Microscopic cluster model calculation in four-nucleons system

Nagaoka Nat. Coll. Tech. Koji Arai

Niigata Univ. S. Aoyama
Niigata Univ. Y. Suzuki
ULB P. Descouvemont
ULB D. Baye

Introduction

- **Conventional microscopic cluster model (RGM)**

⇨ Simple cluster wave function
  (*S*-wave w.fs. for the α, ^{3}H, ^{3}He, d)
  Effective N-N interaction
  (central + LS, no tensor, e.g. Minnesota)

- **Extension of the microscopic cluster model**

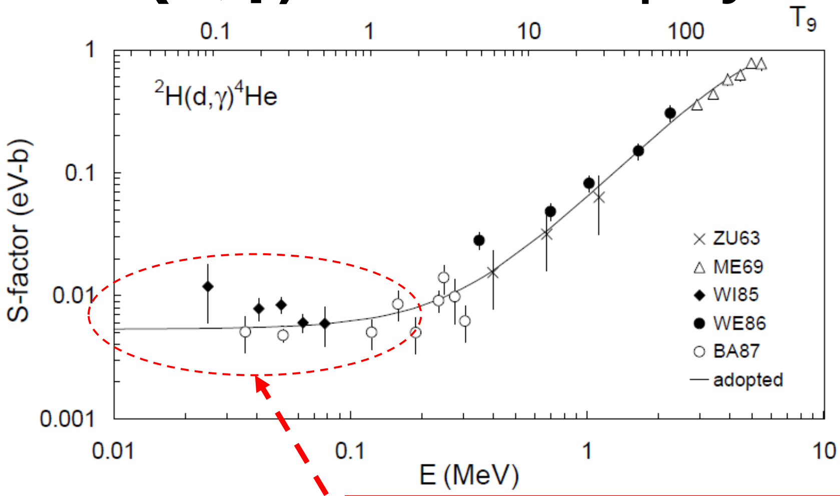
⇨ Precious few-body w.fs. for the clusters
  Realistic N-N interaction

(e.g. ^{3}He+p, *Arai et al. PRC81(2010)037301*
d+d, *Aoyama et al. FBS in press*)

➡ *Ab initio* calculation

Role of the tensor force

- <u>**$^2H(d,\gamma)^4He$**</u> Astrophysical S-factor



Nacre compilation
(C.Angulo et al.
NPA656('99)p.3)

d+d S-wave → D-state in the 0^+ g.s. of ^4He

H. J. Assenbaum and K. Langanke, PRC36('87)p.17

- <u>**^6Li *Q*-moment**</u> $Q_{exp} = -0.064 efm^2$

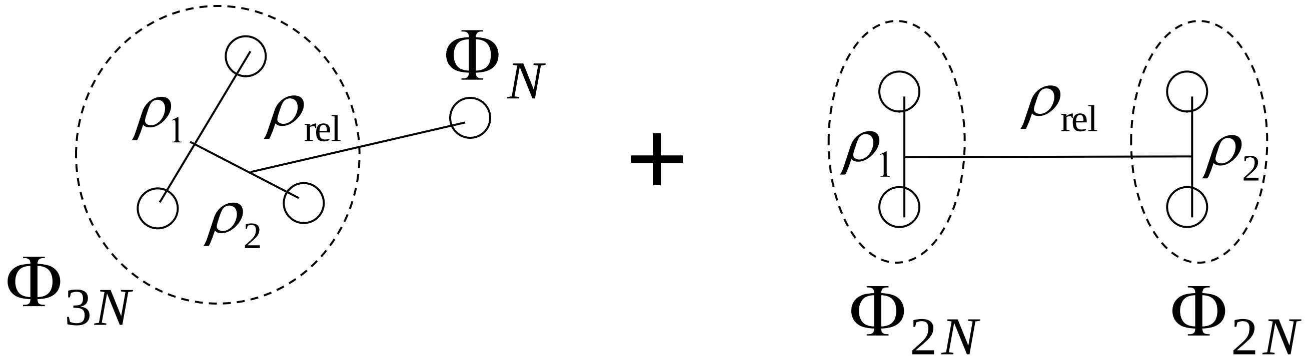
A. Csoto and R.G.Lovas, PRC46(1992)p.576
G.G. Ryzhikh, et al. NPA563(1993)p.247
→ *Cluster model gave positive Q-moment !*

Microscopic Cluster Model (RGM) in 4N system

[3N + N] + [2N + 2N]   two-cluster model

Total wave function

$$\Psi = A\{\Phi_{3N}(\rho_1,\rho_2)\,\Phi_N\ \chi(\rho_{\rm rel})\} + A\{\Phi_{2N}(\rho_1)\,\Phi_{2N}(\rho_2)\,\chi(\rho_{\rm rel})\}$$



$$\ell_1,\ \ell_2,\ \ell_{\rm rel} \le 2$$

N-N interaction

- Realistic N-N pot. (*Central*+*LS*+*Tensor*)

  AV8'

  G3RS (Tamagaki, PTP39('69)91)

  \+ Phenomenological 3BF (Hiyama et al., PRC70('04))

$$\sum_{i=1}^{2} V_i \, e^{-\alpha_i (r_{12}^2 + r_{23}^2 + r_{31}^2)}$$

- Effective N-N pot. (*Central* + *Coulomb*)

  Minnesota pot. (D. R. Thompson, NPA286('77)p.53)

  → 3-range Gaussian potential which reproduces np triplet and pp single s-wave scattering length and effective range

$\Phi_{3N}(\rho_1,\rho_2), \Phi_{2N}(\rho_1)$: Cluster intrinsic wavefunction

Precious three- and two-body wave function.

$$\Phi_{3N}(\rho_1,\rho_2) = A\{[\phi_{ST}[\chi_{\ell 1}(\rho_1)\chi_{\ell 2}(\rho_2)]_L]_J\}$$

W.f. is expanded by the Gaussian basis function.

Including the higher partial wave up to D-wave.

Basis set is selected by

the Stochastic Variational Method (SVM).

V.I. Kukulin and V. M. Krasnopol'sky, JPG3(1977) 795
K. Varga, Y. Suzuki, R. G. Lovas,NPA571(1994)447

Basis dimension ^{3}H, ^{3}He → N=30

^{2}H → N=8

Stochastic Variational Method (SVM)

<u>Basis dimension is fixed as *N*</u>

(1) $\Phi_1, \Phi_2, \cdots\cdots\cdots, \Phi_N \Rightarrow E^*$

(2) Φ_c (Selected in random)

$$\Phi_1,\ \Phi_2,\ \cdots\cdots\ \Phi_i\ \cdots\cdots\ ,\Phi_N$$

$$E_1,\ E_2,\ \cdots\cdots\ E_i\ \cdots\cdots\ ,E_N$$

- If E_i is lowest and $E_i < E^*$, then Φ_i is replaced by Φ_c.

