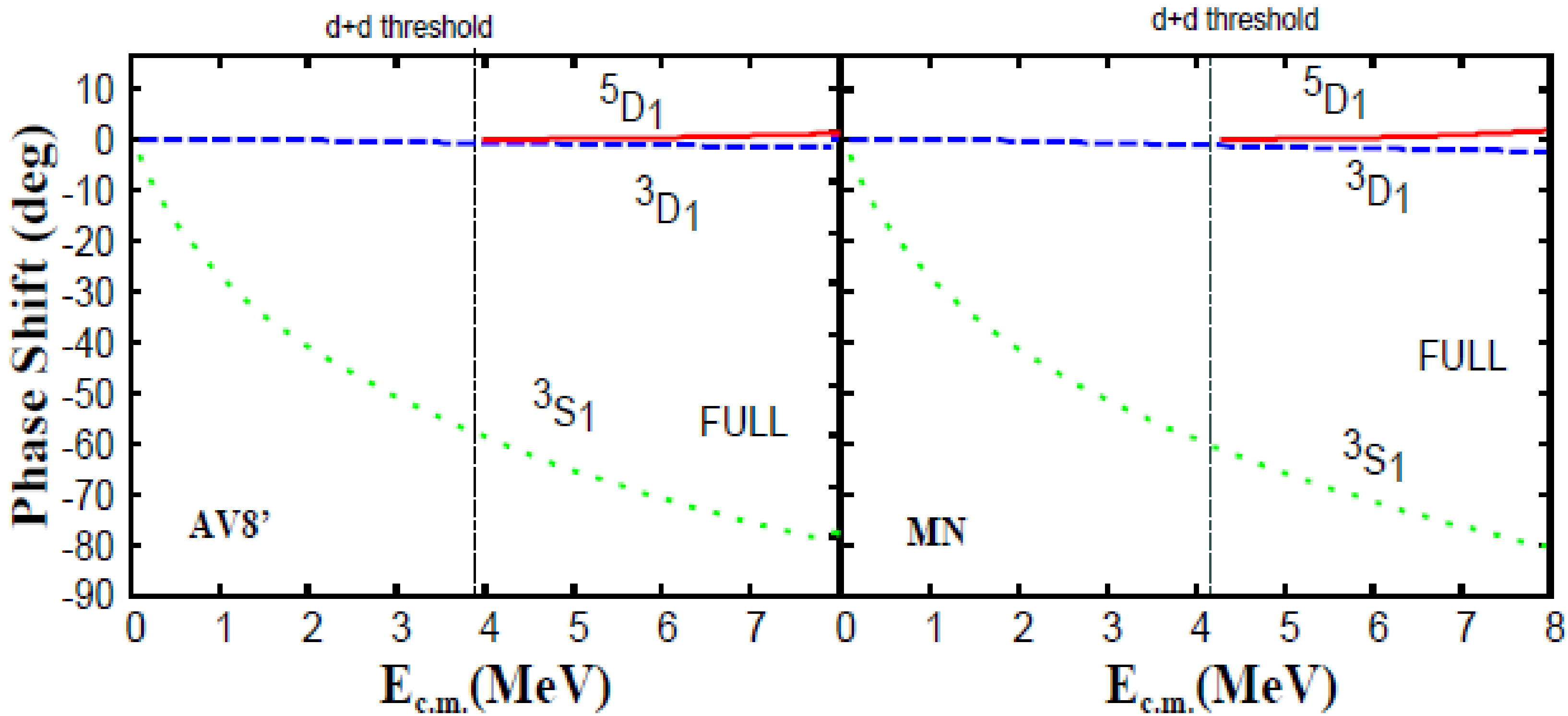
Other elastic phase shifts in 2+



channel \ J^π	0^+	1^+	2^+	0^-	1^-	2^-
$d(1^+)+d(1^+)$	$^1S_0$ $^5D_0$	$^5D_1$	$^5S_2$ $^1D_2$ $^5D_2$	$^3P_0$	$^3P_1$	$^3P_2$
$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$, $h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$	$^1S_0$	$^3S_1$ $^3D_1$	$^1D_2$ $^3D_2$	$^3P_0$	$^1P_1$ $^3P_1$	$^3P_2$

