TEXAS A&M UNIVERSITY COMMERCE

C.A. Bertulani

Direct Reactions with Rare Isotopes

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Low Energy problems

Theories: e.g. radiative capture

Schrödinger equation:

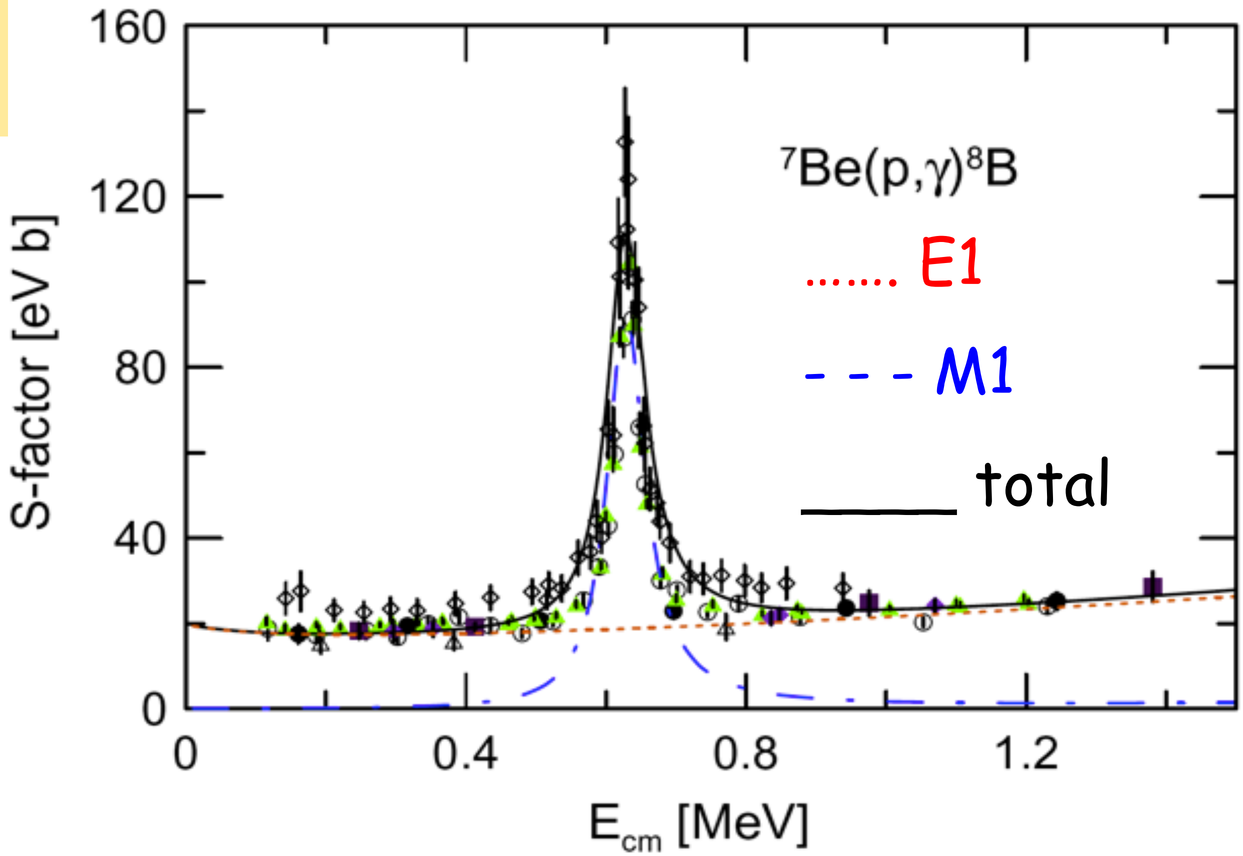
e.g., Woods-Saxon

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}\psi^l(r)+\left[V(r)+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right]\psi^l(r)=E\psi^l(r)$$

Simple to solve numerically for $E < 0$ or $E > 0$

$$\int_0^\infty r^\lambda \psi^l(E_i,r)\psi^{l'}(E_f,r)dr$$

S-factors, X-sections



Bertulani
Z. Phys. A356, 293 (1996)

Cluster models and reactions

What one needs

$$g(r) = \left\langle \chi^{(A)} \middle| \hat{A}\Phi^{(A-a)}\Phi^{(a)}\delta(r - r_{A-a,a}) \right\rangle$$

$$g_{bound}(r) \rightarrow C_{lj}\,\frac{W_{-\eta,l+1/2}(r)}{r}$$

$$g_{scat}(r \rightarrow \infty) \sim I_l(r) - S_l O_l(r)$$

What one does

$$\int dr'\left[H(r,r') - EN(r,r')\right]g(r') = 0 \quad \text{all } r$$

Hill-Wheeler, 1955

$$\begin{Bmatrix} H \\ N \end{Bmatrix}(r,r') = \left\langle \hat{A}\Phi^{(A-a)}\Phi^{(a)}(r') \middle| \begin{Bmatrix} H \\ N \end{Bmatrix} \middle| \hat{A}\Phi^{(A-a)}\Phi^{(a)}(r) \right\rangle$$

ab initio model

Problem:
intercluster effective interactions in continuum

$H = T + \sum v_{ij} + \sum v_{ijk}$

Ab initio models: e.g., no core shell model

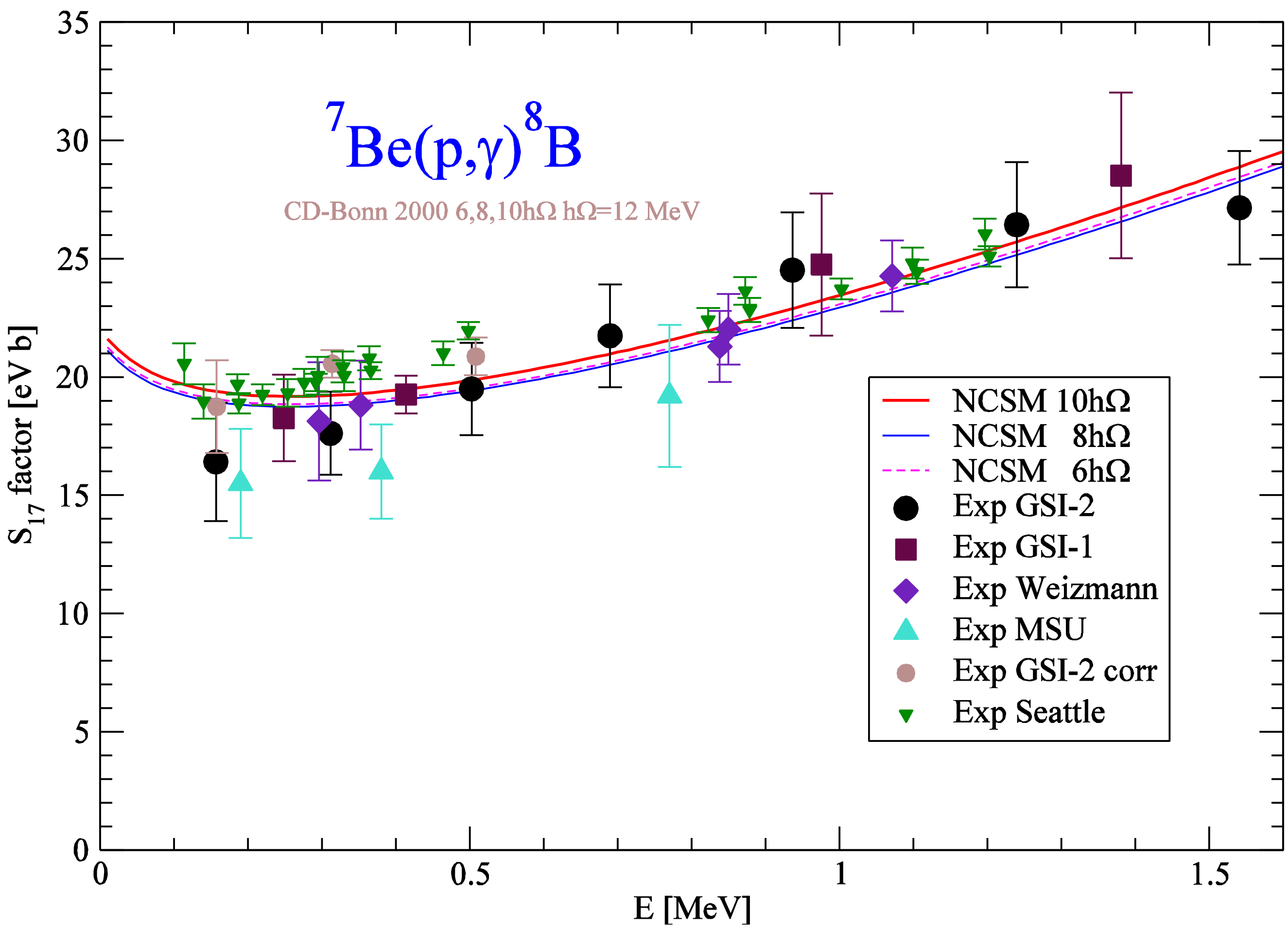
- Realistic interactions: CD-Bonn, INOY
- Accurate wave functions of ^{7}Be, ^{8}B
- Deduce the ANC of ^{8}B

But: ^{7}Be+p scattering states defined in a potential model
"Ab initio" only in the final state

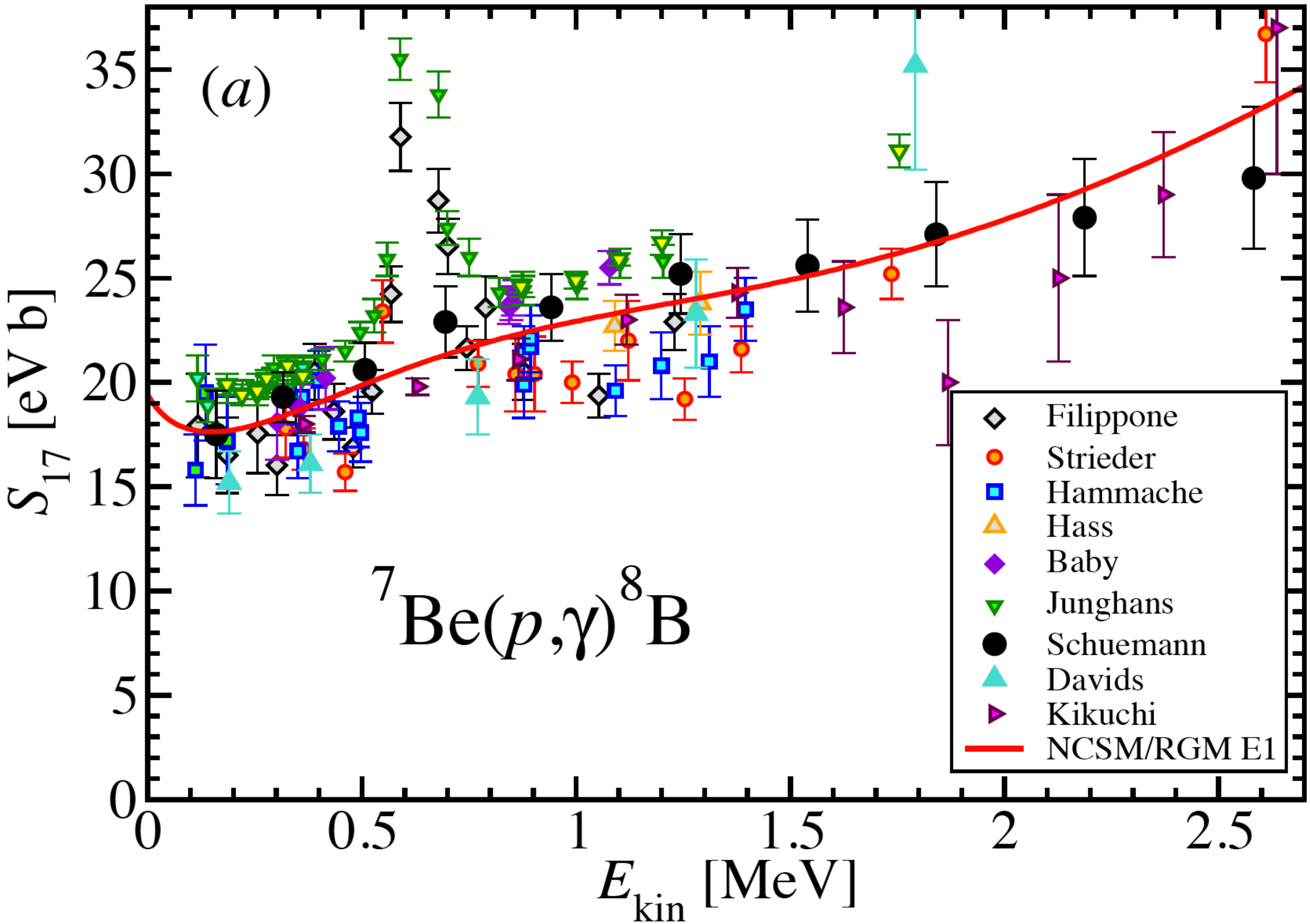
Convergence test with # of oscillator shells

$S_{17} = 22 \pm 1$ eV.b

Navratil, Bertulani, Caurier
PLB 634, 191 (2006)
PRC 73, 065801 (2006)

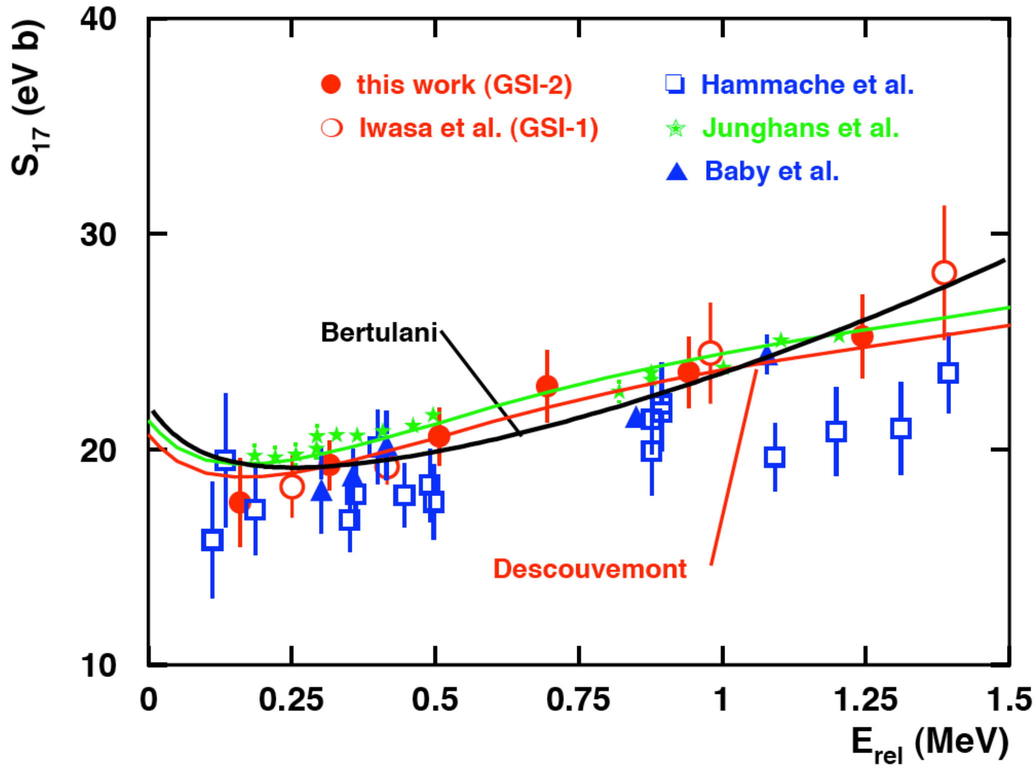


Microscopic models: e.g., no core shell model + RGM

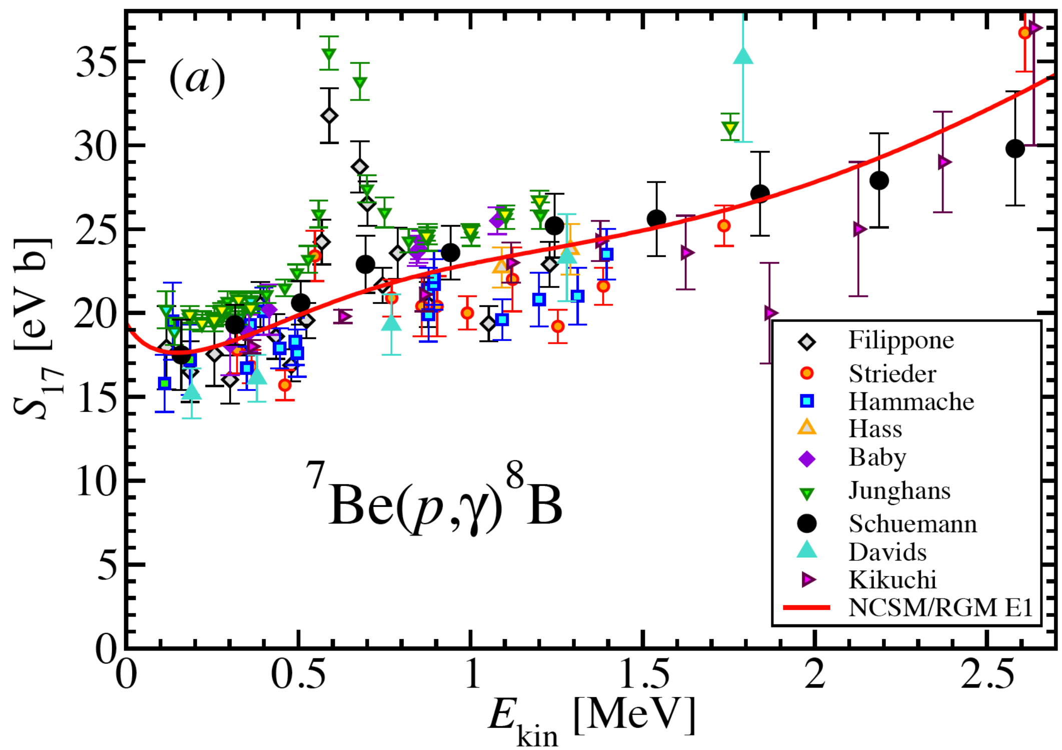


S_{17} = 19.4 ± 0.7 eV.b

Quaglioni, Navratil, Roth
PLB 704 (2011) 379



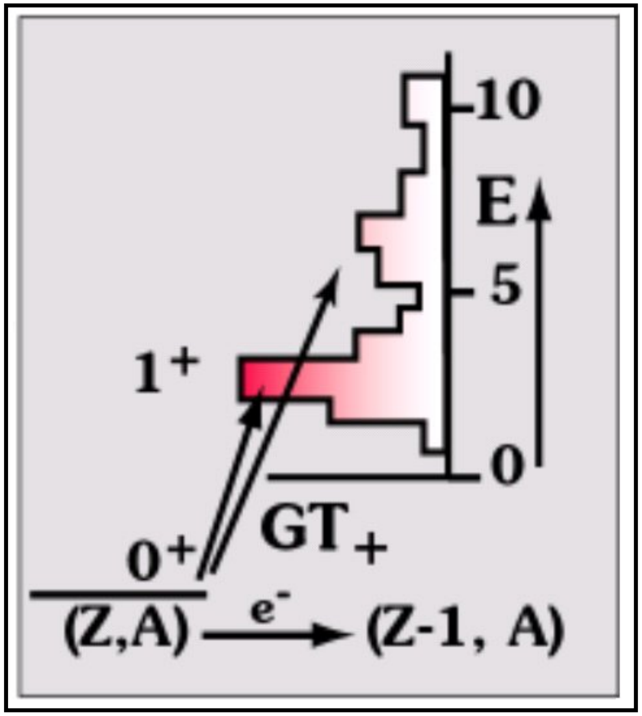
S17 (eV b)
this work (GSI-2)
Iwasa et al. (GSI-1)
Hammache et al.
Junghans et al.
Baby et al.
Bertulani
Descouvemont
E_rel (MeV)

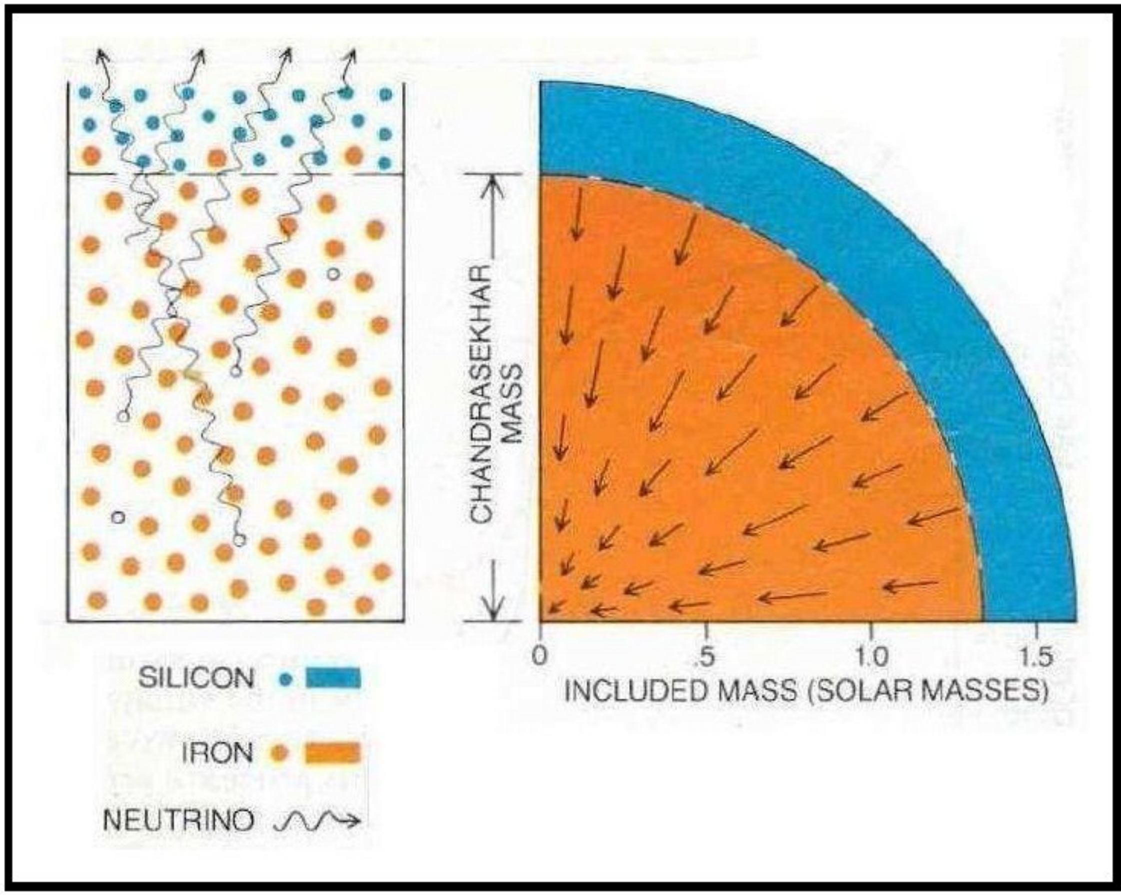


(a)
7Be(p,γ)8B
S17 [eV b]
E_kin [MeV]
Filippone
Strieder
Hammache
Hass
Baby
Junghans
Schuemann
Davids
Kikuchi
NCSM/RGM E1

More energetic problems

Theories:
e.g.neutrinos





e- + (Z,A) ←→ (Z-1, A) + ν_e Needs $\left|\langle B \| \sigma\tau \| A \rangle\right|^2$ for nuclei A > 40

