

# Hypernuclei with a Microscopic Lambda-Nucleon Force

with

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- BHF approach of hypernuclear matter

PLB 355, 21 (1995)

PRC 57, 704 (1998)

- Neutron star properties

PRC 61, 055801 (2000)

PRC 62, 064308 (2000)

- Hypernuclei

PRC 64, 044301 (2001)

PRC 73, 058801 (2006)

PRC 76, 034312 (2007)

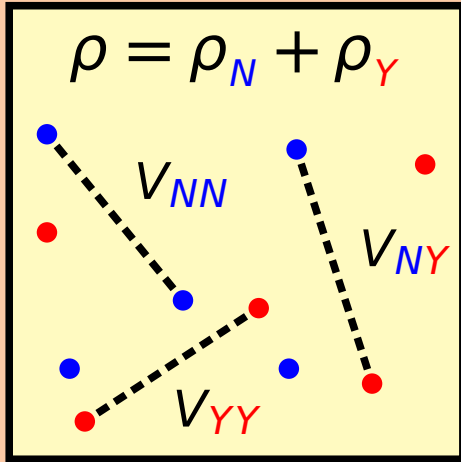
PRC 78, 028801 (2008)

PRC 78, 054306 (2008)

PTP 123, 569 (2010)

PRC 84, 035801 (2011)

# Hypernuclear Matter:



$N = qq\bar{q}$ : n (939 MeV)  
p (939 MeV)

$Y = qq\bar{s}$ :  $\Lambda^0$  (1116 MeV)  
 $\Sigma^{+0-}$  (1193 MeV)

$V_{NN}$  : Argonne, Bonn, Paris, ...

$V_{NY}$  : Nijmegen (NSC89, NSC97, ...)

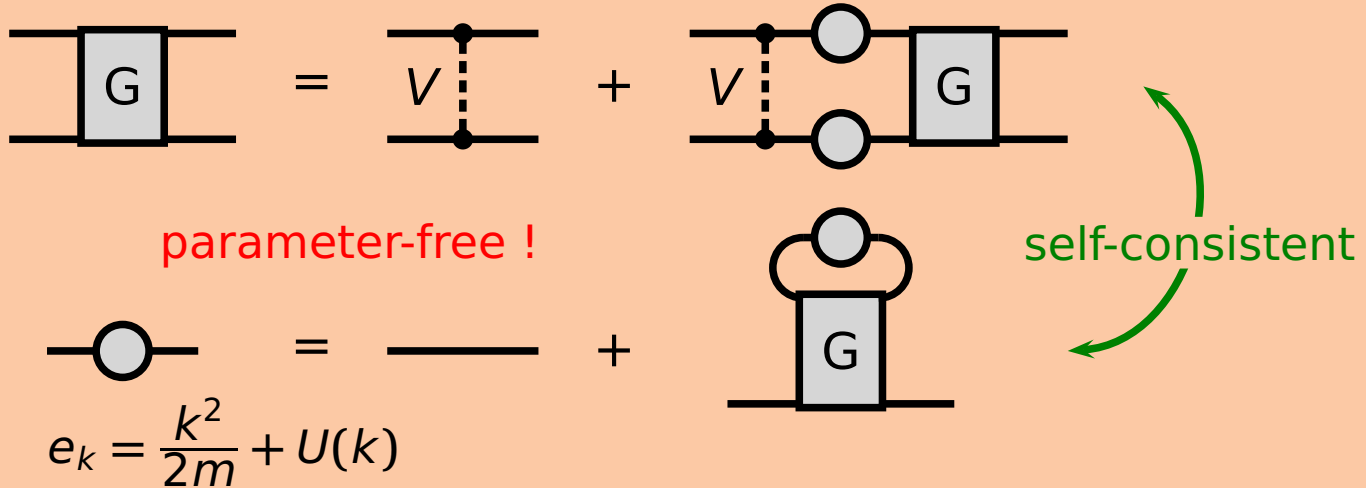
$V_{YY}$  : ? (no scattering data)

In free space weak decay:  $Y \rightarrow N + \pi$  etc. ( $c\tau \approx 8$  cm)

In dense nucleonic medium the decay is Pauli-blocked !

# Brueckner Theory of Nuclear Matter:

- Effective in-medium interaction  $G$  from potential  $V$ :



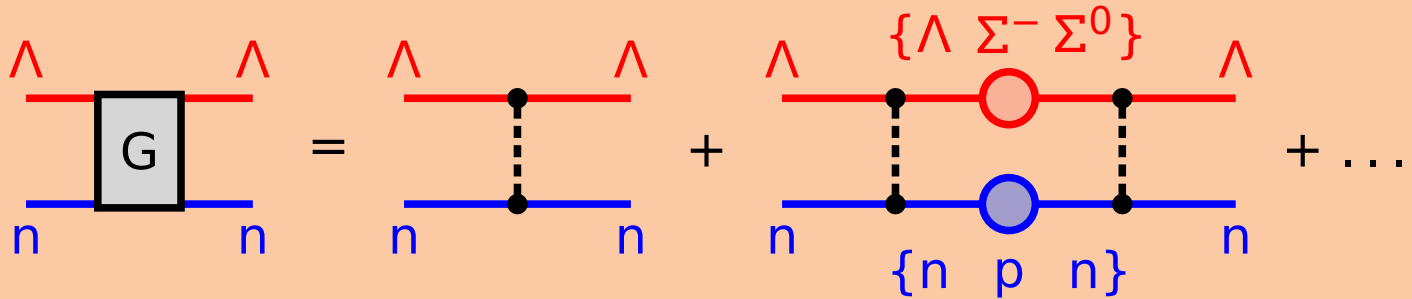
Compute: binding energy, s.p. properties, cross sections, ...

K.A. Brueckner and J.L. Gammel; PR 109, 1023 (1958) for nuclear matter

Extension to hypernuclear matter ...

