Smooth transition from sudden to adiabatic states in deep-subbarrier incident energies

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Steep falloff of fusion cross sections

Standard CC calculations largely deviate from experimental data at below a certain threshold incident energy

$^{16}\text{O} + ^{208}\text{Pb}$ (Mass-asymmetric system)

**Nuclei in Collision**

When two large nuclei collide and fuse—rather than flying apart—some of the credit goes to the internal motions of protons and neutrons that result in excited states of the nuclei. The best models account for these states in calculating the fusion rate. But in the 9 November Physical Review Letters, Australian physicists say their measurements disagree with even these sophisticated models. The researchers suggest that the internal modes get out of sync even while the collision is underway, so that the nuclei behave more like a macroscopic classical object than a tiny quantum one.

Two nuclei can overcome the "Coulomb barrier" of repulsion of like charges and fuse if they approach fast enough. Nuclei with less energy but a barrier process improve with $\text{O} + \text{Pb}$ below 16 MeV. The collision process begins at the top of the potential energy curve. The nuclei move apart at high energy, and they can only come together near the barrier, which can stifle the fusion reaction.

FIG. 3 (color online). Logarithmic slope as a function of energy with respect to the barrier. Calculation with standard parameters fail to match the measurements at low energy.
Steep falloff of fusion cross sections

Energy at touching configuration coincides with threshold incident energy $E_s$

Threshold incident energy $E_s \sim 89$ MeV

Energy at touching point $V_{touch} = 88.61$ MeV(YPE)
Correlation between $E_s$ and $V_{\text{touch}}$

Energy at touching configuration, $V_{\text{touch}}$, strongly correlates with threshold incident energy $E_s$.

- Estimate potential energy at touching configuration $V_{\text{touch}}$ (YPE model)
  
  - $E_s \rightarrow$ Energy at the peak position of the S-factor
  
  - Red curve
    \[ \rightarrow \text{Systematic curve} \]

Potential inversion method

- Extract the lowest eigen-potential for coupled channel calculations from experimental data


Energy at touching point

\[ V_{\text{touch}} = 70.5 \text{ MeV (YPE)} \]

What happens below energy at touching point?
Motivation

Steep fall-off phenomenon can be attributed to dynamics after target and projectile touch with each other

- **Subbarrier energies** \((E > V_{\text{touch}})\)
  - Inner turning point → Outside of touching point
- **Deep subbarrier energies** \((E < V_{\text{touch}})\)
  - Inner turning point → In the overlap region

Sudden approach → Fusion takes place so rapidly
Adiabatic approach → Dynamical change in the density
Sudden and adiabatic approaches

- **Sudden Approach**
  - Shallow potential pocket
  - Frozen density approximation
    Mişicu and Esbensen

Sudden and adiabatic approaches

- **Adiabatic approach**
  → Neck formations

  - Density-constraint time-dependent Hatree-Fock model
    Umar and Oberacker

  - Macrosopic-microscopic model

Sudden picture works well at before touching point

Adiabatic potential energy

Connect smoothly between one- and two-body potential energies

- Total potential energy
  - Lemniscatoid parametrization

\[
E(r) = E_V + E_C(r) + E_N(r)
\]

- Yukawa-plus-Exponential (YPE) model

\[
E_N = -\frac{C_s}{8\pi^2 r_0 a^3} \int \int \left(\frac{\sigma}{a} - 2\right) \frac{e^{-\sigma/a}}{\sigma} d^3r d^3r
\]

\[
C_s = a_s (1 - \kappa_s I^2) \quad I = (N - Z)/A
\]

\[a = 0.68 \text{ fm}, \quad a_s = 21.33 \text{ MeV}, \quad \kappa_s = 2.3785\]

(parameter set: FRLDM2002)
Difficulties in adiabatic approach

How do we describe the total wave function in the one-body system?

- The total wave function is expanded by the asymptotic intrinsic basis of the isolated nuclei

- Require to include all the intrinsic basis in the complete set → Almost impossible in practice

Double counting of CC effects

- Adiabatic one-body potential with neck formations already includes a large part of the channel coupling effects
**Standard coupled-channel model**

\[
\begin{bmatrix}
-n^2 \frac{d^2}{2\mu \, dr^2} + \frac{J(J+1)}{2\mu r^2} + V(r) + \epsilon_n - E
\end{bmatrix} u_n(r) + \sum_n \langle \phi_n | V_{\text{coup}} | \phi_n \rangle u_n(r) = 0
\]

**Vibrational coupling** \( \langle \phi_n | V_{\text{coup}} | \phi_n \rangle \)

\[
V_{\text{coup}}(r, \hat{O}) = V_{\text{coup}}^{(N)}(r, \hat{O}) + V_{\text{coup}}^{(C)}(r, \hat{O})
\]

\[
V_{nm} = \langle I0 | V_N(r, \hat{O}) | I'0 \rangle - V_N^{(0)}(r)\delta_{nm}
\]

\[
= \sum_{\alpha} \langle I0|\alpha \rangle \langle \alpha|I'0 \rangle V_N(r, \lambda_\alpha) - V_N^{(0)}(r)\delta_{nm}
\]