Neutrino flux

Number of target nuclei

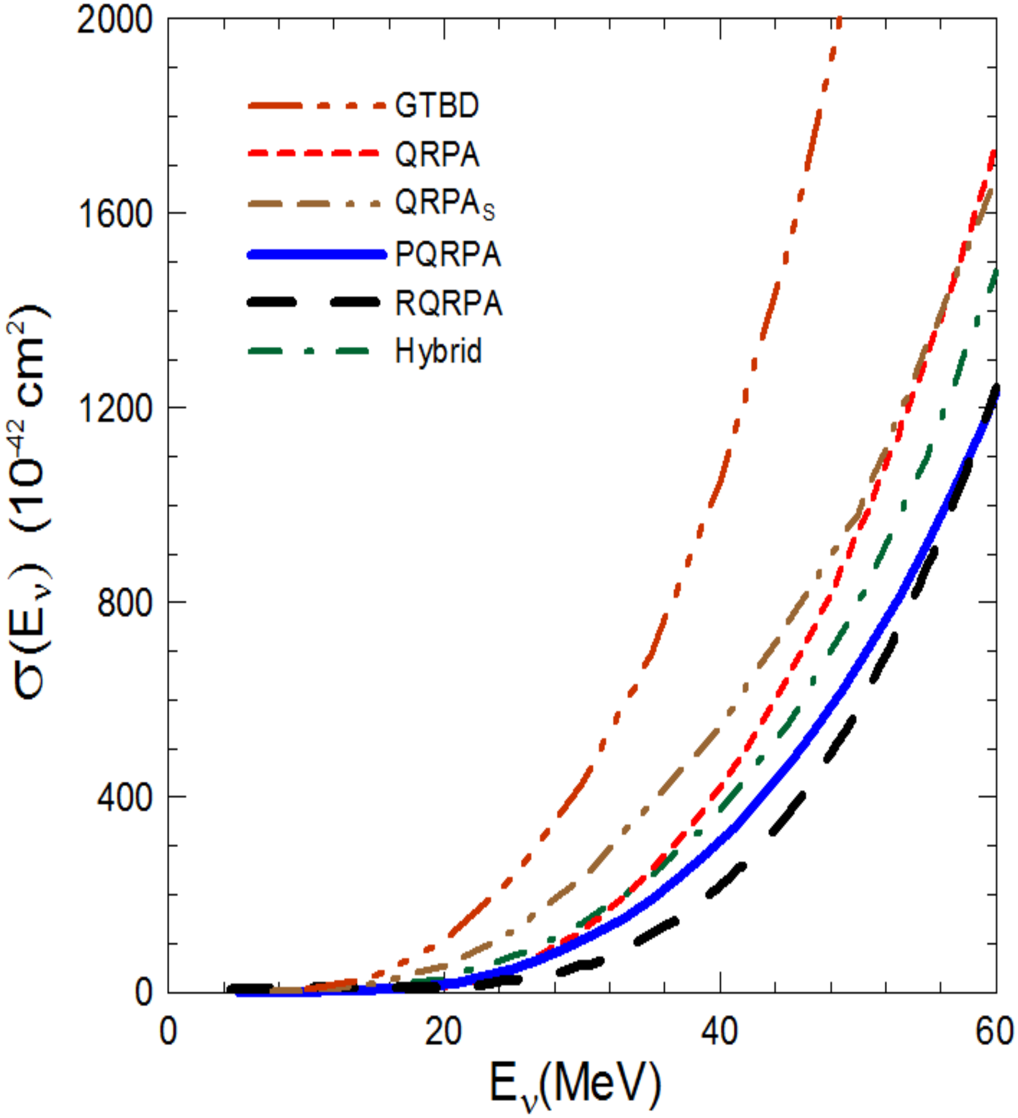
Interaction cross section

Efficiency

$$N_{ev} = N_t \int_0^{\infty} F(E_\nu) \cdot \sigma(E_\nu) \cdot \varepsilon(E_\nu)\, dE_\nu$$

Supernovae neutrinos

$$\nu_e + {}^{56}Fe \Rightarrow {}^{56}Co^* + e^-$$



$$\langle\sigma_e\rangle = \int dE_\nu \sigma(E_\nu) n(E_\nu),$$

$$n(E_\nu) = \frac{96E_\nu^2}{M_\mu^4}(M_\mu - 2E_\nu),$$

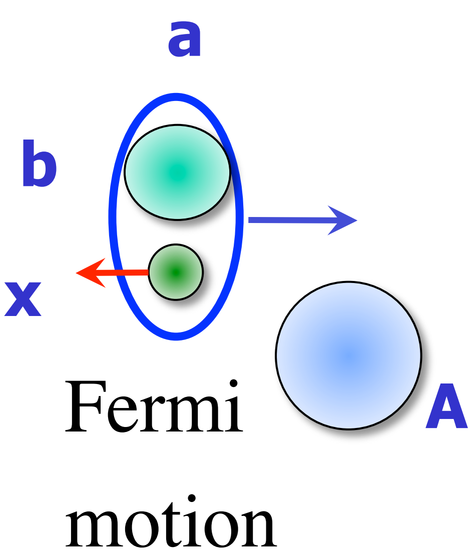
Model	$\langle\sigma_e\rangle$
QRPA	264.6
PQRPA	197.3
Hybrid$^{(a)}$ [14]	228.9
Hybrid$^{(b)}$ [14]	238.1
TM [26]	214
RPA [27]	277
QRPA$_S$ [15]	352
RQRPA [16]	140
Exp[5]	$256 \pm 108 \pm 43$

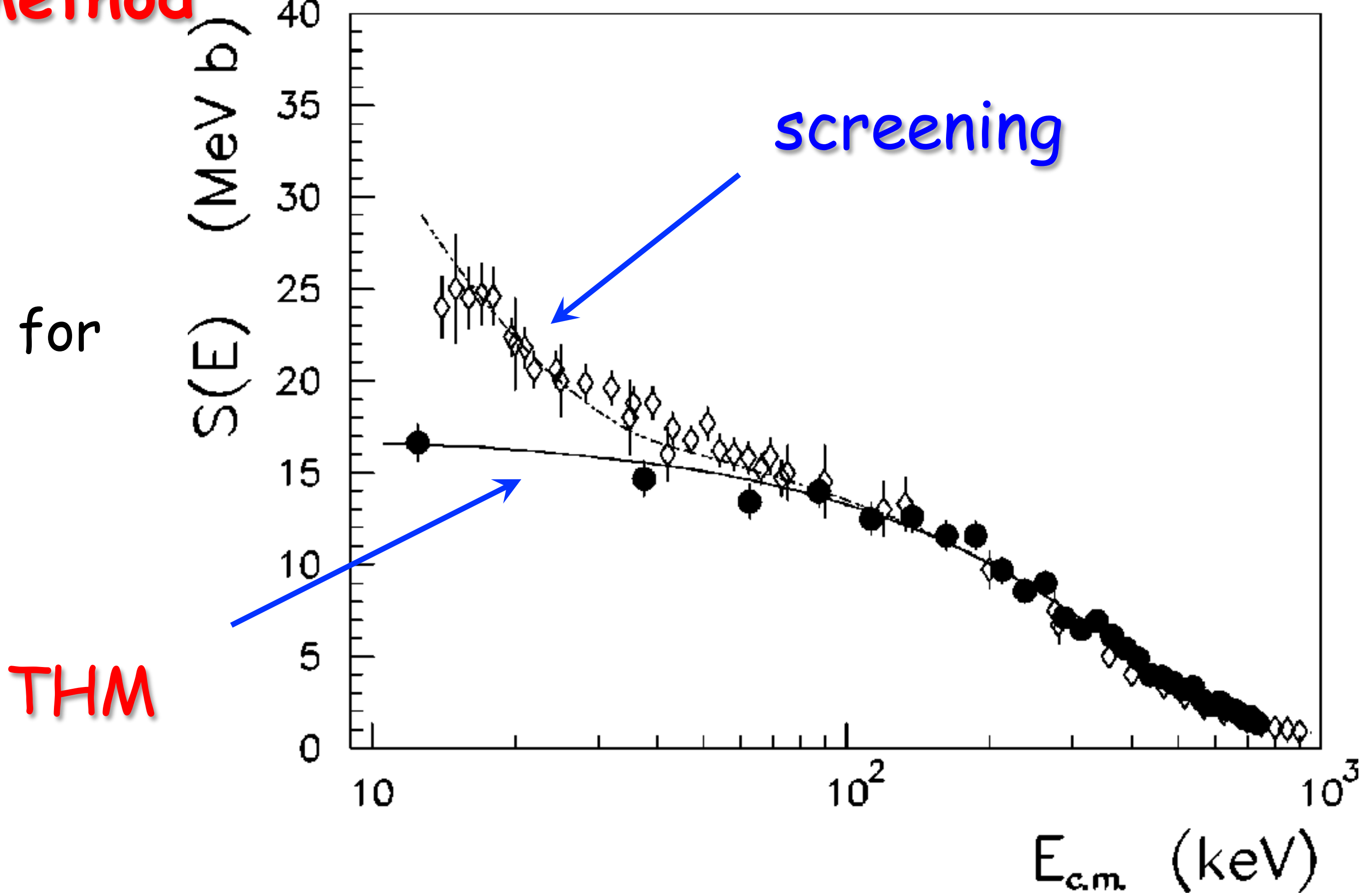
Samana, Bertulani, PRC 78, 024312 (2008)
Samana, Krmpotic, Bertulani, CPC 181, 1123 (2010)


Brilliant ideas

Trojan Horse Method

Astrophysically relevant cross section σ(E) for
B + x → C + D
from transfer
A + B → C + D + s





$^2H(^6Li,\alpha)^4He$
from
$^6Li(^6Li,\alpha\alpha)^4He$

Spitaleri et al., PRC 63, 055801 (2001)

Multi-nucleon transfer (e.g. Pollarolo, Torino)

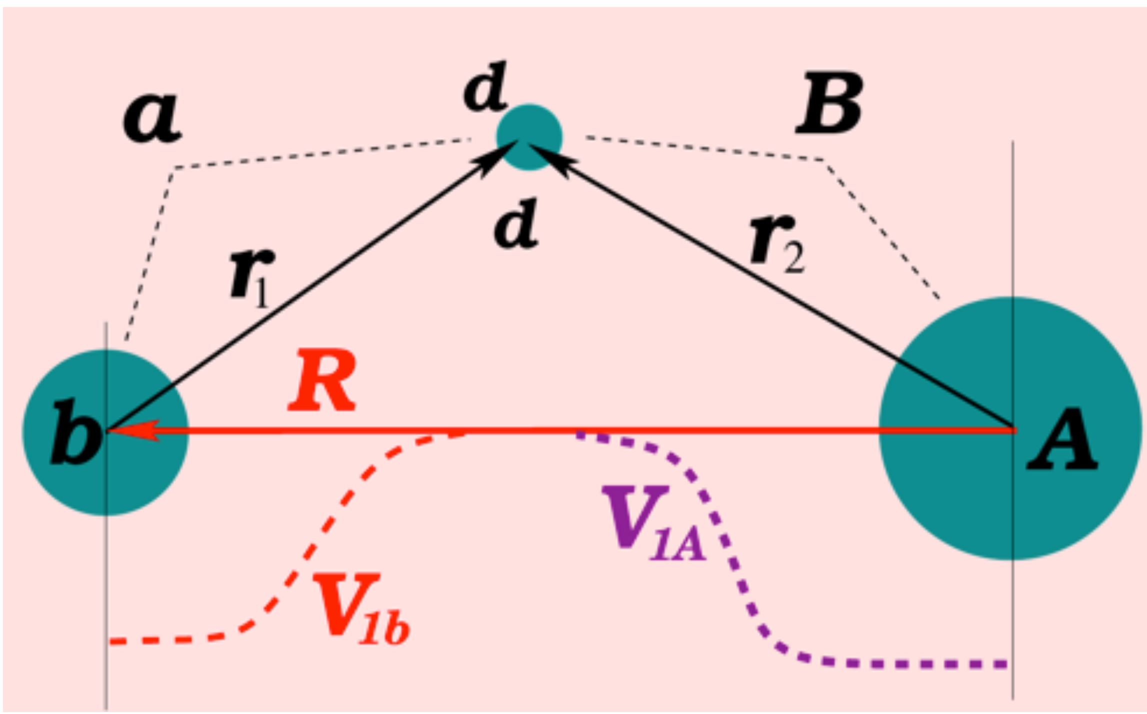
$$P_\beta = \left| \frac{i}{\hbar} \int_{-\infty}^{\infty} dt\, F_{\beta\alpha}(\mathbf{R})\, e^{i(E_\beta - E_\alpha)t/\hbar + (\ldots)} \right|^2 \sim \tau_{coll} \left| F_{\beta\alpha}(D) \right|^2 g(Q_{\beta\alpha})$$

$$F_{\beta\alpha}(\mathbf{R}) \sim \int d^3\mathbf{r}_1\, e^{i\mathcal{Q}\bullet\mathbf{r}_1} \phi_{a_n}^{(A)}(\mathbf{R}+\mathbf{r}_1)\left(V_{1A} - \langle U \rangle\right)\phi_{a_n'}^{(b)}(\mathbf{r}_1)$$

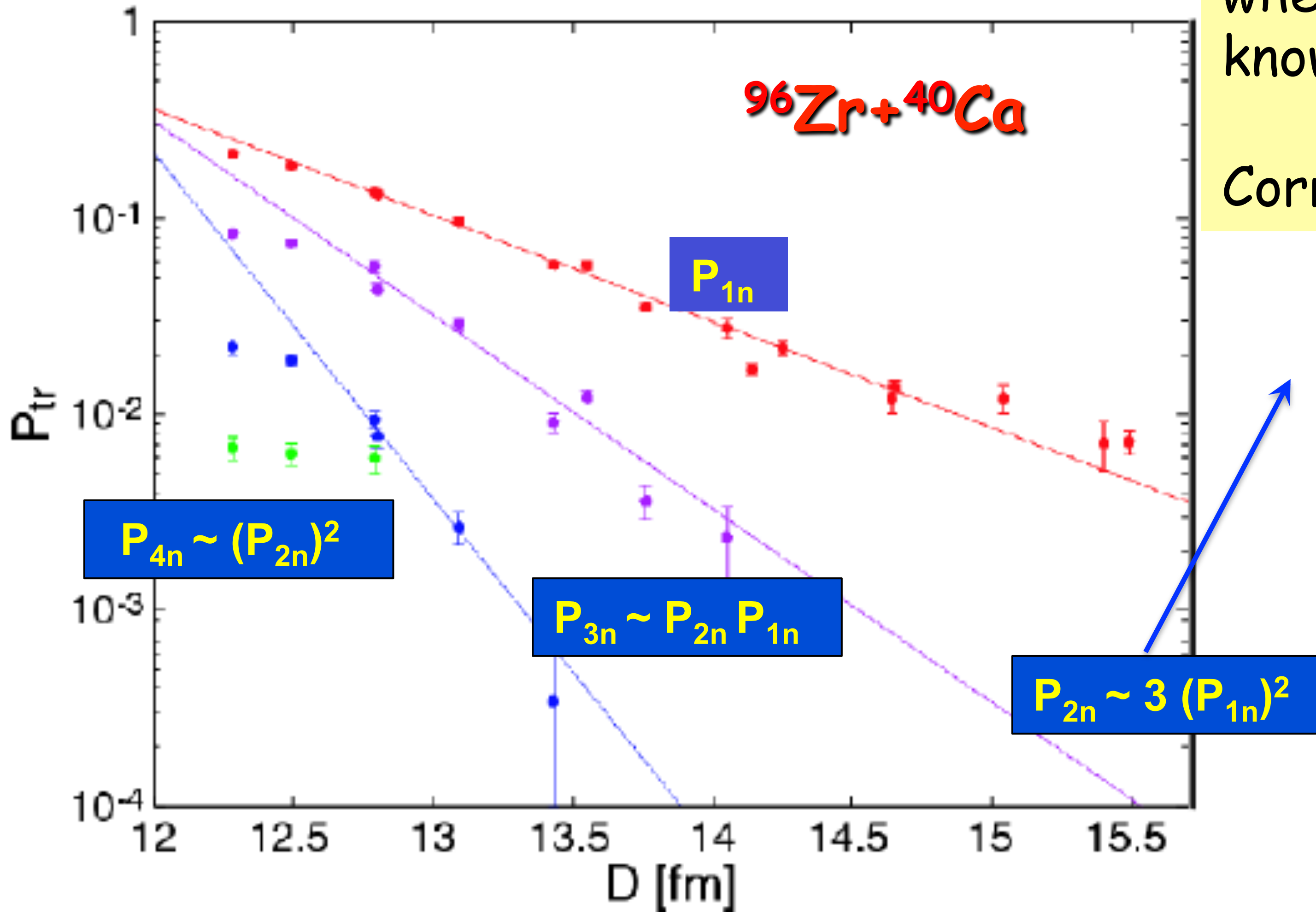
$\mathcal{Q}$ = momentum transfer

V_{1A} transfer interaction.

Why not V_{1b} ?? POST-PRIOR representation



$$\frac{P_{tr}}{\sin(\theta_{c.m.}/2)} \propto \exp(-2\alpha D)$$

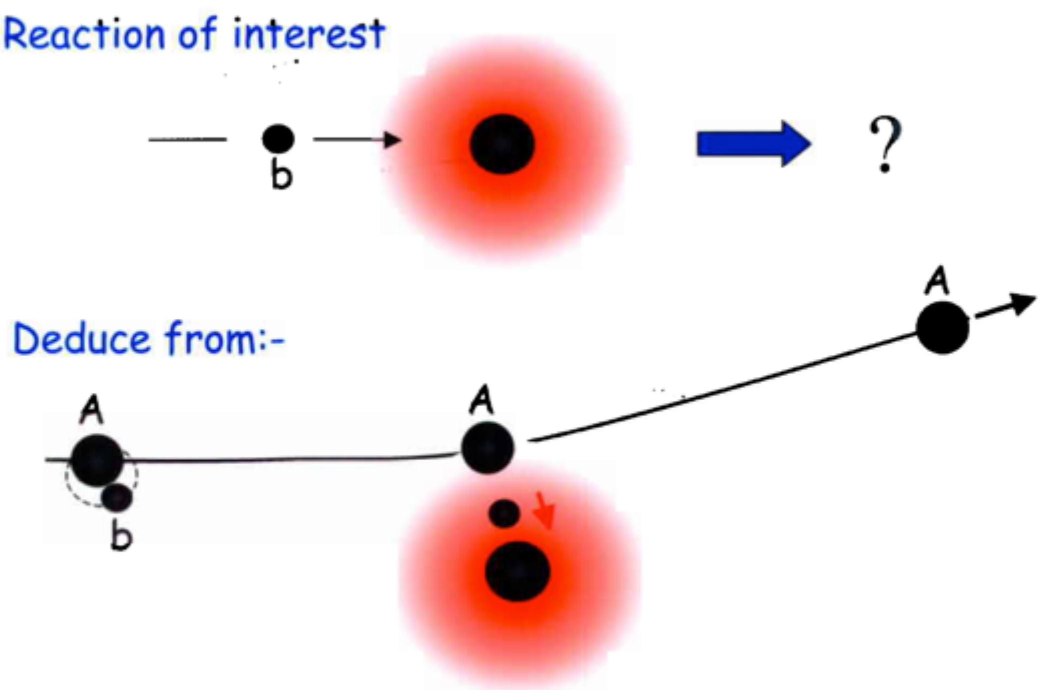


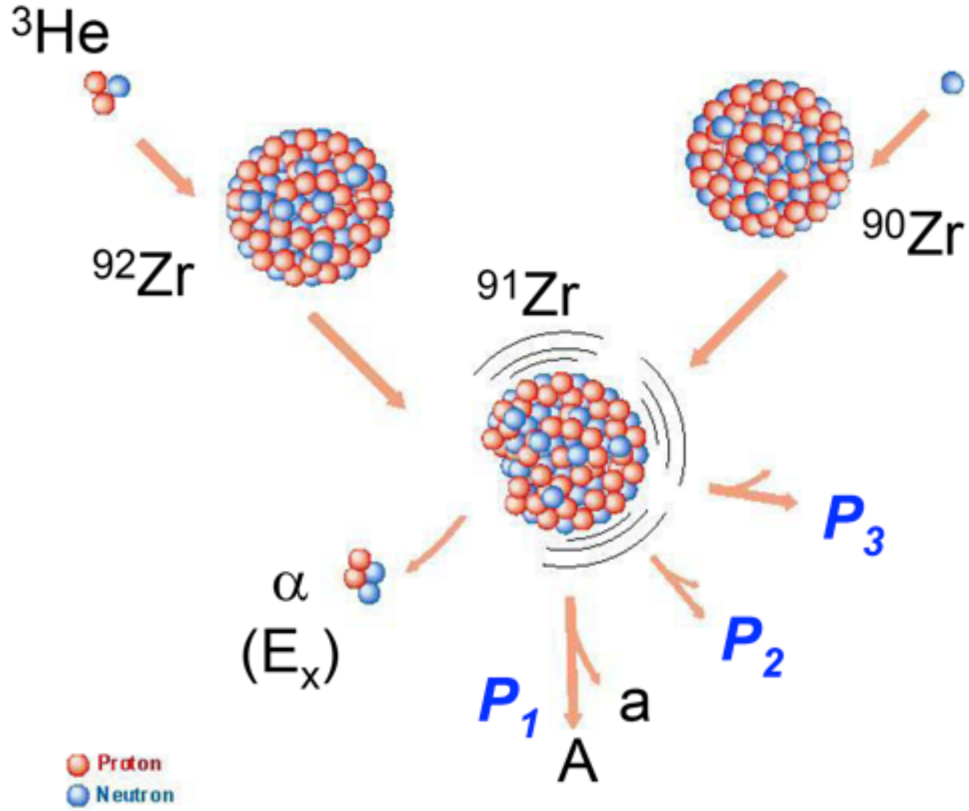
"That is what happens when theorists do not know what to do"

Corradi, Legnaro, 2011

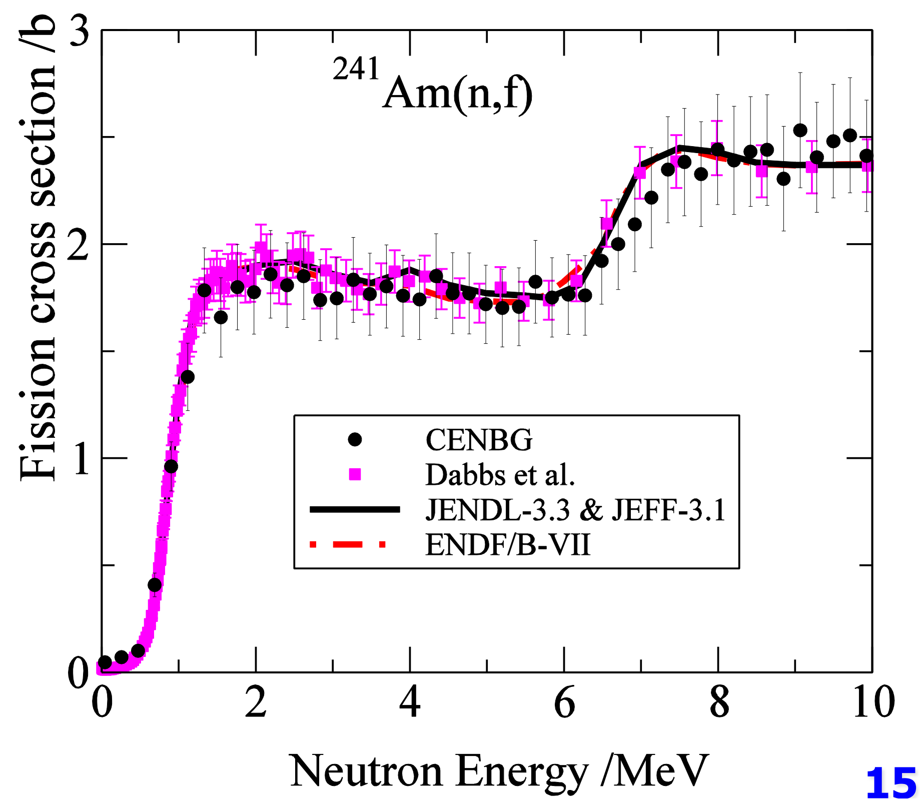
$$D = \frac{Z_1 Z_2 e^2}{2E_{c.m.}}\left(1 + \frac{1}{\sin(\theta_{c.m.}/2)}\right)$$

Surrogate reactions





e.g., (n,f) from transfer reactions
Kessedjian, et al., PLB 692, 297 (2010)



Fission cross sections not sensitive to differences J^{π} distributions!!!

→ **Hauser-Feshbach = Ewing-Weisskopf**

→ Surrogate transfer reactions work

BUT, unfortunately, most often it doesn't work this way.

