Eikonal models for breakup reactions of halo nuclei

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Eikonal models for breakup reactions of halo nuclei

- Introduction
  - Eikonal approximation in potential model
- Principles of eikonal models
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- Applications
  - Breakup of ^{6}He two-neutron halo nucleus
  - Breakup of ^{11}Li two-neutron halo nucleus
  - Breakup of ^{31}Ne and « island of inversion »
  - Near/far decomposition of angular breakup cross sections
- Conclusion

Introduction

Physical motivations

- Exotic nuclei: short-lived and weakly bound (halo nuclei)
- Breakup (dissociation, excitation to the continuum):

  main tool to extract spectroscopic properties during a short lifetime
- Cause of dissociation: differential nuclear and Coulomb forces acting on the projectile components (clusters, nucleons)

Descriptions

- Theoretical models: semi-classical, eikonal approximations, CDCC, …
- Analysis with Near/Far decomposition

Eikonal approximation in potential model

Potential scattering

$$\left(\frac{p^2}{2\mu} + V(r)\right)\psi = E\psi \qquad E = \hbar^2 k^2/2\mu$$

$$\psi(\boldsymbol{r}) = e^{ikz}\hat{\psi}(\boldsymbol{r})$$

$$\left(\frac{p^2}{2\mu} + vp_z + V(r)\right)\hat{\psi} = 0$$

$$v = \hbar k/\mu$$

Approximation (high energy) $\quad |\Delta\hat{\psi}| \ll k|\nabla\hat{\psi}|$

$$\left(-i\hbar v\frac{\partial}{\partial z} + V(r)\right)\hat{\psi} = 0$$

Eikonal wave function $\quad \boldsymbol{r} = (\boldsymbol{b}, z)$

$$\psi(\boldsymbol{r}) = \exp\left[ikz - \frac{i}{\hbar v}\int_{-\infty}^{z} V(\boldsymbol{b}, z')dz'\right]$$

R.J. Glauber, Lectures on Theoretical Physics, vol.1 (Interscience, 1959) p.315

Scattering amplitude (exact if ψ exact)

$$f(\theta) = -\frac{\mu}{2\pi\hbar^2}\int \exp(-i\boldsymbol{k}'\cdot\boldsymbol{r})V(r)\psi(\boldsymbol{r})d\boldsymbol{r}$$

Transfered momentum $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$

Approximation (small angles) $\boldsymbol{k}'\cdot\boldsymbol{r} - kz = \boldsymbol{q}\cdot\boldsymbol{r} \approx \boldsymbol{q}\cdot\boldsymbol{b}$ $\quad \boldsymbol{r} = (\boldsymbol{b}, z)$

$$\begin{aligned} f(\theta) &\approx -\frac{\mu}{2\pi\hbar^2}\int d\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\int_{-\infty}^{+\infty} dz\, V(\boldsymbol{b},z)\exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{z} V(\boldsymbol{b},z')dz'\right] \\ &= \frac{ik}{2\pi}\int d\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\left[1 - e^{i\chi(b)}\right] \end{aligned}$$

Phase shift function $\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} V(\boldsymbol{b},z)dz$

Eikonal scattering amplitude

$$f(\theta) = ik\int_0^{\infty} bdbJ_0(qb)\left[1 - e^{i\chi(b)}\right]$$

Eikonal Coulomb phase

$$\chi_C(b) = -\frac{Z_1 Z_2 e^2}{\hbar v} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{b^2 + z^2}}\, dz$$

Divergence!

Cut off *a* >> *b*

$$\chi_C(b) \approx 2\eta \ln \frac{b}{2a}$$

Eikonal Coulomb scattering amplitude

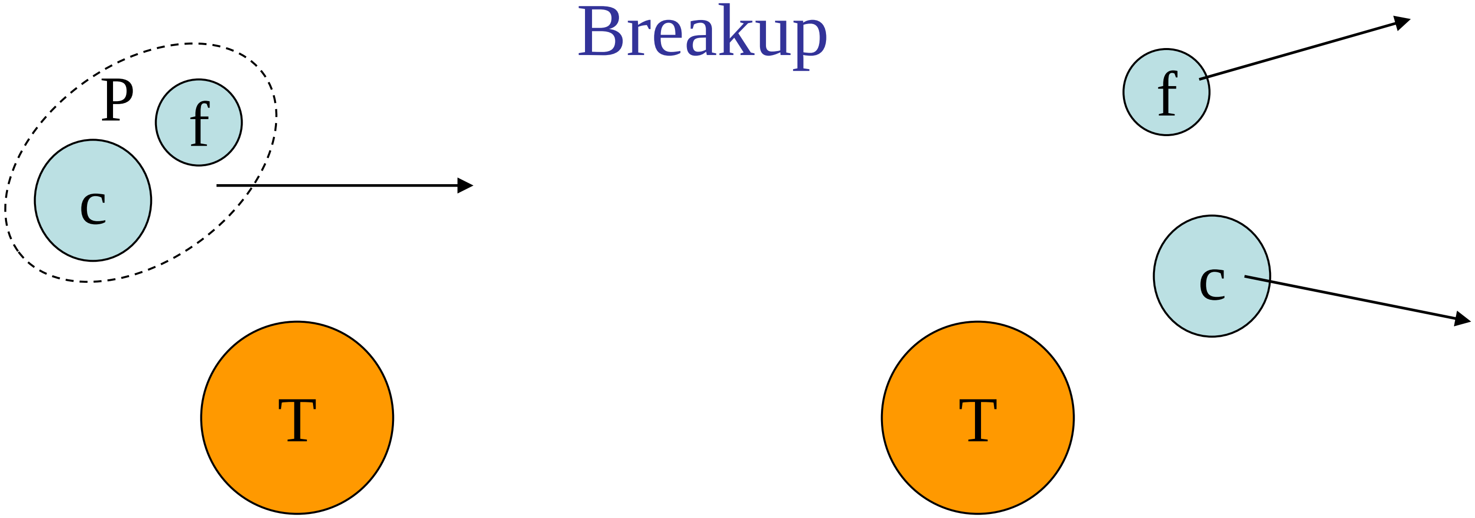
$$f_C^{\text{eik.}}(\theta) = f_C(\theta) e^{-2i\eta \ln 2ka}$$

Exact Coulomb scattering amplitude

$$f_C(\theta) = -\frac{\eta}{2k \sin^2 \frac{1}{2}\theta}\, e^{2i(\sigma_0 - \eta \ln \sin \frac{1}{2}\theta)}$$

R.J. Glauber, Lectures on Theoretical Physics, vol.1 (Interscience, 1959) p.315

Breakup



Projectile (P) = core (c) + fragment(s) (f)
Target (T): no excitation (« elastic » breakup)

Model: 3- or 4-body Schrödinger equation

Ingredients: - projectile bound-state wave function
(exact separation energy, correct asymptotics)
- c-f scattering states at various energies
(3-body scattering states when 3-cluster projectile)

3-body model of breakup (2-cluster projectile)

Target T $\quad m_T,\ Z_T e$

Core c of projectile $\quad m_c,\ Z_c e$

Fragment f of projectile $\quad m_f,\ Z_f e$

3-body Hamiltonian

$$H = \frac{p_f^2}{2m_f} + \frac{p_c^2}{2m_c} + \frac{p_T^2}{2m_T} + V_{cf} + V_{fT} + V_{cT} - T_{\rm c.m.}$$

V_{cf} : core-fragment real potential
(angular-momentum dependent)

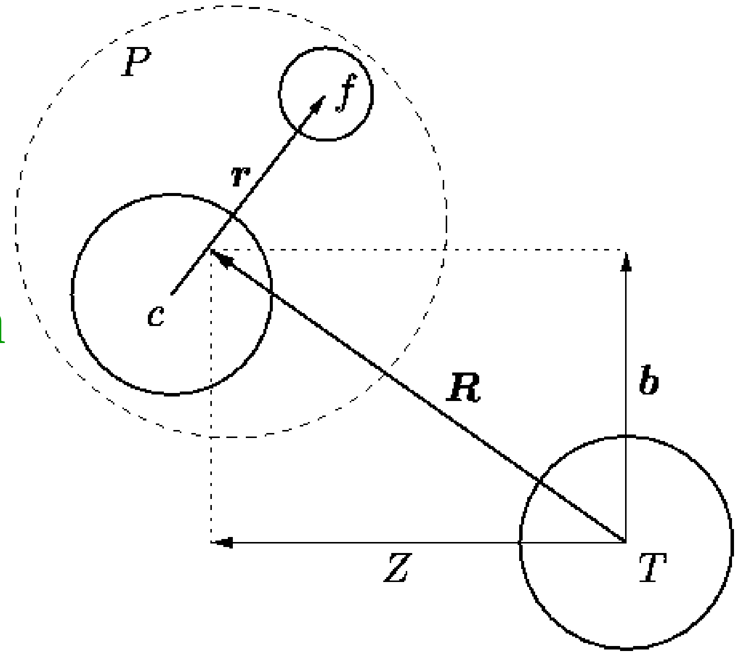
V_{fT} : fragment-target optical potential

V_{cT} : core-target optical potential

$T_{\rm c.m.}$: center-of-mass kinetic energy

Jacobi coordinates and momenta

- internal $(\boldsymbol{r}, \boldsymbol{p})$
- relative motion $(\boldsymbol{R}, \boldsymbol{P})$



Projectile internal Hamiltonian

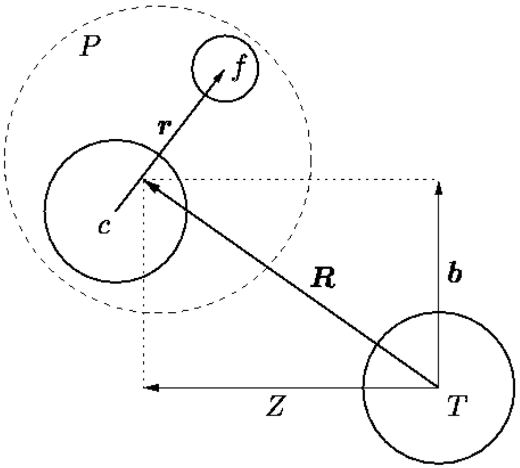
$$H_0 = \frac{p^2}{2\mu_{cf}} + V_{cf}(r)$$

