

# Eikonal reaction theory for neutron removal reaction

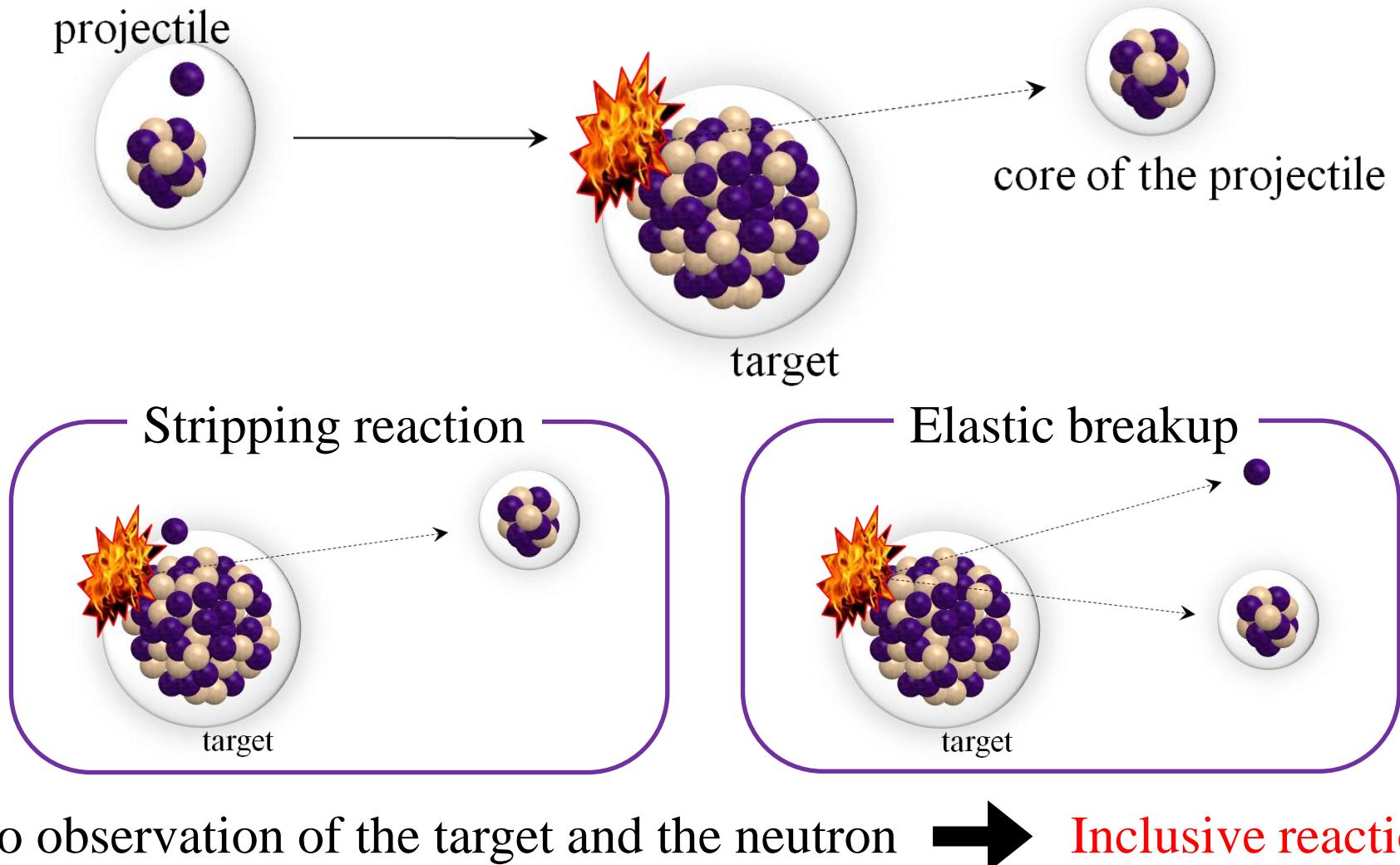
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# One neutron removal

This reaction is an important experimental tool to investigate weakly-bound nuclei.



# Outline

- ✓ A new theory to treat inclusive reactions  
Eikonal reaction theory (ERT)
- ✓ Which quantities should be extracted from neutron removal?  
Spectroscopic factor and asymptotic normalization coefficient  
with theoretical error

I. Introduction ~Studies on “Island of Inversion”

II. Eikonal reaction theory (ERT)

III. Analysis of  $\sigma_{-1n}$  for  ${}^{31}\text{Ne}$

IV. Extension of ERT

V. Summary

# Outline

I. Introduction ~Studies on “Island of Inversion”

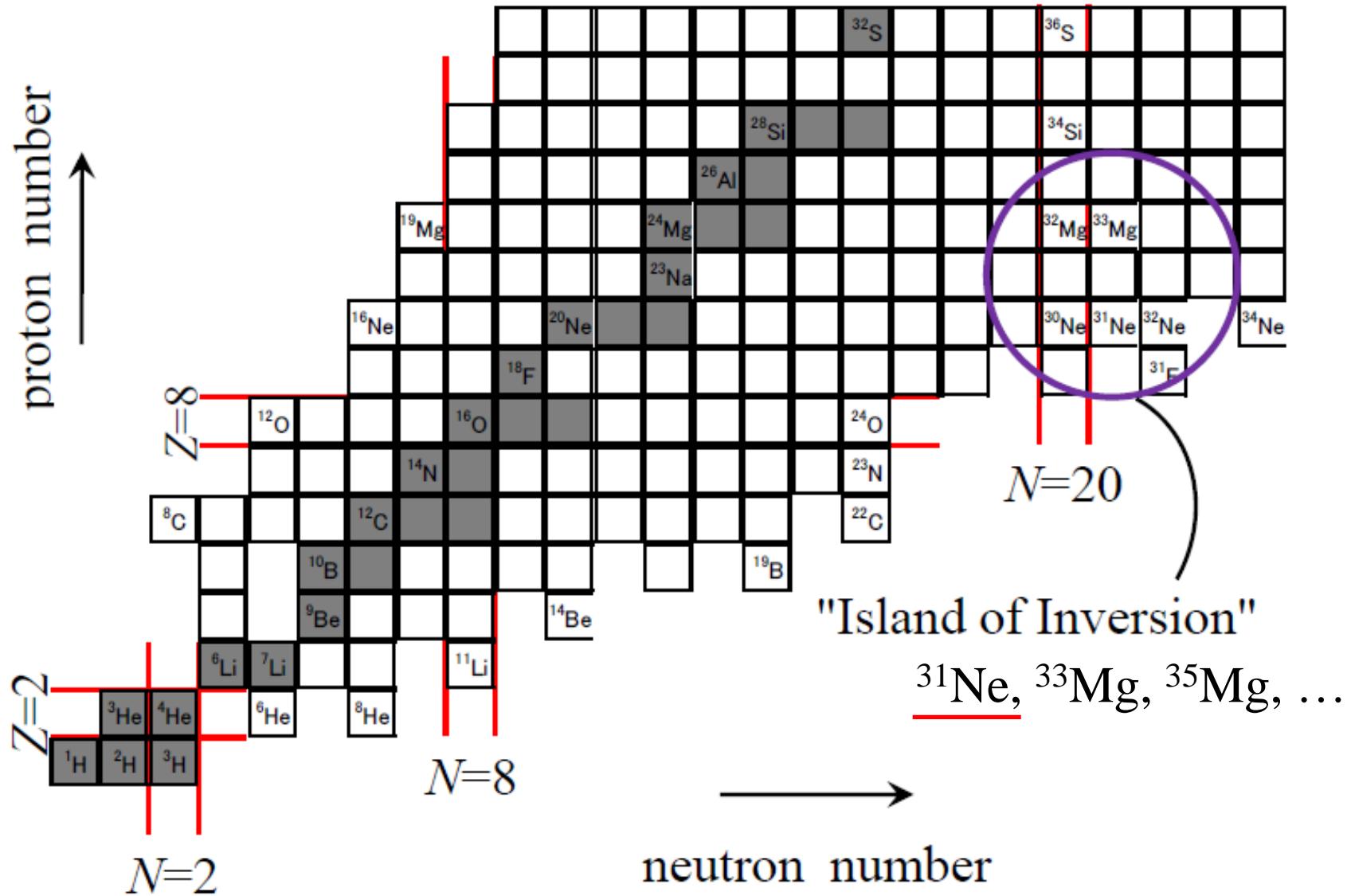
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# Nuclei near the neutron drip line