3N+N and 2N+2N are not coupled

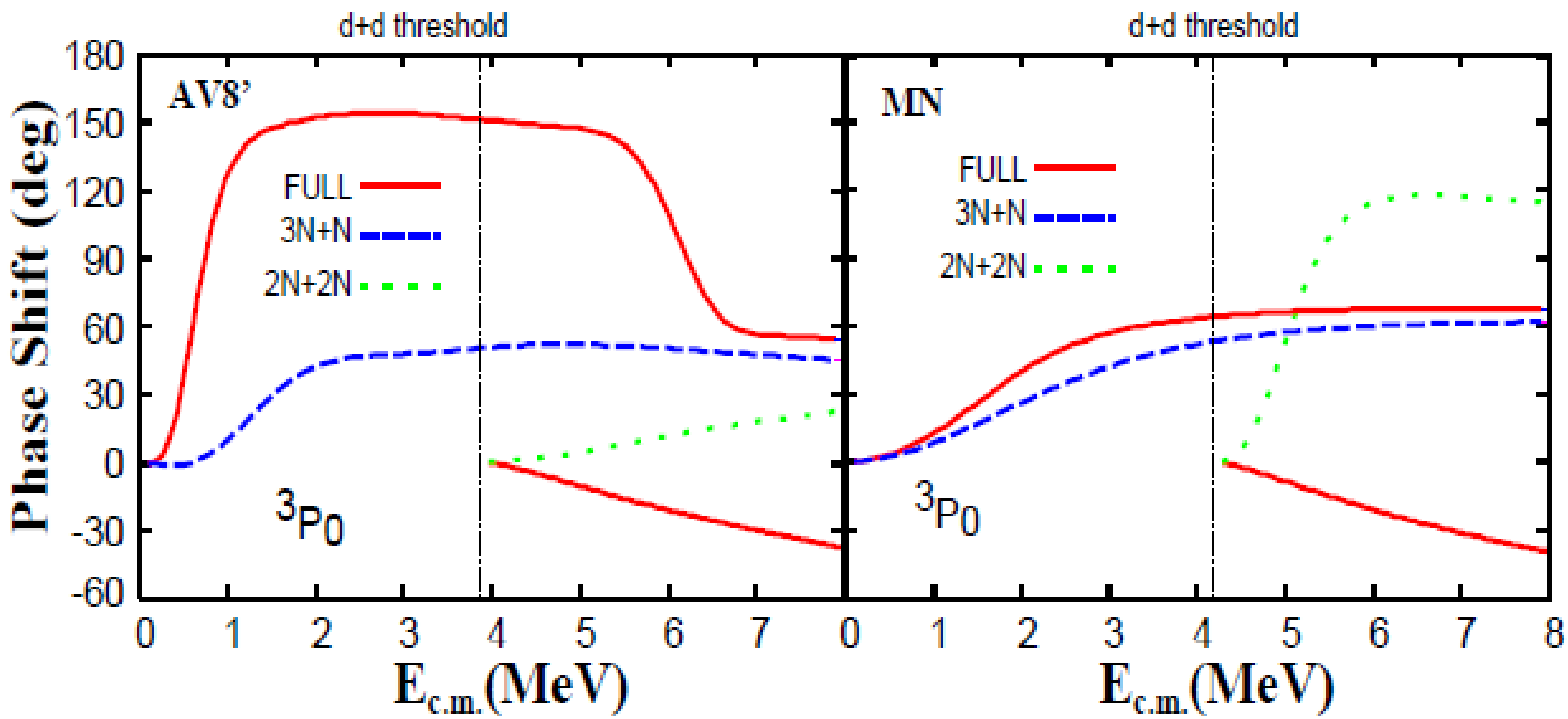
phase shifts in 1+



channel \ J^π	0^+	1^+	2^+	0^-	1^-	2^-
$d(1^+)+d(1^+)$	$^1S_0$ $^5D_0$	$^5D_1$	$^5S_2$ $^1D_2$ $^5D_2$	$^3P_0$	$^3P_1$	$^3P_2$
$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$, $h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$	$^1S_0$	$^3S_1$ $^3D_1$	$^1D_2$ $^3D_2$	$^3P_0$	$^1P_1$ $^3P_1$	$^3P_2$