3-body Schrödinger equation

$$\left(\frac{P^2}{2\mu_{PT}} + H_0 + V_{PT}(\boldsymbol{R}, \boldsymbol{r}) \right) \Psi(\boldsymbol{R}, \boldsymbol{r}) = E_{\text{tot}} \Psi(\boldsymbol{R}, \boldsymbol{r})$$

$$V_{PT}(\boldsymbol{R}, \boldsymbol{r}) = V_{cT}\left(\boldsymbol{R} - \frac{m_f}{m_P}\boldsymbol{r} \right) + V_{fT}\left(\boldsymbol{R} + \frac{m_c}{m_P}\boldsymbol{r} \right)$$

Dynamical eikonal approximation (DEA)



Initial momentum K large

To remove the rapid variation of wave function

$$\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}\hat{\Psi}(\boldsymbol{R},\boldsymbol{r})$$

Tranformed 3-body Schrödinger equation

$$\left(\frac{P^2}{2\mu_{PT}} + vP_Z + H_0 + V_{PT}(\boldsymbol{R},\boldsymbol{r}) - E_0\right)\hat{\Psi} = 0$$

Dynamical eikonal approximation

$$|\Delta_R\hat{\Psi}| \ll K|\nabla_R\hat{\Psi}|$$

$$i\hbar v\frac{\partial}{\partial Z}\hat{\Psi}(\boldsymbol{R},\boldsymbol{r}) = (H_0 + V_{PT} - E_0)\,\hat{\Psi}(\boldsymbol{R},\boldsymbol{r})$$

Dynamical eikonal approximation (DEA)

$$i\hbar v \frac{\partial}{\partial Z} \hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}) = (H_0 + V_{PT} - E_0)\, \hat{\Psi}(\boldsymbol{R}, \boldsymbol{r})$$

Formally identical to a time-dependent Schrödinger equation (TDSE) with straight-line trajectories

$$t = Z/v$$

$$i\hbar \frac{\partial}{\partial t} \tilde{\Psi}(\boldsymbol{b}, \boldsymbol{r}, t) = (H_0 + V_{PT} - E_0)\, \tilde{\Psi}(\boldsymbol{b}, \boldsymbol{r}, t)$$

$$\hat{\Psi}_{\mathrm{DEA}}(\boldsymbol{R}, \boldsymbol{r}) = \tilde{\Psi}_{\mathrm{TDSE}}(\boldsymbol{b}, \boldsymbol{r}, Z/v)$$

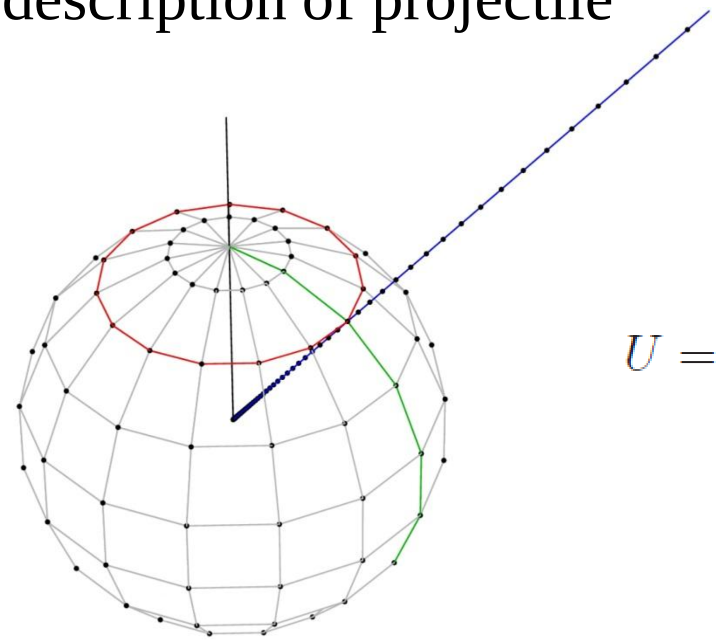
- Uses semi-classical codes but can provide angular distributions
- Numerical resolution
- Dynamical and interference effects included
- No Coulomb problem

D. B., P. Capel, G. Goldstein, Phys. Rev. Lett. 95 (2005) 082502
G. Goldstein, D. B., P. Capel, Phys. Rev. C 73 (2006) 024602

Numerical resolution of time-dependent Schrödinger equation

Projectile frame: moving target

Spherical + radial mesh
for description of projectile



Propagation in time along
semi-classical trajectory

$$\psi(t+\Delta t) = U(t+\Delta t, t)\psi(t)$$

$$U = e^{-i\frac{1}{2}\Delta t V(t+\Delta t)} e^{-i\Delta t H_0} e^{-i\frac{1}{2}\Delta t V(t)} + O(\Delta t^3)$$

No multipole expansion

No partial wave decomposition

V.S. Melezhik, Phys. Lett. A 230 (1997) 203
P. Capel, D.B., V.S. Melezhik, Phys. Rev. C 68 (2003) 014612

Breakup amplitudes

Expansion of solution of TDSE

$$\lim_{Z\to+\infty} \tilde{\Psi}_{\mathrm{TDSE}}(b, \boldsymbol{r}, Z/v) = \frac{1}{r}\sum_{lm} \psi_{lm}(b, r) Y_l^m(\Omega_r)$$

Projection on distorted partial waves

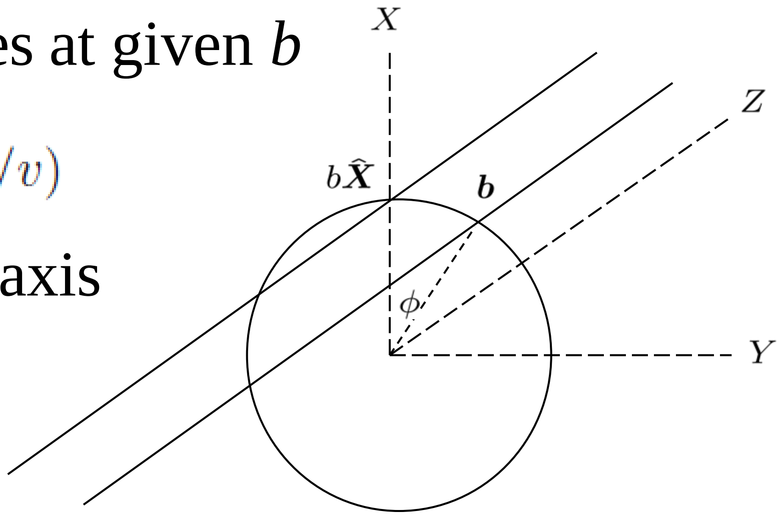
$$S_{klm}(b) = e^{i(\sigma_l+\delta_l-l\pi/2)} \int_0^\infty u_{kl}(r)\psi_{lm}(b, r)dr$$

(performed numerically on the 3D mesh)

Incoherent and coherent eikonal approximations

Incoherent assumption:

- same wave function for all trajectories at given *b*



$$\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}\tilde{\Psi}(b\hat{\boldsymbol{X}},\boldsymbol{r},Z/v)$$

- violates rotational symmetry along *z* axis

Coherent assumption:

- wave functions at given *b* obtained by rotation

$$\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}e^{-i\phi j_z}\tilde{\Psi}(b\hat{\boldsymbol{X}},\boldsymbol{r},Z/v)$$

- ϕ : azimutal angle of ***R*** (or ***b***)
- rotationally symmetric along *z* axis

Also for usual eikonal approximation !

Breakup cross sections

$$\frac{d\sigma}{d\boldsymbol{k}d\Omega} = \frac{1}{(2\pi)^5}\frac{\mu K'}{\hbar^3 v}|T_{fi}|^2$$

Incoherent approximation $q \approx 2K\sin(\theta/2)$

$$\frac{d\sigma}{d\boldsymbol{k}d\Omega} = \frac{2KK'}{\pi k^2}\left|\sum_{lm} Y_l^m(\Omega_k)\int_0^\infty bdbJ_0(qb)S_{klm}(b)\right|^2$$

• depends on φ_k

$\longrightarrow$ not rotationally invariant around *z* axis

Coherent approximation

$$\frac{d\sigma}{d\boldsymbol{k}d\Omega} = \frac{2KK'}{\pi k^2}\left|\sum_{lm} i^{|m|}Y_l^m(\Omega_k)e^{-im\varphi}\int_0^\infty bdbJ_{|m|}(qb)S_{klm}(b)\right|^2$$

• depends on $\varphi_k - \varphi$

$\longrightarrow$ rotationally invariant around *z* axis

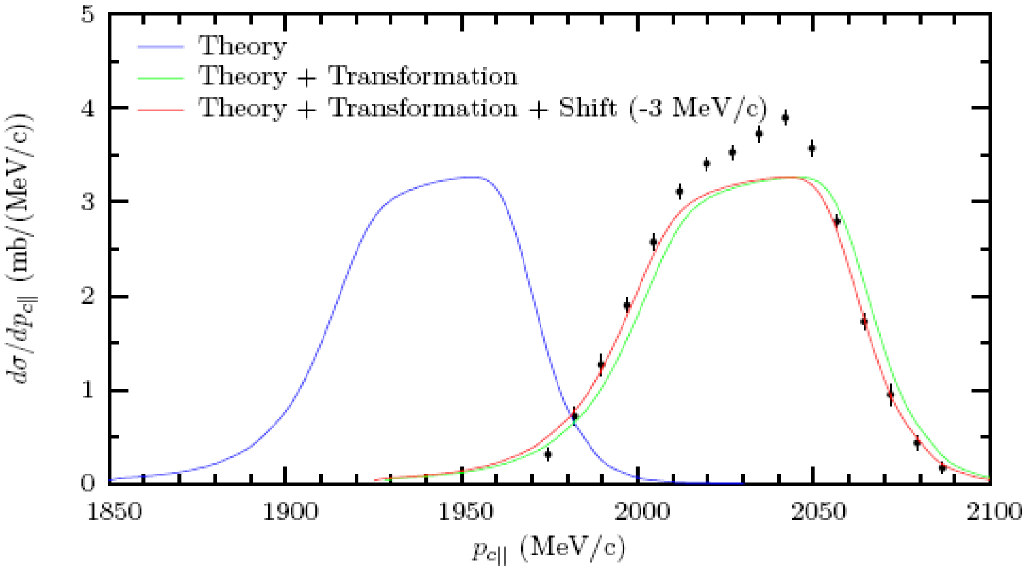
Parallel momentum distribution of ^{7}Be (Lab)

$$\frac{d\sigma}{dp_{c||}} = \frac{2\pi}{m_c}\int_{|p_{c||}|}^{p_c^{\max}} dp_c \int_0^{\pi} d\theta_f \sin\theta_f \int_0^{2\pi} d\Delta\varphi \frac{d\sigma}{dE_c d\Omega_c d\Omega_f}$$

$$p_c^{\max} = p_{c||}/\cos\theta_c^{\max} \qquad \Delta\varphi = \varphi_c - \varphi_f$$