# Studies on “Island of Inversion”

- ✓ One-neutron removal for  $^{31}\text{Ne}$  scattering

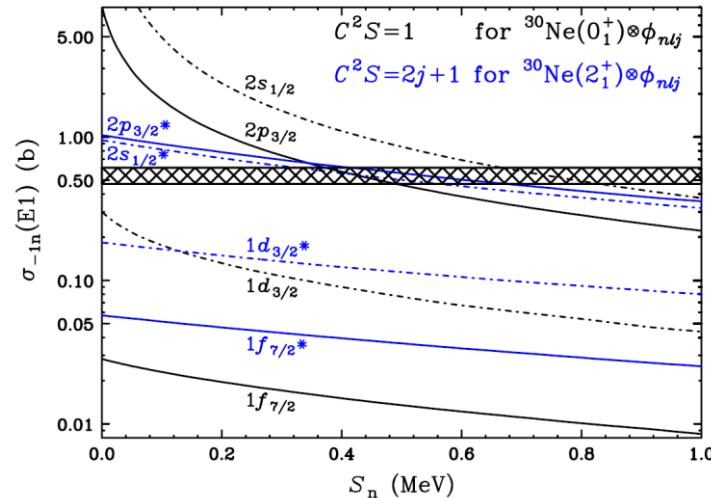
*T. Nakamura, et al., PRL103, 262501 (2009).*

$^{31}\text{Ne} + ^{12}\text{C}$ ,  $E_{\text{lab}} = 230$  (MeV/nucleon)

$^{31}\text{Ne} + ^{208}\text{Pb}$ ,  $E_{\text{lab}} = 234$  (MeV/nucleon)

Large breakup cross section

→ Is  $^{31}\text{Ne}$  halo with the  $p3/2$  configuration?

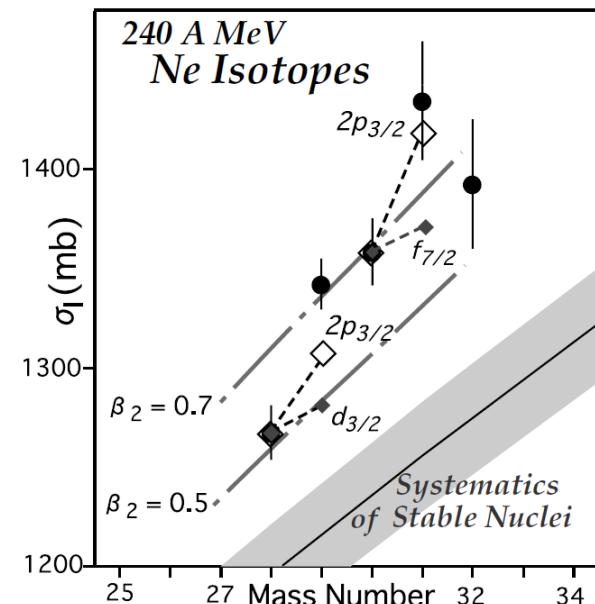


- ✓ Interaction cross sections for Ne isotopes

*M. Takechi, et al., NPA834, 412c (2010).*

$^A\text{Ne} + ^{12}\text{C}$ ,  $E_{\text{lab}} \sim 240$  (MeV/nucleon)

Large interaction cross section of  $^{31}\text{Ne}$



# Studies on “Island of Inversion”

Theoretical analysis of  $^{31}\text{Ne}$

Glauber model

*W. Horiuchi, et al., PRC81, 024606 (2010).*

Particle-rotor model

*Y. Urata, et al., PRC83, 041303(R) (2011).*

Suggested properties of  $^{31}\text{Ne}$

✓ Candidates of  $^{31}\text{Ne}$  spin-parity

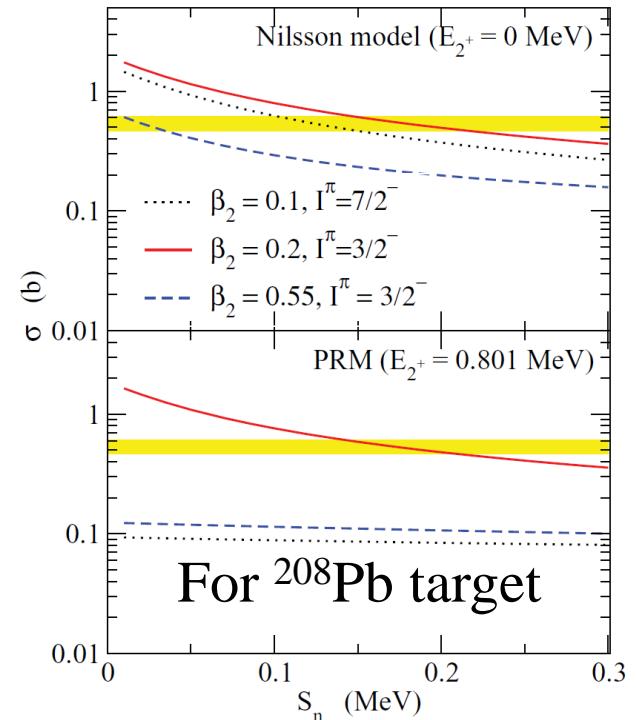
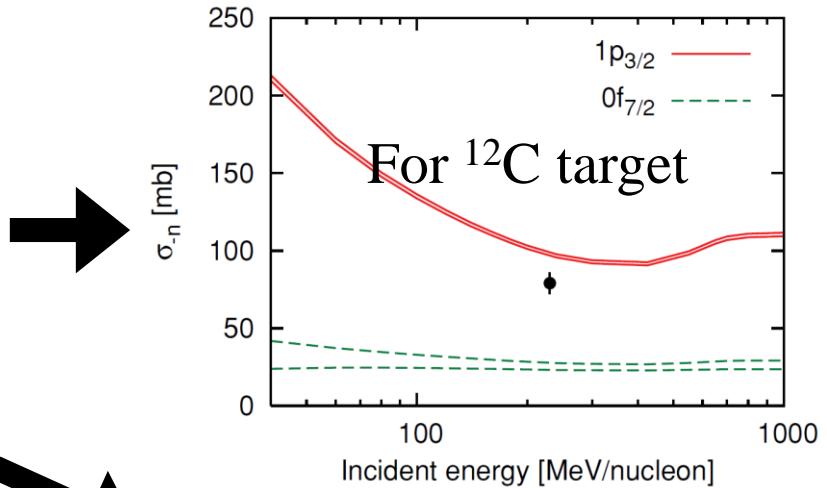
$$J^\pi = \textcircled{3/2}^-, 3/2^+, 7/2^-, 1/2^+$$

✓ Intruder configuration

Super deformation

✓ Halo structure

$$S_n^{(\text{exp})} = 0.29 \pm 1.64 \text{ (MeV)}$$



For  $^{208}\text{Pb}$  target

# Reaction theories

✓ Glauber model

○ Exclusive reaction

○ Inclusive reaction

Eikonal approximation + adiabatic approximation

Breakdown for Coulomb breakup!