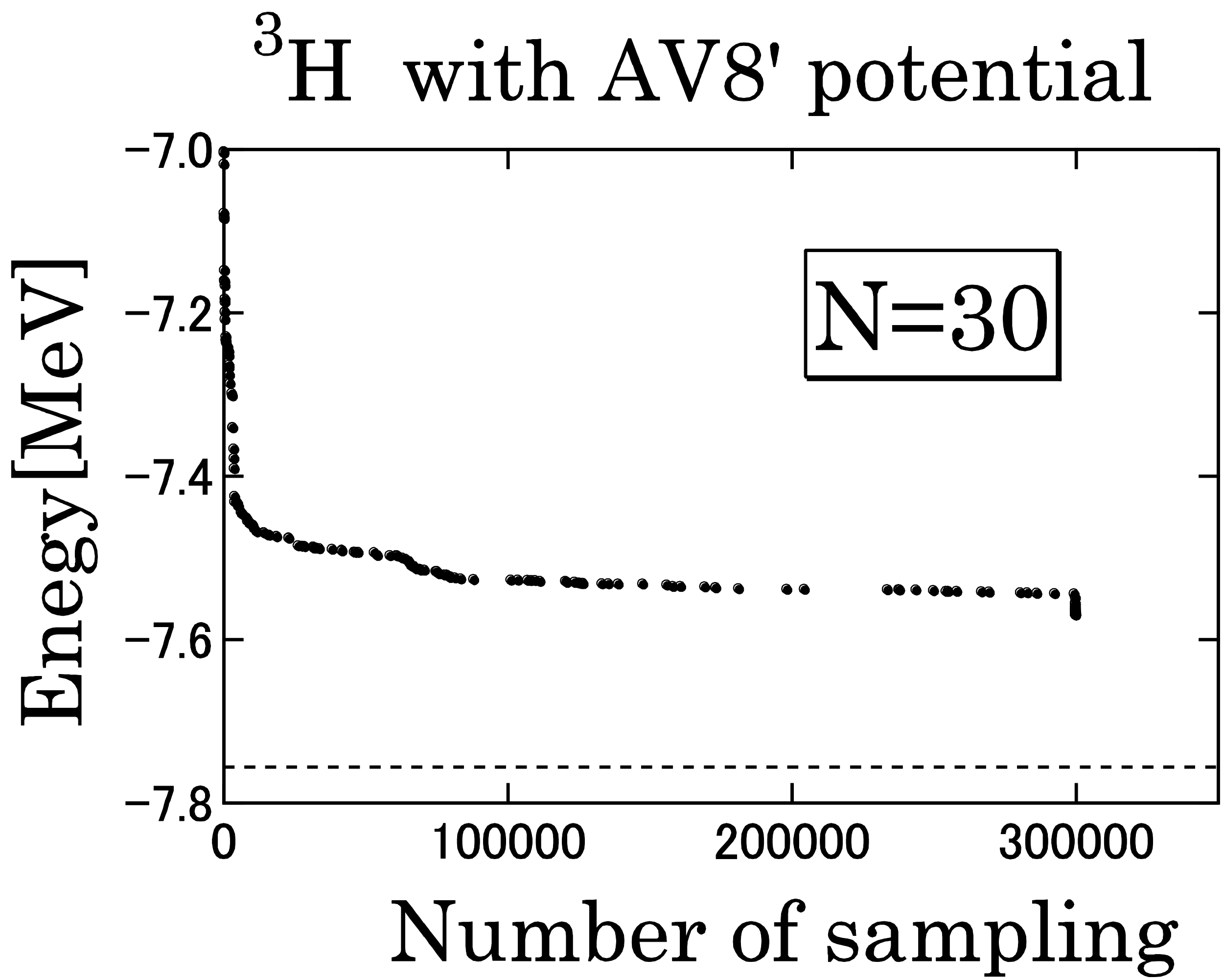
(3) In this new basis function, only width parameters (b_1, b_2, ….) are replaced in the Stochastic way but at a very small range, $b - \delta < b + \delta$ where b is the parameter in $\exp[-(r/b)^2]$.

(4) Go to (2) if the energy is not converged.

⇩ If converged

(5) In all basis function, one by one, only width parameters (b_1, b_2, ….) are replaced in the stochastic way at $b - \delta < b + \delta$

➔ continue until energy is converged !



^{3}H with AV8' potential
N=30
Enegy[MeV]
−7.0
−7.2
−7.4
−7.6
−7.8
0
100000
200000
300000
Number of sampling

- Threshold

AV8' pot.+3BF

d : $E = -2.18$MeV, $P_D = 5.9\%$ $E_{exp} = -2.22$MeV
t : $E = -8.22$MeV, $P_D = 8.4\%$ $E_{exp} = -8.48$MeV
h : $E = -7.55$MeV, $P_D = 8.3\%$ $E_{exp} = -7.72$MeV
α : $E = -27.99$MeV, $P_D = 13.8\%$ $E_{exp} = -28.30$MeV

G3RS pot.+3BF

d : $E = -2.13$MeV, $P_D = 5.0\%$ $E_{exp} = -2.22$MeV
t : $E = -8.24$MeV, $P_D = 6.9\%$ $E_{exp} = -8.48$MeV
h : $E = -7.58$MeV, $P_D = 6.9\%$ $E_{exp} = -7.72$MeV
α : $E = -27.99$MeV, $P_D = 11.2\%$ $E_{exp} = -28.30$MeV

MN pot.（ Cental + Coulomb)

d : $E = -2.10$ MeV, $E_{exp} = -2.22$MeV
t : $E = -8.38$ MeV, $E_{exp} = -8.48$MeV
h : $E = -7.70$ MeV, $E_{exp} = -7.72$MeV
α : $E = -29.94$ MeV, $E_{exp} = -28.30$MeV

$\chi(\rho_{\mathrm{rel}})$: Cluster relative wavefunction

Microscopic R-matrix method (Baye, Descouvemont)

a : channel raidus

$$\begin{cases} \rho_{\mathrm{rel}} < a \text{ --- Gaussian expansion} \\ \rho_{\mathrm{rel}} > a \text{ --- Exact Coulomb function} \end{cases}$$

Microscopic *R*-matrix method

(D.Baye, et al. NPA291 ('77)230)

- Schrodinger e.q. $\left(\hat{H}+\hat{L}-E\right)\Psi^{int}=\hat{L}\,\Psi^{ext}$
- Bloch operator $\hat{L}(E)=\left(\hbar^2/2\mu r\right)\delta(r-a)\left[\frac{d}{dr}r-b\right]$

$$\begin{cases} b=0 & \text{for open channel} \\ b=2kaW'(2ka)/W(2ka) & \text{for closed channel} \end{cases}$$

- W.F ($r<a$)

Gaussian expansion

$$\Psi^{int}=\sum_{\alpha k} f_{\alpha k}u_{\alpha k}\varphi_{\alpha}$$

$$\begin{cases} \alpha:(\ell, I) \\ u_{\alpha k}: \text{Gaussian basis function} \\ \varphi_{\alpha}: \text{Cluster internal function} \end{cases}$$

$$\sum_{\alpha k} f_{\alpha k} \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{H} + \hat{L} - E \middle| u_{\alpha k} \varphi_k \right\rangle = \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{L} \middle| \Psi^{ext} \right\rangle$$

$$C_{\alpha' k', \alpha k} \equiv \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{H} + \hat{L} - E \middle| u_{\alpha k} \varphi_k \right\rangle \qquad W_{\alpha' k'} \equiv \left\langle u_{\alpha' k'} \varphi_{k'} \middle| \hat{L} \middle| \Psi^{ext} \right\rangle$$

$$\Rightarrow \quad \Psi^{int} = \sum_{\alpha k} f_{\alpha k} u_{\alpha k} \varphi_\alpha = \sum_{\alpha k \alpha' k'} C^{-1}_{\alpha k, \alpha' k'} W_{\alpha' k'} u_{\alpha k} \varphi_\alpha$$

$$\Psi^{ext} = \sum_{\alpha_1} r^{-1}_{\alpha_1} \upsilon^{1/2}_{\alpha_1} C_{\alpha_1} \left\{ I_{\alpha_1} \delta_{\alpha_1 \alpha_0} - U_{\alpha_1 \alpha_0} O_{\alpha_1} \right\} \varphi_{\alpha_1} + \sum_{\alpha_2} C_{\alpha_1} W_{-\eta, \ell + 1/2}(2kr)/kr\, \varphi_{\alpha_2}$$

