Eikonal method for halo nuclei

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   - Scattering states

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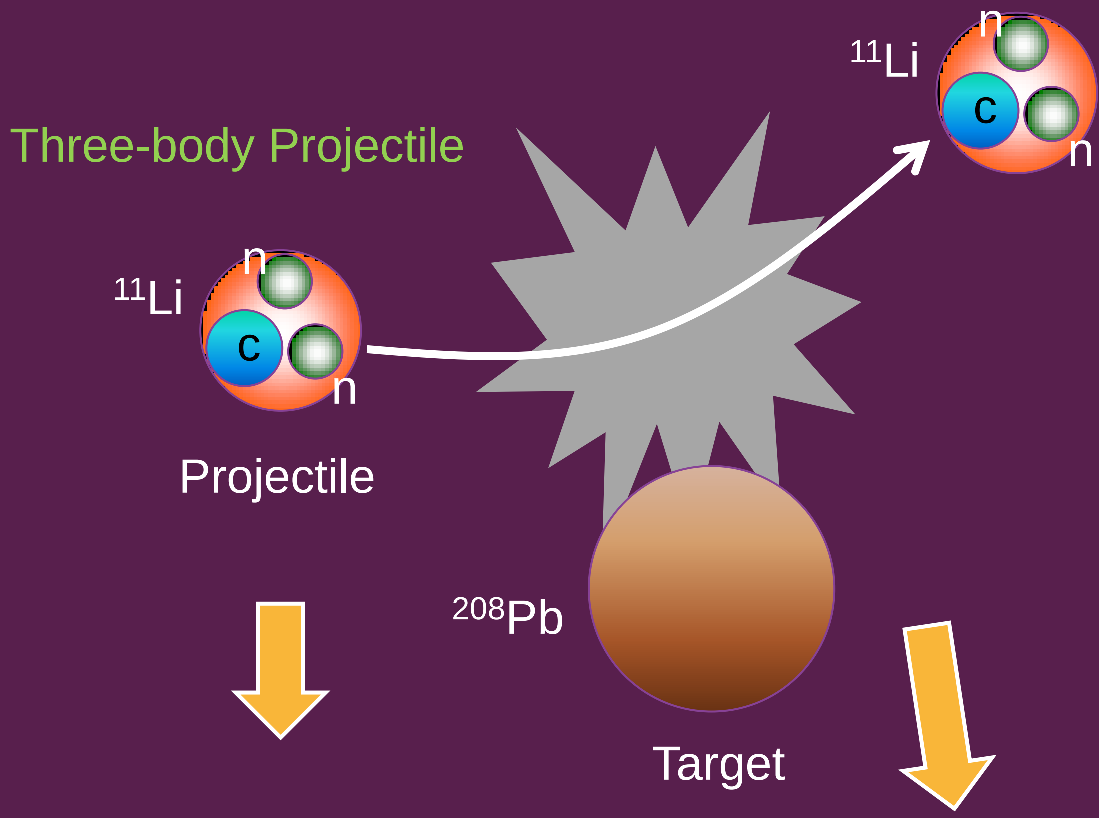
   - ^{6}He
   - Nucleus of our interest ^{11}Li

6. Conclusions

Motivation

To study elastic scattering and breakup cross sections of ^{11}Li in a four-body eikonal model.

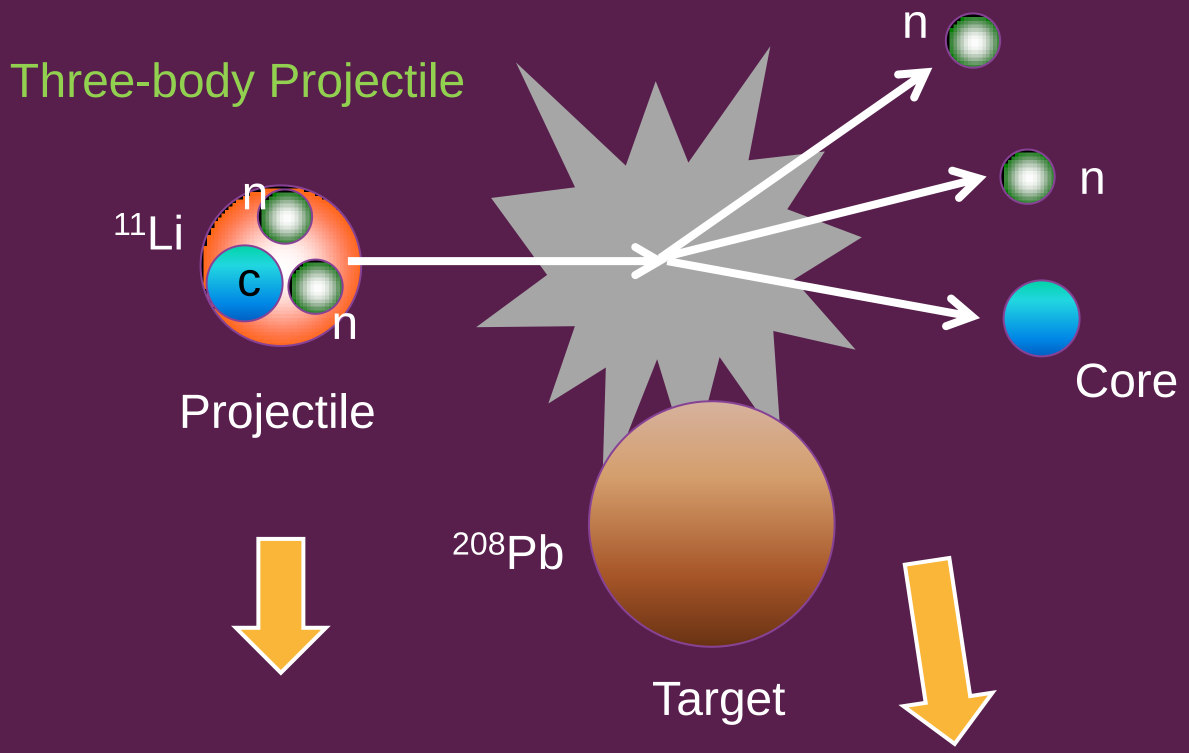


- Bound states
- Continuum states
- Dipole strengths

- Four-body elastic scattering cross sections

Motivation

To study elastic scattering and breakup cross sections of ^{11}Li in a four-body eikonal model.



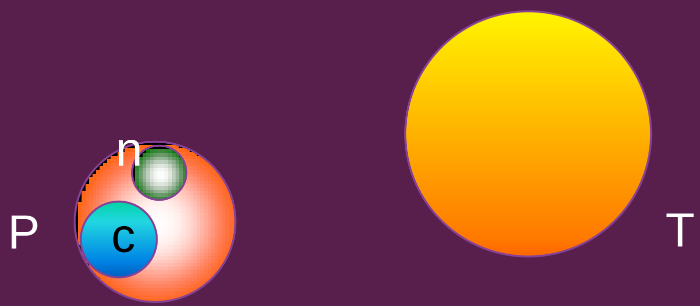
- Bound states
- Continuum states
- Dipole strengths

- Breakup cross sections
- Angular distributions

Introduction

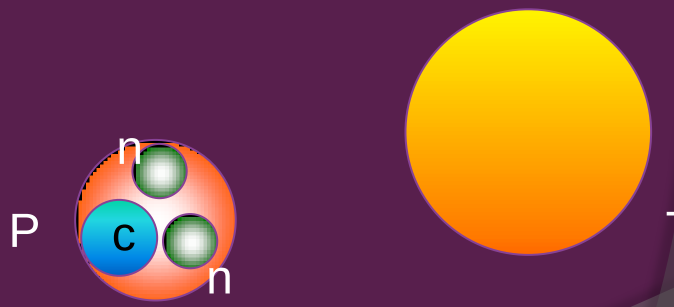
- High-energy reactions are widely used to investigate Halo nuclei.
- High incident energies permits to handle the Schrödinger equation in a simplified way: Eikonal approximation.
- Non-microscopic 2-Body and 3-Body descriptions of the projectile has been introduced in the eikonal method.

Two-body projectile



Elastic scattering, breakup
Ex: $^{11}Be+^{208}Pb=(^{10}Be+n)+^{208}Pb$
G. Goldstein, et. al; Phys. Rev. C 73, 024602 (2006).