✓ Continuum-Discretized Coupled Channels method (CDCC)

○ Exclusive reaction

✗ Inclusive reaction

Reliable calculation

We propose a new theory to treat the inclusive reactions accurately.

Eikonal reaction theory (ERT)

*M. Yahiro, K. Ogata, K. Minomo, PTP126, 167 (2011).*

*S. Hashimoto, K. Ogata, M. Yahiro, K. Minomo, S. Chiba, PRC83, 054617 (2011).*

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**II. Eikonal reaction theory (ERT)**

III. Analysis of  $\sigma_{-1n}$  for  $^{31}\text{Ne}$

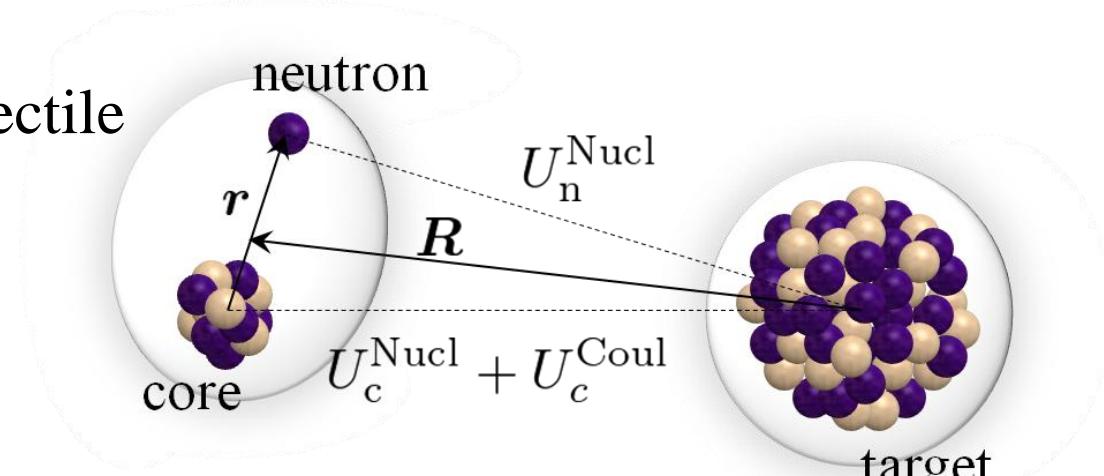
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# Three-body model

One-neutron halo projectile

$^{31}\text{Ne}$



Three-body Schrödinger equation:

$$\left[ -\frac{\hbar^2}{2\mu} \nabla_{\mathbf{R}}^2 + h_{\text{P}} + U(r_{\text{c}}, r_{\text{n}}) - E \right] \Psi = 0$$

Internal Hamiltonian:  $h_{\text{P}} = -\frac{\hbar^2}{2\mu_{\text{P}}} \nabla_{\mathbf{r}}^2 + V(\mathbf{r})$

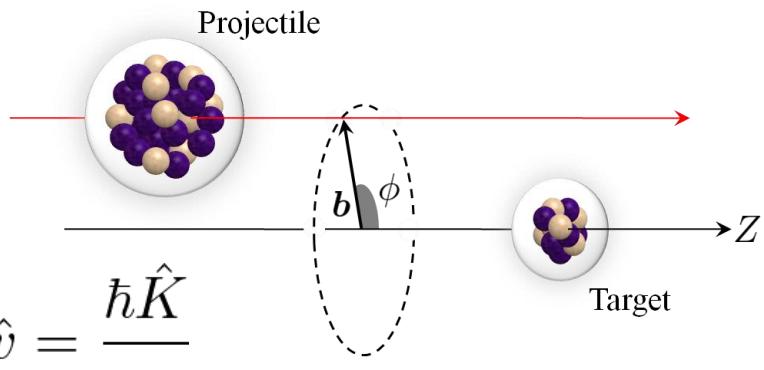
Potentials:  $U(r_{\text{c}}, r_{\text{n}}) = U_{\text{c}}^{\text{Nucl}}(r_{\text{c}}) + U_{\text{c}}^{\text{Coul}}(r_{\text{c}}) + U_{\text{n}}^{\text{Nucl}}(r_{\text{n}})$

# Eikonal reaction theory

✓ Product assumption

$$\Psi = \hat{O}\psi(\mathbf{R}, \mathbf{r})$$

$$\hat{O} = \frac{1}{\sqrt{\hbar\hat{v}}} e^{i\hat{K}\cdot Z} \quad \hat{K} = \frac{\sqrt{2\mu(E - h_P)}}{\hbar} \quad \hat{v} = \frac{\hbar\hat{K}}{\mu}$$



**The eikonal approximation**

(Neglect of  $\hat{O}\nabla_{\mathbf{R}}^2\psi$  in  $-\frac{\hbar^2}{2\mu}\nabla_{\mathbf{R}}^2\Psi$ )

$$i\frac{d\psi}{dZ} = \hat{O}^\dagger U \hat{O}\psi$$

The S-matrix as a formal solution

$\mathcal{P}$ : Path ordering operator  
(Z ordering)

$$S = \exp \left[ -i\mathcal{P} \int_{-\infty}^{\infty} dZ \hat{O}^\dagger \left( U_c^{\text{Nucl}} + U_c^{\text{Coul}} + U_n^{\text{Nucl}} \right) \hat{O} \right]$$

# Eikonal reaction theory

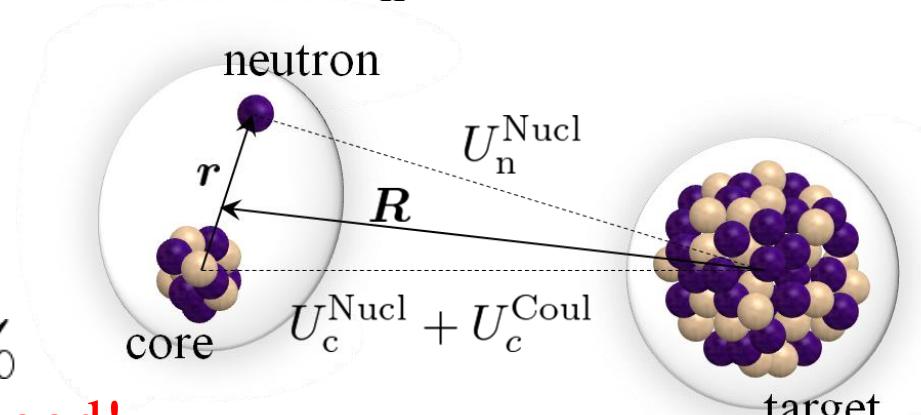
$$S = \exp \left[ -i\mathcal{P} \int_{-\infty}^{\infty} dZ \hat{O}^\dagger \left( U_c^{\text{Nucl}} + U_c^{\text{Coul}} + U_n^{\text{Nucl}} \right) \hat{O} \right]$$