S-matrix

$$U = \left(Z^*\right)^{-1} Z \qquad \because \quad \Psi^{ext}(a) = \Psi^{int}(a)$$

$$Z_{\alpha\alpha'} = I_\alpha \delta_{\alpha\alpha'} - R_{\alpha\alpha'} (k_{\alpha'} a)\, I'_{\alpha'} (k_{\alpha'} a)$$

R-matrix

$$R_{\alpha\alpha'} = \hbar^2 a / 2 \left(\mu_\alpha \mu_{\alpha'}\right)^{-1/2} \left(k_\alpha / k_{\alpha'}\right)^{1/2} \sum_{kk'} u_{\alpha k}(a) C^{-1}_{\alpha k, \alpha' k'} u_{\alpha' k'}(a)$$

$^{3}He+p$ elastic scattering phase shifts

Ref. PRC81(2010)037301

^{3}He+p S- and P- wave pahse shfits

Comparison between two cluster model calculations

- ***Minnesota potential*** ---   An effective N-N potential
  (Central + LS + Coulomb)

  D.R.Thompson and Y.C.Tang, NPA286(1977)p.53
  I.Reichstein and Y.C.Tang, NPA158(1970)p.529

Model space --- [^{3}He($1/2^+$)+p] $\oplus$ [d(0^+,1^+)+2p(0^+)]

^{3}He --- p+p+n w.f but the *S*-wave only

d, 2p --- p+n and p+p w.f. but the S-wave only

- ***AV8' pot.*** ---- A realistic N-N potential
  (Central + LS + Tensor + Coulomb)

  B.S.Pudliner, PRC56(1997)p.1720

Model space ---- [$^3He(1/2^{\pm},3/2^{\pm},5/2^{\pm})+p$] $\oplus$
[$d(0^+,1^+)+2p(0^+)$]

^{3}He --- p+p+n w.f. including higher partial waves
up to D-wave

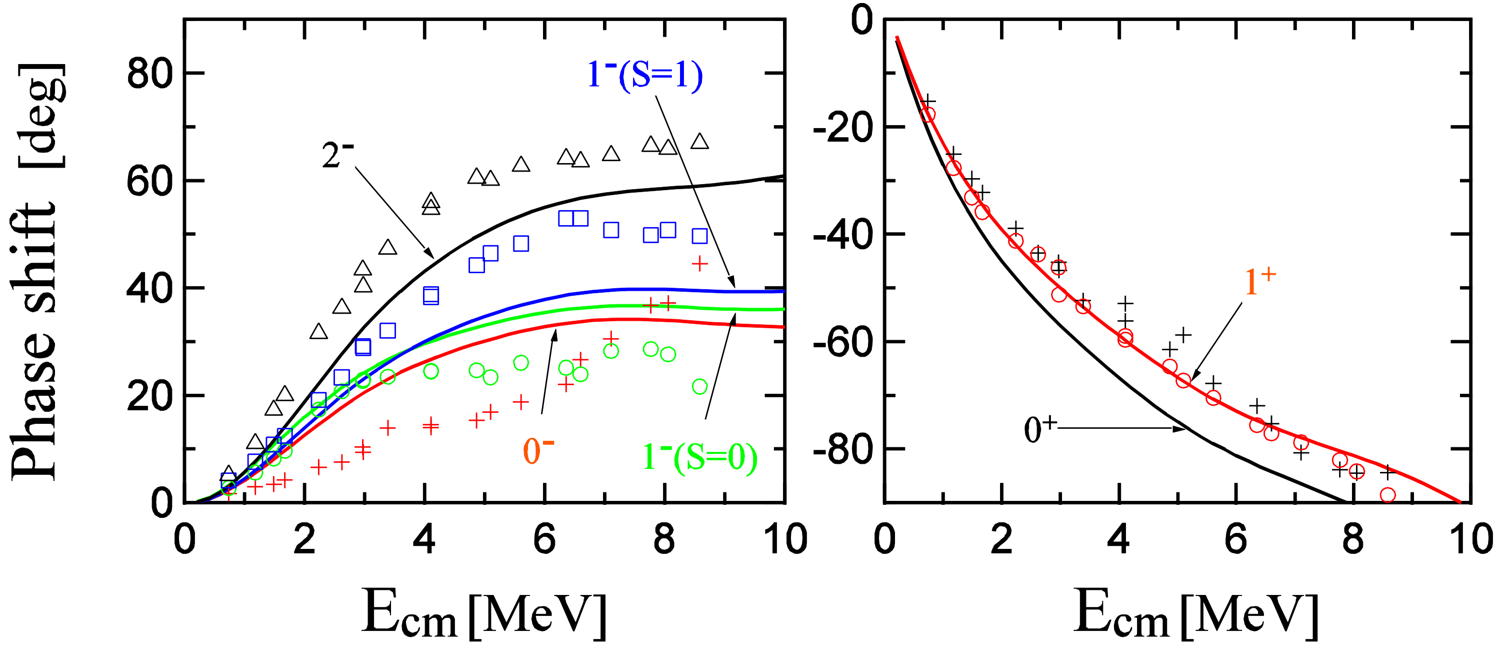
Excited states of ^{3}He --- 10 Gaussian basis
selected randomly

$d(1^+)$ --- p+n S+D wave

d(0+) and 2p(0+) --- p+n and p+p S-wave

^{3}He+p P-wave phase shifts with MN pot.(*No tensor*)

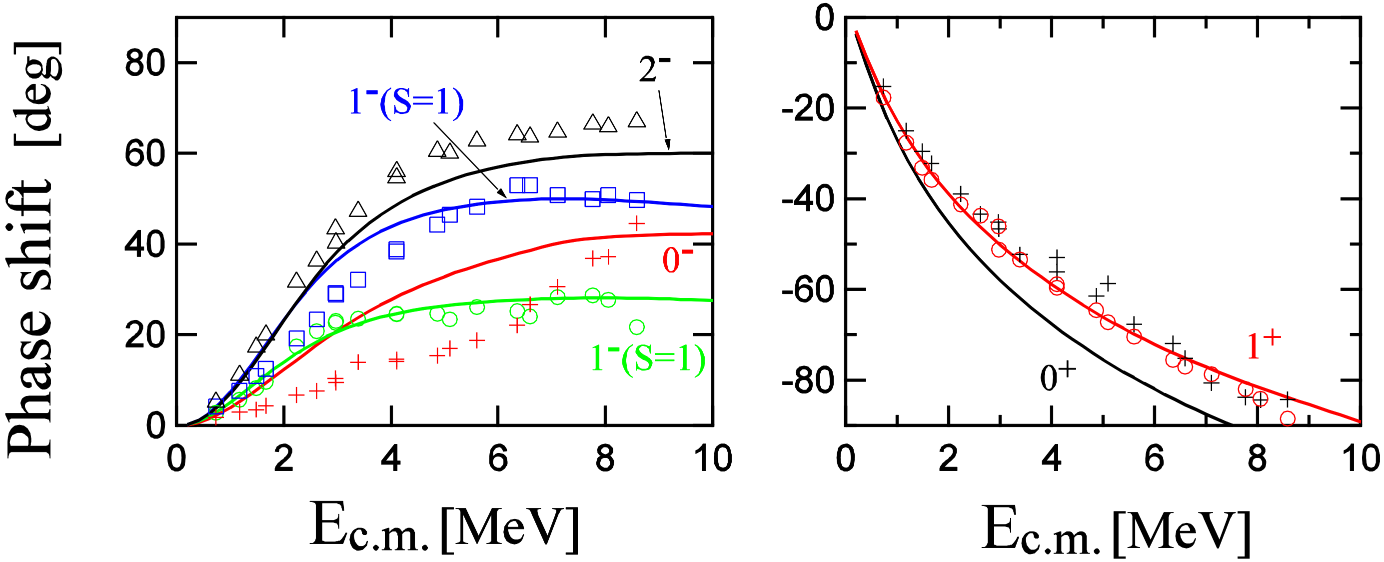
^{3}He → p+p+n S-wave w.f.