Three-body projectile



Elastic scattering, breakup
Ex: $^{6}He+^{208}Pb=(\alpha+n+n)+^{208}Pb$
D. Baye, et. al; Phys. Rev. C 79, 024607 (2009).

Eikonal approximation for one-body projectile

We have to solve the Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu_{PT}}\Delta + V_{PT}(r)\right]\Phi(\boldsymbol{r}) = E\Phi(\boldsymbol{r}).$$

At high-energies the wave function: Smooth deviation from a plane wave

$$\Phi(\boldsymbol{r}) = \frac{1}{(2\pi)^{3/2}} e^{iKZ}\,\hat{\Phi}(\boldsymbol{r}),$$

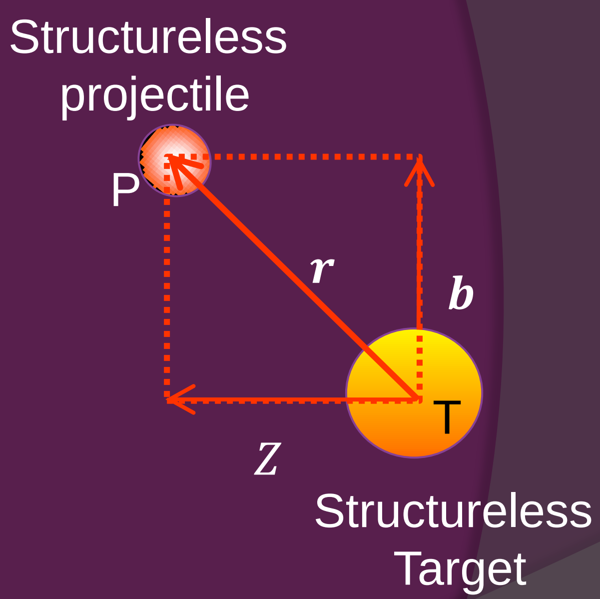
Smooth varying function

we have

$$-\frac{\hbar^2}{2\mu_{PT}}\left[\Delta + 2iK\frac{\partial}{\partial Z} + V_{PT}(r)\right]\hat{\Phi}(\boldsymbol{r}) = 0.$$

At high-energies $|\Delta\hat{\Phi}| \ll K\left|\frac{\partial\hat{\Phi}}{\partial Z}\right|$, then

$$\Phi^{\mathrm{eik}} = \frac{1}{(2\pi)^{3/2}}\exp[iKZ - \frac{i}{\hbar v}\int_{-\infty}^{Z} V_{PT}(\boldsymbol{b}, Z')dZ'].$$



Eikonal approximation for one body projectile

Ex: Elastic scattering of an incident uncharged particle

The elastic amplitude

$$f(\theta) = iK\int_0^\infty J_0(qb)\left[1 - e^{i\chi(b)}\right]bdb; \quad q = 2K\sin\frac{\theta}{2}$$

The eikonal phase

$$\chi(b) = -\frac{1}{\hbar\, v}\int_{-\infty}^{\infty} V_{PT}(b,Z)dZ; \;\; v = \frac{\hbar K}{\mu_{PT}}$$

Extension to charge particles

$$\chi(b) = \underbrace{\chi_N(b)}_{\text{Nuclear}} + \underbrace{\chi_C(b)}_{\text{Coulomb}}$$

Corrected to overcome divergences due to the Coulomb potential.

Elastic cross sections for n+^{208}Pb at different incident energies

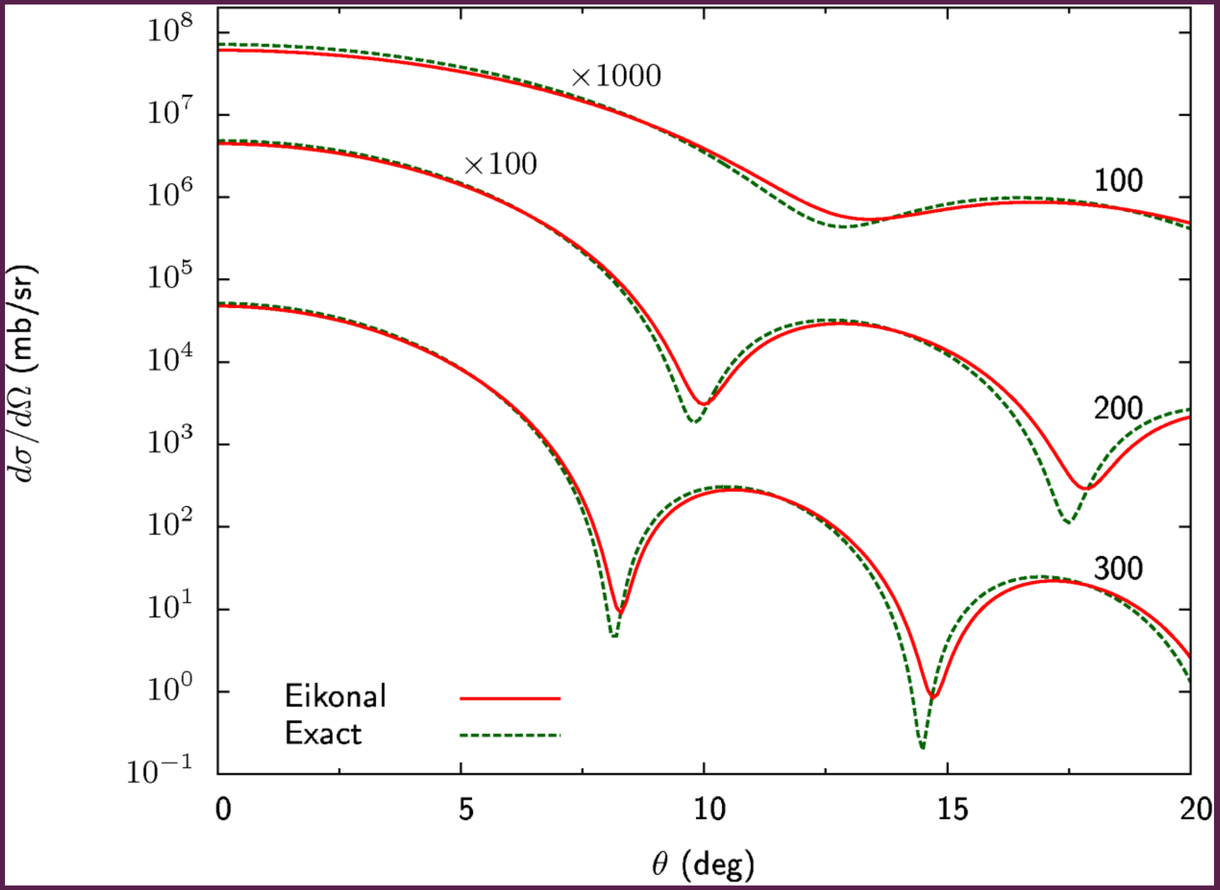
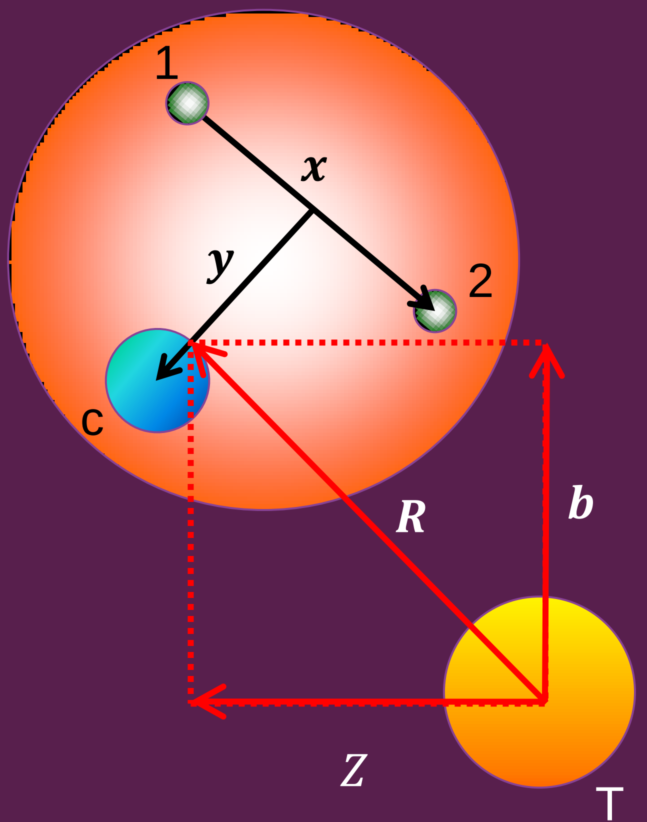


Fig 1. The energies are shown in MeV. The n+^{208}Pb potential is taken from *A. J. Kooning and J. P. Delaroche, Nucl. Phys. A 713, 231 (2003)*.