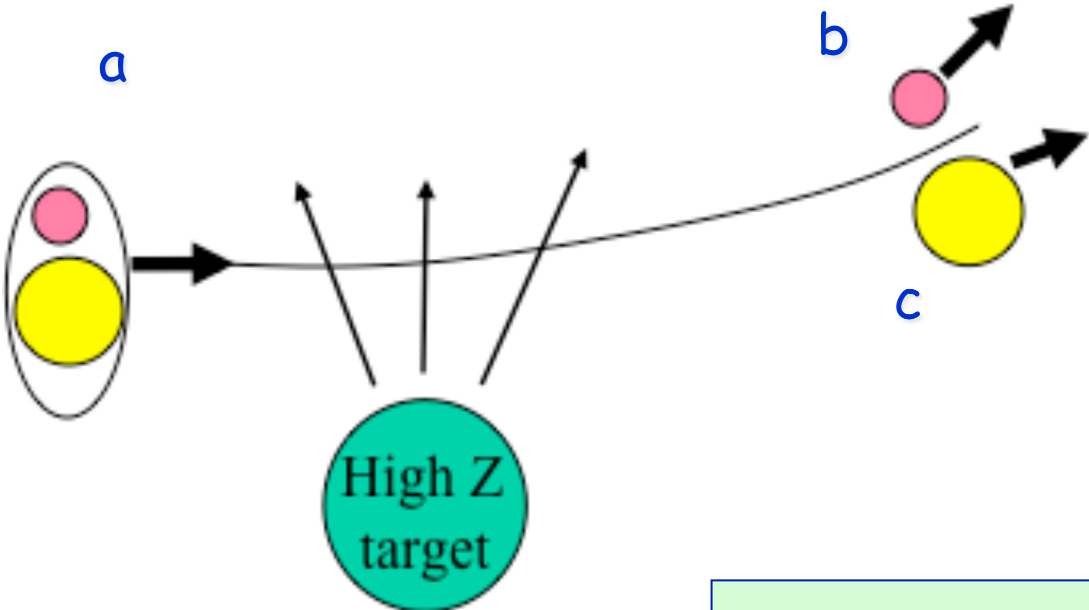

Other Beautiful ideas

Coulomb dissociation method

Baur, Bertulani, Rebel
NPA 458 (1986) 188

$$\frac{d\sigma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma}\sum_l \frac{dn_l(E_\gamma,\Omega)}{dE_\gamma d\Omega}\sigma_{\gamma+\mathrm{a}\rightarrow\mathrm{b}+\mathrm{c}}(E_\gamma)$$

Theory



detailed balance

$$\sigma_{b+c\rightarrow a+\gamma} = \frac{2(2j_a+1)}{(2j_b+1)(2j_c+1)}\frac{k_{bc}^2}{k_\gamma^2}\sigma_{\gamma+a\rightarrow b+c}$$

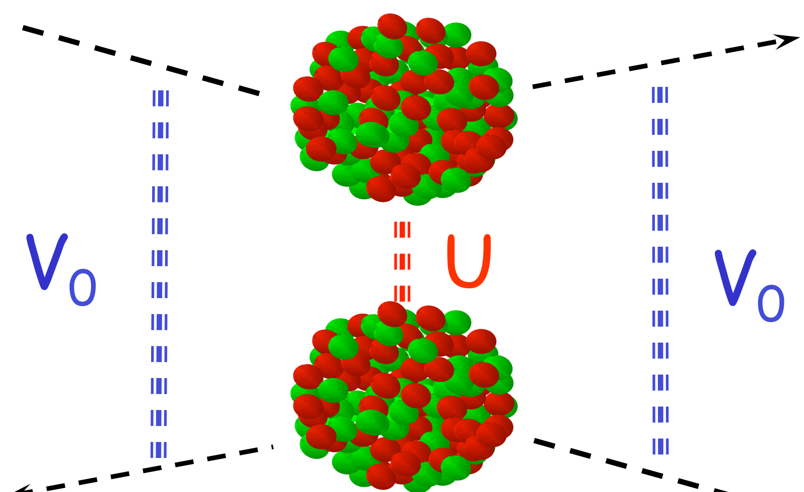
Applications to radiative capture (n,γ) and (p,γ) reactions in nuclear astrophysics.

DWBA

$$f_{inel}(\theta) = -\frac{4\pi^2\mu}{\hbar^2}\int d^3r\, \chi_{\mathbf{k}'}^{(-)*}(\mathbf{r})\, U(\mathbf{r})\, \Psi_{\mathbf{k}}^{(+)}(\mathbf{r})$$

$$\Psi^{\pm} \sim \chi^{\pm} \quad \Longrightarrow \quad f_{DWBA}(\mathbf{k}',\mathbf{k}) = -\frac{4\pi^2\mu}{\hbar^2}\left\langle \chi_{\mathbf{k}'}^{(-)} \middle| U \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

$$T_{DWBA}(\mathbf{k}',\mathbf{k}) = \left\langle \chi_{\mathbf{k}'}^{(-)} \middle| U \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$



Distorted: all orders in V_0

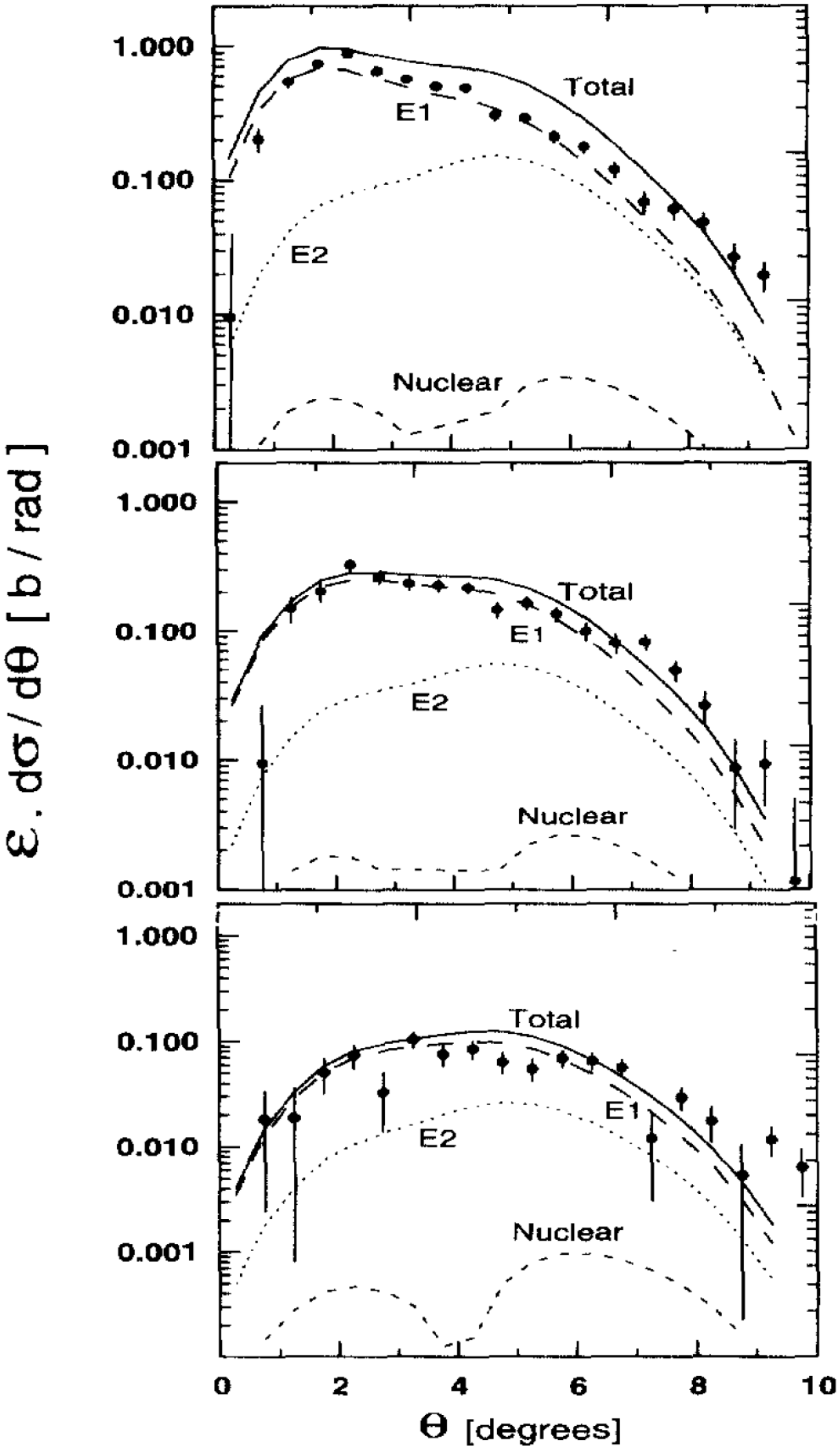
Born: only first order in U

Coulomb excitation + Nuclear excitation

$$f^{C}_{inel}(\theta) \approx \int d^3r\, d^3r'\, \Psi^{(-)*}_{\mathbf{k}'}(\mathbf{r})\,\varphi_f(\mathbf{r}')\,\boxed{V_C(\mathbf{r},\mathbf{r}')}\,\Psi^{(+)}_{\mathbf{k}}(\mathbf{r})\,\varphi_i(\mathbf{r}')$$

$$f^{N}_{inel}(\theta) \approx \int d^3r\, d^3r'\, \Psi^{(-)*}_{\mathbf{k}'}(\mathbf{r})\,\varphi_f(\mathbf{r}')\,\boxed{U_N(\mathbf{r},\mathbf{r}')}\,\Psi^{(+)}_{\mathbf{k}}(\mathbf{r})\,\varphi_i(\mathbf{r}')$$

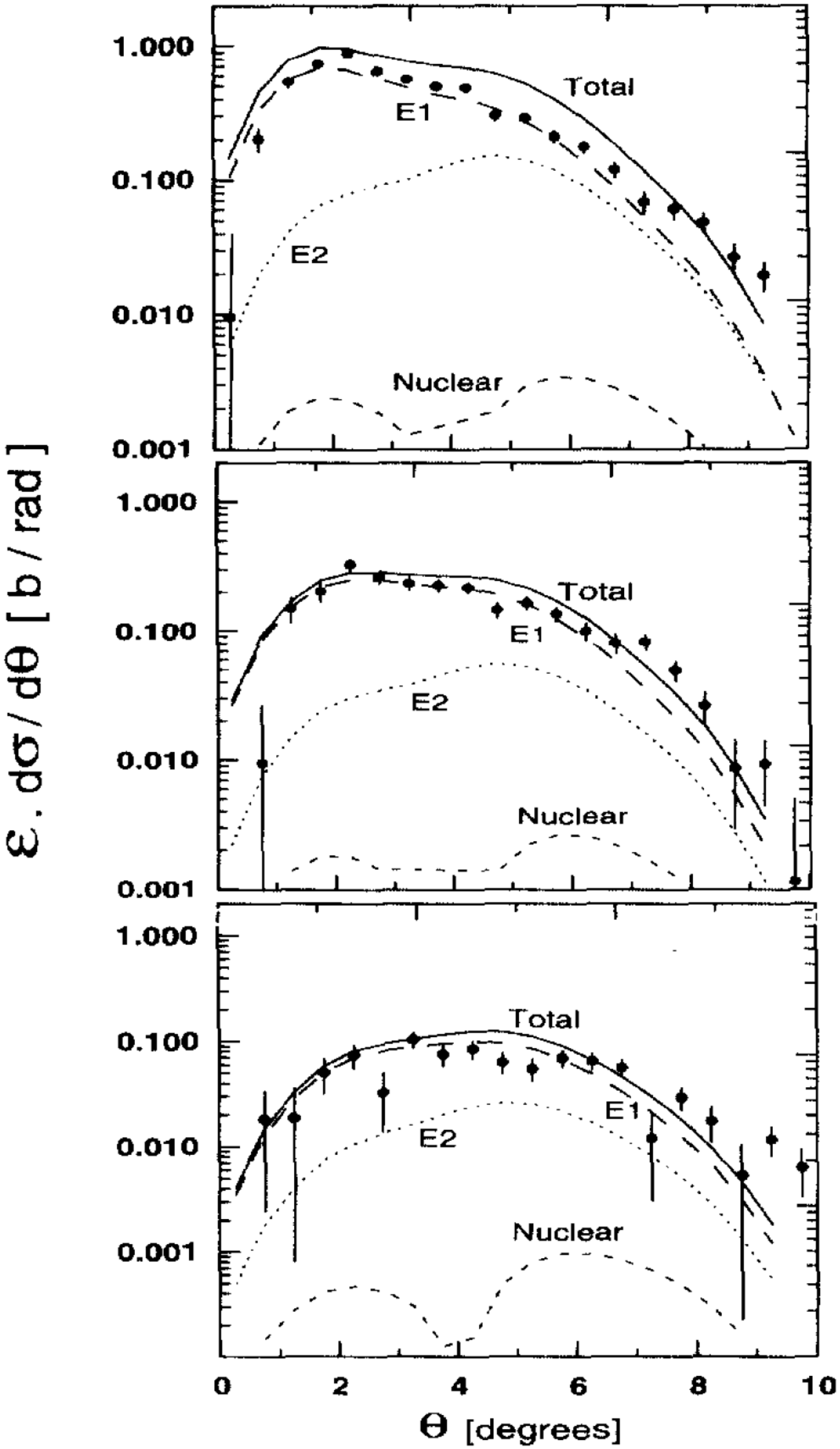
$$\frac{d\sigma}{d\Omega} = \left| f^{N}_{inel}(\theta) + f^{C}_{inel}(\theta) \right|^2$$



$^{8}B + Pb \rightarrow {}^{7}Be + p + Pb$
50 MeV/nucleon

Relevant for $^{7}Be(p,\gamma)^{8}B$ (Sun)

Data: Kikuchi et al, PLB 391, 261 (1997)
Calc: Bertulani, Gai, NPA 636, 227 (1998)



$^8B + Pb \rightarrow {}^7Be + p + Pb$
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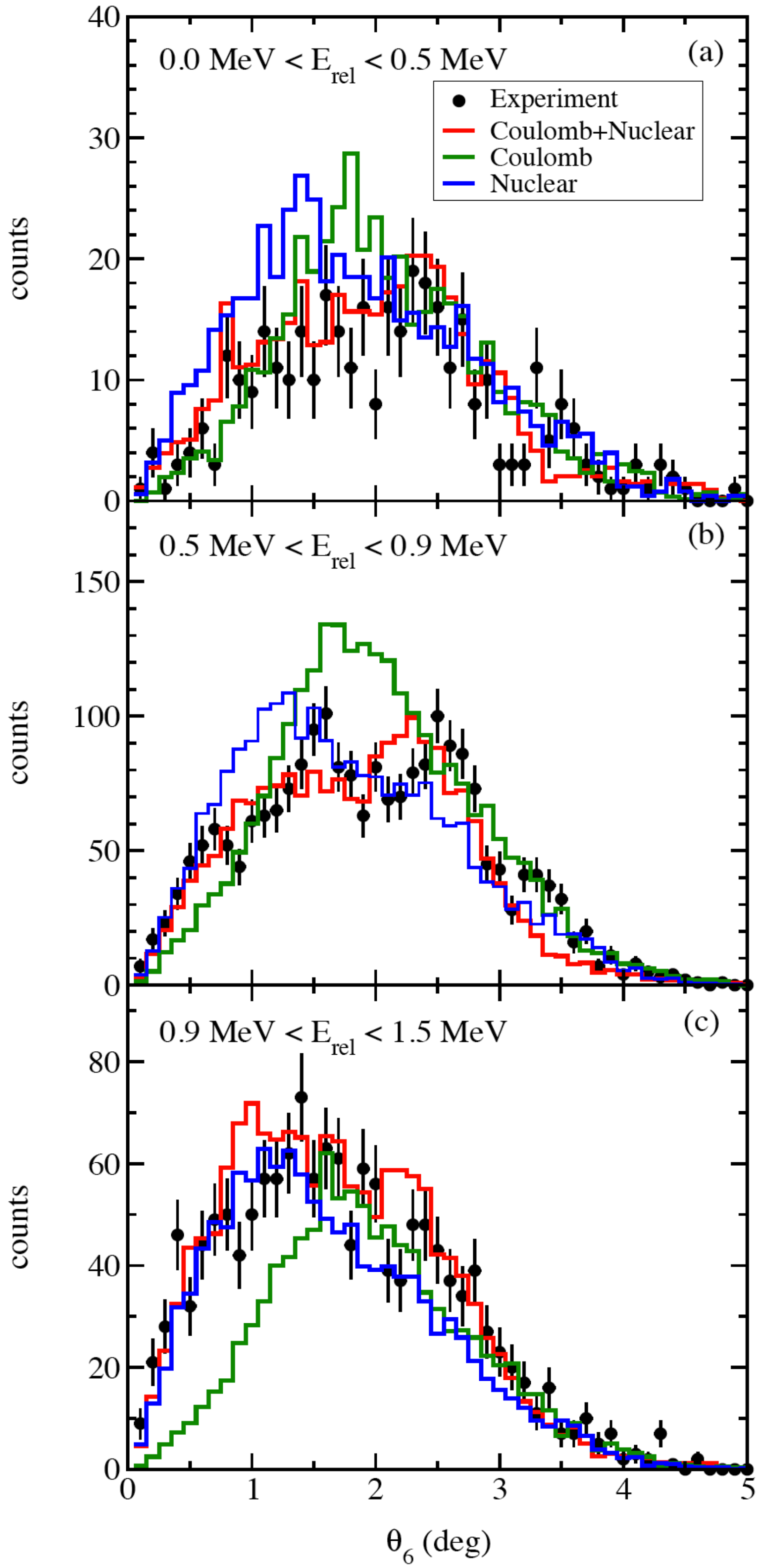
Relevant for $^7Be(p,\gamma)^8B$ (Sun)

Data: Kikuchi et al, PLB 391, 261 (1997)
Calc: Bertulani, Gai, NPA 636, 227 (1998)

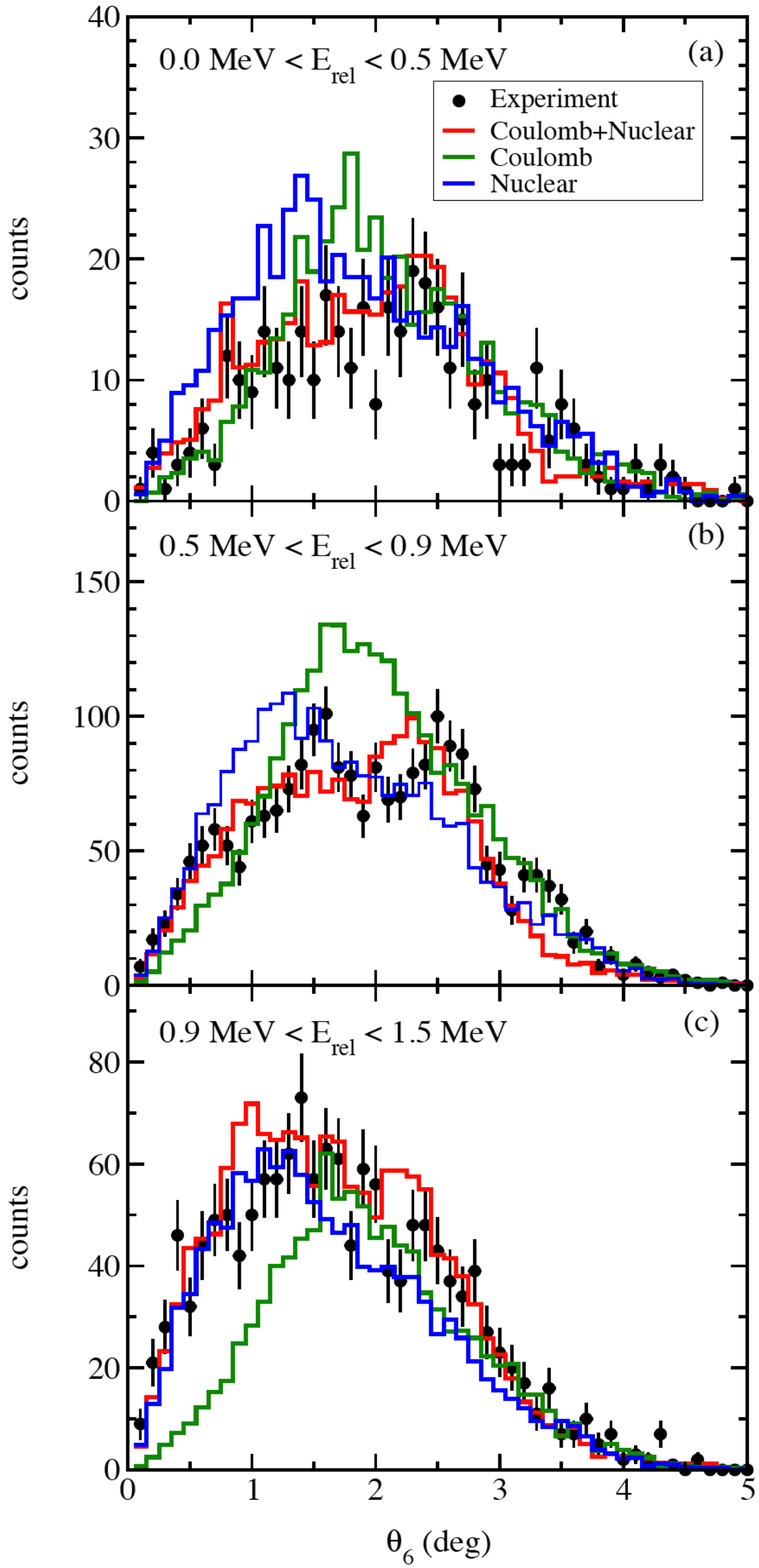
Good wins!



$^6Li + Pb \rightarrow \alpha + d + Pb$
26 MeV/nucleon

Relevant for BBN

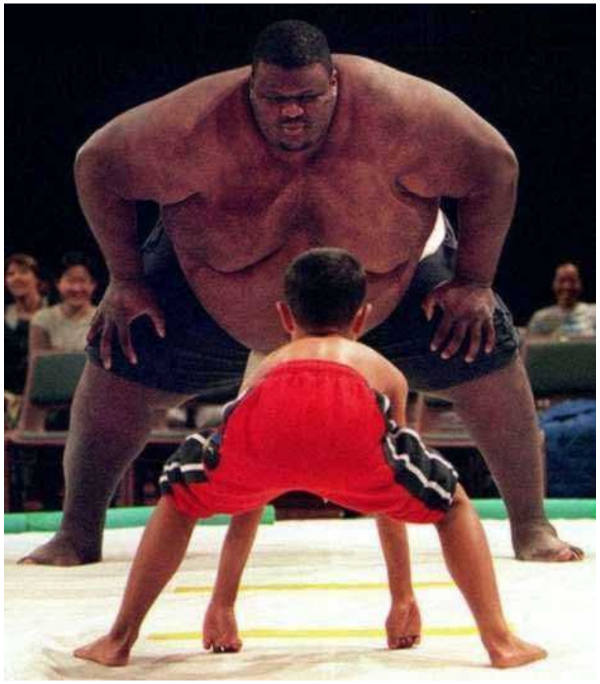
Data: Hammache et al., PRC 82 (2010) 065803
Calc: Stefan Typel



^{6}Li + Pb → α + d + Pb
26 MeV/nucleon

Relevant for BBN

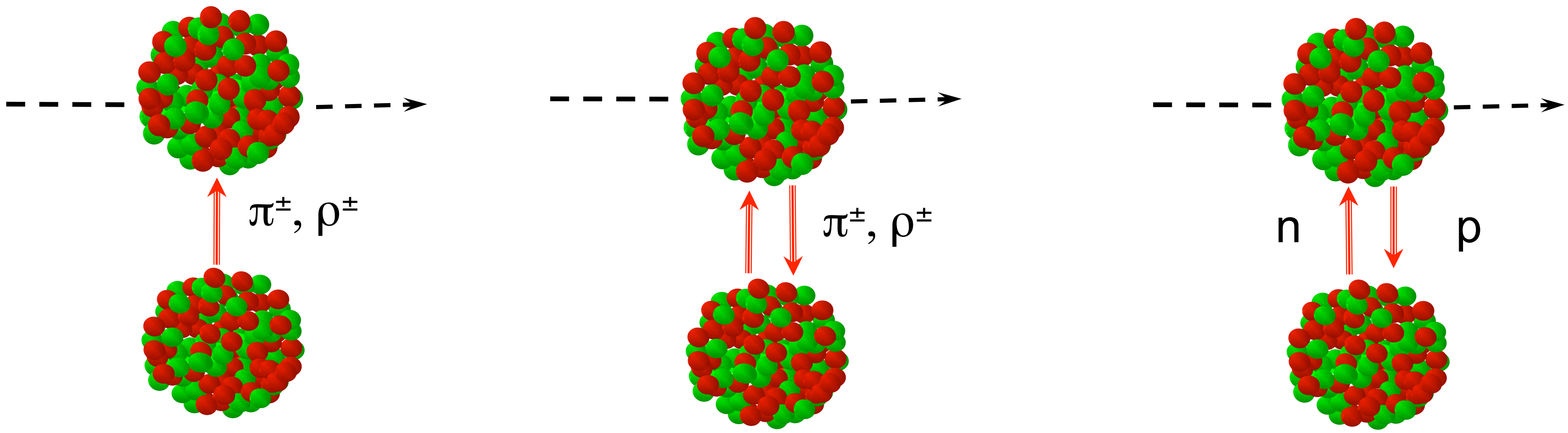
Data: Hammache et al., PRC 82 (2010) 065803
Calc: Stefan Typel



Evil wins!