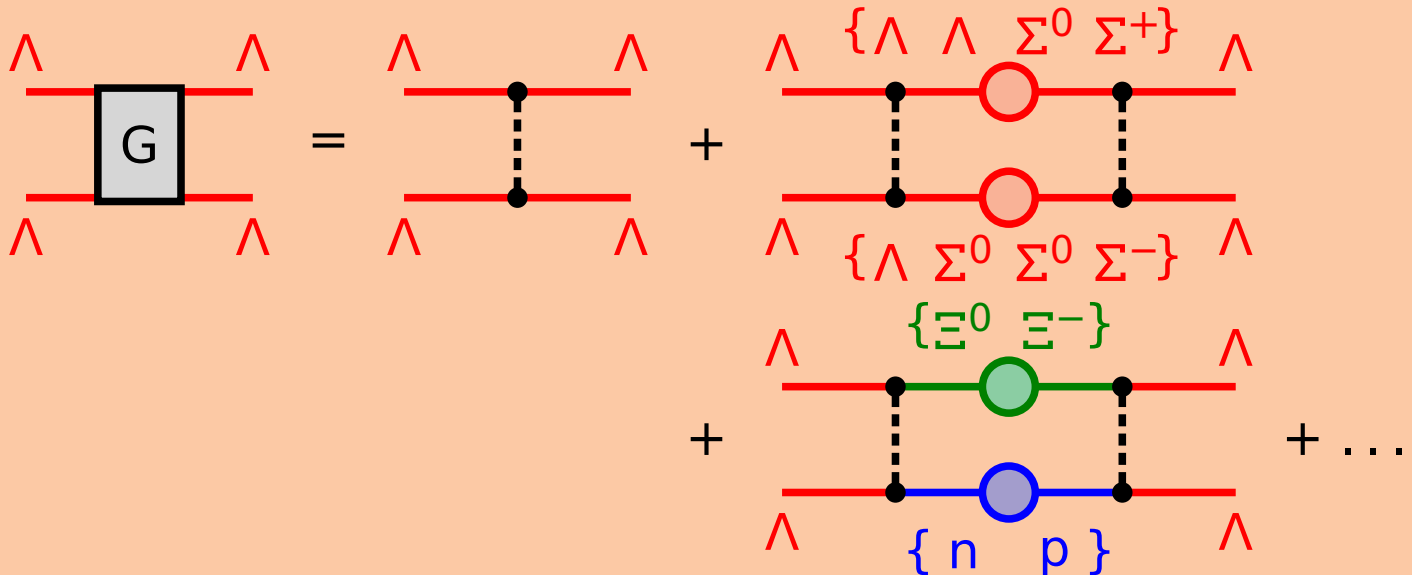
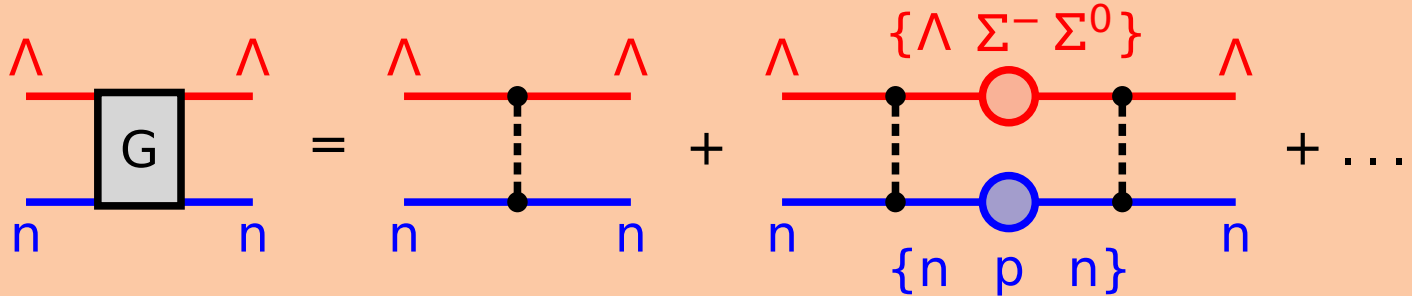
# Include Hyperons:

- Technical difficulty: coupled channels:

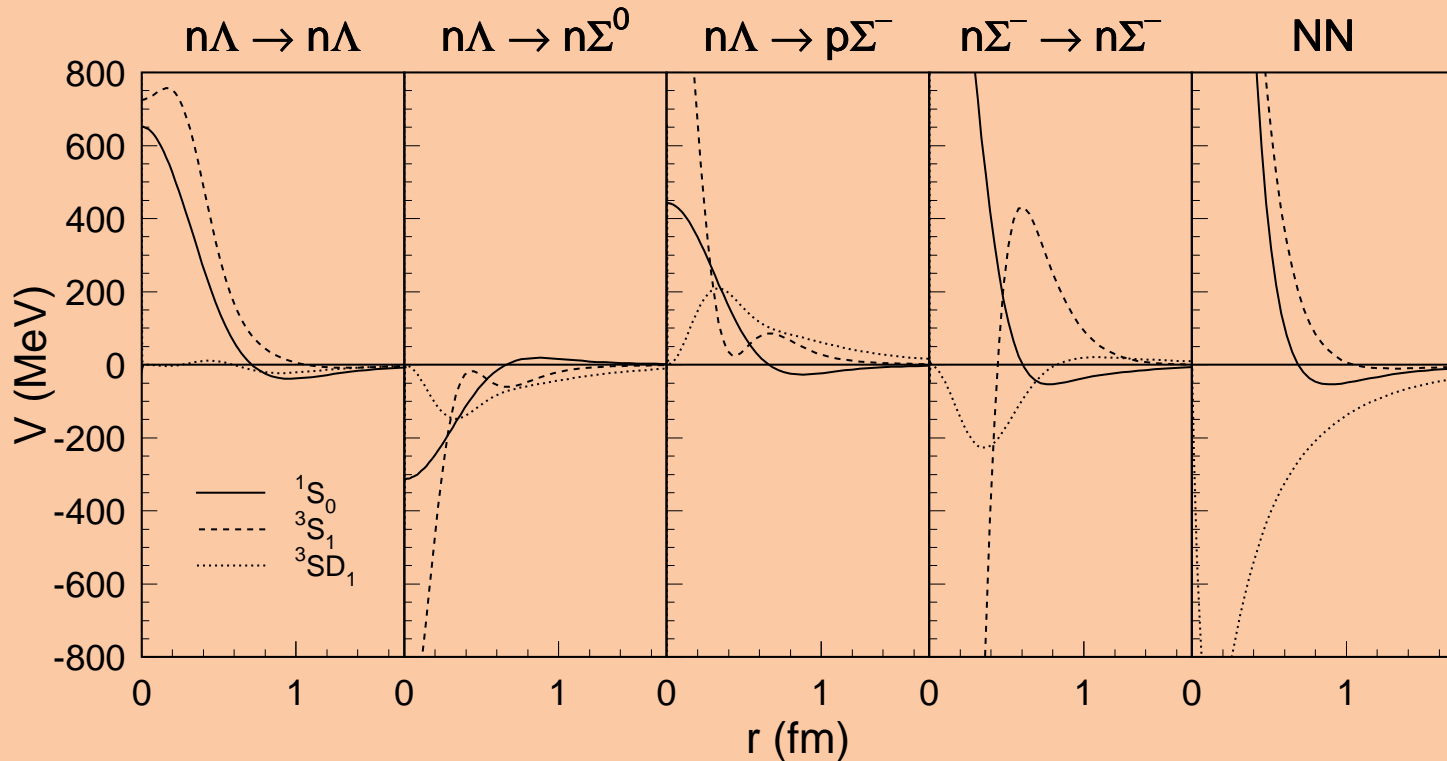


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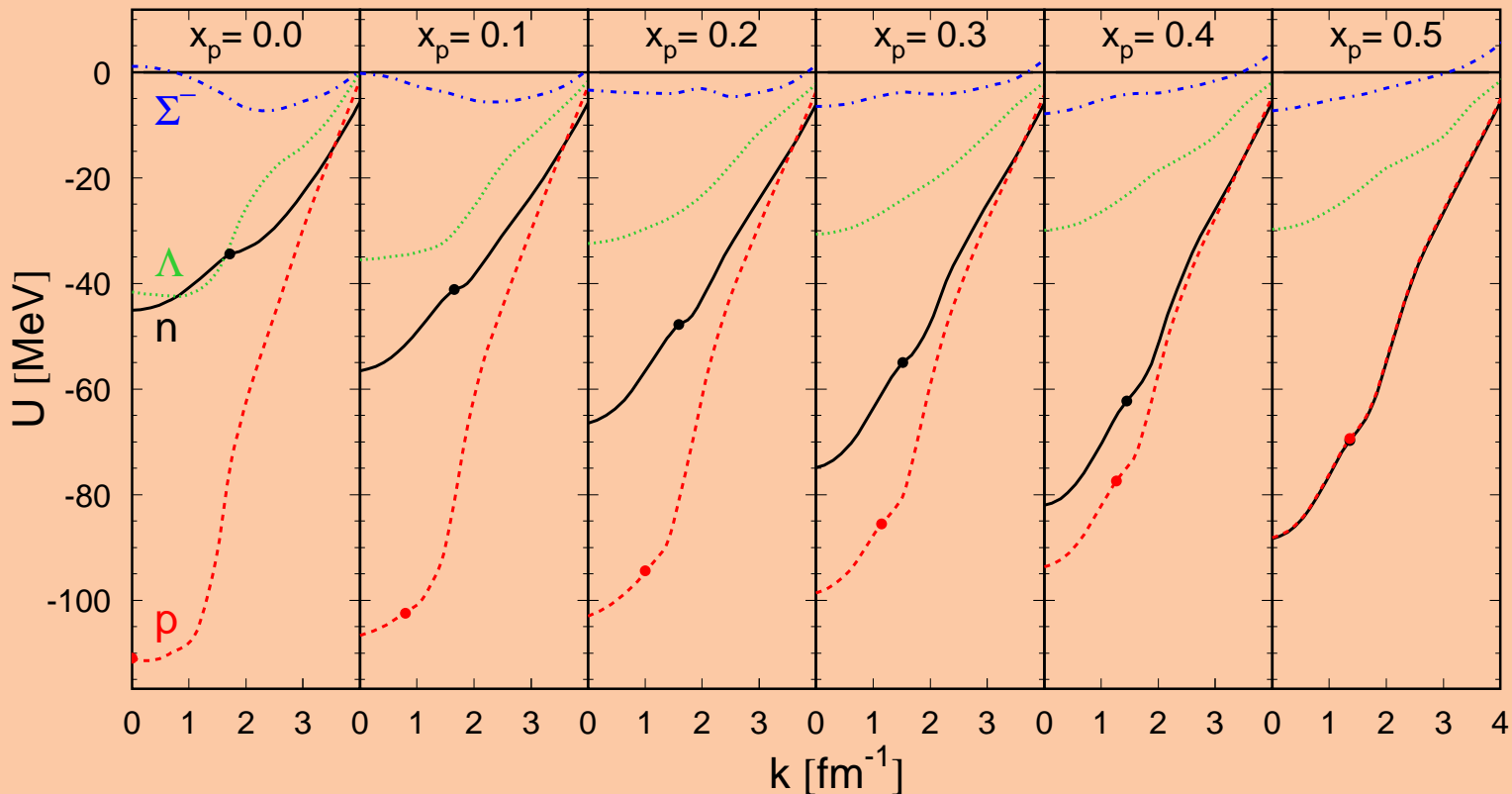
• Hyperon-nucleon potentials (NSC89) vs. Paris NN:



↪ “Soft” cores, Strong coupling  $N\Lambda \leftrightarrow N\Sigma$

● Single-particle potentials in nuclear matter ( $\rho_N = \rho_0$ ):

V18+UIX' NN & NSC89 YN,  $\rho_N = 0.17 \text{ fm}^{-3}$ ,  $\rho_\Lambda = \rho_\Sigma = 0$

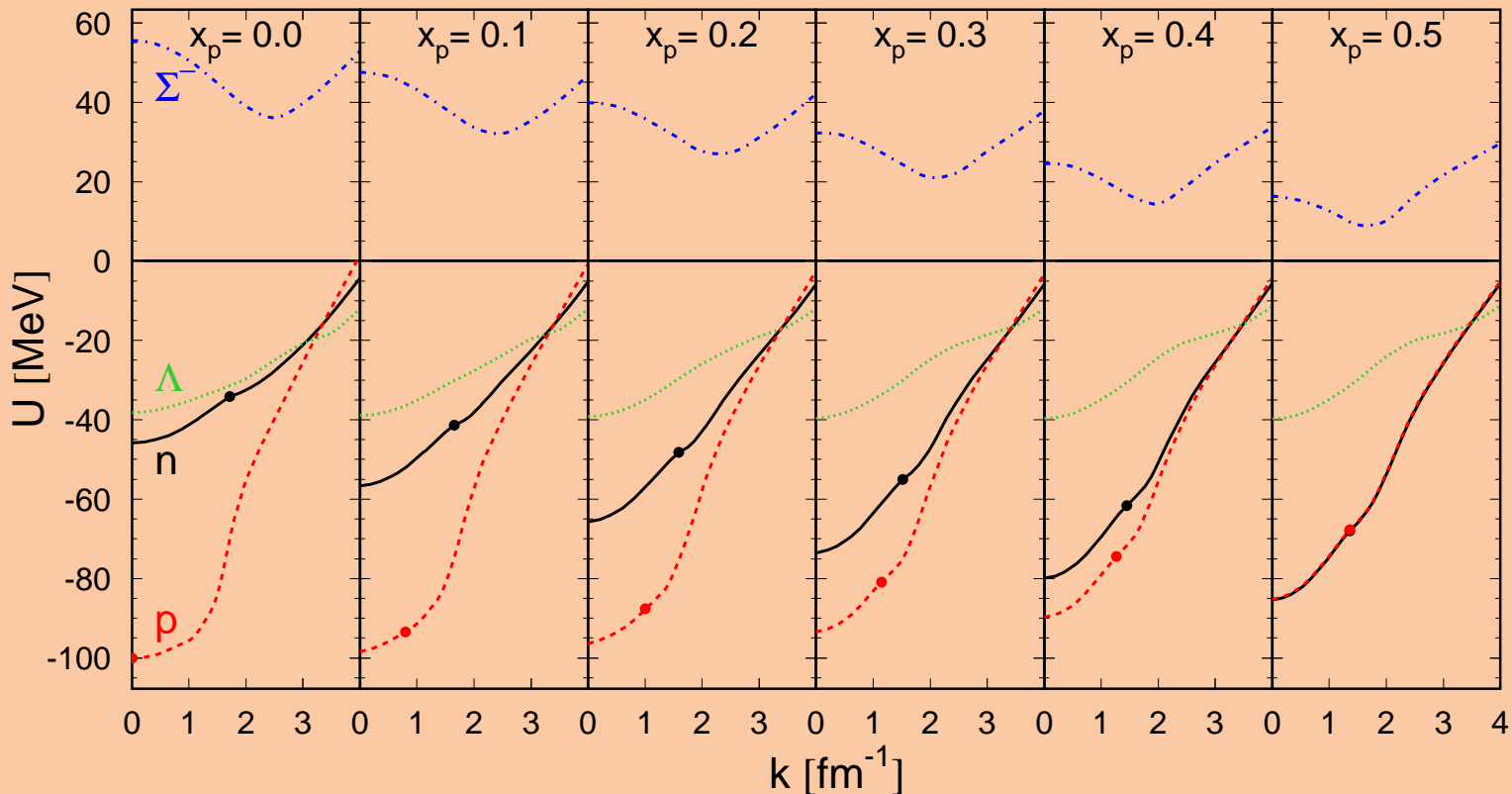


↪ Hyperons are weaker bound than nucleons

Only slight dependence on proton fraction  $x_p = \rho_p/\rho_N$

● Results with ESC08b NY potential:

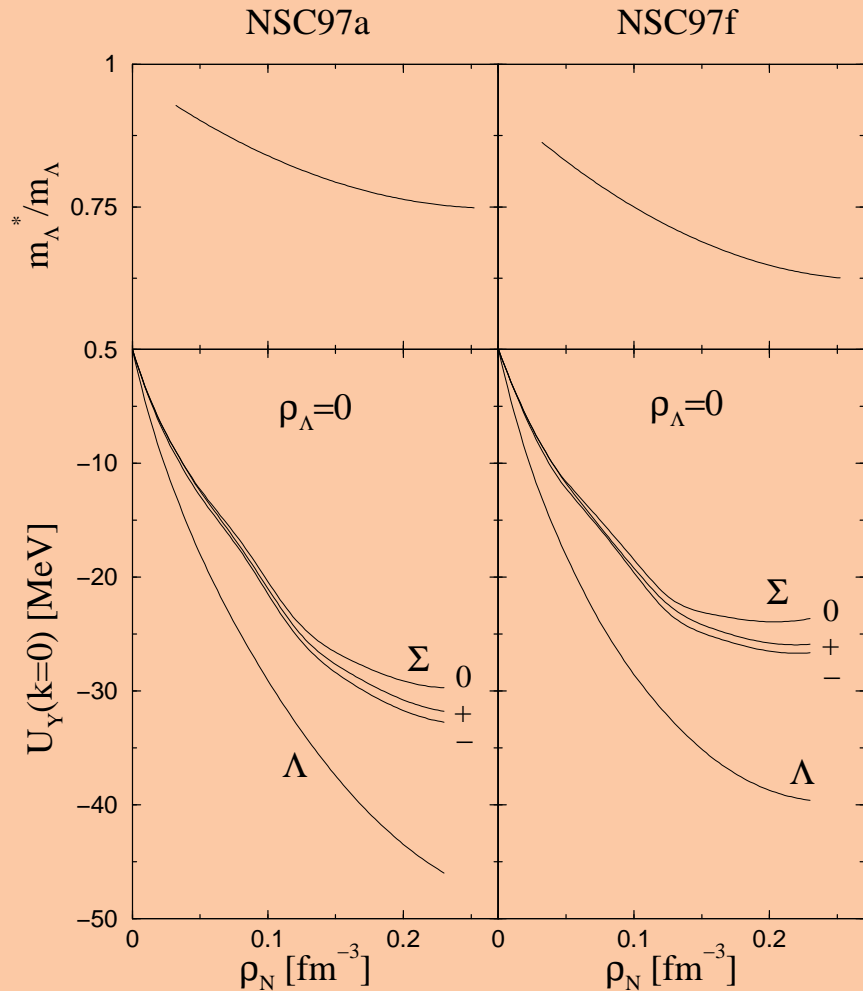
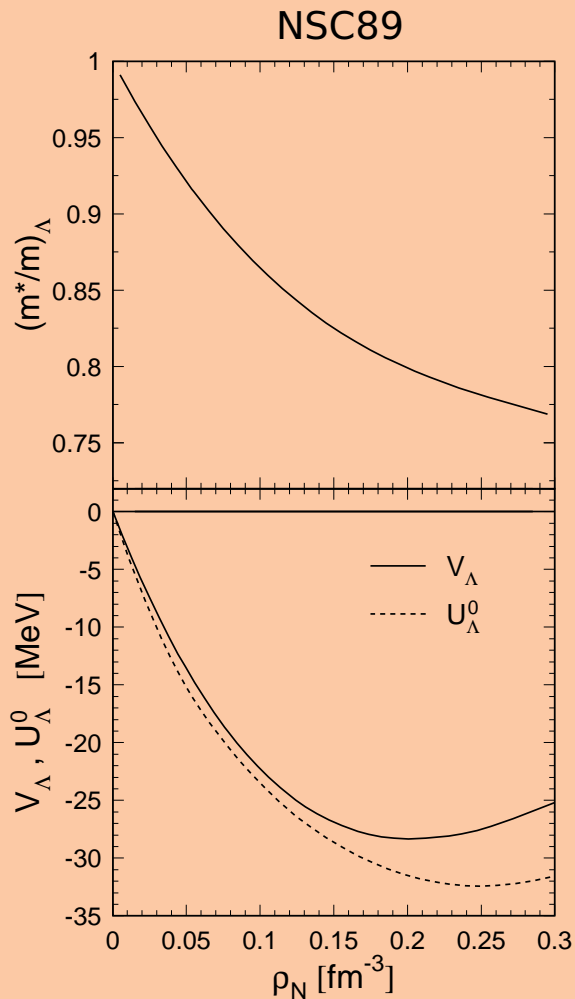
V18+TBF NN & ESC08 YN,  $\rho_N = 0.17 \text{ fm}^{-3}$ ,  $\rho_\Lambda = \rho_\Sigma = 0$



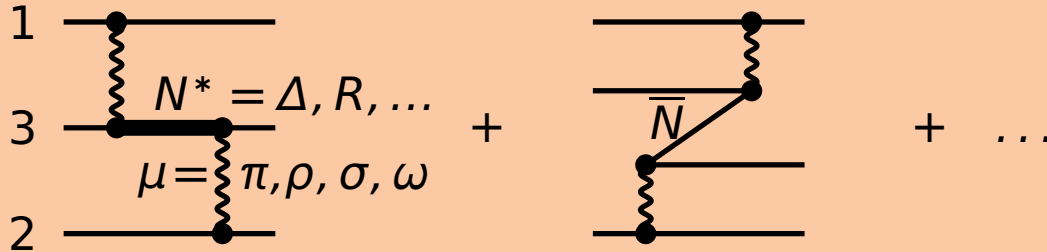
↪  $\Sigma^-N$  interaction is repulsive



● Lambda effective mass and hyperon well depths:



# Three-Nucleon Forces:



- Only small effect required [ $\delta(B/A) \approx 1$  MeV at  $\rho_0$ ]
- Model dependent, no final theory yet
- Use and compare microscopic and phenomenological TBF...
  - Microscopic TBF of P. Grangé et al., PRC 40, 1040 (1989):  
Exchange of  $\pi, \rho, \sigma, \omega$  via  $\Delta(1232), R(1440), N\bar{N}$   
Parameters compatible with two-nucleon potential (Paris,  $V_{18}, \dots$ )
  - Urbana IX phenomenological TBF:  
Only  $2\pi$ -TBF + phenomenological repulsion  
Fit saturation point