❖ The agreement improves when the energy increases and θ decreases.

Four-body eikonal



$$H_{4B}\Phi = E_T\Phi, \qquad E_T = E_0 + \frac{\hbar^2K^2}{2\mu_{PT}}$$

$E_0 \rightarrow$ G. S. energy of the projectile

$\frac{\hbar^2K^2}{2\mu_{PT}} \rightarrow$ Initial relative P.T. energy

$$H_{4B} = -\frac{\hbar^2}{2\mu_{PT}}\nabla_R^2 + V_{PT} + H_{3B},$$

Nuclear optical potentials+Coulomb

$$V_{PT} = V_{CT} + V_{Tn} + V_{Tn}$$

Factorizing: $\Phi(\vec{R},\vec{x},\vec{y}) = e^{iKZ}\hat{\phi}(\vec{R},\vec{x},\vec{y})$

$$\rightarrow \left(-\frac{\hbar^2}{2\mu_{PT}}\nabla_R^2 - i\hbar\partial_Z + V_{PT}\right)\hat{\phi} = 0$$

The eikonal approx.
(High-energies) $|\nabla^2\hat{\phi}| << K|\partial_Z\hat{\phi}|$

Four-body eikonal

Eikonal w. f. $\longrightarrow$ $$\widehat{\Phi}^{\text{eik}}(\boldsymbol{R},\boldsymbol{x},\boldsymbol{y}) \approx \Psi_0(\boldsymbol{x},\boldsymbol{y})\exp\left[-\frac{i}{\hbar v}\int_{-\infty}^{Z} V_{PT}(\boldsymbol{b},\boldsymbol{Z}',\boldsymbol{x},\boldsymbol{y})dZ'\right]$$

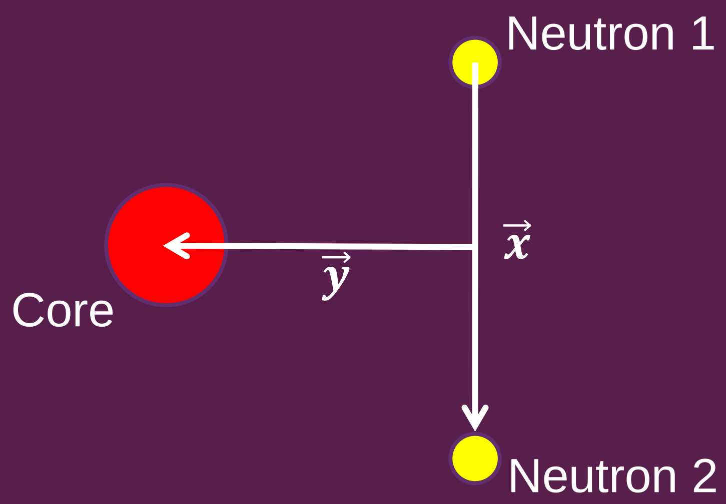
Eikonal elastic amplitude $\longrightarrow$ $$S(\boldsymbol{b}) = \langle \underbrace{\Psi^{J_0 M_0{}'\pi_0}}_{\text{3B bound state}} | e^{i\chi(\boldsymbol{b})} | \underbrace{\Psi^{J_0 M_0 \pi_0}}_{\text{3B bound state}} \rangle$$ $\longrightarrow$ Elastic Cross sections

Eikonal breakup amplitude $\longrightarrow$ $$S(\boldsymbol{b}) \propto \langle \underbrace{\Psi_{k_x K_y}(E)}_{\text{3B scattering State R-matrix}} | e^{i\chi(\boldsymbol{b})} | \underbrace{\Psi^{J_0 M_0 \pi_0}}_{\text{3B bound state}} \rangle$$ $\longrightarrow$ Bup obs.

$1^-, 0^+, 2^+$

Eikonal phase
(Dynamics information) $\longrightarrow$ $$\chi(\boldsymbol{b}) = -\frac{i}{\hbar v}\int_{-\infty}^{\infty}[V_{CT}(\boldsymbol{b}) + V_{nT}(\boldsymbol{b}) + V_{nT}(\boldsymbol{b})]\mathrm{dZ}$$

Three-body model of the projectile



$\vec{x}, \vec{y}$: Jacobi coordinates

ρ,α: Hyperspherical coordinates

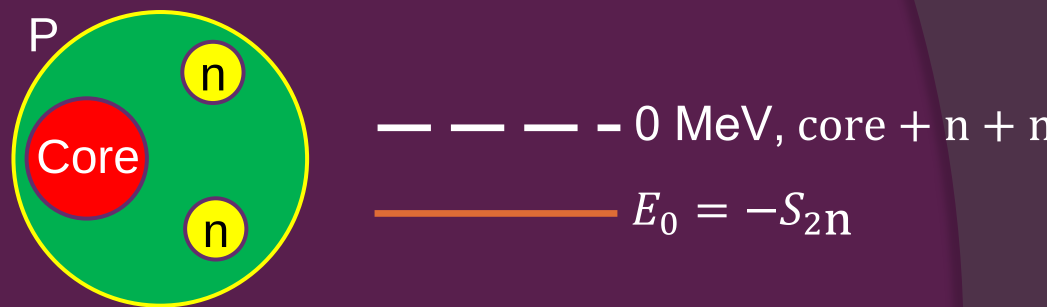
$\rho^2 = x^2 + y^2$: Hyperradius

$\alpha = \arctan\left(\frac{y}{x}\right)$: Hyperangle

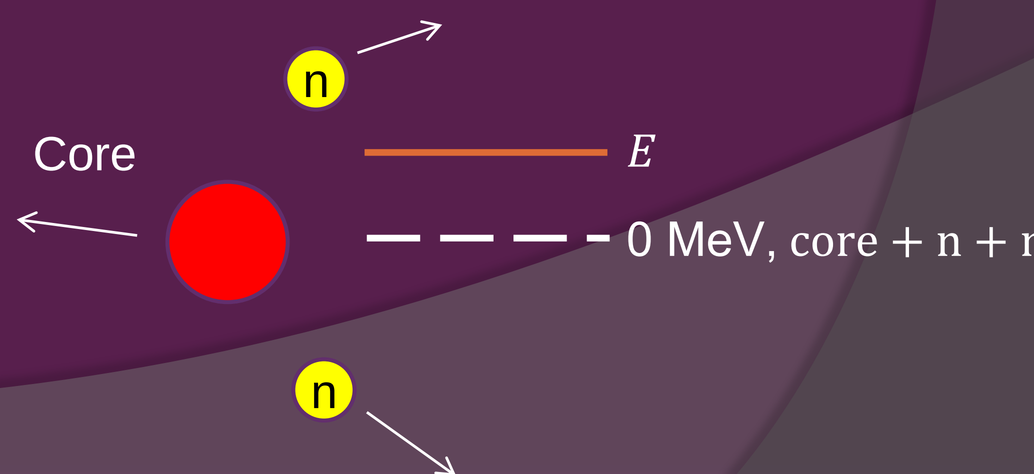
$\Omega_5 = (\alpha, \Omega_x, \Omega_y)$

$$H_{3B}\Psi^{J\pi} = E\Psi^{J\pi}$$

$E < 0 \rightarrow$ Bound state



$E > 0 \rightarrow$ Scattering states



Three-body model of the projectile

$$H_{3B}\Psi^{J\pi} = E\Psi^{J\pi}$$

$$H_{3B} = -\frac{\hbar^2}{2m_n}\nabla_x^2 - \frac{\hbar^2}{2m_n}\nabla_y^2 + T_{c.m.} + \sum_{i<j} V_{ij}$$