More ways

Charge-exchange reactions (e.g., Fujita, RCNP)



One step

(a)

Two step

(meson exchange)

Two step

(nucleon pickup & stripping)

(b)

(a) $$T_{DWBA}(\mathbf{k'},\mathbf{k}) = \left\langle \chi_{\mathbf{k'}}^{(-)} \middle| U \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

(b) $$T_{DWA}(\mathbf{k'},\mathbf{k}) = \sum_{\gamma=0} C_\gamma \left\langle \chi_{\mathbf{k'}}^{(-)} \middle| U\left(G^{(+)}U\right)^\gamma \middle| \chi_{\mathbf{k}}^{(+)} \right\rangle$$

Effective interaction V_{NN} (phenomenological)

$$V_{NN}(\mathbf{r}) = V^C(r) + V_\sigma^C(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + \left[V_\tau^C(r) + V_{\sigma\tau}^C(r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right](\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$
$$+\left[V^T(r) + V_\tau^T(r)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\right] S_{12}(\hat{\mathbf{r}}) + V^{LS}(r)\ \boldsymbol{l}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$$

Antisimetrization: $V_{NN}(\mathbf{r}) = \left[1-(-)^l P_x\right] V_{12}(\mathbf{r})$ $\quad P_x : \mathbf{r} \to -\mathbf{r}$

$V^{LS}(r)\,\boldsymbol{l}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$ small and usually neglected

Notation:

$$V^C(r) = V_{00}^0(r),\quad V_\sigma^C(r) = V_{10}^0(r),\quad V_\tau^C(r) = V_{01}^0(r)$$
$$V_{\sigma\tau}^C(r) = V_{11}^0(r),\quad V^T(r) = V_{10}^2(r),\quad V_\tau^T(r) = V_{01}^2(r)$$

$$V_{12}(\mathbf{r}) = \sum_{\substack{K=0,2\\ ST}} V_{ST}^K(r) C_S^K\ Y_K(\hat{\mathbf{r}}) \left[\boldsymbol{\sigma}_1\otimes\boldsymbol{\sigma}_2\right]^K \left[\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right]^T$$

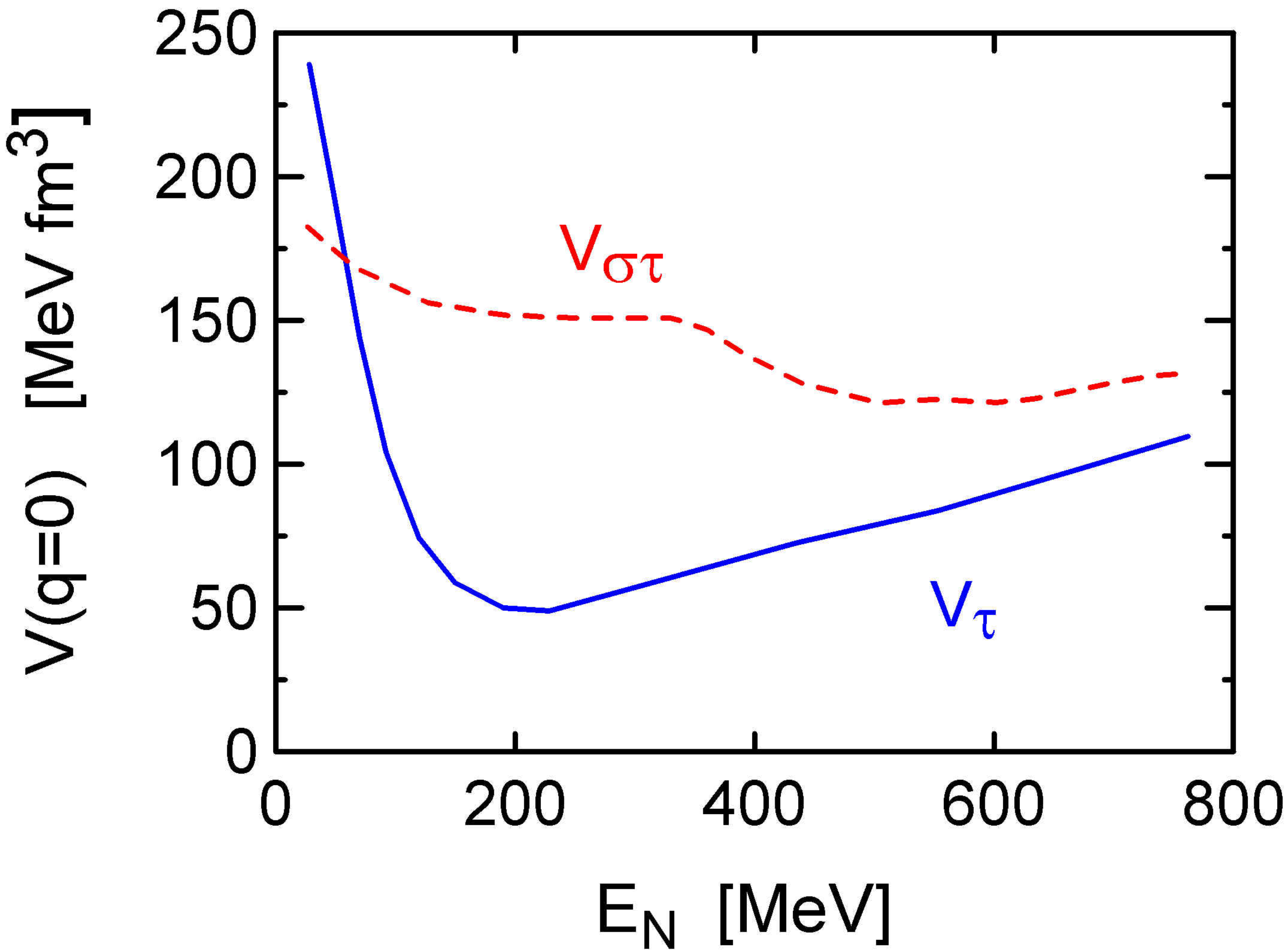
K = 0 : central force

K = 2 : tensor force

$$\boldsymbol{\sigma}^{S=0} = 1,\quad \boldsymbol{\sigma}^{S=1} = \boldsymbol{\sigma}$$
$$\boldsymbol{\tau}^{T=0} = 1,\quad \boldsymbol{\tau}^{T=1} = \boldsymbol{\tau}$$

$$C_0^0 = \sqrt{4\pi},\quad C_1^0 = -\sqrt{12\pi}$$
$$C_0^2 = 0,\quad C_1^2 = \sqrt{25\pi/5}$$

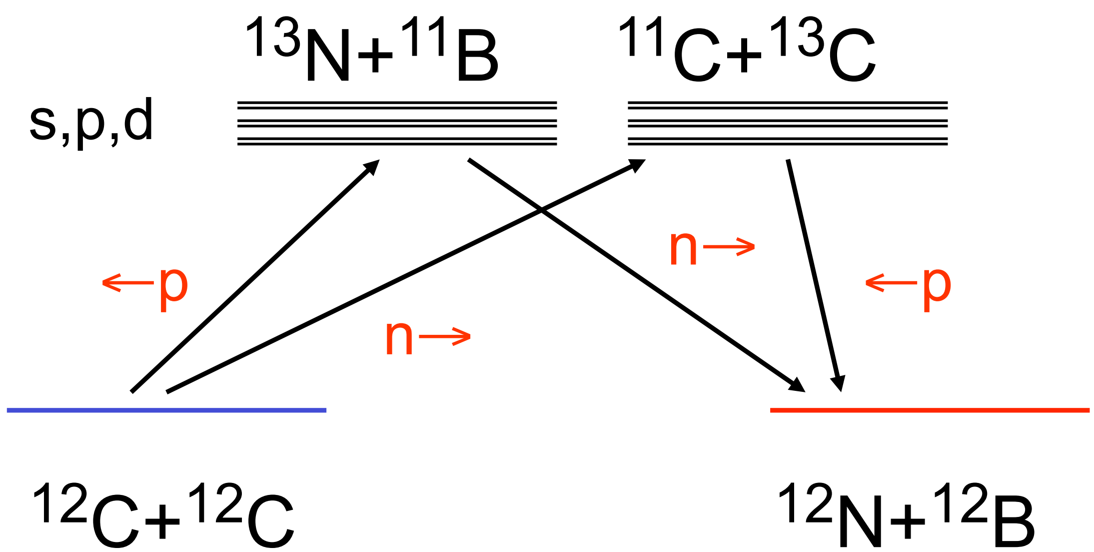
Love, Franey, NPA 1981, 1985

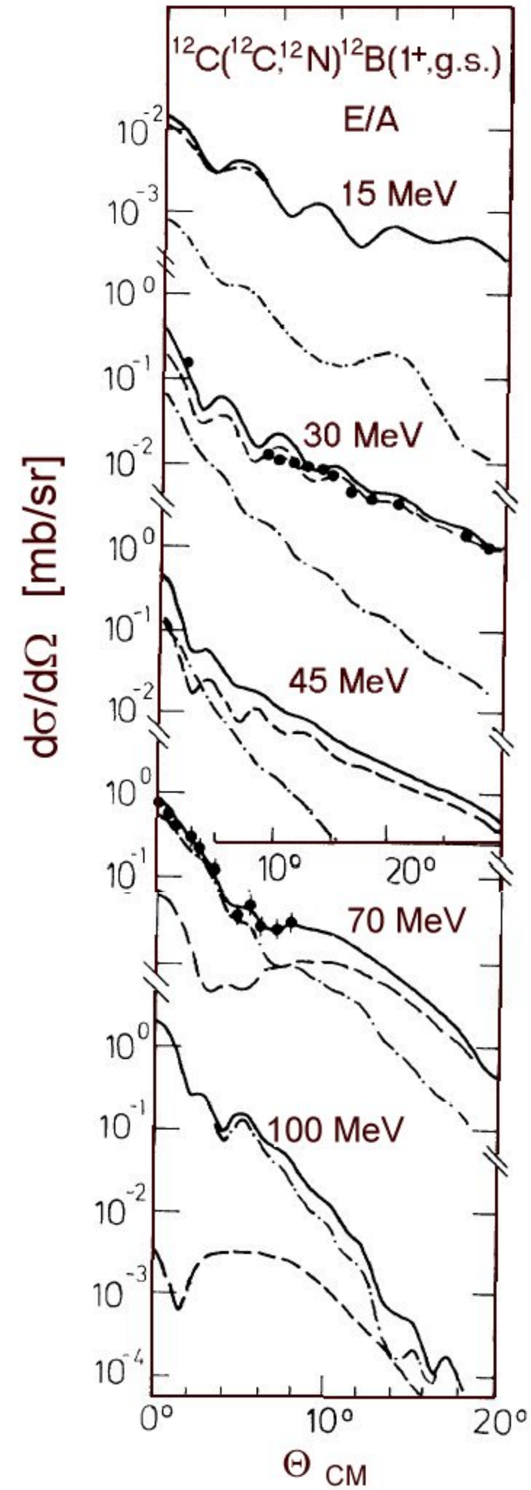


Lenske, Wolter, Bohlen, PRL 62, 1457 (1989)

$^{12}C(^{12}C,^{12}N)^{12}B(1^+,gs)$

Two step (proton pickup & neutron-stripping)





Bertulani, NPA 554, 493 (1993)

$^{13}C(^{13}N,^{13}C)^{13}N$ 70 MeV/nuc.

Two step (double $\pi+\rho$ exchange)

$$\sigma_{2nd} \sim 10^{-4} \times \sigma_{1st}$$

DBWA

$$T_{ch.exch.}(\mathbf{k}',\mathbf{k}) = \int d^3r\ S(b)\exp[i\mathbf{q}\cdot\mathbf{r}]\ \langle bB|U(\mathbf{r})|aA\rangle$$

$$|\,aA\rangle = |\,aA; J_a M_a T_a N_a; J_A M_A T_A N_A\rangle$$

eikonal + few pages of algebra

Bertulani, NPA 554, 493 (1993)

$$T_{ch.exch.}(\mathbf{k}',\mathbf{k}) = \sum_{\substack{K=0,2\\ ST}} \sum_{\substack{LL'JJ'\\ MM'\mu}} C(KS;LL'JJ'MM'\mu)\int db\, b\ S(b) J_0(qb)$$
$$\times \int dp\, p\ J_{M'-M-\mu}(pb)\, \underbrace{\tilde{V}^K_{ST}(p)\, \tilde{\rho}^{aA}_{LJST}(p)\, \tilde{\rho}^{bB}_{L'J'ST}(p)}$$

$$\tilde{\rho}^{aA}_{LJST}(p) = \int dr\, r^2\ j_L(pr)\left\langle J_a T_a \left\| \sum_i \frac{\delta(r-r_i)}{r_i^2} \Im^{LSJ}_M \boldsymbol{\tau}^T \right\| J_b T_b \right\rangle$$

STRUCTURE INPUT
beautifully factorized

$$\Im^{LSJ}_M = \sum_{\mu M_L} \langle L M_L S\mu \,|\, JM\rangle i^L\, Y_{LM_L}(\hat{\mathbf{r}})\sigma^{S\mu}$$

$$T_{aA\to bB}(\mathbf{k}',\mathbf{k}) = \sum_{\ldots}\sum_{\ldots}\cdots\int db\, b\, S(b)\, J_0(qb)\int dp\, p\, J_{\ldots}(pb)\, \tilde{\rho}^{aA}_{\ldots}(p)\, \tilde{\rho}^{bB}_{\ldots}(p)$$

$$S(b)\sim 1 \quad\Longrightarrow\quad p\sim q$$

- $S(b)\neq 1$ but largest value of $T_{aA\to bB}$ occurs when

$J_0(qb)$ oscillates in phase with $J_{\ldots}(pb)$

$\Longrightarrow$ $p\sim q$

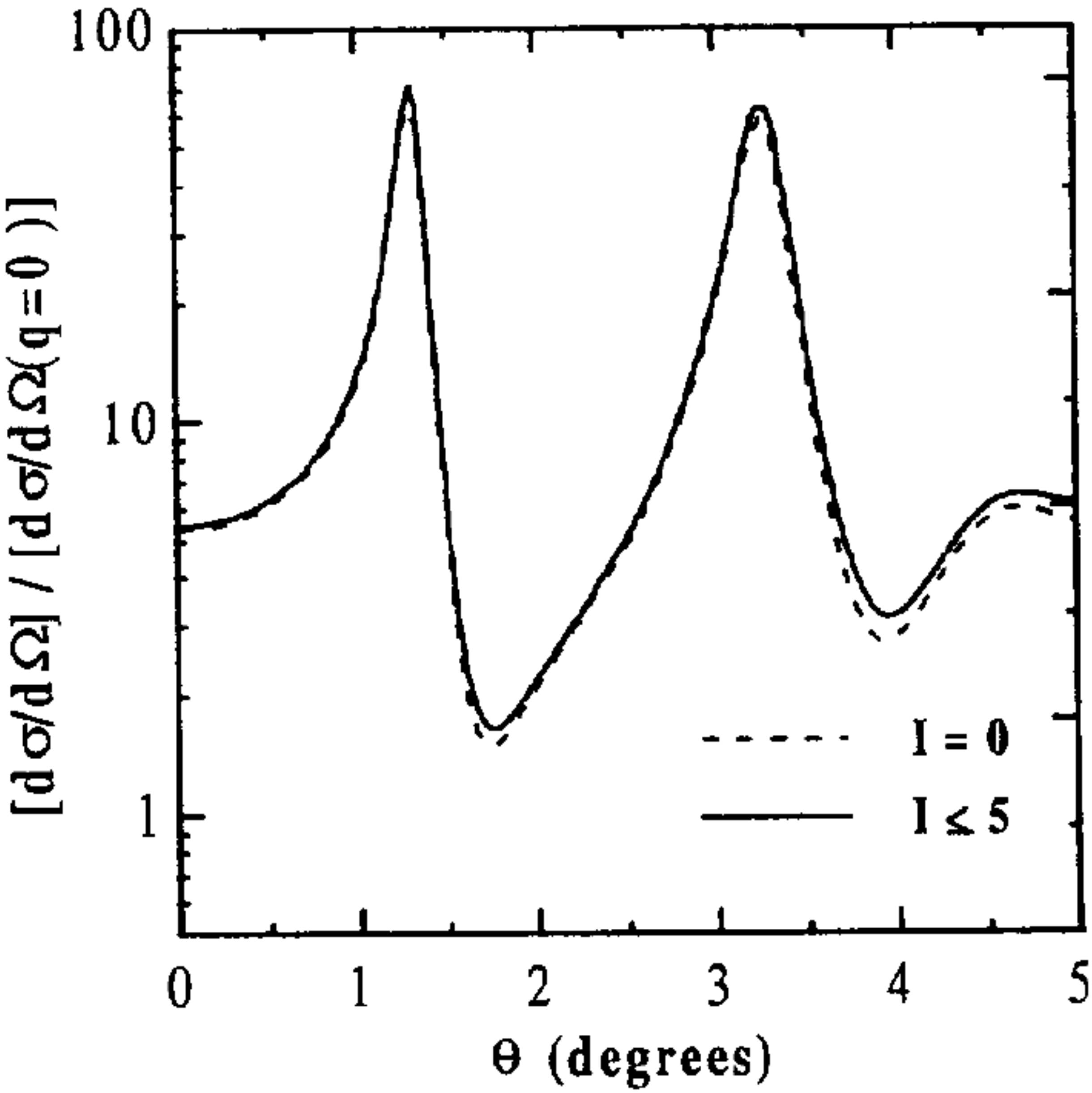
Forward scattering: $q\sim 0$ Bertulani, NPA 554, 493 (1993)

$$f_{aA\to bB}(\theta\sim 0) = \ldots\tilde{\rho}^{aA}_{\ldots}(0)\, \tilde{\rho}^{bB}_{\ldots}(0)\times\int dp\, p\, V^{K}_{ST}(p)\times\int db\, b\, J_0(qb)\, \mathrm{e}^{iX(b)}$$

$$\tilde{\rho}^{aA}_{\ldots}(0) = \cdots\langle A\|\sigma^S\tau\|a\rangle \qquad \frac{d\sigma}{d\Omega}(\theta\sim 0^0) = \cdots\left|\langle A\|\sigma^S\tau\|a\rangle\right|^2 \left|\langle B\|\sigma^S\tau\|b\rangle\right|^2$$

$\Longrightarrow$ • If $\left|\langle A\|\sigma^S\tau\|a\rangle\right|^2$ well known. E.g. (a,A) = (n,p) then

Fermi and Gamow-Teller m.e. READ DIRECTLY from $\frac{d\sigma}{d\Omega}(\theta\sim 0^0)$



$^{13}C(^{13}N,^{13}C)^{13}N$ 70 MeV/nuc.

Bertulani, Lotti, PLB 402 (1997) 237


Relax

Rolling back. We forgot lots of things!

All the way back to 1911 (Rutherford?)

Rutherford was wrong*

Aguiar, Aleixo, Bertulani, PRC 42, 2180 (1990)

$$L = L^{LO} + L^{NLO} + L^{N^2LO} + \cdots$$

$$L^{LO} = \frac{1}{2}\mu c^2 \left(\frac{v}{c}\right)^2 - \frac{Z_1 Z_2 e^2}{r}$$

$$L^{NLO} = \frac{\mu^4 c^2}{8}\left[\frac{1}{m_1^3} - \frac{1}{m_2^3}\right]\left(\frac{v}{c}\right)^4 - \frac{\mu^2 Z_1 Z_2 e^2}{2 m_1 m_2 r}\left[\left(\frac{v}{c}\right)^2 + \left(\frac{\mathbf{v}\cdot\mathbf{r}}{cr}\right)^2\right]$$

$$L^{N^2LO} = \frac{\mu c^2}{512}\left(\frac{v}{c}\right)^6 + \frac{Z_1 Z_2 e^2}{16 r}$$

$$\times\left[\frac{1}{8}\left\{\left(\frac{v}{c}\right)^4 - 3\left(\frac{v_r}{c}\right)^4 + 2\left(\frac{v_r v}{c}\right)^2\right\} + \frac{Z_1 Z_2 e^2}{\mu c^2 r}\left\{3\left(\frac{v_r}{c}\right)^2 - \left(\frac{v}{c}\right)^2\right\} + \frac{4 Z_1^2 Z_2^2 e^4}{\mu^2 c^4 r^2}\right]$$

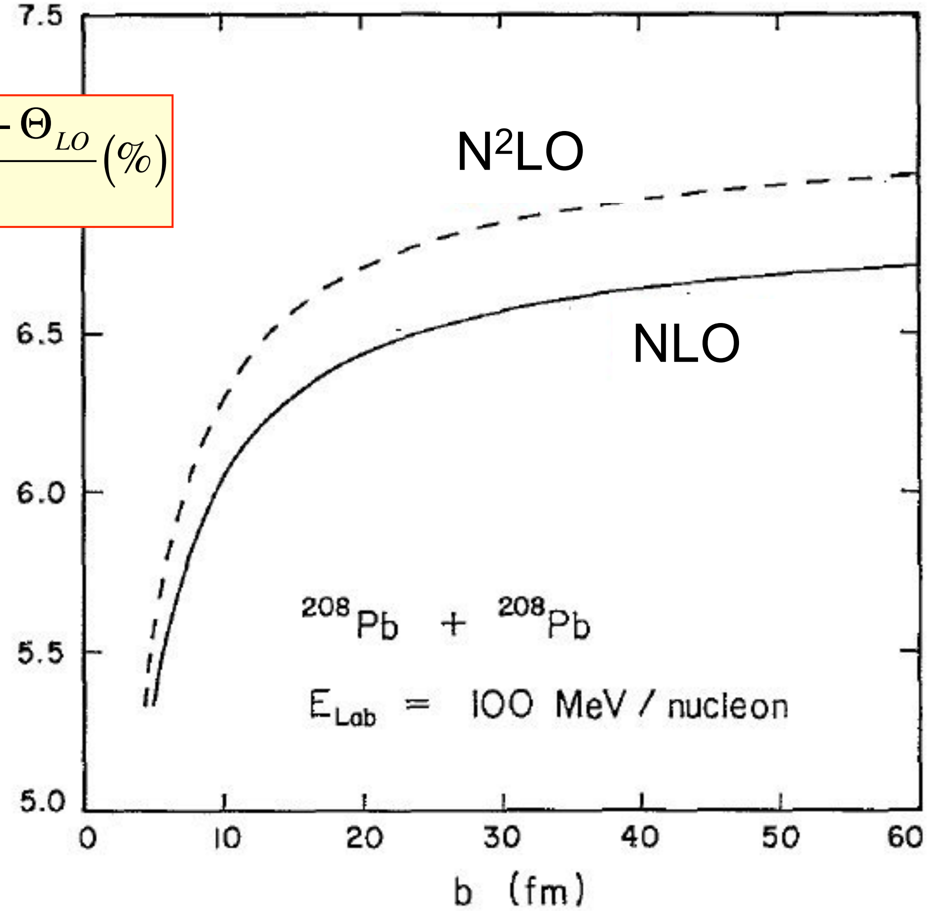
Equations of motion

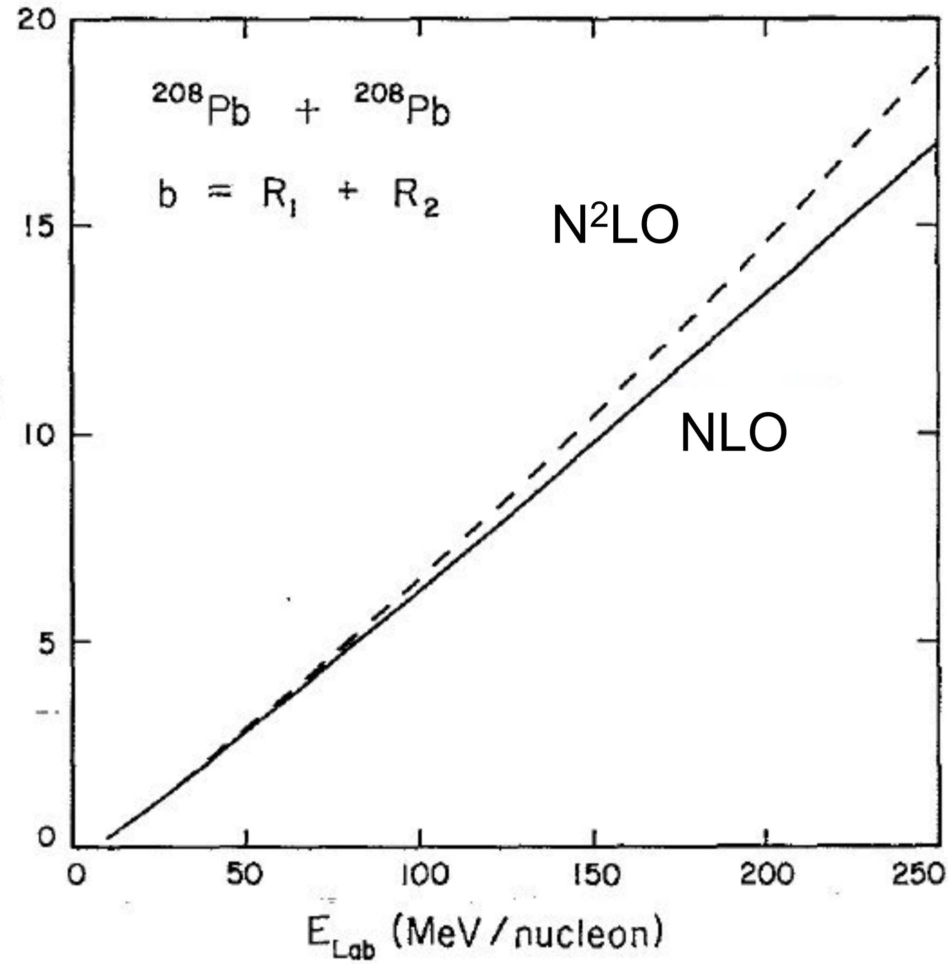
$\Longrightarrow$ **r(t), p(t), Θ(b),** $\dfrac{d\sigma}{d\Omega}$