We apply the adiabatic approximation to only  $\hat{O}^\dagger U_n^{\text{Nucl}} \hat{O}$ .

$$\hat{O}^\dagger U_n^{\text{Nucl}} \hat{O} \rightarrow \frac{U_n^{\text{Nucl}}}{\hbar v_0}$$

The error coming  
from this approximation  $\sim 3\%$

Very good!



$$S \approx \exp \left[ -i\mathcal{P} \int_{-\infty}^{\infty} dZ \hat{O}^\dagger \left( U_c^{\text{Nucl}} + U_c^{\text{Coul}} \right) \hat{O} \right] \exp \left[ -\frac{i}{\hbar v_0} \int_{-\infty}^{\infty} dZ U_n^{\text{Nucl}} \right]$$

$$= S_c S_n$$

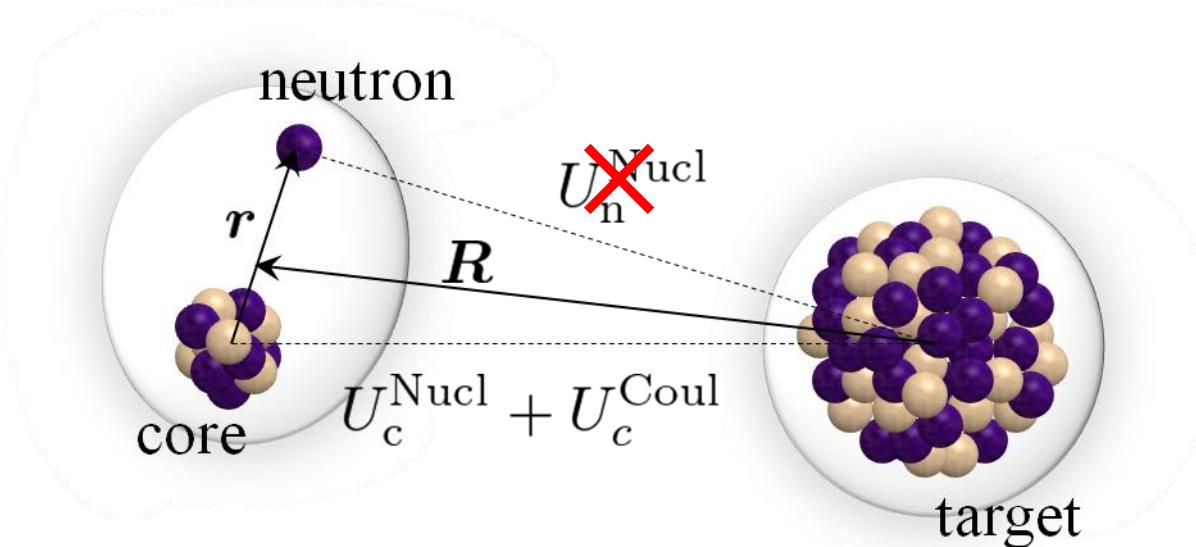
# How to get $S_c$

$$S_c = \exp \left[ -i\mathcal{P} \int_{-\infty}^{\infty} dZ \hat{O}^\dagger (U_c^{\text{Coul}} + U_c^{\text{Nucl}}) \hat{O} \right]$$

If we use CDCC, we should solve the below equation.

$$\left[ -\frac{\hbar^2}{2\mu} \nabla_{\mathbf{R}}^2 + h_P + U_c^{\text{Nucl}}(r_c) + U_c^{\text{Coul}}(r_c) - E \right] \Psi_c = 0$$

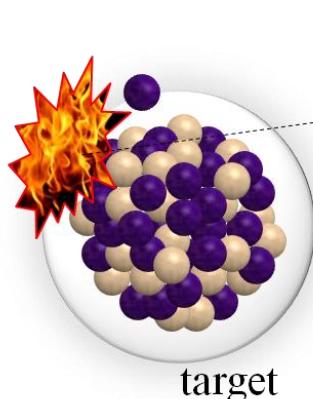
No  $U_n^{\text{Nucl}}$ !



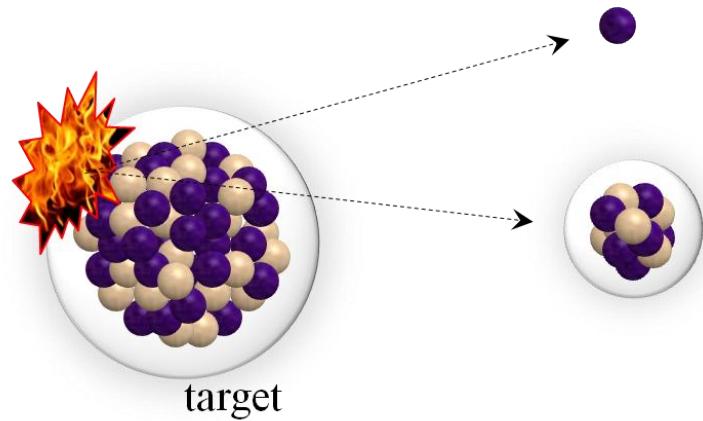
# Cross sections

- ✓ One-neutron removal cross section  $\sigma_{-n} = \sigma_{\text{str}} + \sigma_{\text{bu}}$

Stripping reaction



Elastic breakup



- ✓ Stripping cross section

$$\sigma_{\text{str}} = \int d^2\mathbf{b} \langle \varphi_0 | |S_c|^2 (1 - |S_n|^2) | \varphi_0 \rangle$$

- ✓ Elastic breakup cross section

$$\sigma_{\text{bu}} = \int d^2\mathbf{b} \left( \langle \varphi_0 | |S_c S_n|^2 | \varphi_0 \rangle - |\langle \varphi_0 | S_c S_n | \varphi_0 \rangle|^2 \right)$$

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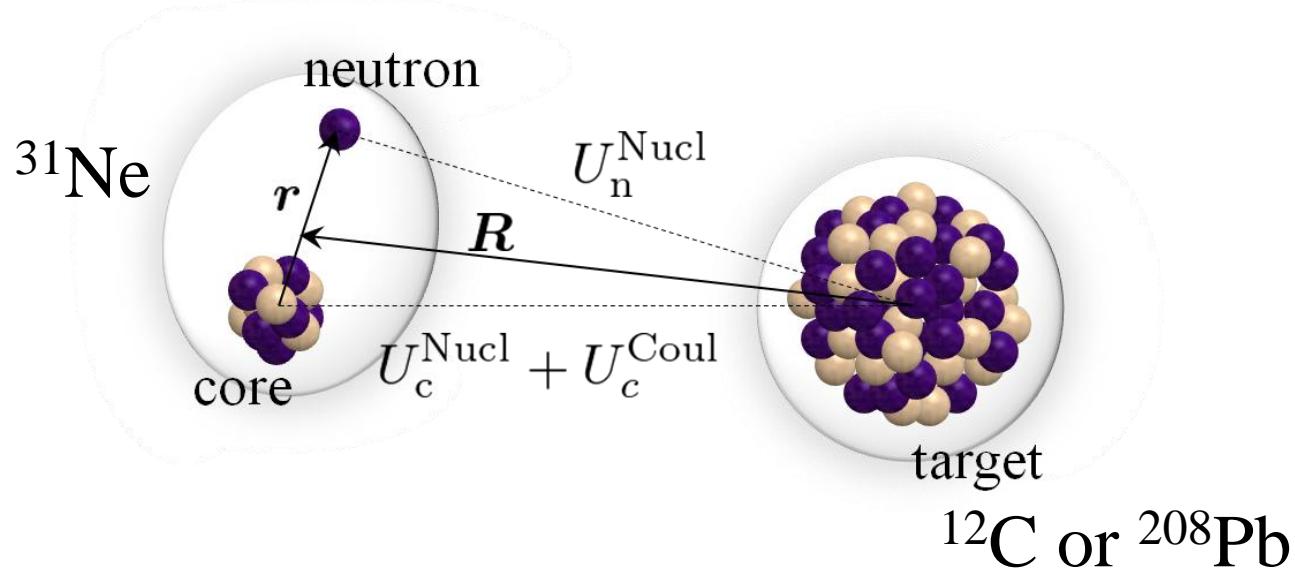
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# Model setting