$\boxed{\text{2B potentials, Vcn Gaussian, W. Saxon}}$

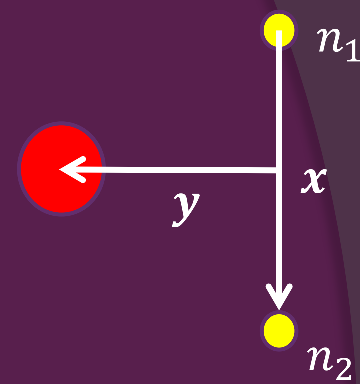
Kmax

$$\Psi^{J\pi} = \rho^{-5/2}\sum_{K=0}^{\infty}\sum_{\gamma}\chi_{\gamma K}^{J\pi}(\rho)\mathcal{Y}_{\gamma K}^{JM}(\Omega_5)$$

Hyperradial Function (Unknown)

Eigenfunction of angular momentum *K* (Known)

$\pi = (-1)^K \rightarrow$ Parity of the relative motion of the 3B



Spinless core

$$\gamma = (l_x, l_y, L, S)$$

$$\hat{L} = \hat{l}_x + \hat{l}_y$$

$$\hat{S} = \hat{S}_1 + \hat{S}_2$$

$$\hat{J} = \hat{L} + \hat{S}$$

Three-body bound states

$$H_{3B}\Psi^{J\pi} = E\Psi^{J\pi}$$

$$\Psi^{J\pi} = \rho^{-5/2}\sum_{K=0}^{\infty}\sum_{\gamma}\chi_{\gamma K}^{J\pi}(\rho)\mathcal{Y}_{\gamma K}^{JM}(\Omega_5)$$

$$\chi_{\gamma K}^{J\pi}(\rho) = \sum_{i=1}^{N} C_{\gamma K i}^{J\pi}\, u_i(\rho),$$

Eigenvalue problem

Lagrange basis

It facilitates the calculations

Three-body continuum states: R-matrix

Internal region

$$\chi_{\gamma K}^{J\pi}(\rho) = \sum_{i=1}^{N} c_{\gamma K i}^{J\pi}\, u_i(\rho)$$

External region

$$\chi_{\gamma K}^{J\pi}(\rho \to \infty)$$

a ρ

Nuclear + Coulomb + Centrifugal potentials

Coulomb + Centrifugal potentials

$$\chi_{\gamma K}^{J\pi}(\rho \to \infty) = A_{\gamma K}^{J\pi}\left[H_{\gamma K}^{-}(k\rho)\delta_{\gamma\gamma'}\delta_{KK'} - U_{\gamma K,\gamma' K'}^{J\pi} H_{\gamma K}^{+}(k\rho)\right]$$

Hankel functions

$U_{\gamma K,\gamma' K'}^{J\pi}$ → Collision matrix → $e^{2i\delta}$ → Eigenphases

→ Large matrix for typical γK values

Dimension of the R-matrix calculations

$J=0^+$		$J=1^-$		$J=2^+$	
Kmax	***γK***	***Kmax***	***γK***	***Kmax***	***γK***
12	28	9	40	12	99
16	45	13	77	16	172
20	66	17	126	20	265

$$\gamma = (l_x, l_y, L, S)$$

$N \rightarrow$ Number of Lagrange basis, typical $N = 40$

$\gamma K \rightarrow$ Channels number

Matrices of $\rightarrow \gamma K N \times \gamma K N$

Example: $J = 2^+$ and $K\max = 20$

Matrices of $\rightarrow \gamma K N \times \gamma K N = 265 \cdot 40 \times 265 \cdot 40 = \underline{10600} \times \underline{10600}$

Applications for ^{6}He: Three-body resonances

- $R^{J\pi} \longrightarrow U^{J\pi} \longrightarrow (S^{-1}US = e^{2i\delta})$
- Information about three-body resonances is contained in the eigenphases δ.

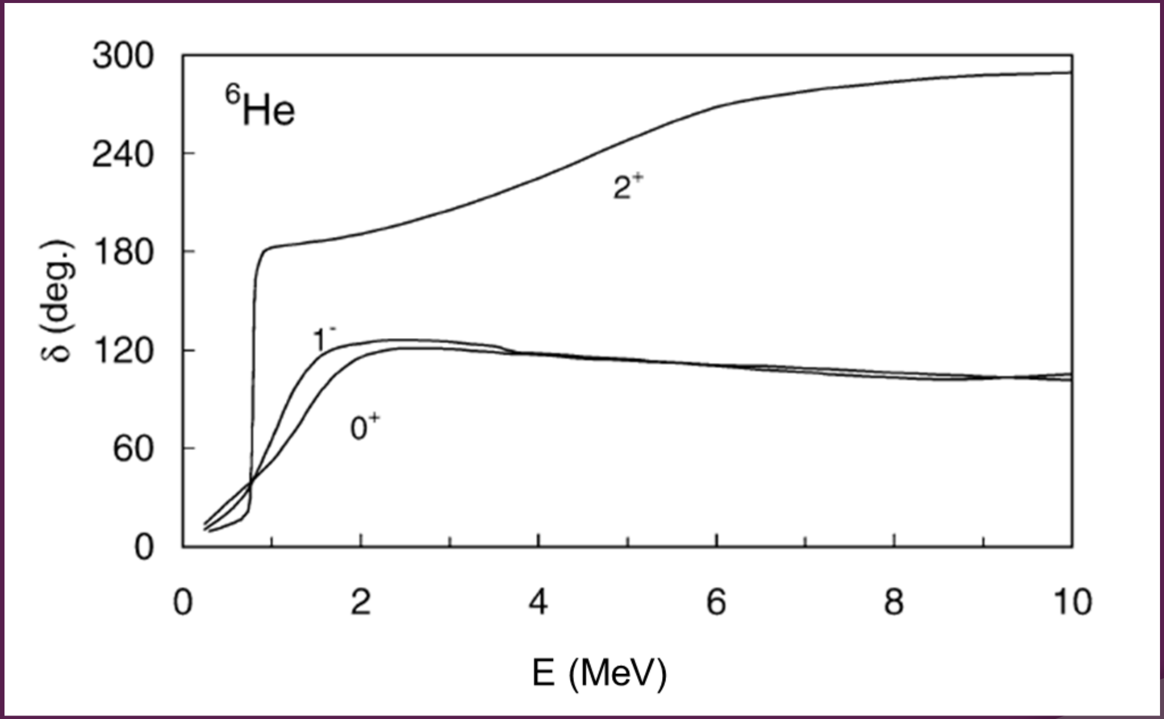


Fig. 2. Eigenphases for ^{6}He for different J values **(**From *P. Descouvemont et al, Nucl. Phys. A 765 (2006) 370*).

Applications for ^{6}He: E1 strength distribution

$$\frac{dB_{E1}}{dE}(E) \propto \Big| \underbrace{\big\langle \Psi_{k_x k_y}(E)}_{1^- \text{ 3B cont. R-matrix}} \big| \mathcal{M}^{E1} \underbrace{\big| \Psi^{J_0 M_0 \pi_0} \big\rangle}_{0^+ \text{ 3B bound state}} \Big|^2$$