* If one includes relativistic corrections

Deviations from Rutherford

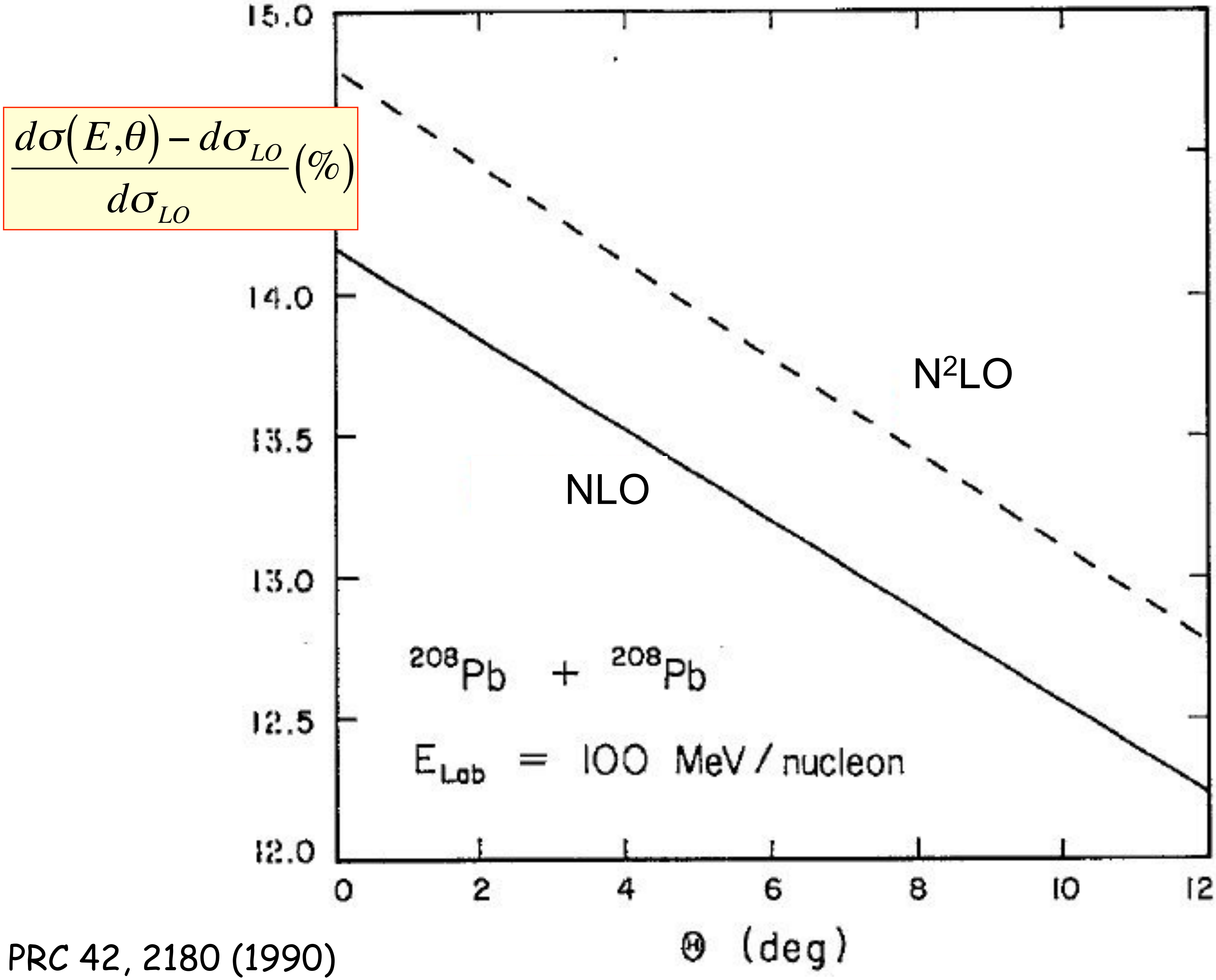
$$\frac{\Theta\,(E,b)-\Theta_{LO}}{\Theta_{LO}}(\%)$$





important for elastic scattering: experimental data often reported as $\dfrac{d\sigma_{elast}}{d\sigma_{Ruth}}$

PRC 42, 2180 (1990)



PRC 42, 2180 (1990)


So what?

Coulomb excitation: orbital integrals with retardation

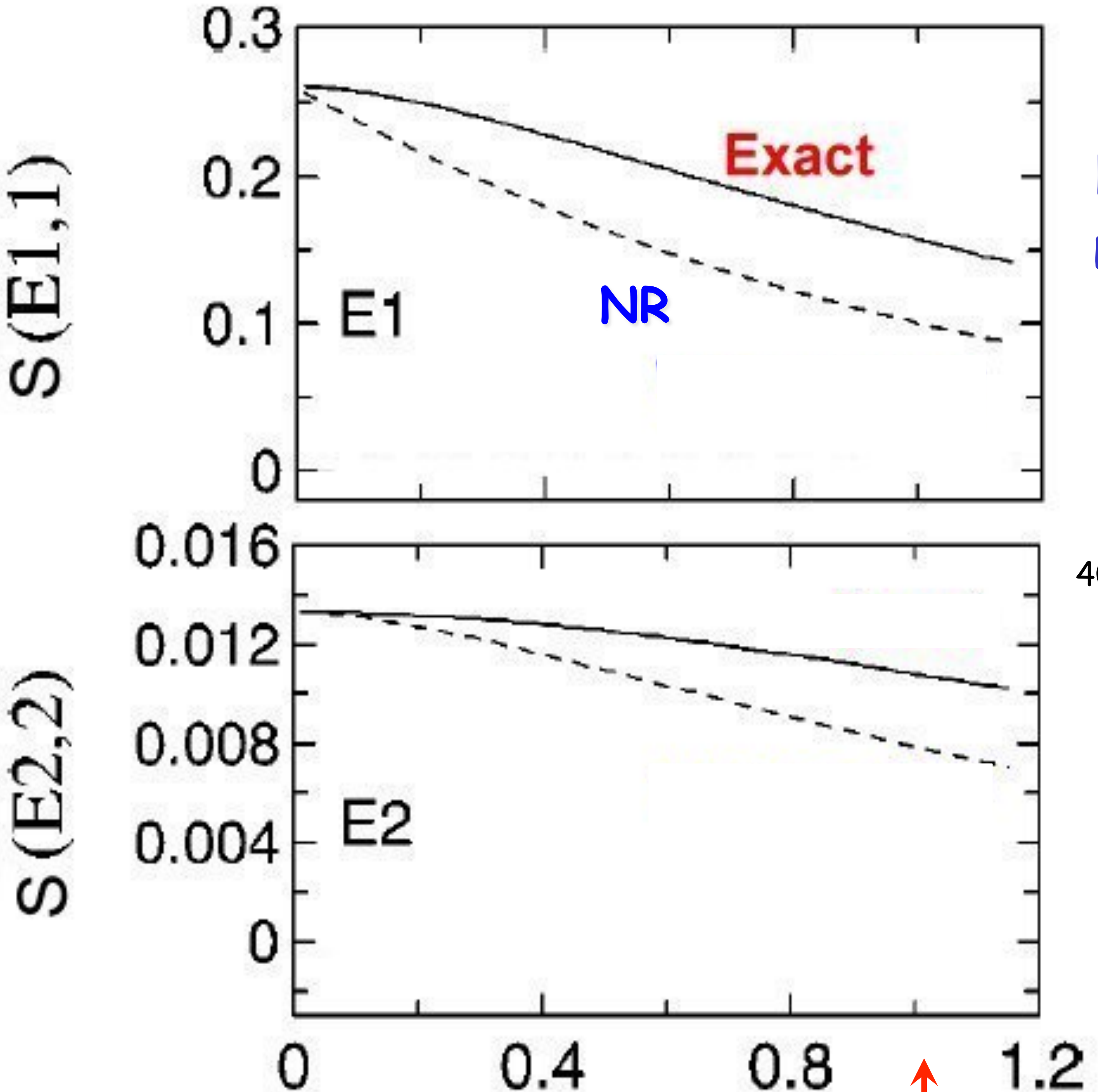
Aleixo, Bertulani, NPA 505, 448 (1989)

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{elast} \times P_{exc}$$

$$= \sum_{\pi\lambda\mu} \left| S(\pi\lambda\mu) \right|^2 \left| M_{fi}(\pi\lambda, -\mu) \right|^2$$

$$M_{fi}(\pi\lambda\mu) = \left\langle f \middle| \mathrm{EM\,Operator}(\lambda\mu) \middle| i \right\rangle$$

$$S(\pi\lambda\mu) = \text{orbital integrals}$$



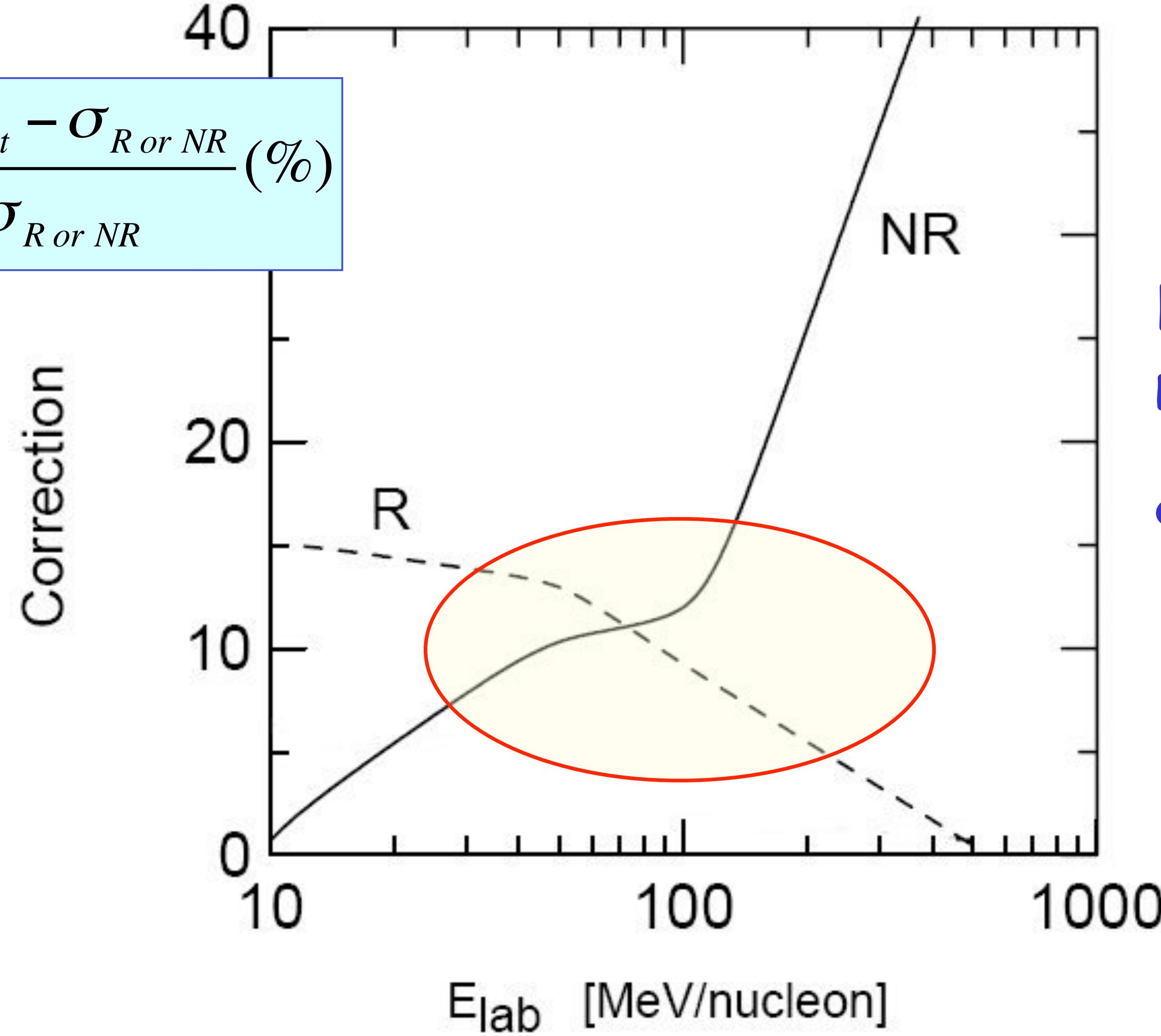
$$\xi = \frac{E_x b}{\gamma \hbar v}$$

Deviations from non-relativistic

^{40}S (100 MeV/nucleon) + Au

Corrections important large b's, large E_x's

$$\frac{\sigma_{exact} - \sigma_{R\ or\ NR}}{\sigma_{R\ or\ NR}}(\%)$$



Deviations from non-relativistic

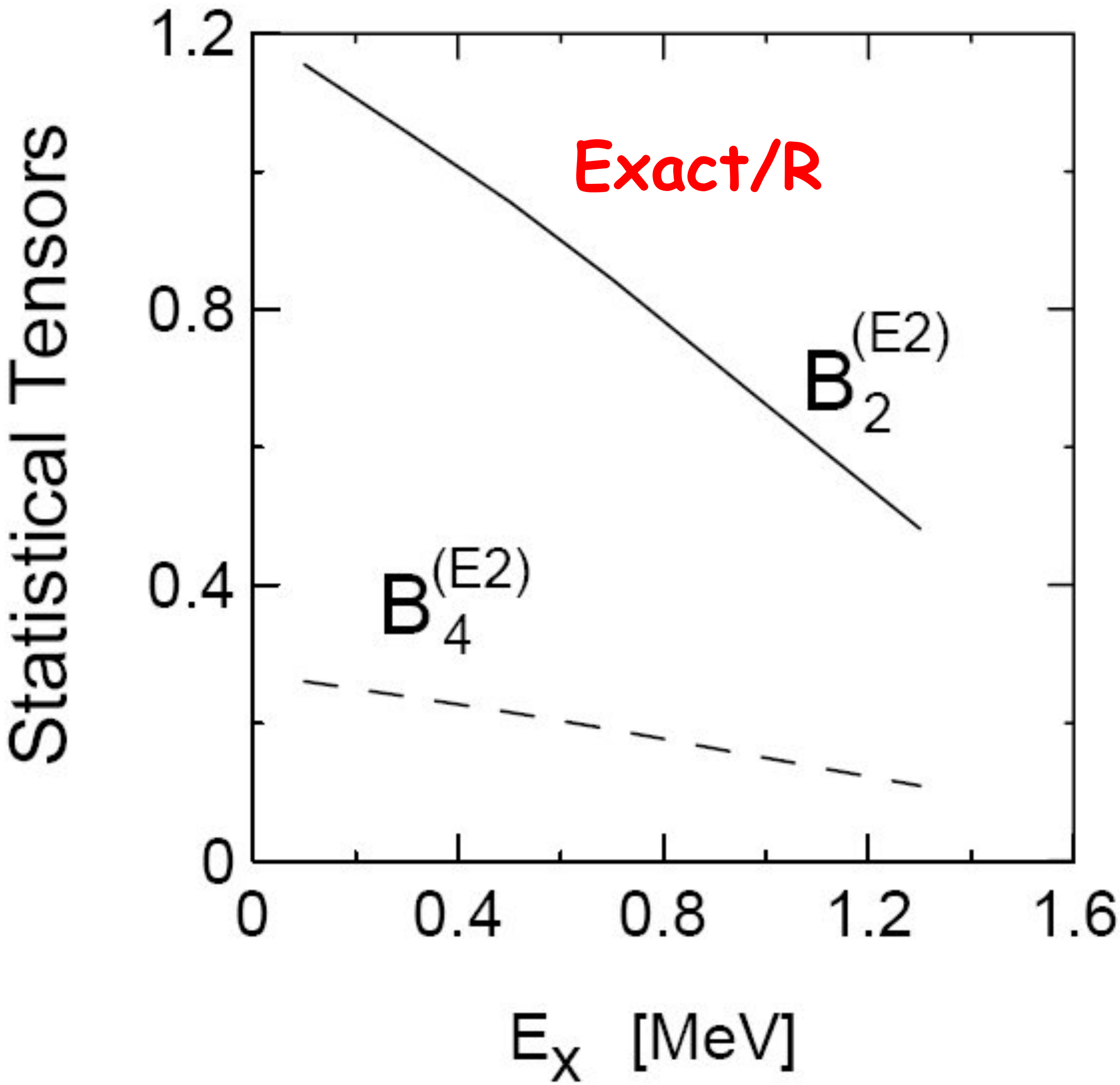
& from relativistic

^{40}S (100 MeV/nucleon) + Au

E_x = 0.89 MeV

De-excitation by γ-ray emission

$$W_\gamma(\theta_\gamma) = 1 + \sum_{\kappa=2,4} B_\kappa Q_\kappa(E_\gamma)\, P_\kappa(\cos\theta_\gamma)$$



Deviations from from relativistic theory

^{38}S (100 MeV/nucleon) + Au

We all know that:

- Relativitiy obviously important at GANIL, GSI, MSU and RIKEN
  (we are talking dynamics)

- 'Rather' easy to include for Coulomb interaction
  (transformation properties of E/M fields well known)

How about nuclear interaction?

- Transformation properties of nucleus-nucleus potentials not exactly known

- Solution has to be based on QFT (QM + relativity)

- Can we save our DWBA, CC, or CDCC knowledge for something practical?


Way out?
奉

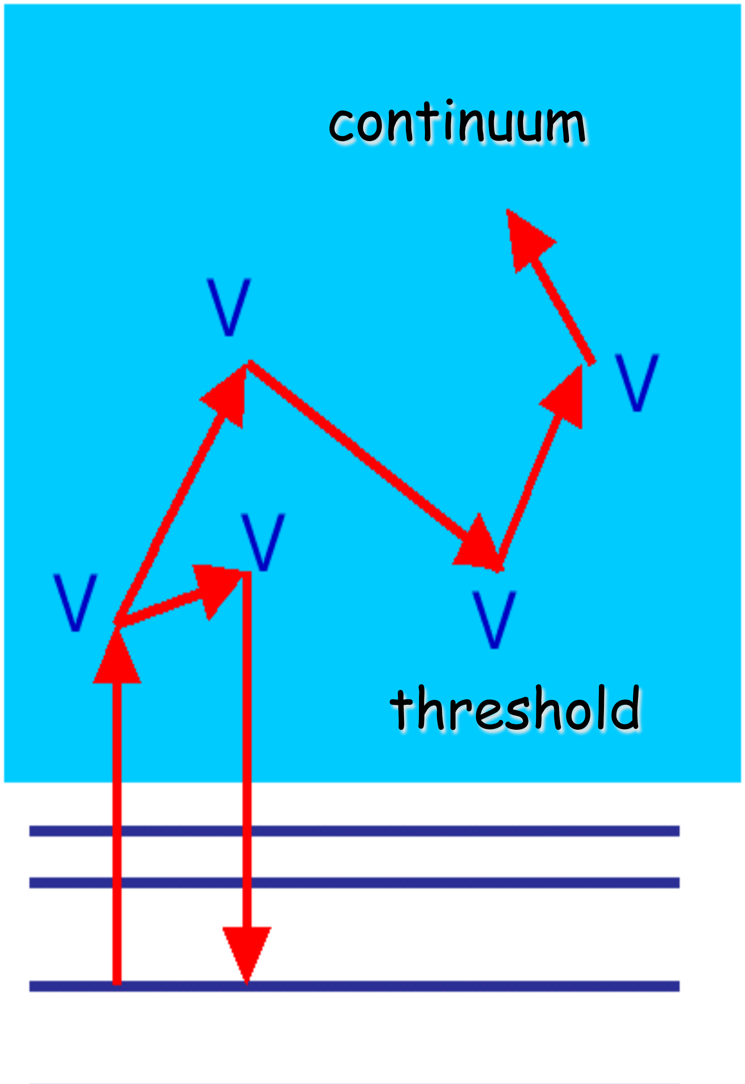
Continuum (CDCC)

$$\left|\varphi_b\right\rangle = \left|E_b, J_b M_b\right\rangle$$

$$\left|\varphi_{jJM}^{(c)}\right\rangle = \int \Gamma_j(E)\left|E, JM\right\rangle dE$$

$$\int \Gamma_i(E)\Gamma_j(E)\,dE = \delta_{ij}$$

continuum discretization



$$V_{\alpha\beta}(\mathbf{R}) = \left\langle \phi_\alpha(\mathbf{r})\middle| U(\mathbf{R},\mathbf{r}) \middle|\phi_\beta(\mathbf{r})\right\rangle$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - E\right]\chi_\alpha(\mathbf{R}) = -\sum_{\beta=0}^{N} V_{\alpha\beta}(\mathbf{R})\chi_\beta(\mathbf{R})$$

Coupled-channels

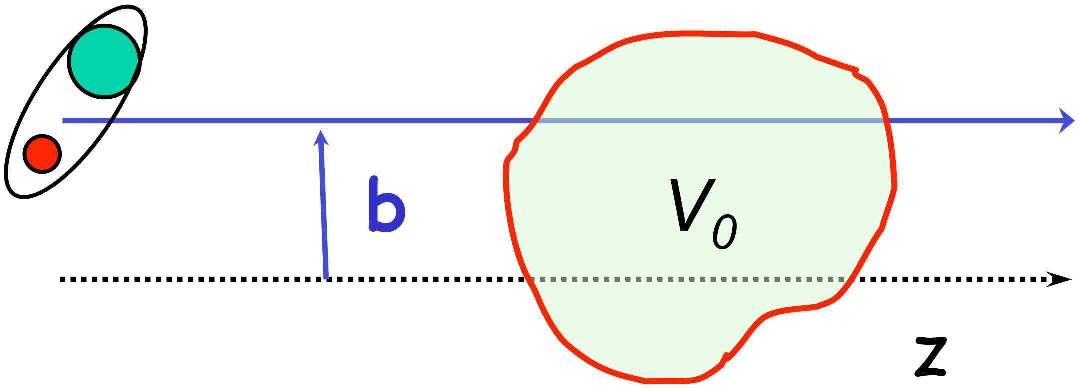
Bertulani, Canto, NPA 539, 163 (1992)
^{11}Li+^{208}Pb (100 MeV/nucleon)