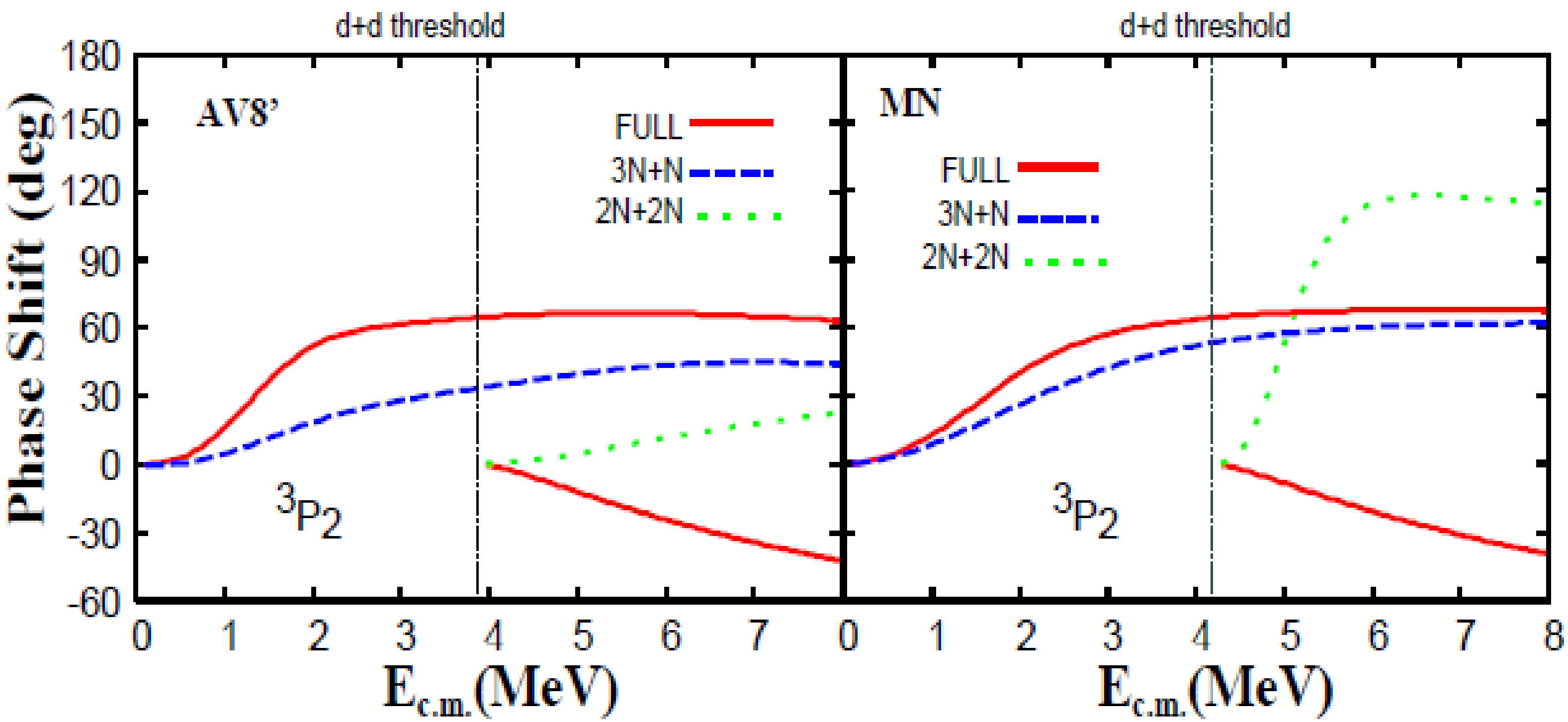
3N+N and 2N+2N are not coupled

$^3P_0$ elastic phase shift (0-)