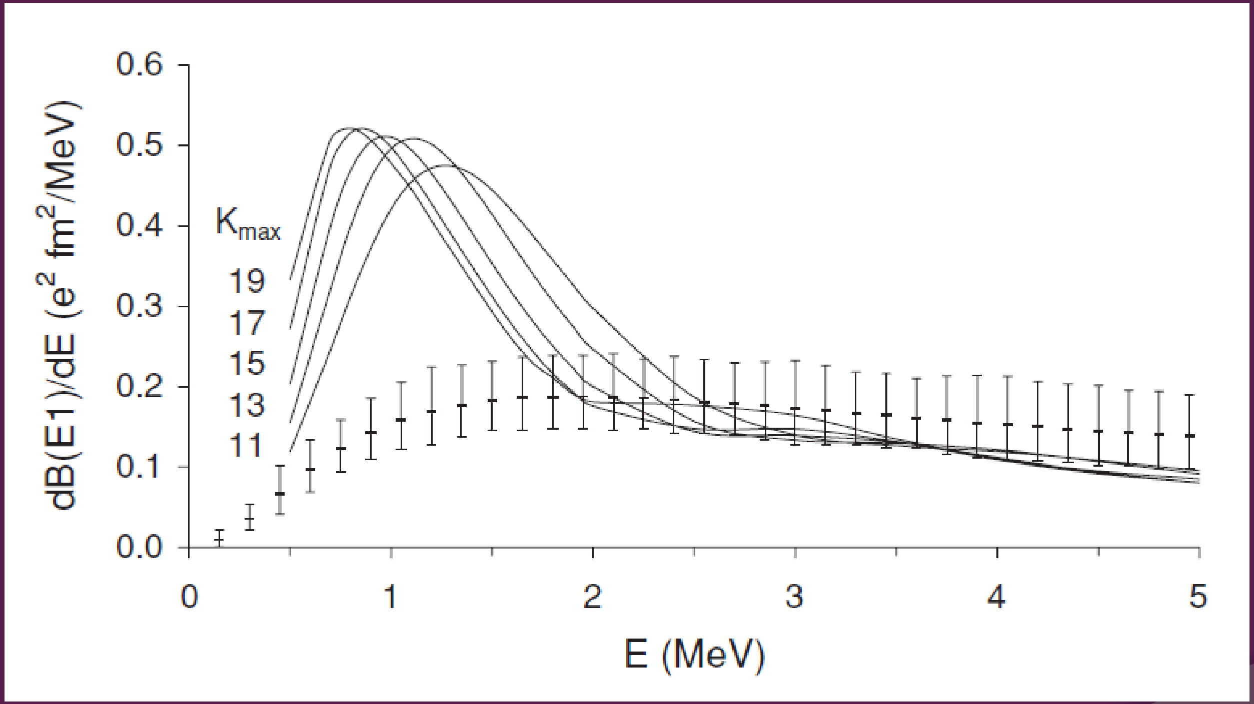


Fig. 3. Electric dipole distribution for different Kmax values. From *D. Baye et al, Phys. ReV. C 79, 024607 (2009)*.

Applications in ^{11}Li: Conditions of the calculations

To calculate bound and scattering states of ^{9}Li+n+n

^{9}Li+n interaction

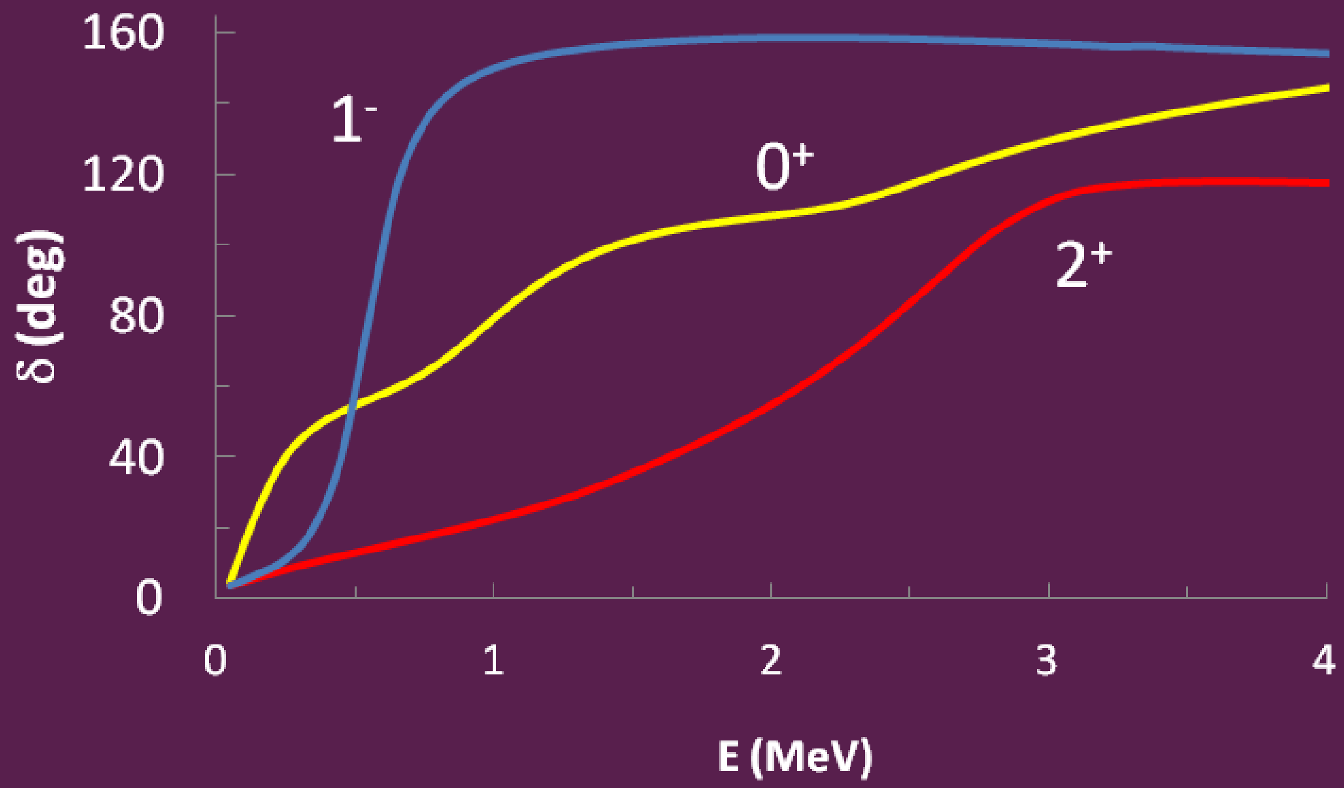
- From *H. Esbensen, et. al, Phys. ReV. C 56, 3054 (97).*
- Non-existent elastic scattering experimental data.
- Fitted to reproduce a presumed $p_{1/2}$ resonance at 540 keV and a s virtual state.
- ^{9}Li-n interaction multiplied by 1.0056 to reproduce **G.S. energy of ^{11}Li = - 0.378 MeV**.

n+n potential

- Minnesota interaction

We those potentials we well reproduce **r.m.s. radius of ^{11}Li : 3.1 fm** (exp. r.m.s of 3.16 ±0.11 fm).

Eigenphases of ^{11}Li in a three-body model



- Like-resonant behavior for 1^- and 2^+ continuum
- Rise of the 0^+ phase shift with energy: “Like a superposition of resonances”

Convoluted E1 strength distribution of ^{11}Li with the detector response

R-matrix (Red curve)

$$\frac{dB_{E1}}{dE}(E) \propto \left| \underbrace{\left\langle \Psi_{k_x k_y}(E) \right|}_{1^-} \mathcal{M}^{E1} \underbrace{\left| \Psi^{J_0 M_0 \pi_0} \right\rangle}_{0^+} \right|^2$$

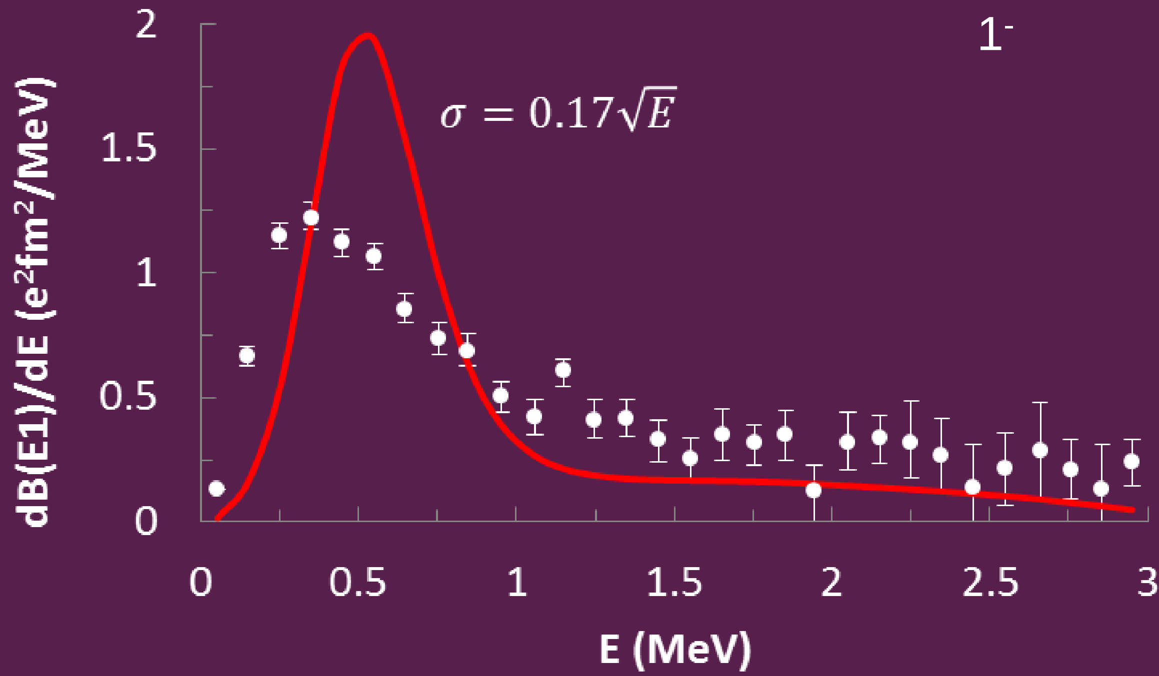


Fig 5. The σ value is in MeV. Experimental Data from *T. Nakamura et. al, Phys. Rev. Lett. 252502 (2006).*

❖ We overestimate the E1 distribution in the peak region.