[^{3}He(1/2$^+$)+p] *single channel calculation*

^{3}He+p P-wave phase shifts with AV8' pot.

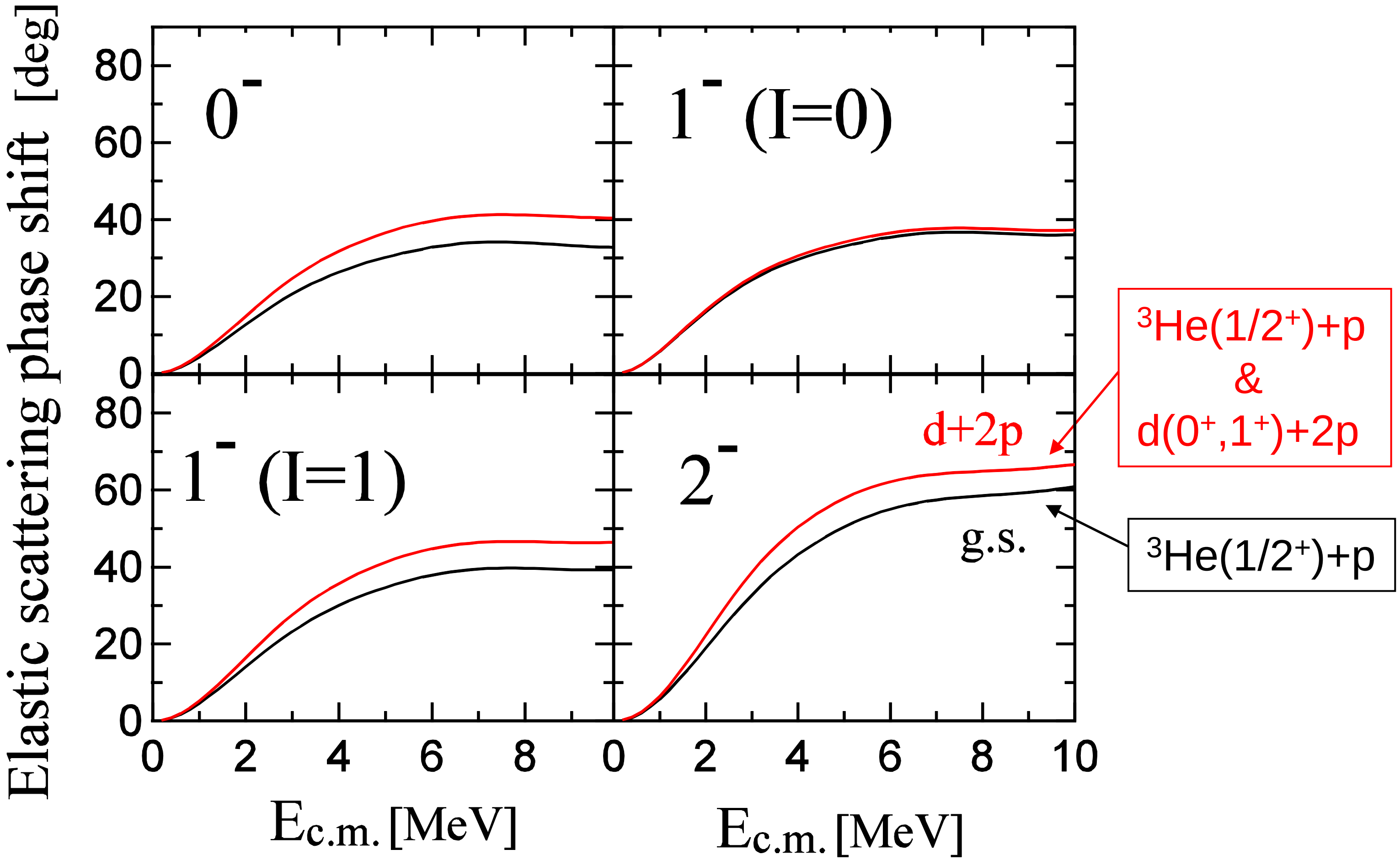
^{3}He → p+p+n w.f. including the higher partial waves



$$[\,^3\mathrm{He}(1/2^{\pm},3/2^{\pm},5/2^{\pm})+\mathrm{p}\,] \oplus [\mathrm{d}(0^+,1^+)+2\mathrm{p}(0^+)]$$

multi-channel calculation

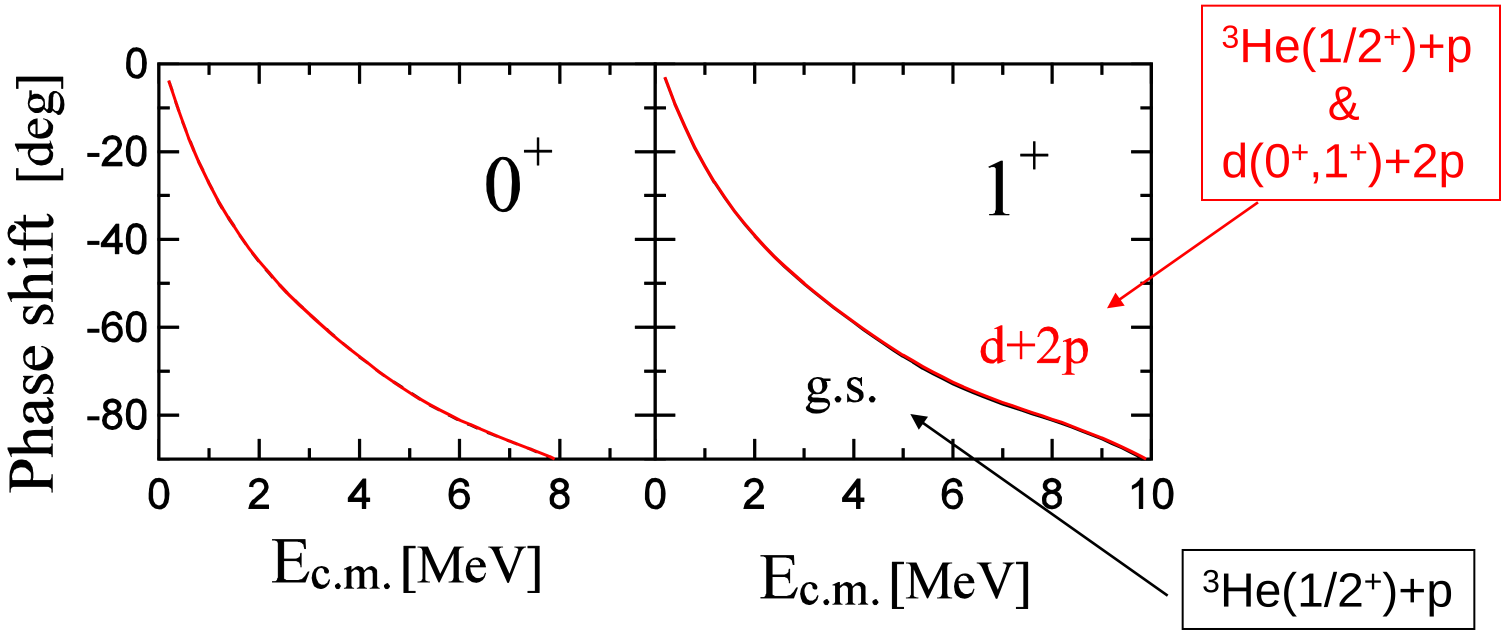
^{3}He+p MN pot. (^{3}He → p+p+n S-wave w.f.)