$$U_n^{\text{Nucl}}(\mathbf{r}_n) = \int d\mathbf{r}' \rho_T(\mathbf{r}') t_{\text{NN}}(\mathbf{r}_n - \mathbf{r}')$$

$$U_c^{\text{Nucl}}(\mathbf{r}_c) = \int d\mathbf{r}' d\mathbf{r}'' \rho_c(\mathbf{r}') \rho_T(\mathbf{r}'') t_{\text{NN}}(\mathbf{r}_c - \mathbf{r}' + \mathbf{r}'')$$

✓ Niigata interaction  $t_{\text{NN}}$

*B. Abu-Ibrahim, W. Horiuchi, A. Kohama, and Y. Suzuki, PRC77, 034607 (2008).*

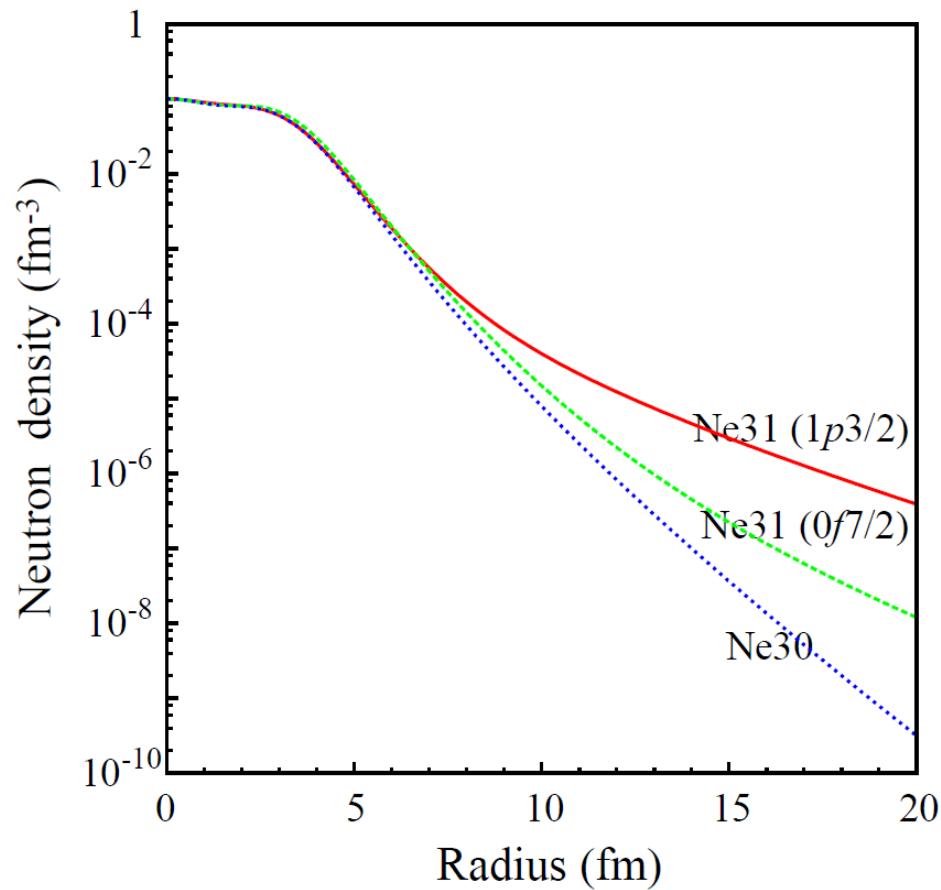
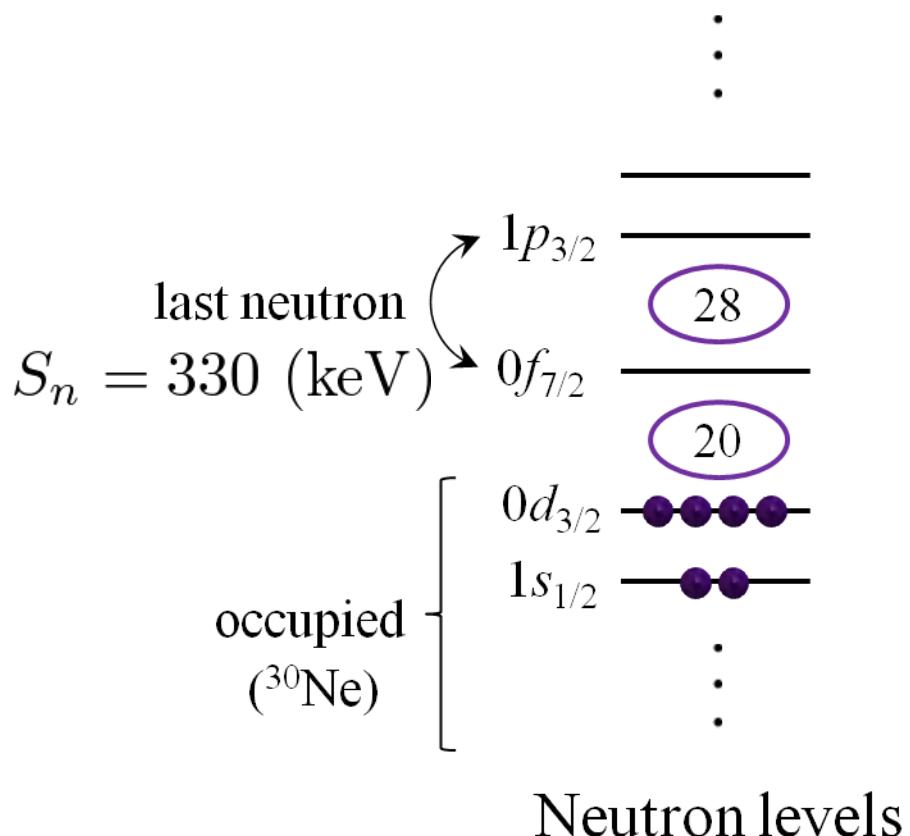
✓ Projectile and target densities

*W. Horiuchi, Y. Suzuki, P. Capel, and D. Baye, PRC81, 024606 (2010).*

# Structure model of $^{31}\text{Ne}$

Core ( $^{30}\text{Ne}$ ) + valence neutron ( $0f_{7/2}$  or  $1p_{3/2}$ )