Four-body breakup eikonal

- Some applications by *D. Baye, P. Capel, P. Descouvemont and Y. Suzuki, Phys. ReV. C 71, 024607 (2009).* They described the elastic breakup cross section of ^{6}He on ^{208}Pb @ 70 A MeV.

- **Qualities of the model:**

✓ Contributions different from the dipole.

✓ It does not require ^{6}He-^{208}Pb potential: α-^{208}Pb potential and n-^{208}Pb potential are well known.

✓ It takes nuclear and Coulomb effects and their interference on the same footing.

✓ There is not adjustable parameter.

Conditions of the calculations for ^{11}Li on ^{208}Pb

To calculate the breakup cross sections of ^{11}Li on ^{208}Pb @ 70 A MeV:

❖ **^{9}Li-^{208}Pb potential (lack of the potential):**
Renormalized ($9^{1/3}$+$208^{1/3}$) α-^{208}Pb interaction @ 70 A MeV of B. Bonin et. al. (Following the same idea of *P. Capel et. al, Phys. Rev. 68, 014612 (2003)* for ^{10}Be on ^{208}Pb).

❖ Variation of the **^{9}Li-^{208}Pb** potential was checked but it did not provide a significant change to the breakup and angular distributions.

❖ **n-^{208}Pb potential**:
Kooning and Delaroche, Nucl. Phys. A 713, 231 (2003).

Breakup cross sections of ^{11}Li on ^{208}Pb @ 70 A MeV

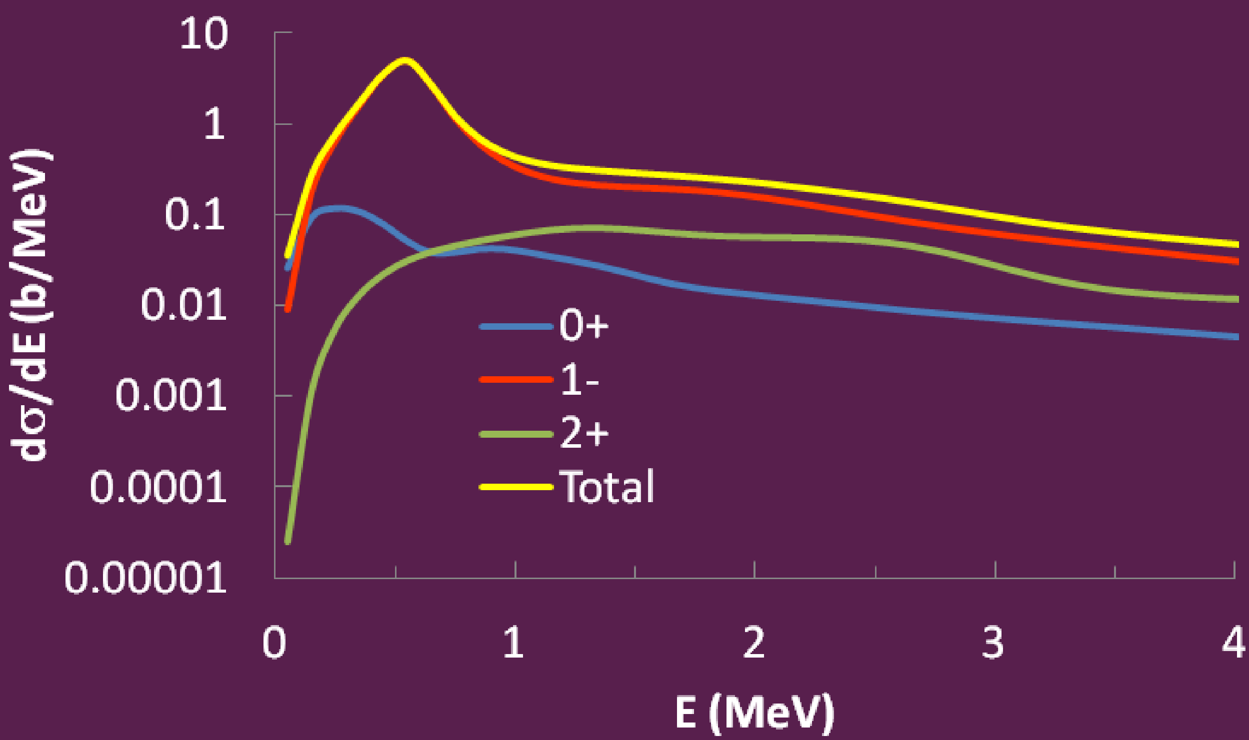


Fig. 6. Partial and total eikonal breakup cross sections.

❖ Small correction of the 0^+ and 2^+ partial waves to the total cross section.