Minor contribution of the d+2p channel !!

^{3}He+p S-wave phase shifts with the MN pot.

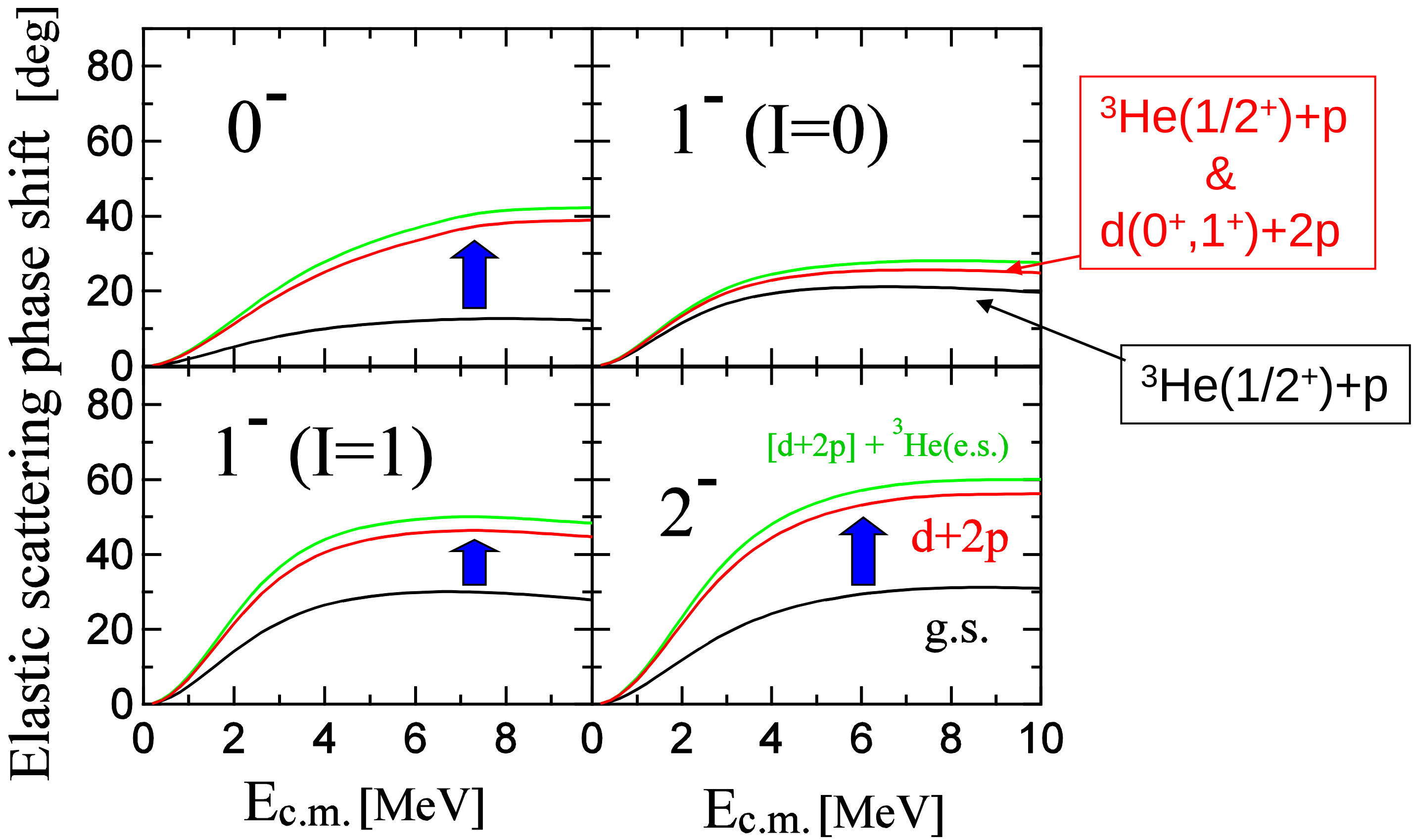
^{3}He → p+p+n S-wave w.f.



Contribution of the d+2p channel is negligible.

^{3}He+p AV8' pot.

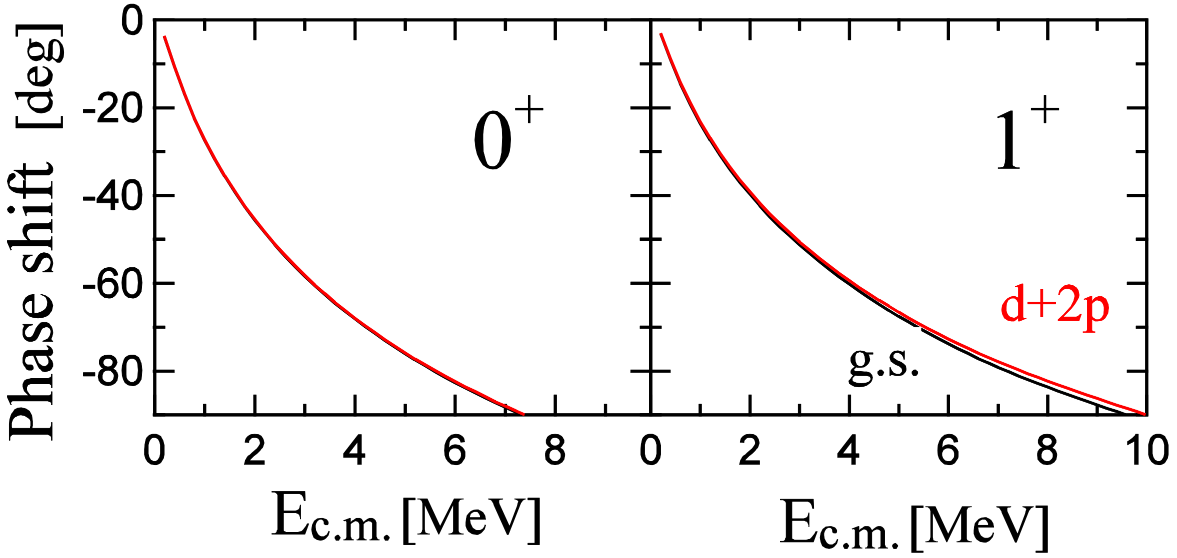
^{3}He → p+p+n w.f. including the higher partial waves



Significant contribution of the d+2p channel !!