# Hypernuclei: Single, Double, Multi-Lambda:

- Created by  $(\pi^+, K^+)$ ,  $(K^-, \pi^-)$ ,  $(e, e'K^+)$  reactions (BNL, CERN, JLAB, KEK, LNF, GSI, J-PARC, ...)
- Experimentally known (heavy)  $\Lambda$  hypernuclei:
  - Single-lambda:  ${}_{\Lambda}^{13}\text{C}$ ,  ${}_{\Lambda}^{16}\text{O}$ ,  ${}_{\Lambda}^{28}\text{Si}$ ,  ${}_{\Lambda}^{40}\text{Ca}$ ,  ${}_{\Lambda}^{89}\text{Y}$ ,  ${}_{\Lambda}^{139}\text{La}$ ,  ${}_{\Lambda}^{208}\text{Pb}$ , ...
  - Double-lambda:  ${}_{\Lambda\Lambda}^6\text{He}$ ,  ${}_{\Lambda\Lambda}^{10,11,12}\text{Be}$ ,  ${}_{\Lambda\Lambda}^{13}\text{B}$  (8 events !)
  - Multi-lambda: **None !**
- Observables:
  - Single-particle levels:  $e_q^i$  ( $q = n, p, \Lambda$ )
  - Binding energy:  $B_{\Lambda} = E({}^{A-1}Z) - E({}_{\Lambda}^AZ)$
  - Rms radii:  $R_q = \sqrt{\langle r^2 \rangle_q}$

# Lambda Hypernuclear Chart:

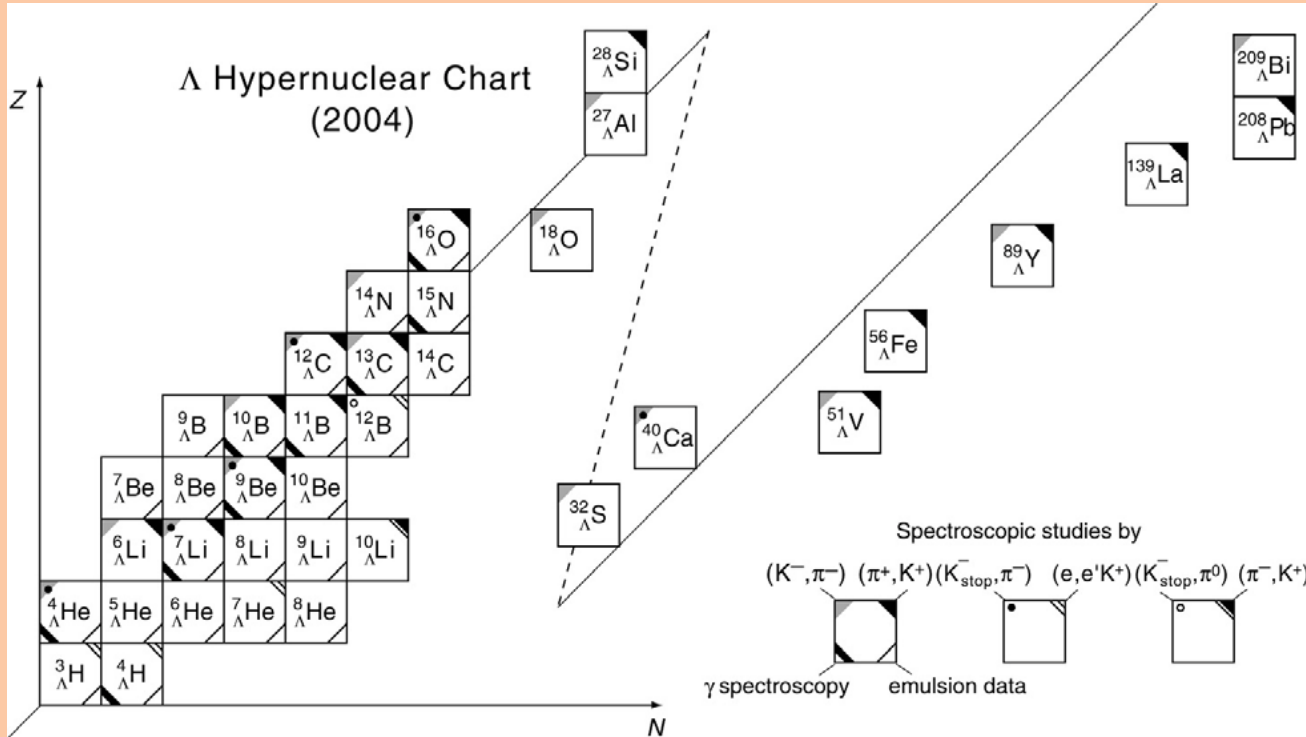
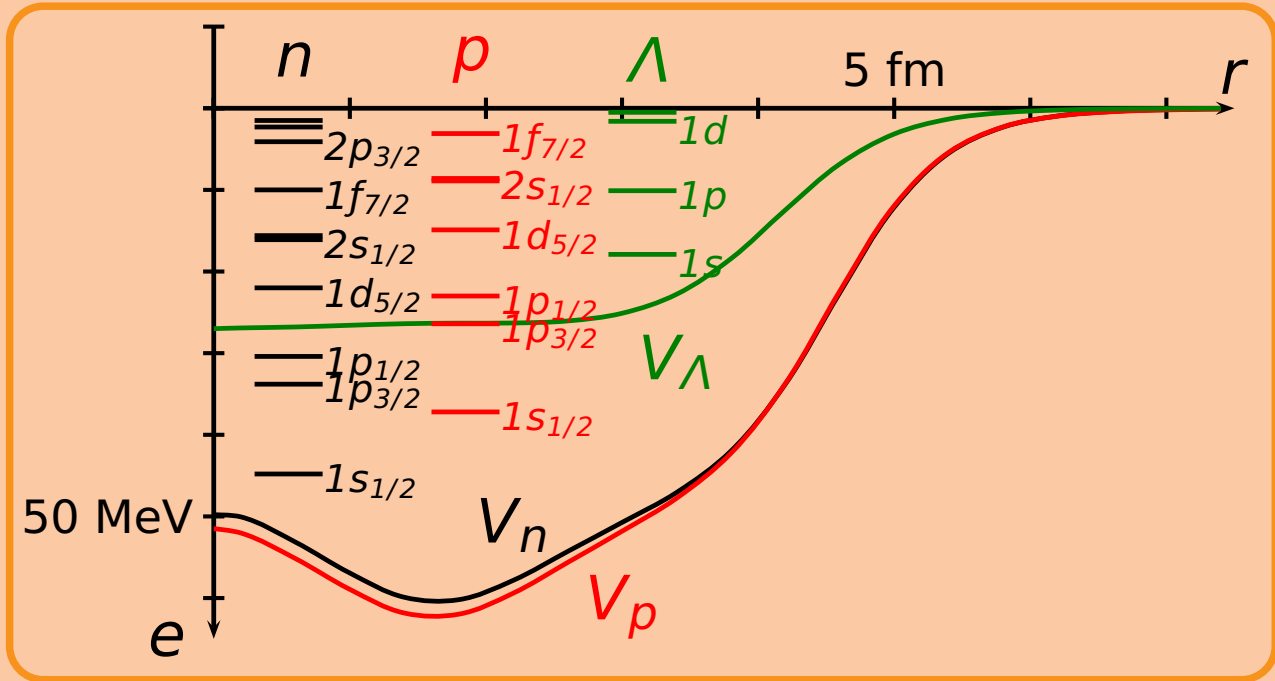


Fig. 1.  $\Lambda$  hypernuclear chart. The experimentally identified  $\Lambda$  hypernuclei and the experimental methods used to study them (reaction spectroscopies of  $(K^-, \pi^-)$ ,  $(\pi^+, K^+)$ ,  $(e, e'K^+)$ , etc.,  $\gamma$  spectroscopy, and the emulsion method) are shown.