Relativistic momentum transformation

$$p_i \xrightarrow{\mathrm{R}} v \xrightarrow{\mathrm{NR}} \text{dynamical eikonal calculation} \xrightarrow{\mathrm{NR}} m_c\, v_{c\parallel} \xrightarrow{\mathrm{R}} p_{c\parallel}$$



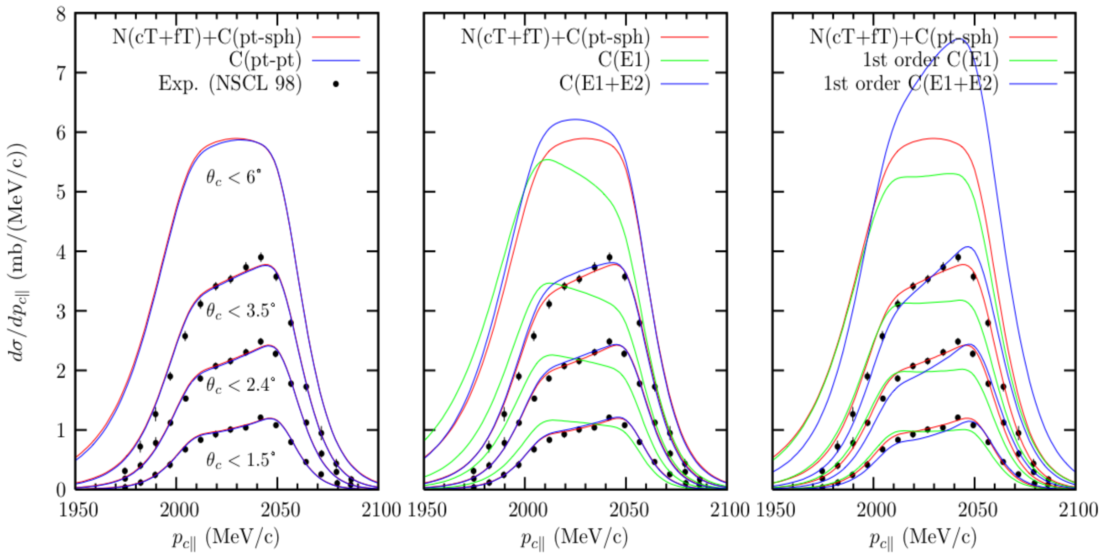
44 MeV/nucleon

$^{8}B + ^{208}Pb \rightarrow ^{7}Be + p + ^{208}Pb$

44 MeV/nucleon :

θ < 1.5, 2.4, 3.5, 6º

Dynamical eikonal approximation (red lines)



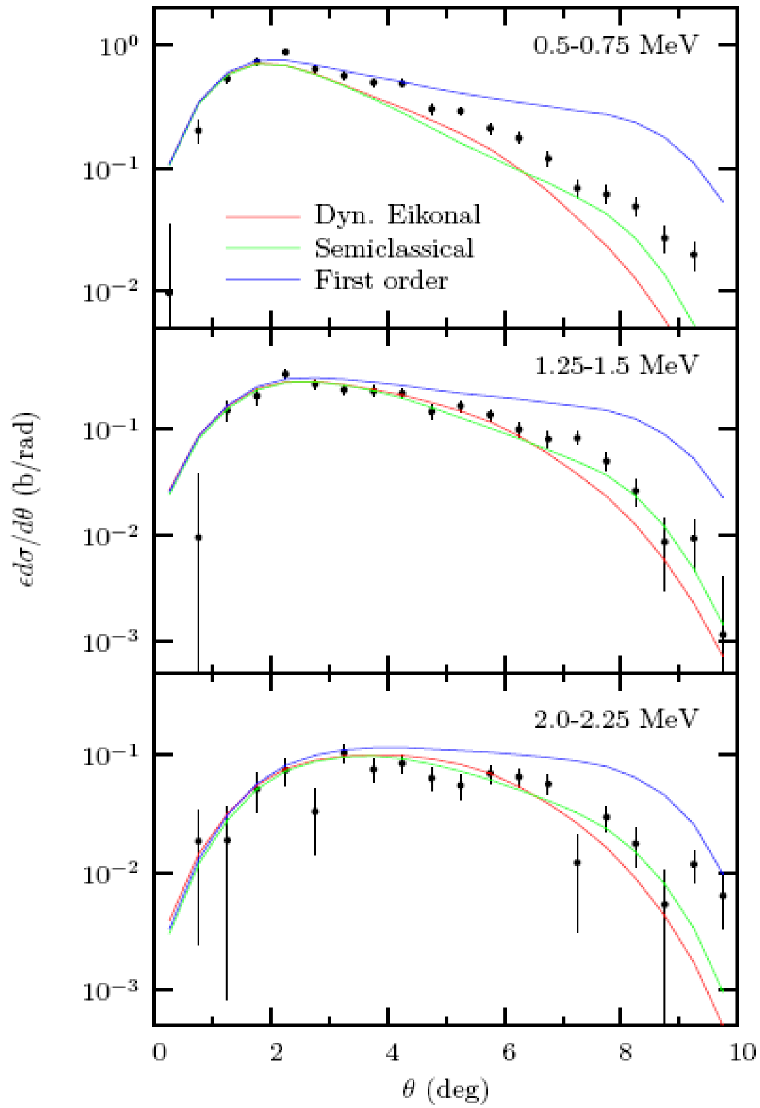
No parameter !

- Nuclear force negligible
- E1-E2 interference (asymmetry)
- Higher-order effects

Exp: B. Davids et al, Phys. Rev. Lett. 81 (1998) 2209

DEA: G. Goldstein, P. Capel, D.B., Phys. Rev. C 76 (2007) 024608

$^{8}B + ^{208}Pb \longrightarrow ^{7}Be + p + ^{208}Pb$



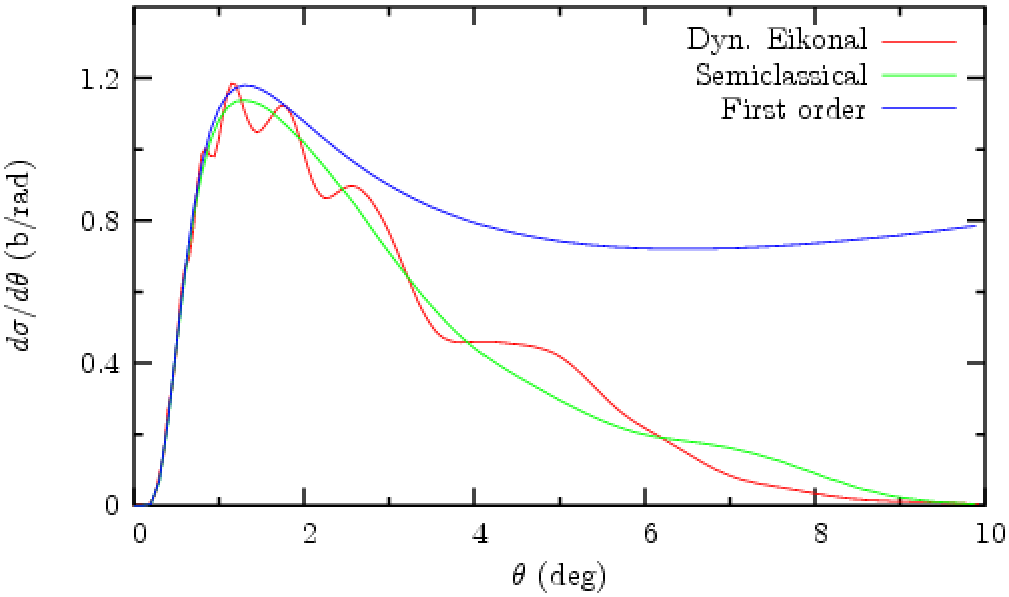
52 MeV/nucleon

Exp: T. Kikuchi et al, Phys. Lett. B 391 (1997) 261

Theor: G. Goldstein, P. Capel, D.B. Phys. Rev. C 76 (2007) 024608

Angular distributions of ^{8}B on ^{208}Pb

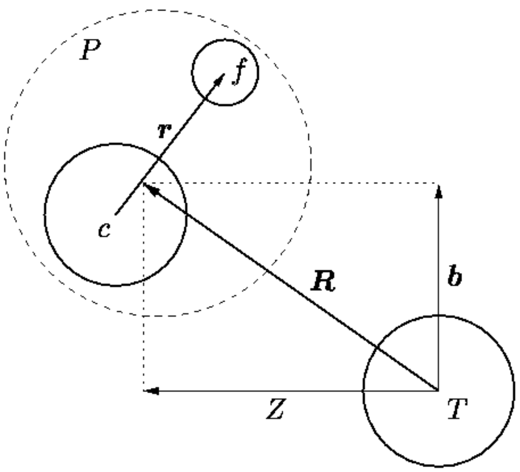
Comparison of reaction models



52 MeV/nucleon

Eikonal approximation for breakup

Additional adiabatic approximation $H_0 \approx E_0$



(fast collision: projectile wave function has no time to change)

$$\left(vP_Z + \cancel{H_0} + V_{PT}(\boldsymbol{R},\boldsymbol{r}) - \cancel{E_0}\right)\hat{\Psi} = 0$$

$\Rightarrow$ Standard eikonal approximation

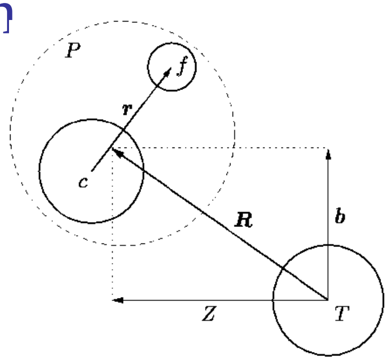
$$\left(-i\hbar v\frac{\partial}{\partial Z} + V_{PT}(\boldsymbol{R},\boldsymbol{r})\right)\hat{\Psi}_{\text{eik.}} = 0$$

$$\hat{\Psi}_{\text{eik.}}(\boldsymbol{R},\boldsymbol{r}) = e^{i\chi}\phi_0(\boldsymbol{r}) \qquad \chi = -\frac{1}{\hbar v}\int_{-\infty}^{Z} V_{PT}(\boldsymbol{b},Z',\boldsymbol{r})dZ'$$

Simple but problems with Coulomb (divergence)