# Single particle levels in Woods-Saxon potential



# Results

	1p3/2 orbit		0f7/2 orbit	
	$^{12}\text{C}$	$^{208}\text{Pb}$	$^{12}\text{C}$	$^{208}\text{Pb}$
$\sigma_{\text{str}}$	90	244	29	53
$\sigma_{\text{bu}}$	23.3	799.5	3.3	73.0
$\sigma_{-\text{n}}$	114	1044	32	126
Exp.	$79 \pm 7$	$712 \pm 65$	$79 \pm 7$	$712 \pm 65$
$\mathcal{S}$	0.693	0.682	2.47	5.65

# Good consistency!

$S_{\text{AMD}} \sim 0.6$  by M. Kimura

*cf. The results with Glauber model*

*W. Horiuchi, Y. Suzuki, P. Capel, and D. Baye, PRC81, 024606 (2010).*

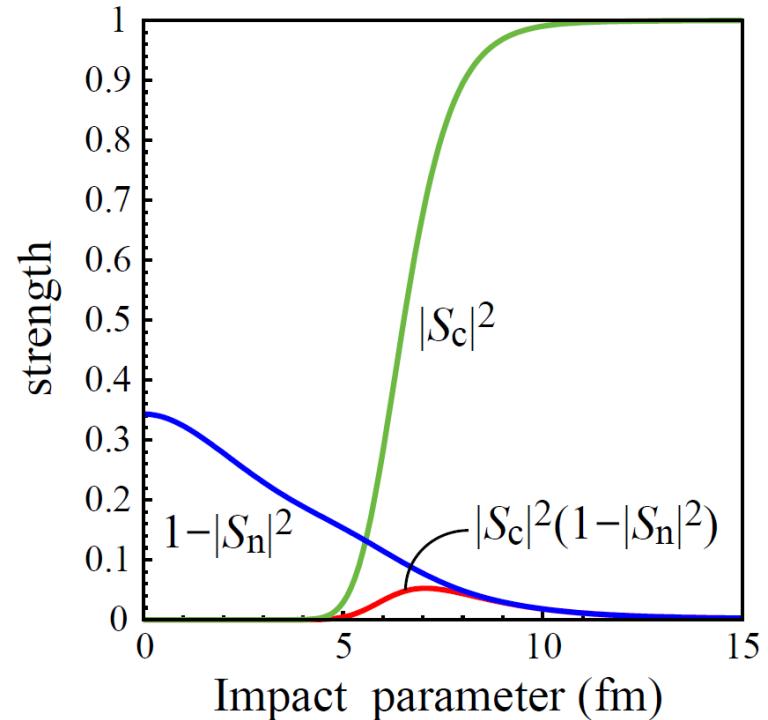
$$\text{For } 1p3/2, \quad \sigma_{-n}^{(12\text{C})} = 96 \text{ (mb)} \quad (\mathcal{S} = 0.823) \quad \sigma_{-n}^{(208\text{Pb})} = 1140 \text{ (mb)} \quad (\mathcal{S} = 0.625)$$

# Reaction mechanism

In the ERT,

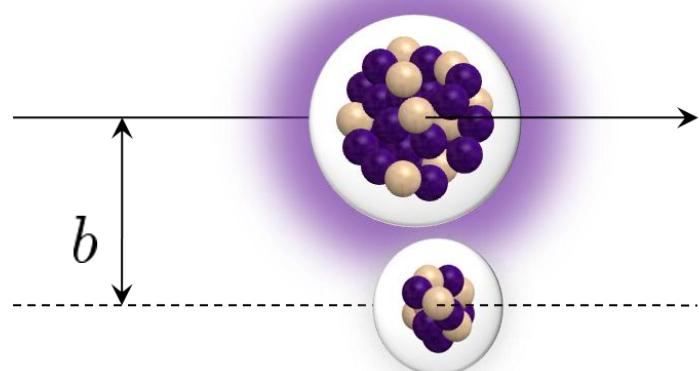
$$\sigma_{\text{str}} = \int d^2\mathbf{b} \langle \varphi_0 | \frac{\text{core}}{|S_c|^2} \frac{\text{valence}}{(1 - |S_n|^2)} | \varphi_0 \rangle$$

- ✓ The case of small  $b$   
Strong absorption
- ✓ The case of large  $b$   
No reaction



Peripheral reaction!

→ Application of the ANC method  
(ANC : Asymptotic normalization coefficient)

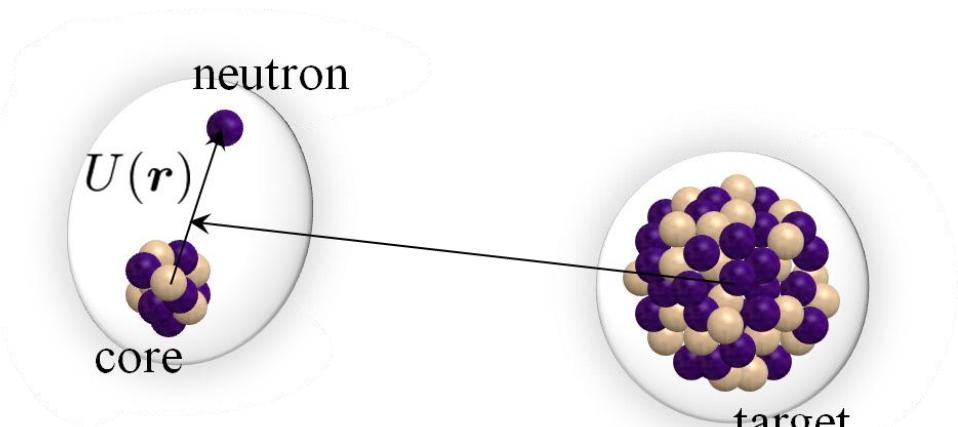
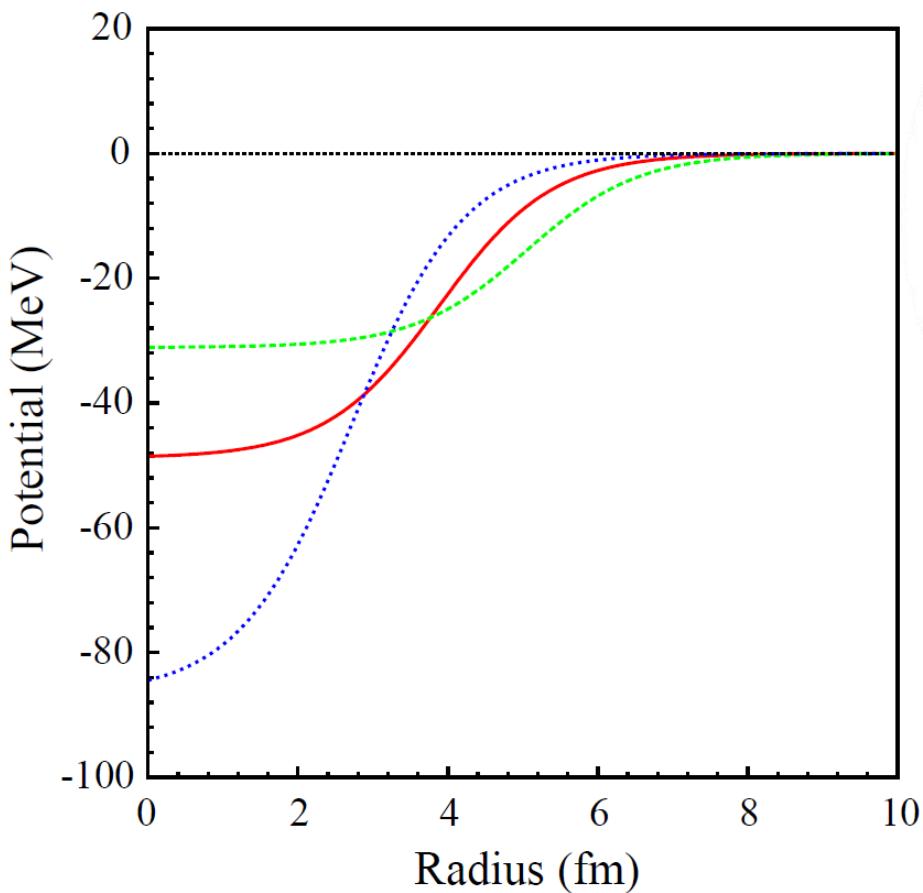