Convoluted breakup eikonal cross section with the detector response

^{11}Li on ^{208}Pb @ 70 A MeV

Theoretical data convoluted with a Gaussian of $\sigma = 0.17\sqrt{E}$ MeV

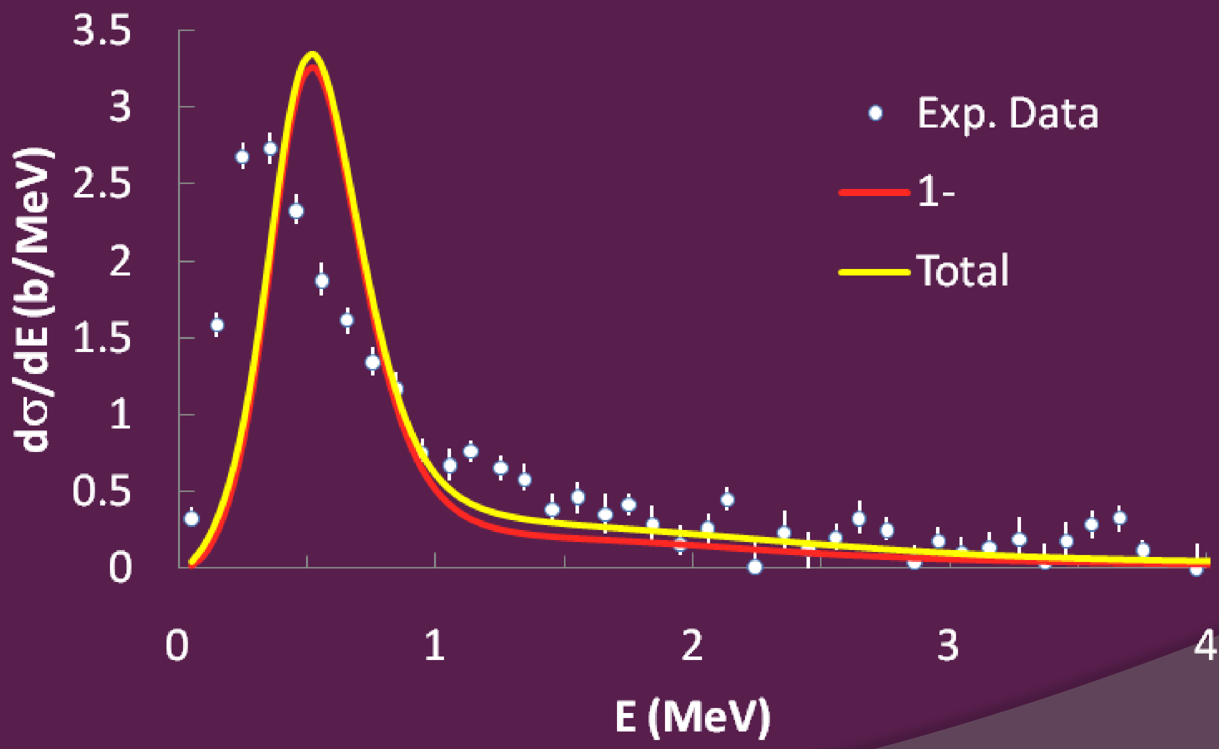


Fig. 7. Exp. Data from *T. Nakamura et. al, phys. Rev. Lett. 252502 (2006).*

Angular distributions of ^{11}Li on ^{208}Pb @ 70 A MeV

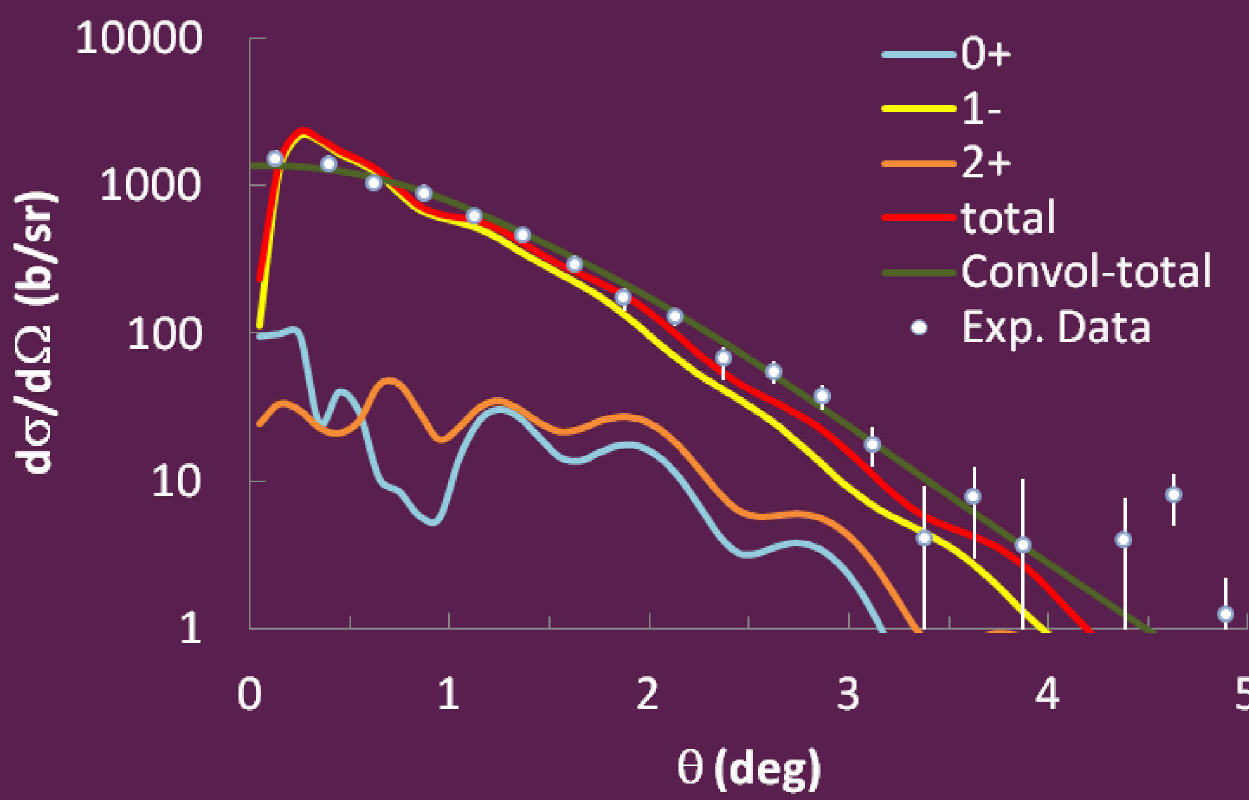


Fig. 8. Partial, total and convoluted total angular distributions. Experimental Data from *T. Nakamura et. al, Phys. Rev. Lett. 252502 (2006).*

- Very good agreement of the total convoluted curve for almost all angles.
- Appreciable 0^+ and 2^+ contributions after $\theta \gtrsim 1$ deg.

Convoluted E1 strength distribution of ^{11}Li with the detector response

R-matrix (Red curve)

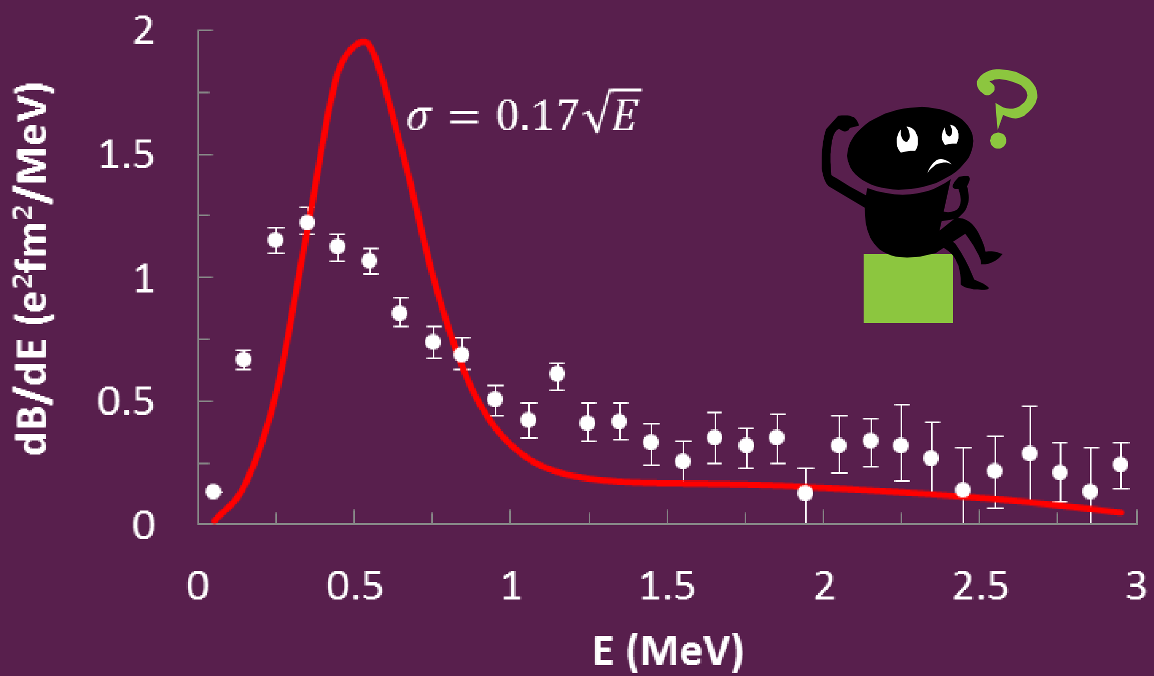


Fig. 9. The σ value is in MeV. Experimental Data from *T. Nakamura et. al, Phys. Rev. Lett. 252502 (2006).*

❖ Why we overestimate the E1 distribution?

Why we overestimate the E1 distribution?

In the breakup reactions of $^{11}Li+^{208}Pb$ @ 70 A MeV

- $\frac{d\sigma}{dE}$ is measured directly → We fit the data
- $\frac{d\sigma}{d\theta}$ is measured directly → We fit the data
- $\frac{dB(E1)}{dE}$ is measured indirectly → We do not fit the data
  (It depends on model assumptions)

How is determined experimentally dB(E1)/dE?

It is extracted from the equivalent photon method as

$$\boxed{\frac{d\sigma^{\mathrm{Exp}}}{dE} = \frac{16\pi^3}{9\hbar c}\frac{dB^{\mathrm{Exp}}(E1)}{dE}\int_{b_{min}}^{\infty} 2\pi db b\, N_{E1}(b,E)}$$

❖ From b_{min} to exclude nuclear excitation.

❖ $N_{E1}(b,E) \rightarrow$ Number of virtual photons incident on ^{11}Li by unit area.

❖ It is assumed to be one step and dominated by a single E1 multipolar transition.

❖ It comes from semi-classical perturbation theory.

^{11}Li is excited by absorption of a virtual photon from the Coulomb field of the target.

