TEXAS A&M UNIVERSITY COMMERCE

Coulomb and medium effects in knockout reactions

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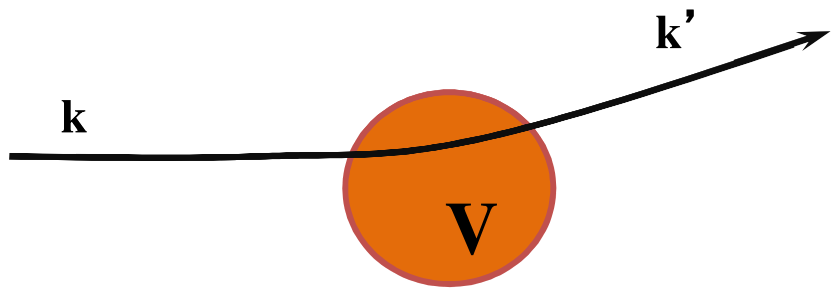
Basic theory: (a) One-body scattering

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V\right]\Psi = E\Psi$$



Partial waves:

$$u_l(r) \xrightarrow[r\to\infty]{} \frac{i}{2}\left\{H_l^{(-)}(kr) - S_l H_l^{(+)}(kr)\right\}$$

Incoming wave

"Survival" amplitude

Outgoing wave

Bertulani & Danielewicz, "Introduction to Nuclear Reactions", IOP Publishing, 2004

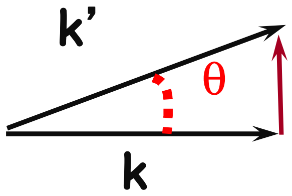
$$S_l = e^{2i\delta_l}$$

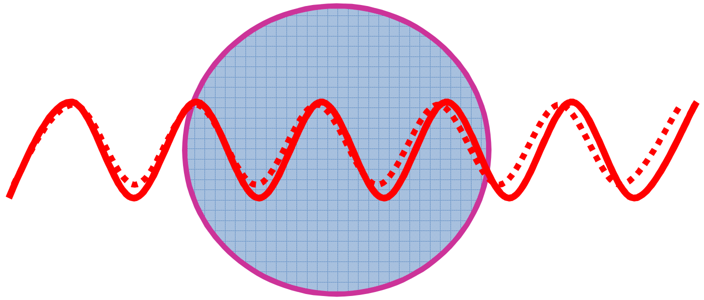
(δ_l = Phase shift)

$|S_l|^2$ = "Survival" probability ≤ 1

(b) Eikonal Waves

$\Delta E \ll E$, $\theta \ll 1$ radian , $|\Delta\psi/\psi|_{\Delta r=\lambda} \ll 1$





$$\Psi(\mathbf{r}) = S(\mathbf{b}, z)\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

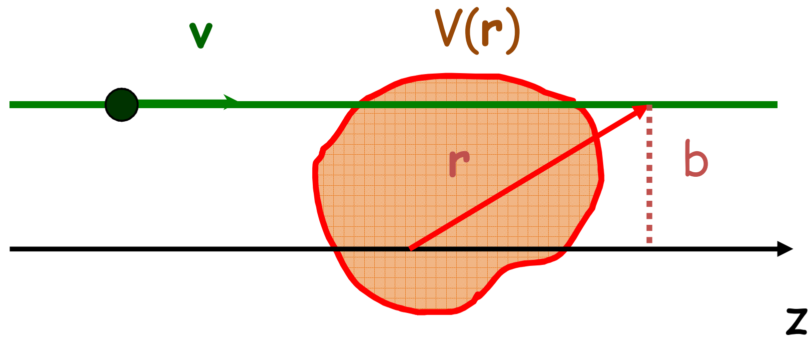
$$S(\mathbf{b}, z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z} V(\mathbf{r}')\, dz'\right\}$$

$$\mathbf{r}' = (\mathbf{b}, z')$$

$z \to \infty$ after the collision:

$$\Psi(\mathbf{r}) = S(\mathbf{b})\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$S(\mathbf{b}) = e^{i\chi(\mathbf{b})} = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{\infty} V(\mathbf{r}')\, dz'\right\}$$

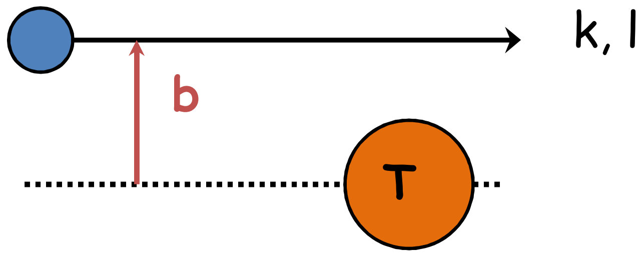


Eikonal waves (reactions)

Harmonic oscillator (structure)

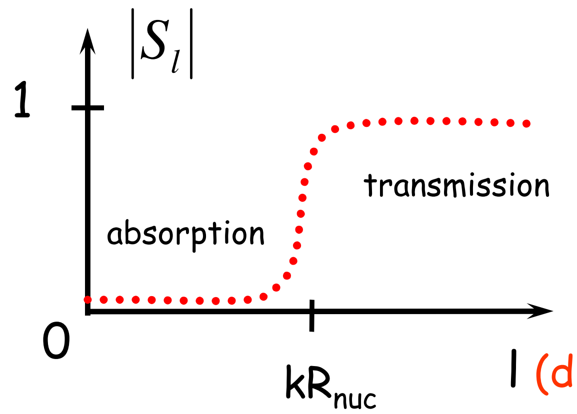
Pearls of quantum mechanics

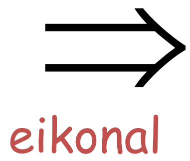
(c) S-matrices ("Survival" Amplitudes)

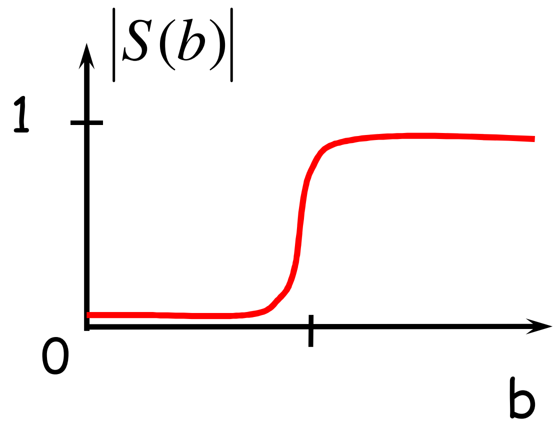


b = impact parameter

l = kb (actually l + 1/2 = kb)







Ex: Elastic Scattering

$$f(\theta) = \frac{i}{k}\sum_l (l+\tfrac{1}{2})(1-S_l)P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

$$f(\theta) = ik\int db\, b\, J_0(kb)\{1-S(b)\}$$

(d) Immediate application:
Probing nuclear structure

Ex: Elastic scattering as a probe of nuclear densities

Need optical potentials:

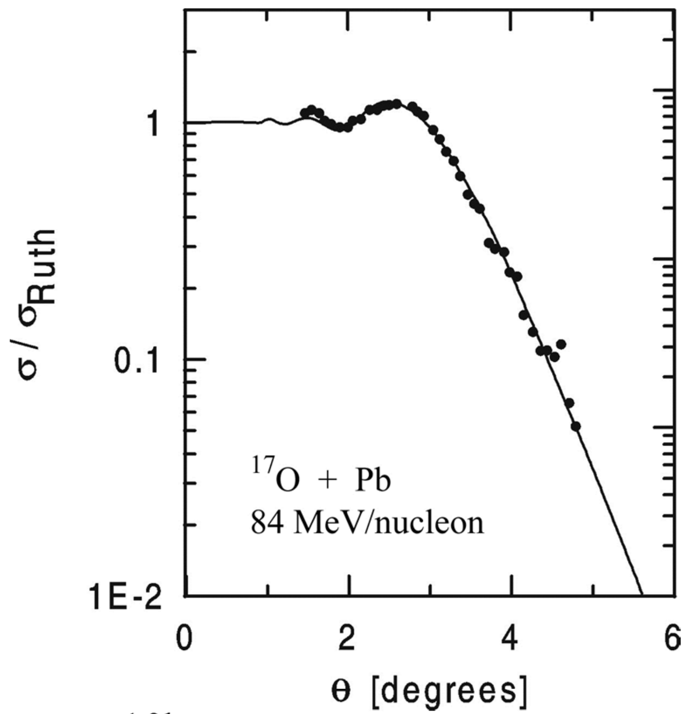
1. Fit to data –phenomenology

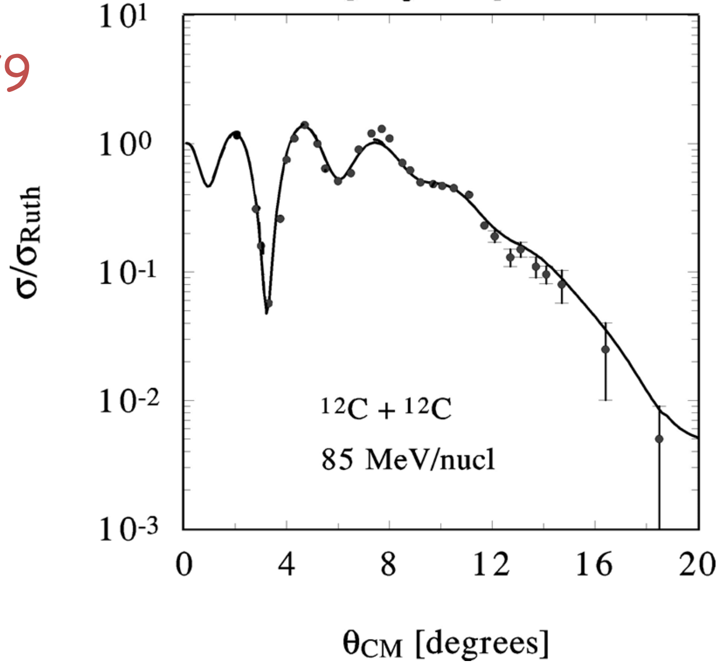
2. Theory, e.g. "tρρ"
   Ray, PRC 20 (1979) 1957
   Hussein, Rego, Bertulani, Phys. Rep. 5 (1991) 279

$$\chi_{AB}^{(N)}(b) = \frac{1}{k_{nn}} \int_0^\infty dq\, q\, \tilde{\rho}_A(q) \tilde{\rho}_B(q)\, f_{nn}(q)\, J_0(qb)$$

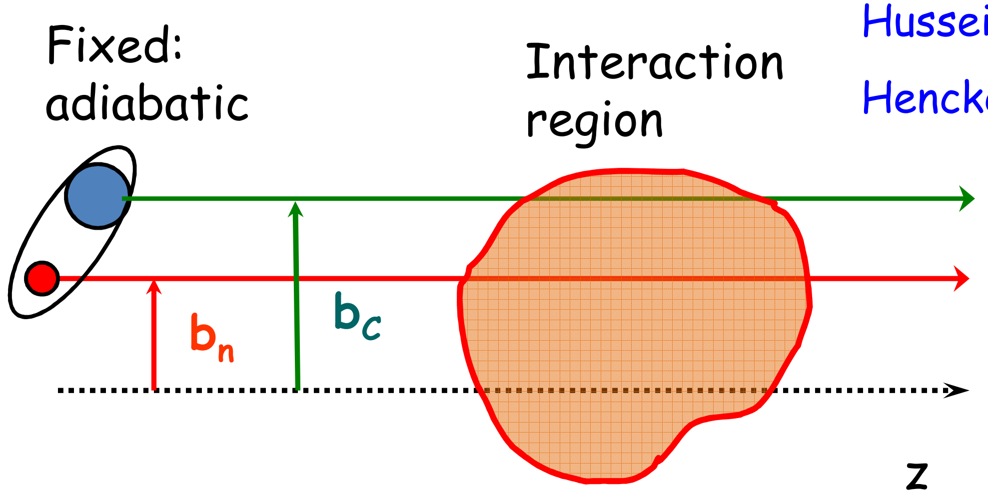
$$f_{nn}(q) = \frac{k_{nn}}{4\pi} \sigma_{nn} \left(i + \alpha_{nn}\right) e^{-\beta_{nn} q^2}$$