^{3}He+p AV8' pot.

^{3}He → p+p+n w.f. including the higher partial waves



Contribution of the d+2p channel is negligible.

Summary

Two microscopic cluster model calculation

- **MN pot. (no tensor) & p+p+n w.f. for ^{3}He**
  (S-wave only)

→ **Minor role of the additional d+2p channel**

- **AV8' pot. (with tensor) & p+p+n w.f. for ^{3}He**
  (higher partial waves)

→ **Important role of the additional d+2p channel**
only for the resonance states

→ **Strong cluster distortion effect**
for the resonance states

$^{2}H(d,\gamma)^{4}He$, $^{2}H(d,p)^{3}H$ and $^{2}H(d,n)^{3}He$ reactions

Ref. *PRL23 (2011) 132502*

Cross section of the capture reaction

$$\sigma_\gamma^{E\lambda}(E)=\frac{2J_f+1}{(2I_1+1)(2I_2+1)}\frac{8\pi}{\hbar}\left(\frac{E_\gamma}{\hbar c}\right)\frac{(\lambda+1)}{\lambda\left((2\lambda+1)!!\right)^2}$$

$$\times\sum_{J_iI_i\ell_i}\frac{1}{(2\ell_i+1)}\left|\left\langle\Psi^{J_f\pi_f}\left\|M_\lambda^E\right\|\Psi_{\ell_iI_i}^{J_i\pi_i}\right\rangle\right|^2$$

Present cal.: E2 transtion ($2^+ \rightarrow 0^+$ g.s.)

Cross section of the transfer reaction

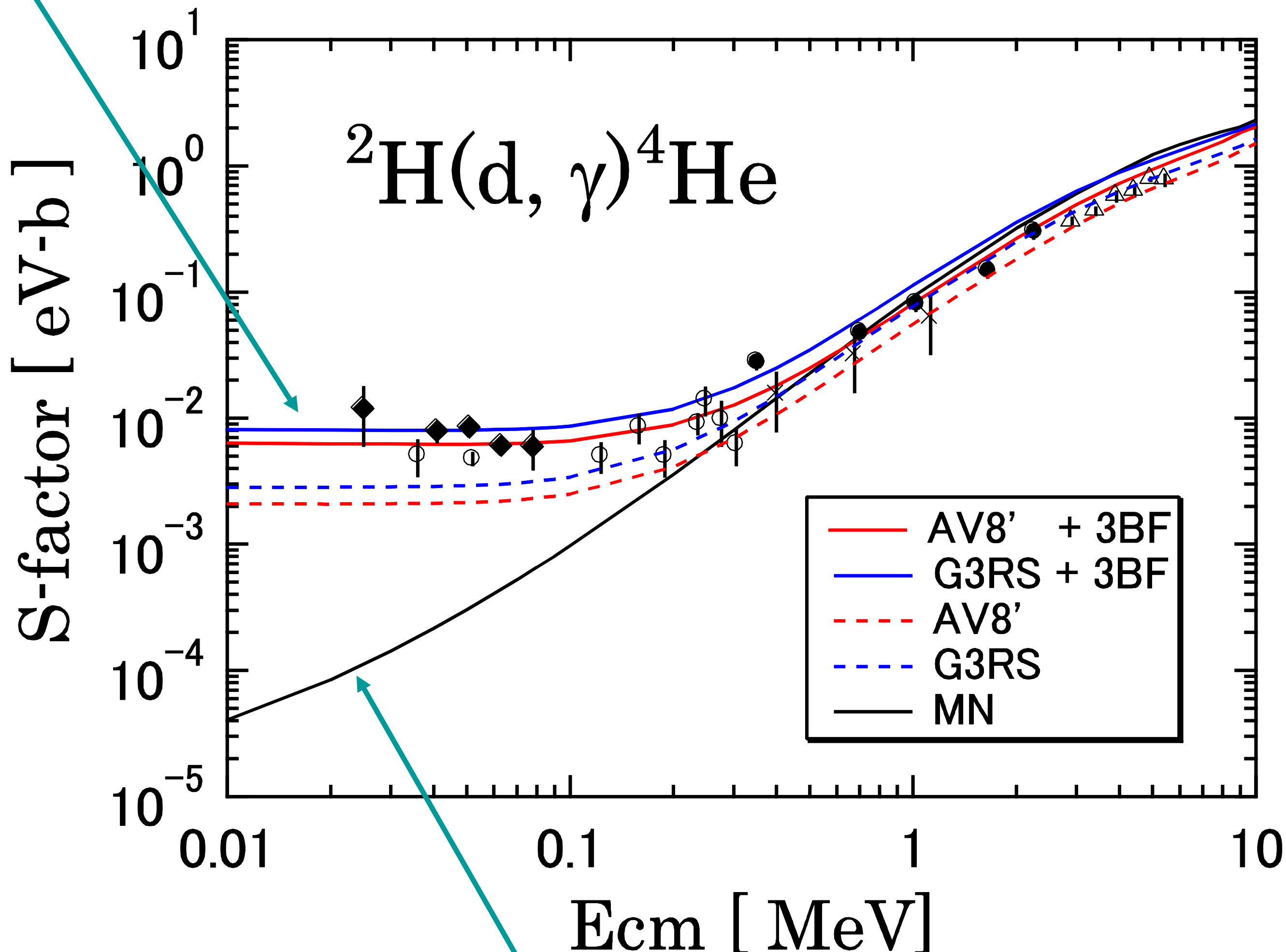
$$\sigma(E)=\frac{\pi}{k^2}\sum_{J\pi}\frac{2J+1}{(2I_1+1)(2I_2+1)}\sum_{\ell_i\ell_fI_iI_f}\left|U_{i\,\ell_iI_i,f\,\ell_fI_f}^{J\pi}(E)\right|^2$$