Relativistic-CDCC

Bertulani, PRL 94, 072701 (2005)

$$\left[\nabla^2 + k^2 - U\right]\Psi(\mathbf{R},\mathbf{r}) = 0$$

$$U = V_0\left(2E - V_0\right)$$



$$\Psi(\mathbf{R},\mathbf{r}) = \sum_\alpha S_\alpha(\mathbf{b},z)\, e^{ik_\alpha z}\, \phi_\alpha(\mathbf{r}),$$

$$\mathbf{R} = (\mathbf{b}, z)$$

$$U \approx 2V_0 E$$

$$\nabla^2 S << ik_z \partial_z S$$

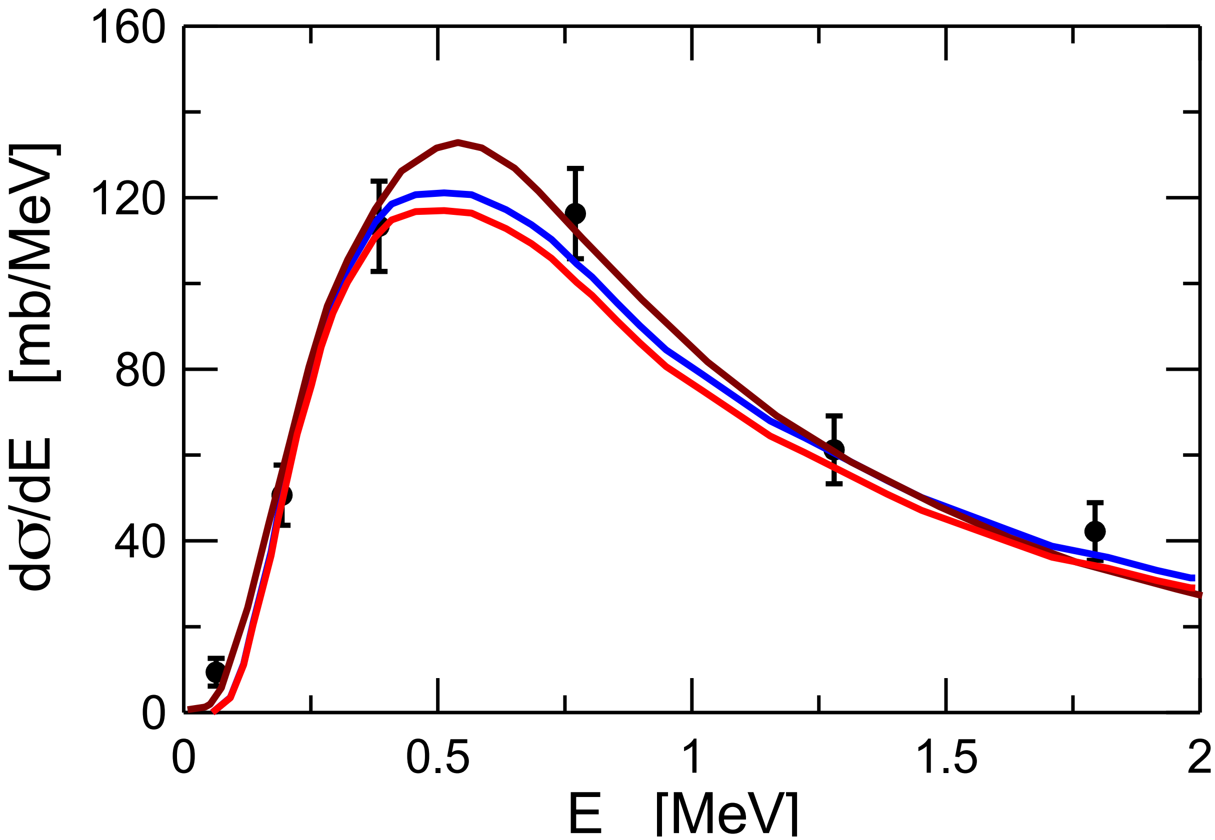
$$iv\partial_z S_\alpha(\mathbf{b},z) = \sum_\beta V_{\alpha\beta}(\mathbf{b},z) S_\beta(\mathbf{b},z)\, e^{i(k_\beta - k_\alpha)z}$$

$$f_\alpha(\mathbf{Q}) = -\frac{ik}{2\pi}\int d\mathbf{b}\, e^{i\mathbf{Q}.\mathbf{b}}\left[S_\alpha(\mathbf{b}, z=\infty) - \delta_{\alpha,0}\right]$$

$$\mathbf{Q} = \mathbf{K}_\perp^{'} - \mathbf{K}_\perp \qquad \alpha = jlJM$$

V_0 = time-like component of 4-vector $\Rightarrow$ Relativistic CDCC = Lorentz invariant

Pb(^{8}B,p^{7}Be) at 83 MeV/nucleon



DATA: Davids et al, 2002

LO

all orders

all orders relativistic

V_0 = Coulomb + nuclear with relativistic corrections

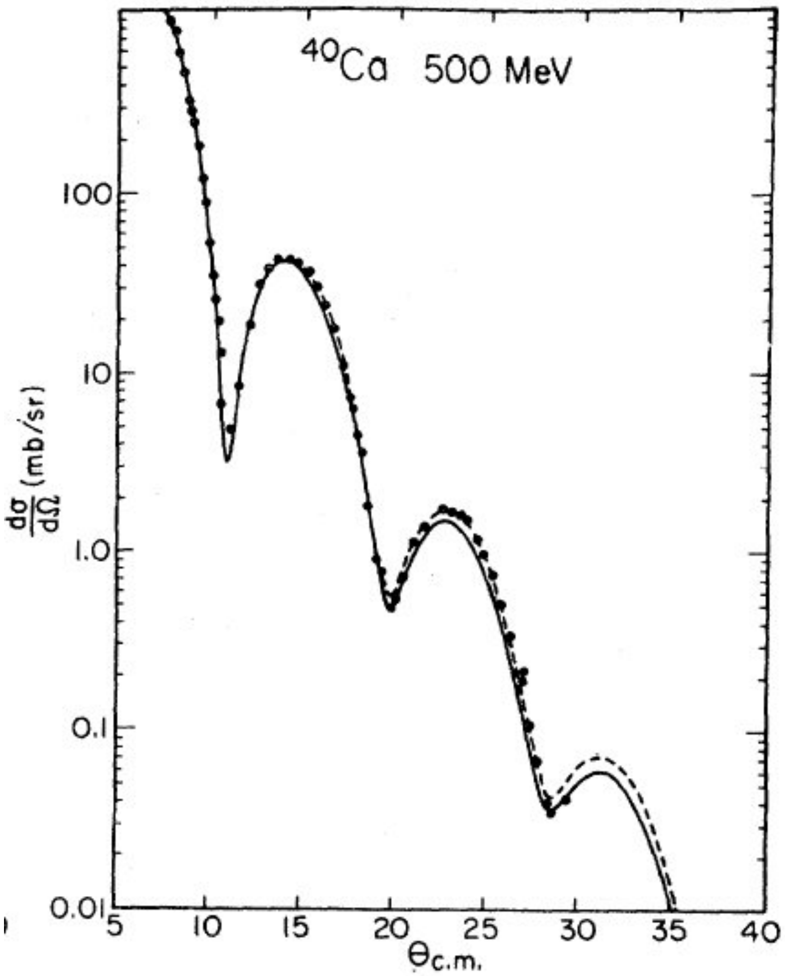
4-10% effect

Nuclear

Clue: Proton-nucleus scattering at intermediate energies

- meson exchange, two-nucleon interaction
- mean field approximation, U_0 (ω exchange), U_S (2π exchange)

$$\left[E - V_C - U_0 - \beta\left(mc^2 + U_S\right)\right]\Psi = -i\hbar c\alpha\cdot\nabla\Psi$$



non-relativistic reduction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U_{cent} + \left(\frac{\hbar}{2mc}\right)^2 \frac{1}{r}\frac{d}{dr}U_{SO}\,\sigma\cdot\mathbf{L}\right]\phi = E\phi$$

$$U_{cent} = m^*\left(U_0 + U_S\right) + \cdots$$

$$m^* = 1 - \frac{U_0 - U_S}{2mc^2} + \cdots$$

$$U_{SO} = U_0 - U_S + \cdots$$

Arnold, Clark, PLB 84, 46 (1979)

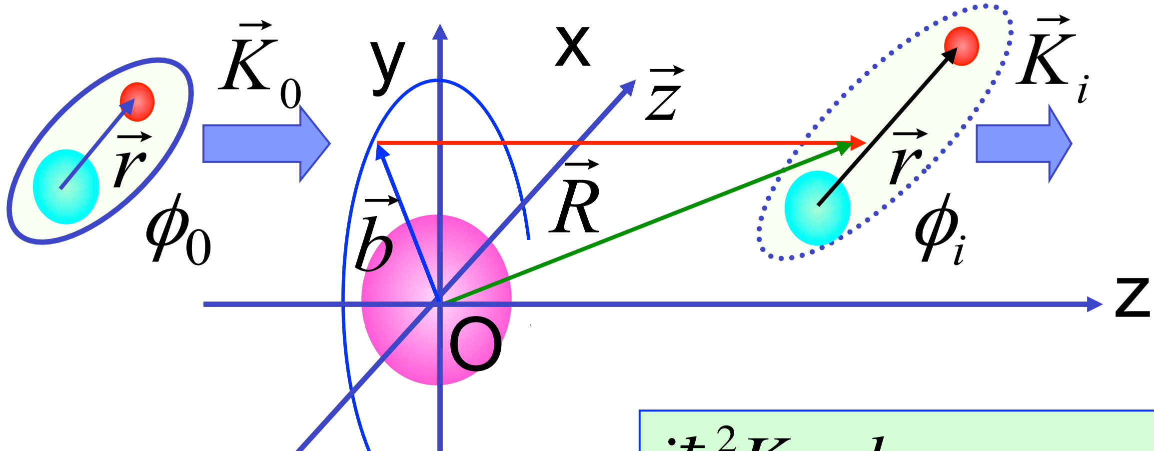
Transition: low to high energies

Eikonal scattering waves $\hat{S}_i(K_i,\vec{R})$

$$\psi^{E-CDCC} = \sum_i \hat{\phi}_i(\vec{r})\ \hat{S}_i(b,z)\exp(i\vec{K}_i \cdot \vec{R})$$

$$K_i = \sqrt{2\mu_R(E-\varepsilon_i)}\,/\,\hbar,$$

Energy conservation



- Boundary condition

$$\hat{S}_i(b,z) \xrightarrow{z \to -\infty} \delta_{i,0}$$

$$\Delta \hat{S}_i(b,z) \cong 0 \Rightarrow \frac{i\hbar^2 K_i}{\mu_R}\frac{d}{dz}\hat{S}_i^{(b)}(z) = \sum_{i'} \mathrm{F}_{ii'}^{(b)}(z)\ \hat{S}_{i'}^{(b)}(z)e^{i(K_{i'}-K_i)z}$$

Eikonal scattering amplitude transformed into QM form

$$f_{i,0}^{\mathrm{E}} = \sum_L f_L^{\mathrm{E}} \equiv \sum_L \frac{2\pi}{iK_i}\sqrt{\frac{2L+1}{4\pi}}\, i^m Y_{Lm}(\Omega)[S_{i,0}^{b(L;i)} - \delta_{i,0}]$$

Hybrid scattering amplitude is given by

$$f_{i,0}^{\mathrm{H}} \equiv \sum_{L=0}^{L_{\mathrm{C}}} f_L^{\mathrm{Q}} + \sum_{L=L_{\mathrm{C}}+1}^{L_{\max}} f_L^{\mathrm{E}}$$

Ogata et al,
PRC68, 064609 (2003)

Relativistic CDCC

Form factor of non-rel. E-CDCC

$$F_{c'c}^{(b)}(Z) = \left\langle \Phi_{c'} \middle| U_{1A} + U_{2A} \middle| \Phi_c \right\rangle_{\mathbf{r}} e^{-i(m-m')\phi} = \sum_{\lambda} F_{c'c}^{(b);\lambda}(Z)$$

Lorentz tranform of form factor and coordinates

$$F_{c'c}^{(b);\lambda}(Z) \to f_{\lambda,m'-m} \gamma F_{c'c}^{(b)\lambda}(\gamma Z)$$

$$f_{\lambda,m'-m}^{\mathrm{Coul}} = \begin{cases} 1/\gamma & (\lambda=1,\ m'-m=0) \\ \gamma & (\lambda=2,\ m'-m=\pm 1) \\ 1 & (\text{otherwise}) \end{cases}$$

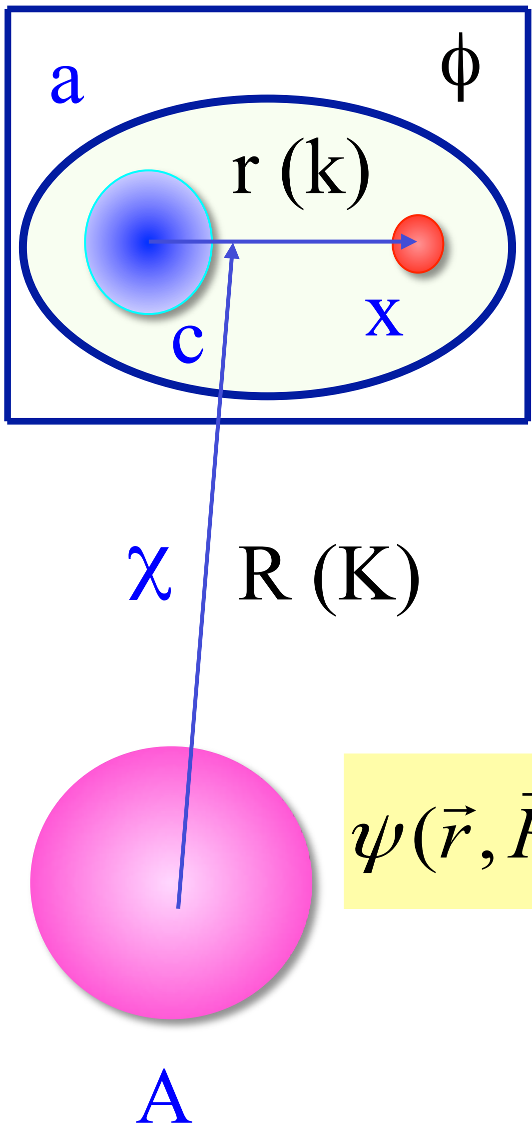
$$f_{\lambda,m'-m}^{\mathrm{nucl}} = 1$$

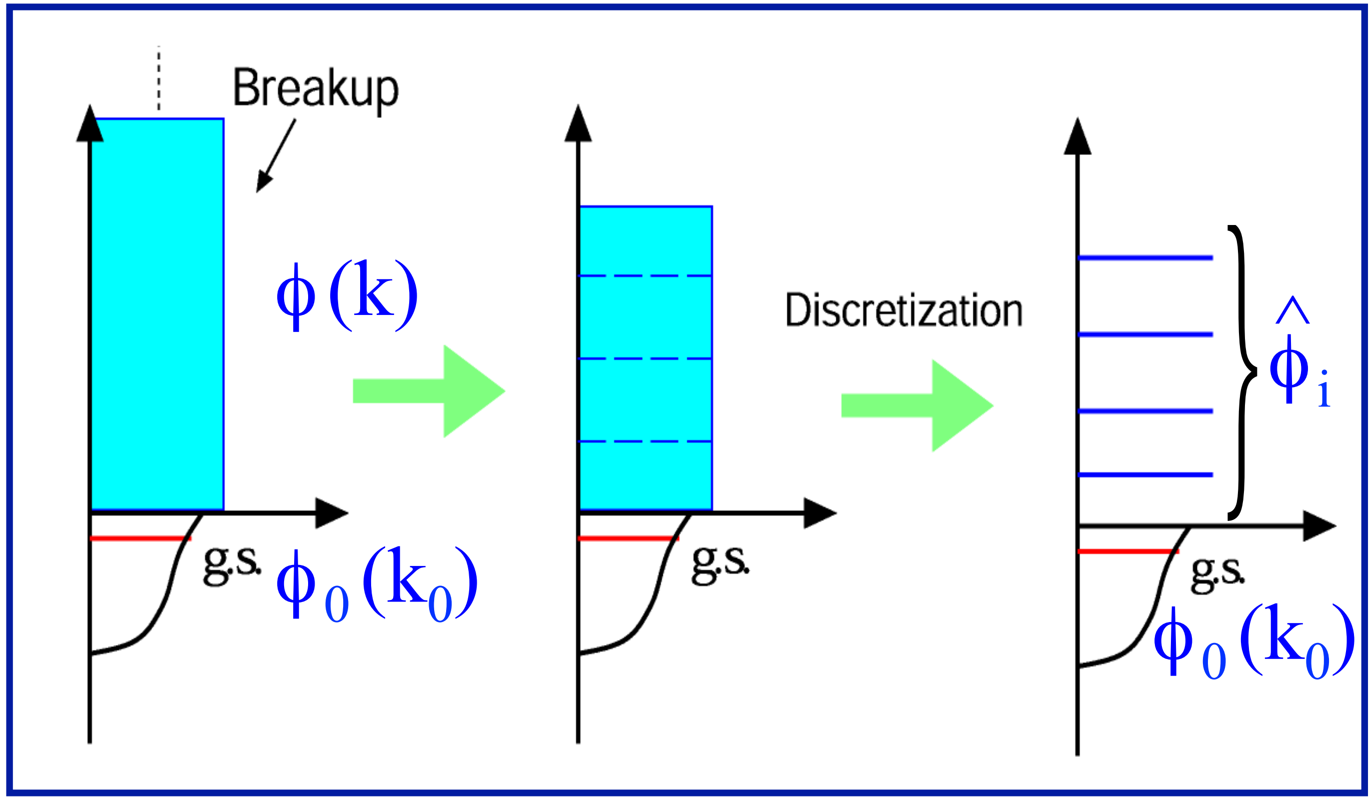
Assumptions

- ✓ Point charges for 1, 2 and A
- ✓ Neglecting far-field ($r_i > R$) contribution
- ✓ Correction to nuclear form factor

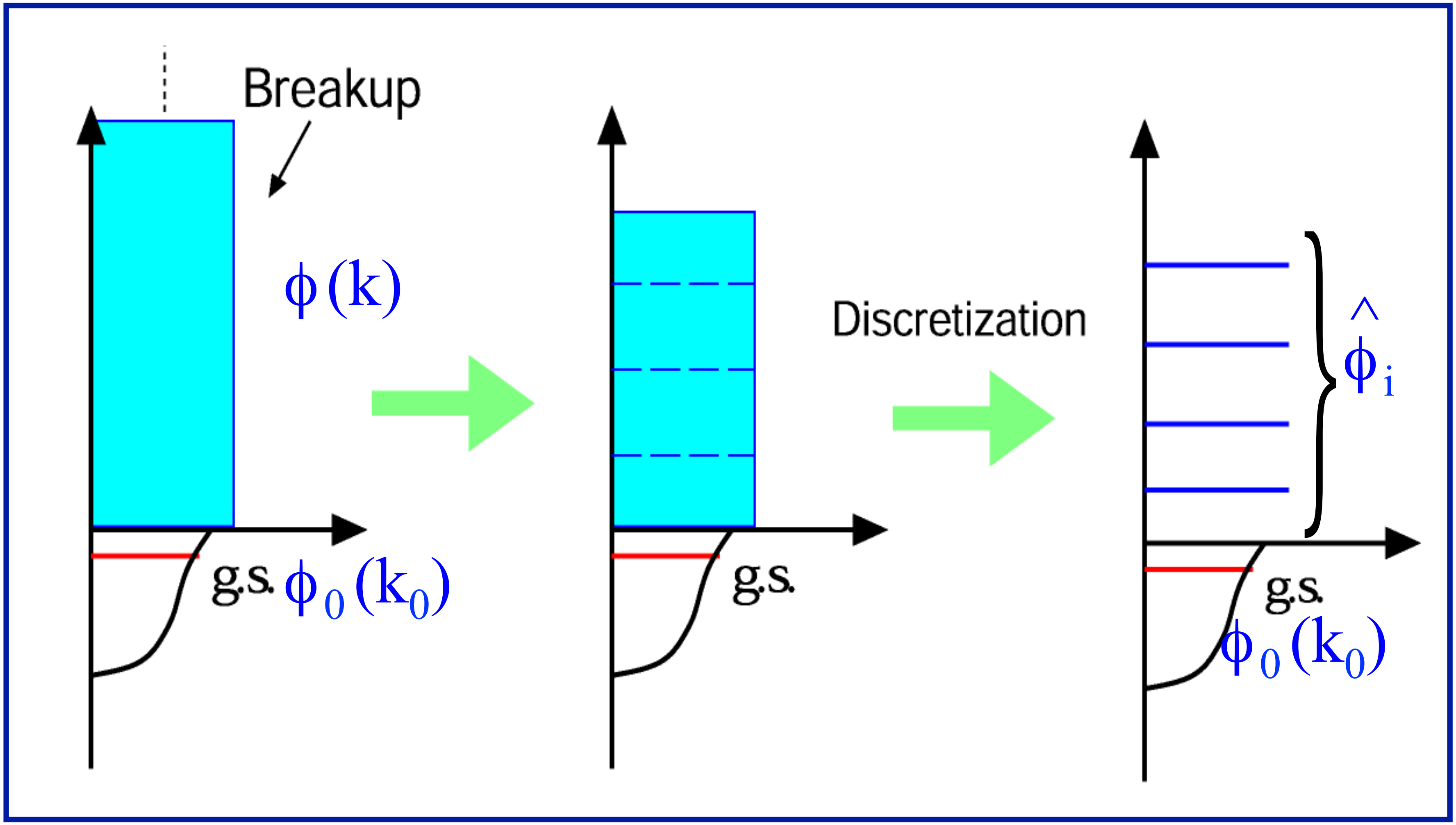
Ogata, Bertulani, PTP 121 (2009), 1399
PTP, 123 (2010) 701