$\rightarrow$ Coulomb-corrected eikonal approximations

Coulomb-corrected eikonal approximation



Eikonal amplitude $$e^{i\chi} = e^{i\chi_{PT}^{C}} e^{i\chi^{N}} e^{i\chi^{C}}$$

Projectile-Target Coulomb phase $$\chi_{PT}^{C} \approx 2\eta \ln b$$

Nuclear phase $$\chi^{N} = -\frac{1}{\hbar v}\int_{-\infty}^{\infty}[V_{cT}^{N}(\boldsymbol{b}, Z, \boldsymbol{r}) + V_{fT}^{N}(\boldsymbol{b}, Z, \boldsymbol{r})]dZ$$

Tidal Coulomb phase (core + neutron)

$$\chi^{C} = -\eta \int_{-\infty}^{\infty} \left(\frac{1}{|\boldsymbol{R} - \frac{m_n}{m_P}\boldsymbol{r}|} - \frac{1}{R} \right) dZ \underset{b\to\infty}{\longrightarrow} \frac{1}{b}$$

$\longrightarrow$ divergence of cross section

Coulomb-corrected eikonal amplitude $$e^{i\chi^{C}} \rightarrow e^{i\chi^{C}} - i\chi^{C} + i\chi^{FO}$$

First-order perturbation $$\chi^{FO} = -\eta \int_{-\infty}^{\infty} e^{i\omega Z/v} \left(\frac{1}{|\boldsymbol{R} - \frac{m_n}{m_P}\boldsymbol{r}|} - \frac{1}{R} \right) dZ$$

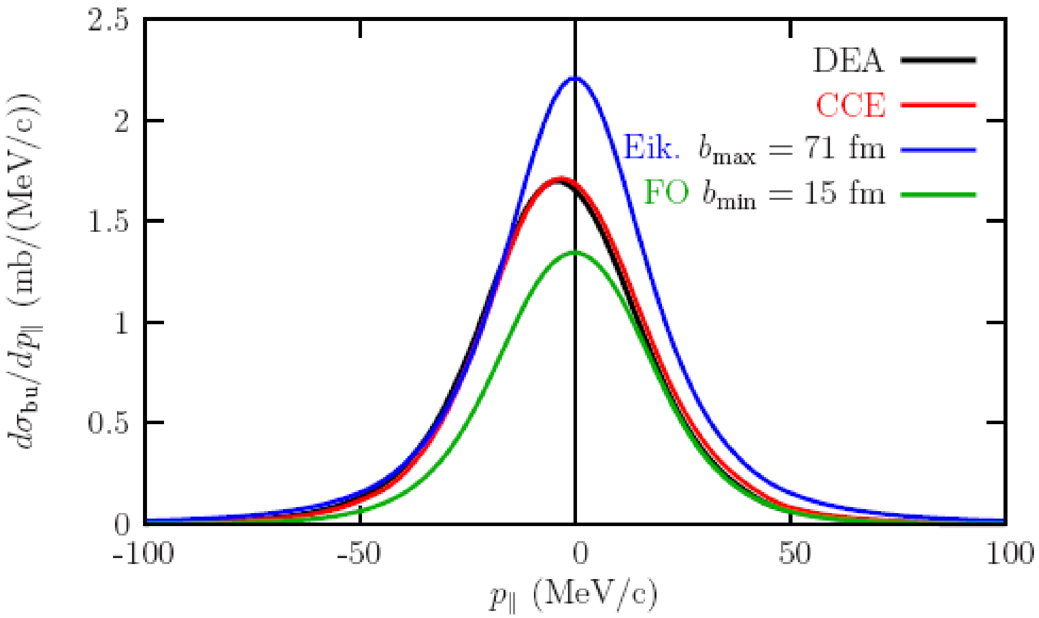
J. Margueron, A. Bonaccorso, D.M. Brink, Nucl. Phys. A 720 (2003) 337
P. Capel, D. B., Y. Suzuki, Phys. Rev. C 78 (2008) 054602

Coulomb-corrected eikonal approximation (CCE)

Elimination of divergence due to first-order term

$\Rightarrow$ replace first-order eikonal term by first-order perturbation term

J. Margueron, A. Bonaccorso, D.M. Brink, Nucl. Phys. A 720 (2003) 337



Comparison of approximations:

^{10}Be-n parallel momentum distributions for ^{11}Be breakup on ^{208}Pb at 69 MeV/u

P. Capel, D. B., Y. Suzuki, Phys. Rev. C 78 (2008) 054602

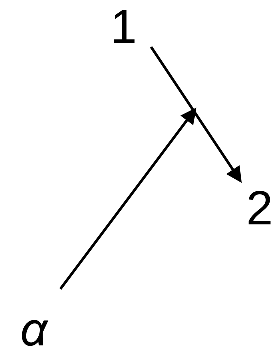
Breakup of ^{6}He two-neutron halo nucleus

- Theoretical description of 3-body projectile more difficult (especially continuum)
- Hyperspherical-harmonics expansion
- Infinite scattering matrix (truncated)

  $\rightarrow$ many entrance channels
- Much longer calculations $\rightarrow$ CCE

- Experiments also more difficult
- GSI: breakup on lead at 240 MeV/nucleon

α + n + n three-body scattering



Hyperspherical coordinates

$$\boldsymbol{x} = \frac{1}{\sqrt{2}}\boldsymbol{r}_{21} \qquad \boldsymbol{y} = \sqrt{\frac{4}{3}}\boldsymbol{r}_{\alpha(12)}$$

$$\rho = \sqrt{x^2 + y^2} \qquad \alpha = \arctan\frac{y}{x}$$

$$\Omega_5 = (\Omega_x, \Omega_y, \alpha)$$

Expansion in hyperspherical harmonics $\gamma = (l_x, l_y, L, S)$

$$\Psi^{JM\pi}(\rho, \Omega_5) = \rho^{-5/2} \sum_{\gamma K} \chi^{J\pi}_{\gamma K}(\rho)\ \mathcal{Y}^{JM}_{\gamma K}(\Omega_5)$$

Infinite system of coupled equations (truncated at $K_{\max}$)

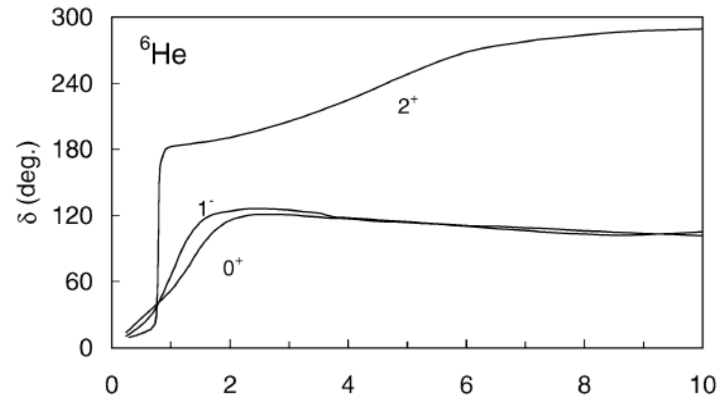
$$\left[-\frac{\hbar^2}{2m_N}\left(\frac{d^2}{d\rho^2} - \frac{(K+3/2)(K+5/2)}{\rho^2}\right) - E\right]\chi^{J\pi}_{\gamma K}(\rho) + \sum_{K'\gamma'} V^{J\pi}_{K\gamma,K'\gamma'}(\rho)\,\chi^{J\pi}_{\gamma' K'}(\rho) = 0$$

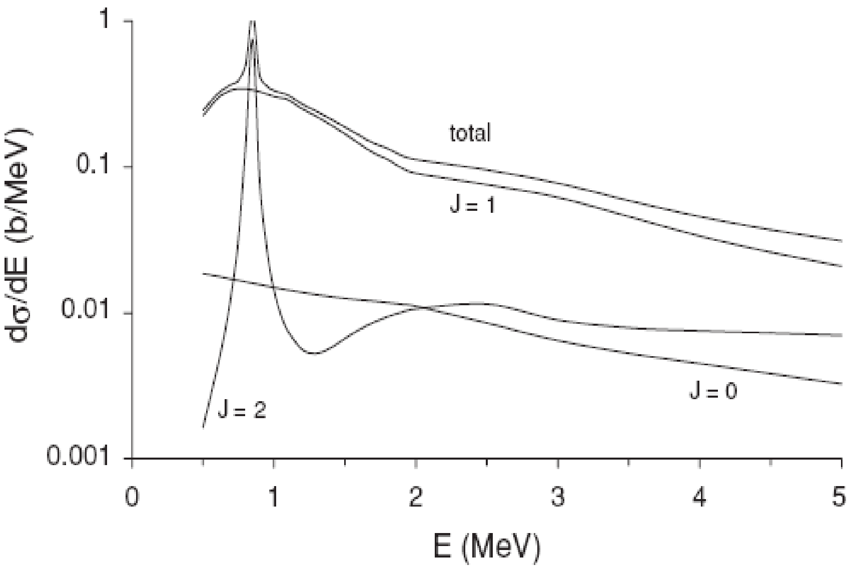
Bound state and continuum (FSI) calculated with *R* matrix + Lagrange mesh

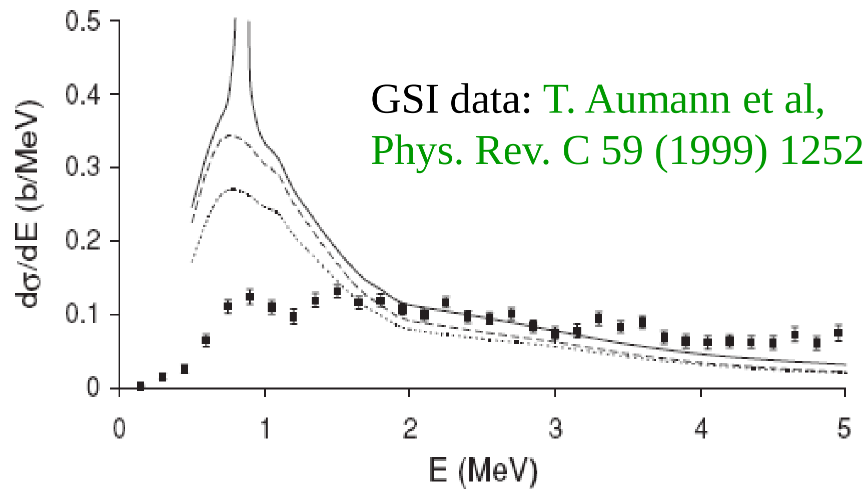
Four-body Coulomb-corrected eikonal

$^{6}He + {}^{208}Pb \rightarrow {}^{4}He + n + n + {}^{208}Pb$ (240 MeV/nucleon)