(from nn scattering)





Breakup Reactions (two-body): (a) elastic



Hussein, McVoy, NPA 445, 124 (1985)

Hencken, Bertsch, Esbensen, PRC 54 (1996) 3043

Elastic:

including breakup effects

$$\Psi^{eik}(\mathbf{r}) = S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, e^{i\mathbf{k}\cdot\mathbf{r}}\, \phi_0$$

Best possible wfs:

(Spectroscopy)

$$S_{elast}(\mathbf{b}) = \left\langle \phi_0 \middle| S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n) \middle| \phi_0 \right\rangle$$

Survival amplitude

for projectile at impact parameter b

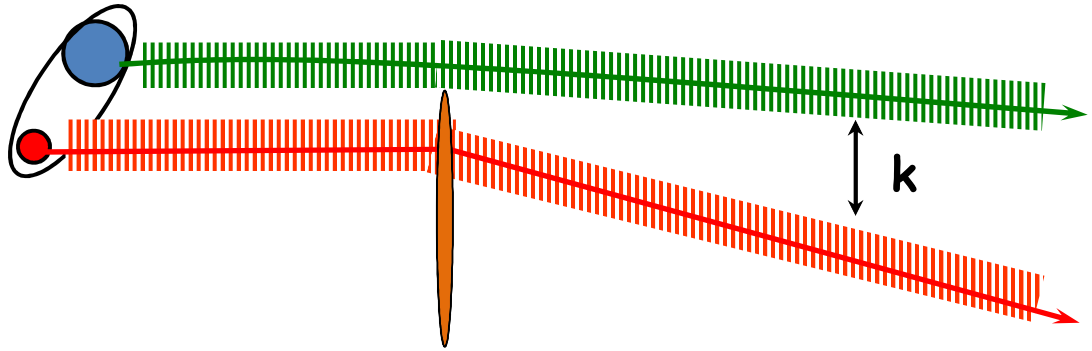
Survival amplitudes

for particles C and n at impact parameters b_C and b_n

(Dynamics)

(b) Diffraction Dissociation:

Breakup amplitude:
to state $\phi_{\mathbf{k}}$



$$1 - S_{dif.dis.}(\mathbf{b}) = 1 - S_C(\mathbf{b}_C) + 1 - S_n(\mathbf{b}_n) - [1 - S_C(\mathbf{b}_C)][1 - S_n(\mathbf{b}_n)]$$

$\varphi_{\mathbf{k}} \perp \phi_0$ $\Longrightarrow$

$$S_{dif.dis.}(\mathbf{b}) = \langle \phi_{\mathbf{k}} | S_C(\mathbf{b}_C) S_n(\mathbf{b}_n) | \phi_0 \rangle$$

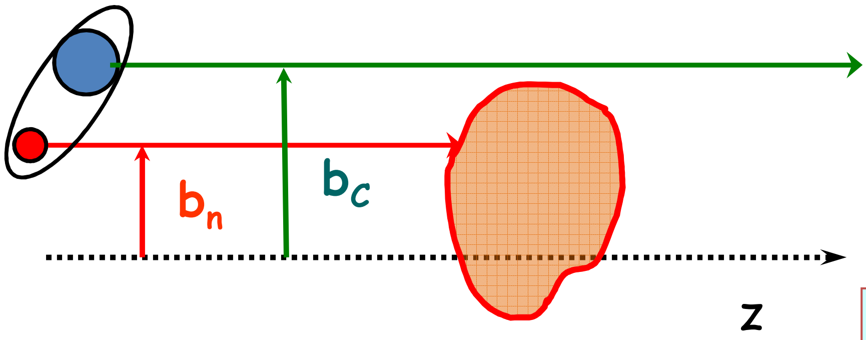
Closure:

$$\int d\mathbf{k} | \phi_{\mathbf{k}} \rangle \langle \phi_{\mathbf{k}} | = 1 - | \phi_0 \rangle \langle \phi_0 | - | \phi_1 \rangle \langle \phi_1 | - \cdots$$

$\Rightarrow$ Breakup X-section (only one bound state):

$$\sigma_{dif.dis.}(\mathbf{b}) = \int d\mathbf{b} \left[\langle \phi_0 | \left| S_C S_n \right|^2 | \phi_0 \rangle - \left| \langle \phi_0 | S_C S_n | \phi_0 \rangle \right|^2 \right]$$

(c) Stripping:

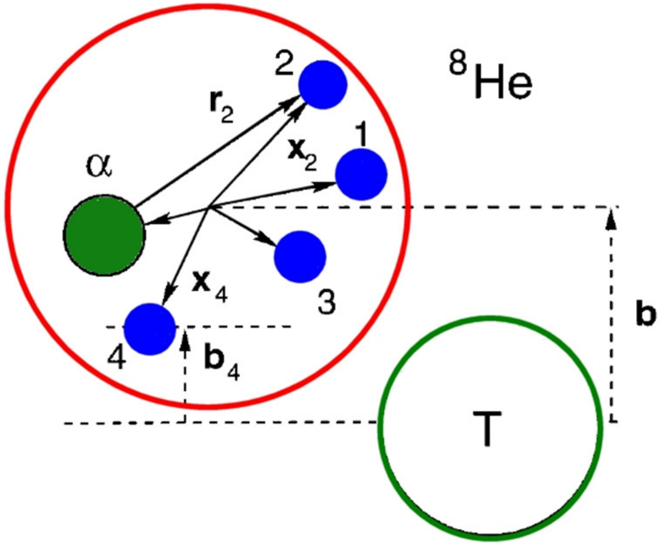


$$\left|S_C(\mathbf{b}_C)\right|^2\left(1-\left|S_n(\mathbf{b}_n)\right|^2\right)$$

C survives, n absorbed

$$\sigma_{strip}(\mathbf{b}) = \int d\mathbf{b} \left\langle \phi_0 \left| \, |S_C|^2 \left(1-|S_n|^2\right) \right| \phi_0 \right\rangle^2$$

(d) Composite particles:



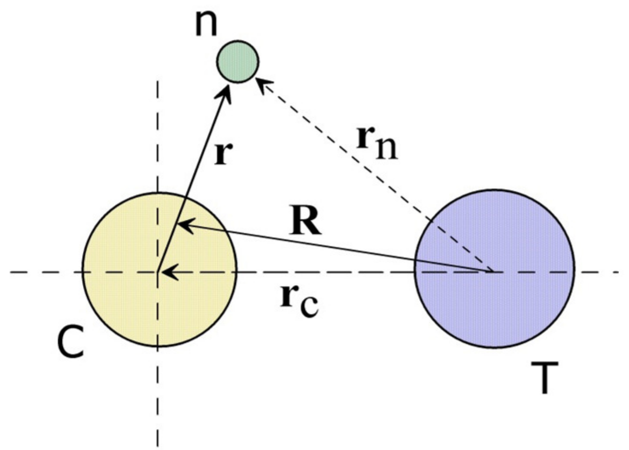
$$S_{dif.dis.}(\mathbf{b}) = \left\langle \phi_8 \left| S_\alpha(\mathbf{b}_\alpha) \prod_{i=1}^{4} S_i(\mathbf{b}_i) \right| \phi_8 \right\rangle$$

$$\prod_{j\ survive} \left|S_j(\mathbf{b}_j)\right|^2 \prod_{k\ absorbed} \left(1-\left|S_k(\mathbf{b}_k)\right|^2\right)$$

Momentum Distributions: (a) Stripping

C scatters elastically and C+n breaks up:

$$\left|\left\langle \phi_{Continuum}(\mathbf{r}) \middle| S_C(\mathbf{b}_C)\phi_{l_0,m_0}(\mathbf{r})\right\rangle\right|^2$$

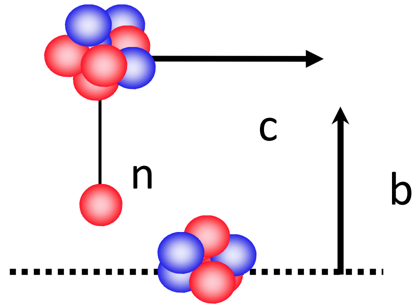


$$\mathbf{r}_n = \mathbf{R} + \frac{m_C}{m}\mathbf{r}, \qquad \mathbf{r}_C = \mathbf{R} - \frac{m_n}{m}\mathbf{r}$$

$$\mathbf{K} = \mathbf{k}_C - \mathbf{k}_n, \qquad \mathbf{k} = \frac{m_n}{m}\mathbf{k}_C - \frac{m_C}{m}\mathbf{k}_n$$

n is absorbed:

$$1 - \left|S_n(\mathbf{b}_n)\right|^2$$



$$\phi_{Continuum}(\mathbf{r}) \sim e^{i\mathbf{k}.\mathbf{r}}$$

$$\frac{d\sigma_{strip}}{d^3k_C} = \frac{1}{(2\pi)^3}\frac{1}{(2l_0+1)}\sum_{m_0}\int d^2b_n \left[1 - \left|S_n(\mathbf{b}_n)\right|^2\right]\left|\int d^3r\, e^{-i\mathbf{k}_C\cdot\mathbf{r}}\; S_C(\mathbf{b}_C)\,\phi_{l_0,m_0}(\mathbf{r})\right|^2$$

Momentum Distributions

Serber model:
PR 72 (1947) 1008

$$S_C(\mathbf{b}_C) \approx 1 \quad \Rightarrow \quad \frac{d\sigma_{strip}}{d^3k_C} = C_{geometry} \left| \tilde{\phi}_{l_0 m_0}(\mathbf{k}_c) \right|^2$$

(b) Diffraction dissociation

C and n scatters elastically:

$$S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, \phi_{l_0,m_0}(\mathbf{r})$$

Project onto continuum CM and relative coordinates:

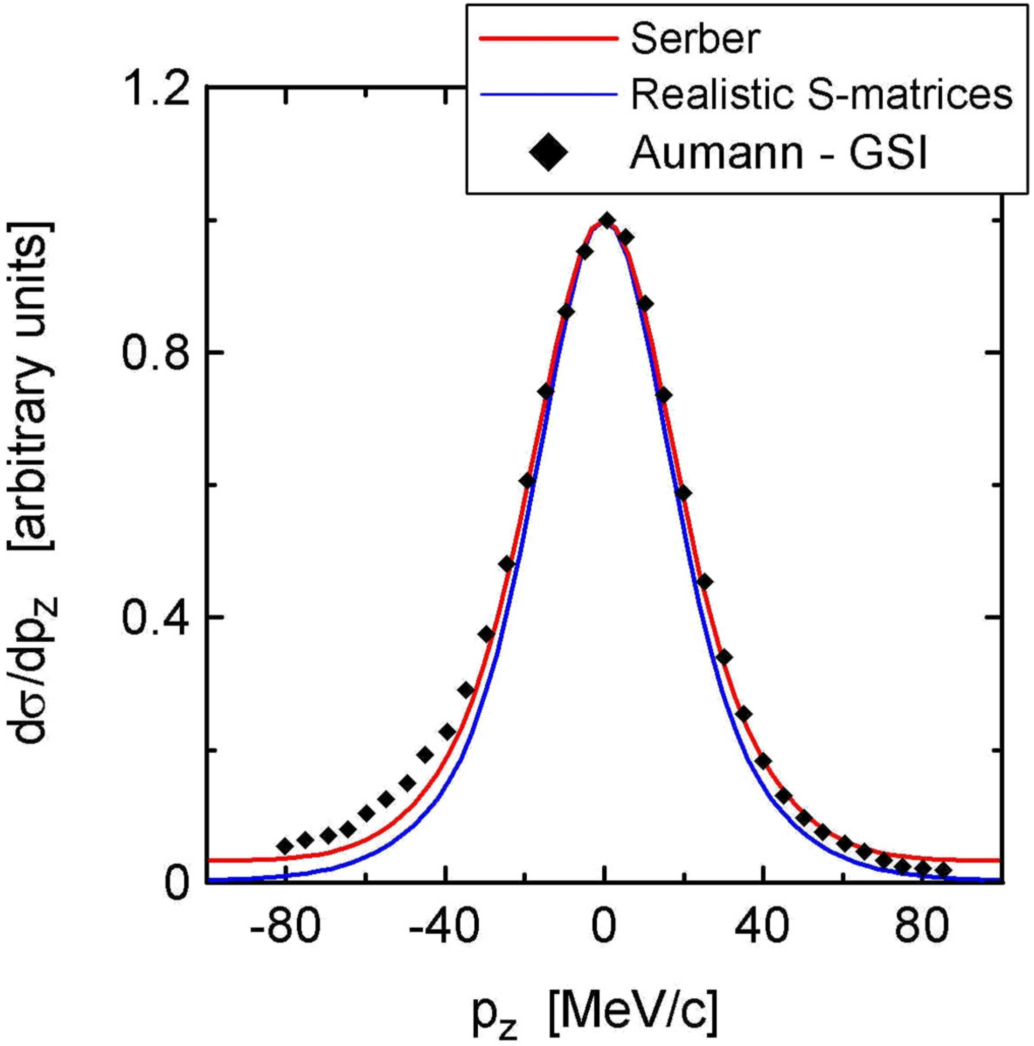
$$\left\langle e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \phi_{\mathbf{k}}^*(\mathbf{r}) \middle| S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, \phi_{l_0,m_0}(\mathbf{r}) \right\rangle$$

CM; Relative motion

$$\frac{d\sigma_{dif.dis.}}{d^2K_\perp\, d^3k} = \frac{1}{(2\pi)^5} \frac{1}{(2l_0+1)} \sum_{m_0} \left| \int d^3r\, d^2b\, e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \phi_{\mathbf{k}}^*(\mathbf{r})\, S_C(\mathbf{b}_C)\, S_n(\mathbf{b}_n)\, \phi_{l_0,m_0}(\mathbf{r}) \right|^2$$

Applications: (a) Longitudinal Momentum Dist. (LMD)

$$\frac{d\sigma_{strip}}{dk_C^z} = \frac{1}{2\pi}\frac{1}{(2l_0+1)}\sum_{m_0}\int d^2b_n\left[1-\left|S_n(\mathbf{b}_n)\right|^2\right]\int d^2b_C\left|S_C(\mathbf{b}_C)\right|^2\left|\int dz\, e^{-ik_C^z\cdot z}\ \varphi_{l_0,m_0}(\mathbf{r})\right|^2$$



$^{9}\mathrm{Be}\left(^{11}\mathrm{Be},\,^{10}\mathrm{Be}+\gamma\right)\mathrm{X}$

One neutron-removal

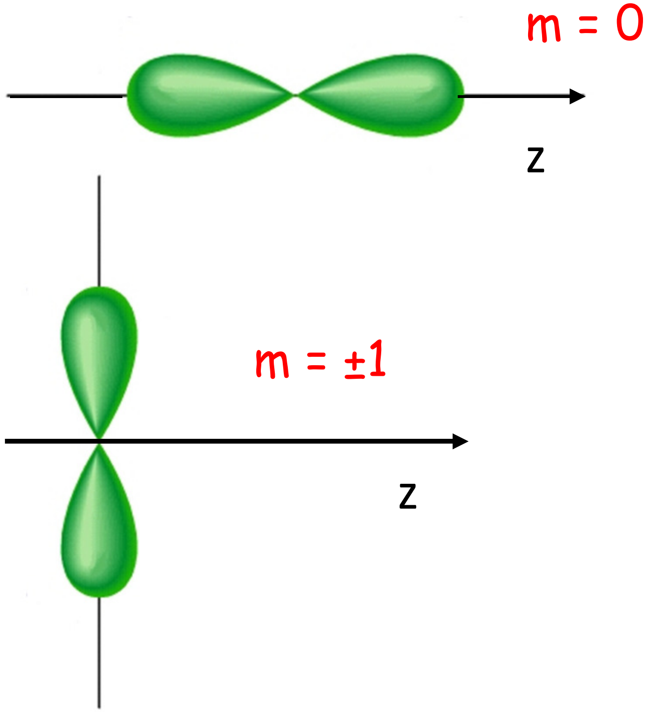
60 MeV/nucleon

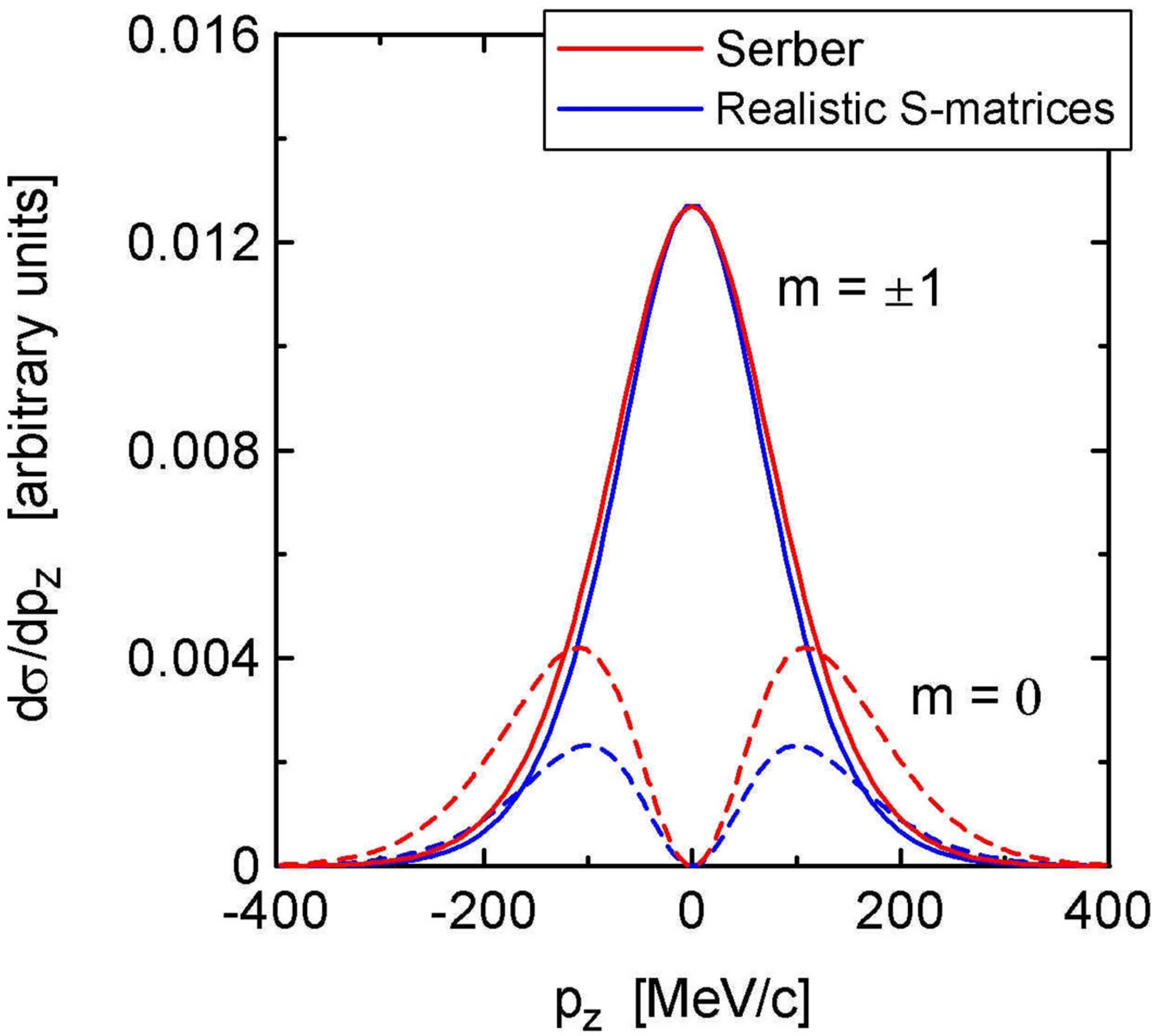
$1s\frac{1}{2}$ neutron, S_n = 0.503 MeV

Tails & asymmetry:
higher order corrections

Tostevin et al, PRC 66, 024607 (2002)

<u>LMD:</u> Orbital Alignment





$^{9}Be(^{7}Li, ^{6}He)X$

One proton-removal

80 MeV/nucleon

0p3/2 proton, S_p = 9.98 MeV

LMD: Black-Disk Model

"Wounded" wavefunction

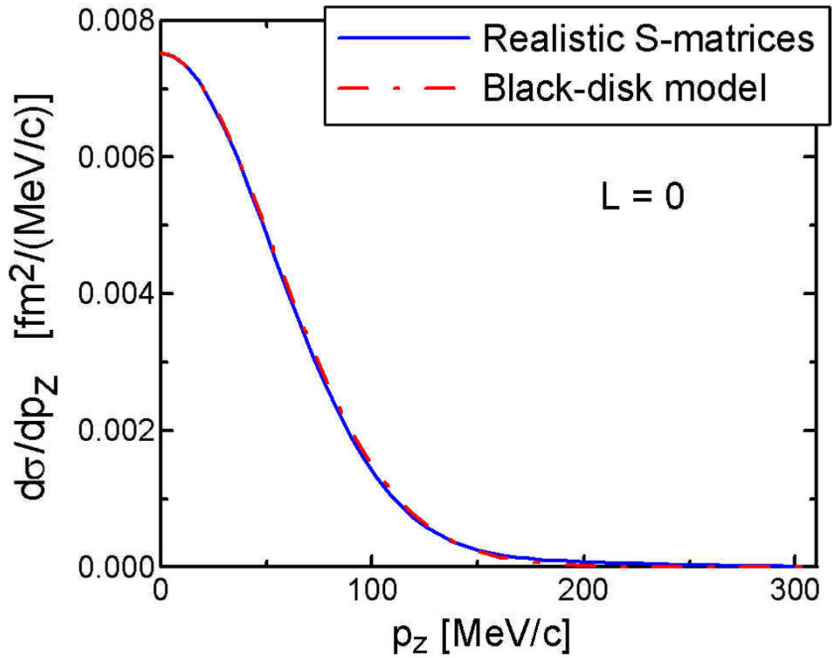
Hansen, PRL 77, 1016 (1996)