Theory movie in next 5 transparencies (enjoy!)©

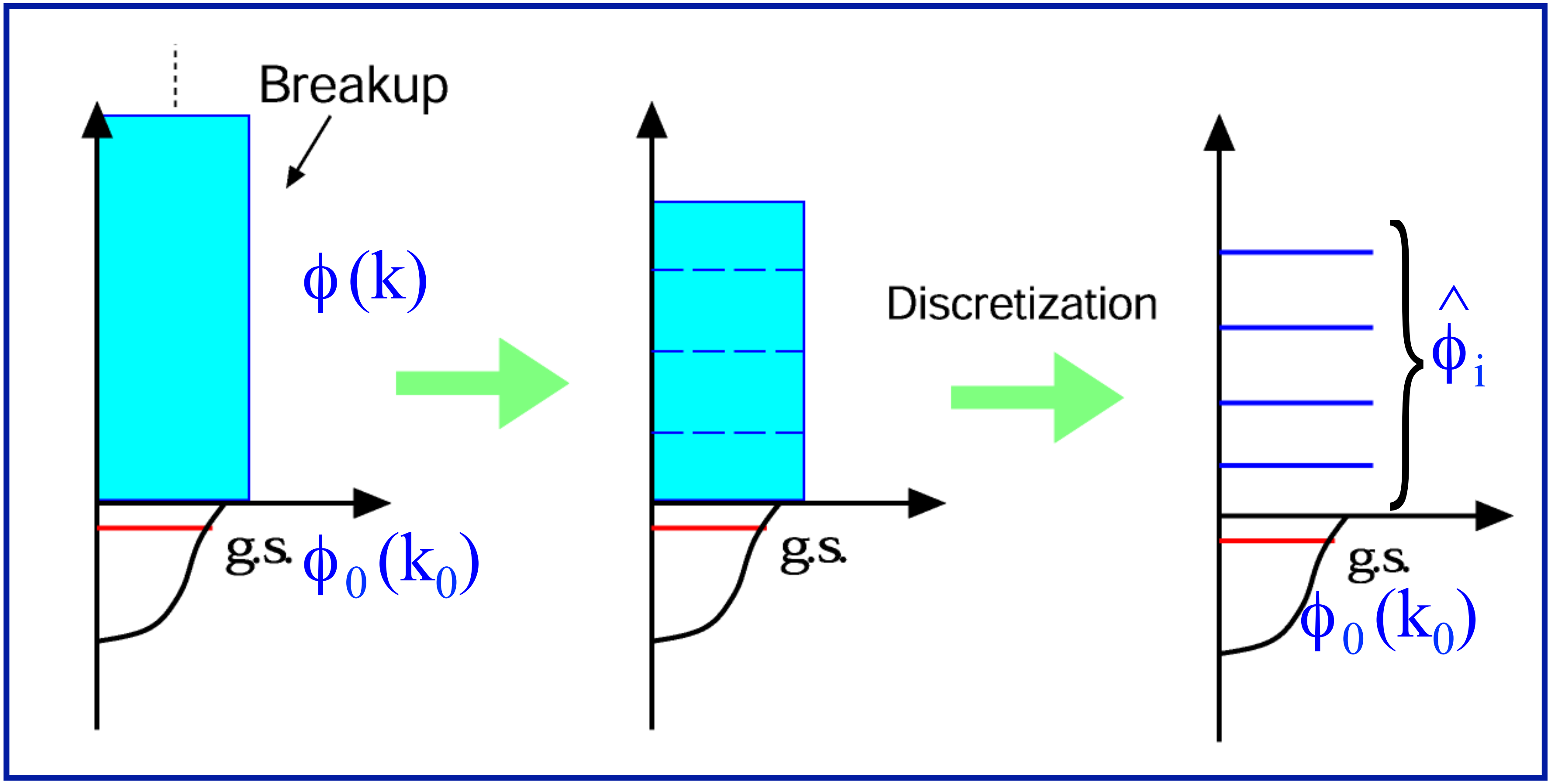




$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$

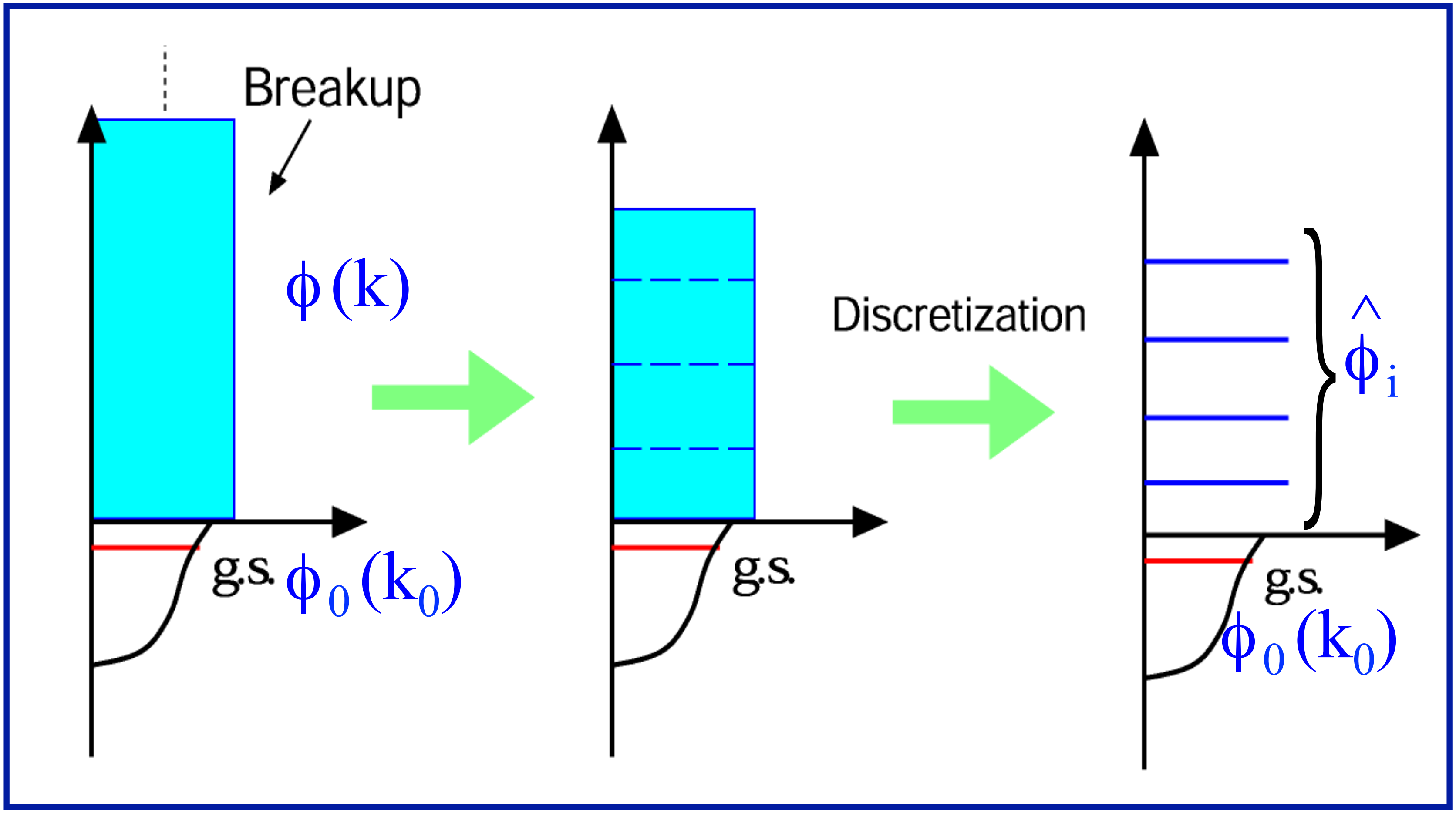


$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$



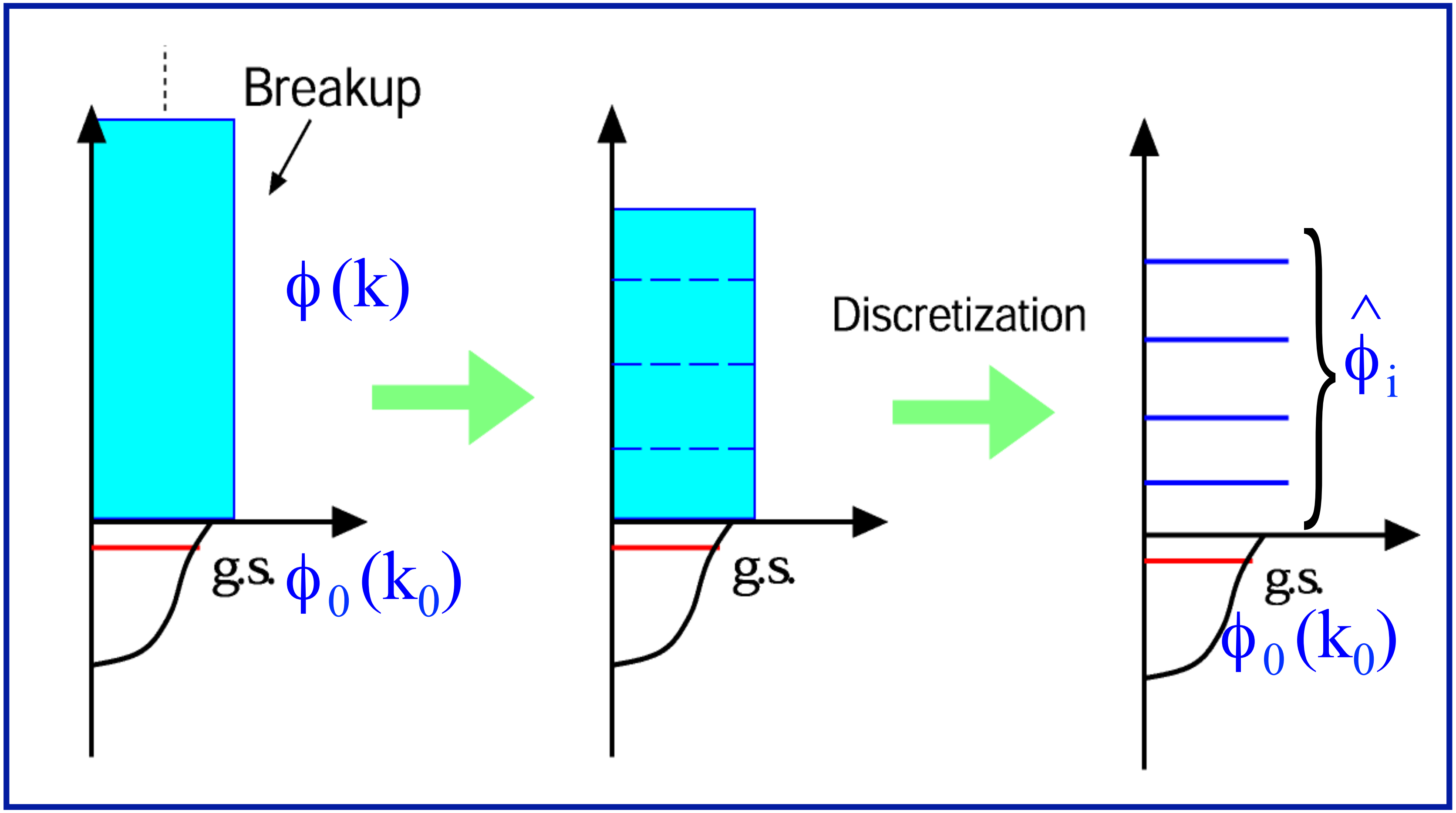
$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$

Truncation and Discretization



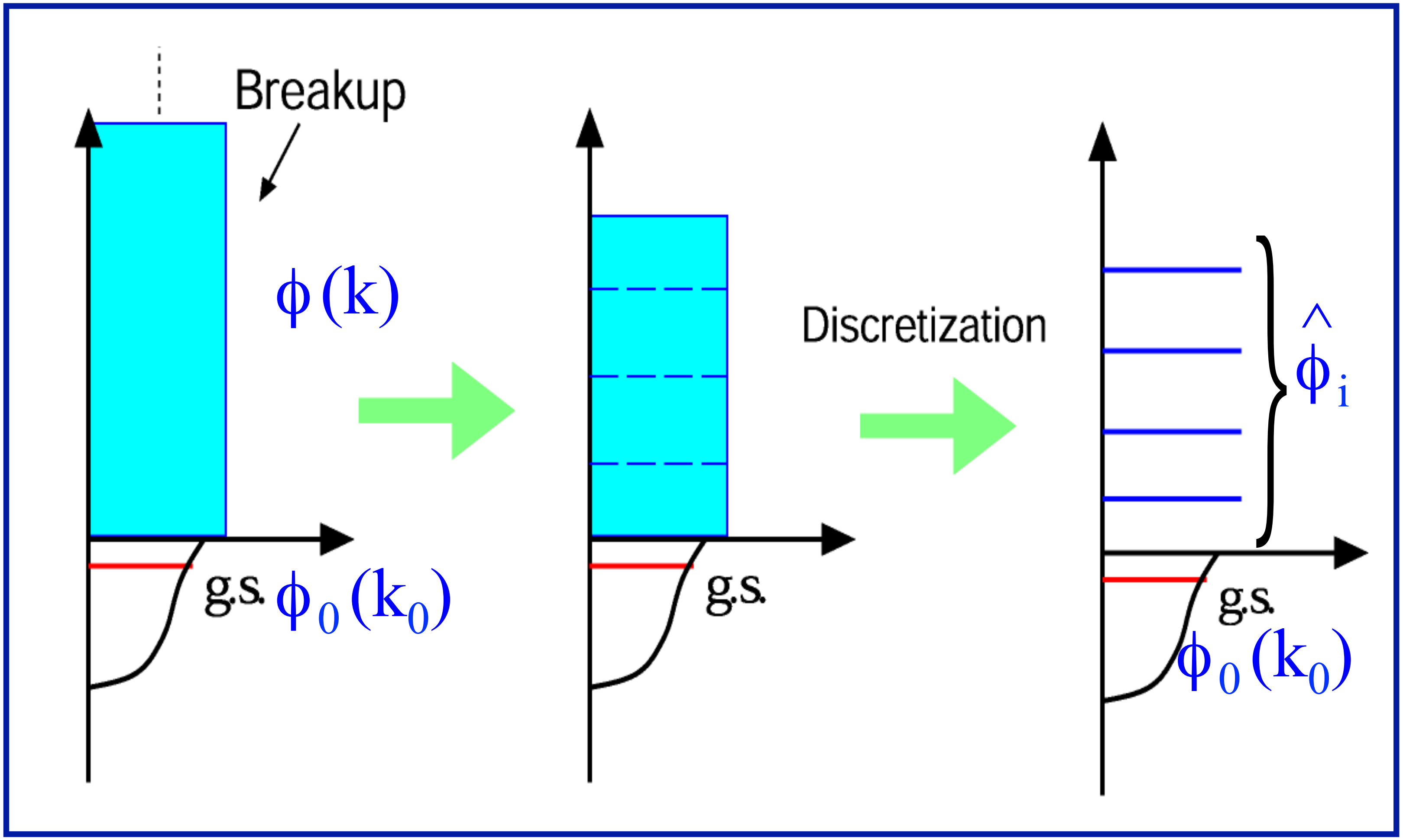
$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \int_{k_{i-1}}^{k_i} \phi(k,\vec{r})\chi(K,\vec{R})dk$$

Truncation and Discretization



$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \chi(K_i,\vec{R})\int_{k_{i-1}}^{k_i} \phi(k,\vec{r})dk$$

Truncation and Discretization



$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \chi(K_i,\vec{R})\int_{k_{i-1}}^{k_i} \phi(k,\vec{r})dk$$

Truncation and Discretization

THE END.

$$\psi^{\mathrm{CDCC}}(\vec{r},\vec{R}) = \sum_{i=0}^{i_{\max}} \hat{\phi}_i(\vec{r})\hat{\chi}_i(K_i,\vec{R})$$

Reaction

$^{208}Pb(^{8}B,\ ^{7}Be+p)$ at 250 A MeV and 100 A MeV

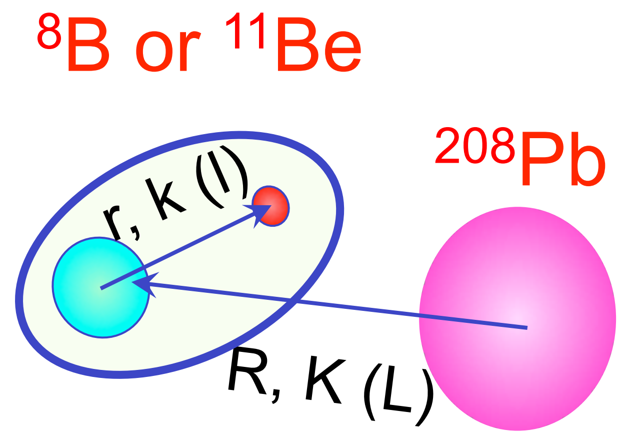
$^{208}Pb(^{11}Be,\ ^{10}Be+n)$ at 250 A MeV and 100 A MeV

Projectile wave function and distorting potential

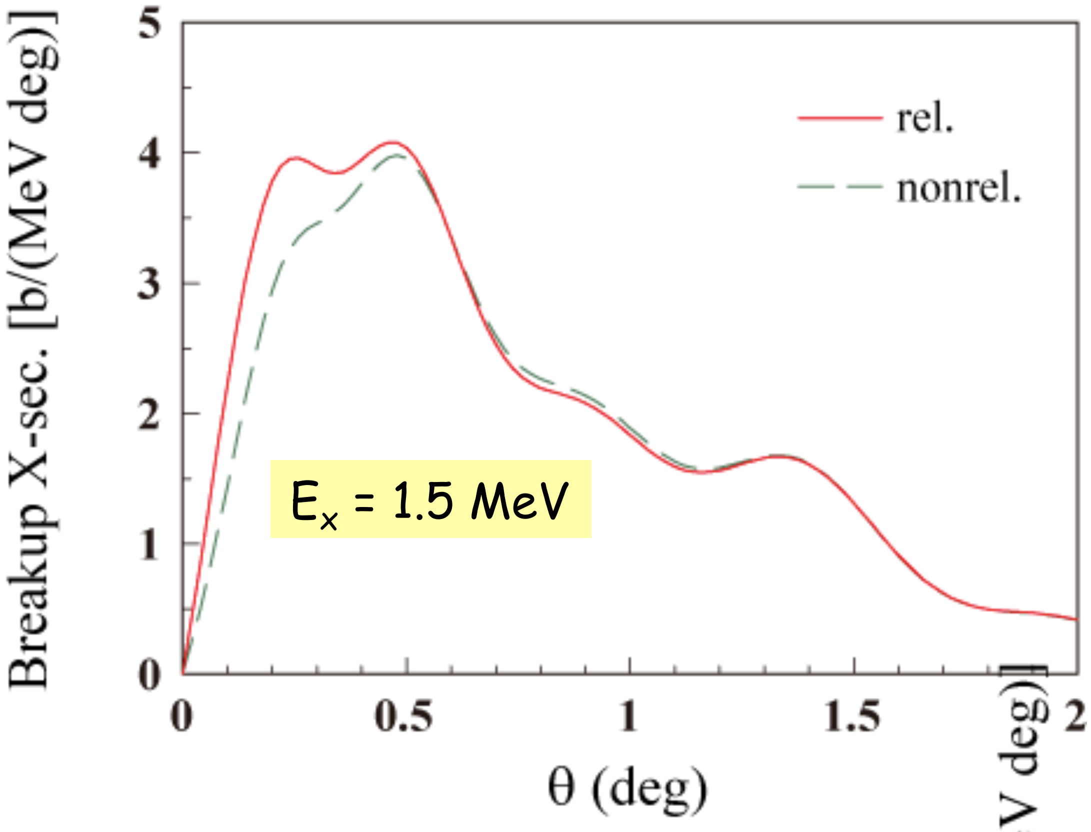
Standard Woods-Saxon

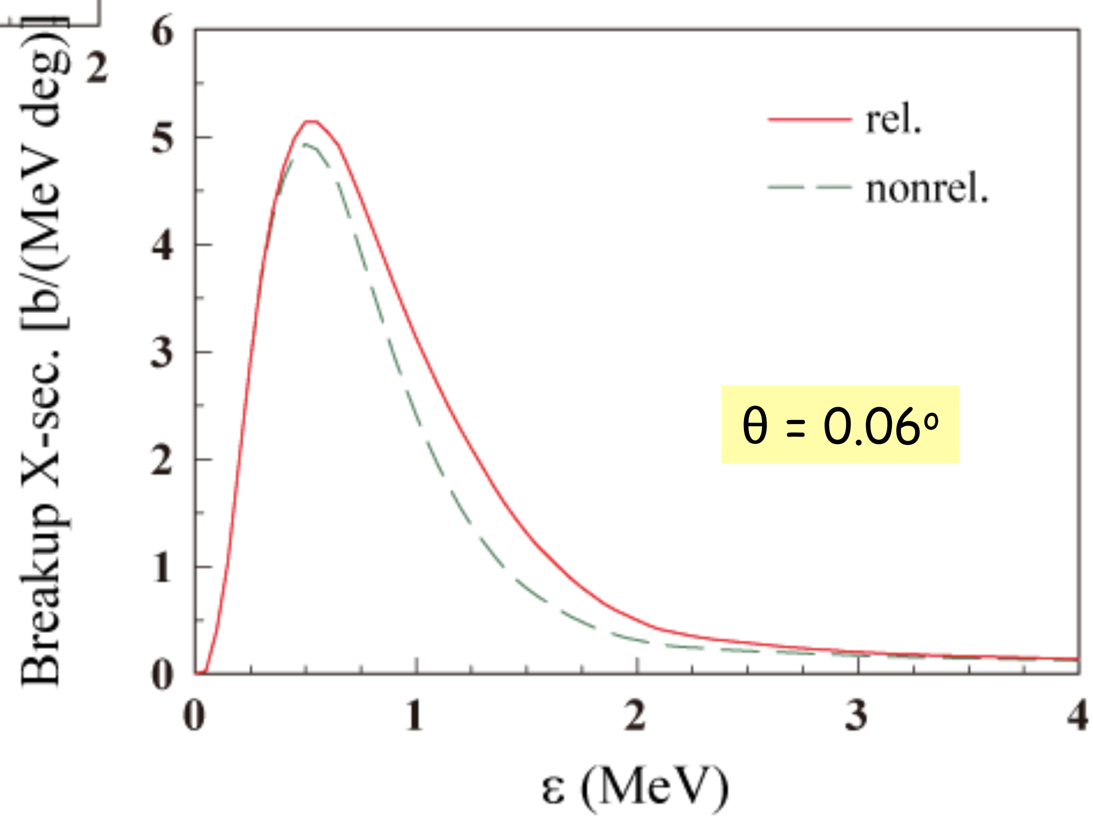
Modelspace

^{8}B	^{11}Be
$l_{max}= 3$	$l_{max}= 3$
$N_s=20$, $N_{p\text{-}d}=10$,	$N_{s,p}=20$, $N_d=10$,
$N_f=5$	$N_f=5$
$\varepsilon_{max}=10$ MeV	$\varepsilon_{max}=10$ MeV
$r_{max}= 200$ fm	$r_{max}= 200$ fm
$R_{max}= 500$ fm	$R_{max}= 450$ fm
$N_{ch} = 138$	$N_{ch} = 166$