# Spectroscopic factor and ANC

$$S = 0.693 \pm 0.133 \pm 0.061$$

$$C_{\text{ANC}} = 0.320 \pm 0.010 \pm 0.028 \text{ (fm}^{-1/2}\text{)} \text{ for } 1p_{3/2} \text{ orbit and } ^{12}\text{C target}$$

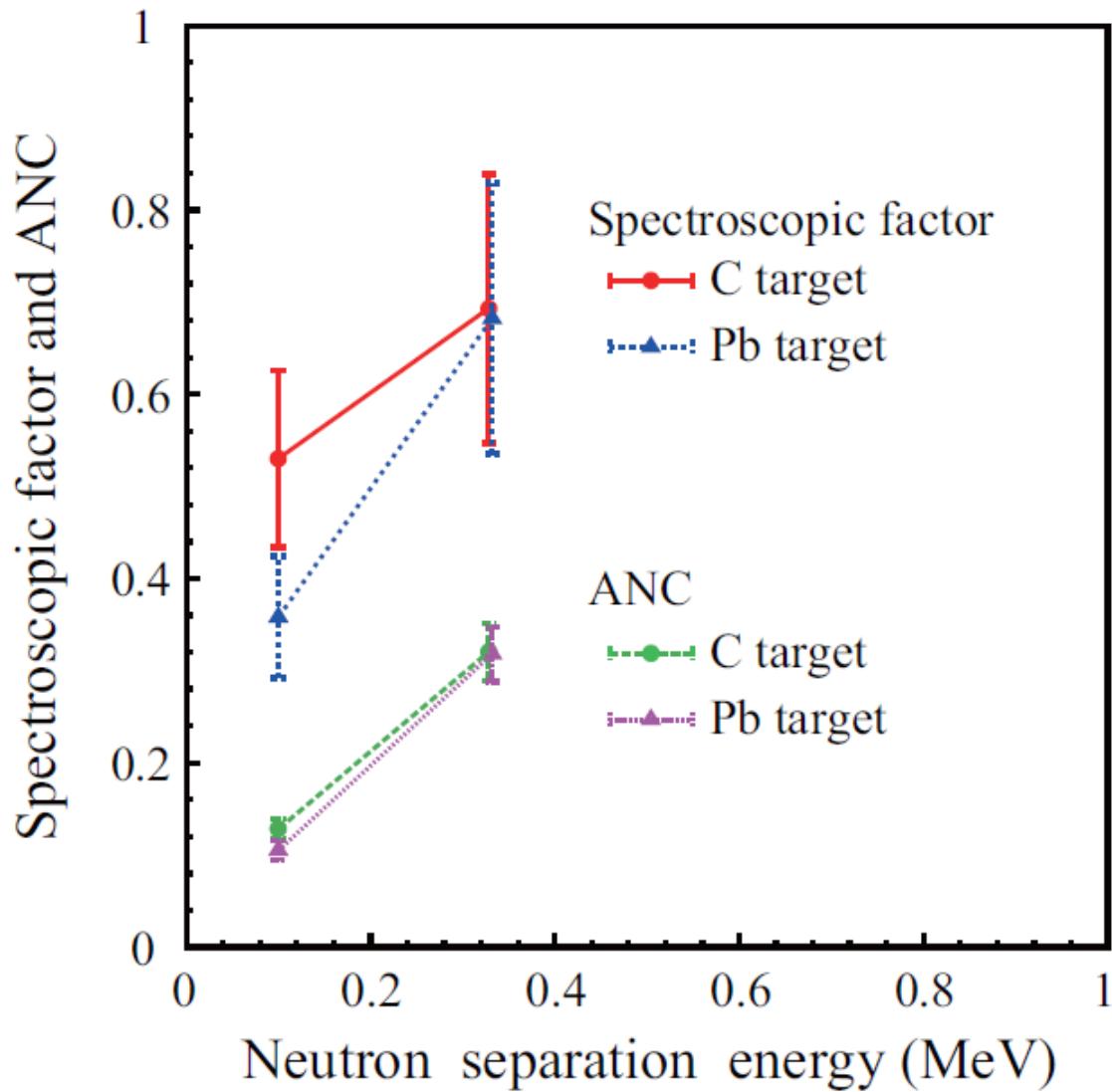
Theoretical error      Experimental error



$$U(\mathbf{r}) = V_0 \frac{1}{1 + \exp[(r - r_0)/a_0]}$$

Changing  $r_0$  and  $a_0$  by 30 %  
( $V_0$  is determined from  $S_n$ .)

# Spectroscopic factor and ANC



- ✓ Target dependence  
**Good agreement**
- ✓ Spectroscopic factor  
**Large theoretical ambiguity**  
(about 20%)
- ✓ ANC  
**Small theoretical ambiguity**  
(about 3%)

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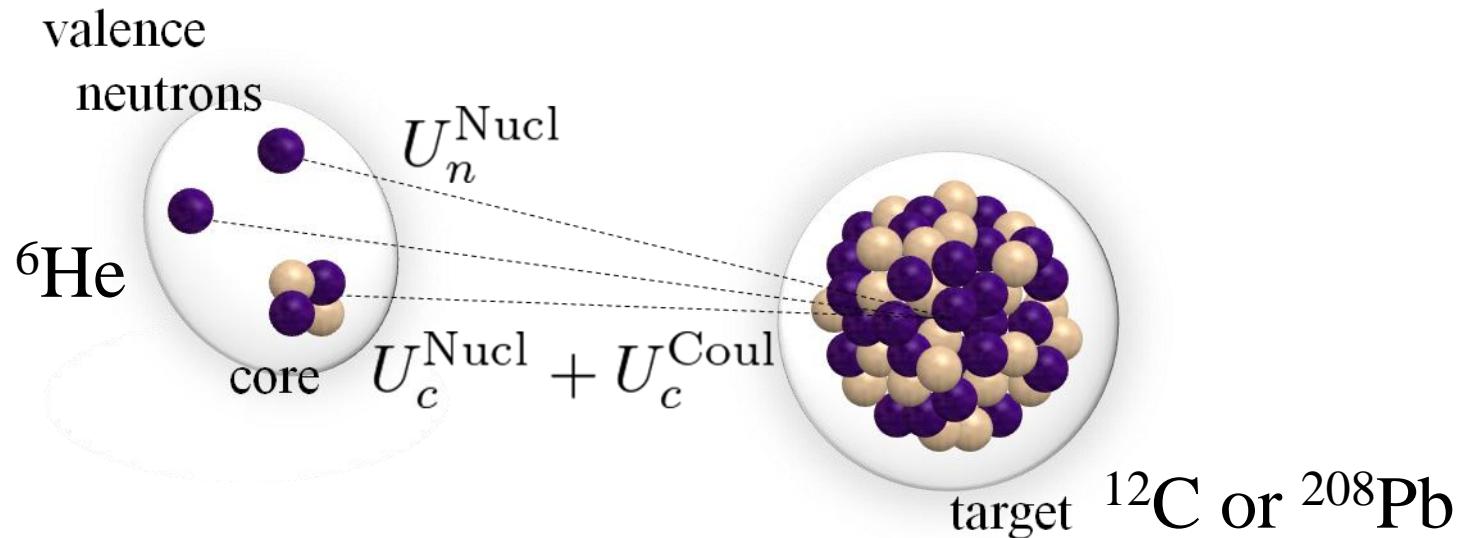
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# One- and two-neutron stripping