**Intrinsic state:** \( \hat{h} |\alpha \rangle = \epsilon_{\alpha} |\alpha \rangle \)

\[
\hat{O} = \frac{\beta_{\lambda}}{\sqrt{4\pi}} R_T \left( a_{\lambda 0}^\dagger + a_{\lambda 0} \right)
\]

\[
\hat{O} |\alpha \rangle = \lambda_\alpha |\alpha \rangle
\]

\[
V_N(r, \lambda_\alpha) \sim V_N^{(0)}(r) - \frac{dV_N^{(0)}(r)}{dr} \lambda_\alpha + \frac{1}{2} \frac{d^2V_N^{(0)}(r)}{dr^2} \lambda_\alpha^2
\]
Extension of coupled-channel model

**Damping factor**

\[ \Phi(r, \lambda_\alpha) = \begin{cases} 
1 & (r \geq R_d + \lambda_\alpha) \\
\exp\left[-(r-R_d-\lambda_\alpha)^2/2a_d^2\right] & (r < R_d + \lambda_\alpha) 
\end{cases} \]

\[ R_d = r_d \left( A_T^{1/3} + A_P^{1/3} \right) \quad a_d: \text{Damping factor} \]

\[ V_N(r, \lambda_\alpha) \sim V_N^{(0)}(r) + \left[ -\frac{dV_N^{(0)}(r)}{dr} \lambda_\alpha + \frac{1}{2} \frac{d^2V_N^{(0)}(r)}{dr^2} \lambda_\alpha^2 \right] \Phi(r, \lambda_\alpha) \]

Different touching point in each eigenchannel

Two body

Touching point

One body

\[ \left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)}{2\mu r^2} + V(r) + \epsilon_n - E \right] u_n(r) + \sum_n \left\langle \phi_n | V_{\text{coup}} | \phi_n \right\rangle u_n(r) = 0 \]

\[ V_{nn}^{(N)} = \left\langle I0 | V_N(r, \hat{O}) | I'0 \right\rangle - V_N^{(0)}(r) \delta_{nn} \to 0 \]

\[ \left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)}{2\mu r^2} + V(r) + \epsilon_n - E \right] u_n(r) = 0 \]
Input parameters

- **CC calculation CCFULL (K. Hagino)**
    - $2^+$: $E_x = 1.35$ MeV, $\beta_c = 0.165$, $\beta_N = 0.185$, 2ph
    - $3^-$: $E_x = 3.56$ MeV, $\beta_c = 0.193$, $\beta_N = 0.20$, 1ph
    - $2^+$: 3ph, $3^-$: 1ph
    - $^{208}\text{Pb} \rightarrow 3^-$: $E_x = 2.615$ MeV, $\beta_c = 0.161$, $\beta_N = 0.733$, 2ph
    - $^{16}\text{O} \rightarrow 3^-$: $E_x = 6.13$ MeV, $\beta_c = 0.733$, $\beta_N = 0.733$, 2ph

- **Damping factor**
  - $^{64}\text{Ni} + ^{64}\text{Ni}$: $r_d = 1.298$ fm, $a_d = 1.05$ fm
  - $^{58}\text{Ni} + ^{58}\text{Ni}$: $r_d = 1.3$ fm, $a_d = 1.3$ fm,
  - $^{16}\text{O} + ^{208}\text{Pb}$: $r_d = 1.28$ fm, $a_d = 1.28$ fm

Radius parameters are almost the same as each system
Calculated results: fusion cross section

Drastic improvements are achieved by damping factor
First derivative of fusion cross section

Reproduce the saturation at extremely low incident energies
Astrophysical S-factor

Differs considerably from sudden model
Adiabatic potential

Reproduce the thickness of the CC adiabatic potential

\[16_{\text{O}} + 208_{\text{Pb}}\]

Energy (MeV)

\[r (\text{fm})\]
Difference between two approaches

- Both the sudden and adiabatic models provide similar results for the fusion cross sections
  - What is a difference between these two models?
    - Average angular momentum of compound nuclei

By measuring average angular momentum, we can discriminate the two approaches
Summary

- We have proposed a novel extension of the standard CC calculations based on the adiabatic approach
  - Energy at touching point strongly correlates with threshold incident energy for steep-falloff of fusion cross sections
  - Introduce the damping of CC forme factor inside touching point, to simulate transition from sudden to adiabatic states
  - Sudden approximation works well before touching of two nuclei
  - Smooth transition from two-body to adiabatic one-body potential is responsible for steep falloff of fusion cross sections