- Typical example:  ${}^{40}_{\Lambda}\text{Ca}$  :



- Theoretical model:

- Skyrme-Hartree-Fock (SHF) [Vautherin & Brink, PRC 5, 626 (1972)]
- Standard  $NN$  force: SIII, SGII, SkI4, SLy4, ...
- Effective microscopic  $\Lambda N$  force from BHF results ...

- SHF Schrödinger equation:

$$\left[ -\nabla \cdot \frac{1}{2m_q^*(r)} \nabla + V_q(r) - i\nabla W_q(r) \cdot (\nabla \times \boldsymbol{\sigma}) \right] \phi_q^i(r) = -e_q^i \phi_q^i(r)$$

- SHF mean fields:

$$V_N = V_N^{\text{SHF}} + \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_N} \quad , \quad V_\Lambda = \frac{\partial \epsilon_{N\Lambda}}{\partial \rho_\Lambda} \quad , \quad W_\Lambda = 0$$

- Effective mass  $m_\Lambda^*(\rho_N, \rho_\Lambda)$  and

Energy density due to  $N\Lambda$  interaction: no free parameters

$$\begin{aligned} \epsilon_{N\Lambda}(\rho_N, \rho_\Lambda) = \\ (\rho_N + \rho_\Lambda) \frac{B}{A}(\rho_N, \rho_\Lambda) - \rho_N \frac{B}{A}(\rho_N, 0) - \rho_\Lambda \frac{B}{A}(0, \rho_\Lambda) \end{aligned}$$

- Coupled equations for eigenvalues  $e_q^i$

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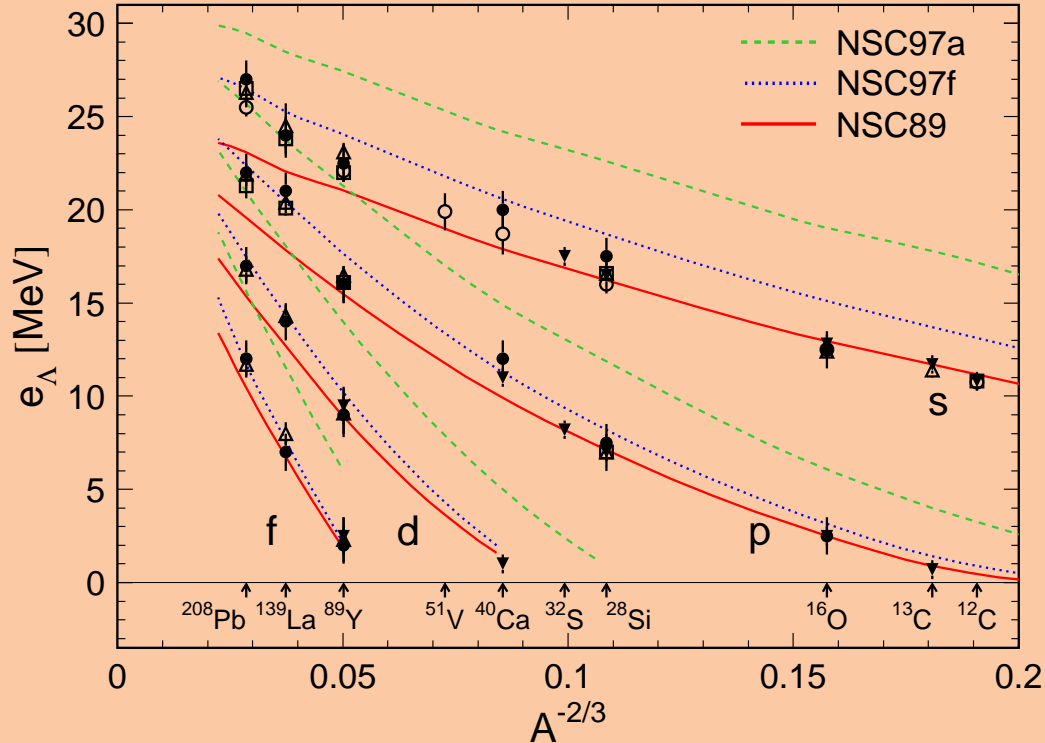
- Effective mass  $m_\Lambda^*(\rho_N, \rho_\Lambda)$  and  $\epsilon_{N\Lambda}(\rho_N, \rho_\Lambda)$  from BHF  
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- Coupled equations for eigenvalues  $e_q^i$

# Results: Single- $\Lambda$ Hypernuclei:

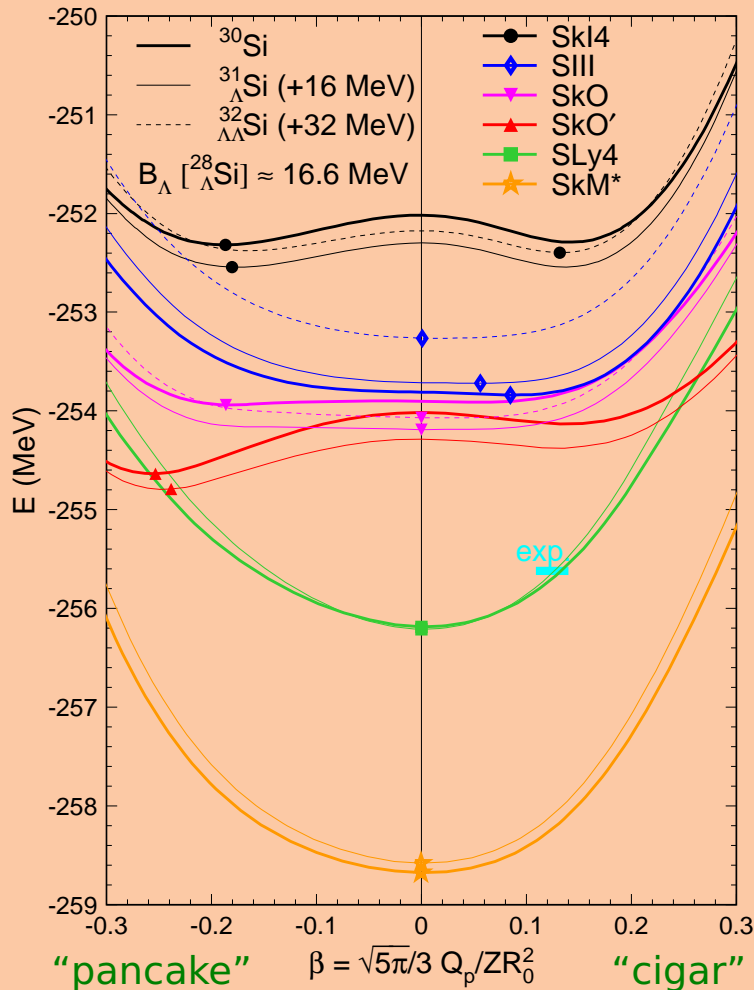
- Lambda single-particle levels:



↪ Fair agreement with NSC89 and NSC97f potentials  
No indication of strong hyperon TBF