3-body α+n+n phase shifts
Hyperspherical-harmonics expansion
of bound and scattering states
P. Descouvemont, E. Tursunov, D.B.,
Nucl. Phys. A 765 (2006) 370





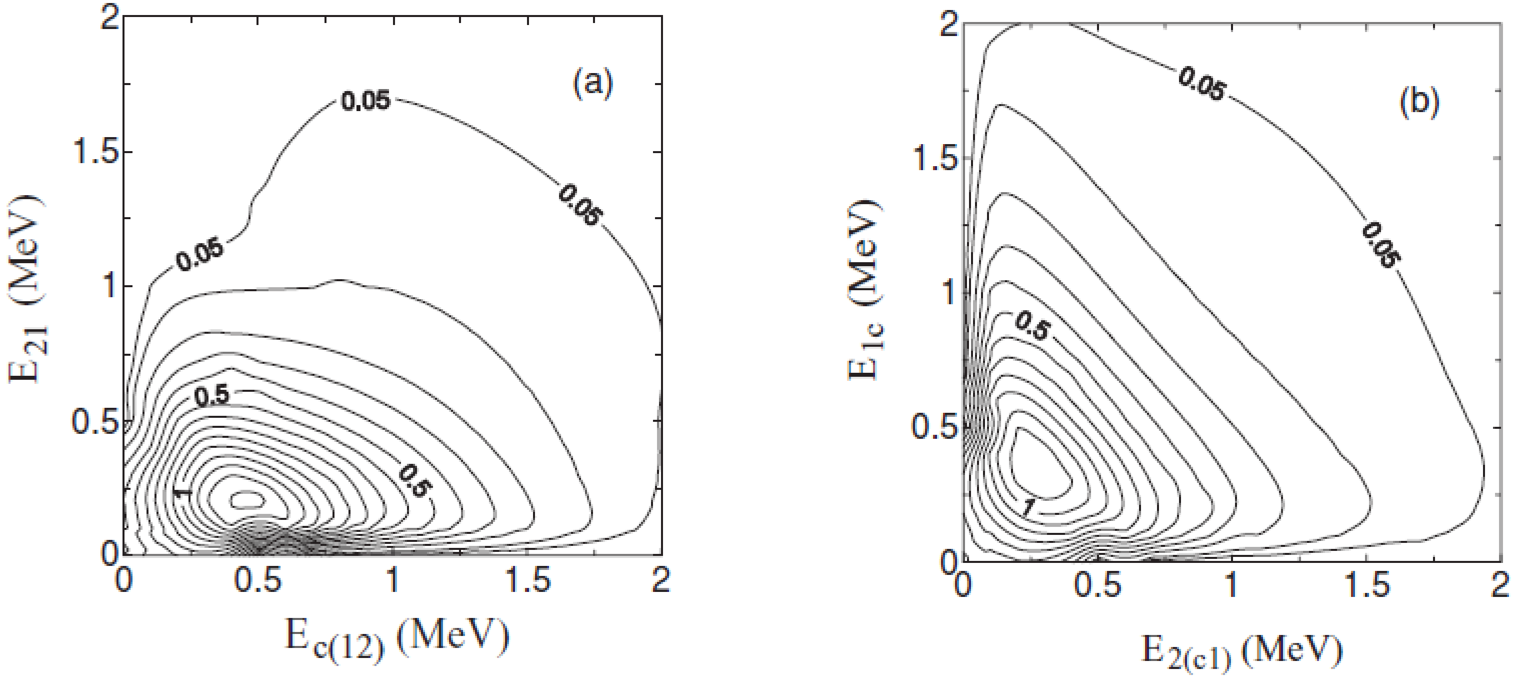


GSI data: T. Aumann et al,
Phys. Rev. C 59 (1999) 1252

Discrepancy in cross sections: existence of 3-body 1^- resonance?

D.B., P. Capel, P. Descouvemont, Y. Suzuki, Phys. Rev. C 79 (2009) 024607

1^- component of double-differential cross sections



$d\sigma / dE_{21}\, dE_{c(12)}$ $\qquad$ $d\sigma / dE_{1c} dE_{2(c1)}$

Data on the correlations between the emitted neutrons would allow discriminating between theoretical models of ^{11}Li

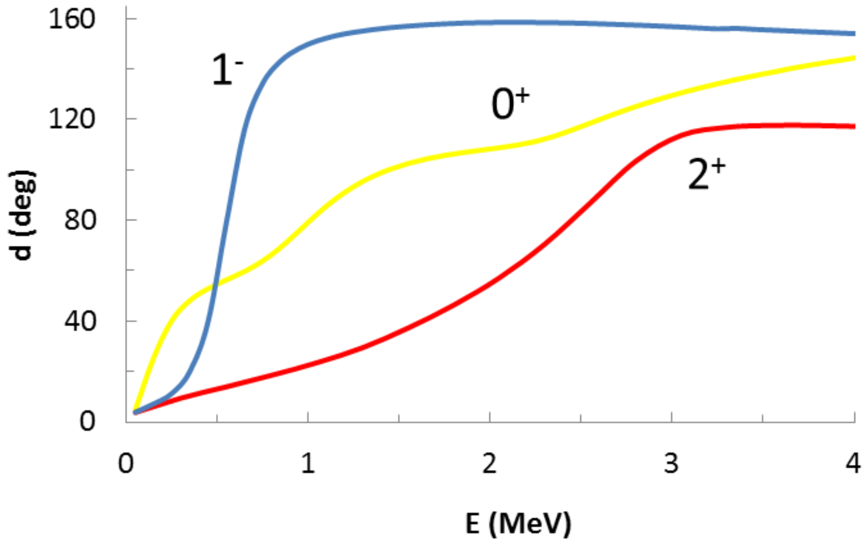
Breakup of ^{11}Li two-neutron halo nucleus

- Smaller 2n separation energy (0.378 MeV)
- Slower convergence of hyperspherical-harmonics expansion
- CCE

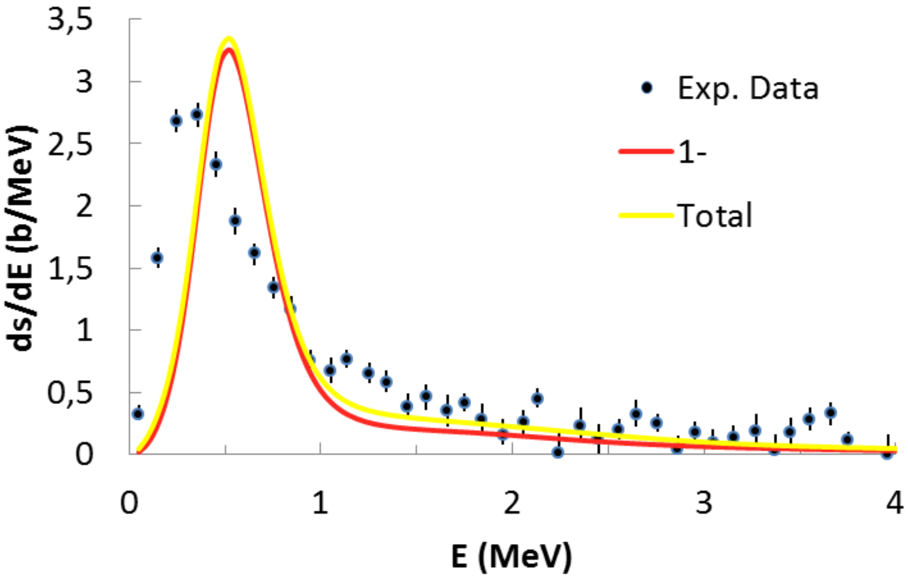
- Contradictory experiments
- Better data at low energies in RIKEN experiment

T. Nakamura et al, Phys. Rev. Lett. 96 (2006) 252502

^{9}Li+n+n phase shifts

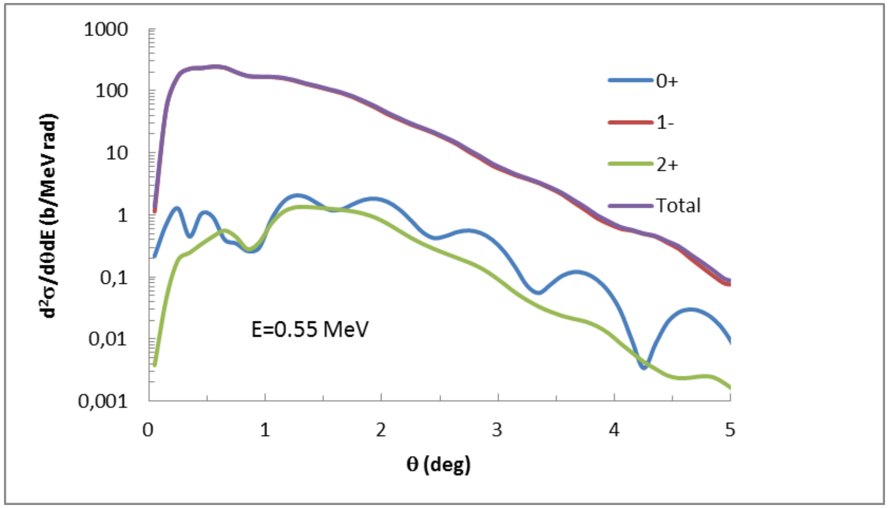


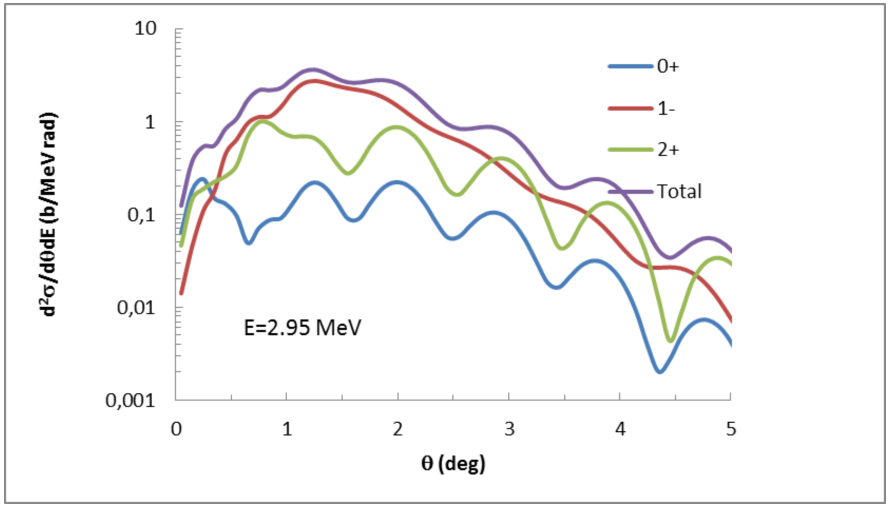
CCE breakup cross section and RIKEN data