Presnt cal : $J^\pi = 0^\pm,\ 1^\pm,\ 2^\pm$

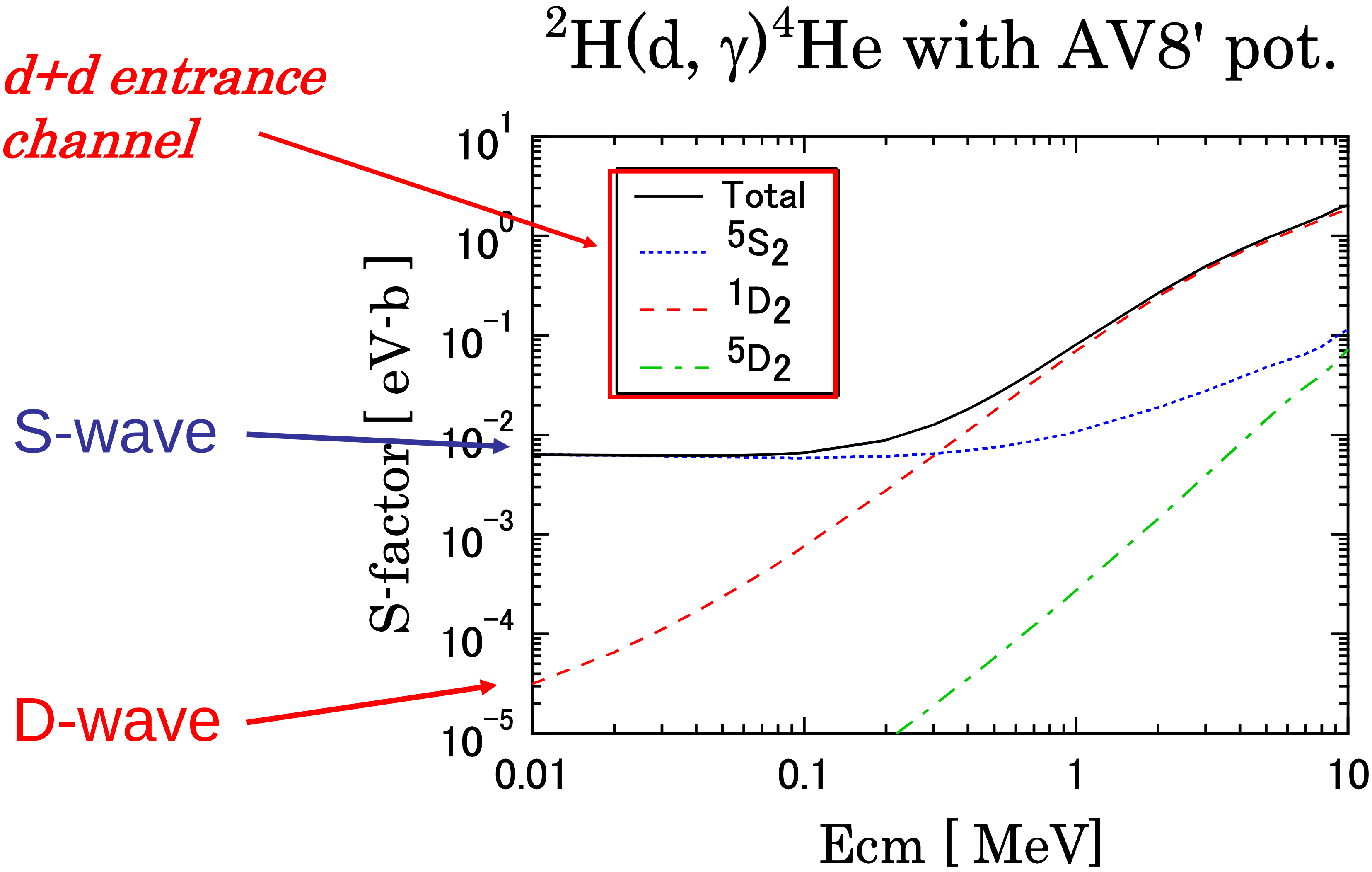
- K.Arai, D.Baye, P.Descouvemont, NPA699(02)p.963

Capture reaction

Realistic force

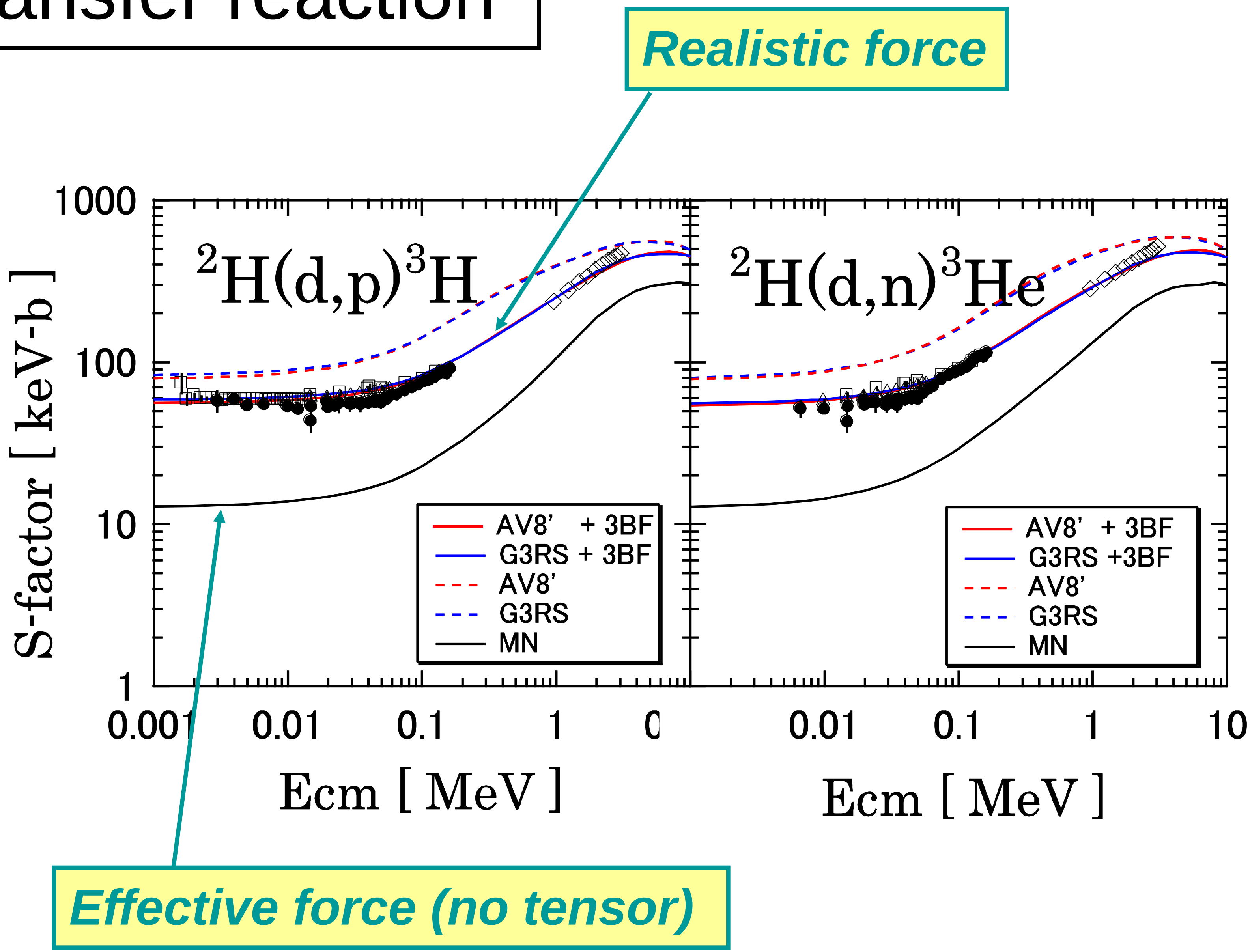


Effective force (no tensor)