Estimation of the θ_c dependence in the dipole distribution of ^{11}Li

In non-relativistic regime

$$\boxed{\frac{dB^{\mathrm{Exp}}(E1)}{dE} = \frac{9}{32\pi}\left(\frac{\hbar v}{Z_T e}\right)^2 \frac{1}{\xi_{min} K_0(\xi_{min}) K_1(\xi_{min})} \frac{d\sigma^{\mathrm{Exp}}}{d\Omega}}$$

$v \rightarrow$ Projectile-target relative velocitiy, $\xi_{min} = \dfrac{E - E_0}{\hbar v} b_{min}$,

$E \rightarrow$ Excitation energy of ^{11}Li, $E_0 \rightarrow$ G. S. energy of ^{11}Li

$b_{min} = \dfrac{Z_P Z_T e^2}{2\tan\left(\frac{\theta_c}{2}\right)} \rightarrow$ Min. Impact parameter for the semi-calssical Coulomb trajectory

$\theta_c \rightarrow$ maximum scattering angle (beyond θ_c nuclear interaction is important.)

Estimation of the θ_c dependence in the dipole distribution of ^{11}Li

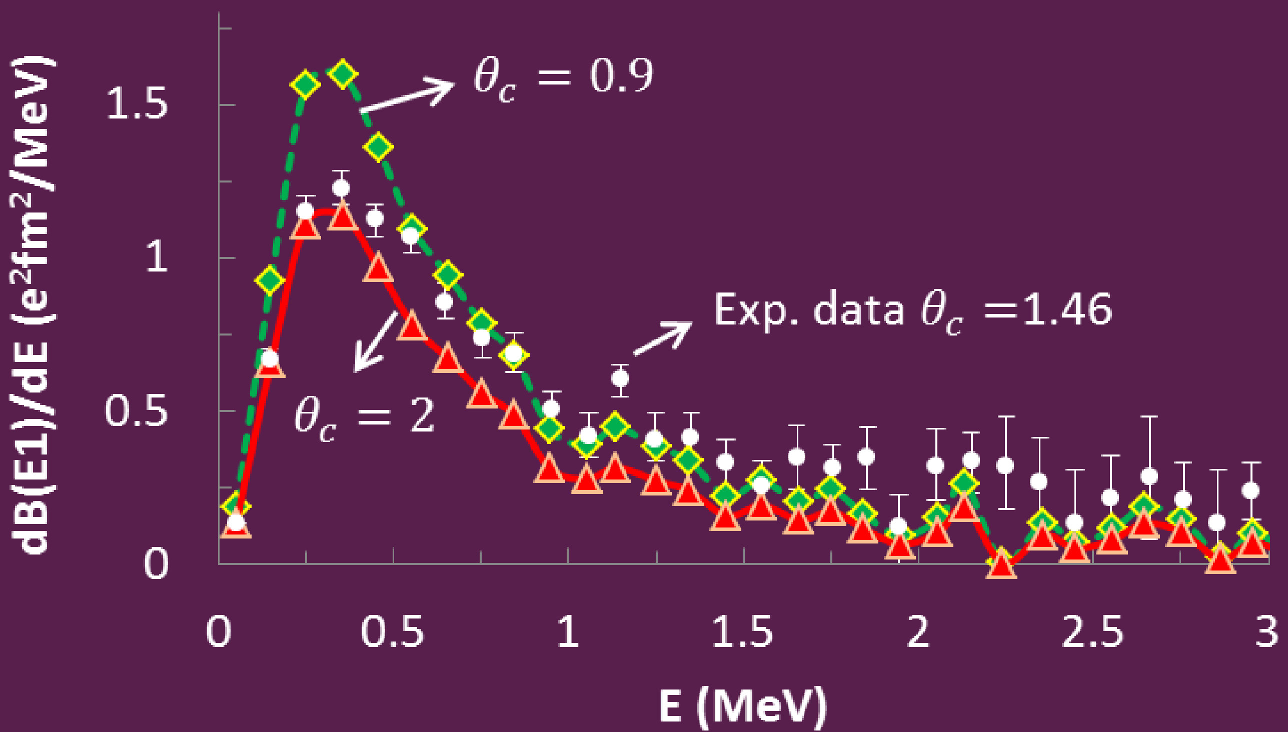
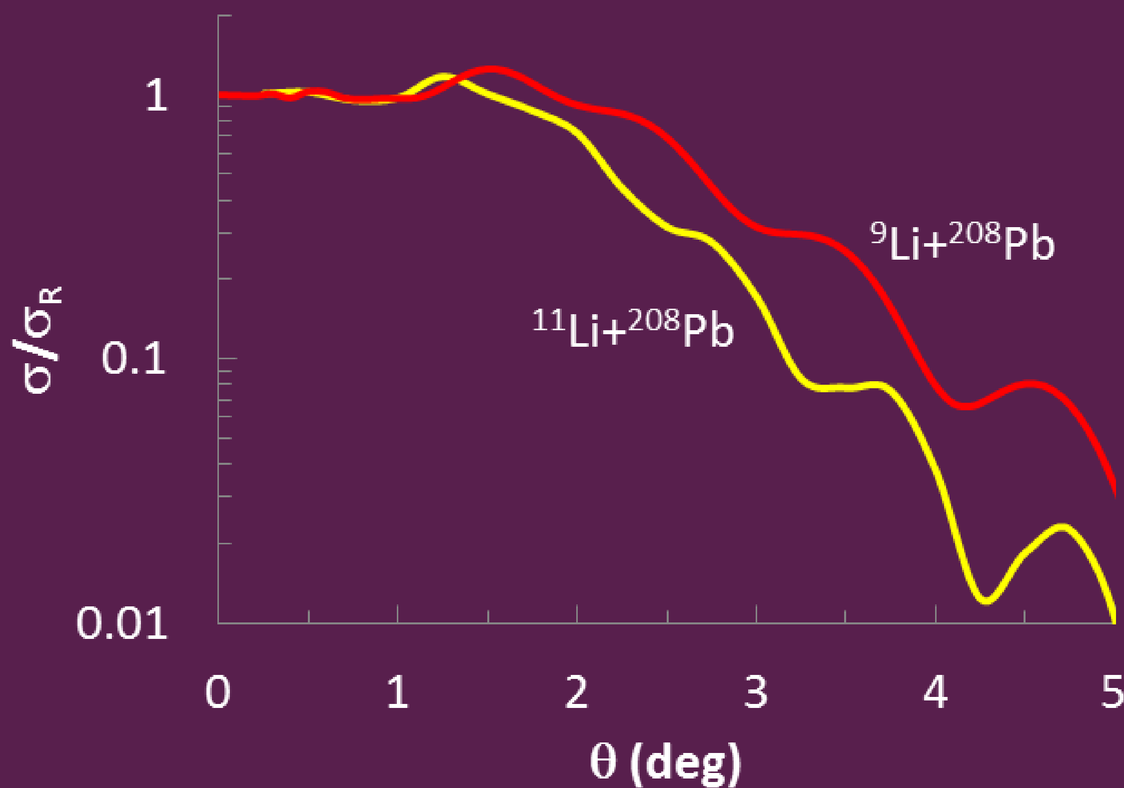


Fig. 10. The θ_c values of 0.9, 1.46 and 2 deg correspond to b_{min} of 31, 19 and 14 fm respectively.

- Small θ_c provides a larger dipole distribution at low excitation energies.

Elastic scattering of ^{11}Li on ^{208}Pb @ 70 A MeV in the Eikonal method

Three-body projectile (yellow curve) One-body projectile (red curve)



- Reduction in the $^{11}Li+^{208}Pb$ elastic scattering due to flux going to breakup.
- $0 \lesssim \theta \lesssim 1 \rightarrow$ Rutherford scattering.

Conclusions

❖ We have predicted a 1^- resonant eigenphase for ^{11}Li.

❖ The maximal contribution for the total breakup cross section is coming from the 1^- partial wave.

❖ The breakup cross sections and angular distributions of ^{11}Li on ^{208}Pb are in good agreement with the experimental data.

❖ To test our model we suggest to experimentalist to measure elastic scattering of ^{11}Li at high-energies.

❖ We need to clarify why we overestimate the dipole strength distribution of ^{11}Li with the same ^{11}Li wave functions that we had successful results for the breakup and angular distributions. Ideas are welcome!

Thank you for your attention