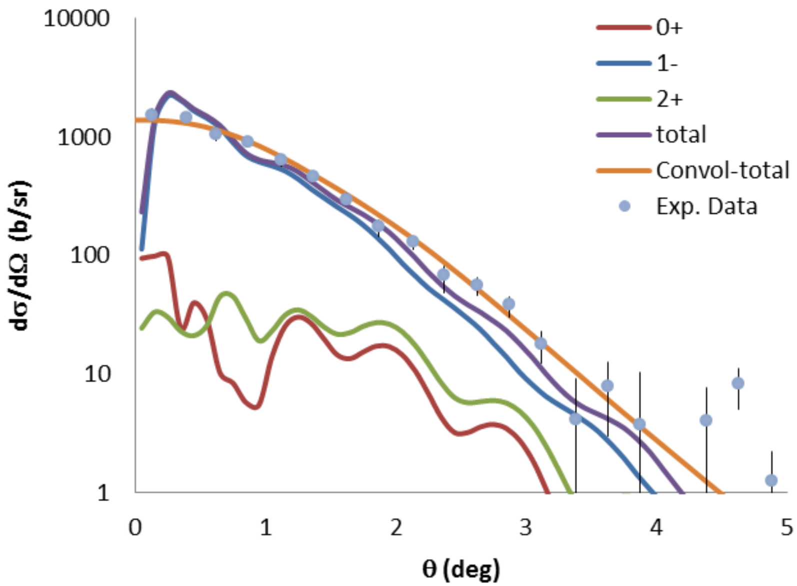
E. Pinilla (preliminary)

Angular breakup cross sections





Integrated over energy (0 – 4 MeV)



E. Pinilla (preliminary)

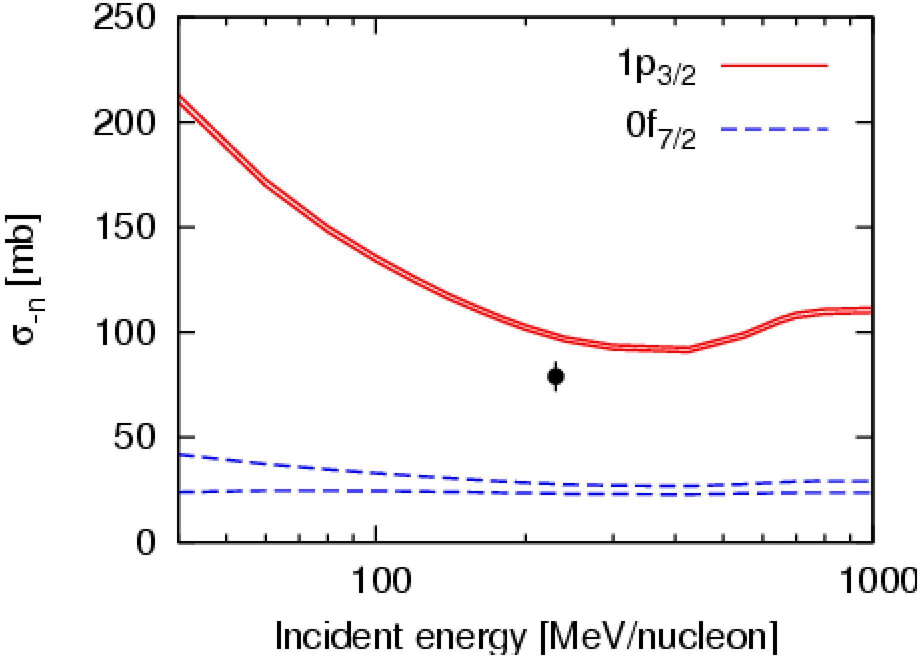
Breakup of ^{31}Ne and « island of inversion »

- Ground state: one neutron beyond closed *sd* shell
  *f*7/2 (naive shell model) or *p*3/2 (intruder)?
- Poorly known separation energy
- Measurements of one-neutron removal and reaction cross sections: comparison with Glauber model
- No potential, but « profile functions » based on nucleon-nucleon cross sections

  B. Abu Ibrahim, W. Horiuchi, A. Kohama, Y. Suzuki,
  Phys. Rev. C 77 (2008) 034607
- DEA for momentum distributions

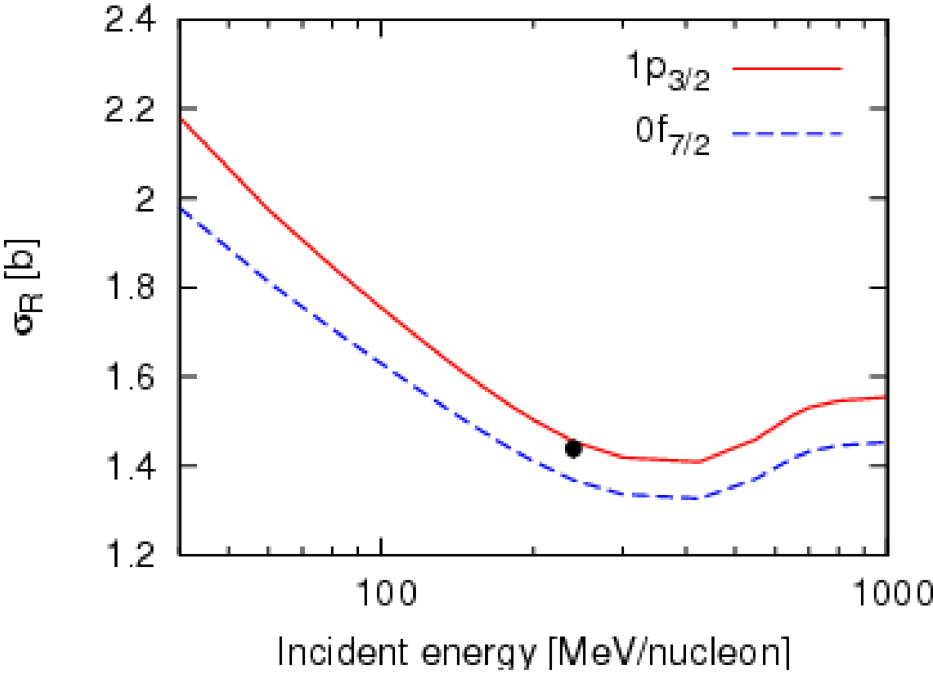
Eikonal Glauber model

One-neutron removal
cross section



Exp: T. Nakamura et al, Phys. Rev. Lett. 103 (2009) 262501

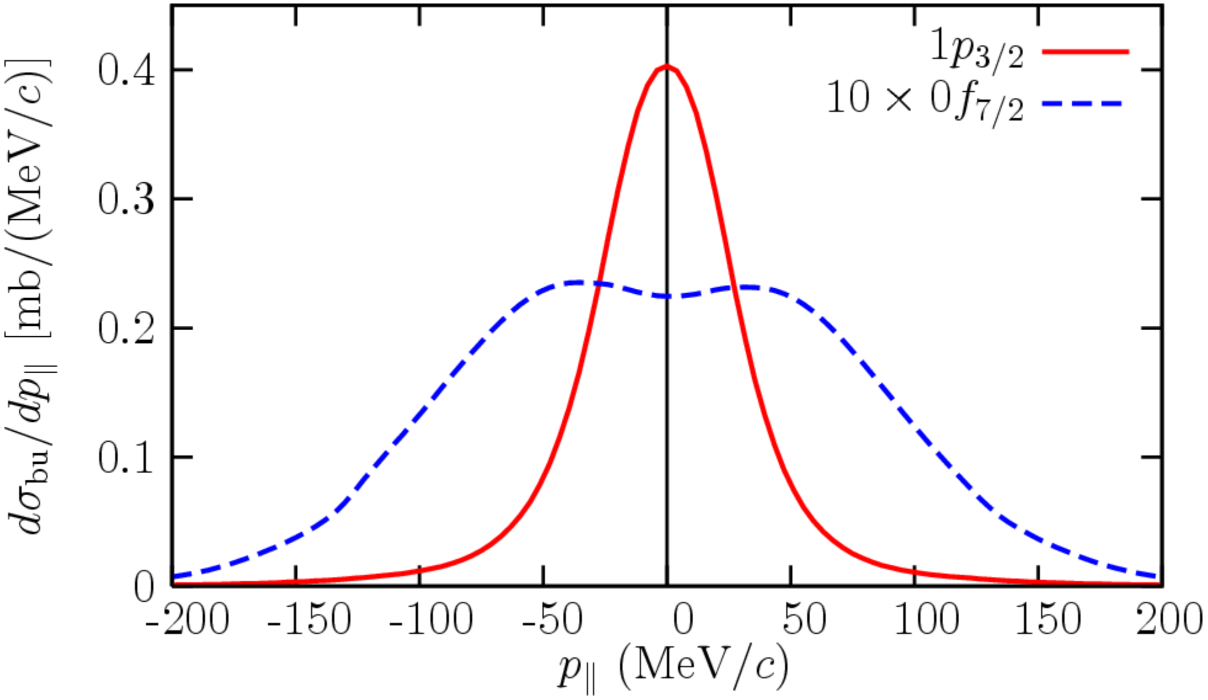
Reaction cross section



Exp: M. Takechi et al, Nucl. Phys. A 834 (2010) 412c

Th: W. Horiuchi, Y. Suzuki, P. Capel, D.B., Phys. Rev. C 81 (2010) 024606
P. Capel, W. Horiuchi, Y. Suzuki, D.B., Mod. Phys. Lett. A 25 (2010) 1882

Parallel-momentum distribution for the elastic breakup of ^{31}Ne on carbon at 240 MeV/nucleon (DEA)



Clear differences (shape, magnitude) → need for experimental data

W. Horiuchi, Y. Suzuki, P. Capel, D.B., Phys. Rev. C 81 (2010) 024606

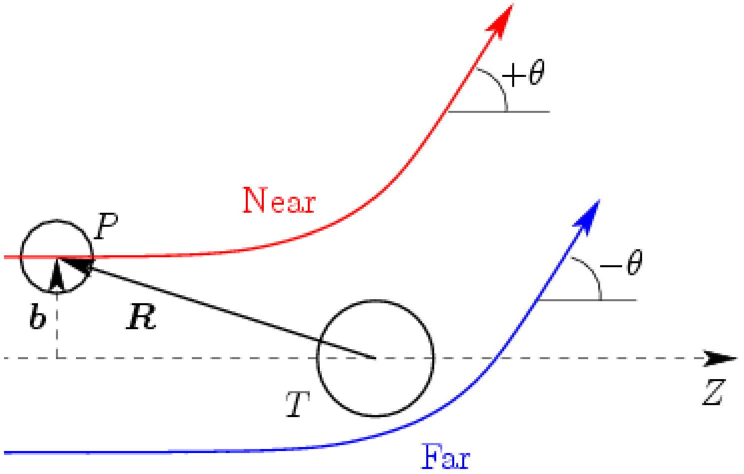
Influence of halo on angular breakup cross sections