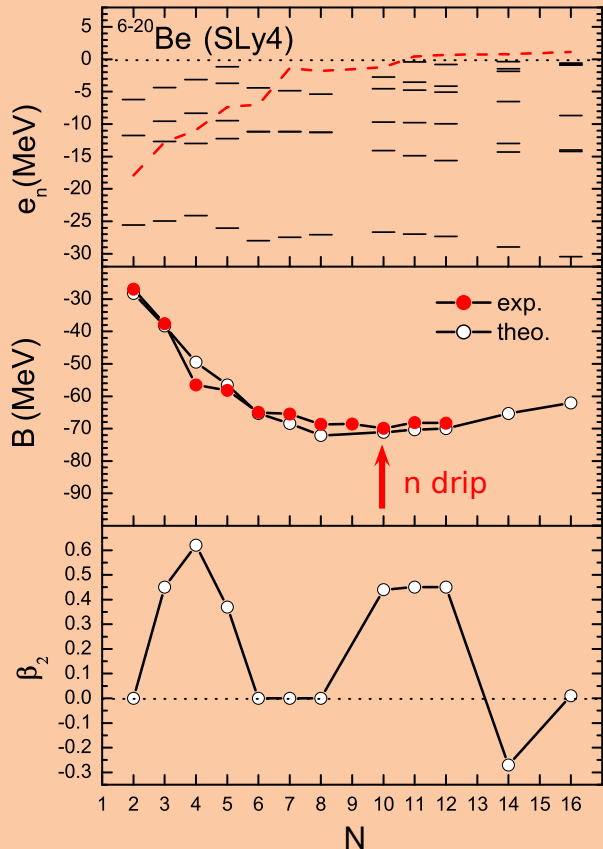
● Deformed (hyper)nuclei, e.g.,  $^{30}\text{Si}$ :



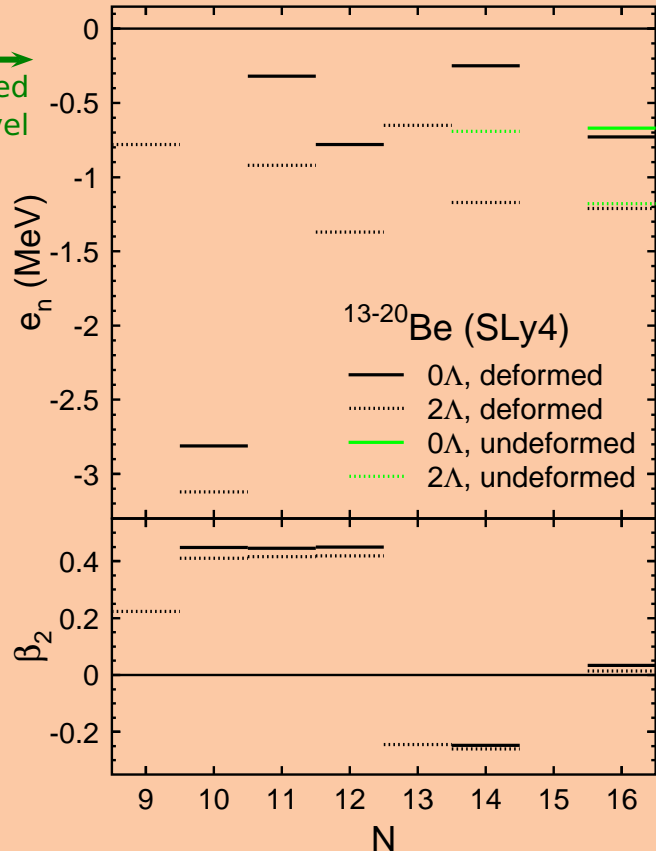
↪ Strong dependence on the NN Skyrme force, not predictive

↪ The  $\Lambda$ 's might 'pull' together a nucleus with a weak deformation minimum

• Neutron-rich (halo) hypernuclei, e.g., Be isotopes:

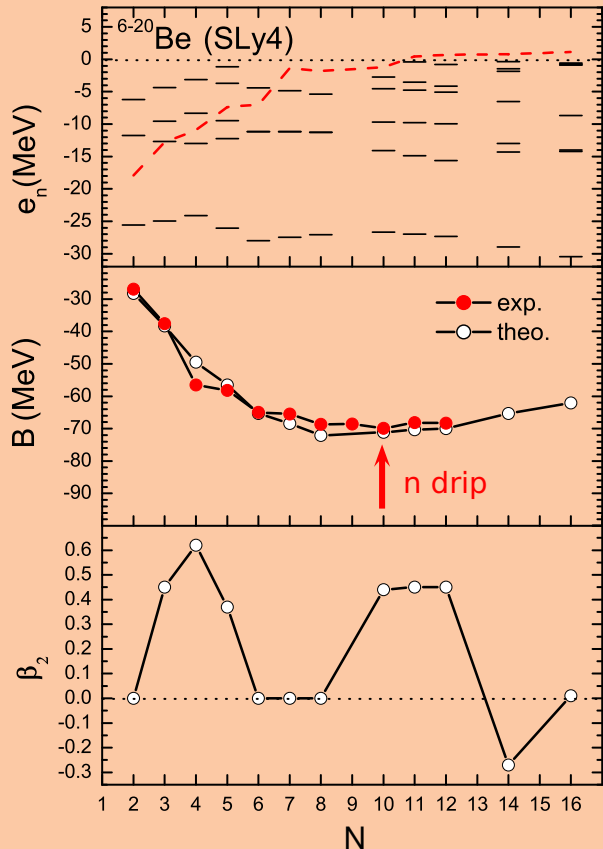


highest occupied neutron s.p. level ( $1d_{5/2}$ )

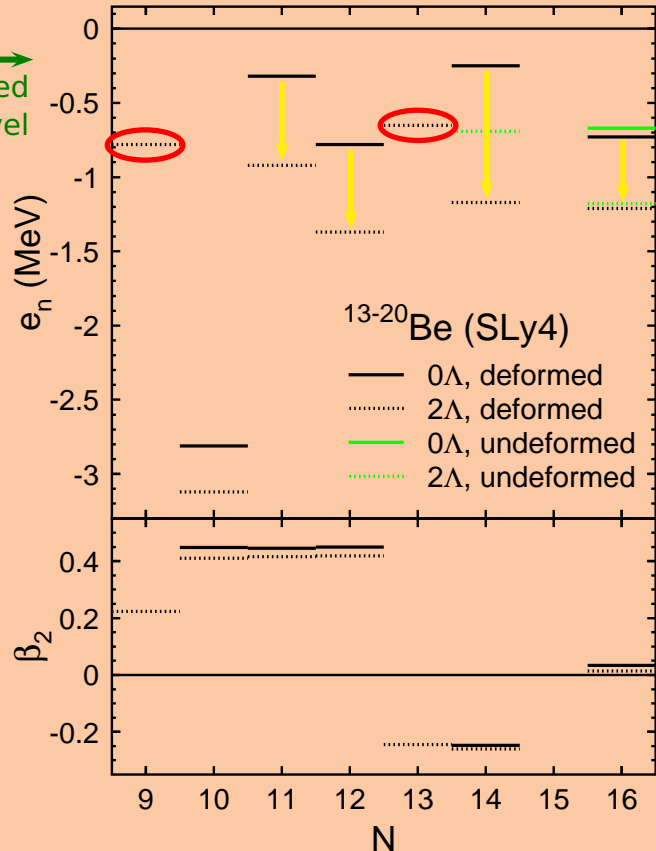


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# Results: Double- $\Lambda$ Hypernuclei:

- Important observable: Bond energy:

$$\Delta B_{\Lambda\Lambda} = 2E({}^{A-1}_{\Lambda}Z) - E({}^{A-2}Z) - E({}^A_{\Lambda\Lambda}Z)$$

- Experimental: 3+5 events:

- 3 (1960's):  ${}^6_{\Lambda\Lambda}\text{He}$ ,  ${}^{10}_{\Lambda\Lambda}\text{Be}$ ,  ${}^{13}_{\Lambda\Lambda}\text{B}$  :  $\Delta B_{\Lambda\Lambda} \approx 5 \text{ MeV}$

- 5 (1991...):  ${}^6_{\Lambda\Lambda}\text{He}$ ,  ${}^{10,11,12}_{\Lambda\Lambda}\text{Be}$ ,  ${}^{13}_{\Lambda\Lambda}\text{B}$  :  $\Delta B_{\Lambda\Lambda} \approx -1.5 \dots 4 \text{ MeV} \text{ !?}$

- Theoretical:

$$\Delta B_{\Lambda\Lambda} \approx U_{\Lambda}^{(\Lambda)}(\bar{\rho}_{\Lambda}) - U_{\Lambda}^{(\Lambda)}(2\bar{\rho}_{\Lambda}) \approx -U_{\Lambda}^{(\Lambda)}(\bar{\rho}_{\Lambda}), \quad \bar{\rho}_{\Lambda} \approx \rho_0/A$$

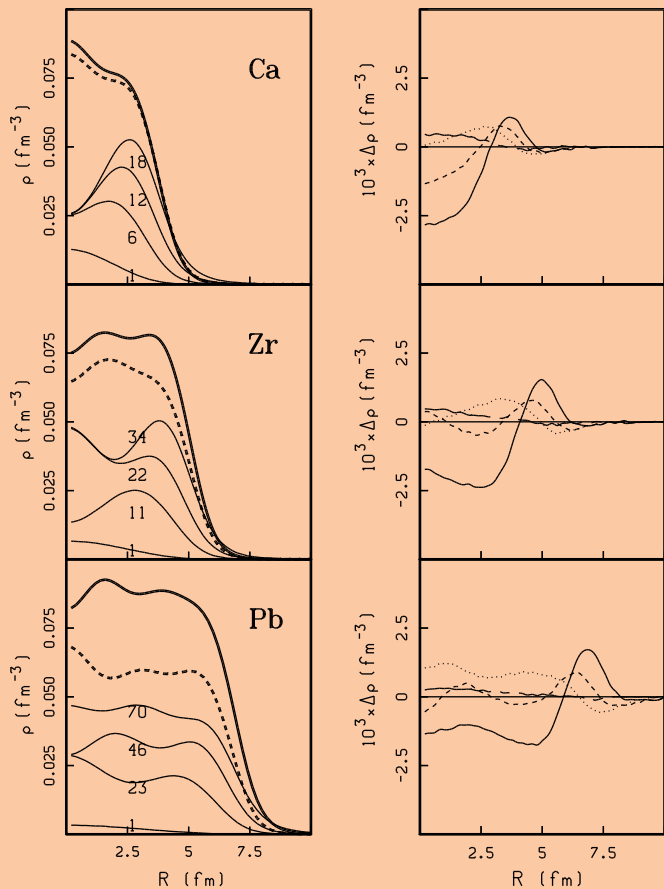
- Results with Nijmegen potentials:

	$\Delta B_{\Lambda\Lambda}$ [MeV]		
	NSC89	NSC97a	NSC97f
$^{10}_{\Lambda\Lambda}\text{Be}$	-0.34	+0.37	-0.35
$^{14}_{\Lambda\Lambda}\text{C}$	-0.41	+0.32	-0.47
$^{18}_{\Lambda\Lambda}\text{O}$	-0.41	+0.32	-0.41
$^{30}_{\Lambda\Lambda}\text{Si}$	-0.33	+0.25	-0.35
$^{42}_{\Lambda\Lambda}\text{Ca}$	-0.31	+0.19	-0.32
$^{92}_{\Lambda\Lambda}\text{Zr}$	-0.21	+0.09	-0.24
$^{142}_{\Lambda\Lambda}\text{Ce}$	-0.14	+0.05	-0.18
$^{210}_{\Lambda\Lambda}\text{Pb}$	-0.12	+0.01	-0.15

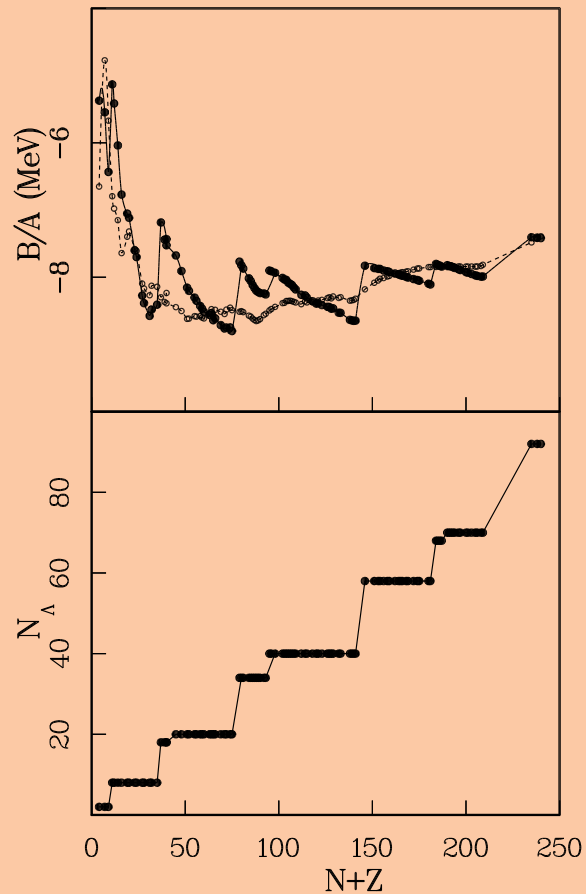
No YY components: core rearrangement effect

↪ NSC89,97 potentials predict too small  $\Lambda\Lambda$  binding

# Results: Multi- $\Lambda$ Hypernuclei:



Density profiles



$\Lambda$  drip line: Shell effect (no  $\Lambda\Lambda$  force)

## Outlook:

- Future work on  $\Lambda$  hypernuclei:
  - New NY, YY potentials (ESC08 ...)
  - Spin-orbit force
  - Hyperonic TBF
  - Constrained  $\Lambda N$  Skyrme force
  - HFB for dripline (hyper)nuclei

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    - HFB for dripline (hyper)nuclei
  - What about  $N\Sigma^-$  interaction,  $\Sigma^-$  hypernuclei ? :
    - Older data indicate attraction
    - New experiment repulsion, quantitatively unknown
- ➡ ESC08 preferred; NSC97, ESC04 excluded; **need data !**