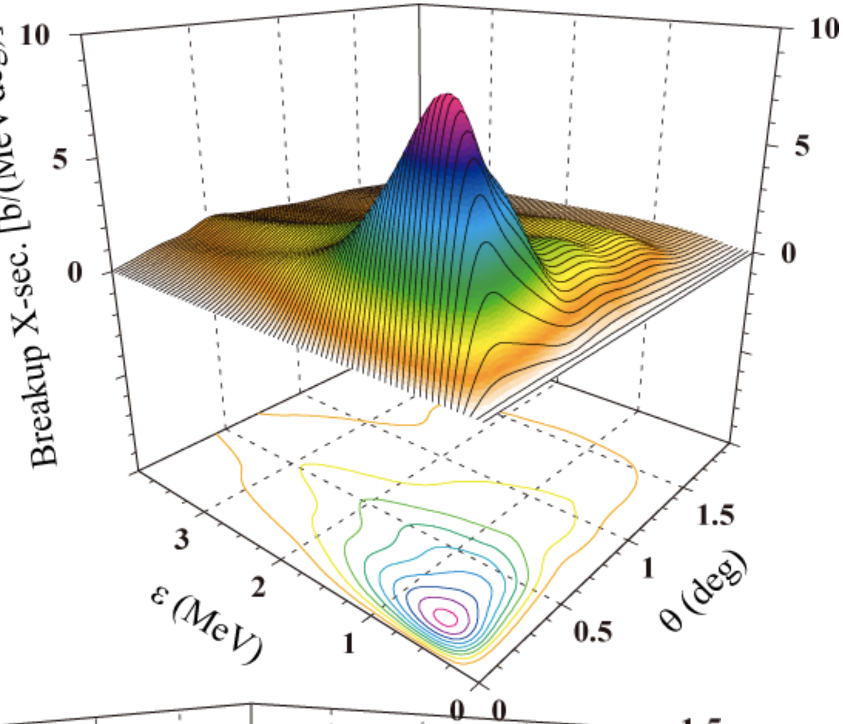
Pb(^{8}B,p^{7}Be) at 250 MeV/nucleon



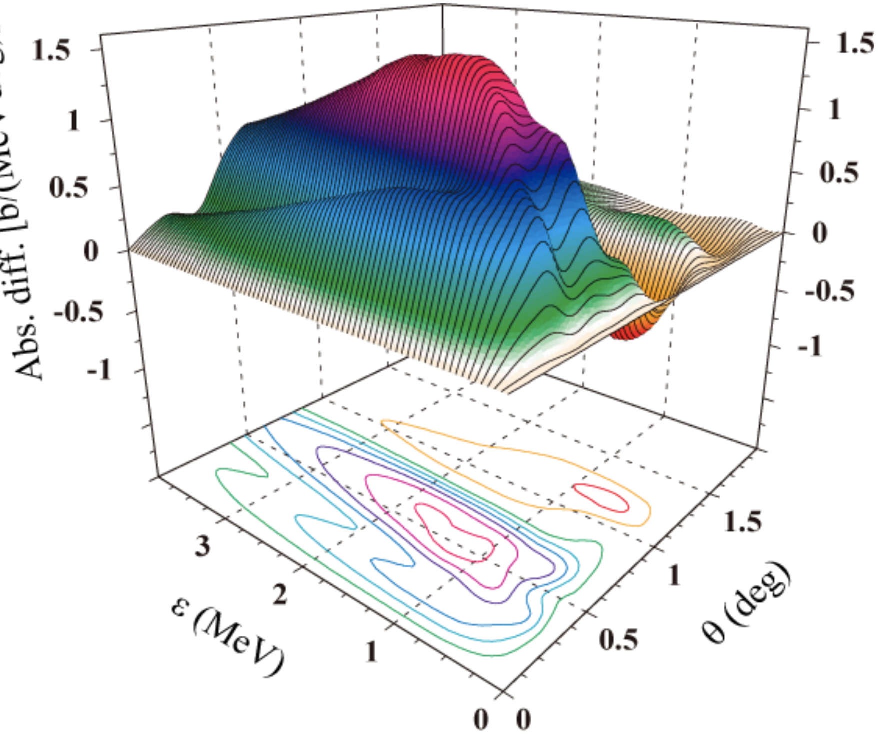


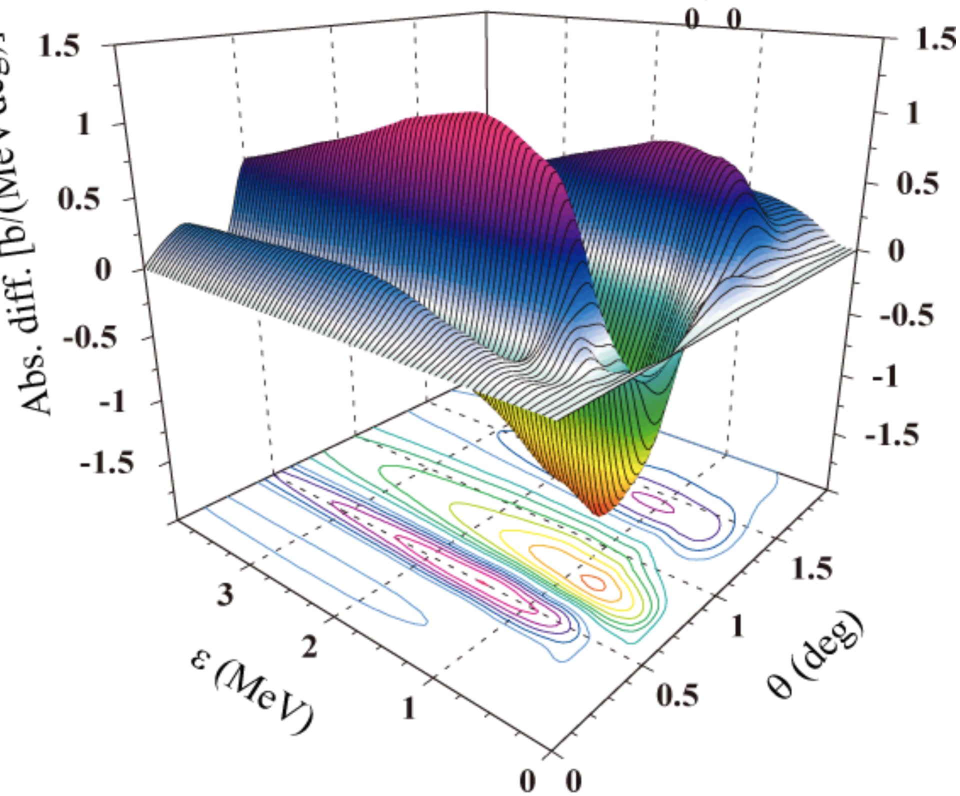
Pb(^{8}B,p^{7}Be) at 250 MeV/nucleon

all orders



$|\sigma_{all} - \sigma_{NR}|$





$|\sigma_{all} - \sigma_{no\text{-}nuclear}|$

Things can get worse

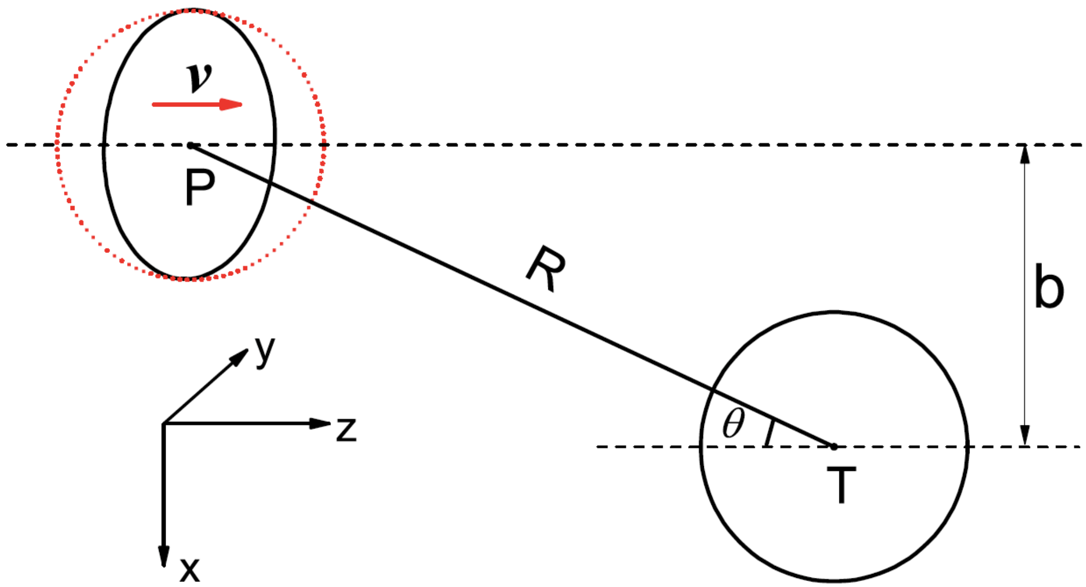
Relativistic MF nucleus-nucleus potential

Long, Bertulani, PRC 83, 024907 (2011).

σ, ω, ρ and γ exchange

$$E = \int d^3r \sum_a \bar{\psi}_a(-i\gamma\cdot\nabla + M)\psi_a$$
$$+\frac{1}{2}\sum_{\phi=\sigma,\omega,\rho,\gamma}\int d^3r d^3r' \sum_{ab}\bar{\psi}_a(\mathbf{r})\bar{\psi}_b(\mathbf{r}')\Gamma_\phi(\mathbf{r},\mathbf{r}')D_\phi(\mathbf{r}-\mathbf{r}')\psi_a(\mathbf{r})\psi_b(\mathbf{r}')$$

$$\Gamma_\phi(\mathbf{r},\mathbf{r}') = -g_\sigma(\mathbf{r})g_\sigma(\mathbf{r}')$$
$$\Gamma_\omega(\mathbf{r},\mathbf{r}') = -\left(g_\omega\gamma^\mu\right)_{\mathbf{r}}\cdot\left(g_\omega\gamma_\mu\right)_{\mathbf{r}'}$$
$$\Gamma_\rho(\mathbf{r},\mathbf{r}') = -\left(g_\rho\gamma^\mu\vec{\tau}\right)_{\mathbf{r}}\cdot\left(g_\rho\gamma_\mu\vec{\tau}\right)_{\mathbf{r}'}$$
$$\Gamma_\gamma(\mathbf{r},\mathbf{r}') = \frac{e^2}{4}\left[\gamma^\mu(1-\tau_z)\right]_{\mathbf{r}}\cdot\left[\gamma_\mu(1-\tau_z)\right]_{\mathbf{r}'}$$



$$D_\phi = \frac{1}{4\pi}\frac{e^{m_\phi|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$
$$D_\gamma = \frac{1}{|\mathbf{r}-\mathbf{r}'|}$$

Lorentz transform

$$x_p = x_t + b, \qquad y_p = y_t$$
$$z_p = \gamma(z_t + R\cos\theta)$$

$$E(A_t, A_p, v) = E(A_t) + E(A_p, \mathrm{v}) + \mathrm{E}\,(A_t, A_p, \mathrm{v})$$

$$\mathrm{E}\,(A_t, A_p, \mathrm{v}) = \sum_{\phi=\sigma,\omega,\rho,\gamma} \int d^3r \int d^3r' \sum_{ab} \overline{\psi}_{t,a}(\mathbf{r}) \overline{\psi}_{p,b}(\mathbf{r}')\, \Gamma_\phi(\mathbf{r},\mathbf{r}')\, D_\phi(\mathbf{r}-\mathbf{r}')\, \psi_{t,a}(\mathbf{r}) \psi_{p,b}(\mathbf{r}')$$

Ex: σ and ω contributions

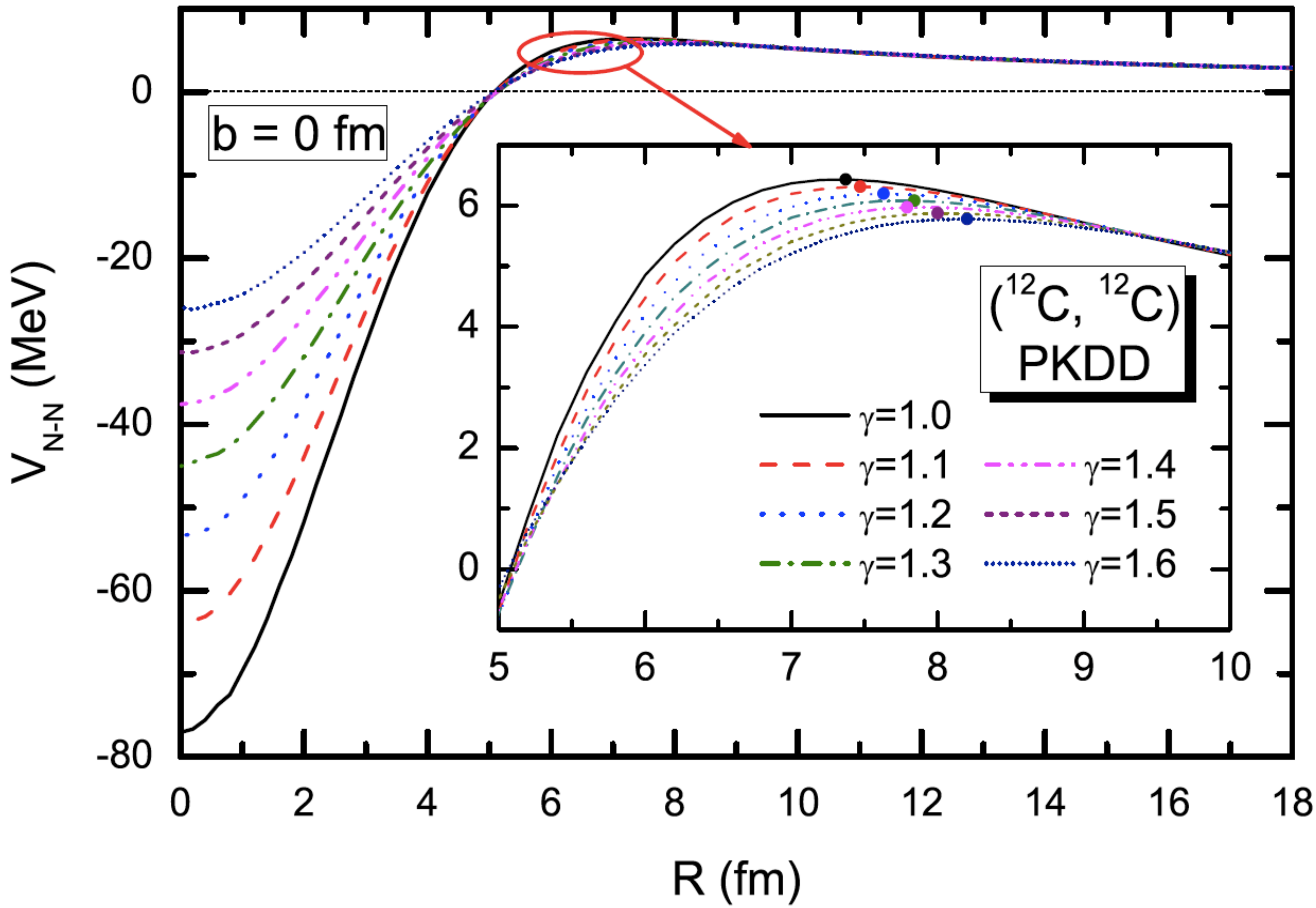
$$\mathrm{E}_\sigma = -\frac{1}{\gamma} \int d^3r_t \int d^3r_p'\, g_\sigma(\mathbf{r}_t)\, \rho_{s,t}(\mathbf{r}_t)\, D_\sigma(\mathbf{r}-\mathbf{r}')\, \rho_{s,p}(\mathbf{r}_p') g_\sigma(\mathbf{r}_p')$$

$$\mathrm{E}_\omega = \int d^3r_t \int d^3r_p'\, g_\omega(\mathbf{r}_t)\, \rho_{b,t}(\mathbf{r}_t)\, D_\omega(\mathbf{r}-\mathbf{r}')\, \rho_{b,p}(\mathbf{r}_p') g_\omega(\mathbf{r}_p')$$

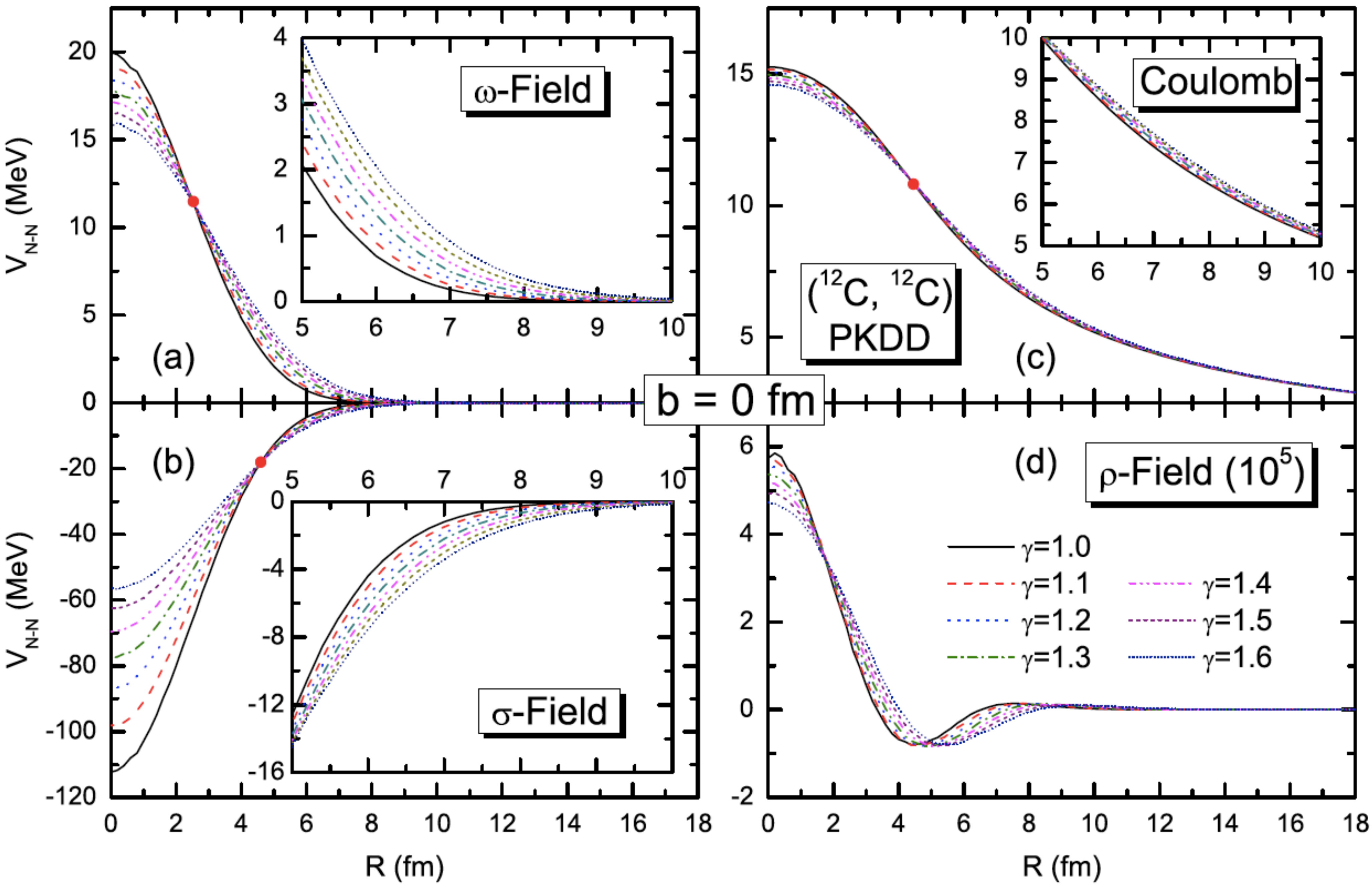
$$\rho_s(\mathbf{r}) = \sum_a \overline{\psi}_a(\mathbf{r}) \psi_a(\mathbf{r}), \qquad \rho_b(\mathbf{r}) = \sum_a \overline{\psi}_a(\mathbf{r}) \gamma^0 \psi_a(\mathbf{r})$$

Projectile densities boosted to the target frame

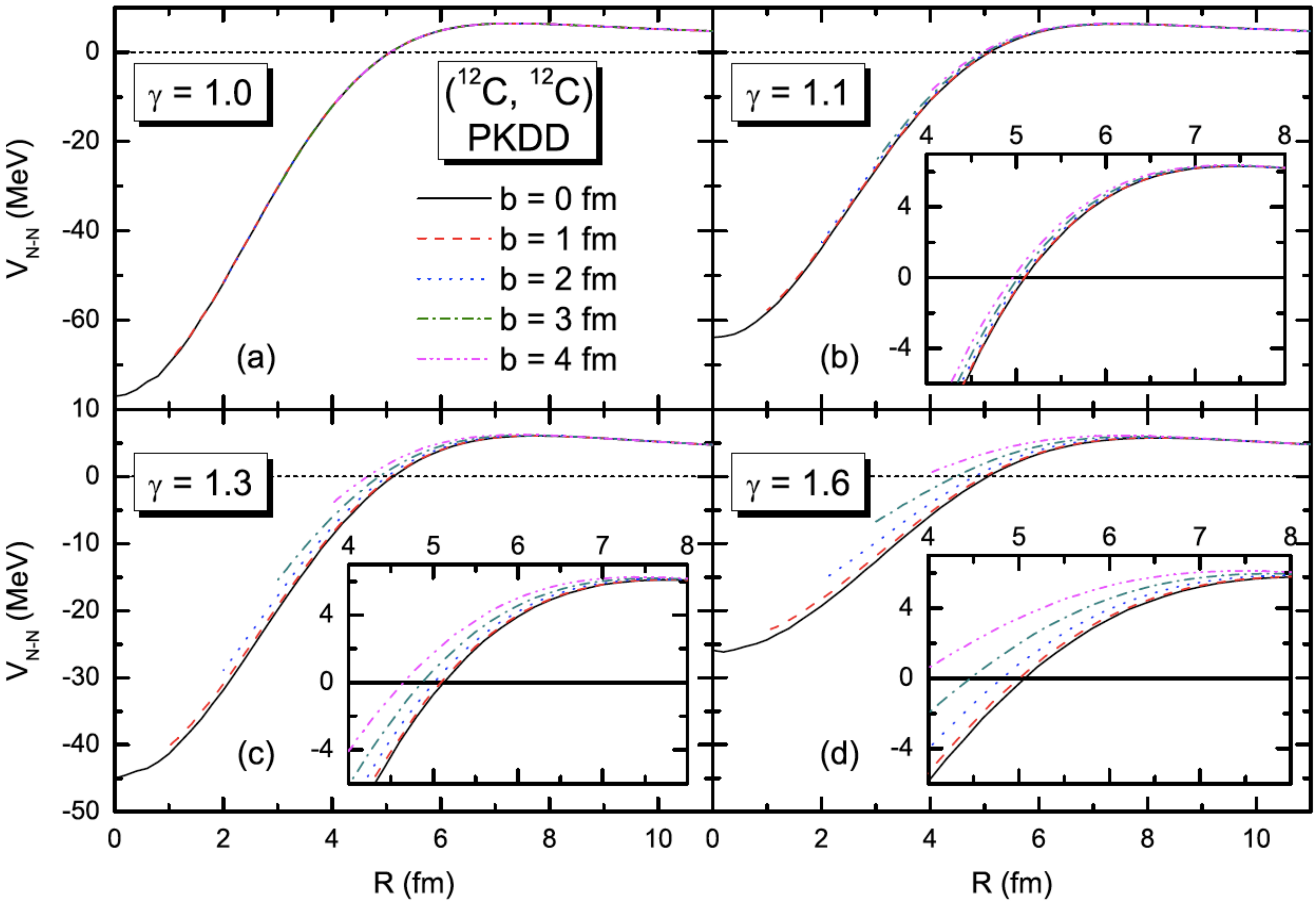
Results for ^{12}C + ^{12}C

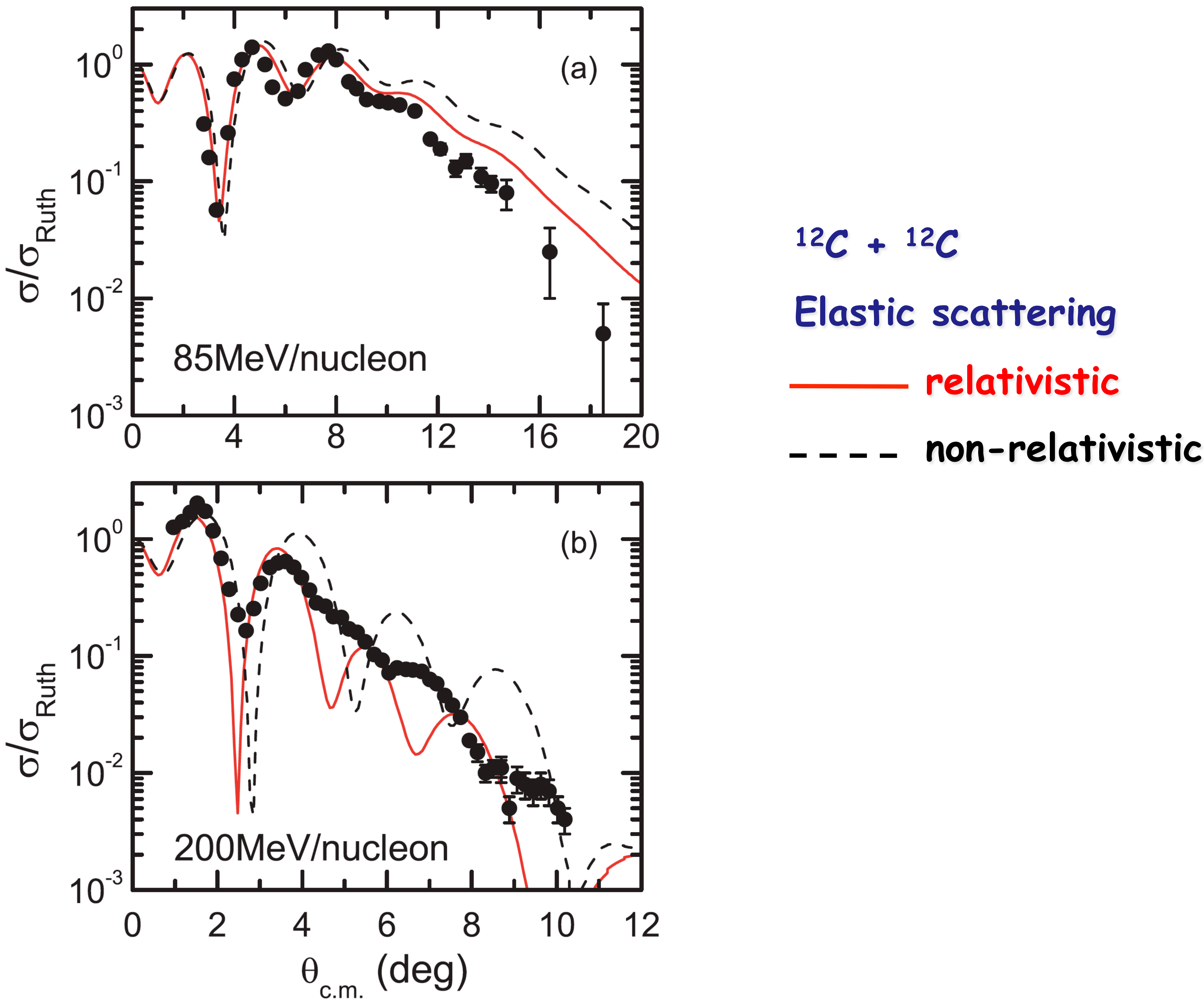


Contribution of different fields



Dependence on energy and impact parameter




(a)
85MeV/nucleon
(b)
200MeV/nucleon
σ/σ_Ruth
θ_c.m. (deg)
^{12}C + ^{12}C
Elastic scattering
relativistic
non-relativistic

OUT OF TIME