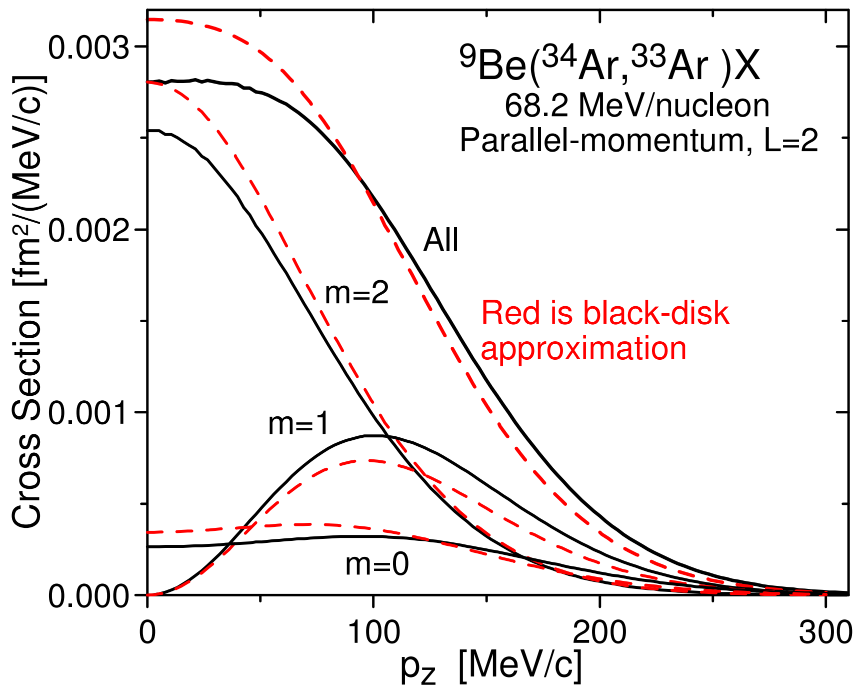
$^{9}Be(^{34}Ar, ^{33}Ar)X$

One neutron-removal

68.2 MeV/nucleon, 0d5/2 ($1s\frac{1}{2}$) neutron,

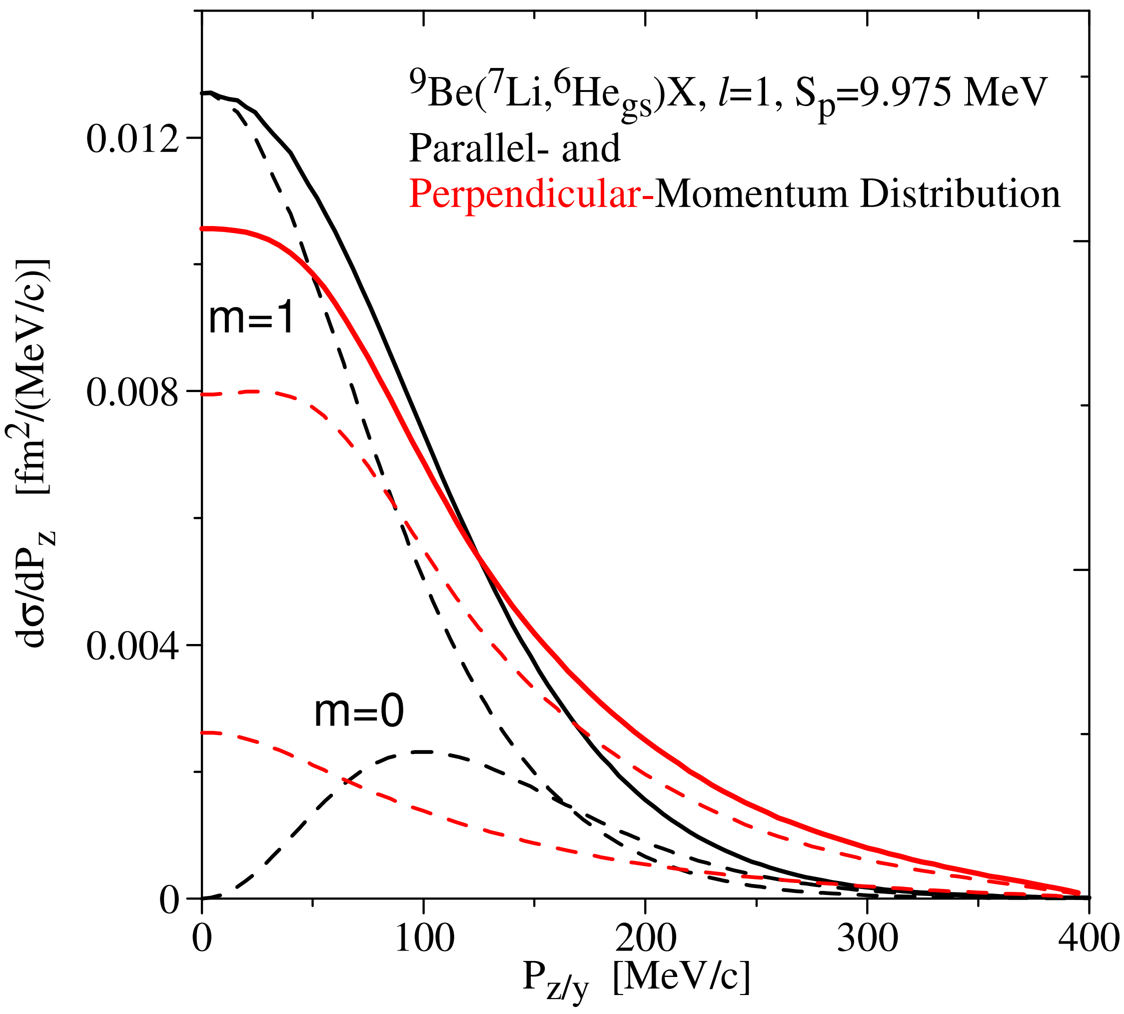
$S_n^{(L=2)}$ = 18.43 MeV, $S_n^{(L=0)}$ = 17.07 MeV



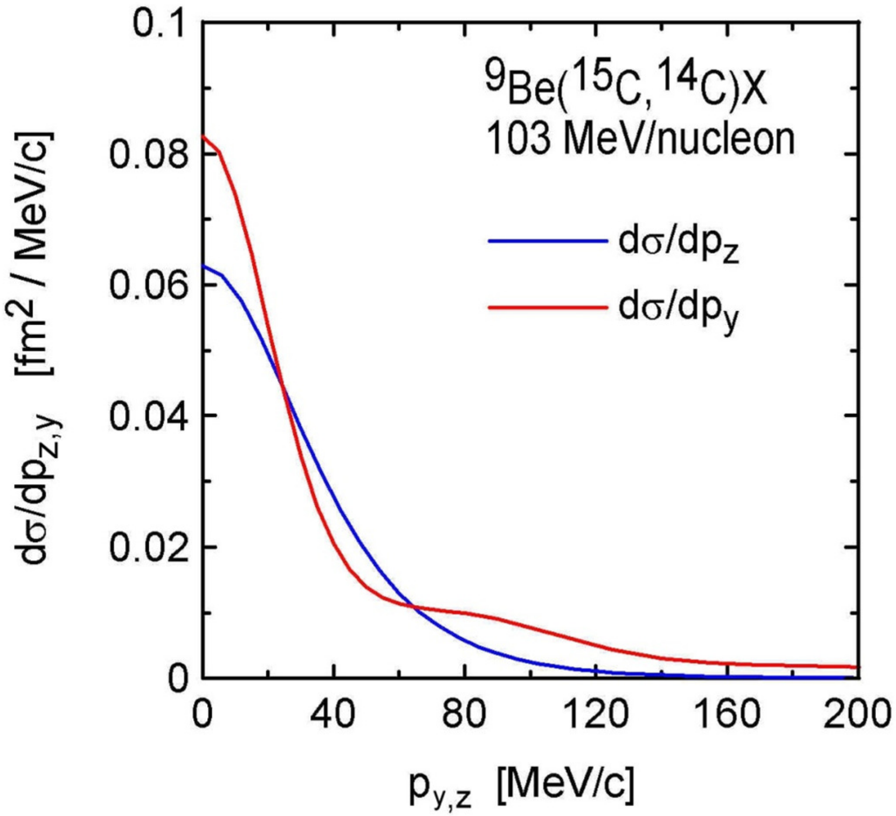


(b) Transverse Momentum Distribution (TMD)

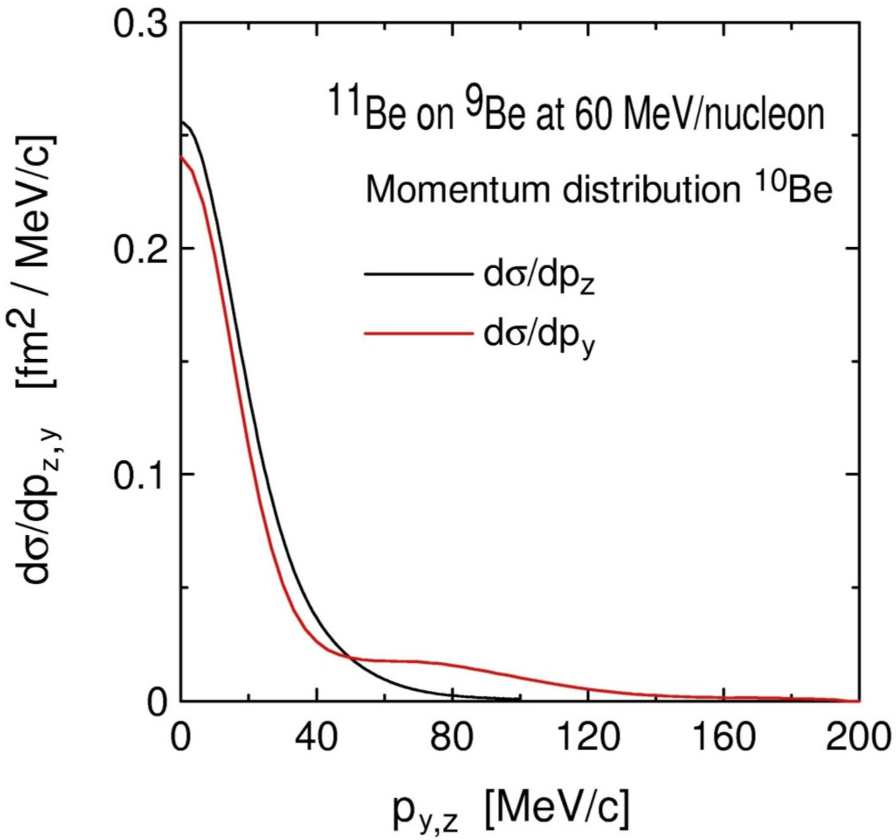
$$\frac{d\sigma_{strip}}{d^2 k_C^{\perp}} = \frac{1}{2\pi} \frac{1}{(2l_0+1)} \sum_{m_0} \int d^2 b_n \left[1 - \left| S_n(\mathbf{b}_n) \right|^2\right] \int dz \left| \int d^2 b_C \, e^{-i\mathbf{k}_C^{\perp}\cdot\mathbf{r}} \, S_C(\mathbf{b}_C) \, \varphi_{l_0,m_0}(\mathbf{r}) \right|^2$$



TMD x LMD:

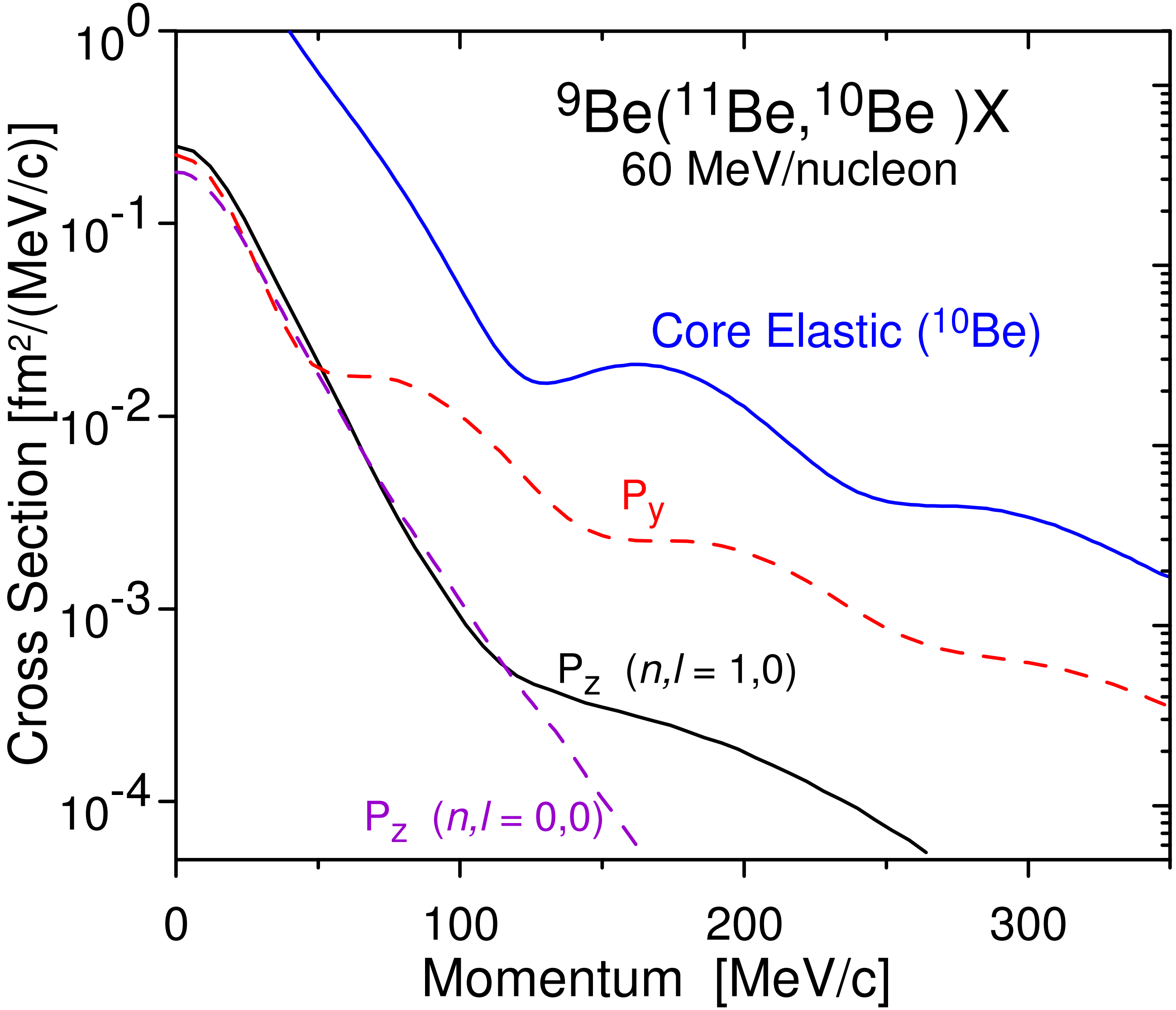


$^9Be(^{15}C,^{14}C)X$
103 MeV/nucleon
$d\sigma/dp_z$
$d\sigma/dp_y$
$d\sigma/dp_{z,y}$ [fm^2 / MeV/c]
$p_{y,z}$ [MeV/c]
0.1
0.08
0.06
0.04
0.02
0
0
40
80
120
160
200



^{11}Be on ^9Be at 60 MeV/nucleon
Momentum distribution ^{10}Be
$d\sigma/dp_z$
$d\sigma/dp_y$
$d\sigma/dp_{z,y}$ [fm^2 / MeV/c]
$p_{y,z}$ [MeV/c]
0.3
0.2
0.1
0
0
40
80
120
160
200

TMD: final state elastic core-target scattering

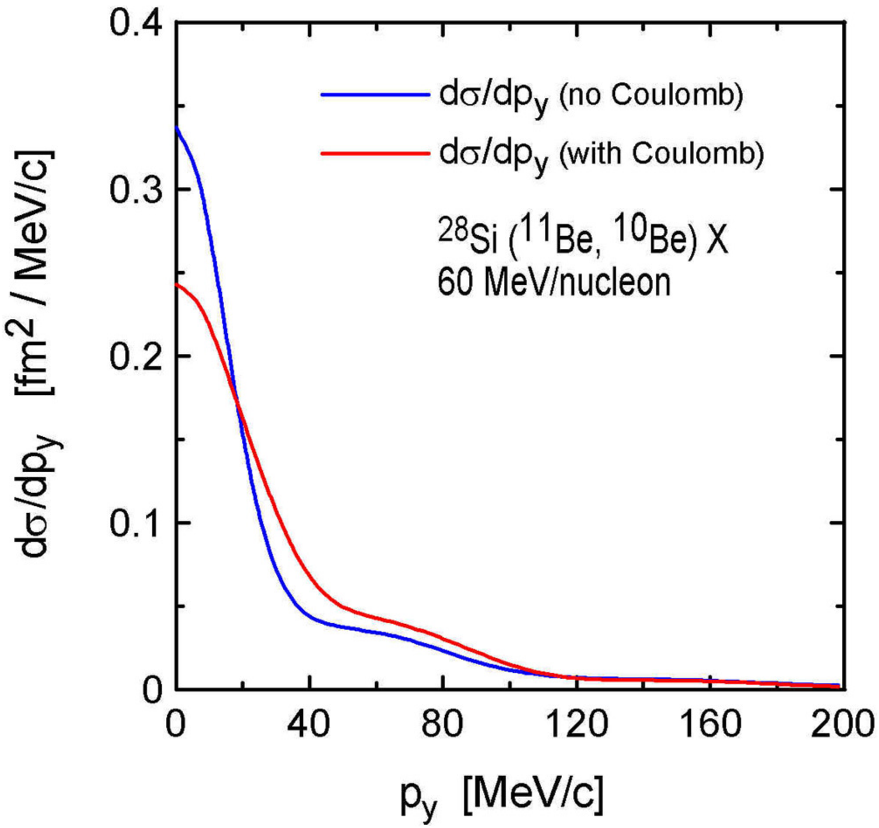


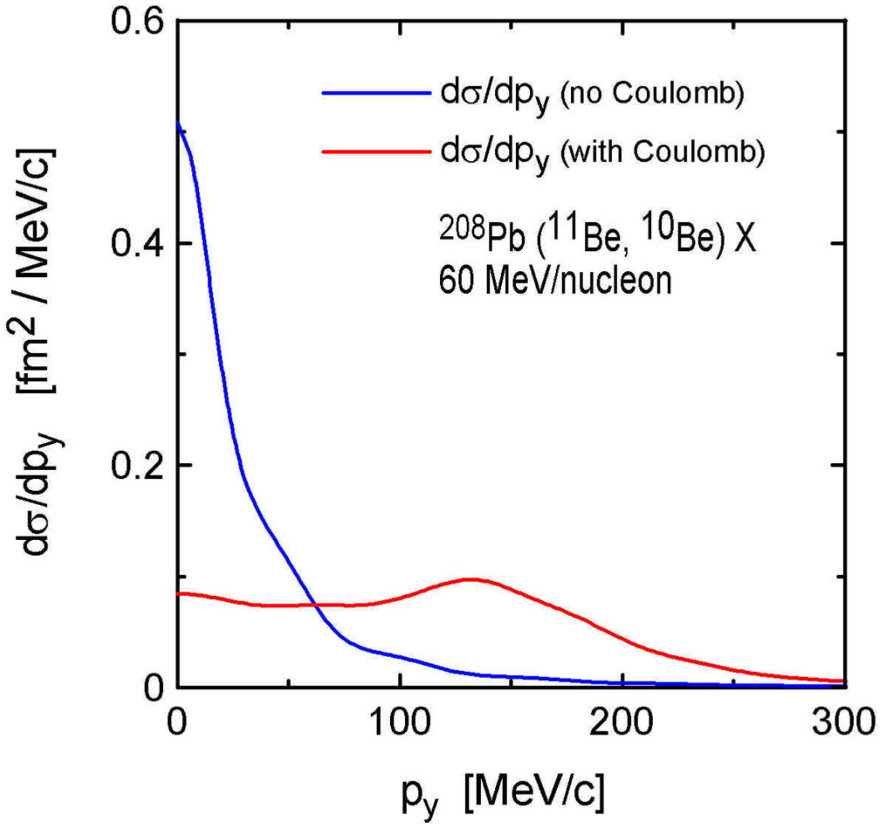
Bertulani, Hansen, PRC 70, 034609 (2004)

Coulomb and medium effects in knockout reactions

TMD: final state core-target Coulomb scattering

Karakoc, Bertulani, to be published





Medium effects in σ_{NN}

$$\langle \mathbf{k}|G|\mathbf{k}_0\rangle = \langle \mathbf{k}|V_{NN}|\mathbf{k}_0\rangle - \int \frac{d^3k'}{(2\pi)^3} \frac{\langle \mathbf{k}|V_{NN}|\mathbf{k'}\rangle Q(\mathbf{k'}) \langle \mathbf{k'}|G|\mathbf{k}_0\rangle}{E(\mathbf{k'}) - E_0 - i\varepsilon}$$

$$E(\mathbf{P},\mathbf{k}) = e(\mathbf{P}+\mathbf{k}) + e(\mathbf{P}-\mathbf{k})$$

e = single-particle energies

E_0 = E on-shell

$$Q(\mathbf{P},\mathbf{k}) = 1, \quad \text{if} \quad k_{1,2} > k_F$$
$$= 0, \quad \text{otherwise}$$

$$\mathbf{k}_{1,2} = \mathbf{P} \pm \mathbf{k}$$

In real calculations:

$$\overline{Q}(P,k) = \frac{\int d\Omega\, Q(\mathbf{P},\mathbf{k})}{\int d\Omega}$$

$$e(p) = T(p) + v(p)$$

$$v(p) = \langle p|v|p\rangle = \mathrm{Re} \sum_{q \le k_F} \langle pq|G|pq - qp\rangle$$

- e depends on v
- v depends on G
- G depends on v

$\Rightarrow$ Solve self-consistently (Brueckner theory)

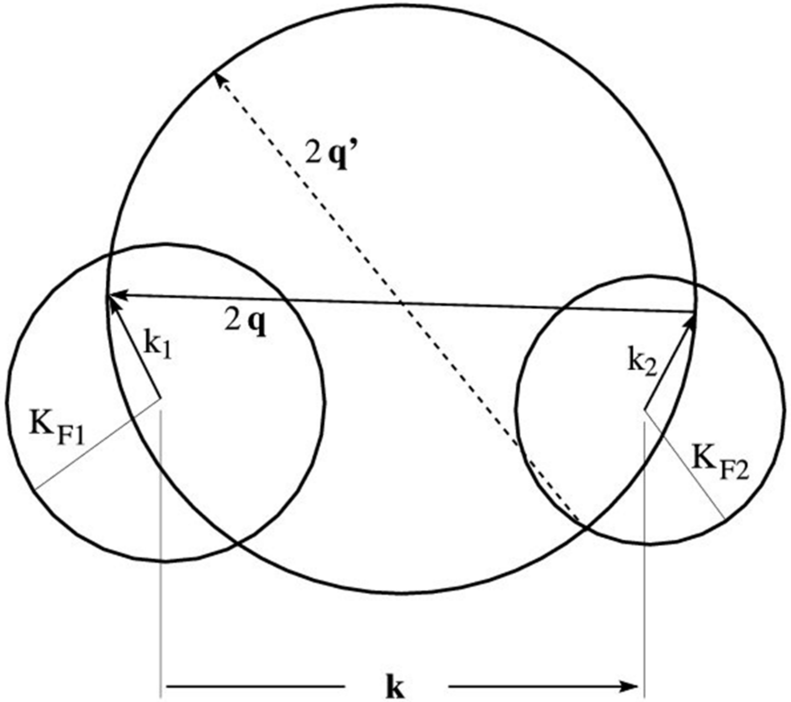
Geometric approximation + LDA

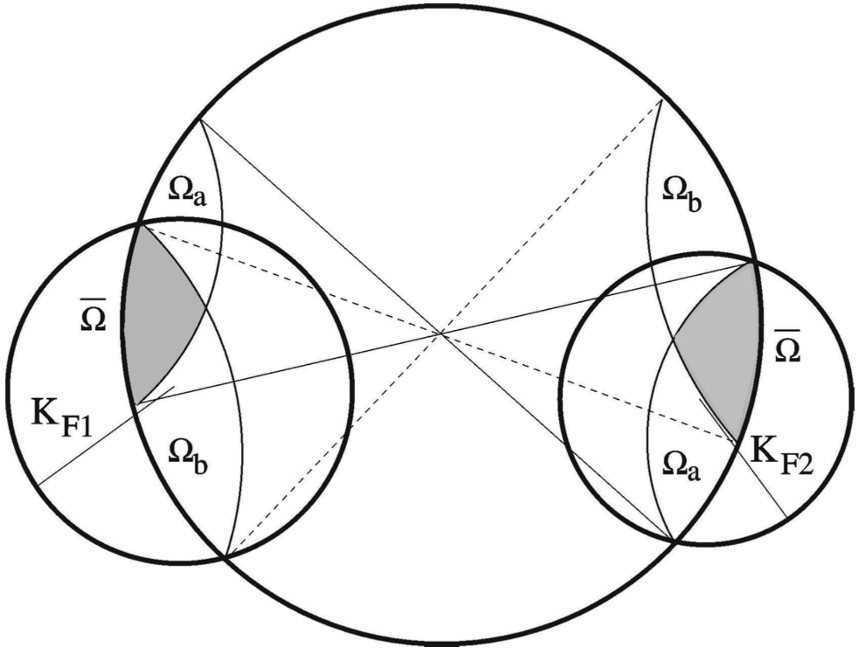
Bertulani, Phys. Rep. (1991), JPG 27, L67 (2001)