- Near/Far decomposition

$$\frac{d\sigma}{d\Omega} \propto \left| \int_0^\infty J_0(qb) S_0(b) b db \right|^2$$

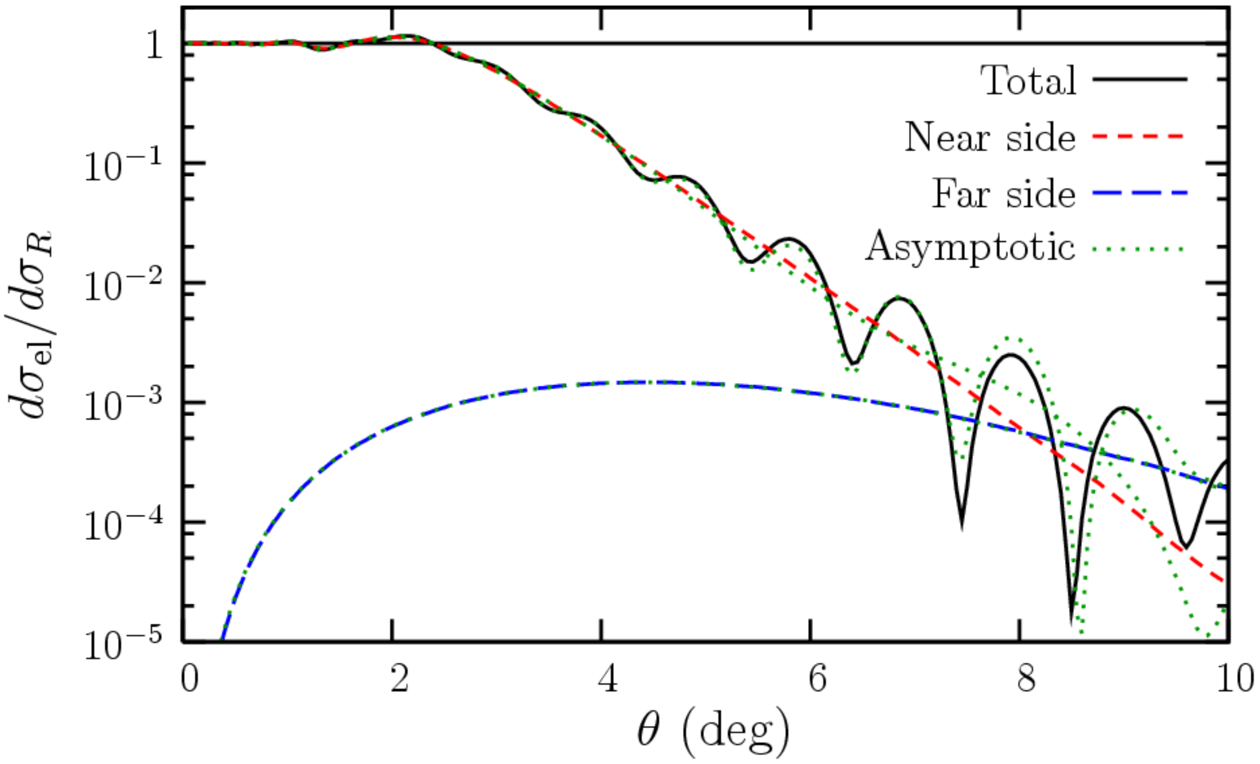
$$J_0 = \tfrac{1}{2}\left(H_0^{(1)} + H_0^{(2)}\right) \qquad H_0^{(1,2)}(z) \underset{z\to\infty}{\longrightarrow} \sqrt{\frac{\pi}{2z}}\, e^{\pm i(z-\pi/4)}$$

$$\frac{d\sigma^{N,F}}{d\Omega} \propto \left| \int_0^\infty H_0^{(2,1)}(qb) S_0(b) b db \right|^2$$



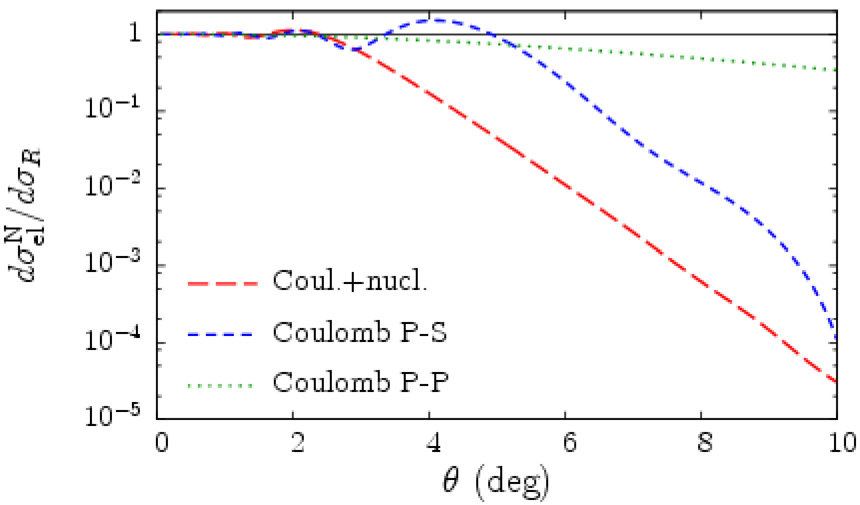
Elastic scattering of ^{11}Be on ^{208}Pb at 69 MeV/nucleon

Near/Far decomposition



P. Capel, M.S. Hussein, D.B., Phys. Lett. B 693 (2010) 448

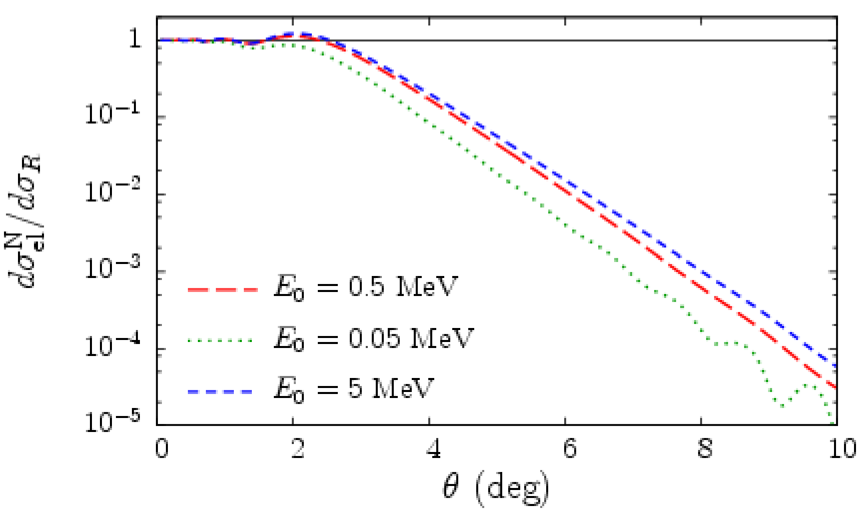
Influence of projectile-target interaction



V_{PT}	σ_{bu}
C.+N.	1.70 b
PS	2.10 b
PP	2.58 b

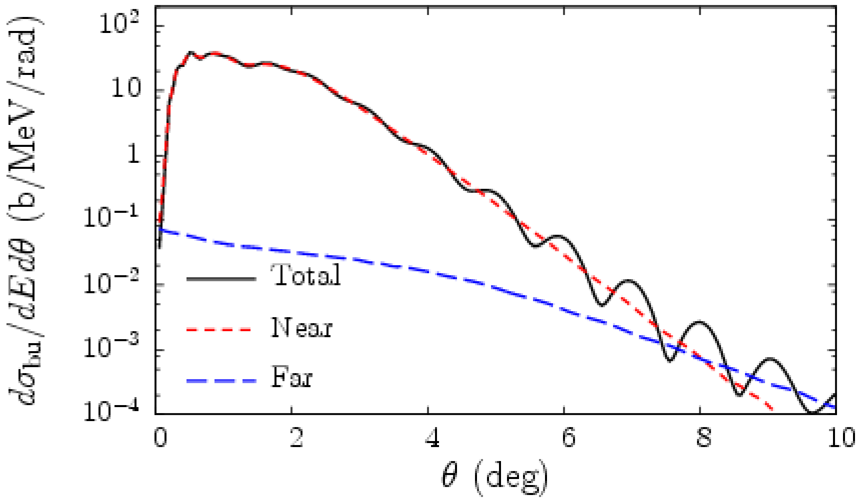
Decrease not due to loss towards breakup!