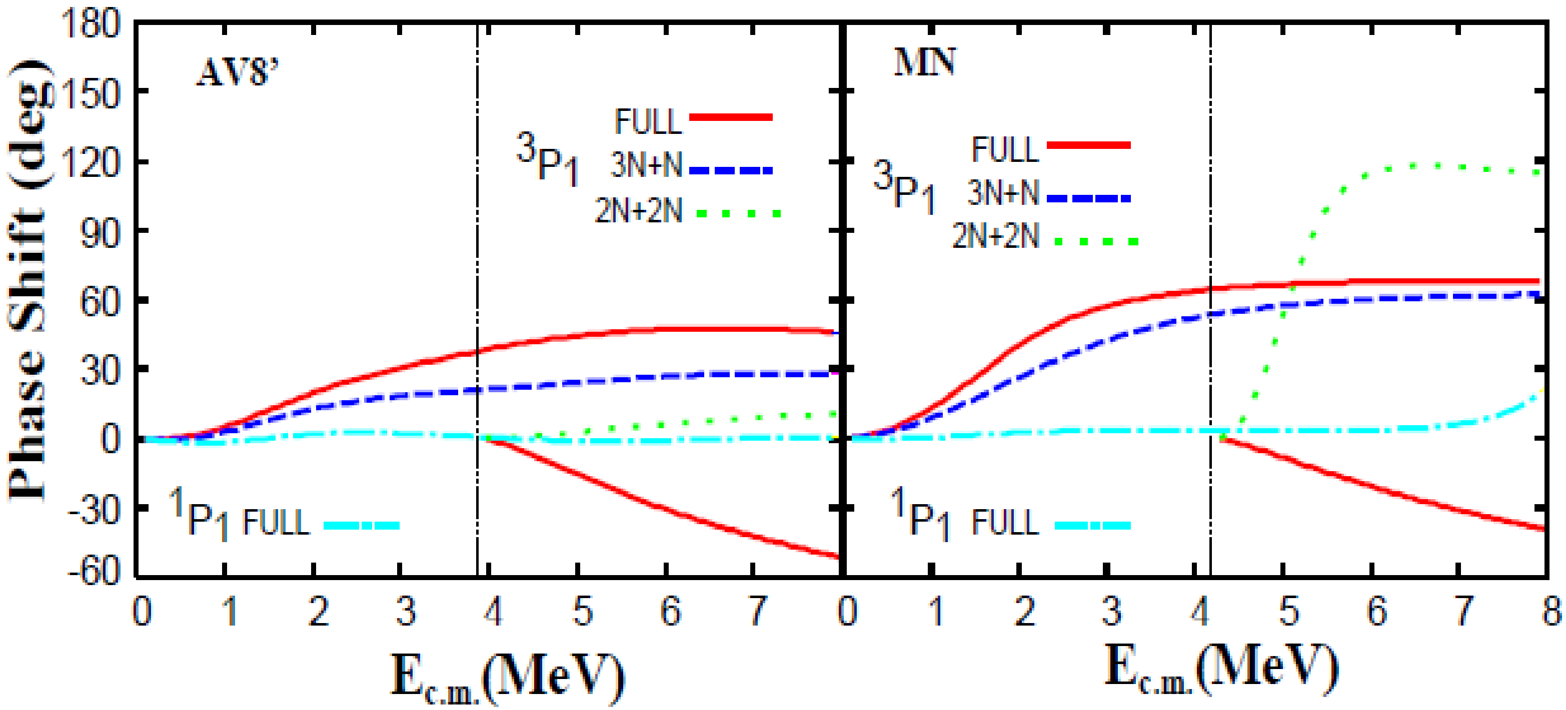
channel \ J^π	0^+	1^+	2^+	0^-	1^-	2^-
$d(1^+)+d(1^+)$	$^1S_0$, $^5D_0$	$^5D_1$	$^5S_2$, $^1D_2$, $^5D_2$	$^3P_0$	$^3P_1$	$^3P_2$
$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$, $h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$	$^1S_0$	$^3S_1$, $^3D_1$	$^1D_2$, $^3D_2$	$^3P_0$	$^1P_1$, $^3P_1$	$^3P_2$

$^3P_2$ elastic phase shift (2-)