Bertulani, De Conti, PRC 81, 064603 (2010)

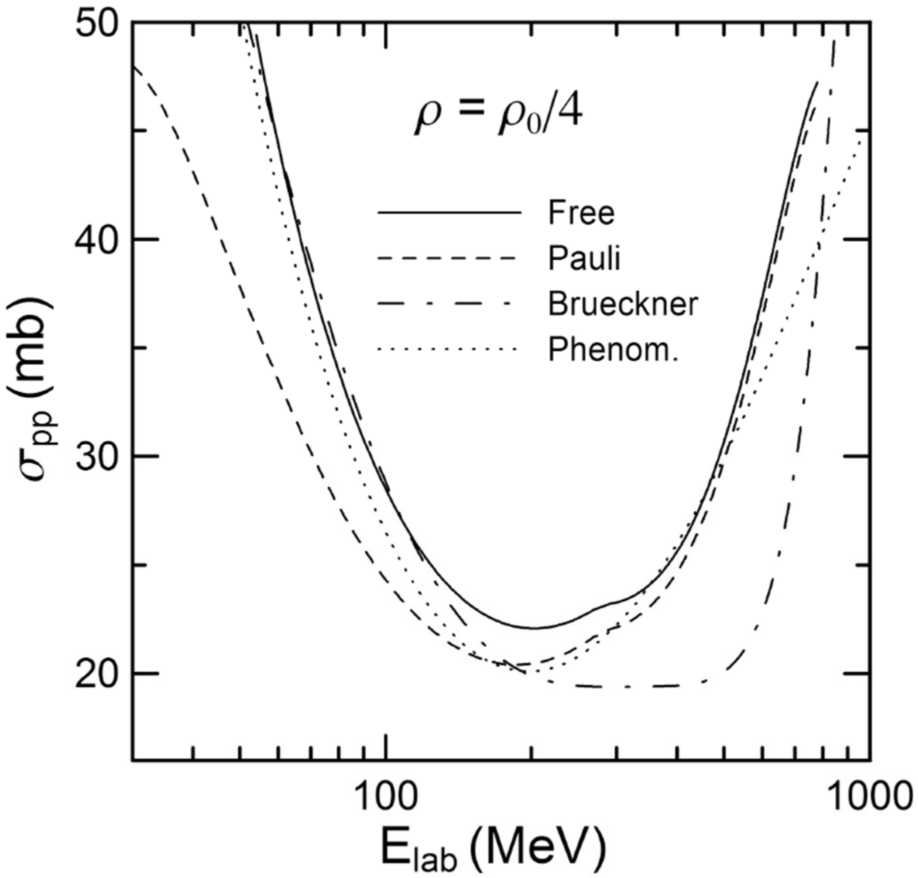
$$\overline{\sigma}_{NN}(E) = \int \frac{d^3k_1 d^3k_2}{\left(4\pi k_{1F}^3/3\right)\left(4\pi k_{2F}^3/3\right)} \frac{2q}{k} \sigma_{NN}(q) \frac{\Omega_{Pauli}}{4\pi}$$

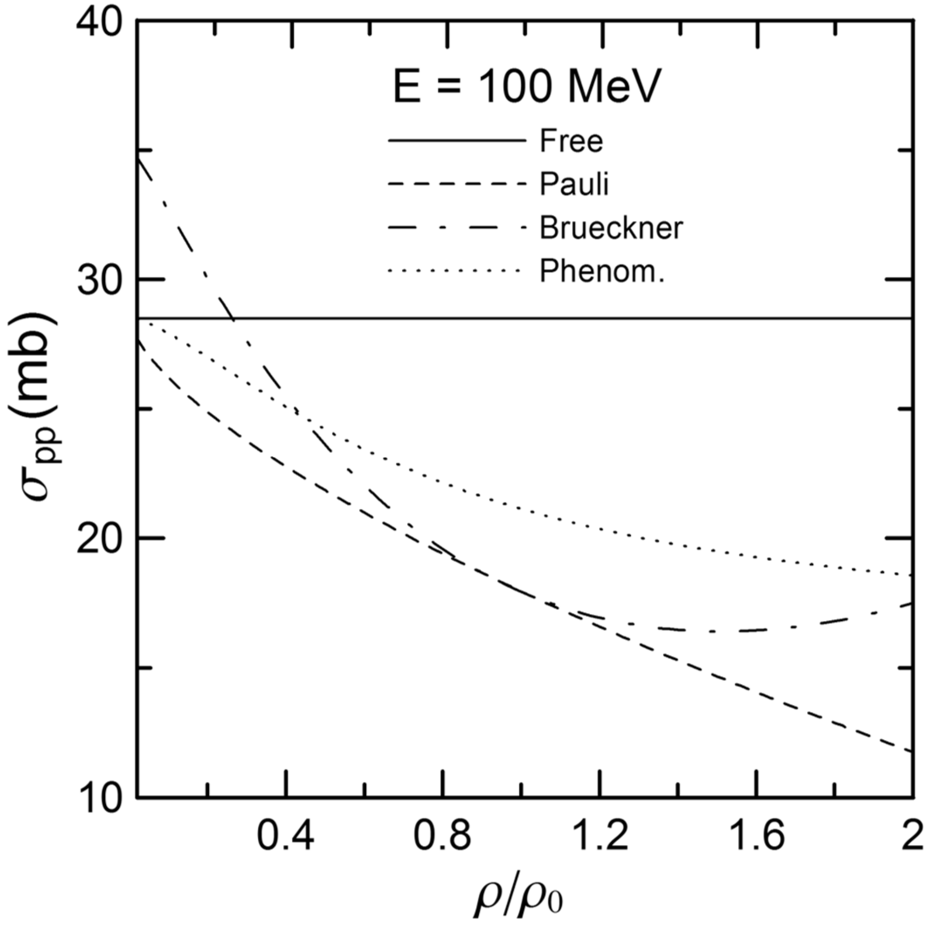
$$\Omega_{Pauli} = 4\pi - 2\left(\Omega_a + \Omega_b - \overline{\Omega}\right)$$ = analytic





Medium effects in σ_{NN}



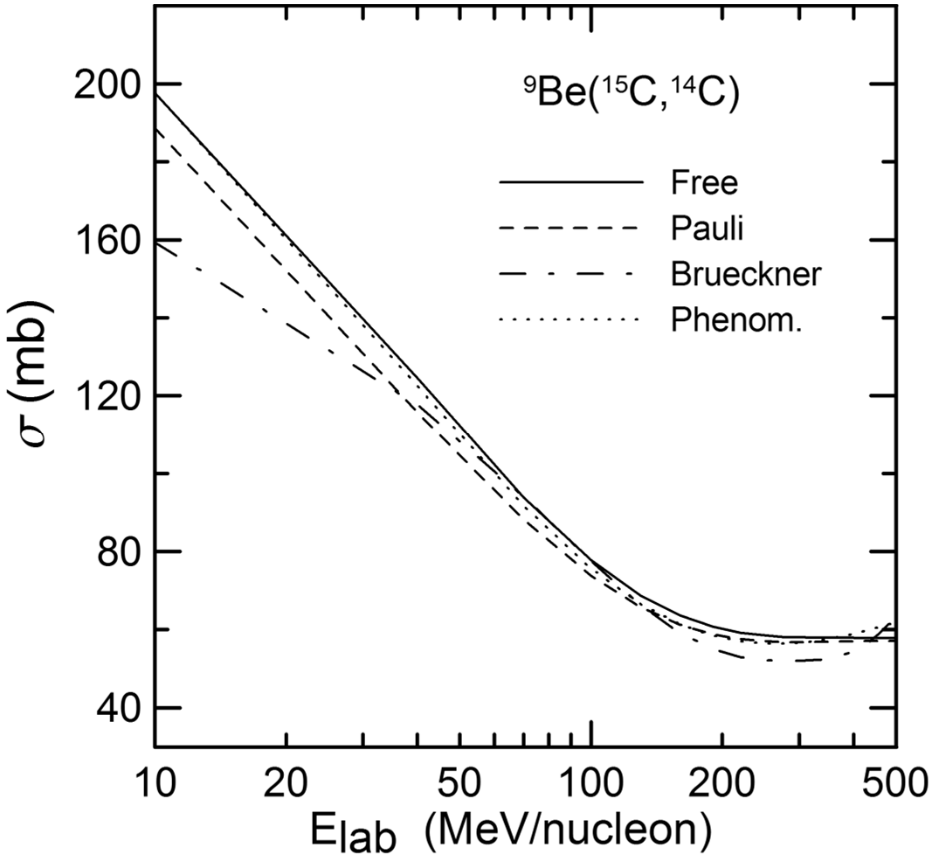


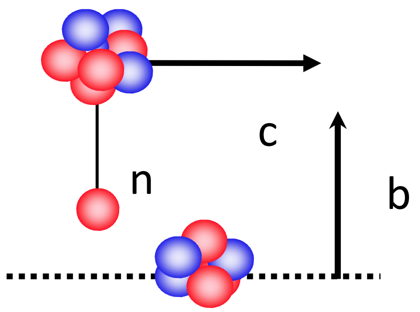
Bertulani, De Conti,
PRC 81, 064603 (2010)