Influence of size of the halo

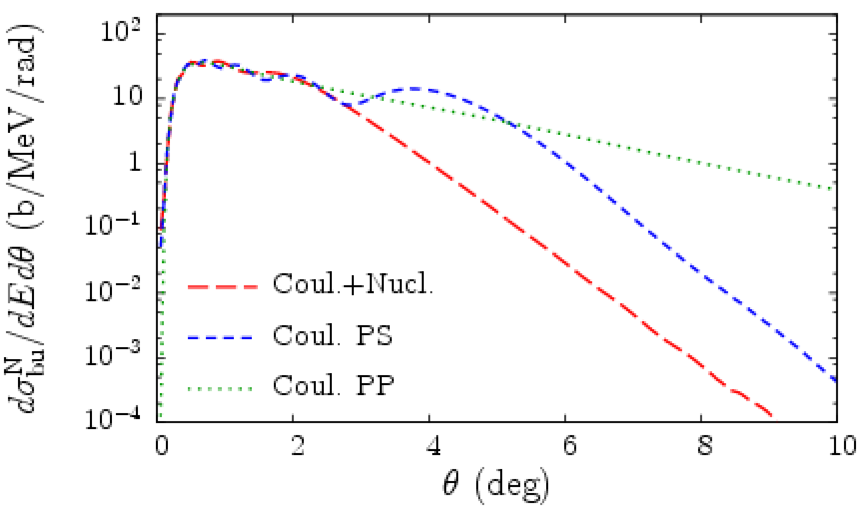


E_0	σ_{bu}
0.5 MeV	1.70 b
0.05 MeV	23.6 b
5 MeV	0.07 b

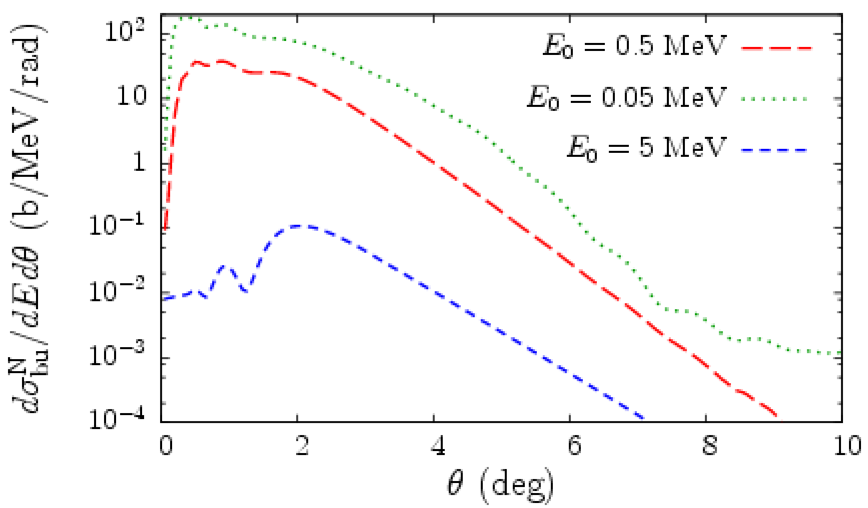
Breakup of ^{11}Be on ^{208}Pb at 69 MeV/nucleon



Influence of projectile-target interaction



Influence of size of the halo



Conclusion

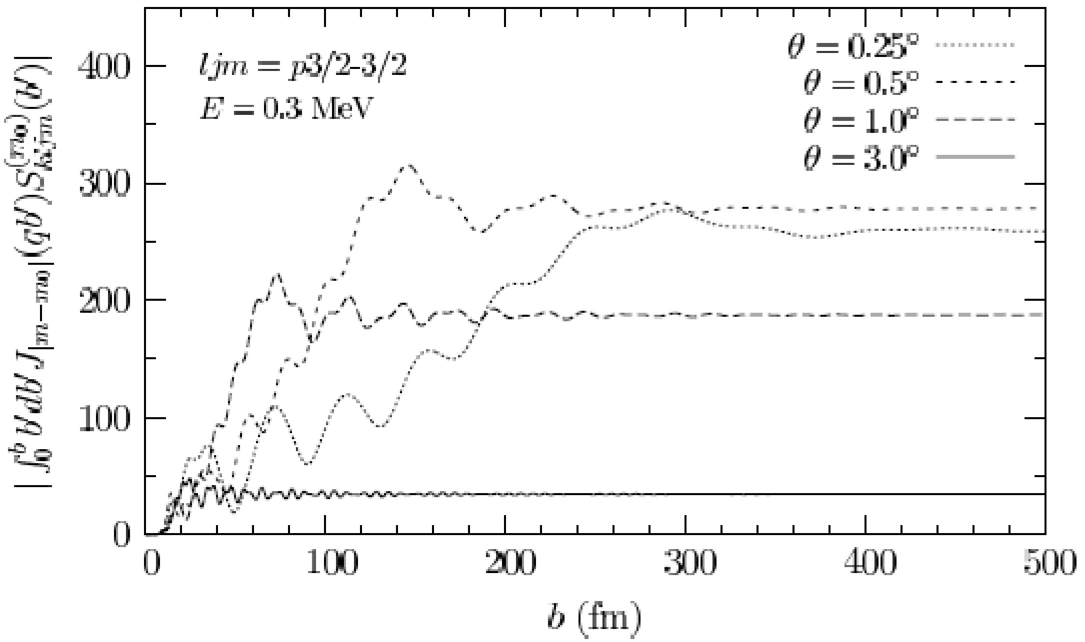
- Reaction models
- DEA accurate at intermediate energies
- Simpler CCE corrects divergence problem of eikonal approximation
- Not valid at low energies (→ CDCC)
- Good results with simple projectile models (What do we learn?)
- Importance of final-state interactions

- Breakup of ^{6}He
- Difficult treatment of continuum
- 1^- resonance in conflict with GSI experiment (in all model calculations!)
- Breakup of ^{11}Li
- Good agreement with experiment (including angular dist ribution)
- Breakup of ^{31}Ne
- $3/2^-$ ground state (intruder): "island of inversion"

- Near/far decomposition of elastic and breakup cross sections
- Decrease of elastic scattering is not a loss towards breakup
- Reflects the range of the core-target interaction

Coulomb effects with straight-line trajectories

$$\int_0^\infty b\,db\,J_{|m|}(qb)S_{klm}(b)$$



Matching of asymptotic frequencies *q* and $\frac{d\chi_C}{db} = \frac{2\eta}{b}$

$$b \approx \frac{2\eta}{q} \approx \frac{2\eta}{K\theta} \qquad b_{\mathrm{cl.}} = \frac{\eta}{K\cot(\theta/2)}$$

Relativistic kinematics

Initial velocity

$$\frac{v}{c} = \sqrt{1 - \frac{1}{(1 + \frac{T_i}{m_P c^2})^2}}$$

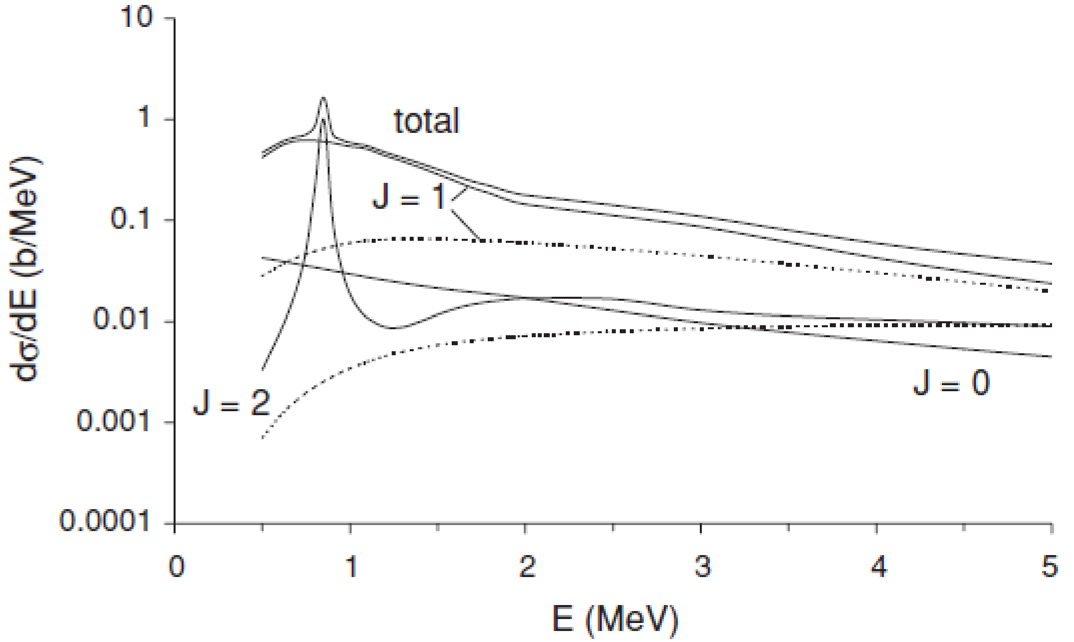
Final non-relativistic core parallel momentum

$$p_{c\|}^{NR} = \left(\frac{1}{(m_P c)^2} + \frac{1}{p_{c\|}^2} \right)^{-1/2}$$

Final non-relativistic core emission angle

$$\tan \theta_c^{NR} = \frac{p_{c\|}}{p_{c\|}^{NR}} \tan \theta_c$$

Importance of final state interactions



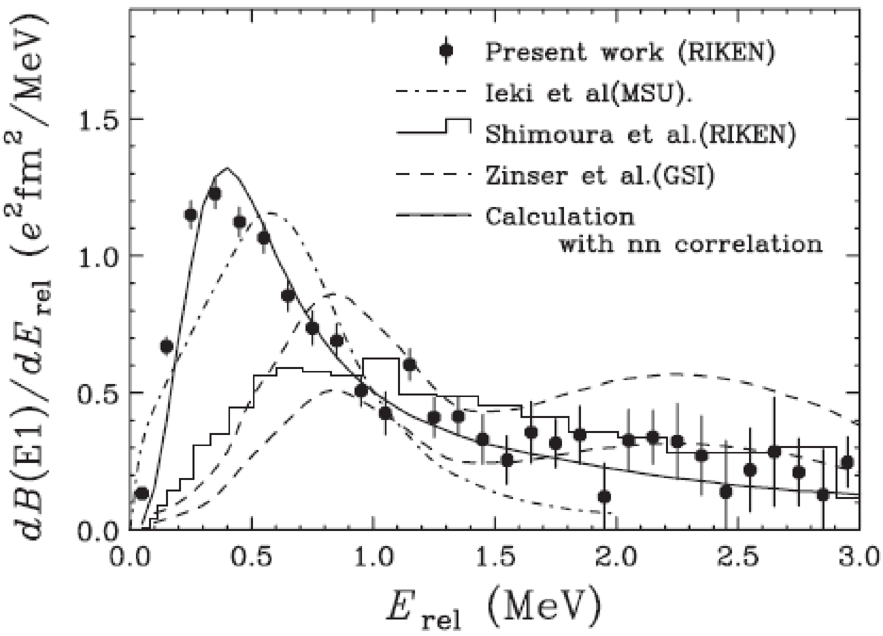
Full lines: distorted final states
Dotted lines: final plane waves

D.B., P. Capel, P. Descouvemont, Y. Suzuki, Phys. Rev. C 79 (2009) 024607

Dipole strength of ^{11}Li

$^{11}\text{Li} + {}^{208}\text{Pb} \rightarrow {}^{9}\text{Li} + n + n + {}^{208}\text{Pb}$

Contradictory experimental results for $dB(\text{E1})/dE$



« Soft dipole » strength
$B(\text{E1}) \approx 4.5$ W.U.
($E < 3$ MeV)

Problems of several breakup experiments:

- Poor efficiency at low relative energies
- Validity of first-order perturbation theory

Observation of low-energy peak (breakup at 70 MeV/nucleon) :

T. Nakamura et al, Phys. Rev. Lett. 96 (2006) 252502