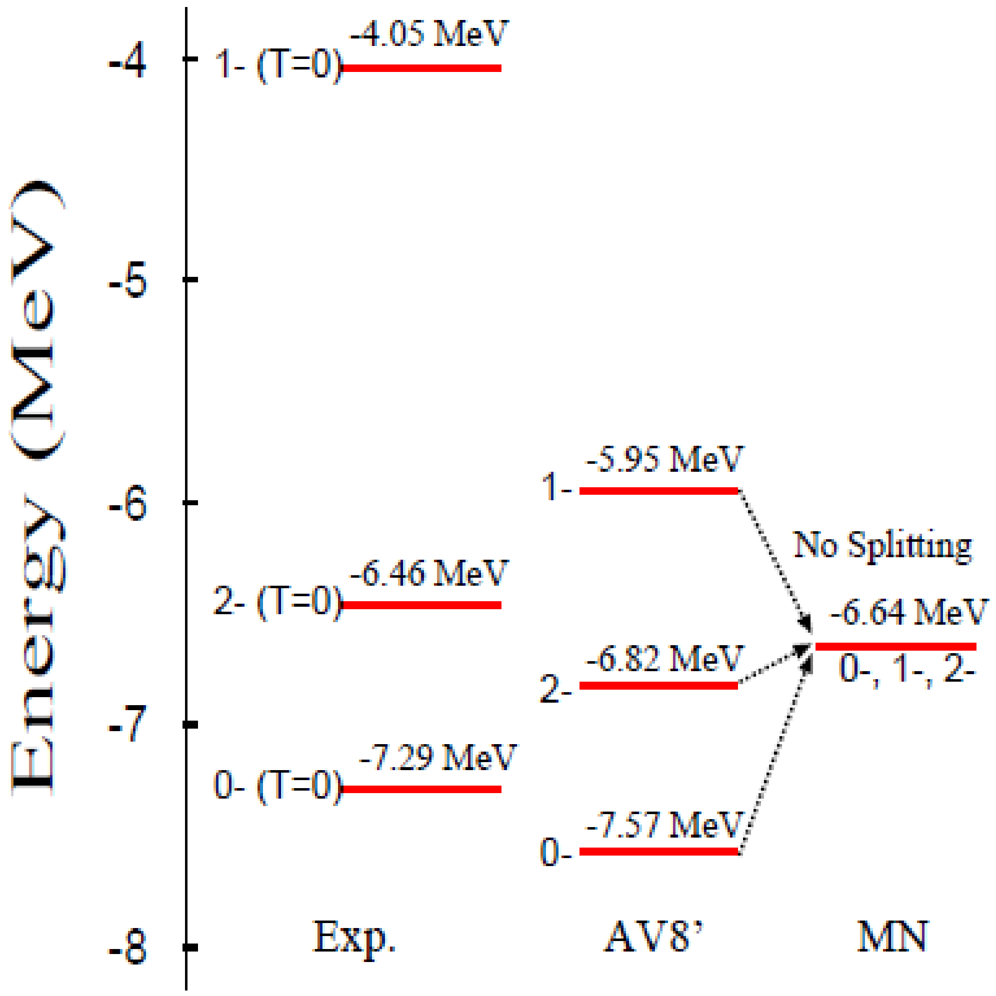
J^π / channel	0^+	1^+	2^+	0^-	1^-	2^-
$d(1^+)+d(1^+)$	$^1S_0$, $^5D_0$	$^5D_1$	$^5S_2$, $^1D_2$, $^5D_2$	$^3P_0$	$^3P_1$	$^3P_2$
$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$, $h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$	$^1S_0$	$^3S_1$, $^3D_1$	$^1D_2$, $^3D_2$	$^3P_0$	$^1P_1$, $^3P_1$	$^3P_2$

$^3P_1$ elastic phase shift (1-)



channel \ J^π	0^+	1^+	2^+	0^-	1^-	2^-
$d(1^+)+d(1^+)$	$^1S_0$, $^5D_0$	$^5D_1$	$^5S_2$, $^1D_2$, $^5D_2$	$^3P_0$	$^3P_1$	$^3P_2$
$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$, $h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$	$^1S_0$	$^3S_1$, $^3D_1$	$^1D_2$, $^3D_2$	$^3P_0$	$^1P_1$, $^3P_1$	$^3P_2$

Energy levels for negative parity states



Effective interaction (MN) gives same phase shift for 0-.1-.2- !

Summary

By using **the triple global vector representation method** with MRM, we calculated the **four nucleon scattering phase shifts** with a realistic interaction (AV8'+3NF) and an effective interaction (MN).

The distortion of the deuteron cluster for $^1S_0$ due to the tensor interaction is large.

For negative parity states, the energy splitting of ^3P_J is very large for the realistic interaction, but they are degenerating for the effective interaction.

Next

5-nucleon scattering