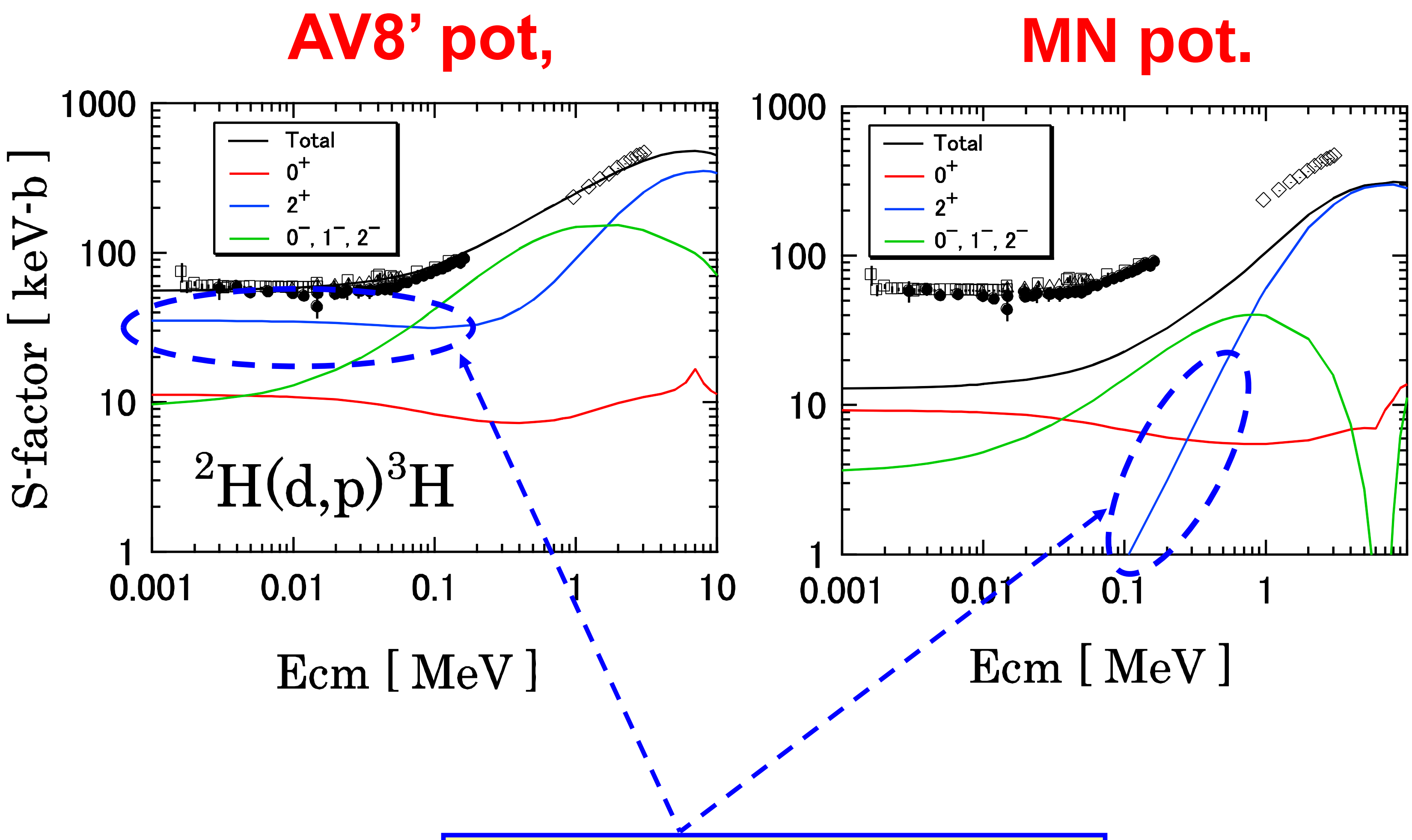


- d+d ***S-wave*** → ^{4}He 0^+ (***L=2, S=2***) ***D-wave*** component
- d+d D-wave → ^{4}He 0^+ (L=0, S=0) S-wave component

Transfer reaction

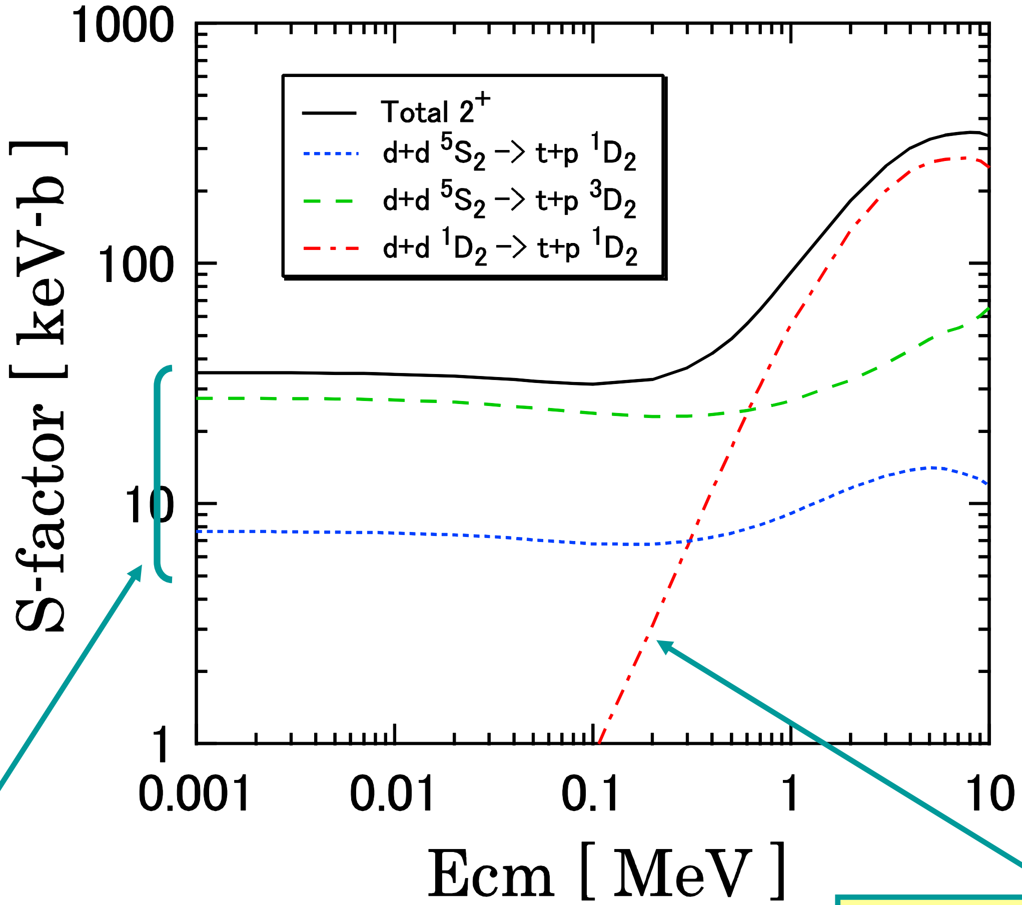


Contribution of each spin parity state in $^2H(d,p)^3H$



S-factor by the 2+ state

2^+ contribution in $^2H(d,p)^3H$ is decomposed according to the entrance & exit channel



AV8' pot.

d+d S-wave → t+p D-wave

d+d D-wave → t+p D-wave

Summary

- $^2H(d,\gamma)^4He$

D-wave component

$$\begin{cases} E_{cm} < 0.3\text{MeV} & d+d\ \ S\text{-wave} \rightarrow (\text{L},\text{S}) = (2,2) \\ E_{cm} > 0.3\text{MeV} & d+d\ \ D\text{-wave} \rightarrow (\text{L},\text{S}) = (0,0) \end{cases}$$

S-wave component

- $^2H(d,p)^3H$, $^2H(d,n)^3He$

$J^{\pi}=2^{+}$ contribution

Coupled by tensor force

$$\begin{cases} E_{cm} < 0.3\text{MeV} & d+d\ \ S\text{-wave} \rightarrow t+p\ D\text{-wave} \\ E_{cm} > 0.3\text{MeV} & d+d\ \ D\text{-wave} \rightarrow t+p\ D\text{-wave} \end{cases}$$

- Tensor force plays an essential role to reproduce the astrophysical S-factor not only in the capture reaction, $^{2}H(d,\gamma)^{4}He$, but also in the transfer reaction, $^{2}H(d,p)^{3}H$ and $^{2}H(d,n)^{3}He$.