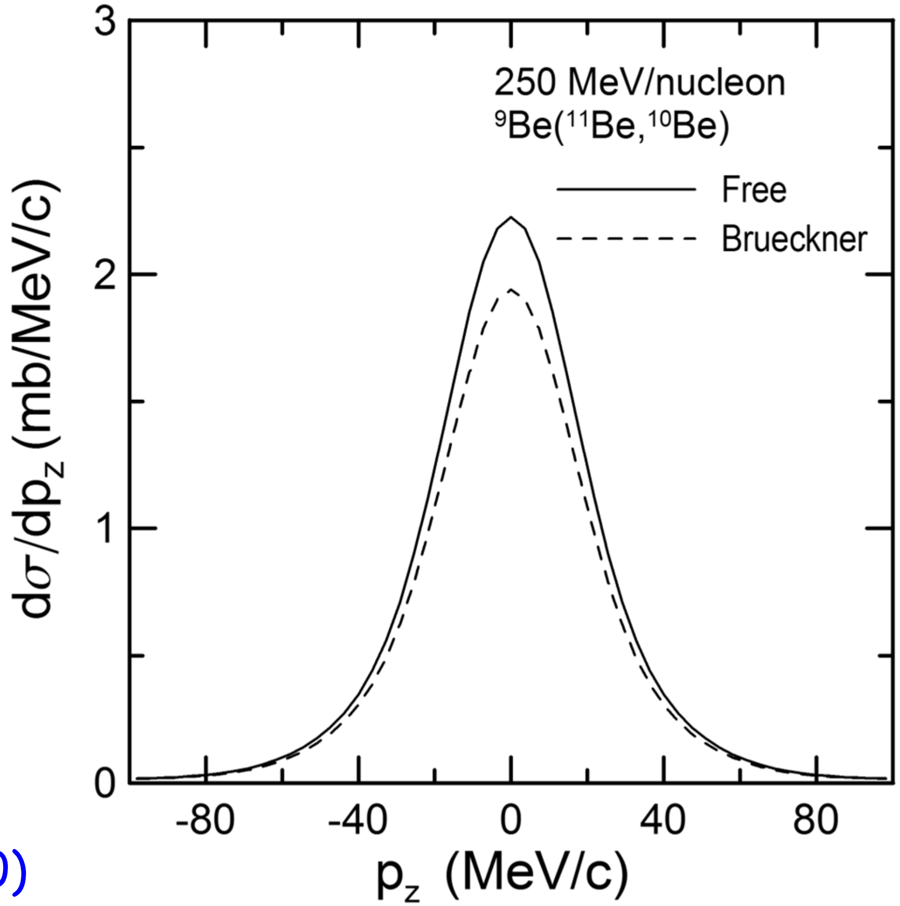
Medium effects in knockout reactions

and

momentum distributions

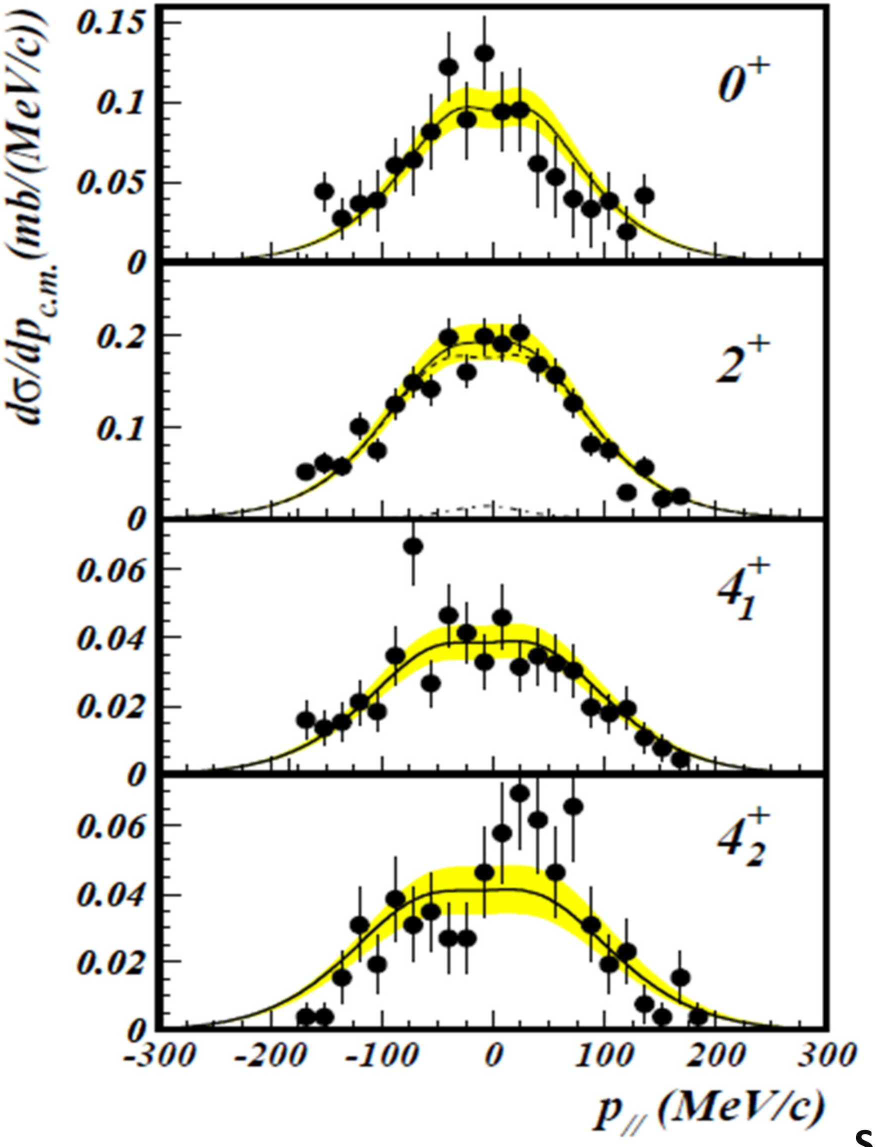


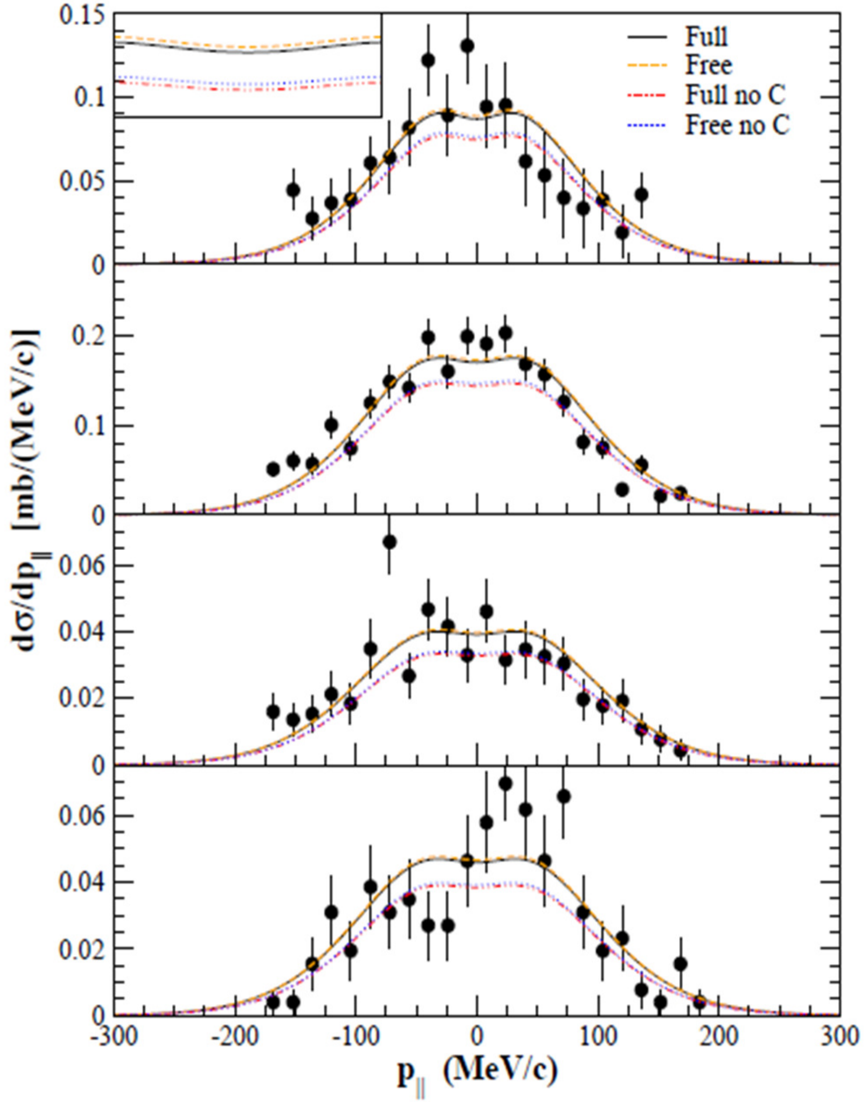




Bertulani, De Conti, PRC 81, 064603 (2010)

$^{12}C(^{23}Al,^{22}Mg)X$ @ 50 MeV/u



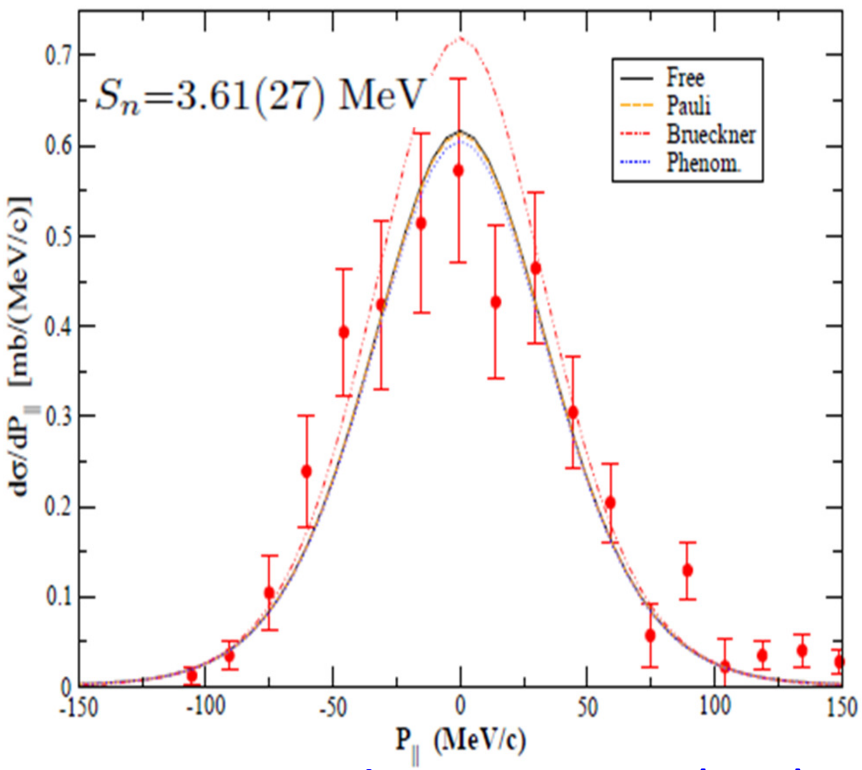


S_n=0.141 MeV

Banu et al, PRC 84, 015803 (2011)

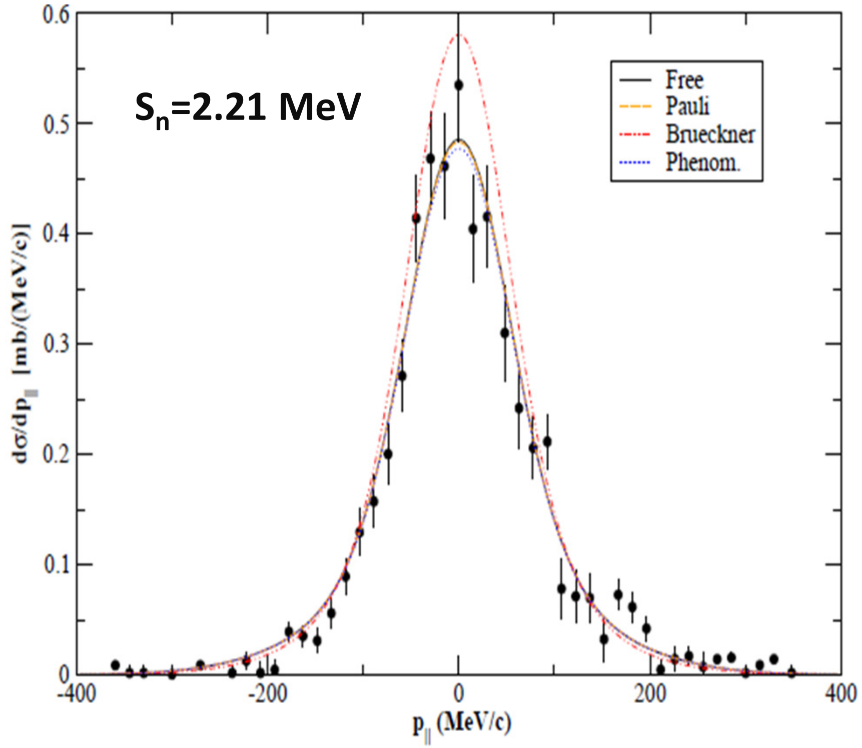
Karakoc, Bertulani, to be published

$^{12}C(^{24}O,^{23}O)X$ @ 920 MeV/u



Data: Kanungo et al, PRL 102, 152501 (2009)