- ✓ ERT for 3-body projectile



$$\sigma_{1n \text{ str}} = 2 \int d^2 \mathbf{b} \langle \varphi_0 | |S_c|^2 |S_{n_1}|^2 (1 - |S_{n_2}|^2) | \varphi_0 \rangle$$

Adiabatic      Adiabatic

$$\sigma_{2n \text{ str}} = \int d^2 \mathbf{b} \langle \varphi_0 | |S_c|^2 (1 - |S_{n_1}|^2) (1 - |S_{n_2}|^2) | \varphi_0 \rangle$$

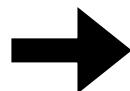
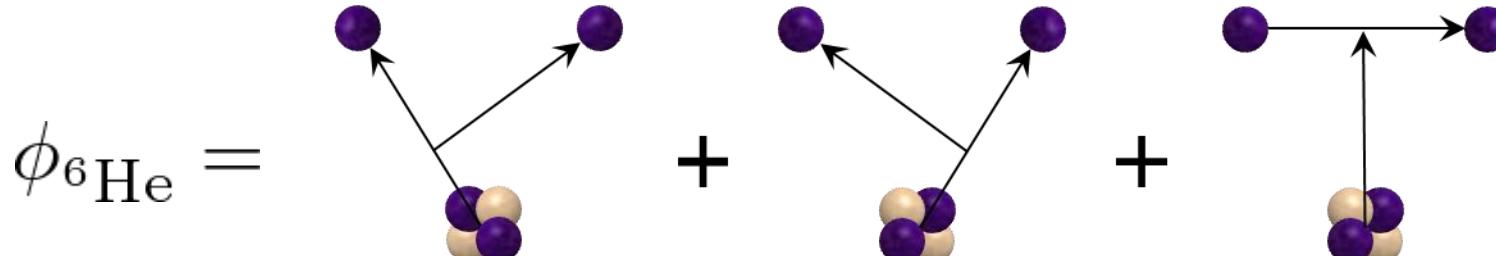
Adiabatic      Adiabatic

# 4-body CDCC

- ✓ 3-body projectile

## Gaussian Expansion Method (GEM)

*E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part Nucl. Phys. 51, 223 (2003).*



Discretization with pseudostates

*T. Matsumoto, et al., PRC70, 061601(R) (2004).*

*M. Rodriguez-Gallardo, et al. PRC80, 051601(R) (2009).*

- ✓ Melbourne interaction  $g_{NN}$

*K. Amos, et al., Adv. Nucl. Phys. 25, 275 (2000).*

- ✓ Core and target densities

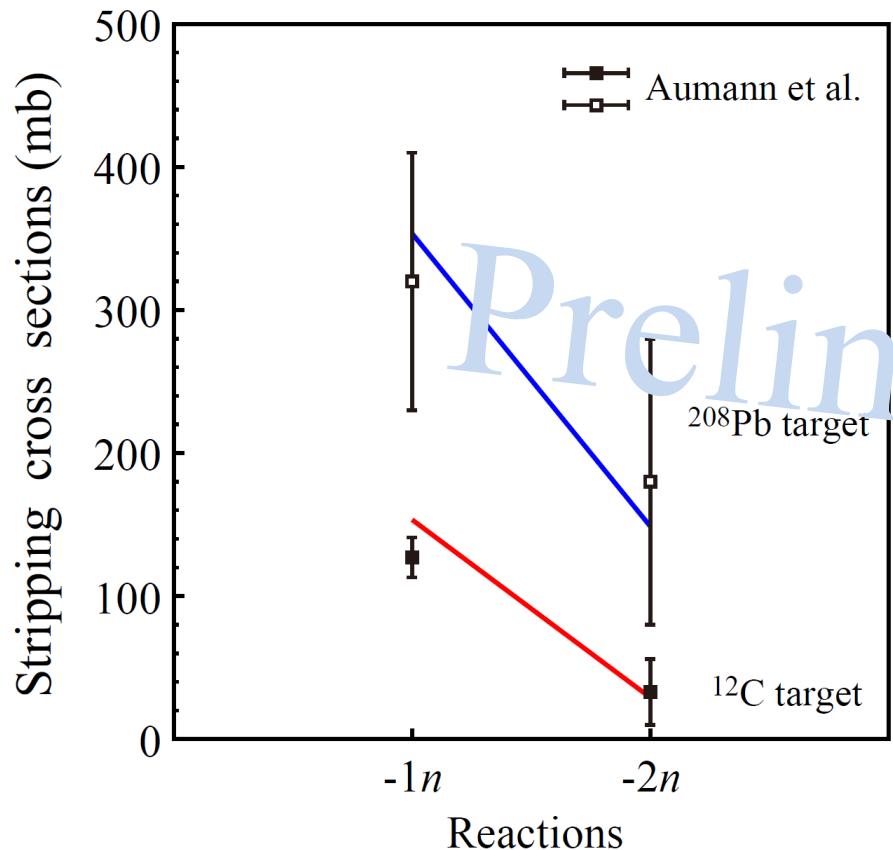
Hartree-Fock calculation with Gogny D1S interaction

# One- and two-neutron stripping

Preliminary results of ERT + 4-body CDCC

${}^6\text{He} + {}^{12}\text{C}$  ( ${}^{208}\text{Pb}$ ) ,  $E_{\text{lab}} = 240$  (MeV/nucleon)

T. Aumann, et al. PRC59, 1252 (1999).



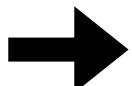
2n removal

${}^{12}\text{C}$   $\sigma_{2n \text{ rmv}}^{(\text{calc})} = 198$  (mb)  
 ${}^{12}\text{C}$   $\sigma_{2n \text{ rmv}}^{(\text{exp})} = 190 \pm 18$  (mb)

${}^{208}\text{Pb}$   $\sigma_{2n \text{ rmv}}^{(\text{calc})} = 1016$  (mb)  
 ${}^{208}\text{Pb}$   $\sigma_{2n \text{ rmv}}^{(\text{exp})} = 1150 \pm 90$  (mb)

Good agreement!

Future work



Role of dineutron correlation?

# Summary

- ✓ Eikonal reaction theory

ERT is an accurate method to treat neutron removal reaction.

It's an extension of the CDCC.

- ✓ One-neutron removal for  $^{31}\text{Ne}$  scattering

The major component of the  $^{31}\text{Ne}_{\text{g.s.}}$  is  $^{30}\text{Ne}(0^+) \otimes 1p3/2$ .

We showed the applicability of the ANC method.

The s-factor should be extracted from the experimental data  
with the theoretical error bar.

- ✓ One- and two-neutron stripping for  $^6\text{He}$  scattering

Good agreements for  $^{12}\text{C}$  and  $^{208}\text{Pb}$  targets