$^{12}C(^{33}Mg,^{32}Mg)X$ @ 898 MeV/u



Data: Kanungo et al, PLB 685, 253 (2010)

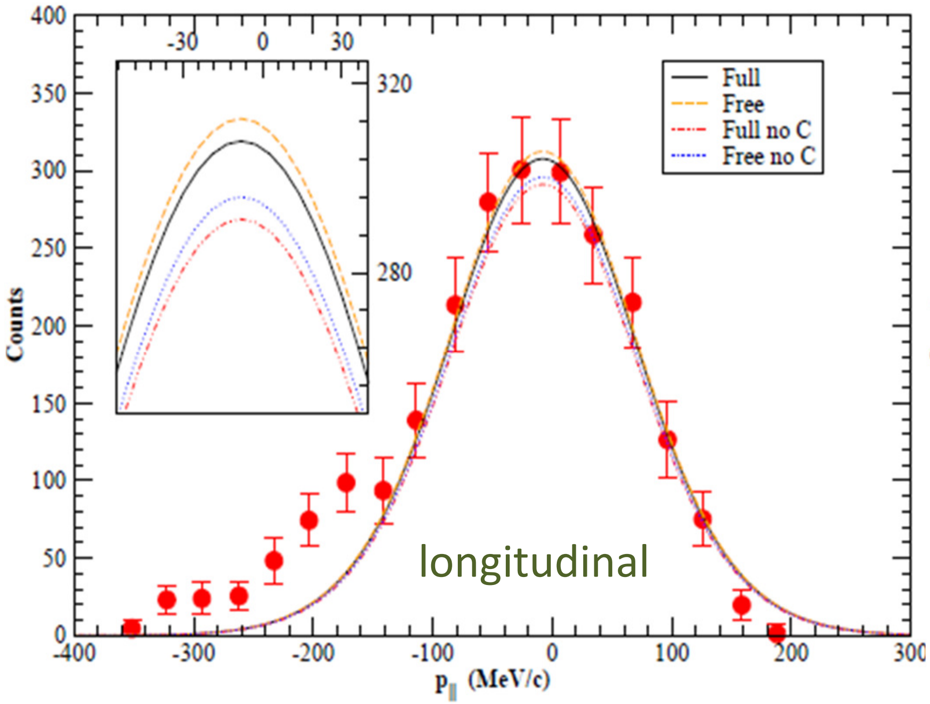
Brueckner calculations are limited by the pion production threshold, and should only be valid for projectile nucleon energies below 300 MeV.

$$^{23}\mathrm{O}_{gs}(1/2^+) + \mathrm{n\ in\ } 2s_{1/2}$$

$$C^2S_i = \alpha_i^2 \text{ is the spectroscopic factor}$$

$$|^{33}\mathrm{Mg}_{gs}(3/2^+)\rangle = \alpha_1|^{32}\mathrm{Mg}(3^+)\otimes \nu 2p_{3/2}\rangle$$
$$+\ \alpha_2|^{32}\mathrm{Mg}(2_2^+)\otimes \nu 2s_{1/2}\rangle + \alpha_3|^{32}\mathrm{Mg}(3^+)\otimes \nu 1f_{7/2}\rangle$$
$$+\ \alpha_4|^{32}\mathrm{Mg}(gs)\otimes \nu 1d_{3/2}\rangle$$

$^{9}Be(^{15}O,^{14}N)X$ @ 56 MeV/u

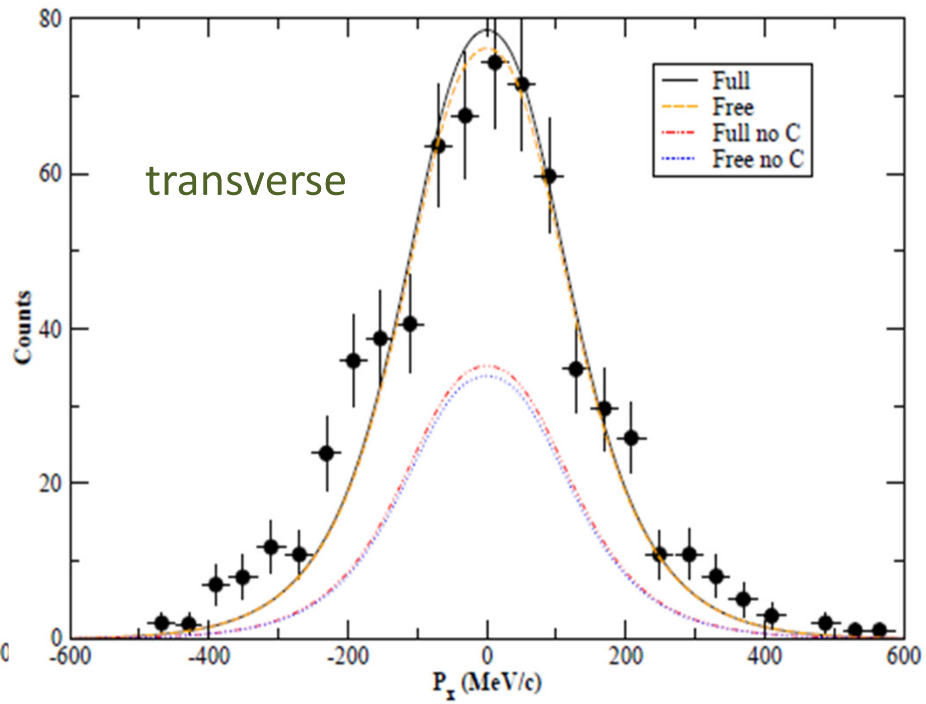


Data: Jeppesen et al, NPA 739, 57 (2004)

Medium effects and Coulomb distortion do not have an impact on the extraction of spectroscopic factors.

^{15}O, S_p=7.30 MeV
^{17}C, S_p=23.0 MeV

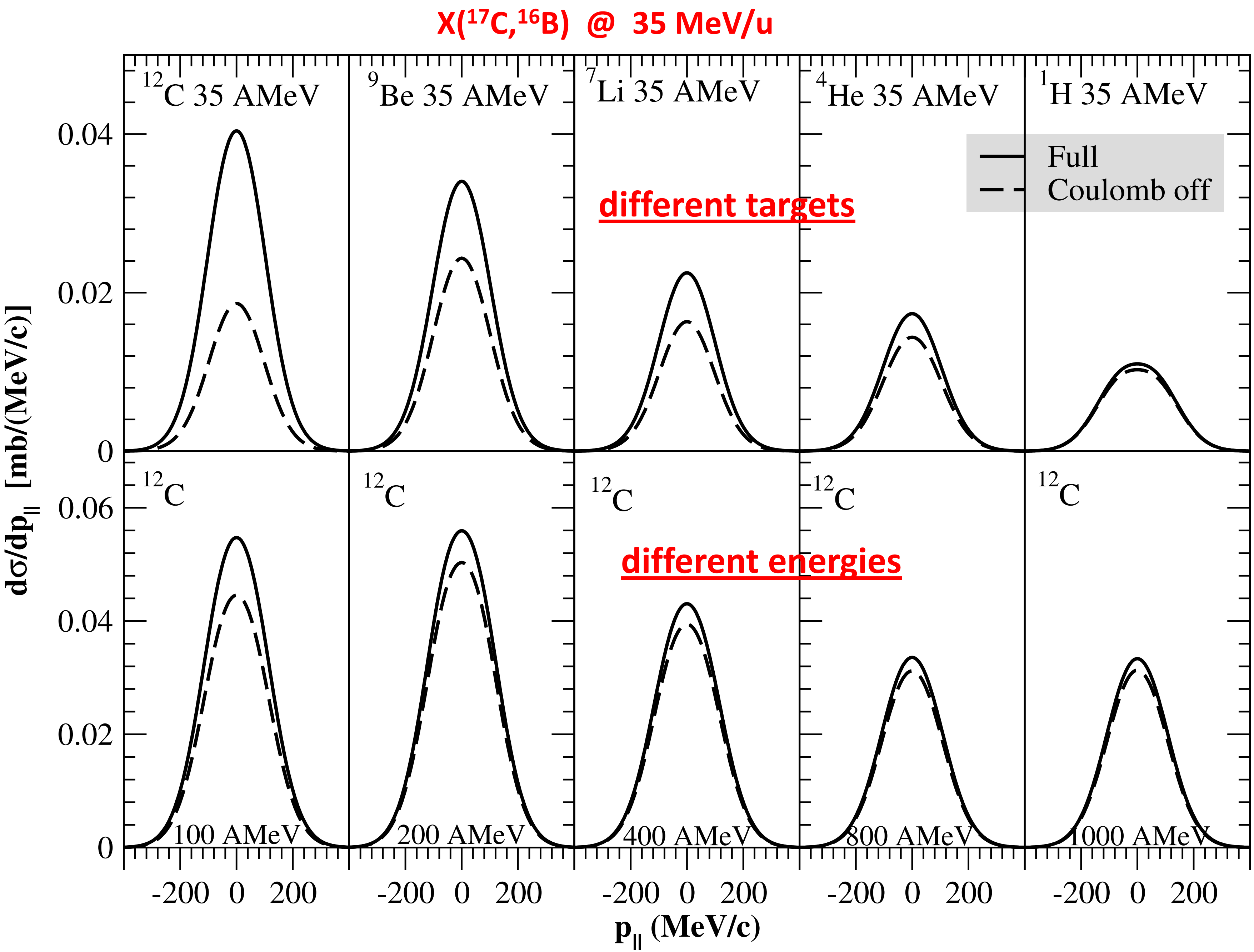
$^{12}C(^{17}C,^{16}B)X$ @ 35 MeV/u



Data: Lecouey et al, PLB 672, 6 (2009)

Medium corrections change the x-sections by about 3% but, the Coulomb corrections has a huge effect which is almost 54%.

$$
\begin{aligned}
|^{17}\mathrm{C}\rangle &= \alpha_1|^{16}\mathrm{B}(0^-)\otimes\pi 1p_{3/2}\rangle \\
&+ \alpha_2|^{16}\mathrm{B}(3_1^-)\otimes\nu 1p_{3/2}\rangle + \alpha_3|^{16}\mathrm{B}(2_1^-)\otimes\nu 1p_{3/2}\rangle \\
&+ \alpha_4|^{16}\mathrm{B}(2_2^-)\otimes\nu 1p_{3/2}\rangle + \alpha_5|^{16}\mathrm{B}(1_1^-)\otimes\nu 1p_{3/2}\rangle \\
&+ \alpha_6|^{16}\mathrm{B}(3_2^-)\otimes\nu 1p_{3/2}\rangle,
\end{aligned}
$$



X(^{17}C,^{16}B) @ 35 MeV/u
^{12}C 35 AMeV
^{9}Be 35 AMeV
^{7}Li 35 AMeV
^{4}He 35 AMeV
^{1}H 35 AMeV
Full
Coulomb off
different targets
different energies
^{12}C
100 AMeV
200 AMeV
400 AMeV
800 AMeV
1000 AMeV
0.04
0.02
0
0.06
-200
200
dσ/dp$_{||}$ [mb/(MeV/c)]
p$_{||}$ (MeV/c)